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THE CITY UNIVERSITY, LONDON
The Graduate Business Centre

D Y N A M I C A P P R O A C H E S T O C A P I T A L
I N V E S T M E N T D E C I S I O N S
(E M P H A S I S O N R E P L A C E M E N T I N V E S T M E N T)

by

Aviam Abraham Heidecker, M.A.

Ph.D. Thesis

January, 1976.

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NOTATION AND TERMINOLOGY.

Mathematical notation when used in this thesis is explained in the text. Separate notations are used for the linear models discussed in Chapter 5 and for the exponential S-model discussed in Chapters 6-7. This was done for two reasons. First, parameters in the two systems are not always compatible so separate notation avoids a confusion of giving two somewhat different interpretations to the same symbol. Secondly, it was done to conform with the references. Most references on the linear models use notations comparable to the one I adopted, and the reference for the exponential S-model: Sir Robert Shone "Price and Investment Relationships" Elek., London 1975; gives the notation I adopted for the S-model.

I tried to avoid using esoteric terminology but I found it useful to adopt some of the terminology found in the main references. Whenever 'esoteric' terms are introduced they are either explained or made clear from the text.

The terms: equipment-replacement, asset-replacement, plant-replacement, machine-replacement are interchangeable in the text.

Somewhat interchangeable are Model, Method, System, in: S-model, S-method, S-system, or in: MAPI models, MAPI Methods, etc.

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S Y N O P S I S

The thesis investigates dynamic modelling in the field of capital budgeting. The essence of the particular type of modelling concerned is to select for immediate investment projects that can either be carried out "now" or else postponed to a later date. If such projects are postponed then the costs associated with them are likely to be affected by various economic forces, notably technological change. An important investment problem that is characterised by such options is replacement investment. The importance is both empirical and methodological, accordingly much of the study is devoted to this problem.

Investigations of the scope for the dynamic modelling is done along the following lines: a survey of various facets of technological change that have a direct bearing on time movement of costs, an analysis of the micro-economic concepts that form the background to dynamic investment models (this background usually is overlooked by "straight" Operational Research modelling), and an investigation of the institutional setting of capital budgeting and investment decision making.

Two dynamic models are at the centre of the study: the underlying model of the MAPI method of G. Terborgh¹⁾ and the new S-model of Sir R. Shone²⁾. They are investigated both separately and in comparison. First, the MAPI model, together with its various derivatives, is presented and is mathematically and methodologically analysed. Secondly, the S-model,

1) The latest version: G. Terborgh, "Business Investment Management", MAPI, Washington, D.C., 1967.

2) Sir R. Shone, "Price and Investment Relationships", Paul Elek, London, 1975.

interpreted as a replacement model, is presented and accordingly analysed. In addition, its 'behaviour' is investigated numerically over a wide range of parameter values; some of the results are given in tables and in graphs. Thirdly, the S-method is methodologically compared with the MAPI method and its derivatives, the comparison showing some of the inherent merits of the new S-method.

The study is concluded with few general observations on the managerial value of dynamic modelling.

CHAPTER 1

INTRODUCTION.

This thesis is a study of a class of normative investment models that may be labelled 'dynamic'. A typical problem with which these dynamic models deal is the replacement of existing assets by newer ones.

A general appreciation of investment modelling is given in Section 1.1, the dynamic concepts used in this study are given in Section 1.2, and overall structure of the contents of the following Chapters is laid down in Section 1.3.

1.1 The Appreciation of Investment Models

Abstract decision models applied to real life problems are a means of reducing the information relevant in the making of decisions. Investment models as a particular case are a means of reducing and organising information relevant in deciding on accepting or rejecting investment proposals. Ideally perhaps the reduction of the relevant information is done down to a single value indication of action: 'accept' or 'reject'. Sometimes, though, the reduction is not so complete.

There are many aspects to the building of 'good' models. Perhaps the most important and most relevant aspects in this study are:

- (1) Assumptions behind the model.
- (2) Logic of development and inference.
- (3) The ability to estimate the model's parameters.
- (4) Robustness of the model.

The assumptions behind a normative investment model may be divided into categories. First there is the 'objective function' specifying what the model is trying to optimize or satisfy. As far as investment models go this usually is the maximization of some measure of profitability. Secondly there are the assumptions about what information is relevant and the way this information should be grouped into parameters. One may distinguish another category of assumption concerning the relations among the parameters in the model - the specification of the model (e.g. linear, exponential, etc.).

The development of models from assumptions and the inference of a 'solution' are a mathematical exercise. There is always an interaction between the assumptions chosen and the mathematical kit with which the model builder is equipped. Usually, and particularly as far as analysis is originated among academics, assumptions are clearly made to suit pre-existing tools of analysis so that a neat solution will emerge.

The third point mentioned above is the ability to estimate or measure a model's parameters. A model may have an operative value in addition to having a conceptual value only if its parameters can be estimated in a manner that is not 'too objectionable'. The term 'too objectionable' is a relative term and apparently there are no universal rules in deciding what can be estimated and what can not. So a model may have reasonably sound assumptions and may be logically developed and yet may not have any practical value because its parameters cannot conceivably be quantified. An example for such models are the Utility Theory models.

A fourth point is the robustness of models. A decision model is said to be robust if the decision it offers is reasonably good even if certain assumptions, specifications, and parameter estimates of the model do not exactly reflect the 'real life' situation which it describes. Thus a robust investment model may indicate 'invest' when the strictly correct answer is 'do not invest'. But it will do so only when the difference in profitability between 'invest' and 'do not invest' is very small.

One may talk about a robust assumption in a model if such an assumption leads to convenient and reasonably good solutions. One may say, e.g., that the constant time discount rate customarily used in investment models is a robust assumption.

Taking a more detached view we may add that the value of a decision model in Economics and Management Science should be judged also by other criteria, not only by those mentioned above. Some such criteria are given here:

- (1) The 'addition' to existing models.
- (2) Communicative values.
- (3) Spin-off values.

It is always a fair question to ask: 'what does a new model 'X' do that existing models do not do'. New models like any other innovation must expect to meet the type of resistance expressed in such question.

Models have communicative values. Their basic task is in reducing information; their parameters are made up by aggregating and segregating original information. If a large number of people are familiar with a

model then the model as well as its components can be a basis for communication. An example is the simple structure of calculating a DCF rate of return of a project. One may like or dislike the DCFROR as an investment criterion, but the DCFROR has become so familiar that it is a convenient way of condensing some information about a project.

The value of some decision models is not in the actual solution they give but is changing people's attitude to a problem and in motivating them in a certain way and making them aware of difficulties or opportunities that they would otherwise ignore. Perhaps the most important value of introducing discounted cash flow analysis in business has been to teach management, engineers and accountants to study in detail all the relevant components of the cash flow matrix of an investment proposal. Something which they would not otherwise do.

Throughout this thesis a very wide base is given for the appreciation of models criticized and promoted. Technological change phenomena and capital budgeting procedures that are the basis for key assumptions in this work are investigated in some detail. Various investment replacement models are examined with particular emphasis put on the S-model in Chapters 6-8.

1.2 The Dynamic Concepts Relevant in Investment Decision.

Dynamic models in economics are those models that explicitly include the impact of time on the relations among their variables. Thus a standard definition of a dynamic economic 'system' is:

"A system is dynamical if its behaviour over time is determined by functional equations in which variables at different points of time are involved in an 'essential' way.¹⁾

With this definition confined within the area of investment decision let us look at an example that is particularly relevant in this work. One possible dynamic system of investment decisions will be a system where the decision whether to invest today or not will be based on the comparison of the best outcome over time that will result from 'invest today' with the best outcome that will result from 'do not invest today'.

More loosely the 'dynamics' considered in this study includes

- (1) The time value of money and the discounting of cash flows.
- (2) The fact that the 'same' project can be undertaken at different points of time.
- (3) The impact of time and time related factors on revenues and costs.
- (4) The notion that a decision now ought to take view of future decisions to be made by the firm itself.

Discounting appears also in non-dynamic investment models; it is nonetheless clearly related to the movement of time. The second point, that the 'same' project can be carried out at different points of time, is of central importance in this study. I am in particular concerned with

1) P.A. Samuelson; "Foundation of Economic Analysis", Harvard University Press, Cambridge, Mass., 1947, p.314.

projects that can be undertaken either 'now' or later.

The impact of time on revenue and cost plays an important part here. This includes the impact of operating deterioration which being a function of use and age is related to the passage of time. Also, and most importantly, it includes the impact of technological progress on costs and revenues.

Future decisions are decisions on future replacements of the new projects considered now or decisions to phase out or to stop altogether certain investment activity.

We can now deduce where a dynamic approach will make a useful contribution to the analysis of investment proposals. This may be summarized as follows.

A dynamic approach to investment decision will be appropriate either in expansion investment problems or in replacement problems where among the operative alternatives to the project (s) examined there are obvious alternatives at different points of time. Such problems are particularly interesting when costs and revenues are affected by change over time of technology and of the level of wages. The use of dynamic models is further enhanced if the current decision depends on (presupposed) future decisions like the dates of replacement of future generations of equipment.

Though dynamic approach as such is suitable for both expansion and replacement problems, the more interesting and to a certain extent the more general case in this context is that of replacement projects (expansion projects can be regarded as replacement of nil capacity).

1.3 The Structure of this Study.

Chapters 2-4 lay down the background for the investigation of dynamic investment models. They develop the various contexts for assessing such models.

Replacement models as said above are a particular extension rather than a narrowing-down of the dynamic investment problems. So, all the more general dynamic models investigated are or can be looked upon as replacement models. Models met in the literature are discussed in Chapter 5. The new S-model, with the development of which I have been involved, is discussed in Chapters 6-7. Chapter 8 makes comparisons between the S-model and the more important other models. Chapter 9 concludes with some general observations on dynamic investment models.

The content of the various Chapters of this study given in some detail is laid down below.

Chapter 2 investigates factors that have systematic effects on movement of cash flows associated with an investment project. The most 'promising' factors to have a near systematic and a predictable impact on cash flows are the various facets of technological change.

Chapter 3 reduces technological change to an economic phenomenon that can be described by neo-classical microeconomic tools. Two analytical frameworks are used. Simple production functions and vintage models. This analysis thus illustrates the difficulties and the merits there are in describing economically the phenomenon of technological change.

Chapter 4 concentrates on the institutional setting of budgets and investment decision making. Investment decision models are not complete decision systems. So the decision making framework given in that Chapter is the setting within which decision models, should be integrated.

The Chapter highlights the processing of investment proposals and the managerial dynamics of the decision making. The Chapter shows, from the managerial point of view, why replacement proposals are particularly suitable for a dynamic approach.

Chapter 5 discusses the literature on replacement investment. Elements of a pure replacement theory are presented, and various approaches to a range of replacement problems are discussed. But the main part of the Chapter is an analysis and synthesis of the MAPI models¹⁾ for the appraisal of replacement proposals and of derived models. Investment appraisal methods based on these models are an interesting combination of theory and practicality.

Chapter 6 presents the S-model, a new model originated by Sir R. Shone²⁾. After a brief introduction, the mathematics behind the model is developed and the model is shown as an integrative pricing-investment-replacement microeconomic model. The mathematics of the model is further developed to show some of its capabilities.

1) The latest such model was given in S. Terborgh "Business Investment Management" MAPI, Washington D.C. 1967.

2) See Sir R. Shone "Price and Investment Relationships" Elek, 1975. There is similarity between sections in Chapters 6, 7 and 8 here and some parts of Sir Robert Shone's book.

Chapter 7 presents a detailed computerized investigation of parameter relationships in the S-model for a wide range of parameter values. The investigation uses a discrete variant and a continuous variant of the S-model. The results of the computerized investigation are given in Tables and Charts which among others can be used in solving investment and replacement problems.

Chapter 8 compares and contrasts the S-model (as a replacement model) with the MAPI models and their derivatives. The Chapter discusses the particular considerations of technological change in the S-model. It also airs some real life difficulties associated with the application of such model.

Chapter 9 sums up some overall observations as dynamic investment models made in this study which are not given in a particular earlier Chapter.

For the convenience of the reader short Chapter summaries are given at the end of Chapters 2-6, 8-9. The Charts at the end of Chapter 7 may serve as the summary of that Chapter.

CHAPTER 2

THE DYNAMIC ENVIRONMENT OF CAPITAL INVESTMENT

2.1 Technological Change and Advance

In this section and in the following one we shall examine some of the external factors that affect the profitability of capital investment and influence the investment decision. In itself technological change covers a wide area of subjects and only a synopsis of the main economic issues will be given here. Since technological change per se is not the object of this thesis we shall limit the discussion and examine it in relation to the dynamic considerations raised in Chapter 1 - the impact of technological change on cash flows and to the timing and nature of investment and replacement.

Technological Change as the name implies is first of all a technological phenomenon not an economic one. It may be revealed in the production of totally new products like recently, pocket calculators. It is revealed in improvements of given products and services. The improvement and the increase in the range of choice among types of commercial computers in the last 15 years are an example, improvement in quality of international telephone communication is another. But above all, technological change is revealed in the use of new techniques in the production of given products and the supply of given services. There are two reasons for the emphasis on the last type of change. First, in a highly complex and interrelated economy as there is, much of the technological change of the first two types is channelled into improved production of further products. Secondly, the two types of change pose insoluble definition and measurement problems if the technological change is to be quantified by economic tools.

It is impossible to define, not to mention quantify, the technological change of the advent of the first computers but it is conceptually possible to quantify technological change attached to new computers today. This is done as Mansfield¹⁾ clarifies: by the effect of technological change - by the average rate of movement of the production function.

2.1.1. Total Productivity Movement

The above definition is unlikely to be to the taste of the technologist. A minor improvement in the design of a tractor such that makes it narrow enough to be used in cultivating a vineyard may introduce great cost reduction and may thus be measured as a "high rate of technological change" in the year of introduction (and symmetrically a high rate of obsolescence of existing cultivating equipment).

Important work into the magnitude of industrial productivity movements was done by J. Kendrick.²⁾ His results are of interest to us because they show the general order of magnitude of the phenomenon. They are given here in Exhibit 2.1, where total productivity movement is compared with labour productivity movement. One has to be careful though, not to draw too many conclusions from the Table. First because the data on which the Table is based are varied in reliability and secondly because the results are smoothed over a period of 54 years (1899-1953); much higher rates can be encountered over shorter periods.

1) E. Mansfield, "The Economics of Technological Change", Longmans, 1969.

2) J.W. Kendrick, "Productivity Trends in the U.S." Princeton, N.J., 1961 also reproduced by E. Mansfield, op. cit., p.28.

Exhibit 2.1:

Estimates of Annual Rate of Increase of Total Productivity
(TP estimate) and labour productivity (LP) in
Various Sectors of the U.S. Private Domestic Economy, 1892-1953

Sector	TP	LP
	ESTIMATE (PERCENT PER YEAR)	ESTIMATE (PERCENT)
Farming	1.1	1.7
Mining	2.2	2.5
Metals	2.2	2.6
Anthracite coal	0.7	0.7
Bituminous coal	1.6	1.7
Oil and gas	3.0	3.4
Nonmetals	2.6	2.9
Transportation	3.2	3.4
Railroads	2.6	2.8
Local transit	2.5	2.4
Residual transport	4.0	4.1
Communications and public utilities	3.6	3.8
Telephone	2.0	2.0
Telegraph	1.8	1.6
Electrical utilities	5.5	6.2
Manufactured gas	4.7	4.7
Natural gas	2.0	3.0
Residual sector	1.3	1.4
Manufacturing	2.0	2.2
Foods	1.7	1.8
Beverages	1.6	1.6
Tobacco	3.5	5.1
Textiles	2.4	2.5
Apparel	1.7	1.9
Lumber	1.0	1.2
Furniture	1.4	1.3
Paper	2.3	2.6
Printing	2.6	2.7
Chemicals	2.9	3.5
Petroleum	2.4	3.8
Rubber	4.1	4.3
Leather	1.2	1.3
Glass	2.6	2.7
Primary metals	1.9	2.3
Fabricated metals	2.6	2.7
Machinery, nonelectric	1.7	1.8
Machinery, electric	2.2	2.4
Transportation equipment	3.5	3.7

Source: E. Mansfield.

Total productivity movements are very well illustrated¹⁾ by A. Phillips's data²⁾ for first year operating costs (including depreciation) per seat-mile in American airplanes (Exhibit 2.2). One notices the underlying reduction in costs. Associated with total productivity movement is the movement of output price.

This is also illustrated very clearly by A. Phillips (Exhibit 2.3), one sees a 55% real reduction in air fares over a period of 33 years (1933-1965³⁾). Another and even more drastic reduction in real prices is that of international telephone calls.

Exhibit 2.4, that I have compiled for this study examines the real price reduction of London-New York telephone call, from the time the service commenced in 1927 until 1973. The Exhibit also shows a number of unquantifiable elements of technological change in that period. The examples given here are from industries that have undergone fast technological change. One would expect real output prices to rise in industries where the rate of technological change is below the all industries average. R. Shone gives the Inland Mail Service in Britain as an example of such industry⁴⁾.

2.1.2. Innovation and Diffusion

Economists cannot be satisfied with the treatment of technological change solely as movements of some synthetic production functions, this is evidenced by the scope of research done on the subject. One important area of research is the relation between the existence of the technological knowledge and the actual implementation (i.e. "innovation"). S. Enos

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- 1) Though not actually presented, because profit and financial charges are not included.
 - 2) A. Phillips, "Air Transportation in the U.S.", in W.M. Capron, Ed., Technological Change in Regulated Industries, The Brookings Institution, Washington D.C. 1971, pp. 123-165.
 - 3) A. Phillips, ibid.
 - 4) Sir R. Shone, "Problems of Investment", Basil Blackwell, Oxford, 1971, p.xii.

Exhibit 2.2: Estimated Operating Costs per Seat-Mile for Aircraft Used by Scheduled Domestic Air Carriers, 1926-65

In cents per seat-mile, 1954 dollars

<i>Manufacturer and model</i>	<i>Year first available for service</i>	<i>Operating costs, including depreciation, in first year</i>
Ford 4-AT	1926	9.50
Fairchild 71	1928	12.82
Fokker Super Universal	1928	13.40
Fokker F-10A	1928	11.80
Hamilton Silver Streak	1928	12.13
Bellanca Pacemaker	1929	12.80
Boeing 40-B1	1929	29.14 ^a
Boeing 80A	1929	10.81
Curtiss Condor Transport	1929	9.75
Fokker F-14	1929	17.24
Ford 5-AT-C	1929	7.92
Lockheed Wasp Vega	1929	10.28
Metal Aircraft Flamingo	1929	11.71
Ryan Brougham	1929	12.22
Stinson SM-6B	1929	10.89
Travel Air 6000B	1929	12.40
Boeing 221	1930	20.10 ^a
Consolidated Fleetster	1930	12.10
Curtiss-Robertson Kingbird D-2	1930	9.39
Fairchild Pilgrim 100	1930	12.19
Fokker F-32	1930	6.55
Northrop Alpha	1930	10.84
Stinson SM-6000A	1930	8.31
Lockheed Orion	1931	10.90
Stinson U	1932	9.43
Boeing 247	1933	7.78
Curtiss Condor T-32	1933	7.30
Douglas DC-2	1934	6.81
Lockheed L-10	1934	4.70
Stinson A	1934	7.92
Douglas DC-3	1936	3.28
Lockheed L-12	1936	5.95
Lockheed L-14	1937	4.77
Boeing 307	1940	3.22
Lockheed L-18	1940	4.74
Douglas DC-4	1946	2.35
Lockheed L-049	1946	2.84
Douglas DC-6	1947	2.17
Lockheed L-749	1947	2.51
Martin 202	1947	2.53
Convair 240	1948	2.51
Boeing 377	1949	2.44
Douglas DC-6B	1951	1.99
Lockheed L-1049	1951	1.84
Martin 404	1951	2.38
Convair 340	1952	2.58
Douglas DC-7A	1953	1.80
Lockheed L-1049C	1953	1.86
Vickers Viscount V-745	1955	1.62
Convair 440	1956	2.22
Douglas DC-7C	1956	2.31
Lockheed L-1649	1957	2.39
Boeing 707-120	1959 ^b	1.70
Douglas DC-8	1959	1.41
Lockheed L-188	1959	2.10
Boeing 720	1960	1.54
Convair 330	1960	1.73
Boeing 720B	1961	1.43
Sud Caravelle SE-210	1961	1.66
Convair 990	1962	1.55
Boeing 727	1964	1.14
British Aircraft BAC-111	1965	1.55
Douglas DC-9	1965	1.55 ^c

Source: See note 5.

a. Cost estimate is high because aircraft was used primarily for mail, to which no costs are allocated.

b. The 707-120 was introduced into domestic service on December 10, 1958. In the cost estimates, 1959 is taken as its first model year.

c. Based on actual costs covering only two weeks of operation in December 1965.

Source: A. Phillips

**Exhibit 2.3 Revenue per Passenger-Mile and Total Passenger-Miles,
Scheduled Domestic Air Carriers, 1932-65**

Year	Revenue per passenger-mile		Total revenue passenger-miles (thousands)
	Current dollars	1957-59 dollars ^a	
1932	0.0610	0.1282	127
1933	0.0610	0.1353	174
1934	0.0590	0.1266	189
1935	0.0570	0.1154	281
1936	0.0570	0.1145	391
1937	0.0560	0.1107	410
1938	0.0518	0.1016	480
1939	0.0510	0.1024	683
1940	0.0507	0.1024	1,052
1941	0.0504	0.0984	1,384
1942	0.0527	0.0946	1,417
1943	0.0535	0.0964	1,632
1944	0.0534	0.0962	2,177
1945	0.0495	0.0894	3,360
1946	0.0463	0.0794	5,945
1947	0.0505	0.0785	6,105
1948	0.0576	0.0804	5,997
1949	0.0578	0.0751	6,768
1950	0.0556	0.0704	8,029
1951	0.0561	0.0668	10,590
1952	0.0557	0.0622	12,559
1953	0.0546	0.0593	14,794
1954	0.0541	0.0595	16,802
1955	0.0536	0.0598	19,352
1956	0.0533	0.0584	22,399
1957	0.0531	0.0550	25,379
1958	0.0564	0.0566	25,375
1959	0.0588	0.0565	29,308
1960	0.0609	0.0587	30,557
1961	0.0528	0.0593	31,062
1962	0.0645	0.0602	33,623
1963	0.0617	0.0572	38,457
1964	0.0512	0.0560	41,141
1965	0.0606	0.0545	51,837

Source: Civil Aeronautics Board, *Handbook of Airline Statistics, 1965 Edition* (1965), pp. 47, 105.

a. Except for the years 1932-34, for which the aggregate consumer price index was used, deflation of revenue data was based on the index for all transportation.

Source: A. Phillips, op. cit.

Exhibit 2.4:

London-New York Telephone Calls - Price and Quality of Service

a) General Developments

1927 - 1973

<u>Year</u>	<u>Details</u>
1927	- London-New York radio telephone service commenced. Basic rate £15 for 3 minutes conversation.
1936	- New York telephone, basic rate reduced to £4.4S.
1945	- Basic rate £3 for 3 minutes.
1946	- Basic rate £2.5S for 3 minutes.
1956	- Opening of the Transatlantic Telephone Cable.
1957	- Introduction of transferred charge facility on calls to U.S.A.
1963	- Introduction of operator dialling on telephone circuits between Britain and U.S.A.
1970	- International Subscriber Dialling opened between London and New York.

b) Telephone Tariffs for 3m day call

<u>Year</u>	<u>Current Tariff (decimal) £</u>	<u>Purchase Power of £ (in 1973 prices)</u>	<u>Tariff in 1973 prices</u>
1927	15.00	5.10	76.50
1936	4.20	5.70	24.40
1945	3.00	3.32	9.96
1946	2.25	3.22	7.90
1973	2.16 ¹⁾	1.00	2.16

1) Direct dialling could be cheaper on shorter calls.

Compiled from: 1. Post Office Telecommunication Statistics 1971,
2. Post Office Dialling Instruction and Call Charges,
1973,
3. The Economist Diary 1974.

(Exhibit 2.5)¹⁾ shows that the time elapsed between invention and innovation is varied and can be very long.

Furthermore even if an invention has become "innovation" it still takes considerable time for users other than the first innovator to follow suit. Considerable effort into investigating this phenomenon of 'technological diffusion' has been made by the National Institute of Economic and Social Research. Exhibit 2.6²⁾ shows the considerable time lag in the application of some new technologies among West European countries where the geographical proximity and the comparable economic conditions are supposed to enhance fast diffusion. Quite so, but it is also true that cost structure for different producers can be very different.

Furthermore, a new technology is not always an economic breakthrough, a number of techniques may coexist for a very long time. An example is found in the Axminster carpet manufacturing. Two absolutely different techniques - the traditional Spool-Axminster and the Gripper-Axminster produce virtually the same type of carpet. This has been so ever since the 'Gripper' was introduced by Brintons of Kidderminster in the 1890's³⁾.

The stark conclusion drawn from the various works on innovation diffusion⁴⁾ is that firms do not introduce the same technology at one point of time. The reasons are lack of information about the best techniques, differences in financial strength and differences in cost structure and factor prices (mainly differences between countries).

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- 1) J. Enos, "Invention and Innovation in the Petroleum Refining Industry", Princeton, N.J., 1962. also reproduced by Mansfield, ibid p.101
 - 2) Following G.F. Ray, "The Contribution of Research" (Symposium paper) in Accelerating Innovation, ASLIB, London, 1970
 - 3) For more about Axminster carpets see G. Robinson, "Carpets", Textile Book Service, London, 1972.
 - 4) E. Mansfield Op. cit., chapter 4 mentions work done by many researchers including himself.

Exhibit 2.5 Estimated Time Interval between Invention and Innovation, Forty-Six Inventions, Selected Industries *

INVENTION	INTERVAL (YEARS)	INVENTION	INTERVAL (YEARS)
Distillation of hydrocarbons with heat and pressure (Burton)	24	DDT	3
Distillation of gas oil with heat and pressure (Burton)	3	Electric precipitation	25
Continuous cracking (Holmes-Manley)	11	Freon refrigerants	1
Continuous cracking (Dubbs)	13	Gyrocompass	56
"Clean circulation" (Dubbs)	3	Hardening of fats	8
Tube and tank process	13	Jet engine	14
Cross process	5	Turbojet engine	10
Houdry catalytic cracking	9	Long-playing record	3
Fluid catalytic cracking	13	Magnetic recording	5
Gas lift for catalyst pellets	13	Plexiglass, lucite	3
Catalytic cracking (moving bed)	8	Cotton picker	53
Safety razor	9	Nylon ^b	11
		Crease-resistant fabrics	14
Fluorescent lamp	79	Power steering ^c	6
Television	22	Radar	13
Wireless telegraph	8	Self-winding watch	6
Wireless telephone	8	Shell molding	3
Triode vacuum tube	7	Streptomycin	5
Radio (oscillator)	8	Terylene, dacron	12
Spinning jenny	5	Titanium reduction	7
Spinning machine (water frame)	6	Xerography	13
Spinning mule	4	Zipper	27
Steam engine (Watt)	11	Steam engine (Newcomen)	6
Ball point pen	6		

* The first eleven inventions in the left-hand column were those that occurred in petroleum refining.

^b Actually, this is the length of time between the beginning of fundamental research by DuPont on superpolymers and the production of nylon on the first commercial unit.

^c This figure pertains to Vickers' booster units, not Davis's system.

Source: E. Mansfield

Exhibit 2.6:

Six European Countries, Ten New Technologies

Time lag of each country behind pioneer country (years)

Technology	Date (and country) of first introduction	Austria	France	Germany	Italy	Sweden	United Kingdom
OXY	1952 (A)	0	4	4	12	4	8
CC	1952 (A)	0	8	2	6	11	8
SP	1963 (S)	3	2	2	2	0	1
NC	1955 (UK)	8	2	7	5	3	0
SL	1953-4 (F)	7	0	1	6	3	4
FG	1958 (UK)	--	8	8	7	--	0
TK	1948 (S)	9	1	11	3	0	*
SCM	1950 (S)	--	10	3	12	0	0
ATL	1947 (F-UK)	--	0	7	3	8	0
GA	1959 (S-UK)	--	7	--	--	0	0
Mean		4.5	4.2	5.1	6.2	3.2	2.3

* Omitted as extreme

- OXY - Basic oxygen process in steel making
- CC - Continuous casting of steel
- SP - Special presses in paper making
- NC - Numerical control of metal working
- SL - Shuttle-less looms in weaving
- FG - Float - glass - new method
- TK - Tunnel kilns in brick making
- SCM - Steel plate marking and cutting, new method
- ATL - Automatic transfer lines in the manufacture of engines
- GA - Use of Gibberellic Acid in malting-brewing

Source: G.F. Ray

One ought not underestimate non-economic factors of management attitude and considerations of industrial relations in tilting the balance towards or against adoption of new technology.

2.1.3. Economies of Scale

Associated with technological change is in many cases a change of the scale of the production unit. The existence of economies of scale in production brings about concentration of production whether within the same firm or further by amalgamation of industrial firms. Sometimes economies of scale are the result of intensified and increased demand. The development of jumbo jets is a result of the fact that there are enough people going from city A to city B on a certain date to fill such an air-carrier.

C.F. Pratten¹⁾ has investigated the subject of economies of scale in British manufacturing industries. His findings are that economies of scale do exist and their magnitude in many cases affects industry structure. A typical example given in Exhibit 2.7 - costs and scale for new general purpose refineries.

The relation between technological change and economies of scale is complex. If they are considered separately they can give two different causal answers to change in productivity. This is

1) C.F. Pratten, "Economies of scale in Manufacturing Industry", Cambridge University Press, London, 1971.

Exhibit 2.7: Costs and Scale for New General Purpose Refineries ^{(a) (b)}

Crude Refinery Capacity (million tons)	Interim Estimates of:		Revised Estimates of:	
	Refinery costs per ton	Ex-refinery costs per ton	Refinery costs per ton	Ex-refinery costs per ton
	(Indices of Costs)			
1	100	100	100	100
2	85	96	75	92
5	66	91	56	86
10	52	87	44	82
20			40	81

(a) Refinery costs include works fuel. It is assumed that there are no economies of scale for fuel and crude oil (included in ex-refinery costs). In practice a large refinery can be supplied by larger tankers or maintain lower stocks of crude in relation to its output.

(b) It is assumed that each refinery carries out the same range of operations. If a small refinery limited its range it could reduce its handicap. This point was discussed in the interim report.

Source: C.F. Pratten

Exhibit 2.8: Changes in Characteristics of Air Travel on Scheduled Domestic Air Carriers, 1932-65

Year	Average seating capacity	Average airspeed ^a (miles per hour)	Total route-miles	Passenger fatalities per 100 million passenger-miles
1932	6.6	109	28,956	14.9
1933	7.6	116	28,283	4.6
1934	8.9	127	28,609	9.0
1935	10.3	142	29,190	4.7
1936	10.7	149	29,797	10.0
1937	12.5	153	32,006	8.3
1938	13.9	153	34,879	4.5
1939	14.7	153	36,654	1.2
1940	16.5	155	42,757	3.0
1941	17.5	160	45,163	2.3
1942	17.9	159	41,596	3.7
1943	18.4	154	42,537	1.3
1944	19.1	156	47,334	2.2
1945	19.7	153	48,516	2.2
1946	25.3	169	53,981	1.2
1947	29.9	170	62,215	3.2
1948	32.4	176	68,702	1.3
1949	34.7	178	72,667	1.3
1950	37.1	180	77,440	1.1
1951	39.1	183	78,913	1.3
1952	42.2	189	77,894	0.4
1953	45.6	196	78,384	0.6
1954	49.6	204	78,294	0.1
1955	51.5	208	78,992	0.8
1956	52.1	210	81,189	0.6
1957	53.7	214	87,550	0.1
1958	55.5	219	89,569	0.4
1959	58.7	223	95,063	0.7
1960	65.4	235	98,008	0.9
1961	72.9	252	102,309	0.4
1962	79.4	274	104,673	0.3
1963	83.4	286	105,003	0.1
1964	85.1	296	105,059	0.1
1965	89.2	314	104,870	0.4

Sources: Civil Aeronautics Board, *Handbook of Airline Statistics, 1965 Edition* (1965); Civil Aeronautics Administration, *Statistical Handbook of Aviation* (1954); *World Airline Record, 1950-51 Edition* (Chicago: Roy R. Rood, 1950); U.S. Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1957* (1960).

a. Based on time from takeoff to landing, or "wheels-off wheels-on" speed.

Source: A. Phillips

exactly the nature of the disagreement between Hart and Chawla¹⁾ and Brechling and Surrey²⁾. The latter, on examining fuel productivity in electricity generating industry, maintain that :

"In the main technical characteristics of generating equipment, comparisons of the generating equipment installed in Britain, France and the United States over the period 1948-63 suggest that Britain has tended to lag up to four years behind France and up to nine years behind the U.S.³⁾"

Hart and Chawla contest these results, mainly on the grounds that:

"Because there are increasing returns to scale, the largest new vintage American plants will tend to be more efficient than the largest new vintage British plants There is no evidence to suggest that recent vintage plants in the U.K. lag behind their counterparts in the U.S. in the sense of having a lower production function⁴⁾".

In reality things are interrelated and an optimal new method entails considerations of both technology and size.

To consider technology alone and not size is to be in error.

1) P.E. Hart & R.K. Chawla, "An International Comparison of Production Functions: The Coal Fired Electricity Generating Industry". Economica Vol. 39 (May 1970) pp. 164-177.

2) F.P.R. Brechling & A.J. Surrey "An International Comparison of Production Techniques. The Coal Fired Electricity Generating Industry" National Institute Economic Review, No. 36 (May 1966) pp.30-42.

3) F.P.R. Brechling & A.J. Surrey op. cit. p.41.

4) P.E. Hart & R.K. Chawla, op. cit. p.174.

W.F. Cartwright¹⁾ criticizes British Industry for being afraid of big units of production, and not for eschewing new technology. He also raises the interesting argument that there is a sequential relation in technological change. First a distinctly new technology is introduced (like LD steel making process). Subsequent to its successful introduction further improvements take place by increasing the size of the unit of production. Many industries experience cost reduction per unit of output concurrently with increased size of production unit. Exhibit 2.8²⁾ illustrates numerically the impact of economies of scale on U.S. domestic air transport. (The cost trend is presented in Exhibit 2.2 above).

It is appealing to consider all cost reduction factors whether directly resulting from technological change, or indirectly through economies of scale, as one single "technological" change factor. There are, though, serious problems in this simplification. First there is the problem of comparison. Instead of comparing a new machine with an old one, one has to compare it to two or three old machines. Second, and more significant, is the fact that in many process industries the optimal size of plant has reached a large percentage of the market³⁾ and it is impossible to gain from actually introducing a new plant. The introduction of the best practice technique is inhibited by the small size or the

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- 1) W.F. Cartwright, "The Growth of Unit Output and its Effect on Works Planning and Management", Iron and Steel Institute Journal, 1969, pp.10-19.
 - 2) From A. Phillips, Op. cit.
 - 3) For a comprehensive list of the relation between size of optimal plant and the size of the British market in many industries see C.F. Pratten Op. cit. pp. 260-277.

low rate of growth of the market. Therefore, the observed technological change is slower than what is possible in a bigger market. Furthermore, it is quite possible that it will be economically justified not to delay introduction of best plant until there will be "a sufficient demand for it" but to saturate demand by introducing plant of less than "optimal size". F.M. Sherer in a recent paper¹⁾ has found that inter-national and inter-industry plant size variations are associated systematically with market size, and with the degree to which sales are concentrated in the hands of few leading producers.

2.1.4. Uneven Factor Saving in Technological Change

Another important aspect of Technological change that affects factor input is the question of "what is changed?" Technological change (and economies of scale) can reduce the requirements of capital, labour, materials, fuel etc. at an uneven pattern. The terms capital saving and labour saving technological changes are frequently met in the literature.

Microeconomic analysis of bias in change confines itself usually to dealing with the technological change that affects the "added value" of the industry, namely change in capital and labour. This is not satisfactory in an analysis that is done on industry level and directed toward decision making. Intermediate products, material fuel etc. account for about 50% of industrial input²⁾ and in many industries a lot more. Technological change that affects labour cost can invariably be regarded as inherent to the industry employing that labour. Technological change that affects materials would be

1) F.M. Scherer, "The Determinants of Industrial Plant Sizes in Six Nations" The Review of Economics and Statistics, Vol. 55. No. 2 (May 1973) pp. 135-145.

2) For some kind of evidence see Central Statistical Office, "Blue Book 1970 (Input-Output Tables)".

related to the user industry when decreasing quantities of it are required for the manufacture of the industry's output. This is the case in electricity generating industry where coal consumption per thermal unit has gone down considerably, since World War II. When the saving is in the price the change is related to 'materials' producing industries. Following the same example as before one could think of the real price reduction of electricity to its users¹⁾.

Differences in the marginal productivity movement (i.e. differences in the impact of technological change and economies of scale) are evidenced by Grieve-Smith and Miles²⁾ who found that over the period 1950-1960 total productivity gains in the British steel industry were of about 1.4% p.a. while labour productivity gains were 3.5% p.a., fuel and materials 0.8% p.a. and capital -1.5% p.a. (negative!).

Perhaps the most serious complication in applying past movements of productivity to future productivity forecasting is the fact that not only technology changes over calendar time but also relative and absolute factor prices. The changes are by no means independent. Especially vulnerable are the cases where industrial wage agreements depend on (the partial) labour productivity. There, the introduction of a new technique increases overall productivity and with it labour productivity. Wage increase follows and as a result part of the advantage of the new technique is lost. At the extreme what is estimated as labour

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- 1) H.J. Pick in Q and in * gives an interesting account on the economy and technical progress in the use of materials in industry especially the engineering industry. Among others he stresses the advent of plastic as a new material and the interaction between material supplier and industrial customer (e.g. material made according to customer specification can save subsequently both labour and capital). Discussion like his underlines the difficulty the economist has in relating technical change to a specific industry.
- 2) J. Grieve-Smith and T.P. Miles, "Total Productivity of the British Iron and Steel Industry, 1950-60", in Iron and Steel Institute Special Report 75 productivity in the iron and steel industry, pp. 18-26.
- Q "How to use Materials", Management Today, June 1969
- * "The Role of Materials in Engineering and in the Economy" Metals & Materials Sept. 196

saving technological change at fixed prices may turn out to be neutral technological change or even capital saving change because of the salary and wage augmentation that is associated with the change. (Note that here we refer to the impact of change on the money outlay on each factor in determining the bias and not to the 'physical outlays').

The other factor in the bias in technological change - the "capital" is a highly elusive Economic concept. Here we shall only stress that "capital" (investment) includes construction and development work carried out by a company's own labour force, and similar assets; thus not only capital goods bought 'outside'. Underground mine development and open pit mine overburden stripping are in this sense capital investment. The crucial test of capital expenditure is that the benefit from it (i.e. profit) is not immediate and that it spreads over a considerable period of time.

On a slightly more 'macroeconomic' level the question of complementarity rather than substitution between labour and capital looms high. Many technologies aimed at utilizing untapped labour resources of developing countries need a preliminary spin-off operation that when looked at on its own is highly capital intensive. An example is a highly mechanized deep level shaft sinking in gold mines that subsequently use 'cheap labour'. G. Pfefferman¹⁾ discusses this underlying characteristic of the 'dual economies' of developing countries. In circumstances like this, measurement of technological change just in terms of substitution is insufficient.

1) G. Pfefferman, "Men and Machines in Africa", Finance and Development, Vol. 11 No. 1, March 1974.

2.1.5 Technological Change - Embodied or Disembodied?

The last important aspect of technological change discussed here is the question whether technological change is embodied or disembodied. Embodied technological change according to the accepted terminology is technological change that follows from the act of capital investment. In other words this type of technological change requires the firm, i.e. the investor, to discard or displace its existing facilities and introduce new ones instead. Disembodied technological change (or productivity increase) takes place without necessitating new investment. In practice it is the kind of improvement that results from improved organisation, increase in workers skill and sometimes from the reduction of price and improvement in quality of intermediate products - fuel and materials. A detailed account on the question of embodiment or disembodiment of technological change is given by Shone¹⁾. His conclusions, which are also generally accepted are that technological change is mainly embodied change. One may add that much of what appears to be disembodied change is embodied change in disguise. Not all the benefits from new investment appear immediately. Some investments require the workers and management to adapt themselves to the new technology, and that takes time. It can be said that the act of capital investment introduces some immediate change and in addition it also determines the framework for subsequent 'disembodied' improvements.

Finally, accepting the assumption that capital investment and reinvestment constitute embodied technical change does not

1) Sir R. Shone, "Technical Progress and Investment" in Sir R. Shone, Problems of investment, Basil Blackwell, Oxford 1971, p.81 ff.

necessarily require that the change is capital-saving change. It can be labour or material saving change just as well.

2.2. Other Types of Environmental Change that Affect Investment

Technological change is generally regarded as the major environmental change that affects investment. It is the most systematic change in the sense that it affects most industries. Many other 'environmental' changes like real increase in labour cost are directly linked to the general phenomenon of technological change. But yet there are other types of time related factors that affect investment decisions. So investment and reinvestment are sometimes a response to social, legal or political changes rather than to technical changes. Donald A. Schon, an American social philosopher, is very interested in social and industrial changes and all their implications. In a recent book¹⁾, he presents the following real-life example concerning a proposed change in lumber standards.

"The literal '2 by 4' had long been out of date. The question now was whether boards marked '2 by 4' should have a thickness of $1\frac{1}{2}$ inches measured at a fixed moisture content, or whether the thickness should be $1\frac{5}{8}$ inches without specification of moisture content.

This seemed one of the less passionate issues of the day, but it ended up by generating approximately 30,000 letters per year - more than any other issue in the recent history of the Department of Commerce. It divided the country into 'wets' and 'drys'. The drys were those few lumber manufacturers large enough to afford a kiln, so that they could make kiln-dried lumber to dimension. And the wets were those tens of thousands of lumber manufacturers too small to afford kiln-drying equipment; because they could not afford it, they would not have been able to meet the new standard.

1) D.A. Schon, "Beyond the Stable State", Temple-Smith, London, 1971.

The standard would, in all likelihood, have eliminated thousands of small producers. It would have shifted the regional balance of lumber production; and it would, it was rumoured, have added approximately \$500 million dollars to the value of Weyerhaeuser's timber holdings, simply by enabling that firm to make a greater number of '2 by 4s' from a single tree.

Schon's own interest is in the area of resistance and response to industrial change but the implications of the above excerpt for investment and profitability are obvious.

The anticipation of political and fiscal change are an obvious factor in investment decision. Some economic factors are dynamic in the sense that they have a cyclical characteristic. Business cycles play, as we shall see in later chapters, an important role in the exact timing of investment. Exhibit 2.9 shows the movement of the Bank of England Rate over 1932-1971. Movement which through its impact on the securities market affects the cost of investment capital.

Inflation is, of course, another important consideration. Its impact on investment comes through its interaction with taxation and through uneven cost escalation of the various production factors.

Exhibit 2.9: Bank of England Rate 1932-1971

	Per cent		Per cent
June 30th, 1932 ..	2	October 5th, 1961 ..	6½
August 24th, 1939 ..	4	November 2nd, 1961 ..	6
September 28th, 1939 ..	3	March 8th, 1962 ..	5½
October 26th, 1939 ..	2	March 22nd, 1962 ..	5
November 8th, 1951 ..	2½	April 26th, 1962 ..	4½
March 11th, 1952 ..	4	January 3rd, 1963 ..	4
September 17th, 1953 ..	3½	February 27th, 1964 ..	5
May 13th, 1954 ..	3	November 23rd, 1964 ..	7
January 27th, 1955 ..	3½	June 3rd, 1965 ..	6
February 24th, 1955 ..	4½	July 14th, 1966 ..	7
February 16th, 1956 ..	5½	January 26th, 1967 ..	6½
February 7th, 1957 ..	5	March 16th, 1967 ..	6
September 19th, 1957 ..	7	May 4th, 1967 ..	5½
March 20th, 1958 ..	6	October 19th, 1967 ..	6
May 22nd, 1958 ..	5½	November 9th, 1967 ..	6½
June 19th, 1958 ..	5	November 18th, 1967 ..	8
August 14th, 1958 ..	4½	March 21st, 1968 ..	7½
November 20th, 1958 ..	4	September 19th, 1968 ..	7
January 21st, 1960 ..	5	February 27th, 1969 ..	8
June 23rd, 1960 ..	6	March 5th, 1970 ..	7½
October 27th, 1960 ..	5½	April 15th, 1970 ..	7
December 8th, 1960 ..	5	April 1st, 1971 ..	6
July 26th, 1961 ..	7	September 2nd, 1971 ..	5

Source: W. Nursaw, "The Art and Practice of Investment",
Hutchinson, London 1972 p. 12.

2.3 Summary

Technological change seems the most systematic time related change affecting the capital investment environment. Technological change affects products - by introducing totally new products - and it affects production - by improving and reducing the costs of production inputs. It is accepted that on an industry or a product level the impact of only the second type of technological change can be meaningfully quantified.

There are many aspects to this production technological change. The most important one is the overall rate of technological advance, the total productivity movement in an industry. Industries vary greatly in this respect. The implementation of a new technology by a pioneer company is the innovation; the subsequent adoption of the technology by other companies is the diffusion of that new technology. There are no strict rules as to the speed of innovation and diffusion. Companies vary greatly in the speed they adopt new technology. An economically important aspect of technological change is economies of scale. Many industries see a reduction of the production cost per unit of output as a result of an increase over the years in the size of the optimal production unit.

Two aspects are particularly relevant in investment planning. First, production technological change may 'save' more on one production factor, e.g. labour, than on others. However, definition and measurement of this bias are very difficult since changes in wage rate concur with technological change. Second, there is the relation between technological change and the actual act of capital investment. It is generally accepted that most production technological changes require investment in new plant, machinery, etc. It is thus 'embodied technological change'.

Many other time related changes affect investment. These are legal, political and social changes, etc. But these usually cannot be quantified. There are also general macroeconomic and financial changes to consider, some of which are cyclical and others are of more persistent nature.

CHAPTER 3
MICROECONOMICS BACKGROUND

3.1 General

3.1.1 Microeconomic Modelling

Perhaps one distressing conclusion to be drawn from the complexity of the phenomenon of change, even technological change alone, is that reality is too complex to be reduced to any representative model. So much so, in the description of the phenomenon of change let alone in any prescription of how to invest in the environment of technological change. But since companies have to make investment decisions, assessing the viability of projects, they have to make assumptions about future movement of technology and prices. These assumptions underlie models of 'the future'.

Theoretical models are logical deductions from a given set of assumptions. Thus assumptions on costs and future technical change may lead to an output pricing model. Clearly there is a need for models in economics but this need is a source of two types of difficulty.

First there is the problem of setting acceptable "realistic" assumptions. It is very difficult to produce a model that does not raise an objection e.g. consider the concepts of demand curve, market, perfect competition, neoclassical production function. Usually, the main justification for a set of assumptions is defensive - that is, there are no better, more defensible assumptions.

Second, there is a tendency to accept a model just because it is logically sound and see the reality "through it", even if the assumptions are totally "unrealistic". This is particularly true

of mathematically sophisticated models. C.W. Churchman¹⁾ has this to say about the suggestive power of the elegance of mathematical modelling:

"Because mathematics has become so revered as a discipline in recent years it tends to lull the unsuspecting into believing that he who thinks elaborately thinks well. Actually one great risk of being able to think rigorously is that we may continue to go down the wrong pathway, forgetting the assumptions that started the thinking process in the first place. The elegant feeling of deriving clean looking theorems may lead us to forget that the assumptions were totally unrealistic."

M. Shubik²⁾ regrets that modern microeconomics texts do not find it necessary to balance a rigorous development of models by a scrupulous weighing and selection of assumptions according to their real life relevance.

"The microeconomic texts in their haste to present the general picture concerning models we know how to handle, give virtually no guidance as to the relevance and importance of factors left out or simplified".

On the question of use of microeconomic models, he stresses that a model is not reality itself. It tries to represent a specific and separate problem. So there is no General Microeconomic Theory but a collection of models each aimed at a particular problem in a specific setting.

1) C. West Churchman, "Reliability of Models in the Social Sciences" in P. Langhoff (ed), Models, Measurement and Marketing, Prentice Hall, 1965.

2) M. Shubik, A Curmudgeon's Guide to Microeconomics, Journal of Economic Literature, June 1970.

"Logical consistency between one theory in microeconomics and another is a luxury and not a necessity. The theories are or should be constructed to answer a limited set of questions. The aggregations and selection of variables for one theory will be different from those of another".

In this chapter we reduce the technological change phenomenon to parameters that can be dealt with by neoclassical 'marginal' microeconomics. This reduction is done with caution and in the light of the above discussion.

I find I am justified in doing so for the following reasons

- (a) The main investment model I operate on, the 'S' model, is constructed as a neoclassical model
- (b) The neoclassical approach has been a source of the related terminology: technological change, capital saving change, etc. An investigation through this approach may sharpen some of the concepts according to our need in investment decision models.
- (c) The analysis reveals certain aspects of technological change that are not otherwise obvious, (e.g. adjustment procedure under change in technological expectation) (described in 3.3.4 below)

There are other Economic approaches to technological change (like that of the so called 'Cambridge School') but they will not be investigated here. This is in line with M. Blaug¹⁾ who, having compared the various approaches strongly recommends the neoclassical approach as the framework for organising one's knowledge of the subject.

1) M. Blaug, "A Survey of the Theory of Process Innovations", Economica, February 1963, pp 13-32.

"The case for the neo-classical approach is that it provides a meaningful framework for organising our knowledge of technical progress and to provide more decisive consideration that no satisfactory alternative approach is in view".¹⁾

More importantly, I must add that at certain times during the work on this dissertation I had doubts as to the value of "channelling" the technological change through any accepted theory at all. The alternative I had was to follow Shubik's "consistency of theories as a luxury not a necessity"²⁾ and consider assumptions of technical change only in the context of investment decision models, as others do (see Chapter 5). Having done the analysis in this Chapter I am of the opinion that familiarity with the microeconomic approach though perhaps not essential in developing 'realistic' investment models is extremely important in analyzing and assessing weaknesses in existing models.

3.1.2 Relevant Models

There are two neoclassical models (or "theories") that will be discussed here. One is the single product production function and the other is the partial equilibrium analysis of a competitive industry. The context of the discussion is that of investment and long term, multiperiod considerations. Therefore the production function will consider capital or investment as a production "factor" in arriving at a best production technique. And the partial equilibrium which is in this context long run partial equilibrium will emphasise the long run marginal cost in pricing and investment appraisal.

The emphasis in this work is on optimal investment decisions, "optimal" in the sense of maximizing the return on investments. The investment decision includes the selection of plant (selection of production technique) and the determination of level of investment

1) M. Blaug, op. cit.

2) M. Shubik, op. cit.

(which is a decision on the level of production). Arguably, these two sides of the investment decision are interrelated, but in practice they usually are separated. On the one hand there is the decision on the best practice plant and on the other hand there is a separate decision on the level of investment.

Moreover, the decision on the optimal level of investment assumes that the best practice technique in a period is known. Otherwise alternative levels of production cannot be compared. Similarly, the selection of equipment is made with the assumption that the level of investment is known (or that it is very large compared with the equipment unit and therefore does not affect the decision). There is therefore a certain "duality" relation between the two problems, the solution of one assumes parameters the "variables" of the other problem.

The modelling of each problem is separate and aimed at highlighting different specific points. The neoclassical model of the selection of technique is the production function and that of the investment level (which can be 0) is the partial equilibrium scheme.

Note that both models are descriptive rather than prescriptive. They assume, within the formidable battery of assumptions that underlie the neoclassical approach¹⁾ that optimal decisions are made "all the time" by many decision makers who make only optimal decisions. They nevertheless and in the absence of a suitable, general prescriptive investment model provide a starting point for prescribing a correct investment decision "how!"

1) See for example. T.H. Naylor and J.M. Vernon, "Microeconomics and Decision Models of the Firm", Harcourt, Brace and World, Inc., New York, 1969.

3.2 Production Functions

3.2.1 Recapitulation of Theory.

Production functions express the relation between output and the possible combinations of input factors. In one of its simplest forms, a production function expresses a single product output Q as a mathematical function of the input factors.

For two input factors a , and b with the respective quantities X_a and X_b the production function is:

$$Q = f(X_a, X_b) \quad \dots (1)$$

$$\text{with } X_a, X_b > 0 \quad ; \quad Q > 0$$

the function 'f' is convex to the origin and as required by the neo-classical "marginal" approach, is a differentiable function.

Q, X_a, X_b are expressed for a period of time (although they can also be expressed as densities for 'moments' of time).

It is convenient to express the function 'f' for a unit of output, thus:

$$1 = f\left(\frac{X_a}{Q}, \frac{X_b}{Q}\right) \quad \dots (2)$$

Replacing Y_a for $\frac{X_a}{Q}$ and Y_b for $\frac{X_b}{Q}$ we obtain

$$1 = f(Y_a, Y_b) \quad \dots (3)$$

Note that input and output are expressed in physical terms

Using input prices as parameters P_a and P_b , we express a unit output price as P :

$$P = Y_a \cdot P_a + Y_b \cdot P_b \quad \dots\dots (4)$$

P is optimized (minimized) subject to the technology constraint of (3) above¹⁾.

An important characteristic of the neoclassical production function is the elasticity of substitution between a , and b . It is defined by the pair (Y_a, Y_b) as:

$$\sigma = - \frac{d\left(\frac{Y_b}{Y_a}\right) / \left(\frac{Y_b}{Y_a}\right)}{d\left(\frac{d Y_b}{d Y_a}\right) / \left(\frac{d Y_b}{d Y_a}\right)} \quad \dots\dots (5)^2)$$

From (4) above we calculate $\frac{d Y_b}{d Y_a}$ for

the optimal P :

$$\frac{\partial P}{\partial Y_a} = 0 = \frac{\partial Y_a}{\partial Y_a} \cdot P_a + \frac{\partial Y_b}{\partial Y_a} \cdot P_b \quad \dots\dots (6)$$

or

$$\frac{\partial Y_b}{\partial Y_a} = \frac{d Y_b}{d Y_a} = - \frac{P_a}{P_b} \quad \dots\dots (7)$$

- 1) The optimization under constraint can be illustrated as follows: Assume the production function for the production of a unit of output in $Y_a \cdot Y_b = 1$ (which expresses the numerical combination of the two factors), and the input prices: $P_a = 3$ $P_b = 2$. The problem is then: $\text{Min } P = 3 \cdot Y_a + 2 \cdot Y_b$
s.t. $Y_a \cdot Y_b - 1 = 0$

The problem can be restated using a Lagrange multiplier λ

$$\begin{aligned} \text{Min } P &= 3 \cdot Y_a + 2 Y_b - \lambda (Y_a \cdot Y_b - 1) \\ \frac{\partial P}{\partial Y_a} &= 3 - \lambda \cdot Y_b = 0; \quad \frac{\partial P}{\partial Y_b} = 2 - \lambda \cdot Y_a = 0; \quad \frac{\partial P}{\partial \lambda} = Y_a \cdot Y_b - 1 = 0 \end{aligned}$$

Solution: $Y_a = \sqrt{2/3}$; $Y_b = \sqrt{3/2}$; $\lambda = \sqrt{6}$ $P = 6 \cdot \sqrt{2/3}$

- 2) Following T. Blumental, D. Levhari, G. Ofer, and E. Sheshinski, "Price Theory", [Hebrew Text], Academon, Jerusalem, 1971.

Using this result we rewrite(5) as

$$\sigma = \frac{d\left(\frac{Y_b}{Y_a}\right) / \left(\frac{Y_b}{Y_a}\right)}{d\left(\frac{P_a}{P_b}\right) / \left(\frac{P_a}{P_b}\right)} \dots\dots (8)$$

So the elasticity of substitution at the optimum can be paraphrased as the ratio of the proportional change in relative input quantities to the proportional change in relative prices, or "the ease at which a change in prices can change the combination of the production factors".

Figure 3.1 illustrates graphically the impact of a change in relative prices. The equilibrium point on the production function is shifted from L to M.

There is some interest in comparing two or more production functions especially in the comparison of the state of technology in consecutive periods. The interest is mainly in a rate of technological change and in the bias of technological change towards a greater saving on one production factor relative to the other.

The comparison is usually restricted to the optimal technologies in the periods considered. But even this comparison is difficult. If the technology in period 0 uses the combination Y_a^0 , and Y_b^0 and the technology in period 1 uses Y_a^1 and Y_b^1 such that $Y_a^0 > Y_a^1$ and $Y_b^0 < Y_b^1$ then we cannot "automatically" rank one technology as superior to the other.

As in other cases of partial ranking we obtain complete ranking and a scale of measuring "technologies" by the use of index numbers. Here we take as weights the prices of the input factors in period 0: P_a^0 , and P_b^0 .

The overall rate of technological progress between period o and period b is measured by:

$$1 - \frac{P_a^0 \cdot Y_o^1 + P_b^0 \cdot Y_b^1}{P_a^0 \cdot Y_a^0 + P_b^0 \cdot Y_b^0} \dots\dots (9)$$

or by similar measures. Graphical presentation of the comparison of two production functions is given in Figure 3.2.

Salter¹⁾ investigates the question of bias in technological change. Using the neoclassical concepts of production function and factor prices he observes three elements in determining the bias;

- a) a shift in the production function "towards" one factor,
- b) a change in relative prices,
- c) a change of the elasticity of the production function as from one period to another.

There are obvious doubts as to the real-world relevance of the elusive concept of "change of elasticity over time". But it has, it seems, some "insight" value. I illustrate the breakdown of the bias in technological change into its components in Figure 3.4 below.

1) W.E.G. Salter, "Productivity and Technical Change", Cambridge University Press, 1966.

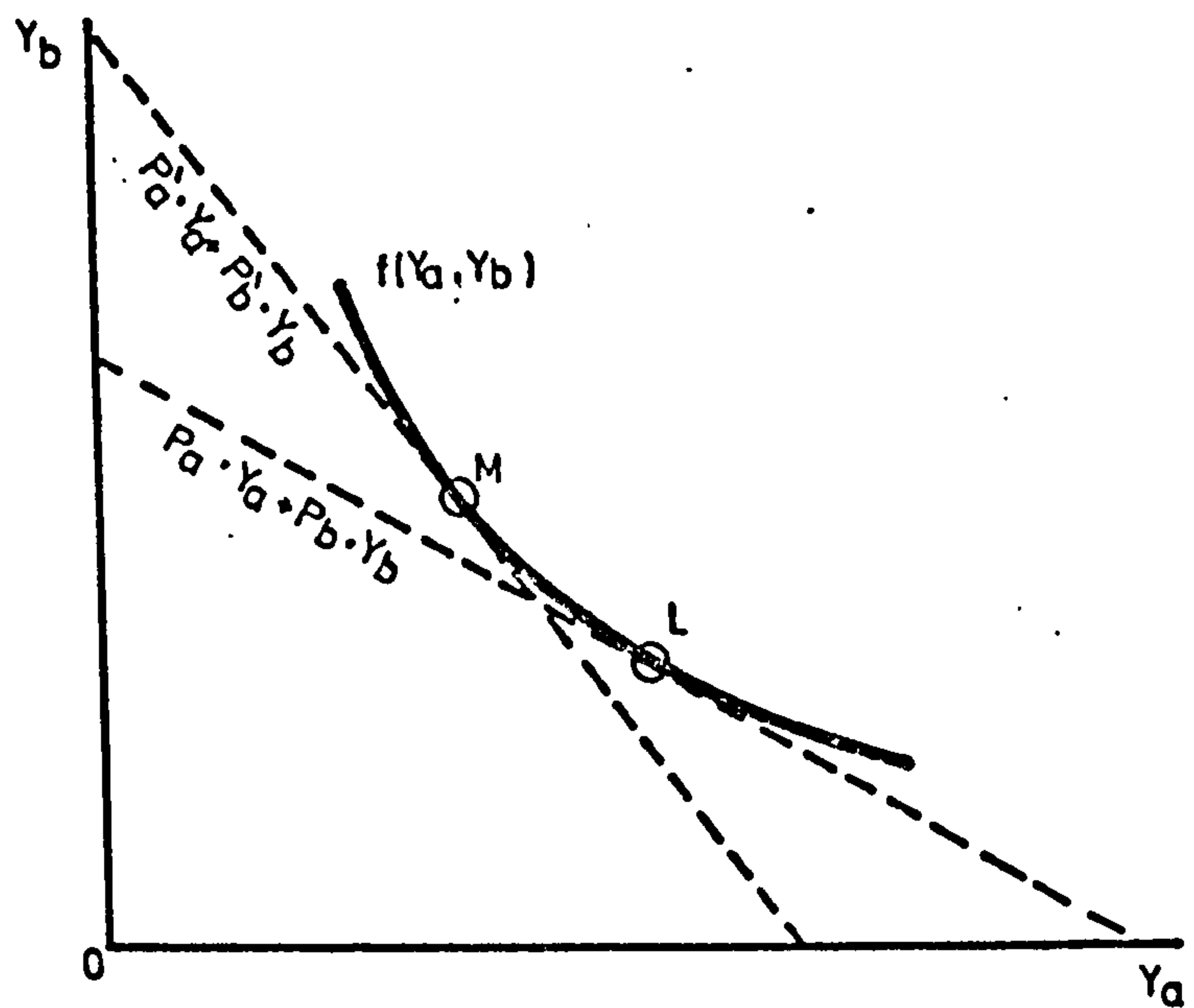


Figure 3.1: The Production Function.

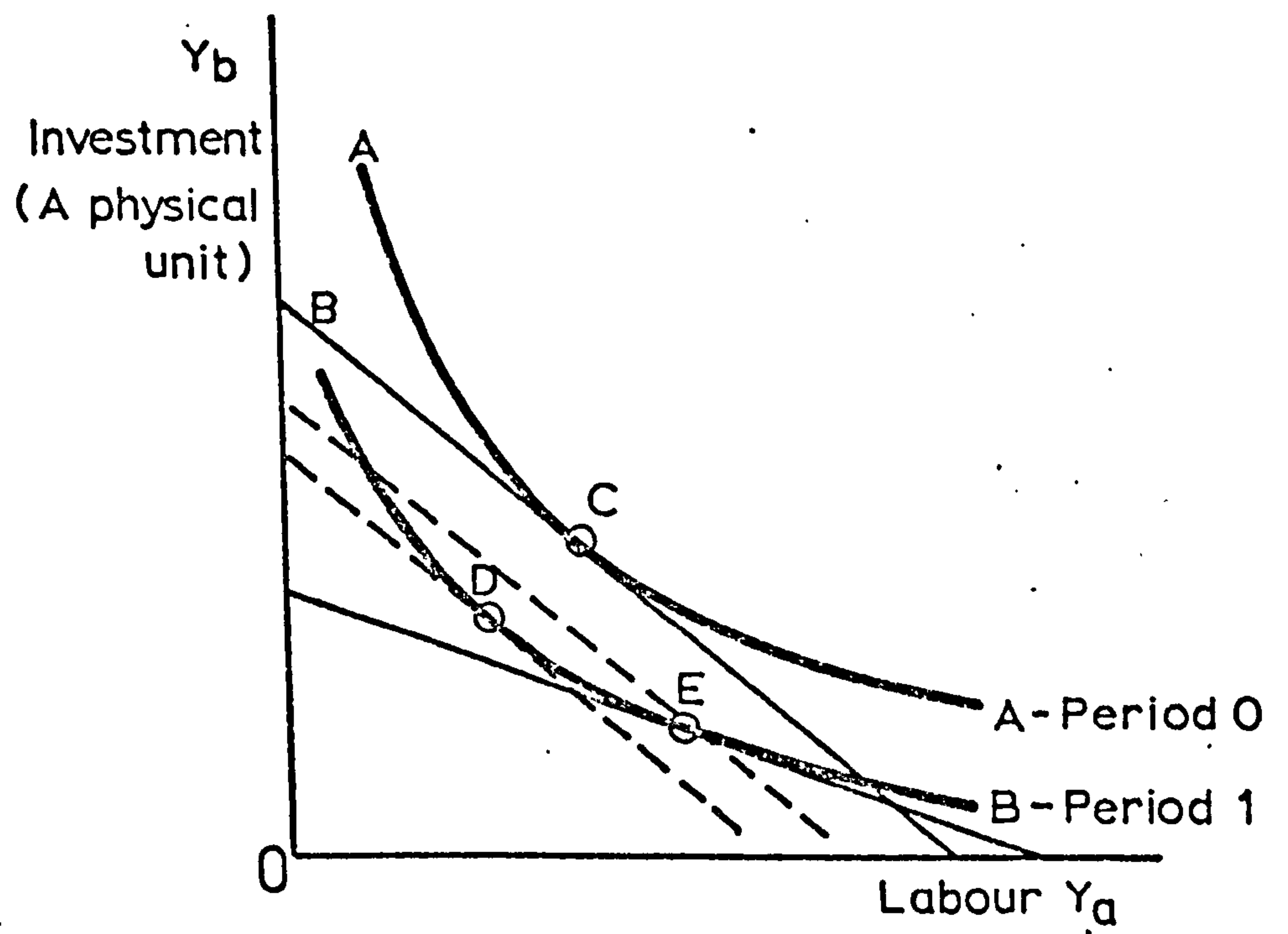


Figure 3.2: "Indexation" of technological progress.

3.2.2 Production Functions for Long Term Alternatives: Discussion

A production function as presented above is an important micro-economic tool in the description of the alternatives that a producer has in the production of a given quantity of output. The alternatives are the various combinations of production factors to produce that quantity. The criterion for decision is cost minimization. The producer chooses the factor combination that minimizes the cost of production. Economically a production function deals solely with one aspect of the general production problem, that is, with "how to produce", it assumes that the question of "what to produce" has already been solved.

Usually production functions deal with short term problems. There the production factors are well defined and their prices are constant. In this sense the construction of a production function and the calculation of optimal solutions is an Operations Research Problem.

Conceptual complications arise once the 'production function' is used in the description of long term production alternatives (i.e. investment alternatives). There, even if we assume that the questions of "what" and "how much" to produce have been answered we still have to consider changes over time, e.g. change in labour cost and deterioration of plant. Also, if different techniques include different machines and not only different quantities of given factors we have a 'multidimensional' production function which, to say the least, is difficult to handle¹⁾. Still the use of production function is not ambiguous because machines are defined in physical terms, labour, say, in man hours, materials and fuel in their respective units and the absolute and relative prices are known.

1) See W.E.G. Salter, "The Production Function and the Durability of Capital" The Economic Record, No. 70, April 1959: "To treat each item of equipment as a separate factor of production ... is precise but unilluminating". (p.47)

So far for a single long term production function. To represent the effect of technological change on production coefficients between two periods we compare two production functions and here we have additional difficulties. First new machines that have not existed before are introduced, secondly new techniques are introduced in the use of existing equipment in the production of the given product and thirdly, both these changes are affected by (as well as create) changes in wages.

A way out of these difficulties, at least for the purpose of describing the nature of technological change, is to assume that there are only two production factors, one is a unique capital factor and the other a unique operating cost factor (for simplicity I would deal here with labour only). Technological change between the two periods would then reduce the physical quantity of at least one of them in the production combination. Because of the special context in which we deal with the production function I would term the capital factor as "investment" (this is in line with Salter¹).

Production functions are usually expressed per unit of output per period (year, month, etc.) In order to justify a two dimensional presentation I would ignore the impact of deterioration and assume a certain 'average' production. I would also assume that the length of life of the alternative projects is constant and equal, otherwise the element of capital recovery in the investment cost cannot be given and comparison of different production functions would be impossible. I would also state that there are no economies of scale in production and therefore a production function calculated per unit of output per period is independent of the size of the production unit.

1) W.E.G. Salter, "Productivity and Technological Change" Cambridge University Press, London, 1966, pp 8-9.

3.2.3 Interpretation of the "Graphical Presentation" of Technological Change

A graphical description of the production function is given in Figure 3.2. Technical advance is represented by the movement of the production function towards the origin. Thus, instead of production function AA that we have in period 0 we have production function BB in period 1. But between period 0 and period 1 there is also a change in relative prices which is represented by the change of the slope of the "budget line". Therefore, the new equilibrium in period 2 is point E and not the point D (which would be the equilibrium point if there was no change in relative prices).

To measure the extent of past technological change in the way described in formula (9) above we compare points C and E and not points C and D. By doing so, and as shown by the budget lines in Figure 3.2, we actually underestimate the movement of the production function towards the origin.

The two period comparison underlines 3 elements in the technological change.

- (a) The technical advance. The total reduction in the required production co-efficients this has to be measured as an index number. As suitable weights for the two factors, would be their prices in the first period (see formula (9) above).
- (b) The bias in the type of technological change. This can be either toward greater saving of investment (capital saving technological change) or labour (labour saving technological change) if there is no bias we would talk about neutral technological change.
- (c) The elasticity of substitution. This factor is important in determining the new equilibrium point. It indicates the sensitivity of the equilibrium point on a production function to changes in relative prices.

Figure 3.3 demonstrates capital saving technological change (BB as compared with AA), labour saving change (CC as compared with AA) and elasticity of substitution (DD is more elastic than AA)

Usually, and always in ex-post analysis, bias in technological change is measured between the two equilibrium points in the two periods compared. This measured bias is affected both by the genuine bias as defined before and by the impact of substitution. To differentiate the two would require a reconstruction of the production functions in the two periods; in practice it cannot be done. An interesting possible combination is demonstrated in Figure 3.4. There we have a genuine capital saving technological change, but it coincides with a relative increase in labour cost. The new equilibrium point requires more capital than the original equilibrium point. This change would be observed as labour saving technological change.

3.2.4 Long Term Cost Movement: "Technological Change" in the Monetary Sense.

The definitions we have used in the description of change are not useful for the businessman because he is interested in the cost reduction effect and not in the technological change itself. He is indifferent to the question whether the technological change is a result of an important discovery in the capital goods industry that leads to reduction in the prices of capital goods or whether the technical change is inherent in his own industry and thus enables him to reduce the number of physical units of investment in the production of the given output. In reality usually this distinction does not even exist.

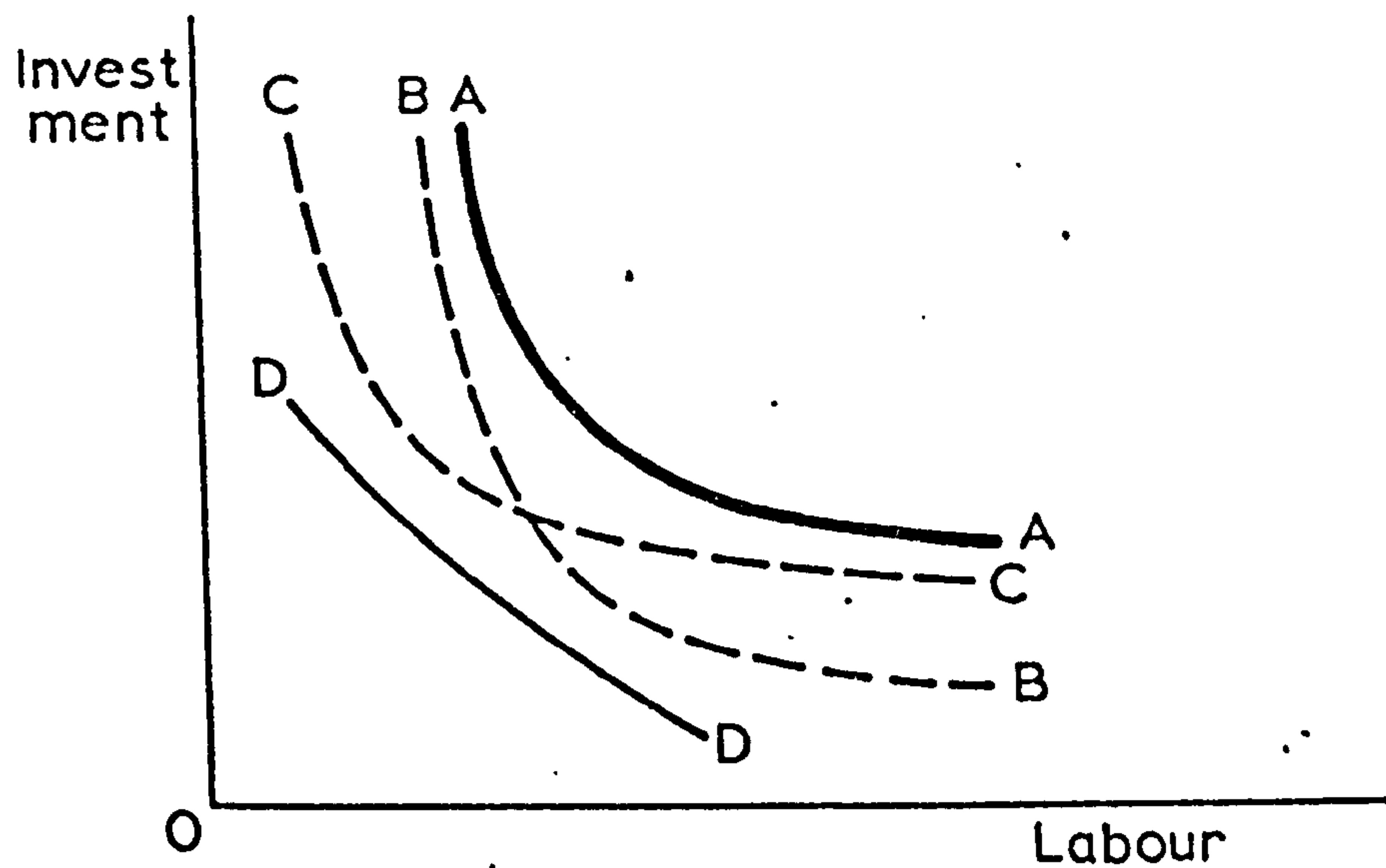


Figure 3.3: Bias in technological change and change in the elasticity of substitution.

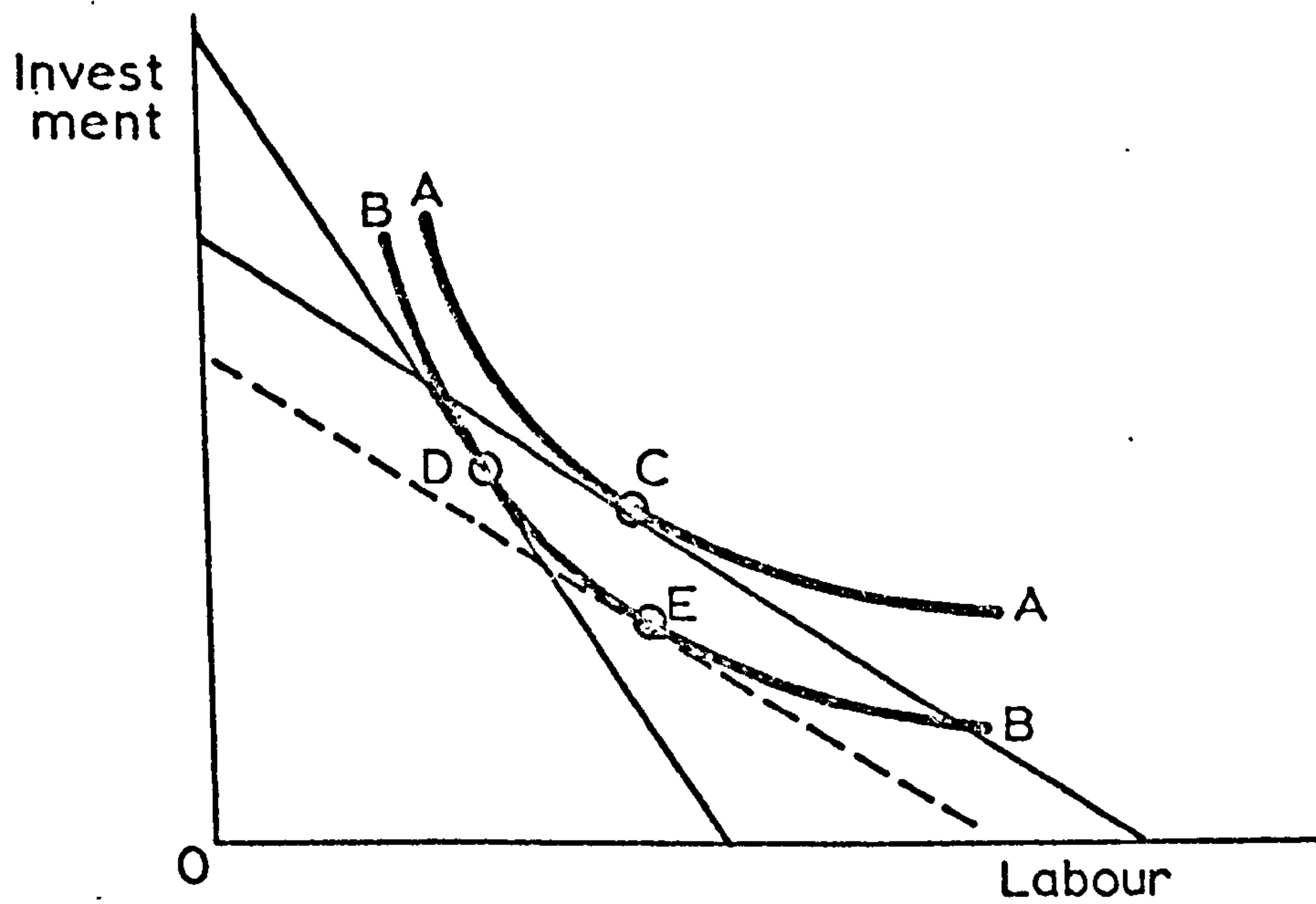


Figure 3.4: "Synthesis" of technological change: A special case.

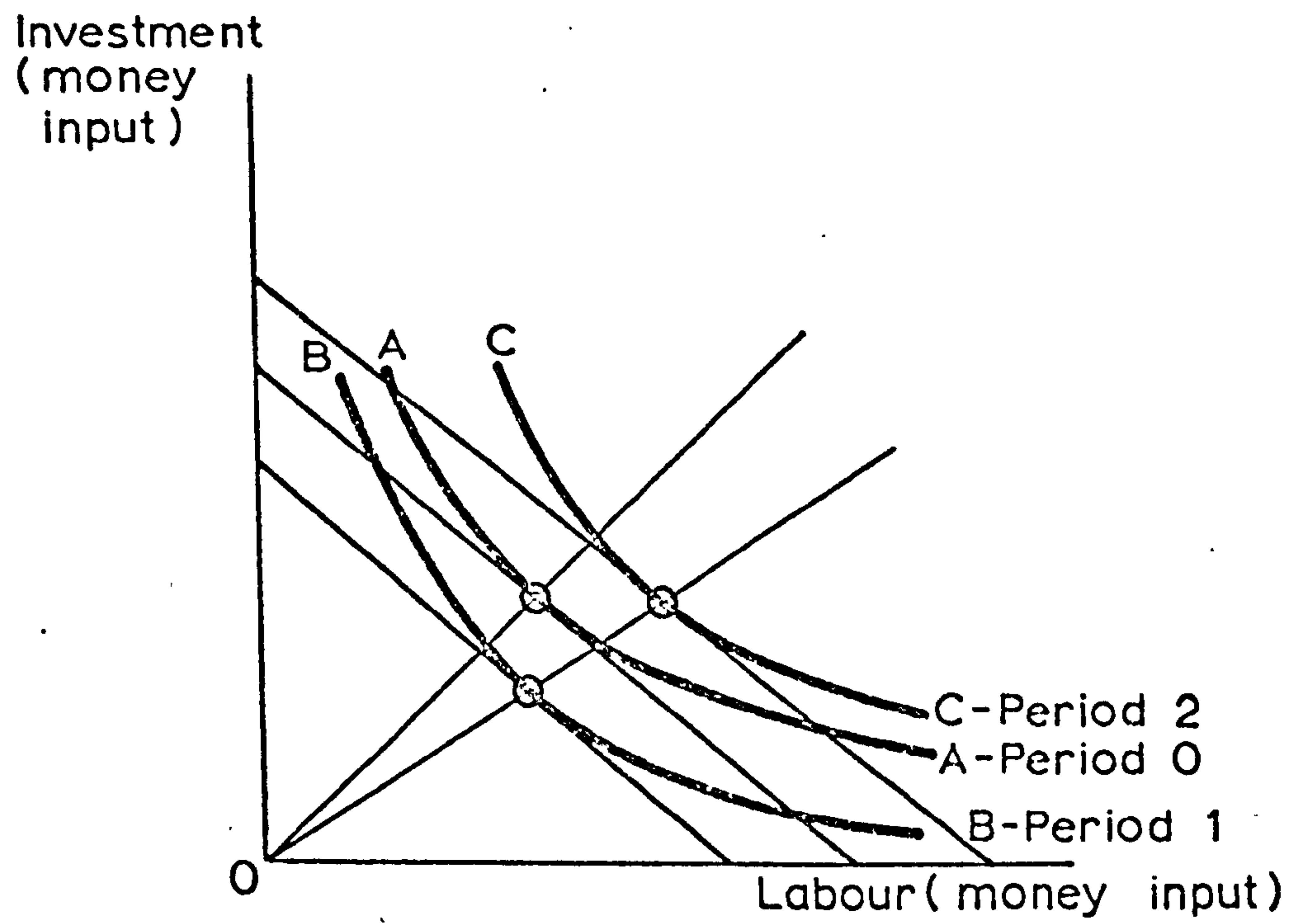


Figure 3.5: "Technological change" in the monetary sense.

Therefore we would now reduce the production function to a more "naive" form. It would simply be the total cost function per unit of output per period. It would be reconstructed from the prevailing prices at a period and from the technological possibilities of the period. Note, that while the earlier graphical presentation considers only relative prices of the two production factors here we also consider the absolute movement of factor prices over time (in fixed money terms).

Figure 35 shows the changes in the equilibrium from 'D' on AA in period 0 to 'E' on BB in period 1. The change is biased, it is a capital saving change in the sense that there is a great reduction in the capital requirement for a unit of output and a lesser reduction in labour requirement. The change from period 1 to period 2, of course, is not a 'technological decline' it is a situation that is found in 'increasing cost industries'. Note though that the cost increase here is 'neutral' the relative money expenditure on each production factor is unchanged. Change as between periods 1 and 2 can be explained as a situation where the technical progress that affects an industry is smaller than that which affects the economy as a whole. Technical progress in the economy is translated into wage increase that affects all industries. The industry in question has to pay more for its production factors than the increase in their productivity.

Some writers, in prescribing investment decision models (discussed later in Chapter 5) deal only with situations of cost reduction over time, i.e. with cost decreasing industries. They lump together the combined impact of wage movement and technological change into one "obsolescence" factor. They thus disregard the components. This may be a weakness in the assumptions behind their models.

The "monetary production function" is presented algebraically in formula (10) below. It is in fact an objective function of the total cost per unit of output per period expressed for a new investment decision taken at period i . It is:

$$L_i \cdot W_i + K_i \cdot C \quad (10)$$

where

- W_i - is the "averaged" wage rate per unit of output, per period over the life of investment launched at period (or year) i ;
- C - is the periodical cost of capital. It is a function of the rate of interest and the length of investment life;
(since they are assumed constant C is constant as well)
- L_i - labour requirement per unit of output per period associated with new investment at period i ;
- K_i - Investment capital required per unit of output per period for new investment at period i .

The algebraic expression is convenient for analyzing the sources, technological and otherwise for cost decrease (or increase) between periods .

The optimum points at periods i and $i + 1$ (i.e. the minimum cost in each period) are denoted:

$$L_i^* \cdot W_i + K_i^* \cdot C, \text{ and } L_{i+1}^* \cdot W_{i+1} + K_{i+1}^* \cdot C \text{ respectively.}$$

(Note, in ex-post analysis we assume these formulas to represent the actual costs).

The optimum in a period i is the pair (L_i^*, K_i^*) . For the understanding of the concept of technological change in an industry it is important to look at the determinants of K^* . Within the assumptions they are:

- (1) Technological change in the industry in question (affecting both L and K) - improved utilization of production factors.
- (2) Technological change in the industry producing the capital goods. This reduces the price of the capital goods for the industry in question.

If we relieve the assumption of constant wage rate through the economy a change in K^* may reflect a change in the relative wage rates of the two industries.

3.3 Vintage Models

The discussion so far has dealt with the choice that a producer has in deciding on the best production combination in each period. We would now assume that the question of the production combination has been resolved and that there is a single best practice technique in each period. Therefore, in this section we would concentrate on the complementary questions of "how much to produce", whether to enter the market, whether to scrap old plant and conversely what price to charge. The discussion will also include an ex-post view of the industry.

3.3.1. Long Run Marginal Cost Concept

There is little formal theory on the Long Run equilibrium of competitive industry. Much of the work there is on the subject has been done in Britain¹⁾.

Standard modern textbooks discuss extensively subjects like neoclassical, linear and other production functions, but they hardly discuss long run equilibrium beyond a mere restatement of short run equilibrium, mentioning that in the long run all costs are variable²⁾. The discussion is usually confined to the graphical presentation of the microeconomic equilibrium.

As a starting point let us look at such presentation bearing in

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- 1) e.g. a) W.E.G. Salter (1966), op. cit.
b) H.R. Fisher, "Obsolescence and Optimal Replacement Timing", The Chemical Engineer, April 1963, pp 86-98.
c) R. Turvey, "Marginal Cost", The Economic Journal Vol. 79 No. 2 (June 1969), pp. 282-299.
d) Sir R. Shone, "Technical Progress and Investment" in Sir R. Shone Ed. Problems of Investment, Basil Blackwell, Oxford 1971.
- 2) e.g. T.H. Naylor and S.M. Vernon "Microeconomic and Decision Models of the Firm" Harcourt, Brace & World, Inc., New York, 1969.

mind the assumptions behind models of perfect competition: complete certainty, free information, universal availability of technology, the single firm or consumer has no influence on prices, free entry to the market and free exit from it.

Figure 3.6(a) shows the simple case of a firm (every firm) in a competitive industry where today's cost structure is expected to prevail in the future and where the price that secures entry to the market equals the Long Run Marginal Cost and the Average Total Cost of production of the firm. Figure 3.6(b) shows the market equilibrium. The profile of the supply curve indicates that new entrants require the market price but that firms already in the industry are satisfied with lower prices. Their capital costs are sunk costs and they need only to cover their operating costs.

Figure 3.6(c) illustrates a multiperiod market in a decreasing cost industry (assuming, for graphical simplicity, that the demand per period is constant). S_0 is the supply curve in the first period. S_1 below it is the supply curve in the next period. The Long Run Marginal Cost of a single firm in this type of market is a slightly more complicated concept than above. Here, for example, the firm has to consider decrease in revenue in subsequent years.

As a fairly general statement one can say that the long run marginal cost equals to the initial price p_0 that a new entrant to the market would require in order to ensure normal profit over the life of a project, assuming no excess profit can be made on delaying

the entry to the market.¹⁾

R. Turvey offers a general definition for the marginal cost in the discrete case ("Annual jumps").

"[Long run] marginal cost for any year is the excess of (a) the present worth in that year of system costs with a unit permanent output increment starting then, over (b) the present worth in that year of system costs with the unit permanent output increment postponed to the following year".

Note that Turvey's marginal cost is a discrete annual equivalent to our p_0 . A firm would bring forward its entry to a market by one year if the excess present worth of costs (Turvey's marginal cost) resulting from this action would be compensated by an equal initial market price (p_0 equivalent for the discrete case).

-
- 1) Here is an illustration. Assume future prices are related to the initial price by the function $p_t(p_0)$, the production cost for today's new entrant is C_t^0 at t , the life of the plant n , initial investment K_0 , and the cost of capital, which equals the rate of normal profit is r . Assume also that plant has no salvage value and that production is constant over the life of the project. Then p_0 is such that

$$\int_0^n (p_t(p_0) - C_t^0) e^{-rt} dt - K_0 = 0$$

The actual determination of P_0, P_t and m is based on future costs per unit of output: capital cost of future plants and operating costs patterns in future plants over their lives.

For a more detailed discussion see Chapter 6.

- 2) R. Turvey Op. cit., p. 289.

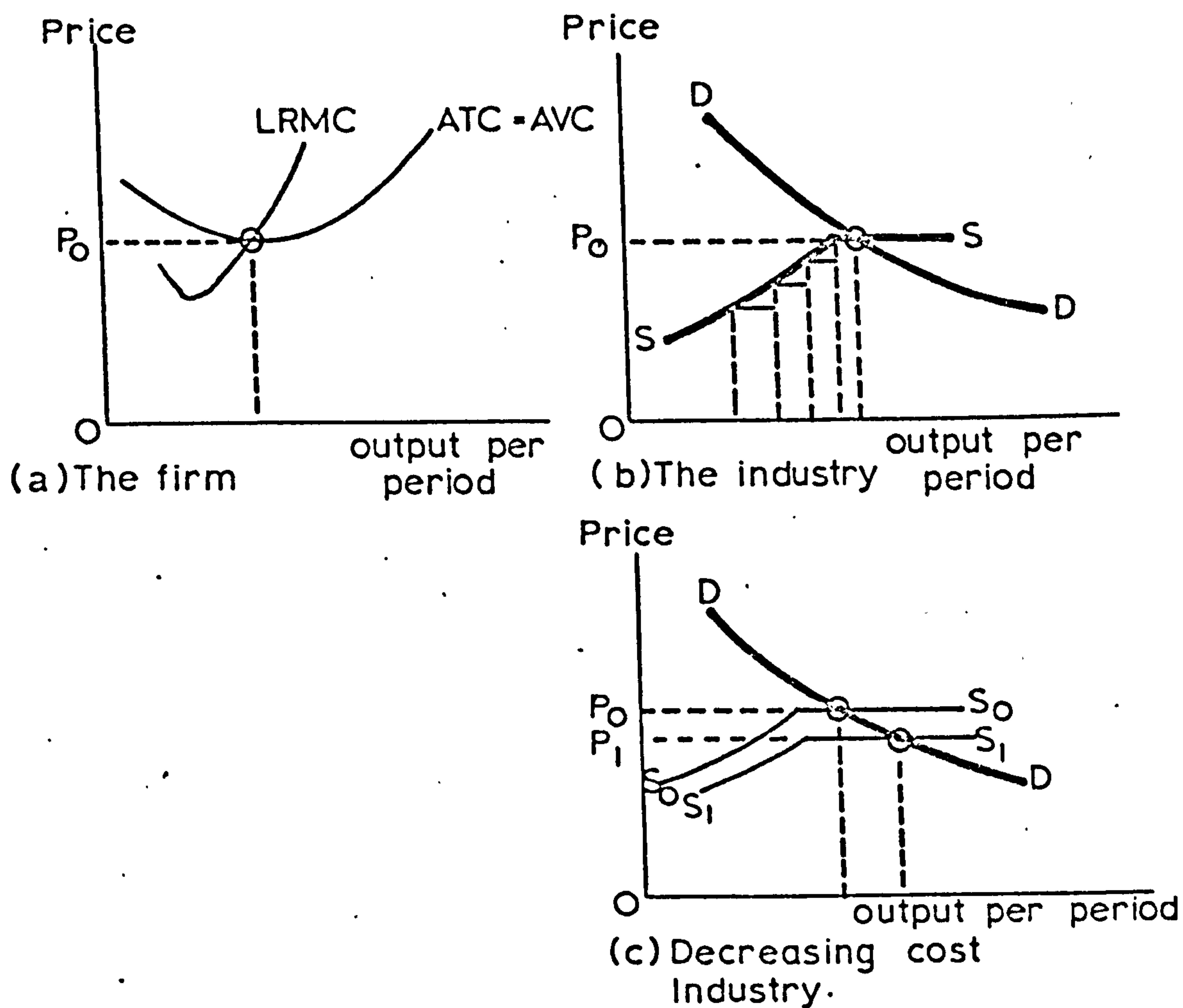


Figure 3.6: The Firm and the Industry in Perfect Competition.

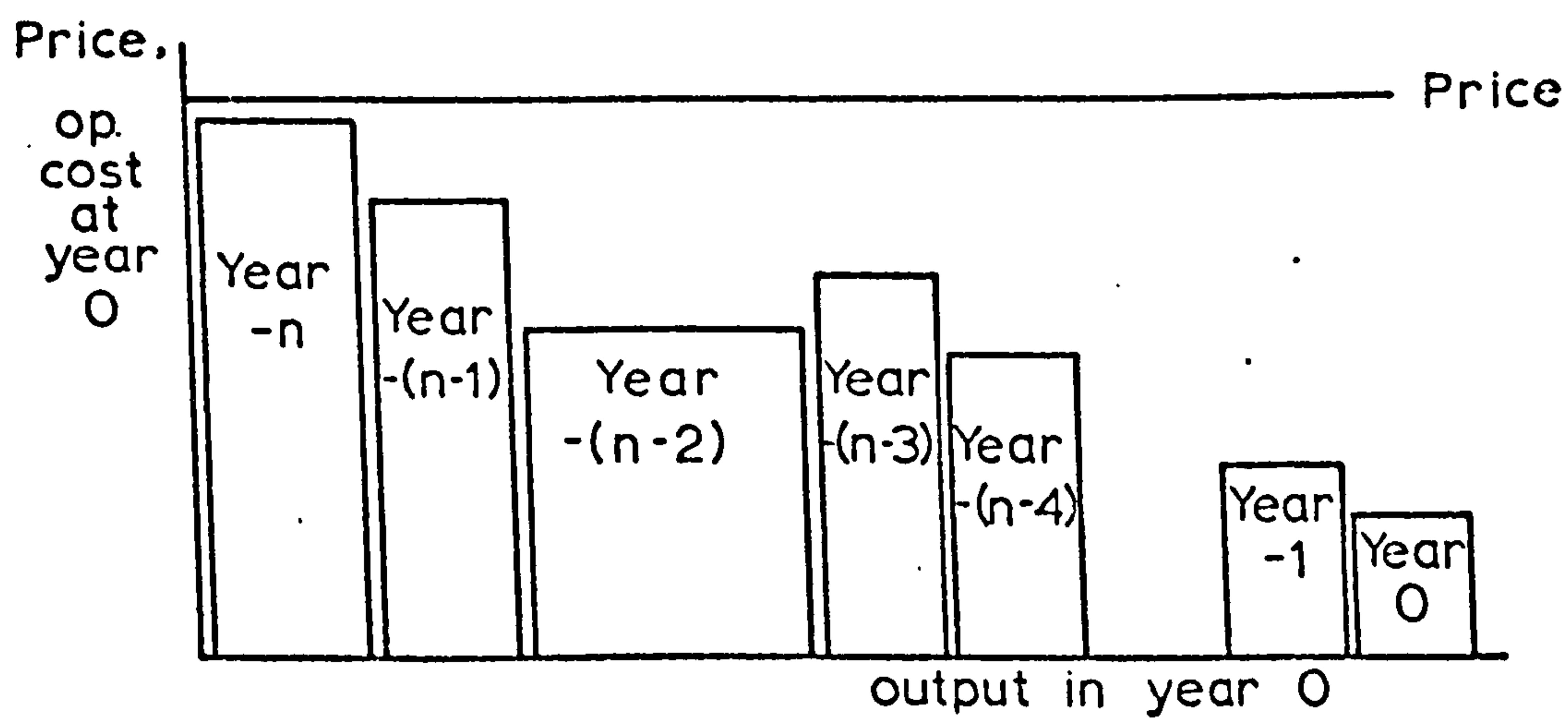


Figure 3.7: Cross section of an industry's output and operating costs according to plant "vintage".

3.3.2 A Cross Section of an Industry: Plant age and Plant Cost

Following Salter¹⁾ we could say that a mature competitive industry consists of an assortment of plants²⁾ of various ages that turn out the product of that industry. Existing plants are regarded economically as "fossils" they are not production variables any more. The earnings that accrue with them are quasi-rents and as long as they have positive operating earning margins - as long as output price covers operating costs - there is no need to discard them.

Figure 3.7 illustrates the age structure of the plants in the industry. Notice the difference in the base width, this usually indicates differences in the quantum of capacity installed in each period. It is a combined result of shift in the demand, technological change at the period, elasticity of demand and the age structure of the industry at that year.

Notice the difference in operating costs between plants that were installed in the years $-(n-2)$ and $-(n-3)$. One explanation for the reverse movement in costs is that in year $-(n-3)$, compared with $-(n-2)$, there was a drastic capital saving bias in technological change and/or change in relative prices that made labour (operating costs) relatively cheaper to capital (e.g. as a result of a rise in the rate of discount). Another explanation is that plant from year $-(n-3)$ deteriorates faster than plant from year $-(n-2)$.

The situation of old plant having lower operating cost than a younger plant can be met in practice, but clearly is not the rule:

For simplicity we shall hence assume that operating costs are a

1) W.E.G. Salter (1966) op. cit.

2) "plants" are used in a generic sense and have the meaning of machines, pieces of equipment, etc.

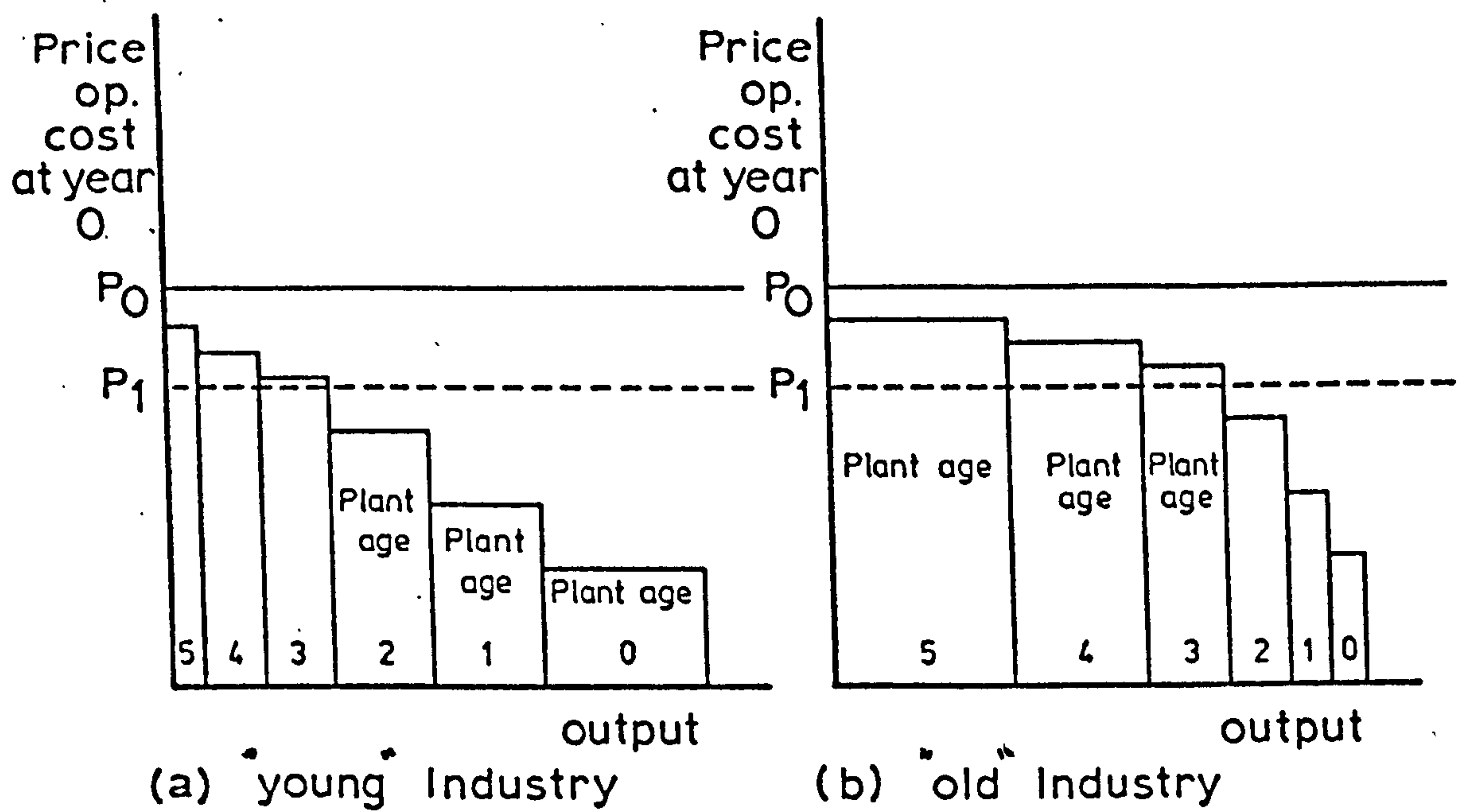


Figure 3.8: Industry Profile, "young" and "old" industries.

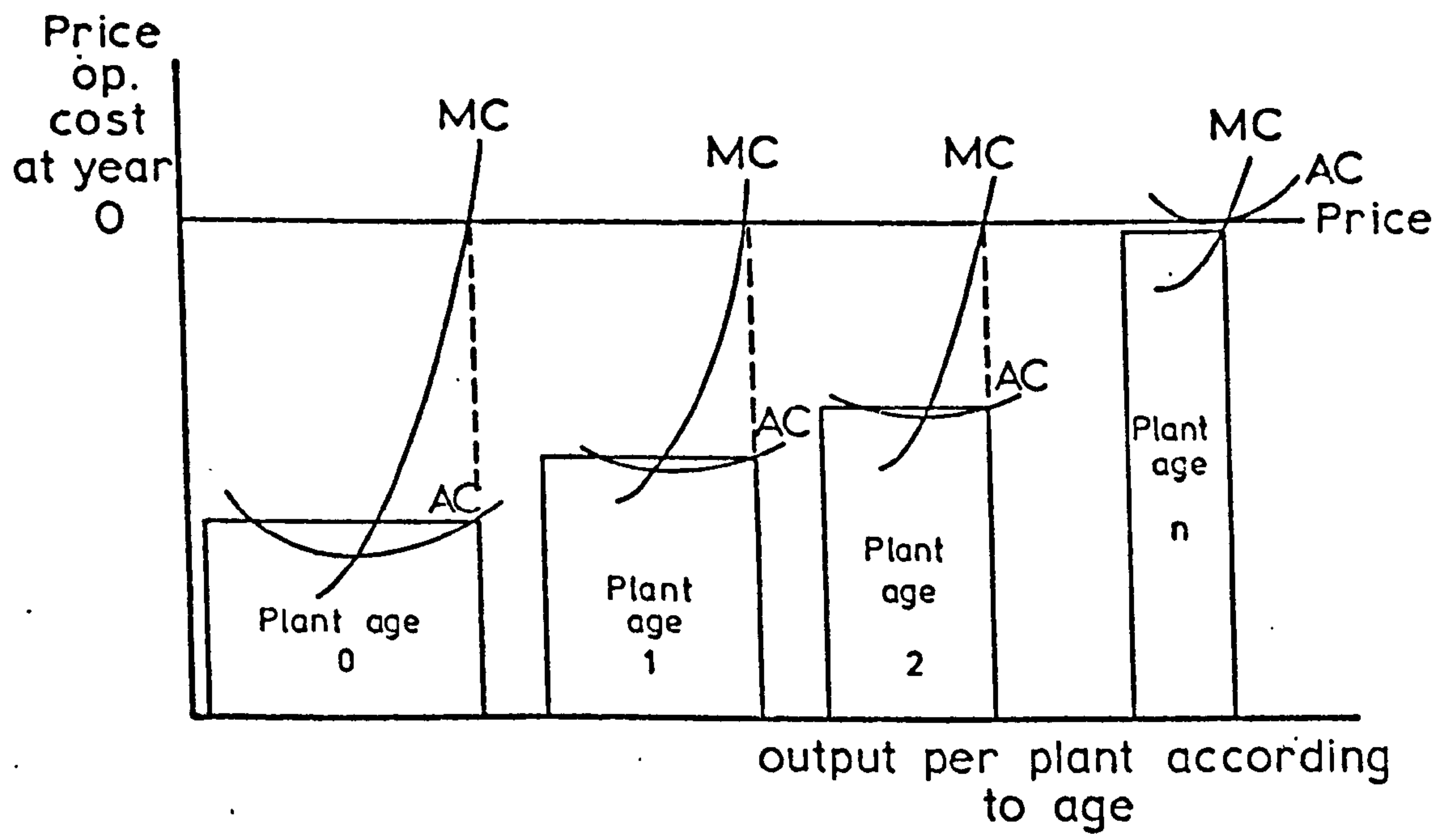


Figure 3.9: Intra plant capacity adjustment, according to output price.

monotonously increasing function of age in a cross section analysis of industry at any given time. So, in general, a growing industry is characterised by a large proportion of new plants with low operating costs, and a small proportion of old plants (Figure 3.8(a)). A declining industry is characterised by a small proportion of new plants (Figure 3.8(b)). The difference between the two industries is demonstrated by their sensitivity to cut price operations. While in the growing industry only a small proportion of the plant will be put out of work (Figure 3.8(a)), a large proportion will be put out of work in the declining industry. A recent example for an industry with a 'young' age structure is the Japanese Steel Industry. Its age structure explains a great deal of its ability to launch price competition for steel over the world. But, the rate of growth of the Japanese steel industry has decreased since 1970 so, inevitably the age structure of that industry is becoming 'older' and the capability to cut prices is being reduced.

"Marginal" analysis reveals yet another point that, though mentioned by Salter¹⁾, is usually overlooked or at least not formalised in decision models. That is the point that economically 'normal capacity' is not a physical characteristic of a plant but is determined by output prices, deterioration and wage increase overtime. This is demonstrated in Figure 3.9. Each plant is utilised up to the point where its marginal cost in the year equals the output price. The average cost at the point indicates the quasi rent gained per unit of output. Note in particular, an n years old plant in Figure 3.9. It has no quasi rent at the going market price and can therefore be discarded. Figure 3.10 shows that the exact quantity produced by each

1) W.E.G. Salter, op. cit. (1966).

plant is determined by the intersection point of the (increasing) marginal cost of operation and the price. Ceteris paribus, reduction in price would reduce the normal capacity of a given plant, increase in price would increase it. In general, deterioration, wage increase, and price reduction (in itself determined by technological and wage increase) combined reduce the 'normal capacity' of plants as they grow older¹⁾.

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- 1) Engineers, especially process engineers, tend to dismiss the 'niceties' of such analysis. They maintain that for every practical purpose capacity of a plant is constant over its life. Within the assumptions, this can be explained by postulating that the MC curve is very steep at the region of normal capacity, therefore none of the changes mentioned above can make an obvious impact on the level of capacity. But if the engineers are correct then this analysis is really superfluous. A more robust approach would be to assume that capacity is constant over the life of plant. This is the approach we take in this work.

3.3.3 Supply and Demand

The elementary supply-demand model is useful in demonstrating and analyzing how the corrective action of investment maintains the long run equilibrium of a competitive industry. It is perhaps worthwhile first to compare the short term equilibrium that supply-demand graphs usually depict with the long run equilibrium.

Figure 3.10(a) shows the short run mechanism. The supply curve consists of the operating costs of the existing plants which are ranked according to their "vintage" or age. It is assumed that in a cross section of the industry in a given period the older the plant is the higher is its operating cost. The demand for the product of the industry is D . As usual, the demand curve represents the product quantity demanded in a period, as a function of the product price in the period. If demand increases from D to D' a stand-by plant will be put to work, if demand decreases to D'' then the marginally worst plants will be put to work. This and similar models are useful in describing changes in demand that are both "fast" and "temporary"; situations where corrective action is done through existing plants and through changes in output price. Long run models use the same graphical structure (see Figure 3.10(b)). The visible or graphical part of the model still represents the supply and demand in a single period but the determinants of both supply and demand are considerations beyond that visible "window". Changes (mainly increases) in demand in the long run model are expected to persist for at least the length of life of a new plant. Still they are "fast" changes as in the short run model since it is assumed that installation and implementation of new plants are instantaneous. For a moment it may seem a paradox that in long run models the market adjustment is done not by the marginally worst plants but by the marginally best plants: by investment in a new plant that has suffered no deterioration and is technologically superior

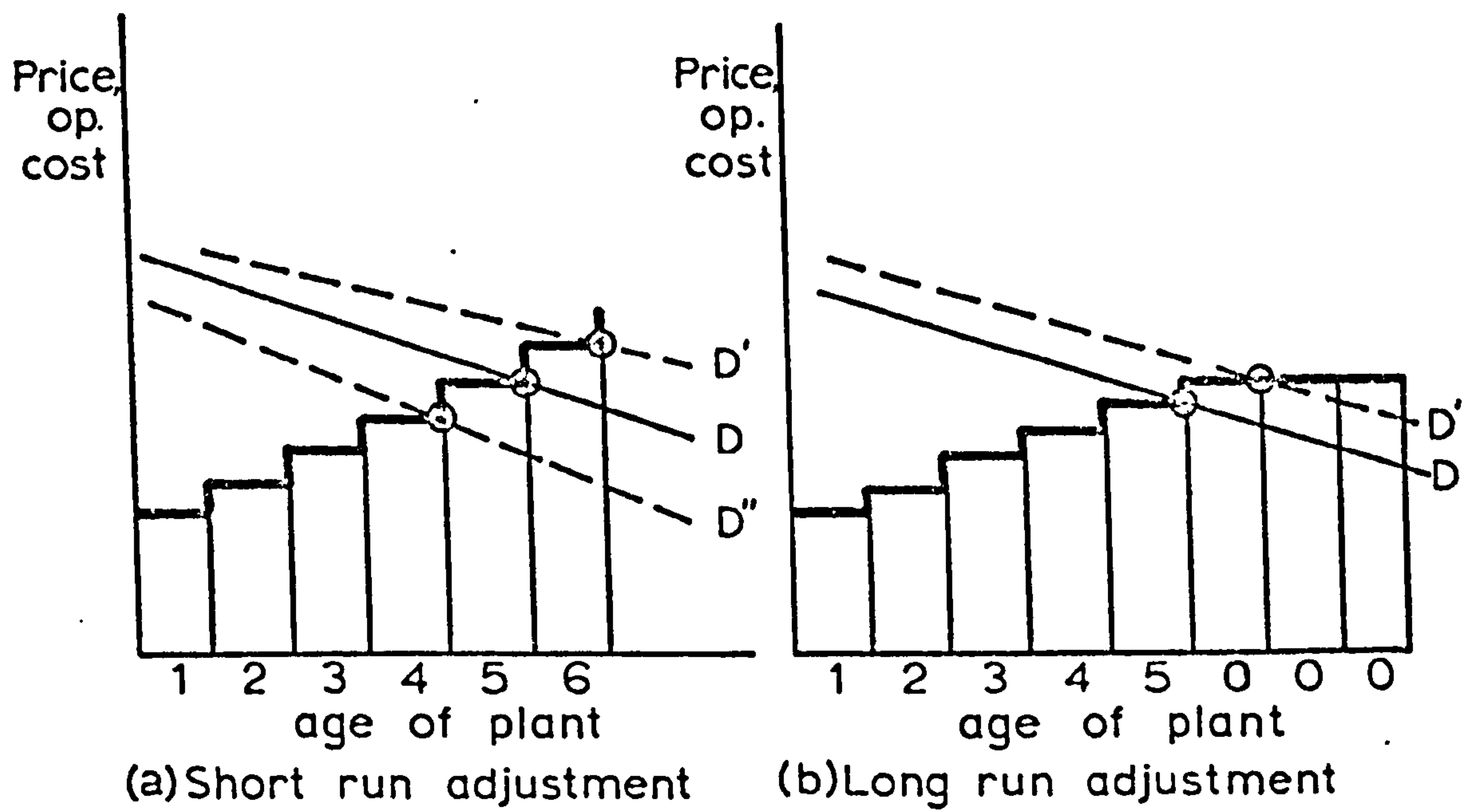


Figure 3.10: Short run and long run supply adjustment.

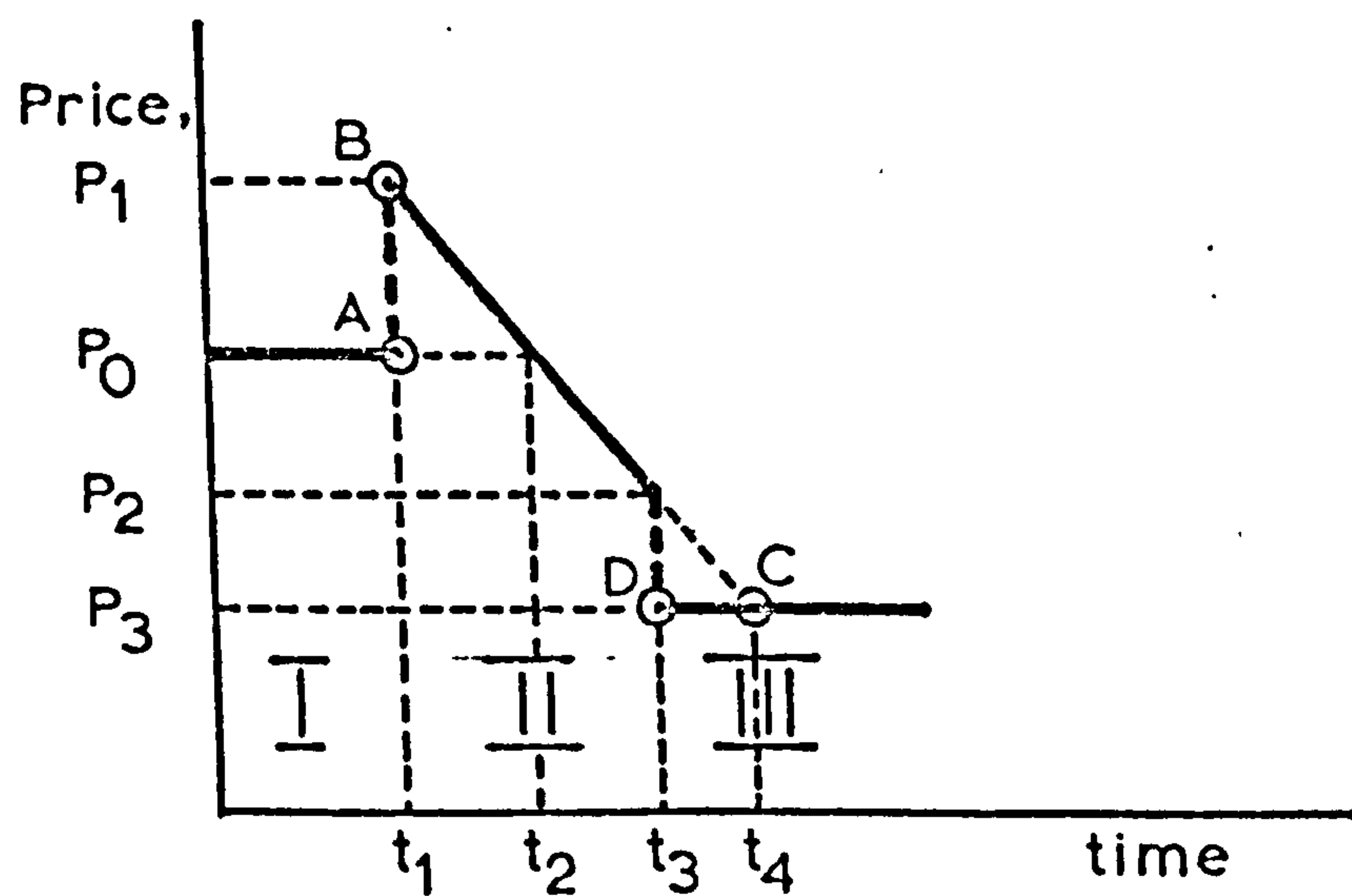


Figure 3.11: Time movement of output price under change of technological expectations.

to existing plants. The explanation, of course, is that in a sense the new plant is "the worst plant". The cost of a new plant includes the fixed cost of investment as a decision variable in addition to the operating cost. In old plant investment or "fixed cost" is a sunk cost and therefore not a decision variable.

The long run model (Figure 10(b)) assumes infinite supply elasticity - in a given period there is no limit on the supply of the given best practice plants. The price in a period is determined solely on cost considerations in new plants (as long as in each future period demand and supply are such that "at least" one new plant is installed each period). This price as stated earlier is the long run marginal cost. Lastly, assuming the long run marginal cost and the product price given, let us list the factors that determine the level of investment in a competitive industry in a period. They are:

- (a) The number of plants that have to be scrapped in the period at the equilibrium price. This number is determined by deterioration and obsolescence and by the age structure of plants in the industry.
(See Figure 3.8).
- (b) The reduction of output in plants that do not require scrapping (if any; see Figure 3.9).
- (c) The elasticity of demand. The new price sets a new figure for the demanded quantity.
- (d) Change in the demand curve. Clearly, if we assume explicit technological change and change in wages over time in the model then a change in the demand function is an obvious corollary.

3.3.4 Shifts in the Expectation of Technological Change

The long run equilibrium model is based on unanimity of expectation of future price movement, today's price level and the volume of investment are related to this expectation. In the following discussion we shall investigate the ex post impact of changes in expectation (unanimous changes!) on price and investment in an industry. In a decreasing cost industry undergoing technological change the change of expectation may be either towards a faster or a slower technological change. The two possible cases can be paraphrased as follows:

- (a) No change, or a slow change has been expected in the past; fast technological change is expected now.
- (b) Expectations for fast technological change have proved to be exaggerated; slow or no change is expected now.

This change of expectation has a somewhat surprising effect on prices. In case (a) the apparent technological break-through will push output prices up. In case (b), the recognition that there is no possible further improvement would reduce output prices. The understanding of this seeming "paradox" is important to the understanding of the observed patterns of technological change in an industry.

The impact of change in expected prices is demonstrated in Figure 3.11. Calendar time is divided into three 'periods' I, II and III. In 'period I' no technological change is expected and price is constant. From a certain point t_1 a rapid technological change is expected, indefinitely. This sort of expectation persists during 'period II' until point t_3 when the expectation is shifted again. In 'period III', from t_3 onwards no more change is expected in the foreseeable future.

The entrepreneur, at point t_1 , suddenly faces the problem that his new potential plant (still exactly as the old one built earlier) would become obsolete and will have to be discarded earlier than would have been expected. The only way to obtain normal return on his investment is, therefore, to raise his price to p_1 . But at a higher price the industry's existing plants will raise their production (equate their MC with the new, higher price). Moreover, some standby plants will re-enter production. Therefore, the price will not rise fully to P_1 . The actual pattern of price increase is determined by deterioration of old plant, age structure of old plant, change in demand, etc. What can be said is that introduction of new plant will not be delayed beyond t_2 (see Figure 5) but nevertheless, there will be some period after t_1 , when the level of new investment will be low. At the other end, at t_3 , the opposite happens. The entrepreneur learns that his new plant will not be affected by obsolescence, therefore, the life expectancy of the plant increases and he is able to charge lower prices for his output. As long as introduction of new plant is instantaneous the price will fall suddenly at time t_3 from P_2 to P_3 . This will be combined with massive investment in new plant and scrapping of old plants unable to cover their operating cost. This is, of course, too abrupt to be realistic, so it can be said that above normal investment activity takes place some time after t_3 . The process will end at t_4 the time when P_3 would have been reached had the technological progress continued¹⁾.

What is the meaning of these price jumps? They are the shadow prices of technological change or the cost of technological change.

1) For a detailed discussion on the forces that determine prices in a situation of changed expectation, see H.R. Fisher, op. cit.

Intuitively they are the extra output price that has to be paid at any point of time in order to enable reduction in future prices. This reduction is possible through the fast rate of scrapping of machines that have inferior performance. At any point of time under technological change a 'premium' has to be paid for the higher rate of capital consumption compared with the hypothetical situation where the process of change and obsolescence could be stopped. Conversely the introduction of technological change for the eventual achievement of price reduction requires at first a rise in the output price to cater for the implied increase in capital consumption. Assume technological progress is a controllable variable, then the decision to embetter the future at the cost of immediate higher prices is highly comparable to macroeconomic capital accumulation decisions where at question is whether to increase saving and improve future welfare or to increase consumption and immediate benefits.

On the decision making level the importance of the phenomenon is that though price reduction is highly related to technological change, the timing patterns of the two do not necessarily coincide. It would be dangerous to qualify that technological change is rapid or slow at a certain period only because price reduction is fast or slow at that period.

3.4 Summary

Any economic modelling of technological change requires very rigid simplifying assumptions. It is widely accepted that the most useful tools of describing the phenomenon of technological change on an industry or a product level are microeconomic neo-classical tools. They clearly are helpful in sharpening concepts and in clarifying difficulties associated with the description of technological change.

One important simplistic tool is the single product production function. Using two production factors this model demonstrates the following: overall productivity increase, bias in technological change, and the impact of change in relative prices on choice of technology. Also it demonstrates the further assumptions required in the measurement of the various aspects of change when information about production factors is given solely in monetary terms, and it demonstrates the actual change in technology that follows from the simultaneous movement of wage increase and the genuine advance in technological know-how.

The second important tool is the vintage models which consider the movement of costs and prices over time, in particular the movement of operating costs in a plant over its life and the movement of the operating costs in the best practice new plants. These models demonstrate how the long-term-marginal-cost-price is determined and show the role of capital investment in satisfying changes in the demand for a product.

Also, vintage models are capable of demonstrating the impact of shifts in technological expectation on the level of output price and on the volume of investment at a particular period of time.

CHAPTER 4

THE PROCEDURE AND THE DYNAMICS OF INVESTMENT

DECISION MAKING

4.1. General

The previous two chapters were devoted to a 'positive' analysis of the dynamic economic environment of investment decision making. That 'positive' approach is important but clearly not sufficient. Our aim is to evaluate and develop investment decision models, and in order to do so we yet have to look at the decision making procedure, or system, itself. We shall look at the actual procedure of investment decision making, both at the way it is carried out in practice and at the way - so far as experts agree - it ought to be carried out. A glimpse at the way the decisions are made by medium-to-large industrial companies is given by a small number of published surveys. These surveys usually highlight differences in procedures and decision criteria which reflect both the conservatism of industry and the fact that new techniques are not a miraculous panacea in a world where investment uncertainty prevails. There are nevertheless certain patterns that hold true for most medium-to-large companies in the private sector and also presumably in the public-nationalized sector.

There are obviously certain basic patterns within which investment decision making procedure should fall. For example - timing. Investment decisions, due to the involvement of many people in making them are made at well prescribed times and not 'at any time'. The usual pattern, nearly without exception, is that investment requires the authorization of the periodical budget, e.g. the annual budget. Investment decision models do not usually consider this point, although as will be seen later the discrete nature of the decision making process itself can enter into a decision model.

A number of stages are associated with every investment proposal.

- a) the origination
- b) the detailed proposal
- c) authorization (acceptance or rejection)

and if accepted also:

- d) implementation (and expenditure control)
- e) post audit.

The degree to which these stages are formalized vary from one company to another and with relation to the type of investment. My interest in this work is in the first three stages which really are the decision making phase, though it must not be forgotten that the existence of clear procedures of expenditure control and post audit themselves affect the amount of effort that is put into the first three stages. A promoter of investment proposals is likely to be more scrupulous in setting his cost estimates if he has to be answerable for excess spending at the implementation stage.

4.2. Procedure: The Organization of Capital Budgeting.

Source materials for this and the following section have been: textbooks on financial management and capital budgeting¹⁾, a number of survey reports and other articles on the subjects, the internal capital budgeting manual of a large mining company, and to some extent my personal experience. Textbooks reflect a serious pedagogic dilemma. They attempt to achieve in their teaching a balance between showing the 'real world', what management does and what management ought to do.

It is extremely difficult to generalize how companies process and implement their capital projects. Companies differ in the nature of their business in their size, in their history, in the personalities involved in running them and in a great many other respects. Procedures, and capital budgeting procedures are no exception, are tailored to fit specific needs. So when writers discuss capital budgeting procedures organization and processing of proposals they describe, above all, one system with which they are familiar, perhaps with some modifications. At the other end there is no such thing as a theory of capital budgeting procedures. Therefore suggestions as to what management ought to do are based on expediency and on common sense. Thus it is significant to note that there is a general agreement among the various writers as to the main ingredients of administration and procedures of capital investment decision making. It is also reassuring that the details of the procedure employed by the large international mining group mentioned above falls neatly within the 'stereotype' of the textbooks.

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- 1) e.g. a) A.J. Merrett & A. Sykes, "The Finance & Analysis of Capital Projects" Longman, London, 1973.
b) H. Bierman & S. Smidt, "The Capital Budgeting Decision" MacMillan, N.Y. 1971.
c) W.T. Morris, "The Capacity Decision System", Richard Irwin, Inc. 1967.

4.2.1. Administration.

A special department in the company deals with all aspects of capital budgeting. The department operates in close conjunction with other relevant management service departments, finance department, management accounting department and planning department, and according to assignments, with line management. The responsibilities of the department include, among others:

- a) to determine procedures for submitting investment proposal forms;
- b) to initiate and assist in the preparation of investment proposals where needed;
- c) to coordinate the capital budget with the operating budget;
- d) to help in setting investment decision criteria;
- e) to take part in the control of capital expenditure;
- f) to conduct post audit on past investments;
- g) to screen and analyze investment proposals;
- h) to analyze and control quasi-investment activities, leasing, hire-purchase, etc.

4.2.2. Types of Investment.

Investment projects are divided into a number of classes according to the complexity of the decision making process associated with them and according to the risk involved. The exact classification is unique to every company¹⁾ though the classification employed by the mining group is not untypical. Here it is:

"Projects would be broadly distinguished as follows:

- (A) Those which must be made in order to continue the business e.g. replacement of worn out or obsolete equipment;

1) Many examples are found in D.F. Istvan, "Capital Expenditure Decisions: How they are made in large corporations", Indiana University, 1961 (pp. 101-104).

- (B) Those intended to reduce costs;
- (C) Those intended to expand production;
- (D) Those required for the production of new or improved products;
- (E) Those necessitated by legislation or by welfare requirements".

Category E obviously requires a simple decision process. There is no question of rejecting or delaying a proposal. All that category E requires is that projects will be carried out at minimum cost. Among the other classes B is more complicated and uncertain than A, C more than B, and D usually entails the greatest risk and therefore requires the greatest amount of judgement in the decision making process. Usually the amount of investment and the length of time involved in carrying out the project increase in the same order, class D being the more expensive and more involved.

The different types of projects call for different treatment and companies employ different procedures for different classes. Thus replacement projects may require different forms than expansion projects and perhaps the decision on them will be made at different levels of management. Nevertheless it is the responsibility of both the capital budgeting and the finance department that a clear and consistent decision criterion will be employed in both cases.

4.2.3. Long Range Planning and Capital Budgeting.

Long range planning department on all its aspects: policy planning, strategic planning, corporate planning, etc., is now part of the household of most large companies. In general the "planning" activity is a combination of 5 - 10 year economic forecasts, 3 - 5 years tentative action plans that comply with corporate policy, and 1 - 2 year detailed action programme which includes not only investment

but many other activities. This finally is related to a one year operating budget. It is the only part of the 'plan' that entails a firm commitment for action; all other forecast, plans and programmes are therefore updated according to need as a matter of annual activity. The stereotype of these forward looking managerial activities is sometimes referred to as PPBS - Planning programming budgeting system.

The importance of such a system to prudent capital budgeting cannot be overstated. It gives a framework for assessing investment projects that are to be carried out in the coming budget period and bear fruition in later years. It also shows how investment projects may fit in the future shape of the company.

4.2.4. Time.

The time dimension in the organization of capital budgeting is very important. Proposals are usually prepared as part of the preliminary stages of the annual budget. An example are the guidelines set out for the subsidiaries of the mining company:

"Each subsidiary company prepares and sends
by the end of September to its country head-
quarters its plan of operation for the follow-
ing year..."

This budget includes the capital budget. According to Istvan¹⁾ this is the standard procedure in most companies. Some of the firms in his survey go so far as to make capital budget completely concrete.

1) Op. Cit., pp 30-31

In these firms proposals have been completely analyzed so that inclusion in the budget is the same as final approval; no resubmission is necessary. Istvan objects to this practice but his general finding is that even though budgets are "blue prints" and not final authorisation there is a clear indication in them as to what will be authorized in the budget period.

"Proposals included in either the rationing or the financial short range budget [the two types of budgets] have been accepted in concept although final approval usually depends upon the submission of additional detailed data, followed by rigorous screening and evaluation".¹⁾

and he adds that:

"In most firms, actual submissions are limited to those proposals included in concept in the budget".²⁾

I have found by way of personal communication that this generally is the picture in overseas mining companies and in the National Coal Board in this country.

Continuous budgeting - continuous submission and approval of investment proposals "as they come" - admittedly has the theoretical appeal of avoiding delays of investment and thus the incidence of opportunity losses. But it has serious practical drawbacks. It requires that decision makers are available all the time, that the cost of new finance can be continuously assessed and that projects can always be judged on their own merits without reference to

1) Istvan, ibid. p.26

2) Istvan, ibid. p.28

competing projects. The practical advantages of annual discrete budget in planning and control clearly outweigh the presumed correctness of continuous budgeting. In particular the annual budgeting enables neat annual reassessment points for rejected and postponed projects. 'Continuous reassessment' of rejected projects is, of course, meaningless.

4.3. Procedure: The Stages in Processing Investment Proposals.

A PEP report¹⁾ published ten years ago highlights a yawning gap between prudent capital budgeting and actual investment decision making in some manufacturing companies. Here are a few examples for management attitude to equipment replacement:

1. [The works manager of a domestic appliance firm]

"Even if the machine was performing well and had been depreciated there wouldn't be a case for replacement, would there?.... It's earned it's keep at work.... There's no case if it is producing effectively to say, 'Well, look it's been a faithful servant. We've had it many years. We must get rid of it'. I should say, 'No. If it is a good performer we'll keep it'."

2. [The works director of 'a very large' earthmoving equipment firm]

"We very rarely chuck out a machine tool because there is something better available. We would normally go to the normal length of that machine tool which is, say, ten years, and we will then replace it willy-nilly. If we find something better to replace it with, obviously we will, but if there isn't anything better then we will replace it with the same again".

3. [A shipyard managing director]

"I think modernisation in shipbuilding has become a fashion rather than a factual need..."

Attitudes as expressed here are still abundant.²⁾ There is a vast scope for improving the methods of initiating, presenting and selecting investment proposals. Surveys like this also show how far from the realities of many companies sophisticated investment decision models can be. Such companies first need some kind of rudimentary capital budgeting organization. Only then can the selection of a 'correct' investment decision criterion be contemplated. It is nevertheless believed that most medium-to-large

1) Published as: "Thrusters and Sleepers", George Allen & Unwin London, 1965.

2) The discussion in this section is restricted to the 'formal' managerial attitude to investment and replacement. Intra-organizational and interpersonal considerations are discussed in: R.M. Cyert & J.G. March, "A Behavioural Theory of the Firm", Prentice-Hall, 1963, especially pp. 48-54.

companies today (employing more than 500 people) have better tools of processing investment proposals and evidence to that effect will be presented in this section.

There are a number of stages in processing investment proposals and companies that employ a comprehensive capital budgeting system formalize these stages into separate activities. For the purpose of discussion these stages are divided into three groups: proposals, selection and subsequent activities.

4.3.1. Proposals

Investment proposals are submitted to the decision makers in documents that include monetary information in their support. The general approach is to present the information in annual cash flows or at least in a format that will enable immediate deduction of annual cash flows. There are however differences in the presentation of investment proposals from different classes. Routine proposals, e.g. equipment replacement and other cost reduction proposals, lend themselves to the use of standard forms while major proposals are unique and are better treated on an ad hoc basis. But what is a major project for one company is a routine project for another larger company. The detailed example given in Groundwater¹⁾ is nothing short of a standard procedure employed by an international oil company in the evaluation of major oil fields. A similar example showing forms used in the evaluation of new mining projects is given by Merrett and Sykes²⁾. Simple proposal forms are given here as Exhibit 4.1 and in Chapter 5 as Exhibit 5.1. The internal manual of the mining

1) T.R. Groundwater, "Role of Discounted Cash Flow Methods in the Appraisal of Capital Projects". IMM Transactions Vol. 76 (1967) pp. A67-A82.

2) Op. cit. pp. 352-364.

Exhibit 4.1:

Example of a
"Capital Expenditure
Sanction"(Net cash
Flow or net profit)
Form

(Source:
J. Batty, Op. Cit.)

Proposal No. Date of Proposal			
CAPITAL EXPENDITURE SANCTION (REPLACEMENT/NEW* PROJECTS)			
To: Capital Expenditure Committee/General Manager* From: Capital Budget Department			
Application is submitted for authority to incur the following capital expenditure:			
Project: (Full details to be inserted)		Estimated Life	
Expenditure	Cost of Asset Delivery Charges	£ £	Own Labour Own Material Own Overhead
Total £ (A + B)	A £	B £	£
Reasons for Expenditure:			
Expected Receipts: (Give details of products to be sold summarized from Sales Forecast attached)		Sales Values £	Present Value of Future Sales Values £
Products: YEAR 1 at Standard Price " 2 " " " " 3 " " " " 4 " " "		£	£
Expected Running Costs: (Per Detailed Statement Attached)		Costs £	Present Values of Costs £
YEAR 1 " 2 " 3 " 4		£	£
Net Cash Flow (or Net Profit) Expected Return on Investment = ...		£	£
Capital Expenditure Committee		APPROVAL/REJECTION*	
Reason for Rejection (if applicable) Recommendations (if applicable including recommended accounting year for purchase*)		Date	
Signed (Budget Controller)		Signed (Chairman)	
Board of Directors' Approval			
Date		Secretary	
(Note: All figures to be supported by detailed schedules) * Delete where appropriate			

Exhibit 4.2:

Ruling for Plant
and Machinery
Register

(Source:
J. Batty, Op.cit)

PLANT & MACHINERY REGISTER								Page No.
Fixed Asset Description				Department				Project Sheet No.
Maker's Number				Departmental Number				
Supplier				Depreciation Plan				
Purpose				Residual Value £				
Floor Space Occupied square feet				Horse-Power				
Insurance Cover £								
Costs Purchase Price Installation Modifications	£	Year	Depreciation	Book Value	Year	Depreciation	Book Value	
Maintenance Record		Year	Details	Cost	Year	Details	Cost	
Replacement Cost Assessments		Year	Method	Estimated Replacement Cost	Year	Method	Estimated Replacement Cost	

Exhibit 4.3: A check List of Points to Consider in Preparing a Capital Expenditure Proposal.

(Source: The internal capital budgeting manual of a large mining company).

COMPANY NAME	CAPITAL EXPENDITURE PROPOSAL	SCHEDULE L
The following is a minimum check-list of points to be considered in preparing Schedule L:— (1) Reference number of proposal, including type of proposal. (2) Brief description of proposal including total cost, suggested timing (commencing date), construction period, and likely useful life. (3) Reasons for proposal, and consequences if postponed or rejected. (4) Markets for new products or additional output involved. (5) Availability of materials and services required. (6) Capital cost (amount and timing) split between materials, equipment, contracted work, own labour, materials and services, including own equipment transferred from other uses at resale value, etc. (The capital cost can be reduced by the net of tax sales value of any equipment which will be scrapped if the proposal is approved, but this figure should be shown separately.) (7) Additional working capital (amount and timing) and the detailed basis of computation. (8) Special tax allowances or grants. (9) Net cash flow arising from proposed capital expenditure (i.e. additional profit before depreciation, less tax, etc., or net of tax cost savings). (10) Discounted return on capital (i.e. item (9) compared with item (6) plus item (7)) for proposals over £25,000 each, for a range of assumptions where appropriate. For items under £25,000 it may suffice to compute the pay-back period, but the total estimated useful life should also be given for comparative purposes. (11) Patent position. (12) Insurance cover, including fire, pollution hazards, etc. (13) Accounting allocation between buildings and equipment, and between capital and revenue, with justification, and proposed depreciation treatment. (14) Details of alternatives considered, including, for items over £25,000 each, the relevant data under (6) to (10), inclusive. (15) Particulars of those responsible for preparing and submitting the proposal.		
NOTES TO SCHEDULE L (1) Proposals should be made not only for capital expenditure, but also for all long-term commitments, where the total amount involved exceeds, say, £25,000. Instances of such commitments are leases of buildings, leases of plant, machinery and equipment, and long-term contracts for the supply of materials or services. (2) These proposals cover neither New Ventures nor Major Capital Projects which are dealt with in Part Two. (3) The points included in Schedule L are not exhaustive. On the other hand, there will be many cases where some of the points are irrelevant, particularly in the case of expenditure to replace worn-out equipment or to meet legal or welfare requirements. Schedule L is thus intended mainly as a check-list. (4) Once mines have been brought into production it is Group practice to charge off all subsequent Development Expense (other than shaft deepening and ancillary services) as it is incurred. The removal of overburden is treated in the same way. Where this practice is followed, development work will be dealt with in Schedules A and N (the Annual Operating Profit Plan and Monthly Profit Statement).		

group that was available to me gives an extremely useful check list for the preparation of investment proposals - (Exhibit 4.3). Note that this check list is employed in the preparation of routine projects. Major new projects require a much more detailed check list.

Inception of proposals - the origination of investment ideas can derive from many parts of the company. Survey results show that most proposals, in particular cost reduction and expansion projects, originate at plant or other peripheral levels rather than at the head office. Istvan¹⁾ discovered this in 1959 in his survey of 48 large American Corporations. His results were generally confirmed in a more recent survey, by Petty et al²⁾, of large American corporations from the "Fortune 500" list. There is however a clear indication in this last survey that a significant proportion of the projects originate at levels higher than plant level, i.e. divisional (or regional) levels.

Literature of more prescriptive nature discusses origination of projects at the capital budgeting department or its divisional off-shoots in addition to origination at various line management levels. Taussig³⁾ in his work on replacement of vehicles has devised a certain decision making procedure that was based specifically on relevant information in vehicle records. He claims to have achieved a clear cost reduction compared with methods previously employed. Not surprisingly, perhaps, his work dealt with military vehicle fleet. Batty⁴⁾ in a textbook for accountants cautiously comes out with

1) Istvan, op. cit.

2) J.W. Petty, D.F. Scott and M.M. Bird, "The Capital Expenditure Decision Making Process of Large Corporations", The Engineering Economist, Vol. 20 No. 3 (spring 1975) pp. 159-172.

3) R. Taussig, "Information Requirement of Replacement Models", Journal of Accounting Research, Spring 1964 pp. 67-79.

4) J. Batty, "Management Accountancy", MacDonald & Evans Ltd., London, 1966.

similar ideas. Exhibit 42 gives his example of Plant and Machinery Register that gives, together with the historical cost information, some maintenance and replacement cost figures to be used in replacement decision. But the most forceful crusader for an in-built headquarters origination appears to be Terborgh¹⁾.

"[Role of Investment Analyst]

The object of special staffing is not merely to provide competent analysis of individual proposals.... He should have the authority to originate studies and proposals on his own initiative".

To a certain extent the whole Terborgh's MAPI method (discussed in Chapter 5) is aimed at formalizing procedures by which minor investment proposals can be originated by the "analyst".

A point of interest is that companies do not usually have an explicit mechanism that ensures that sufficient proposals are set forward. 47 out of the 48 companies studied by Istran indicated that investment proposals received no direct stimulation. It was assumed by most companies that somehow the competitive nature of business was a sufficient stimulus for that.

"Contrary to the theories generally advanced the organization of capital expenditure proposals receives no special stimulation among the firms studied." 2)

Istvan was astounded by the clear-cut result and believing in the dominance of market forces he hurried to conclude that sufficient proposals of merit arise out of the normal manner of

1) G. Terborgh, "Business Investment Management", MAPI, Washington D.C., 1967, P.21.

2) Istvan, op. cit.

doing business. This clearly is not the case in the companies of PEP survey. I would rather share Terborgh's view that there is no sufficient stimulus to the submitting of investment proposals. This is all the more so as far as replacement projects are concerned; many of which become overdue long before being proposed. This I believe is an underlying weakness in the capital investment decision making system. Generation of investment proposals by the "analyst" in addition to the generation of proposals by line management is a step to remedy that weakness. (The question of encouraging investment proposals is revisited in 4.3.3).

4.3.2. Screening and Project Selection.

The screening of a project is usually a review of the investment proposal done prior to the authorization stage. There can be a number of phases in screening. First a certain managerial level may be required to give its approval to a project before it goes to the head office. Second and perhaps more important is a review of the project done by the budgeting department itself. This includes a verification that the proposal conforms with the requirements laid down for its class of investment (new product, expansion, cost reduction, replacement), certain checks on the financial estimates and, in most cases, some comments on the proposal. The proposal form in Exhibit 4.1 allocates some space for these comments. The capital budgeting department, being better informed than peripheral management in aspects of the company's investment policy, finance, and the availability of alternative competing projects, is in a strong position in making its opinion a dominant one, especially regarding replacement and cost reduction projects.

After an investment proposal has received the consent of the reviewers and proved to conform with the necessary procedures it goes, together with the other capital budget proposals, through a selection-authorization stage. It is appealing to think of this stage as a clear cut single act of selection at which projects are either selected and then forthrightly implemented or rejected and then abandoned. But it is not; this stage can be divided into significant substages and the verdict can be more complicated than a simple accept-reject type.

The standard practice of a periodical capital budgeting means that the majority of projects are 'authorized' twice. Once as a tentative authorization as a part of the periodical capital budgets. This is then followed by a detailed investment proposal i.e. application for funds which require a final authorization. Most projects follow this pattern. It is usually difficult to introduce within the budgeting year an application for a project that has not previously been included in the annual budget. Considering the amount of time that is required both in the preparation and in the implementation of projects there is usually little need for substantial 'emergency' or 'extra budgetary' investment. Small emergency projects are usually covered by contingency budgets or by the discretionary budgets that are available to peripheral managers and which they can spend without further authorization from head office. But again the general pattern is that the vast majority of investment funds are allocated to projects that are authorized individually at a certain head office level. The exact managerial level that has the statutory authority to select projects from a given class is not important in this discussion.

Companies

vary a lot in the pattern of delegation of investment decision making¹⁾.

The importance of the stage of tentative budgetary authorization can be seen from another angle. It is most unusual for companies to irretrievably authorize capital expenditure to be spent in future periods. Unless perhaps, when this expenditure is part of a multiperiod project that is started earlier. The usual procedure is to wait and consider the project only in the relevant periodical budget.

Final authorization of investment projects is a human decision; but impersonal objective investment criteria play an important role in this last decision making stage. The objective criteria are measures of profitability: 'the Payback Period, Return On Investment', DCF rate of return, NPV, etc.²⁾ In routine projects the importance of the 'objective' criteria is so great that it can be assumed that investment projects are accepted if they pass the profitability test and rejected if they do not. Statutory adoption of an objective criterion, especially one of the more rigorous DCF criteria, clearly enhances the status and the importance of the capital budgeting department and gives it certain decision making powers that otherwise remain in the hand of line management.

A good deal of literature has been written on the subject of investment criteria mostly promoting the use of discounted cash flow methods. It is interesting to note that the profile of investment

1) Istvan, Op. cit., passim.

2) I assume the terms are familiar and I do not explain them. Explanation can be found in textbooks, e.g. Bierman & Smidt, Op. cit.

criteria used by U.S. industry as found by Fremgen¹⁾ in 1973 (Exhibit 4.5:(a)and(b)) is markedly different from the one found by Istvan in 1959 (Exhibit 4.4). Over the years there has been a clear trend towards the adoption of discounting techniques.

Exhibit 4.4: SUMMARY OF EMPLOYMENT OF VARIOUS MEASURES OF ACCEPTABILITY

Measure of Acceptability	Number of Firms Using as the Primary Measure	Number of Firms using in a supplementary manner
Time adjusted rate of return	5	9
MAPI formula	2	0
Simple rate of return	24	8
Pay-back	13	21
Subjective judgement	4	44
Total	48	

Source: Istvan, ibid p 96.

It is fair to believe that the development in Britain has generally been the same as in U.S. though adoption of discounting techniques has been somewhat slower over here. Unfortunately I know of no widely based U.K. survey similar to those of Istvan, Fremgen and Petty et al. There have been however a number of more modest surveys that deal with investment replacement. Mentioned above is the PEP survey. Others are that of R. Neild²⁾ and that of H.H. Scholefield.³⁾ The last two conducted questionnaires. Considering their dates they indicate an important shift towards discounting techniques. 2 companies (2%) of Neild's 1963 survey used DCF methods while 14 companies (70%) of Scholefield survey employed these

- 1) J.M. Fremgen, "Capital budgeting: A Survey", Management Accounting (U.S.) May 1973, pp. 19-25.
- 2) R.R. Neild, "Replacement Policy" National Institute Economic Review No. 30 (1964).
- 3) H.H. Scholefield, "Replacement of Equipment", Accounting & Business Research, Autumn 1972.

Exhibit 4.5:

Capital Budgeting Practices, summary of Fremgen's survey

(Source J.M. Fremgen, Op. cit.)

(a)

METHODS IN ACTUAL USE

Size of annual capital budget	Discounted rate of return	Net present value	Present value index	Payback period	Simple rate of return	Other methods
Over \$100 million	78%	34%	9%	72%	60%	14%
\$50-\$100 million	79%	21%	10%	62%	55%	3%
\$10-\$50 million	64%	14%	2%	68%	44%	11%
Under \$10 million	67%	0%	5%	52%	33%	0%
No size given	67%	33%	0%	67%	0%	33%
All respondents	71%	20%	6%	67%	49%	10%

(b)

MOST IMPORTANT METHOD

Size of annual capital budget	Discounted rate of return	Net present value	Present value index	Payback period	Simple rate of return	Other methods
Over \$100 million	34%	5%	0%	2%	31%	7%
\$50-\$100 million	38%	7%	3%	7%	14%	0%
\$10-\$50 million	39%	3%	0%	23%	18%	5%
Under \$10 million	47%	0%	5%	24%	24%	0%
No size given	0%	0%	0%	33%	0%	33%
All respondents	38%	4%	1%	16%	22%	5%

(c)

CAUSES OF CAPITAL RATIONING

Basically external causes:

Limit on new debt imposed by some agreement with outside parties (e.g., a bond indenture)	41%
Limit imposed by higher authority outside the reporting organization (e.g., corporate management, when the respondent is a division or subsidiary)	36%
Lack of free access to capital markets for some other reason	6%

Basically internal causes:

Limit on new borrowing imposed by management	67%
Management's desire to maintain a regular dividend policy and, thus, to restrict retained earnings available for new investment	29%
Management's goal of maintaining some specific earnings per share or price-earnings ratio and, thus, a policy of restricting issuance of additional shares of common stock	21%
Some other restriction on issuance of new shares of common stock (e.g., a desire to maintain close control of a corporation)	15%
Inadequate cash flow from operations to finance new investments	3%
Other causes	8%

(d)

ADJUSTING FOR RISK AND UNCERTAINTY

Requirement of a higher-than-normal index or profitability	64%
Requirement of a shorter-than-normal payback period	40%
Adjustment of estimated cash flows by use of quantitative probability factors	32%
Purely subjective, nonquantitative adjustment	26%
Other methods	6%

(e)

THE MOST CRITICAL AND DIFFICULT STAGES

	Most critical	Most difficult
Project definition and estimation of cash flows	51%	44%
Financial analysis and project selection	27%	12%
Project implementation and review	23%	41%
No response	2%	1%

methods in 1971. Yet an extension of their results to a general industrial picture must be done with caution. 65% of Neild's sample are firms that employ less than 1000 people and Scholefield's sample is very small (20 companies) comprising of small companies and probably highly localized to the Sheffield area.

That the question of investment criterion receives more than its fair share of the literature on investment is indicated in a number of papers. Foremost are Fremgen's results given in Exhibit 4.5:(e). The research into the relevant cash flow is regarded more important and more difficult than the financial analysis. The calculation of the DCF rate of return itself boils down to a simple computational exercise if the cash flows are available. Groundwater¹⁾ too presses this point in an article aimed mainly at engineers:

"... the elements of cash flow form the basic requirements of any method of venture appraisal. It is hoped that this paper has stressed the point that a well prepared cash flow can be of great value to a management even if it were never to be discounted.... Provided the basic cash flow of a venture is accepted, all manner of approaches and all types of criteria can be applied at very little extra cost and effort".

The message is not so much that discounting techniques can be discounted²⁾ but that more research and thought should be given to the search for and forecast of future cash flows. The S method to which much of the latter part of this thesis is devoted is clearly an attempt in this direction.

1) T.R. Groundwater, Op. cit.

2) As in R.M. Adelson: Discounted Cash Flow - Can we discount it? A Critical Examination". Journal of Business Finance Vol. 2. No. 2 (1970) pp.50-66.

In addition to a simple profitability criterion - "cost of capital" type of yardstick, the theory and the practice are aware of two other "formal" considerations, capital rationing and uncertainty of investment. Both introduce additional rules of selection. Capital rationing is a self imposed restriction on investment spending. Uncertainty and in particular the difference in the riskiness of various projects cannot usually be ignored by assuming a certain expected profitability. Exhibit 4.5(c) gives the typical observed causes of capital rationing and 4.5(d) the methods of adjusting for risk in the process of project selection. Both exhibits are Fregman's survey results.

4.3.3. Expenditure Control and Post-Audit.

As far as the actual decision making is concerned the procedure ends with an acceptance of the investment proposal or with a rejection or postponement of it. But as far as the investment decision making system is concerned the very existence of formal subsequent stages of expenditure control and post audit in itself is a major consideration in the way decisions are made. Thus applications for investment funds have to include genuine and realistic estimates of costs. If not then responsibility for serious gaps between budgets and actual cost can be traced back to the person behind an investment proposal.

Obviously expenditure control during the implementation of a project and post audit subsequent to its life are geared more towards checking errors of wrong acceptance of a project than towards checking errors of avoiding a project's (errors of omission). It is perhaps important to note that there are two types of errors of omission, one of rejecting projects when they should be accepted and the other of simply overlooking good investment opportunity as they come. There can be no expenditure control on 'avoided projects'. Similarly post audit cannot be expected to trace managerial responsibility for errors of omission. Post audit (also called post mortem) is usually carried out in order to detect basic errors in implementation or in approaches to decision making, like, say a systematic overstatement of profitability. Post mortem is not carried out to evaluate the quality of decisions made by a specific manager. At the time of post mortem anyway he is not likely to be in the same position as when the decision was taken.

It is difficult to assess how much of the investment proposals is discouraged by expenditure control and post audit. On the other hand and as mentioned earlier, there seems to be no explicit counter-mechanism to encourage submission of investment proposals Istvan¹⁾ who discovered this in his survey comes out with no explanation except to saying that it seems that "sufficient proposals of merit arise out of the normal manner of doing business".

1) Istvan ibid p.13.

4.4 The Dynamics of Investment Decision Making

The investment decision making system is a dynamic system. Decisions take place consecutively; in the case of discrete, imperative budget, decisions are taken year by year and the selected projects for each year are implemented during the budget year. Assuming that a company is 'profit maximizer' then its investment decision making procedure can be seen as a control system that maintains a maximum profit 'trajectory' over time. This control system is fed annually by environmental information: changes in technology, changes in the general economic atmosphere, newly arising investment opportunities, etc. And then the investment decision is taken, i.e. the investment projects for the year are selected.

Evidently there is some clear analogy between an investment decision system and a missile control system. But here the analogy ends. The standard missile control system is based on the momentary information regarding the clear spatial relationship between the missile and its moving target. No consideration is taken of irregular obstacles that might be met by the missile on its way and the future corrective action that will have to be taken to avoid them. Actually, it is assumed that such obstacles do not exist. The information regarding profitability is based on future multi-period forecast and not on accurately measured information. Also the action, namely the investment carried by a company today, has to be seen in the light of the future 'corrective action'. That is, in the light of future investment that the company will make. It would be disastrous for a company to select its investment programme for today without considering what investment projects it will have to take or likely to take tomorrow.

Interdependence of possible immediate and future investments and uncertainty in forecasts are too important to ignore. In this section we shall try to relate the investment decision procedure discussed earlier with these considerations.

4.4.1. The Underlying Dynamics.

The strong connection between long-medium term plans and investment budgets has been stressed in many places. The usual theme being: non-committing long range plans constitute the scenario, or background, for immediate investment plans and the basis for their evaluation.

Much of the writing in this line is related to observation of real life investment decision making situations. R. Turvey,¹⁾ having acted as Chief Economist of the Electricity Council, proposes a two stage method of planning investment in electricity generating plant. In this particular industry he faces problems of increasing demand, technological change, and system load. In addition he faces a problem of long gestation period. It takes five years from authorizing the construction of a power station to commissioning it. Thus authorization in 1970 means commissioning in 1975. Here are some comments he makes on the planning strategy.

"First the broad outline of the pattern of system development over the twenty-five to thirty years beginning in 1975 must be determined. Second, in the light of the background plan so obtained, more detailed consideration is given to the plant programme for 1975 and firm decisions are made. The background plan thus does not in itself constitute a set of decisions but is an essential prerequisite for the second stage which does lead up to decision making."

1) R. Turvey, "Optimal Pricing and Investment in Electricity Supply" George Allen & Unwin Ltd., London, 1968.

It is a natural thought that the two stages might be linked in an iterative analysis... It is clear that a back ground plan can be anything between the quantitative expression of expert guesstimates on the one hand and the product of elaborate and computerised calculation on the other. The uncertainty concerning future factors can be dealt with by hunch or by refined parametric analysis of the optimum." 1)

In another place he stresses that the planning exercise is repeated every year and only the annual "budget" commits the Electricity Council to a definitive action.

"... if planning, authorisation and construction of new nuclear and conventional plant takes up to five years and if planning is an annual exercise, this means that what is at issue is the plant programme for plant to be commissioned during the year beginning five years ahead." 2)

The British Steel Corporation comes up with similar ideas. In its Annual Report of 1971-72 it emphasizes the tentativity of its plans

"It should be emphasised that the investment planning of the Corporation is flexible. There is no need for a single major and irretrievable decision to be taken at any one moment in time on the level of total capacity at some future date. On the contrary, the plans need constant reviewing and up-dating, both for commitment to fresh capacity and the closure of the obsolete and high-cost plants. These two factors, together with the Corporation's ability to adjust the timing of decisions, should enable it to avoid the worst pitfalls of over-capacity or lack of profitability even when the inevitable down-turns in the economic and steel cycle occur."

So, in order to properly do the capital budgeting a company has to weigh the possible investment proposal for immediate implementation vis a vis a scenario which inevitably includes

1) ibid. pp. 27-28.

2) ibid. p.12. Note how Turvey takes 'discrete' budgeting for granted.

future investments of the company itself. But of course the eventual(future) decisions on these future investments will depend on assets that the company will have at the time of decision, i.e. today's investment. Since it is impossible for both the budget and the scenario to be determined simultaneously there must be a certain iterative process by which the budget will finally be determined.

In reality one must not be too specific about the iterative process as it can never be satisfactorily approximated by mathematical techniques. This is so partly because investment criteria and guidelines cannot always be translated to a clear "objective function" that is to be optimized under, just as clear, "constants". Usually companies tend to decide as if they satisfice a great number of constraints rather than optimize anything.¹⁾

Partly and more importantly, mathematical description of the iterative process is unsatisfactory because of uncertainty regarding the future. It is difficult to express what is likely to happen and even more difficult to weigh all that can happen. So really what is sought is a robust investment policy, one that will ensure satisfactory profitability under many possible lines of future development.

An important class of future possible investments is that of today's rejected projects. Projects that are not selected for immediate implementation can be either scrapped forthrightly or postponed for later years. The distinction between totally rejected projects and ones that are postponed is ignored by the "non-dynamic" writing on capital budgeting. There the treatment of projects is strictly dichotomous - projects are either accepted or rejected.

1) See, for example, S. Eilon "Goals and Constraints", Journal of Management Studies, Oct. 1971, pp. 292-303.

Istvan admits that in his survey he originally overlooked the possibility of postponement and only halfway through his survey he recognizes this significant omission¹⁾.

"There are two possible dispositions for a proposal that has been rejected by the decision-making authority. It can be completely voided or it can be held for reconsideration at future time. ... Unfortunately the importance of this aspect of the decision making process was not recognized until after several firms had been interviewed. As a result only 24 firms out of 48 contributed information about their approach to this problem".

His survey result was that 9 out of the 24 made use of rejected i.e. postponed, proposals whereby resubmission was greatly simplified. Istvan explains the relative limited use of postponed proposals by the fact that technological and other change make monetary estimates in a proposal irrelevant at the time of resubmission. But this view of totally identifying projects with laid down proposals is very narrow. "Build a bridge in location X" is basically the same project whatever year it is actually carried out, and so is "Replace an existing truck by the best truck available". Even when some or all of the cost estimates have to be corrected every year the engineering and legal information are likely to remain nearly the same and so are relevant cost categories. It would still be practically as well as conceptually justified to talk of postponing 'the project' and of postponing 'the replacement'.

4.4.2. Replacement Projects.

Replacement investment projects are particularly suitable for dynamic analysis. And, looking at it the other way a replacement

1) Istvan Op. cit. pp 23-24

project very clearly requires a dynamic analysis. It has to be compared with replacement in future dates.

Here are some points to show that replacement projects, perhaps more realistically than others, lend themselves to a specified "dynamic" analysis.

- a) A replacement project is basically a cost minimization project. Its investment (optimization) criteria can be defined more easily than those of a more complex project.
- b) Risk is comparatively small. The scenario of replacement projects is particularly simple because of the underlying assumption behind a replacement decision that the relevant operation or service must be carried on.
- c) The number of alternative investments to consider is small and limited to the timing of investment. At each point of time (i.e. each period) the best alternative is considered (this is not as restricting as it may sound, usually in replacement projects the choice is limited).
- d) The comparison of the alternative cash flows is very simple because one compares very similar alternatives - similar capital costs, labour, fuel, etc.

Because decision is made periodically (in annual jumps), the investment proposal is either accepted and then implemented or else rejected which means that replacement is postponed for a later period. But postponement does not commit a company to action at any particular time. Thus postponement of replacement is also a postponement of the replacement decision itself.

Replacement decision in a future period will of course take into account any new information regarding both existing plant and possible replacements. If we assume the examination of existing equipment in the light of possible replacement to be an annual exercise then we can depict a dynamic decision making system in a flow diagram (see Figure 4.1).

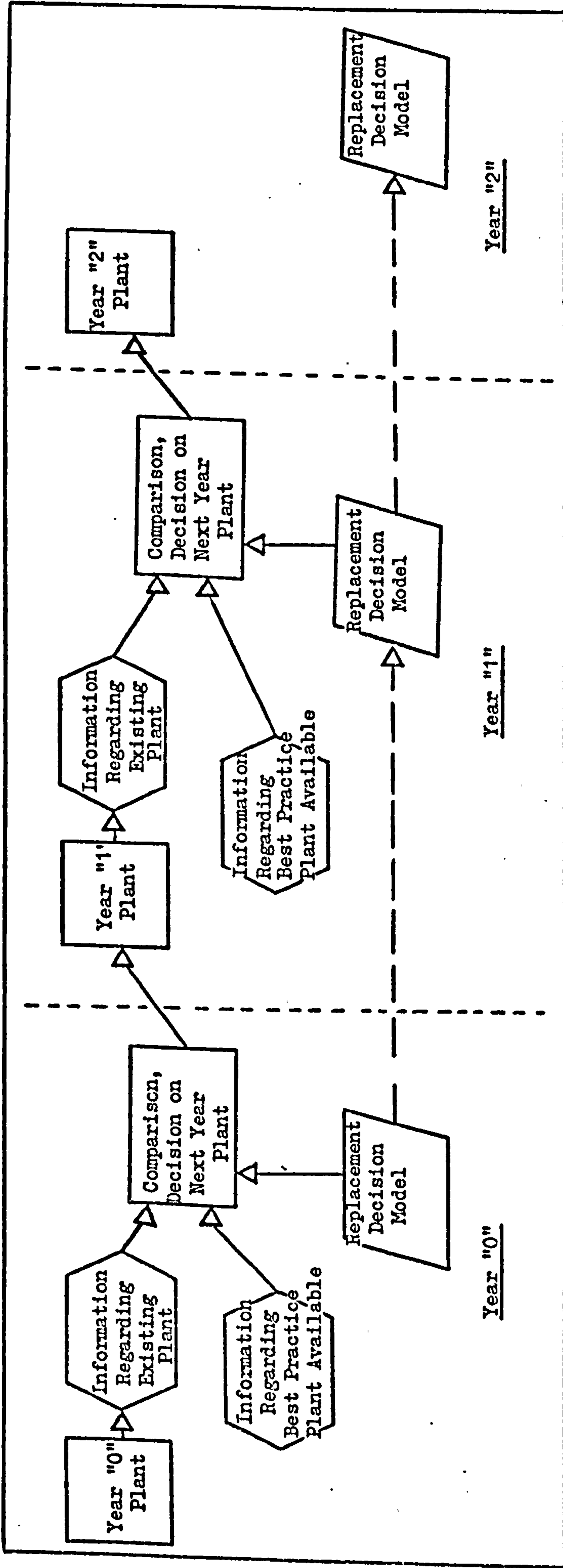
A replacement decision system as presented in Figure 4.1 emphasizes ways of over-coming both errors of omission and errors of commission. Errors of omission are overcome by the periodical examination of existing plant. Errors of commission are overcome by considering the possibility of postponement.

4.4.3. Transitory aspects.

Here I would like to add another aspect of the dynamics of investment decision making, one usually reserved for macro-economics. This is the cyclical nature of investment. An aspect that, perhaps surprisingly, is missing from the prescriptive writings on investment.

Transitory considerations are particularly important in replacement where the question is whether investment should be carried out this year or in later years and not strictly whether

FIGURE 4.1: A Flow Diagram, Showing the Dynamics and the Recurrence of Replacement Decision Making.



in principle investment is desirable or not. So, even though the general pattern of investment replacement is determined by the trends of technological change and deterioration, the exact year of replacement is determined by certain short term and cyclical factors, e.g. the availability of funds in a particular year, opportunity for expansion investment, capacity utilisation, etc.

Some work on the timing of replacement has been done by Feldstein and Foot¹⁾. They analyzed the annual aggregate figures of "MacGraw-Hill survey of planned investment" in the U.S. together with various additional background financial information. Although faced with the limitation of the aggregate analysis they came out with the clear conclusion that:

"There is substantial variation from year to year in the ratio of planned replacement investment to capital stock ... constant proportional replacement may be true on average in the long run but it is not true on a year by year basis".

Econometrically they explain the variation by:

- (a) internal availability of funds,
- (b) the level of expansion investment
- (c) the utilization rate of capital stock.

((b) negatively correlated with replacement investment, (a) and (c) positively correlated).

If dynamic models consider action at different points of time then it is desirable that they should somehow cater for the difference in 'criteria' for investment at different points of time.

1) M.S. Feldstein & D.K. Foot, "The Other half of Gross Investment, Replacement and Modernization Expenditure" The Review of Economics & Statistics, Feb. 1971, pp 49-58.

4.5 Summary

There is increasing evidence that medium-large industrial companies, as well as other organisations, follow highly comparable procedures in the processing of capital investment proposals. The most important aspects of this similarity are the existence of a certain staff department that is responsible for the processing of investment proposals and the periodicity in the authorisation of capital investment usually through the annual budget.

Apart from proposals for major investment projects most investment proposals follow a well defined routine. Proposals are presented on standard forms indicating some aspects of the costs and the benefits of a proposed project. The evidence is that increasingly this information is presented as cash flows. There is evidence too that DCF criteria have emerged as major criteria of investment evaluation.

This capital budgeting system nevertheless is not perfect and leaves open important questions like: how does the system encourage sufficient number of proposals?, Where should origination of proposal start?, What is the purpose and value of post audit?, etc.

Though actual investment budget in a company is prepared for only one year cash flows associated with an investment project are forecast for many years ahead, so inevitably the assessment of projects is interlinked with a longer range planning. This includes among others a forecast of the future economic environment and of the possible future

activities and decisions to be made by the company. An interesting and crucial aspect of such planning-programming-budgeting activity is that most investment projects can be undertaken at either of a number of points of time. This may be particularly true of projects proposed for the current budget. Such considerations are the essence of the analysis of proposals for investment is asset replacement: the decision whether to "replace today" has the obvious alternative of "replace tomorrow", and it can dynamically be looked upon as a decision on "when" to replace an existing asset? It is suggested that this approach to the analysis of replacement proposals can fit in well within a budgeting system.

CHAPTER 5

INVESTMENT APPRAISAL TECHNIQUES AND REPLACEMENT

MODELS - A CRITICAL PRESENTATION

5.1. General.

The problem of equipment and plant replacement is an important managerial problem in industry. In many cases, as shown in Chapter 4, the problem is not being tackled as well as it should be. This however is not the direct concern of Chapter 5. Here I shall present, somewhat critically, what writers on the subject of replacement have to offer in the way of formalizing the equipment replacement problem and in providing aids to proper replacement decisions. Particular emphasis will be put here on technological change as a factor in determining replacement¹⁾. Attention will also be given to the practical relevance of various theoretical models: the modelling of a replacement problem, the robustness of parameters, the ability to collect the required data and the corporate significance of 'answers'.

Theoretical literature dealing with the replacement of a deteriorating asset by a new one starts with the two papers written by J.S. Taylor (1923)²⁾ and by H. Hotelling (1925)³⁾. In the main these papers seek the optimal length of life of a new piece of equipment - the length of life that maximises the present worth of the investment. G.A.D. Preinrich⁴⁾ writing 15 years later has added that the optimal economic life cannot in general be determined in isolation from future replacements. G. Terborgh⁵⁾ was the first author to introduce, explicitly, considerations of technological change, into the assessment procedure.

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- 1) It is important to mention that most of the microeconomics literature on technological change and decision making centres on the problem of plant replacement. A notable exception being R. Turvey's Marginal Cost Economic Journal 1969 (see discussion in Chap. 3).
 - 2) J.S. Taylor, "A Statistical Theory of Depreciation", Journal of the American Statistical Association, Dec. 1923 pp.1010-1023.
 - 3) H. Hotelling, "A General Mathematical Theory of Depreciation", J. Am. Stat. Assoc., September 1925 pp.340-353.
 - 4) G.A.D. Preinreich, "The Economic Life of Industrial Equipment", Econometrica, Jan. 1940 pp. 12-44.
 - 5) G. Terborgh, "Dynamic Equipment Policy", McGraw-Hill, New York 1949.

In that work Terborgh put the emphasis on the decision to replace existing capacity rather than on the assessment of the new project - "the challenger". The technological change (obsolescence), which he introduced to the 'theory', meant that optimal replacement was not only a function of age, but also of calendar time. What Terborgh observed in 1949 was that if technological change were to be recognised as an important factor in determining the life of plant, then quantifiable technological expectations at the time of decision would be of the greatest importance in deciding whether to replace the existing plant or not.

From that point on, the field of research has developed along a few distinctively separate alleys. Some writers deal with the problems of optimal length of life, under various assumptions - Grinyer (1971)¹⁾, Stapelton (1972)²⁾ and others. Some researchers, e.g. Vernon L. Smith (1960)³⁾, have developed a more rigorous theoretical base for optimal replacement decision of the kind that Terborgh initiated. Yet, others, notably Terborgh himself (1958)⁴⁾, (1967)⁵⁾ and Merrett & Sykes (1966)⁶⁾ and Connor and Evans (1972)⁷⁾, have tried to develop efficient practical methods of assessing replacement projects, taking account of taxation and other additional factors. To do this the relative merits of various theoretical factors were re-evaluated, some factors were found relatively unimportant and were excluded from the decision framework. Others were added. Very little theoretical analysis and grounding has been given to these 'practical' methods and of course, there is still a wide range of real-life replacement problems that these methods do not cover satisfactorily.

1) P.H. Grinyer, "The Effect of Technological Change on the Economic Life of Capital Equipment" (unpublished). The City University 1971.

2) R.C. Stapleton, D.B. Hemming and H.H. Scholefield, "Technical Change and the Optimal Life of Assets. ORQ, March 1972, pp. 45-59.

3) V.L. Smith, "Investment and Production", Harvard University Press, 1966.

4) G. Terborgh, "Business Investment Policy", MAPI, Washington D.C. 1958.

5) G. Terborgh, "Business Investment Management", MAPI, Washington D.C. 1967.

6) A.J. Merrett & A. Sykes, "Capital Budgeting and Company Finance", Longmans, 1966.

7) J. Connor & J.R. Evans, "Replacement Investment", Gower Press, London, 1972.

In the meantime an independent line of treatment of replacement problems was developed by R. Bellman (1955)¹⁾. It is a presentation of the replacement problem as a discrete Dynamic Programming problem. It has both theoretical and computational value, but it is unlikely to develop into an important practical tool for reasons that will be presented further on in 5.3.3.

Before actually discussing the literature I shall present, in section 5.2., the main criteria used in replacement decision. These are all discounting criteria and therefore mathematically analogous, but they offer different degrees of flexibility. The differences in criteria are important in the understanding of differences and similarities between models (s-model described in the next Chapter included). Later in Section 5.3., I shall present elements of the replacement 'theory'. The 'classical' theory of replacement will be demonstrated by its latest and perhaps final presentation that of V.L. Smith, (5.3.1.). I shall then discuss the problem of optimal length of life emphasising work done by Prof. P.H. Grinyer of the City University (5.3.2). A different approach to the replacement problem is that of the Dynamic Programming which will be discussed in 5.3.3.

Closely linked with the "theory" are works that produce "charts" and "tables" to be used in actual replacement decisions. These works in addition have a distinct theoretical value. Section 5.4 surveys the very important MAPI approach and its development over the period 1949-1967. Section 5.5 discusses Merrett & Sykes' ORM method and the Connor & Evans adaptation.

To achieve a somewhat better coverage of the field of Investment Replacement Literature, including aspects that are not within the main line of discussion in this dissertation I present in 5.6 a glossary of various replacement problems and literature that tackles them.

1) R. Bellman, "Equipment Replacement Policy", J. Soc. Indus. Appl. Math., 1955 pp 133-136.

5.2 Discounting Techniques Used in Replacement Analysis.

5.2.1. General.

Practically all the literature on replacement analysis uses discounted cash flow techniques in the analysis of the relevant cash flows. Usually the use of some variant of a discounting technique is taken for granted and the relevance of the procedure is not questioned. Discounting is done at a single discount rate¹⁾ and most practical presentations are based on discrete cash flows and discrete discounting. The more theoretical presentations prefer continuous cash flows and continuous discounting. Some papers combine the two presentations e.g. discrete, annual cash flows whose value is determined by an exponential function. In this section I shall emphasise the discrete discounting technique. This is done because of the emphasis that I put on the "Chart methods" of sections 5.4 and 5.5 which singularly use discrete discounting.

Three discounting techniques are used in equipment-selection and replacement analysis problems.

1. Present Value methods (PV or NPV).
2. Internal Rate of Return (IRR, also labelled DCFROR which stands for Discounted Cash Flow Rate of Return).
3. Equivalent annual charge (EAC).

It is taken for granted that the reader of this dissertation is familiar with these techniques. Therefore they will not be formally presented here and the discussion of them will evolve only around a few

1) a) One exception is P. Masse, "Optimal Investment Decisions" Prentice Hall, 1962 who occasionally uses a time related discounting factor $r(t)$.
b) Panic and Close "Profitability of British Manufacturing Industry", Lloyds Bank Review, July 1973, No. 109, and Sir Robert Shone, "Price and Investment Relationships", Elek. 1975, indicate a systematic decline in return on industrial investment as a function of time. But there are difficulties both of estimation and of interpretation in using a time related discounting factor in investment appraisal.

points that I consider particularly important for this Chapter and which are perhaps less emphasised in a standard presentation.

There has been some argument as to the relative merits and demerits of the various discounting techniques¹⁾ in various types of decisions especially when comparing NPV and IRR²⁾.

The EAC is nothing more than an annuity equivalent of a PV, yet it is popular among engineers and accountants. Management likes to appreciate an annual cost of operating a machine over its life. The IRR method has the appeal to financially oriented managers. It indicates the merits of an investment from their particular point of view. The PV is perhaps the simplest to compute but it has little "corporate appeal". It is arguable that the rate of return methods used in replacement analysis problems, are in fact PV in disguise. These rates of return are Terborgh's 'relative return'(Section 5.4) and Merrett & Sykes's "rate of return on extended yield"(Section 5.5).

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- 1) Evidence for this argument can be found in R. Turvey, "PV vs IRR - An Essay in the Theory of the Third Best", The Economic Journal, March 1963, pp 93-98, is also found in all text books that discuss discounting techniques in investment appraisal.
 - 2) In the analysis of a single project there are usually no differences between them, they simultaneously "accept" or "reject" a project. The differences really appear in extensions of the simple problem. This is particularly true in cases of 'rationing' when in addition to the cost of capital constraint one has to select a subset of projects out of a larger number of projects e.g. every case of selecting one out of these 'viable' alternatives. Outside the scope of our discussion there are differences when "risk" is added to the budgeting problem.

5.2.2. The Discounting of a Uniform Gradient Series (Triangular Series) of Cash Flow.

Much of the literature on replacement in particular the 'Chart Methods' makes use of discounting 'a gradient series'. The series consists of annual sums and it has the following pattern:

Year:	1,	2,	3,	4,		N,	...
Annual sum:	0,	G,	2G,	3G,		(N-1) G,	...

The series is used to illustrate linear obsolescence and deterioration. The discounted values of the cumulative sum of the series can be converted into a relatively simple expression of PV equivalent, or into EAC. Thus the PV of a uniform gradient series of length N with a gradient factor 'G', discounted at 'r' is:

$$PV = \frac{G}{r} \left[\frac{(1+r)^N - 1}{r(1+r)} - \frac{N}{(1+r)^N} \right] \quad \dots\dots (1)$$

The annuity equivalent of the series, the EAC is:

$$EAC = G \left[\frac{1}{r} - \frac{N}{(1+r)^N - 1} \right] \quad \dots\dots (2)$$

Further economic interpretation of the series is deferred to section 5.4 but it can be said already here that the discounted cumulative sum of a gradient series is the basic computational tool of Terborgh's method, of Merrett & Sykes's method and of Connor and Evans's method.

The derivation of equations (1) and (2) is given in Section 1 of the Appendix 5.1 at the end of this Chapter

5.3. Theoretical Approaches to the Replacement Problem.

5.3.1. The Capital Replacement Theory of Vernon L. Smith¹⁾

Smith gives a wide theoretical background to the replacement problem. He stresses that the pure replacement problem is a cost minimisation problem rather than a profit maximisation one.

His basic model can be described verbally as: "Find the time L_0 such that, keeping the existing equipment till L_0 and then replacing it, would give the minimum present value of the relevant stream of costs". If L_0 is "now" then immediate replacement is indicated.

In his mathematical formulas various assumptions and approximations are used, all of them discussed meticulously in his book. Formally, he equates the discounted costs to a minimal perpetuity (perpetual annuity) stream - A, rather than to a present value figure:-

$$\min A = r \left\{ \int_0^{L_0} \phi(t) \cdot e^{-rt} dt - S_0(L_0) \cdot e^{-r L_0} \right\} + \\ + r \cdot e^{-r L_0} \sum_{k=0}^{\infty} e^{-rk} \left\{ \int_0^L \phi(L_0 + kL, t) e^{-rt} dt + W - S(L) e^{-rL} \right\} \dots (3)$$

r - discount rate

ϕ - operating expense function

$S_0(L_0)$ - salvage value of existing (incumbent) equipment

L - length of life at all future replacement (this is regarded as a sufficiently good assumption under a wide range of circumstances).

$S(L)$ - salvage value of L years old future replacements

W - investment cost of a future replacement

(Smith's own notation is used here)

1) V.L. Smith, op. cit.

Though W and $S(L)$ are not necessarily independent of calendar time Smith deliberately uses these simplifications since they cover the majority of the models prior to his work. On equation (3) he says

"Equation [(3)] or some variant thereof forms the substructure of all contemporary approaches to applied replacement analysis"¹⁾

It is difficult to dispute this statement however strong it may sound. Referring to a discrete variant of equation (3) Smith comes out with a general statement.

"The net cost of holding the incumbent equipment in service for one additional year is the next year operating cost of the equipment plus the decline in salvage value due to another year's service plus the interest on the salvage proceeds foregone for another year minus the interest on the present value of all future cost savings resulting from the fact that a delay in replacement now causes all future replacements to be delayed and thereby to be effected with improved equipment".

And the right time to replace is

"When this net cost is equal to [or greater than] the uniform equivalent of all future equipment expenses, assuming optimal future replacement policies, it is time to discard the incumbent equipment".²⁾

Subsequently Smith specifies linear ϕ and $S(L)$ functions.

Later he discusses situations where replacement decisions are not independent of output and pricing policies. Yet his main contribution is the above formulation.

1) Smith Op. Cit. p.142.

2) Smith Op. Cit. p. 140.

5.3.2. The Economic Length of Life of Plant and Equipment

Finding the economic life of equipment is closely associated with the more general replacement problem. The subject of calculating the length of life of new equipment is well documented in the literature. All textbooks on Engineering Economy show how to calculate the economic life of new equipment, assuming increasing operating costs of existing plant. More elaborate calculations include obsolescence. The optimal life of equipment, calculated with reference to future generations of replacement is that which maximises the profitability, as measured by NPV, by ROR, etc.

The problem as an optimisation problem is important. Economic length of life should not be determined arbitrarily or as the physical life of equipment. However it can be shown that profitability of investment in equipment measured over a wide range of lengths of life around the 'optimum' is highly insensitive to the length of life parameter used. Also the precise length of life of new equipment has in itself little informative value. Companies do not plan or budget in a way that specifies the exact year in which a newly-purchased equipment will be replaced.

An intriguing point is that the economic life calculated from a mathematical model, is highly sensitive to the model's parameter values¹⁾ and to the model's specification (linear, exponential, etc.). This is all the more true if one begins to investigate optimal life of future replacements, in succession to the immediate replacement.

1) Parameters like obsolescence, deterioration and salvage value.

Some replacement models use a convenient approximation of equal lives of future replacements. In many replacement models this is a robust assumption even when not strictly precise. It is therefore somewhat surprising to come across an article that "proves" that the optimal lengths of life of future generations are not equal¹⁾. It is even more surprising to come across an article that considers the calculated lengths of life of future generations as a decision rule for the future, requiring actual replacements at the fixed given intervals.²⁾

Nevertheless there is a justification for investigating the length of life of equipment particularly in order to assess the main determinants of 'life'. This was done by Fetter and Goodman (1956)³⁾ and more recently by Grinyer and Toole⁴⁾ and by Grinyer⁵⁾ at the City University. The work of Grinyer and Toole, particularly that of Grinyer, is of interest as it extends the analysis of the length of life to analysis of optimal length of life of an existing plant. Here is an outline of their work.

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- 1) Stapleton et al, Op. Cit.
 - 2) R.A. Meyer, "Equipment Replacement Under Uncertainty". Mgt. Sci. July 1971 pp. 750-758.
 - 3) R.F. Fetter & T.P. Goodman "An Equipment Investment Analog" Operations Research, 1957, pp 657-669.
 - 4) P.H. Grinyer & D.G. Toole "A Note on the Theory of Replacement", INFOR, Vol.10 No. 2; June 1972, pp 107-128.
 - 5) P.H. Grinyer, "The Effects of Technological Change on the Economic Life of Capital Equipment" (unpublished), The City University, 1971.

Grinyer and Toole first give a brief summary of the replacement literature then they proceed and construct a 'basic model' that corresponds to much of that literature. The model is a simple replacement model that does not allow for obsolescence. It includes a linearly increasing time function of operating costs and an exponentially decreasing time function of scrap value.

The basic model enables them to make sensitivity analyses of the 'length of life' as in relation to other parameters. In particular they use the model to investigate specification errors. For example they calculate the impact on the economic life of assuming a linearly increasing cost function in cases where the correct specification is exponential, or parabolic.

The findings are that 'life' is sensitive to both parameters within the model, i.e. to errors of estimation, and to errors in the specification of the model.

P.H. Grinyer, in the latter paper¹⁾ extends the work to include obsolescence and the analysis of the actual question of replacing existing plant. This work is thus in line with Smith's work, discussed earlier. The paper examines the sensitivity of the economic life to the rate of obsolescence. In the analysis Grinyer uses various values of structural parameters (cost of capital, operating costs, etc.) and two types of obsolescence - linear and exponential.

The paper demonstrates a number of important points that are beyond the scope of the previous work. First it demonstrates that actual decisions in the near future, i.e. replacement decision,

1) P.H. Grinyer (1971) op. cit.

depend on the type of obsolescence assumed. Secondly it shows a rule that relates the calculated length of life under the two assumptions of linear and exponential obsolescence. Thirdly it highlights the intricacy of the optimal length of life problem by showing special cases where obsolescence may increase the life of plant - where high obsolescence make it desirable to keep existing plant longer in order to enjoy further improvement in later stages.

Grinyer's work is particularly relevant in situations where the actual first replacement cannot be carried out immediately but where orders for replacement have to be set "now". He gives the example of ships. So if it takes a period of 5 years from the setting of an order to the actual replacement of the existing asset it is important to investigate the impact of obsolescence and deterioration on existing asset over that period, before a decision can be made.

Most works on replacement (in particular the 'Chart Methods' discussed in Section 5.4 and 5.5) are in a sense more limited. They implicitly assume that an immediate replacement is possible and that no final date for replacement has to be fixed "now" if any postponement of replacement seems desirable. As a result these works do not use a specified time pattern of obsolescence and deterioration for the existing plant and specify in detail only the obsolescence and deterioration in new plant.

5.3.3. The Dynamic Programming Approach

The Dynamic Programming procedure unlike Classical Methods for modelling the replacement problem does not require that the time functions of costs and income be smooth, analytic functions.

The method, basically, is to relate the immediate or initial decision not only to future conditions, but also and explicitly to future decisions¹⁾. By using various tentative future decisions earlier tentative decisions are arrived at. The final solution is the set of decisions (and actions) that optimizes as objective function - the maximisation of NPV of future earnings.

The first application of DP to replacement problems was made by R. Bellman²⁾. In his paper the basic functional equation of the Dynamic Programming formulation is (Bellman's notation):

$$f(t) = \max \left[\begin{array}{l} P: r(o) - u(o) - c(t) + (1+r)^{-1}f(1) \\ K: r(t) - u(t) \quad \quad \quad + (1+r)^{-1}f(t+1) \end{array} \right] \quad \dots (4)$$

P - stands for "purchase a new machine",

K - for "keep the old one" ,

t = age of the machine, r = discounting factor ,

r(t) = yearly return on a machine of age t ,

u(t) = yearly up-keep of a machine of age t ,

c(t) = cost of replacing a machine of age t.

As all costs are function of age, not time, Bellman obtains a stationary solution. But where technological change is added, as in Bellman and Dreyfus³⁾, a terminal time N_0 for the process is required,

1) This apparently is in line with the general formulation of "dynamic" problems see discussion in Chapter 4.

2) R. Bellman: "Equipment Replacement policy" J. Soc. Indus. Appl. Math., Sept. 1955 pp. 133-136.

3) R.E. Bellman, E.S.E. Dreyfus. "Applied Dynamic Programming" Princeton University Press, 1962.

otherwise there would be no general algorithm for solution. This time limit is not a serious restriction as the discounted value of earnings beyond it is of little significance.

An important feature of the Dynamic Programming concept is that it can easily introduce more operational alternatives: overhaul incumbent equipment, purchase secondhand machines, etc. The Dynamic Programming approach is an important tool for assessing replacement where there is a combined effect of deterioration - obsolescence and a steady state stochastic failure of machines. A large number of recent articles deal with this problem.

Another attraction of the DP Approach is that it presents an analytic process that is comparable to the dynamic way by which human decisions are arrived at. The decisions made today presuppose future decisions as part of the information on which they (today's decisions) are based. It is valuable in a number of sensitivity analyses - the Dynamic Programming process itself can produce a list of a number of 'best alternatives' and show their respective values. Also it can produce an assessment of the sensitivity of 'the best action' to changes in circumstances - how that 'best action' will favour in other circumstances; which is perhaps the most important sensitivity analysis in long range assessments.

The main argument against the Dynamic Programming approach is that it requires explicit numerical data for a large sequence of stages. In equipment replacement the stages are future years, some of which far ahead in the future. The information is very uncertain

and, therefore, the forecast is done by the use of an analytic function, in which case the conventional methods can solve the problem. Another serious argument against the DP approach is that if more alternatives, in particular more complex alternatives, are introduced the computation process becomes prohibitive, so that the advantage compared with conventional methods that deal with a very restrictive number of types of alternatives is limited in practice.

5.4. G. Terborgh's MAPI Models

5.4.1. General

On balance the most important contribution to the literature on replacement decisions and associated cost reduction decisions is the work of George Terborgh and his associates at the Machines and Allied Products Institute in Washington. The various 'MAPI' systems have been published in four books-manuals¹⁾.

Terborgh writes in a forceful persuasive style rather than in the standard formal academic style. This is perhaps a reason why economists and operational researchers sometimes overlook his work²⁾. The underlying analysis and approach though are quite rigorous and require time to study.

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- 1)
 - a) G. Terborgh (1943) op. cit.
 - b) G. Terborgh "MAPI Replacement Manual" MAPI, Washington D.C. 1950.
 - c) G. Terborgh (1958) Op. Cit.
 - d) G. Terborgh (1967) Op. Cit.
 - 2) Here is an excerpt from a very complimentary book review of Terborgh's 1967 book, published in Management Science Oct. 1968 p B.106.
"This excellent book ought to be a required reading for all junior quantitative analysts ... And many senior analysts could profit from its study. These statements need urging because quantitative analysts might well dismiss such a non mathematical book as this one with disdain".

Thus, in some of the textbooks of 'Engineering Economy' the section that deals with the MAPI method is prepared not by the author but by a "guest writer". Terborgh it must be added, uses a rather specialized terminology and to complicate matters further he changes the terminology he uses from one book to the next. Each one of his books is written as if it were a separate and isolated book, but all the same I would accept Grant and Ireson's comment that:

"Anyone who wishes to understand the later models should first study Dynamic Equipment Policy which is the only one of the four volumes that contains a step by step exposition of the underlying theory"¹⁾.

So, the following discussion will be a presentation of the development and evolution of the MAPI method as a replacement decision model. Emphasis will be put on Terborgh's approach to the replacement problem which is broader than what is usually appreciated. In order to keep the discussion reasonably short I shall leave out considerations of salvage value (covered by all MAPI models) and Taxation (covered by the last two models). I shall concentrate on the MAPI decision rules. For this the simple case of replacement of equipment without salvage value in a world-without-tax will suffice.

5.4.2. The Development of Cost-Behaviour Assumptions.

Terborgh's problem is to examine whether replacement 'now' of an existing plant - "the defender" by the immediately best available new plant - "the present challenger" is desirable. To be able to come out with a meaningful solution he had to make some specifying assumptions about relevant future cash flows. One assumption employed in most of his

1) E.L. Grant and W.G. Ireson, "Principle of Engineering Economy" Roland Press, N.Y., 1970

work is concerned with the operating cost (or the operating income in a more general case) associated with the "defender" in future years as compared with operating cost of newly available plants as they emerge. Terborgh's implicit assumption is that this operating difference - "operating inferiority of the 'defender'" - will not decrease with time. Considering the impact of deterioration and obsolescence in increasing the performance gap between the defender and ever-improving "future challengers" this is a highly appealing assumption. This or a similar assumption is required to simplify the actual replacement problem. It also appears to be a very robust assumption¹⁾.

Yet, the more fundamental assumptions that Terborgh makes, those which are the basis for the MAPI Charts, concern costs behaviour in "challengers" only. These are spelled out as the "Standard Assumptions" in his 1949 books.

- "(1) Future challengers will repeat the adverse minimum of the present one.
- (2) The present challenger will accumulate operating inferiority at a constant rate."²⁾

In the remaining part of subsection 5.4.2.I will try to explain the way these assumptions were constructed and to explain the meaning of and the need for these assumptions. Terborgh himself has never given an account of how he had arrived at his standard assumptions.

1) This point is discussed again in Chap. 9.

2) Terborgh (1949), Op. Cit., p. 73.

Terborgh's initial problem in analyzing "challengers"(in 1949) was to calculate the optimal infinite cash flows of capital costs and of operating costs associated with an infinite chain of plant replacements headed by the present challenger. An admissible pattern of the operating costs series is given in Figure 5.1. (A technical point to mention here is that Terborgh was concerned with discrete annual cash flows not with continuous flows as appear in Figure 5.1.).

Having looked at a graph like that in Figure 5.1 Terborgh concluded that a certain element in the operating costs - that in the cross-hatched area was not relevant to the replacement decision. Assuming the service provided by the particular asset was to be continued, the area below the minimum operating cost line in Figure 5.1 indicates a "negative rent" - the minimum possible flow of operating cost (achieved only if the asset is replaced every year). So only cash flow 'above that line', whatever profile it may have, counts for the replacement decisions.

The capital costs are certainly relevant, and if "no salvage value" is assumed the total investment sum of each successive replacement is considered. Generally speaking the less frequent is the replacement the lower is the capital charge.

Terborgh's aim, as Smith's,¹⁾ was to calculate the equivalent annual charge of his relevant costs. He knew that there would be a simple solution to his optimization problem if he could devise a set of assumptions concerning the "challengers" such that would give the infinite series of costs an optimal annuity equivalent that would be independent of the starting point of the replacement chain. He realized that if he succeeded he would be able to calculate the optimal annuity

1) V.L. Smith, Op. cit. see above.

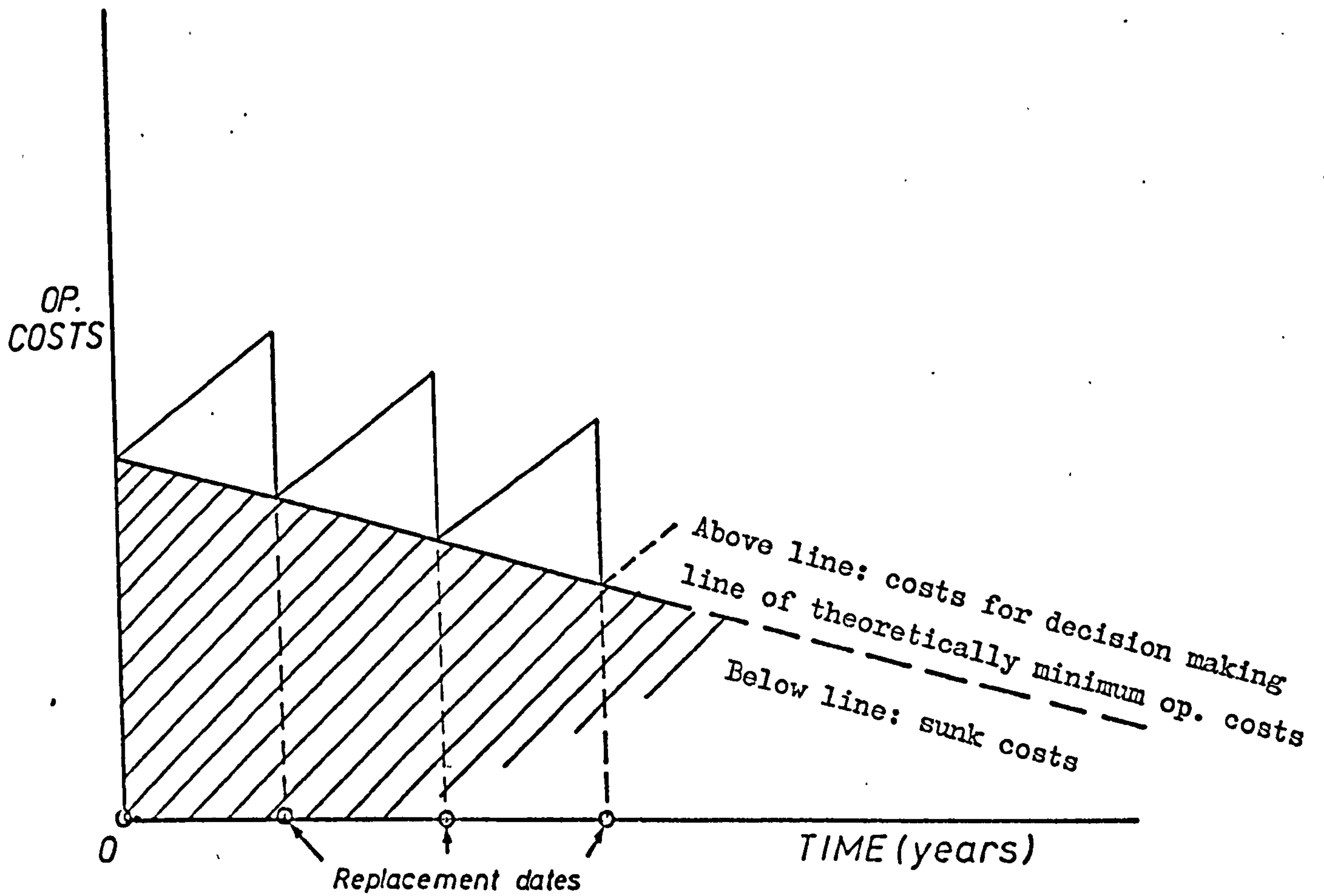


Fig. 5.1. Terborgh's Approach (1949). An infinite chain of replacements; only costs 'above the line' are relevant in deciding the optimal dates of replacement.

equivalent from the cost information of the 'present challenger' only. (Because, in particular, the optimal annuity equivalent of the first replacement to the 'present challenger' and its subsequent replacements will have the same optimal annuity equivalent as the 'present challenger').

The optimal annuity equivalent of relevant costs is the "adverse minimum" and the corresponding assumption is the "first standard assumption" above. On the other hand the clue to the actual calculation of the "adverse minimum" of the present challenger is given in the "second standard assumption".

From the discussion above we see that only operating cost 'above the line' in Fig. 5.1 enters into the calculation of the adverse minimum. The assumption states that operating cost of the present challenger moves away from the 'line' at a constant annual rate, namely, the operating costs of the challenger minus the operating cost of the best available alternative will increase by a constant gradient element every year.

Now Terborgh was able to calculate the annuity equivalent of the sum of the relevant costs - "capital costs" and "operating inferiorities" for every tentative length of life. He labelled this sum by the somewhat confusing term "adverse average" - it is the sum of the annual capital charge and the annuity equivalent of the gradient series expressing the inferiority. (A detailed example of this calculation is given in Table 5.1.). The minimum point of the 'adverse average' is the 'adverse minimum'. This point is achieved at the "optimal length of life" or the "economic life" of the 'present challenger'.

Of the two 'standard assumptions' the second clearly is not as essential as the first one. Thus in his 1958 book Terborgh offered, in addition to the

pattern of inferiority just discussed two alternative patterns of the accumulation of inferiority. One variant corresponds to decelerating inferiority and the other variant to accelerating inferiority. However in his 1967 book Terborgh abandoned these variants as having little practical value. It must be added that Terborgh unlike many of his followers does not assume that each of the deterioration and obsolescence factors are linear, he only assumes that their sum is linear. The first and the more important assumption is theoretically very elegant in that it does not require anything regarding future challengers beyond what is stated. Thus length of life of future challengers is not necessarily assumed constant, and ratios of capital to operating costs and the behaviour of future deterioration and obsolescence can be anything provided the adverse minimum of future challengers is kept constant.

Nevertheless in practice the MAPI approach can be treated 'as if' it assumes constant capital costs of future challengers and linearity in each of the deterioration and obsolescence for all future challengers.

5.4.3. The Decision Criterion.

Terborgh's replacement decision was now very simple indeed. He compared the next year operating inferiority of the existing plant with the challenger's adverse minimum. If the former was higher it was an indication for immediate replacement.

This criterion is simple enough to be used in practice but one should be aware of the assumptions within which the comparison is valid. In general though the comparison is between an infinite series of optimal replacement headed by the present challenger and a series starting with one year retention of the defender followed by the optimal replacement series as headed by next year challenger.

The challenger's adverse minimum in the example given in Table 5.1 below is £336. The initial operating costs of the challenger are £500. Assume now that the operating costs of the defender are £900. Will replacement be justified? The answer is "yes"; the challenger's adverse minimum, £336, is less than next year's operating inferiority of the defender, £900 - £500 = £400.

Table 5.1. Determination of Economic Life of a £1,000 Asset with a £50. Gradient Assuming Zero Salvage Value at All Times and Interest at 10%.

Year n	Excess of op.costs for Year Indicated Over First Year's op.costs for New Asset (inferiority gradient)	Equivalent Annual Cost - N Year Life		
		Capital Recovery Cost	Excess of Annual op. costs (inferiority gradient)	Total (adverse average)
(1)	(2)	(3)	(4)	(5)
1	0	1100	0	1100
2	50	576	24	600
3	100	402	97	449
4	150	316	69	385
5	200	264	90	354
6	250	230	111	343
7	300	205	131	336*
8	350	188	150	338
9	400	174	169	343
10	450	163	186	349
11	500	154	203	357
12	550	147	219	366
13	600	141	235	376
14	650	136	250	386
15	700	132	264	396
16	750	128	277	405
17	800	125	290	415
18	850	122	303	425
19	900	120	314	434
20	950	118	325	443

* adverse minimum

To complete the cost information in the Table we assume that first year's op. costs are £500 and that the inferiority gradient of £50 consists of £40 annual deterioration (increasing costs in an existing asset) and £10 annual obsolescence (decreasing first year operating costs in new plants as they become available).

5.4.4. The Conversion of the MAPI Decision Criterion in the 1950, 1958, 1967 Books.

One conversion made already in the 1950 manual was to use the length of life 'N' as a parameter instead of 'G' the gradient, or the annual inferiority factor. So in the latter MAPI methods G is computed from 'N' and not vice versa.

The reason for the conversion as I understand it is doublefold first as may be seen from Table 5.1 Column (5) the total cost is fairly insensitive to the length of life. But this, I believe, was not the main reason for the conversion. The main reason was computational. Calculating adverse minimum from tables like Table 5.1 or by corresponding optimisation procedures is tedious¹⁾. By using 'N' as a parameter Terborgh could calculate an adverse minimum from a simple formula. He could even calculate the "inferiority" from that formula. The development of Terborgh's formula is given in Appendix 5 Section 2. The formula has some limitations, they too are discussed.

An additional conversion was made in 1958. To understand it we must recapitulate our understanding of the adverse minimum. The adverse minimum by definition is an annuity equivalent over the optimal length of life of the sum of capital cost and the inferiority over the optimal life. In Figure 5.2 this is the area AB $\overline{E}$ F. So if the inferiority is expressed as AFE then the remaining area ABE should be equivalent to the capital cost. Specifically it means that the discounted value of ABE is exactly the original capital cost of the present challenger. (This 'plausible' explanation of the constituents of the adverse minimum is

1) The difficulty is a particular feature of the discrete model. The problem does not exist in a corresponding continuous model, See Appendix 5.2.

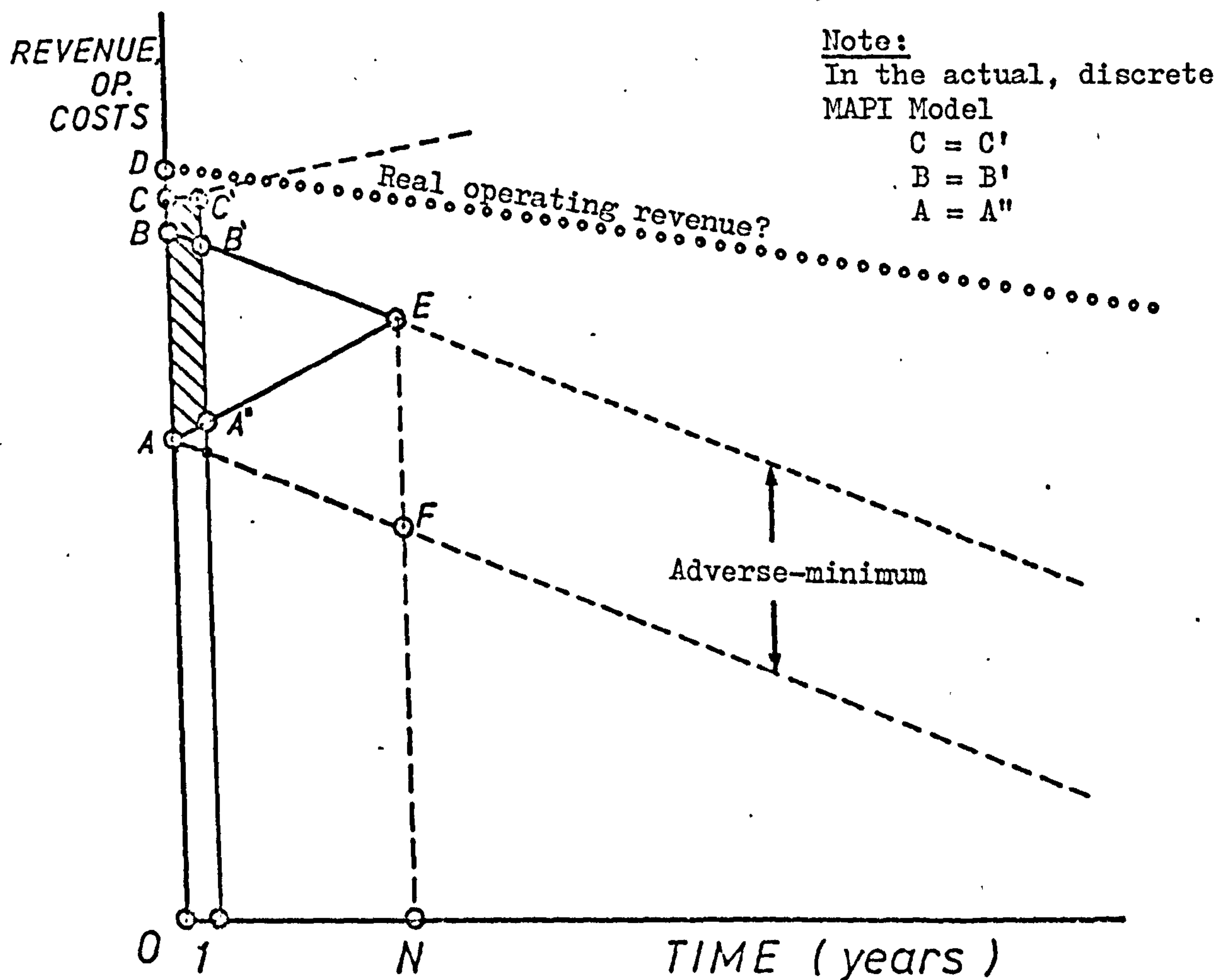


Fig. 5.2 Terborgh's Decision Rule based on comparing the operating advantage of new asset, the cross thatched area $ACC'A''$, with the adverse minimum over the first year, $ABB'A''$, (See Note above). In the later MAPI models the same rule is used though with the slight difference that the area $ABB'A''$ is looked upon as part of the triangle ABE - the "capital recovery", and as such determines the first-year-capital-consumption.

supplemented by the discussion in Section 3 of Appendix 5.1 and aided by Fig. 5.4).

The line BE ... in Fig. 5.2 expresses a notional long term marginal cost "price" (in the sense discussed in Chap. 3) and the area ABE indicates the normal return on the investment. The "price" can be the actual output price but this is not necessarily so, and anyhow in most cases it is impossible to relate a market price to the output of a single asset. The difference between the price and the op. costs is the notional profit, is therefore just a way, though a meaningful way, of "spreading" the original capital cost over the life.

The decision on whether to replace 'now' or not originally was based on comparing the defender's operating inferiority with the challenger's adverse minimum. The comparison in the new MAPI version is still basically as before but the figures were manipulated in such a way as to produce the answer in the form of a DCFROR.

The 'first year capital consumption' - a key concept in the new decision making process - is the difference between the discounted value of a triangle like ABE in Figure 5.2 and the discounted value, a year later of the remaining area of that triangle (A'B'E). Allowing for the fact that in a discrete model we have a 'step function' and not smooth straight lines, the area that is taken out is exactly the adverse minimum. If we denote the adverse minimum by A, the capital cost by C and the discounting factor by r the first year capital consumption will be:

$$c = C - (C(1+r) - A) = A - Cr \quad (5)$$

(The retention value of the equipment after one year C' is

$$C' = (C(1+r) - A), \text{ so } c = C - C'$$

So if the first year operating advantage is 'a' Terborgh could then translate the previous criterion into a DCFROR type of criterion

$$\text{DCFRO}R = \frac{a - c}{C} = \frac{a - (A - Cr)}{C} = \frac{a - A}{C} + r \quad (6)$$

So if the operating advantage exactly equals A, then $\text{DCFRO}R = r$.

In fact Terborgh added another minor change to his CF assumptions. In devising his new decision criterion Terborgh assumed the first year benefit from replacement to start immediately after the actual investment (typical in investment in machinery) so that on average the benefit would be half year after investment and not a whole year after it. So the $\text{DCFRO}R = r'$ in the 1958, 1967 versions actually equates

$$C = \frac{a}{1 + \frac{r'}{2}} + \frac{C'}{1+r'} \quad (7)$$

His final expression is of the form

$$r' = \frac{a - c}{C - \frac{c}{2}} \quad (8)$$

1) Here is the development of the formula instead of (7) we write

$$C(1+r') = a \frac{1+r'}{1+\frac{r'}{2}} + C'$$

using the approximation $\frac{1+r'}{1+\frac{r'}{2}} = \frac{1}{1 - \frac{r'}{2}}$ we write

$$C - C' + Cr' = \frac{a}{1 - \frac{r'}{2}}$$

$$(c + Cr')(1 - \frac{r'}{2}) = a$$

approximately

$$c - c\frac{r'}{2} + Cr' = a$$

$$(C - \frac{c}{2})r' = a - c \quad \text{so } r' = \frac{a - c}{C - \frac{c}{2}}$$

where $C - \frac{C}{2}$ is the "average net investment" in the year.

The r' is called in the 1958 book "the urgency rating" and in the 1967 book the "(MAPI) rate of return" with the emphasis that it is a 'relative return', return on an incremental project.

5.4.5. Some Practical Aspects of the MAPI Method

Exhibit 5.1, below includes the standard MAPI Summary Form on which a user of this method collects and compiles all the relevant information of costs (and benefits) of a particular investment proposal¹⁾. The Exhibit also includes a sample "chart" for the calculation of the first year capital consumption (or capital retention value).

Practical aspects of the MAPI method are discussed by Shone²⁾. Here I shall only outline them.

1. Salvage Value. The length of life of new asset is assumed known and a figure for the salvage value at the end of life is estimated as a parameter. (For treatment of salvage value of existing equipment see Exhibit 5.1).
2. Financial considerations. The MAPI Charts were constructed for a single value of cost of capital - 8.25%, which is the combination of 0.75 equity capital at 10% and 0.25 loan capital at 3%.
3. Taxation. Corporate income tax rate was assumed 0.50. Charts showing the first year capital retention value were drawn for a number of depreciation methods (Chart 1A in Exhibit 5.1 is drawn for a sum-of-digits depreciation method).

1) It is interesting to compare the MAPI's highly specialised form with the more open-ended form of Exhibit 4.1.

2) Shone(1975). Op. cit.

The inclusion of salvage value and tax consideration requires the expansion of the relative return formula (formula (8)).

It is, in practice:

$$r' = \frac{a + b - c - d}{c - \frac{c}{2}} \quad (9)$$

('b' is the gain from disposing existing asset 'now' over disposing it 'next year'. d is tax adjustments associated with 'b'.)

There are some differences between the 1958 and the 1967 books. For one, the 1967 book includes more tax depreciation methods. A 1967 Chart shows retention values of assets while a 1958 Chart shows the complementary value: first year capital consumption. As mentioned earlier the 1958 Charts in addition to "straight line inferiority" included an accelerating inferiority and a decelerating inferiority variants. These were dropped in 1967 as having little incremental value over the standard model.

It is important to note that although the main usage of all MAPI methods is in replacement analysis they can be used in many other applications where a selection is made of one project out of two mutually exclusive projects. This wider range of application is particularly emphasised in the 1967 book. This is perhaps the reason why Terborgh's instrumental DCFROR is labelled there as a 'relative return' and not "urgency rating" as in 1958 (implying the urgency to replace).

Nevertheless, the underlying model is the same in all four MAPI works.

Exhibit 5.1 MAPI Summary Form and an Example MAPI Chart

a. MAPI Summary Form

PROJECT NO. _____

SHEET 1

MAPI SUMMARY FORM
(AVERAGING SHORTCUT)

PROJECT	Box Machine and Stitcher
ALTERNATIVE	Continuing as is
COMPARISON PERIOD (YEARS)	(P) 1
ASSUMED OPERATING RATE OF PROJECT (HOURS PER YEAR)	1,200

I. OPERATING ADVANTAGE

(NEXT YEAR FOR A 1-YEAR COMPARISON PERIOD; * ANNUAL AVERAGES FOR LONGER PERIODS)

A. EFFECT OF PROJECT ON REVENUE

	INCREASE	DECREASE
1 FROM CHANGE IN QUALITY OF PRODUCTS	\$	\$
2 FROM CHANGE IN VOLUME OF OUTPUT	\$	\$
3 TOTAL	\$	\$

B. EFFECT ON OPERATING COSTS

4 DIRECT LABOR	\$ 900	\$
5 INDIRECT LABOR	150	
6 FRINGE BENEFITS	190	
7 MAINTENANCE	200	
8 TOOLING	80	
9 MATERIALS AND SUPPLIES		16,800
10 INSPECTION		
11 ASSEMBLY		
12 SCRAP AND REWORK		
13 DOWN TIME		
14 POWER	40	
15 FLOOR SPACE		1,000
16 PROPERTY TAXES AND INSURANCE	320	
17 SUBCONTRACTING		
18 INVENTORY		1,100
19 SAFETY		
20 FLEXIBILITY		
21 OTHER		
22 TOTAL	\$ 1,880	\$ 18,900

C. COMBINED EFFECT

23 NET INCREASE IN REVENUE (3X-3Y)	\$
24 NET DECREASE IN OPERATING COSTS (22X-22Y)	\$ 17,020
25 ANNUAL OPERATING ADVANTAGE (23+24)	\$ 17,020

* Next year means the first year of project operation. For projects with a significant break-in period, use performance after break-in.

SHEET 2

II. INVESTMENT AND RETURN

A. INITIAL INVESTMENT

26 INSTALLED COST OF PROJECT	\$ 29,800	
MINUS INITIAL TAX BENEFIT OF	\$ 2,100	
INVESTMENT IN ALTERNATIVE		(Net Cost) \$ 27,700
CAPITAL ADDITIONS MINUS INITIAL TAX BENEFIT		\$
PLUS: DISPOSAL VALUE OF ASSETS RETIRED		\$ 4,000
BY PROJECT*		\$ 4,000
27 INITIAL NET INVESTMENT (26-27)		\$ 23,700

B. TERMINAL INVESTMENT

29 RETENTION VALUE OF PROJECT AT END OF COMPARISON PERIOD
(ESTIMATE FOR ASSETS, IF ANY, THAT CANNOT BE DEPRECIATED OR EXPENSED. FOR OTHERS, ESTIMATE OR USE MAPI CHARTS.)

Item or Group	Installed Cost, Minus Initial Tax Benefit (Net Cost) A	Service Life (Years) B	Disposal Value, End of Life (Percent of Net Cost) C	MAPI Chart Number D	Chart Percentage E	Retention Value (A x E / 100) F
Box Machine and Stitcher	\$ 27,700	13	10	1A	89.4	\$ 24,760

ESTIMATED FROM CHARTS (TOTAL OF COL. F)

PLUS: OTHERWISE ESTIMATED

30 DISPOSAL VALUE OF ALTERNATIVE AT END OF PERIOD*

31 TERMINAL NET INVESTMENT (29-30)

C. RETURN

32 AVERAGE NET CAPITAL CONSUMPTION $\left(\frac{28-31}{P}\right)$	\$ 2,940
33 AVERAGE NET INVESTMENT $\left(\frac{28+31}{2}\right)$	\$ 22,230
34 BEFORE-TAX RETURN $\left(\frac{25-32}{33} \times 100\right)$	% 63.3
35 INCREASE IN DEPRECIATION AND INTEREST DEDUCTIONS	\$ 4,190
36 TAXABLE OPERATING ADVANTAGE (25-35)	\$ 12,830
37 INCREASE IN INCOME TAX (36 x TAX RATE)	\$ 6,415
38 AFTER-TAX OPERATING ADVANTAGE (25-37)	\$ 10,605
39 AVAILABLE FOR RETURN ON INVESTMENT (38-32)	\$ 7,665
40 AFTER-TAX RETURN $\left(\frac{39}{33} \times 100\right)$	% 34.5

* After terminal tax adjustments.

Exhibit 5.1 (Cont'd) b. An example MAPI Chart

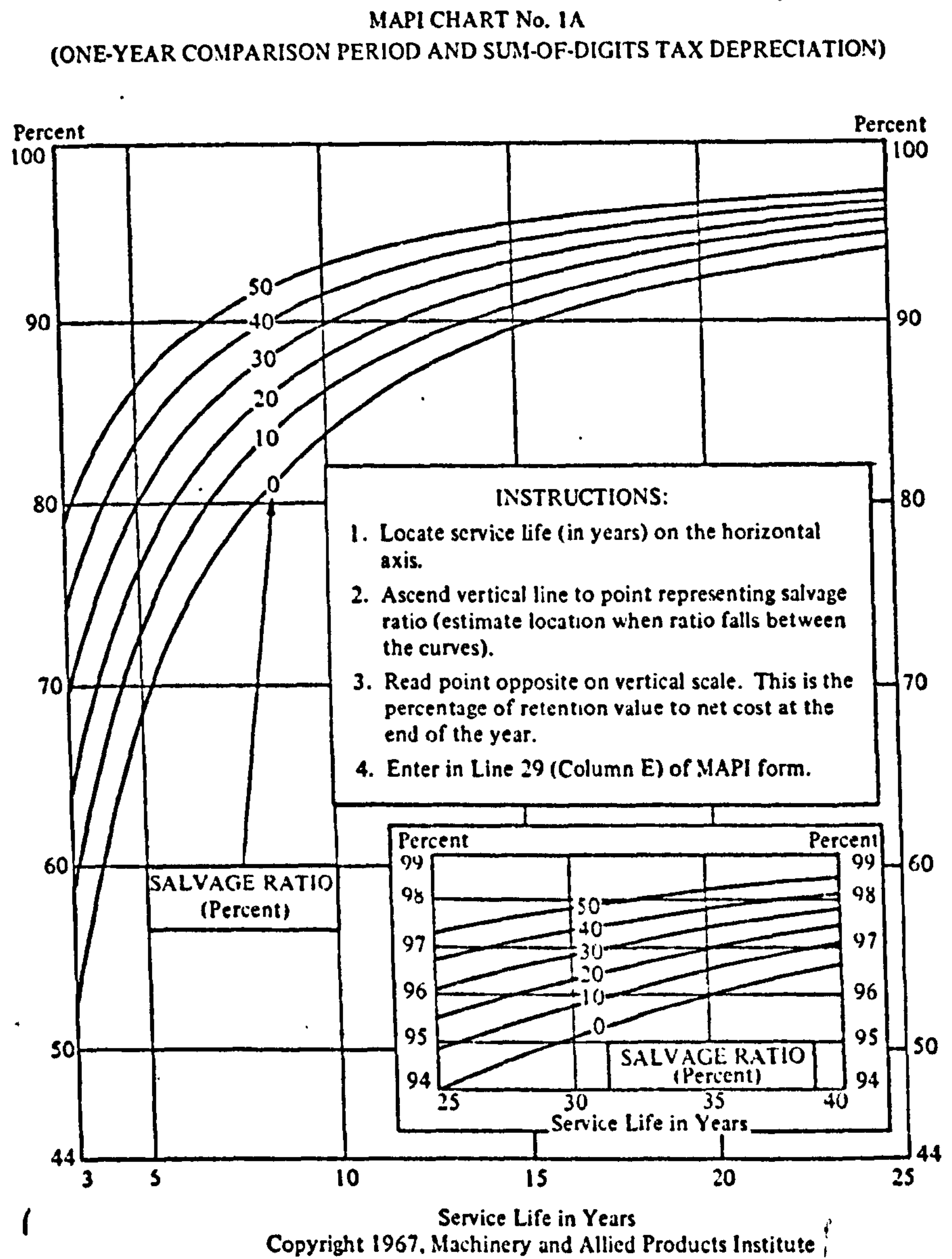
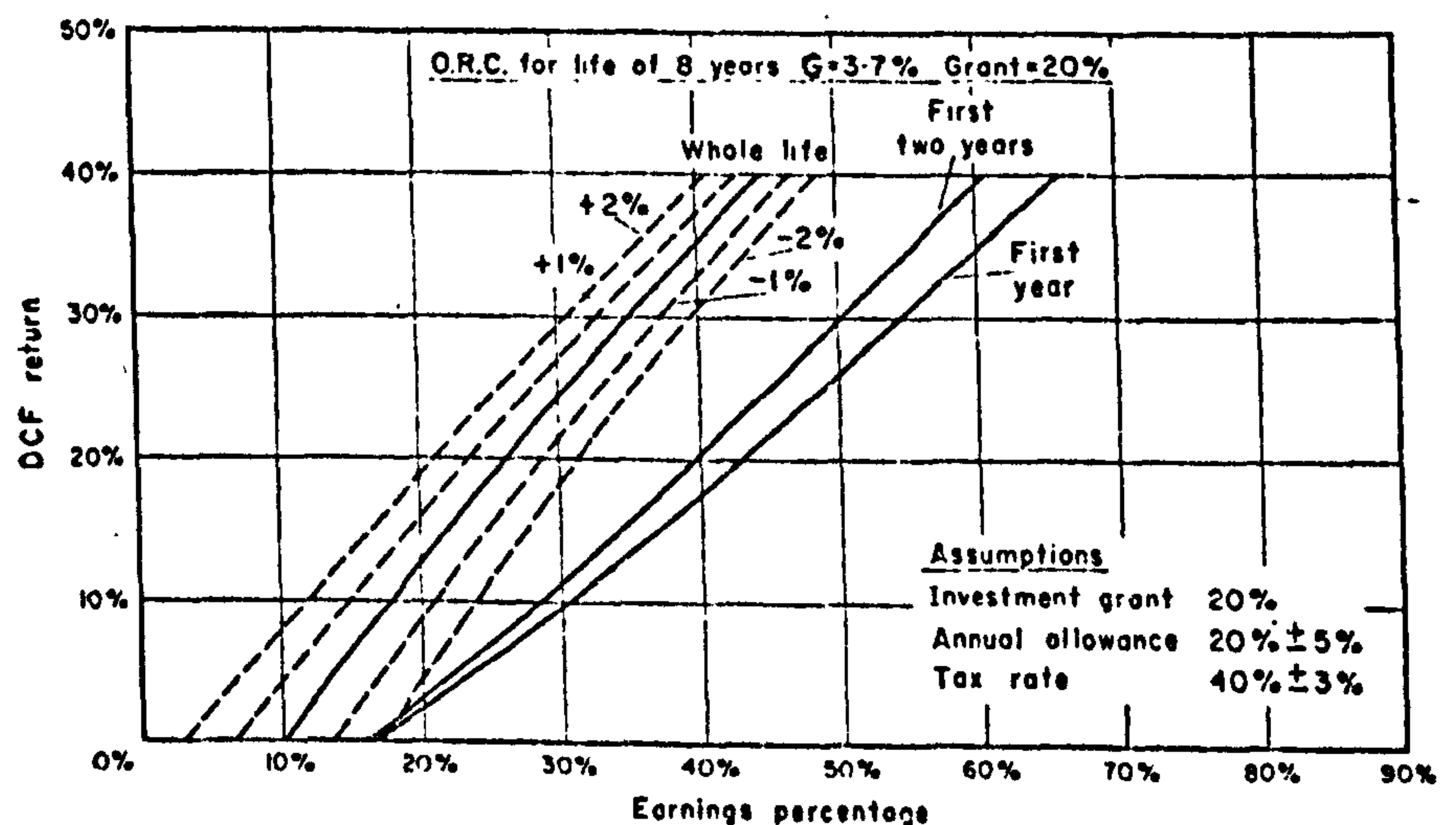


Exhibit 5.2 An Example Optimal Replacement Chart of the ORM Method.



Source: Merrett & Sykes (1966) Op. cit.

5.5 Practical Methods Developed in U.K.

5.5.1. Merrett & Sykes' ORM Method¹⁾

Merrett & Sykes' Optimal Replacement Method is highly comparable to the MAPI method but is adjusted to the U.K. taxation of the 1960s.

Just as the MAPI decision procedure is, the procedure they develop is to calculate a relative return on the incremental project of "immediate replacement minus postponed replacement". They label this rate of return as rate of return on extended yield (see Appendix 5.1 Section 4).

There are some differences between their method and that of Terborgh. With hindsight we may conclude that Terborgh's general approach is superior to theirs, particularly from the computational point of view. Yet their approach has the merit of approaching the replacement problem from "first principles" which is useful in structuring the replacement problem.

They start by specifying the cost stream of an infinite series of replacements. This is given in Table 5.2.

1) Published in a) A.J. Merrett, "Investment in Replacement: the Optimal Replacement Method", J.Mgt. Stds. May 1965, pp 153-166.

and b) A.J. Merrett and A. Sykes, "Capital Budgeting and Company Finance", Longmans, London, 1966.

The method was regarded at the time as a major contribution to proper investment decision making. It was heralded in advance in: R.R. Neild, "Replacement Policy", National Institute Economic Review 30, 1964 pp 30-43,

but recently was excluded altogether from A.J. Merrett & A. Sykes' "Capital Budgeting and Company Finance" Second Edition, Longmans 1974.

Table 5.2 Merrett & Sykes Model: Costs Stream of an Infinite Series of Replacements

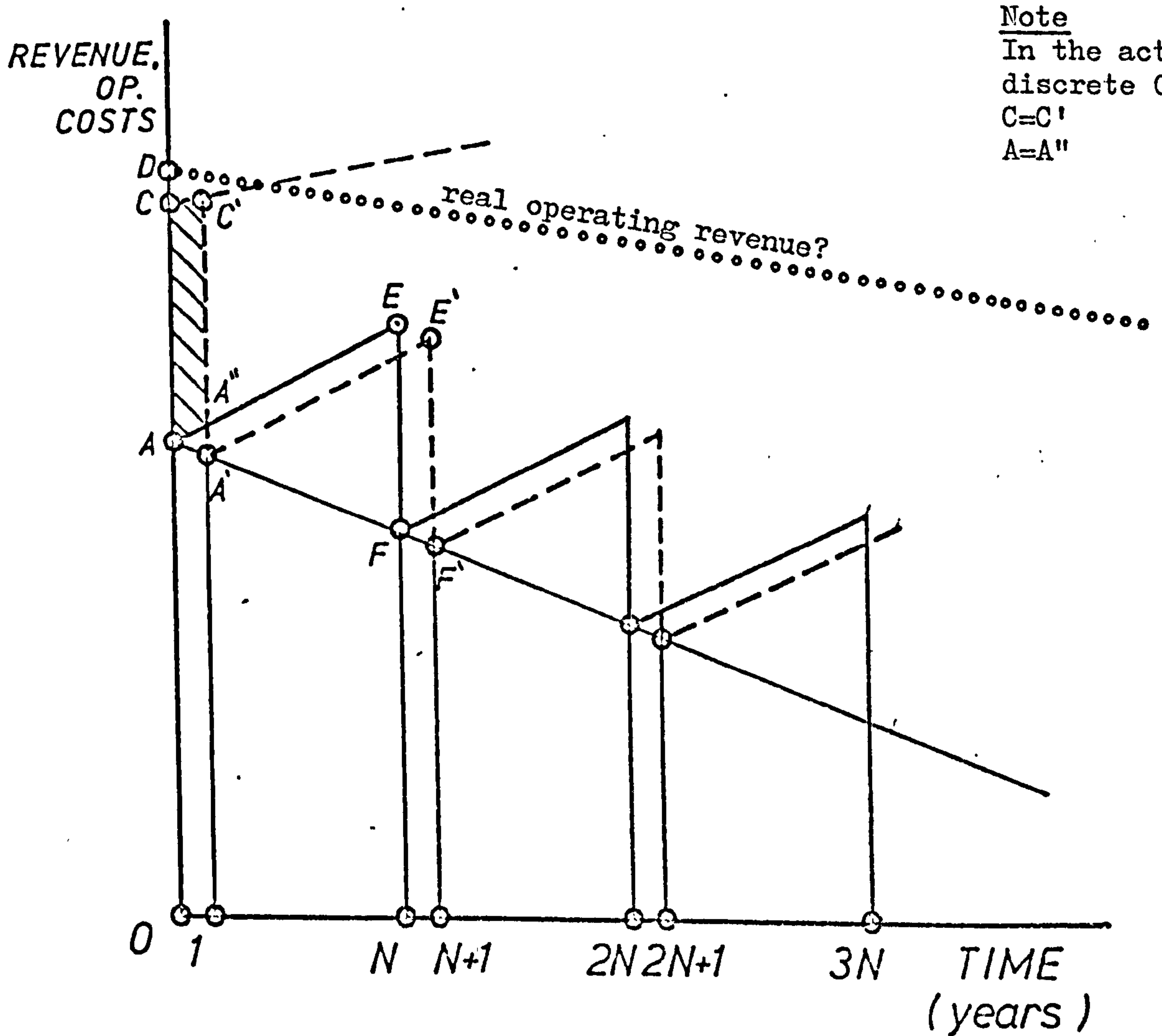
Year	0	1	2...	T	T+1	T+2	...	T+N	T+N-1	...
Cash Flow (Cost)	-	D	D+d...	D+d(T-1) + C-S	R-aT	R-aT+p...	R-aT+p(N-1) +C-M	R-a(T+N)...		
								<u>Old Asset</u>	<u>New Asset</u>	
Capital Cost:			..	..	..	..		-	C	
Resale value:			..	..	..	..		S	M	
Net operating cost:				..	..	..		D	R	
Increase in op. cost p.a.:					..	..		d	p	
Life:			..	..	..	..		T	N	
Obsolescence rate:				..	..	..		a	a	
(Also $p + a = G$; $d + a = f$)										

First replacement may take place at T ($T = 0, 1, 2, \dots$)

Looking at the capital and operating cost pattern of new assets we see that the 'behavioural' assumptions are exactly covered by the MAPI method. In fact they constitute a subset of Terborgh's assumptions¹⁾. Table 5.1 is an example for their behavioural model.

The procedure they use in assessing a replacement proposal is to see whether immediate replacement ($T = 0$) is superior to a first replacement at a later date $T \neq 0$. They compare the cash flows associated with the two alternatives by looking at the difference series (See Figure 5.3).

1) One may therefore say that their model is overspecified, they have unnecessarily excluded from their model cost patterns that could be dealt with by it. See 5.4.2.



Note
In the actual
discrete ORM model
 $C=C'$
 $A=A''$

Fig. 5.3 Merrett and Sykes's Decision Rule (for the specific case of 1 year comparison) compares two alternative chains of replacements. The operating cost portion of the alternative headed by immediate replacement is given by a heavy continuous line. The operating cost portion of the alternative headed by one year retention of existing equipment is given by a heavy broken line.

There is an obvious difficulty in calculating a DCFROR on a difference series (it is the alternating sign of the cash flows, which may produce a multiple answer). Merrett & Sykes modify the DCF calculation by what they call DCF return on extended yield (see Appendix 5.1 Section 4).

What they do is to discount all cash streams beyond a given year T (the alternative date of replacement, say $T = 2$ years) back to that year at a fixed rate (they use 8%) and calculate DCFROR on the modified incremental series.

It is clearly arguable that calculating incremental series for various values T though in principle correct is quite superfluous. It is sufficient to compare a project of immediate replacement with a one year postponement. If this comparison indicates immediate replacement, then in most conceivable models immediate replacement will be considered at least as favourable when compared with later possible replacement. This is particularly true of Merrett & Sykes' own model. As it turns out their ORM Charts (one example given in Exhibit 5.2) provide a visual proof that, provided the required DCF is higher than the 8% cutoff they use, their first year comparison always gives the lowest DCF return.

It is strange then why unlike Terborgh they did not reduce the Charts to a one year comparison. A more efficient presentation of their findings would be to plot the various curves of one year comparison (each for a given G or N) in one single Chart.

As said earlier Merrett & Sykes behavioural assumptions are covered by Terborgh's model. But the computational procedure that the "first year comparison" of two infinite series requires is far more tedious. The optimal life computed by Merrett and Sykes not surprisingly is the same optimal life computed by Terborgh's 1949 procedure (see footnote Appendix 5.1 Section 2). Merrett & Sykes overlooked altogether the advantage of computing the adverse minimum of a series of replacements¹⁾. If they had done this they would have noticed that the difference between their infinite replacement series starting at year 0 and their infinite series of replacement starting at year 1 is exactly the adverse minimum in year 1.

This can be computed easily from the information on 'G', the net capital cost, and the discounting factor using a formula like (14) in Appendix 5.1.

5.5.2. Connor & Evans' Tables²⁾

In a short and useful book (actually a manual) Connor & Evans present Tables that are in fact an adaptation of Terborgh's method to U.K Tax (1970). They readily admit that³⁾ This is to say that they calculate first year capital consumption rather than use Merrett & Sykes' next year replacement comparison. They do not include financial gearing considerations and use a flat 10% cost of capital. They compute operating advantage one year after investment (and not half a year as Terborgh).

1) I admit to having overlooked such considerations myself at various stages of working on this dissertation. Much of the replacement literature overlooks this major contribution by Terborgh.

2) J. Connor & J.B. Evans "Replacement Investment" Gower Press, London, 1972.

3) Ibid p.60.

The decision criterion is similar to that of Terborgh 1949-50 work showing the minimum cost saving that would justify replacement.

Another feature of their work is tables for "deferred" deterioration in addition to uniform deterioration. This is highly comparable to Terborgh's (1958) accelerated depreciation that was dropped later (1967) ¹⁾.

¹⁾ See Comment: Terborgh (1967) Op. Cit. p.92.

5.6 Other Work on Replacement

I shall complete this Chapter with a synopsis of various replacement problems that are not usually covered by the models presented earlier. I shall mention formal methods - if there are - that tackle them and I shall present some of the general observations on replacement decisions found in the literature.

First to mention are stochastic models, aiming to tackle 'risk'. Some confusion may arise here.¹⁾ There is a body of statistical models that deal with replacement of equipment or part of equipment that is liable to 'catastrophic failure': - 'one-horse-shay' type of failure. In its pure form this statistical theory deals with items that do not suffer from either deterioration or obsolescence. Thus there are statistical models that calculate expected life and variation of the expected life of a machine that is made up of a large number of components, such that the failure of a single one of them may terminate the life of that machine. As a decision tool this literature seeks optimal inspection policy, optimal equipment maintenance policy, optimal batch replacement policy (of, say, electric bulbs) etc.

Some of the literature considers combination of deterioration and stochastic failure with parameters that cannot be estimated in practice²⁾. Even though such problems seem to be very realistic this literature using Dynamic Programming approach, is highly theoretical.

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- 1) e.g. D. Jorgensen, J. McCall, R. Radner, "Optimal Replacement Policy", North Holland, 1967, is in fact a book on maintenance policy.
 - 2) Much of this literature combines discrete Dynamic Programming and Markov-Chain stochastic processes that determine failure and deterioration, e.g.: R.A. Howard "Dynamic Programming and Markov Process", The MIT Press, 1960; Yu-Ku Li, 'Optimal Equipment Replacement Policy, with Technological Change', (Unpublished Ph.D. California Berkeley, 1970); B. Kalyon, "Machine Replacement with Stochastic Costs" Management Science Jan. 1972 pp. 288-298.

The subjects discussed are both single machine replacement and fleet replacement¹⁾. There are some writers who try to introduce risk (or variability) in the rates of technological change and deterioration. Solution of such problems is done either mathematically or by simulation²⁾.

A problem oriented approach would reveal a large number of replacement decision making situations. They will be discussed below.

Limited service period. There are situations where the expected service period is considerably limited. The decision whether to replace existing equipment has to be calculated over the service period rather than over the optimum length of life of equipment³⁾.

Inadequate capacity. A reason for replacement may be a need to service a larger range of clients e.g. replacement of an internal telephone exchange by a larger one, a small computer by a large one, etc. It should be stressed that this is an additional reason to replace

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- 1) Modelling of army vehicles replacement based on their repair cost is given by: R.W. Drinkwater & N.A. Hastings; "An Economic Replacement Model", Operational Research Quarterly Vol. 18 (1967) pp. 121-139 & N.A. Hastings: "Some Notes on Dynamic Programming and Replacement". Operational Research Quarterly Vol. 19 (1968) pp. 453-464.
 - 2) e.g. R.A. Meyer, op. cit., R.C. Stapleton et al. op. cit., P.J. Kalman "A Stochastic Constrained Optimal Replacement Model, The Case of Ship Replacement Operations-Research, March-April, 1972. pp. 327-334.
 - 3) The subject is discussed by A.A. Alchian, "Economic Replacement Policy", Rand 224, 1952 and by G.A. Taylor, "Managerial and Engineering Economy" Van Nostrad, 1964

and not a result of 'economies of scale'. Replacement theory does cope with economies of scale by stipulating that it deals with the replacement of capacity by the same capacity (e.g. 3 old trucks by 2 new ones).

Inherent extra capacity. This also is related to replacement in the face of a growing demand. It is sometimes worthwhile to replace existing plant by one that will have excess capacity for a certain period, until demand 'catches up'. There are obviously difficulties in quantifying a replacement proposal in these circumstances¹⁾.

Improvement of present equipment and replacement by second hand equipment are both fair alternatives to going on with existing equipment, so, not only replacement by new equipment should be considered²⁾.

Retention for stand by. This is an alternative use for the existing equipment, once replaced. Such possibility, of course enhances replacement³⁾.

Replacement combined with expansion. This problem cannot be solved purely as a cost minimisation problem (replacement) or as a profit maximisation problem (expansion). The problem is mentioned but not solved⁴⁾.

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- 1) See e.g. M. Adelson "Discounted Cash Flow - Can we Discount it? A Critical Examination". J.O. Business Finance Vol. 2. (1970).
 - 2) This is discussed by Bellman and Dreyfus, op. cit. in the context of Dynamic Programming and by C. Morgan, "The Ultimate Selection of Mine Equipment by Economic Evaluation" (Unpublished M.Sc. Dissertation Imperial College, London 1973).
 - 3) This is discussed by J.L. Meij "Depreciation and Replacement Policy" North Holland, 1961 p.4-5.
 - 4) e.g. Connor & Evans op. cit.

In practice the problem really arises when neither replacement nor expansion dominates the investment problem and when one of them seems "viable" and the other not.

Capital budgeting constraints.

An attempt to compare replacement proposals and expansion proposals in situations of capital rationing was made recently by B. Shore¹⁾. It is difficult to produce a single ranking rule that will be suitable for both replacement and expansion projects, on top of the overall 'cost of capital'.

Business Cycles. These complicate and sometimes invalidate the use of a formal simple investment decision rule. Business cycles have an impact on cash flow and liquidity, on the relative merits of expansion vs replacement, and on fluctuations of the cost of capital over time²⁾.

A large proportion of articles approach the replacement problem from considerations of vehicle replacement. This is true of J. Dean seminal work on capital budgeting³⁾, of theoretical stochastic work discussed above and of a number of accounting paper that consider the information required and the records required in vehicle replacement decision.⁴⁾ Vehicles examined range from 85 ton off-highway-trucks

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- 1) B. Shore. "Replacement Decisions Under Capital Budgeting Constraints," The Engineering Economist 1975 pp. 243-256.
 - 2) See discussion in Chapter 4 and also: L.H. Klassen, L.M. Koyck, J.L. Bouma "The Theory of Depreciation and Entrepreneurial Behaviour" in J.L. Meij. Ed. Op. cit.
 - 3) J. Dean "Capital Budgeting" Columbia University Press 1951.
 - 4) a) R. Tausigg: "Information Requirements of Replacement Models," J. of Accounting Research 1964 pp. 67-77.
b) M.J. Barron; "Replacement Analysis: A Practical Approach," Accountancy 1971 pp. 575-580.

(In an article that emphasises the actual selection of alternatives¹⁾)
to fork lift trucks (in an article that examines replacement decision
rules vis-a-vis actual maintenance costs of trucks in a fleet).²⁾

The validity and the usefulness of models like Terborgh's is queried by a number of articles, R. Mayer throws doubts about the ability of industry to collect reliable data to furnish such models, yet he admits that the theory is valuable in identifying the factors that should govern replacement decision³⁾. In a more recent article Tersine and Crouthers come out with a conceptual model of replacement decision making⁴⁾. They present an important practical observation:

"Good times to consider replacement analysis are:

- 1) When major repairs to the old unit are needed,
- 2) When a major new operation is about to start,
- 3) When new models come on the market,
- 4) Routinely, at least once a year".⁵⁾

Another overall description of replacement considerations is given by E. Vassilatou-Thanopoulos in a work commissioned by the American National Association of Accountants.⁶⁾

Criticism of the theory and of Terborgh's work is often based on misconception. Meyer (discussed in 5.3.3 above) assumes that Challenger's life should be used for the actual replacement of the Challenger.⁷⁾

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- 1) S.A. Rathbun "Equipment Replacement - Evaluation by Return on Investment" C.I.M. Special Volume No. 12 (1971) pp 321-330.
 - 2) S. Eilon, J.R. King & D.E. Hutchinson. "A Study in Equipment Replacement", Operational Research Quarterly Vol. 17 (1965) pp. 59-71.
 - 3) R.R. Mayer, "Problems in the Application of Replacement Theory" Management Science, Vol. 6 (1960) pp 303-310.
 - 4) R.J. Tersine & W.E. Crouthers, Equipment Replacement Policy: A Realistic Synthesis". Journal of Purchasing May 1972, pp 55-72.
 - 5) Ibid., p. 62.
 - 6) E. Vassilatou-Thanopoulos. "Financial Analysis Techniques for Equipment Replacement Decisions". NAA, Research Monograph 1, May 1965.
 - 7) R.A. Meyer, "Equipment Replacement Under Uncertainty", Management Science Vol. 17 pp. 750-758.

Gentry and Johnson assume, erroneously, that the numerical advantage of the "Challenger" over the "Defender" conveys more than just an indication to replace and they "discover" situations where ex-post the annual saving is different from that expected¹⁾.

Text books on the subject of Equipment replacement are mostly books on Operational Research and on Managerial and Engineering Economics. The first group emphasise the modelling of the problem not the problem itself. The second group emphasise the solution of given numerical examples for various situations. Most of the literature in both groups though mentioning Terborgh's work makes no use of the 'adverse minimum', only little reference is given to the 'next year alternative', and none at all to the implied "notional profit" and "capital consumption" presented in 5.4 above²⁾.

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- 1) D.W. Gentry & T.B. Johnson, "An Investment Policy for Equipment Replacement" Mineral Industry Bulletin, Jan. 1974 pp. 1-13.
 - 2) There is a large number of books that discuss replacement at some length. Here is a small selected list, covering a wide range of approaches.
 - a) J.C.T. Mao, "Quantitative Analysis of Financial Decisions", MacMillan, 1971.
 - b) W.T. Morris, "The Analysis of Management Decisions", Richard Irwin, 1964.
 - c) W.J. Fabrycky & P.E. Torgensen: "Operations Economy", Prentice-Hall, 1966.
 - d) D.J. White et al. Operational Research Techniques Vol. 2., Business Books London, 1974.
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 - i) R.V. Oakford, "Capital Budgeting", Roland Press, N.Y., 1970.
 - j) G.A. Taylor, Op. cit.
 - k) E.L. Grant & W.G. Ireson, Op. cit.
 - l) P. Masse, Op. cit.

5.7 Summary

An outline of this Chapter is given in Section 5.1, so this summary presents only the main observations made in the Chapter.

Pure theories (models) of replacement as their name implies are there to illustrate certain points reflecting real life replacement problems. This is done with some success but nearly always through a gross simplification of any conceivable real replacement problem. In most cases the illustration is done without quantifying the model's parameters and always the question of an investor's ability to estimate such parameters is left untouched.

A stage in the bridging of the theory and practical decision making is provided by the MAPI method of G. Terborgh and by analogous methods, notably the ORM method. The final product of Terborgh's approach is a manual that contains a number of charts and a form to be filled in by a manager applying for funds or by an analyst in the capital budgeting department.

There are a number of assumptions or stipulations that enable Terborgh to mould the theory and reduce it into a manual. Of these four are the most important. First assumption (actually a group of assumptions) is a well prescribed movement of relevant costs over time in new investments. Second assumption is that the comparison of replacement today with a replacement next year is a sufficient criterion to decide on replacement today (this is a mute assumption not specified in Terborgh's Manuals). Third is the use of an estimated length of life as a parameter and fourth is the use of a single constant discounting factor.

Terborgh's model has many interesting aspects, theoretical as well as practical. The various points mentioned above, as well as other points, are investigated mathematically in the Chapter. A continuous model analogous to Terborgh's discrete model has been constructed and it too is investigated.

The ORM method of Merrett and Sykes is shown to have the same cost assumptions as the MAPI method has. The way this method tackles the replacement problem, though of interest, appears in some respects to be inferior to the MAPI method.

Appendix 5.1 Computational Notes.

Section 1. The Discounted Value of The Cumulative Sum of a Uniform Gradient Series.

An expression of the general uniform gradient series is given in Table 5.3 below. An alternative expression of the series, convenient for computation is also given there.

Table 5.3: The Uniform Gradient Series

Year	Uniform Gradient Series	Un. Gr. Series Alternative Expression.
1	0	NG - NG
2	G	NG - (N-1)G
3	2G	NG - (N-2)G
⋮	⋮	⋮
N	(N-1)G	NG - G

The computation with a discounting factor r makes use of the standard discounting formula that expresses the present value P of an annuity A :

$$P = A \cdot \sum_{i=1}^N \frac{1}{(1+r)^i} = A \cdot \frac{(1+r)^N - 1}{r(1+r)^N}$$

and its inverse:

$$A = P \cdot \frac{r(1+r)^N}{(1+r)^N - 1}$$

Following Table 5.3 the present value P of gradient series will be:

$$P = \sum_{i=1}^N \frac{(i-1)G}{(1+r)^i} = \sum_{i=1}^N \frac{NG}{(1+r)^i} - \sum_{i=1}^N \frac{(N-(i-1))G}{(1+r)^i} \quad (10)$$

$$P = NG \cdot \sum_{i=1}^N \frac{1}{(1+r)^i} - G \sum_{j=1}^N \sum_{i=1}^j \frac{G}{(1+r)^i}$$

$$P = NG \cdot \frac{(1+r)^N - 1}{r(1+r)^N} - G \cdot \sum_{j=1}^N \frac{(1+r)^j - 1}{r(1+r)^j}$$

$$P = \frac{NG}{r} \cdot \frac{(1+r)^N - 1}{(1+r)^N} - \frac{NG}{r} + \frac{G}{r} \cdot \frac{(1+r)^N - 1}{r(1+r)^N} \quad (11)$$

$$P = \frac{G}{r} \left[\frac{(1+r)^N - 1}{r(1+r)^N} - \frac{N}{(1+r)^N} \right] \quad (12)$$

which is the same as equation (1) in section 5.2.

The EAC of equation (12) will be obtained by equation

$$A = \frac{G}{r} \left[\frac{(1+r)^N - 1}{r(1+r)^N} - \frac{N}{(1+r)^N} \right] \cdot \frac{r(1+r)^N}{(1+r)^N - 1}$$

$$A = G \left[\frac{1}{r} - \frac{N}{(1+r)^N - 1} \right] \quad (13)$$

Section 2: The Conversion of the MAPI Method;

Parameter 'N' replaces parameter 'G' so that in "Adverse Minimum" is computed directly.

The adverse minimum is an equivalent annual charge. It is the sum of the EAC of a gradient series, the inferiority series (roughly the triangle AEF in Fig. 5.4) and of the annual capital charge at the optimal length of life of an asset.

If the relevant capital cost is C, the annual inferiority G, and the discounting factor r, then the adverse minimum A is:

$$A = \min_N \left\{ \underbrace{C \cdot \frac{r(1+r)^N}{(1+r)^N - 1}}_{\text{capital charge}} + \underbrace{\frac{G}{r} - \frac{NG}{r} \cdot \frac{r}{(1+r)^N - 1}}_{\text{inferiority series charge}} \right\} \dots (14)^1)$$

1) Note that the N value that minimizes (14) also minimizes X such that

$$X = \left(A - \frac{G}{r} \right) \cdot \frac{1}{C}$$

or

$$X = \frac{(1+r)^N - \frac{NG}{Cr}}{(1+r)^N - 1} \dots (15)$$

Merrett & Sykes (1966) Op. Cit. p. 146 take 'N' as the value which minimizes an expression identical to (15). So the length of life they use for every generation of 'challengers' is exactly the length of life that Terborgh (1949) attributes to the "present challenger".

The adverse minimum given in Table 5.1 is £336 for $N = 7$ years looking at the Table we see that this £336 is greater than the inferiority at that optimal length of life $(N-1)G$ but it is less than the 'inferiority' in the following year - NG . So clearly it is worthwhile to replace the equipment at the end of the 7th year to avoid operating costs at the 8th year that are higher than the adverse minimum.

'N' is a discrete variable while 'G' a continuous variable. So there must be a range of 'G's that corresponds to a given 'N' (assuming C the capital cost and r the discounting factor given). The lowest value 'G' of a range corresponding to N will be the G value that equates the adverse minimum A to NG .

So we may, for a given N, restrict (14) to

$$\min A = NG = C \cdot \frac{r + (1+r)^N}{(1+r)^N - 1} + \frac{G}{r} - \frac{NG}{r} \frac{r}{(1+r)^N - 1} \quad (16)^1$$

or

$$NG - \frac{NG}{rN} + NG \cdot \frac{1}{(1+r)^N - 1} = C \cdot \frac{r(1+r)^N}{(1+r)^N - 1}$$

or

$$NG \left[1 - \frac{1}{rN} + \frac{1}{(1+r)^N - 1} \right] \left(\frac{(1+r)^N - 1}{(1+r)^N} \right) \cdot rN = Cr^2N \quad (17)$$

Equation (17) may be rewritten as:

$$NG \left[\frac{(1+r)^N rN}{(1+r)^N} - \frac{rN}{(1+r)^N} - \frac{(1+r)^N}{(1+r)^N} + \frac{1}{(1+r)^N} \frac{rN}{(1+r)^N} \right] = Cr^2N$$

and so:

$$NG = \frac{Cr^2N}{rN - 1 + \frac{1}{(1+r)^N}} \quad (18)$$

1) It is clear from the discussion above that this equality when exists is possible only at the optimal length of life. At any other length of life NG is much too small or much too big.

or

$$G = \frac{Cr^2}{rN - 1 + \frac{1}{(1+r)^N}} \quad (19)$$

Example:

Using $C = £1000$ $r = 10\%$ (as in Table 5.1) we obtain:

a) for $N = 7$

$$\text{min. } A = NG = \frac{£1000 \cdot 0.1^2 \times 7}{0.1 \times 7 - 1 + 0.5132} = £328; \quad G = £77$$

b) for $N = 8$ $NG = £300, \quad G = £37$

c) for $N = 6$ $NG = £371, \quad G = £62.$

So, in effect Terborgh by calculating 'G' from 'N' has reduced 'G' to a discrete variable. With given assumptions about C and r there is a single G corresponding to a given N.

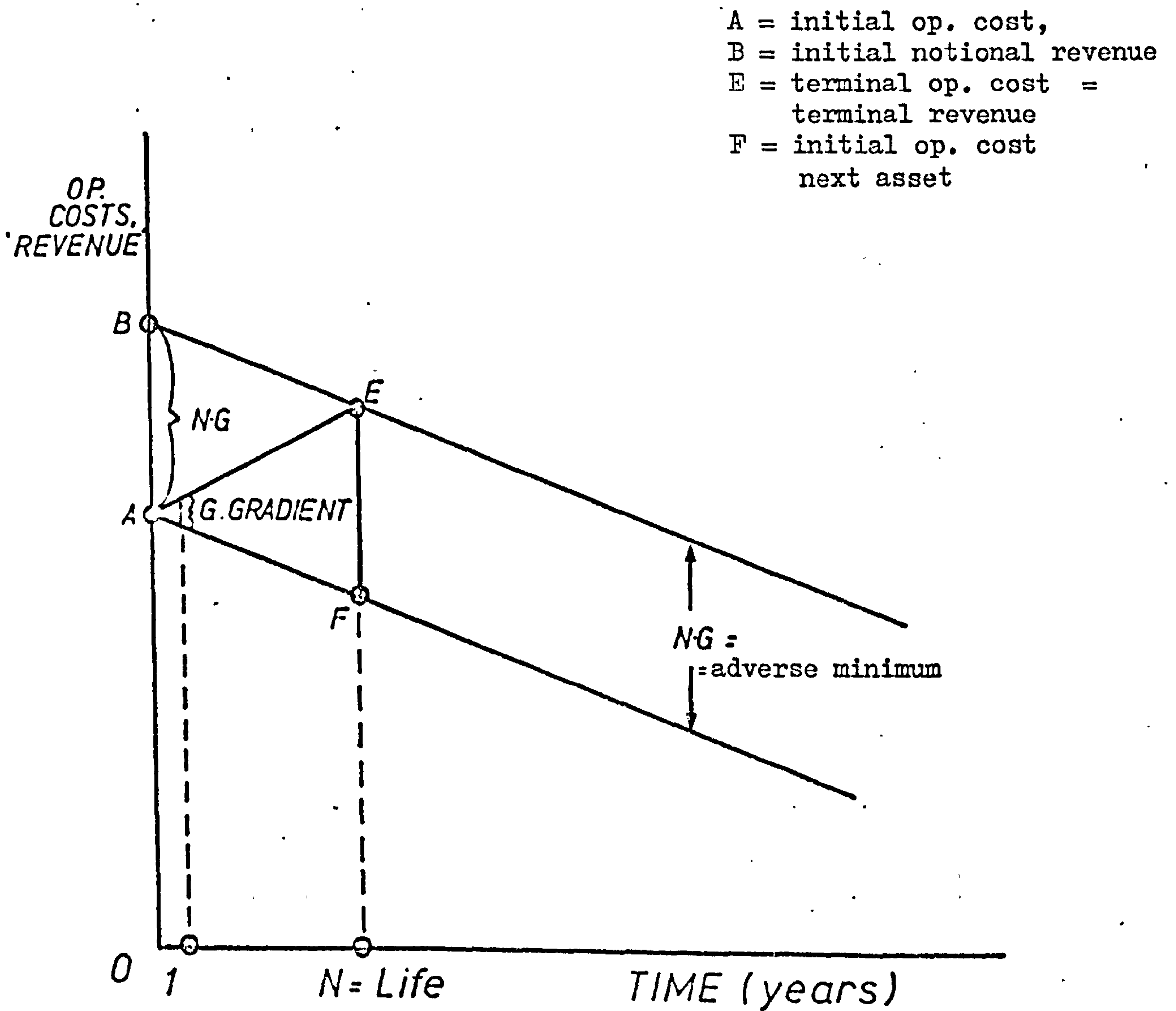


Fig. 5.4 Terborgh's Challenger Analysis: Challenger's adverse minimum over the life N is ABEF. It may be divided into an operating inferiority part AEF, and a capital recovery part ABE.

Section 3: The Conversion of the MAPI Method From "Adverse Minimum" Approach to a "Capital Consumption" Approach.

In section 2 above I showed that the MAPI method was reduced to consider only those 'G' values that produce adverse minimum strictly equal to NG. The discussion in 5.4 may be illustrated in Fig. 5.4. The adverse minimum over the life of an asset (AFEB) is made of two parts: cumulative inferiority (AEF) and capital recovery (ABE). Note though that the basic model is discrete and employ step functions, not strictly linear as Figure 5.4 suggests.

Now that the adverse minimum is reduced to NG I can show directly that the triangle ABE in its discrete interpretation is exactly equivalent to the capital cost¹⁾.

Year	:	1	2	3	...	N
Capital recovery series	:	NG	(N-1)G	(N-2)G		G

The discounted value of this capital recovery series is derived from Section 1 of this Appendix, Equation (10) and (11). It is

$$P = \frac{NG}{r} - \frac{G}{r} \cdot \frac{(1+r)^N - 1}{r(1+r)^N} \quad (20)$$

From Equation (19) above we extract

$$C = \frac{NG}{r} - \frac{G}{r^2} + \frac{G}{r^2(1+r)^N}$$

So, $P = C$, as we wanted to show.

1) This can be shown accurately only for the 'special' values of G calculated by equation (19). For other the adverse minimum is not equal to NG.

The first year capital consumption following equation(5)is

$$c = NG - Cr \quad (21)$$

With NG calculated from (18) above.

Section 4: Merrett & Sykes's Approach: The DCF Return on Extended Yield

The Net Present Value method gives a clear criterion for selecting one investment project out of two mutually exclusive investment projects. It accepts the one project whose NPV is greater.

In formal presentation it is:-

A is preferable to B if:

a_i - cash flow of project A in year i

$$\sum_{i=0}^{\infty} \frac{a_i}{(1+r_0)^i} \geq \sum_{i=0}^{\infty} \frac{b_i}{(1+r_0)^i}$$

b_i - cash flow of project B in year i

r_0 - the cost of capital

or

$$\sum \frac{a_i - b_i}{(1+r_0)^i} \geq 0 \quad (22)$$

The DCFROR method's equivalent for this selection is the "DCF rate of return on incremental project". The project "A - B" will be accepted if:-

$$\sum_{i=0}^{\infty} \frac{a_i - b_i}{(1+r)^i} = 0 \quad (23)$$

for $r \geq r_0$

This in practical terms means that A is preferable to B. The difference $(a_i - b_i)$ for any given year may be positive or may be negative. In the following year the difference may be in either the same direction as in year i or it may "change sign" and be in the opposite direction. If the number of changes of 'sign' in the whole series is more than one then there may be a multiple solution, i.e. more than one rate of return r may satisfy equation (23). To avoid this Merrett and Sykes divide the expression in formula (23) into two parts:

$$\sum_{i=0}^{\infty} \frac{a_i - b_i}{(1+r)^i} = \sum_{i=0}^T \frac{a_i - b_i}{(1+r')^i} + \frac{\sum_{i=T+1}^{\infty} \frac{a_i - b_i}{(1+r_0)^{i-T}}}{(1+r')} \quad (24)$$

This method is applicable in finding a unique DCF solution to their replacement problems. It gives a clear single value for r' . Merrett & Sykes first compute the difference between an infinite cash flow series associated with "immediate replacement" and an infinite cash flow series associated with a first replacement at year T in the future, ($T = 1, 2, 3, \dots$). Then they discount by r_0 , back to year T all cash flows beyond T of that 'difference series'. In this way they ensure a unique solution i.e. a single 'rate of return'. Merrett & Sykes raise a financial argument in favour of this mixed treatment. They regard the 'tail' beyond T as the cost of the commitment of the firm to future replacements and as such it should be capitalized at year T by the firm's cost of capital. They call their rate of return "DCF return on the extended yield".

Appendix 5.2. The Linear Model: Continuous Mode

In this Appendix I extend Terborgh's basic linear model and "translate" it into its continuous mode. I shall calculate the 'adverse minimum' of the continuous mode and demonstrate the various aspects of this mode.

There is in this dissertation a clear interest in developing such parallel continuous mode. First there is a genuine interest in showing analytically the basic relations amongst the model's variables. Secondly the difficulties encountered in the discrete model and discussed in Sections 2 and 3 in Appendix 5.1 do not exist in the continuous mode. It is possible in the continuous mode to show directly for any Gradient factor G , that the same length of life that produces the 'adverse minimum' also produces the 'capital recovery triangle' (ABE in Figure 5.4). Thirdly and most importantly the exponential 'S-model' developed and discussed in the following Chapters is presented in both continuous and discrete modes. The basic linear model of this Chapter is highly comparable to the S-model. It appears that the continuous modes of both models lend themselves better to meaningful numerical comparisons than the discrete modes. One such comparison is given in 8.1.9 below.

a) Calculation of 'adverse minimum' in the continuous mode

The parameters in the continuous mode are:

C - capital investment

r - discounting factor

G - annual gradients

also X - adverse average.

The problem is first to find out of all possible lengths of life N the particular N that minimizes the adverse average.

By definition for every N (here a continuous variable!)

$$X \int_0^N e^{-rt} dt = C + G \int_0^N t e^{-rt} dt \quad (25)$$

$$X = \frac{C + G \int_0^N t e^{-rt} dt}{\int_0^N e^{-rt} dt}$$

$$X = \left[C - \frac{NG}{r} \cdot e^{-rN} \right] \cdot \left(\frac{1 - e^{-rN}}{r} \right)^{-1} + \frac{G}{r} \quad (26)$$

differentiating (26) with respect to N ,

$$\begin{aligned} \frac{dX}{dN} = & - \left[C - \frac{NG}{r} \cdot e^{-rN} \right] \cdot \left(\frac{1 - e^{-rN}}{r} \right)^{-2} \cdot e^{-rN} \\ & - \frac{G}{r} e^{-rN} \left(\frac{1 - e^{-rN}}{r} \right)^{-1} \\ & + NG e^{-rN} (1 - e^{-rN})^{-1} \end{aligned} \quad (27)$$

We obtain at the optimum

$$\frac{dX}{dN} = - \left[C - \frac{NG}{r} e^{-rN} \right] \left(\frac{1 - e^{-rN}}{r} \right)^{-1} - \frac{G}{r} + NG = 0$$

$$C = \frac{G}{r^2} (1 - e^{-rN}) + \frac{NG}{r} (1 - e^{-rN}) + \frac{NG}{r} \cdot e^{-rN}$$

or

$$C = \frac{G}{r} \left(N - \frac{1 - e^{-rN}}{r} \right) \quad (28)$$

Optimal N is extracted from (28) by approximation methods.

The adverse minimum itself is obtained by substituting (28) for 'C' in (26). Note that the adverse minimum X^* is

$$X^* = NG \quad (29)$$

b) Calculating N that exactly recovers the capital investment

Using the parameters C, r and G we would now separately try and find 'N' the asset's life that exactly recovers the capital investment. The relevant cash flow is expressed in Figure 5.4 as the triangle ABE so the cash flow starts as NG at time 0 decreasing linearly to 0 at time N. So,

$$C = \int_0^N G (N-t) e^{-rt} dt \quad (30)$$

$$= \int_0^N GN e^{-rt} dt - \int_0^N Gte^{-rt} dt$$

$$C = \frac{NG}{r} (1-e^{-rN}) + \frac{NG}{r} e^{-rN} - \frac{G}{r^2} (1-e^{-rN})$$

or

$$C = \frac{G}{r} \left(N - \frac{1-e^{-rN}}{r} \right) \quad (31)$$

Equation (31) is identical to equation (28).

This may be paraphrased: the length of life that produces the adverse minimum is also the length of life that is required to recover the capital investment.

The important conclusions are that the adverse minimum or the 'initial earning' of the continuous linear model is always NG and that there is a fairly convenient formula for computing N (formula (28) above).

CHAPTER 6

S - METHOD - THEORETICAL AND OPERATIONAL DESCRIPTION

6.1 Introduction

6.1.1. Background

In his 1971 article¹⁾, Sir Robert Shone develops a model of 'price and investment'. (The model and its further development will be collectively called here 'the S-model'). The model should be seen in the context of the range of economic issues raised by the author there and in other publications²⁾. These issues include: pricing policy in regulated industries - nationalised and otherwise, investment decision in the steel industry, international trade in steel, economic efficiency, the economic growth performance of Britain, disruptive powers of unions, poor management decisions, and many others. Of focal importance is the author's criticism of British governments of systematically discriminating against the industrial sector of the economy vis-a-vis the domestic sector ever since the war.

".[attention will be drawn]once again to the damage that can be done to economic performance by policies which frustrate the cost reducing impact of technological development involving investment. An example of such policies is the attempt of successive British governments to keep down the general price level by artificially holding down prices paid by direct consumers at the expense of costs that enter into industrial activity generally.

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- 1) Sir Robert Shone, "Technical Progress and Investment" in Sir Robert Shone ed. Problem of Investment, Basil Blackwell, Oxford, 1971.
 - 2) a) Sir Robert Shone, "Economic Development of the United Kingdom Steel Industry", Paper delivered to the Royal Society of Arts, 1961.
b) Sir Robert Shone, "Investment and Economic Growth", University of London, The Athlone Press, 1966.
c) Sir Robert Shone, "Price and Investment Relationship", Paul Elek, London, 1975.

... The distortions are not only adverse to Britain's export performance but are one of the factors responsible for her sluggish industrial development" 1)

Thus, says Shone, the apparent subsidisation of the domestic sector at the expense of the industrial sector led to a relatively low level of gross capital formation (as a percentage of real national output) in Britain. This was subsequently reflected in a lower economic growth rate than in most other industrial countries. Thus, in effect, subsidisation of the yesterday's British consumer was done at the expense of today's British consumer.

Shone's criticism is by no means confined to government policy. Excessively defensive management attitudes have led, he says, "to securing less benefit than have competing industrial countries" 2). Trade unions attitude too was short-sighted and damaging. Trade Unions either obstructed technological change directly or destroyed the price reducing effect of technological change by amassing excessive "productivity gains".

A common ground to these and other ills is a disregard for the long run relation between price and investment in a "social market economy". This is either a disregard in principle or a misconception of the relevant costs to consider in both pricing and investment decisions. In many industries today, prices are not determined by impersonal short term market forces. Rather, they are explicitly determined by regulatory bodies or by well established "price leader". Such prices, maintains Shone, should be related to costs and in particular they should properly cover the cost of investment.

1) Sir Robert Shone (1975) op. cit., pp. xiii, xiv.

2) Sir Robert Shone (1975) op. cit., p. xiv.

Investment decision in turn should be made in the light of the price and expected future price movements.

This is not a summary in a nutshell of Shone's economic critique. His critique and his suggested line of correction in the areas of pricing and investment decisions would require a much larger space than that allocated here¹⁾. The discussion above is only meant to present the aura of Shone's economic message and the context in which he develops and promotes the price and investment model. Awareness of this context, and of the audience the model is aimed at are indispensable for the appreciation of the S-model.

6.1.2. The Scope of Investigation

This and the two following Chapters, however, have a more limited purpose. It is a structural analysis of the model. To do this I shall look at the model as a mathematical structure and as a decision model. As such I shall relate it where possible to comparable 'theories', 'models' and 'tools'.

The model itself is not presented until the next section (6.2), so only a rough sketch of the lines of the structural analysis of it will be portrayed here. The analysis includes the following areas'(not strictly in this order):

- (a) The use of the mathematical structure of S-models in a number of "decision models".

1) See Shone's own discussion in Sir Robert Shone (1975) op. cit.

- (b) Developing the model as a particular case from the Microeconomic analysis in Chapter 3. This way of looking at the model reveals some interesting relations among its parameters.
- (c) Comparing the model with the "Business Administration" ad hoc investment decision models (MAPI's and Merrett & Syke's models).
- (d) Development of variants of the basic exponential model; inclusion of "tax" and "inflation" and development of a discrete variant.
- (e) Tabulation of computer results for various parameter values and graphical presentation of the results.
- (f) Various analyses which show how sensitive is the indicated decision to the use of different parameter values.

The basic model is a 'price and investment model', so the basic mathematical structure can be looked upon both as an investment 'model' and as a pricing 'model'. This dissertation is concerned primarily with investment decisions, so the emphasis in the analysis will naturally be put on the S-model as an investment model. A special emphasis will be put on the use of the model in replacement decisions.

6.2 Structural Description of the S-System

6.2.1. Mathematical Structure

The basic S-system is developed and discussed by Shone¹⁾ as the 'price and investment model'. A brief presentation of the system is given here. The emphasis in my presentation is on relating the system to the neoclassical approach of Chapter 3 and on developing three decision models (pricing, expansion investment and replacement) from the basic S-system.

The system is a mathematical identity, showing the capital outlay of a project as equal to the discounted net cash inflow from the project over its life. For simplicity and assuming the product 'annual' output constant over the life of the project the identity is expressed per unit annual output. Cash flows are assumed 'continuous', therefore the 'density' of the gross revenue at every point of time is the price of the product and the 'density' of the operating cost (wages) is operating cost per unit of output. See graphical presentation in Figure 6.1.

The actual price movement over time is determined solely by the initial price P_0 and by the time rates of technological change and wage increase. The operating cost in a going plant is determined by the initial wages W_0 and by the time rates of plant deterioration and wage increase.

The exact mathematical relations are

$$P_t = P_0 \cdot e^{(a-b)t} \quad (1)$$

$$W_{o,t} = W_0 \cdot e^{(a+c)t} \quad (2)$$

1) Sir Robert Shone (1971) op. cit.

2) This is one of the stricter interpretations of the S-Model parameters. Looser interpretation is in effect used later on, particularly where we discuss the investment and replacement models of the S-System. Thus instead of price (or even more narrowly the value added component of output price) we would have revenue, instead of wages per unit of output we would have operating costs and the capital investment volume will be considered in total and not per unit output capacity.

where:

t - time variable (initial value $t = 0$ meaning "now"),

P_t - price at time t ,

P_0 - price at time 0,

a - rate of wage increase,

b - rate of technological change (embodied in new investments),

c - rate of deterioration,

$W_{0,t}$ - wage cost of a unit of output at time t in
plant installed at time 0,

W_t - abbreviated symbol for $W_{t,t}$ (thus in particular W_0 is
wage rate per unit output at time 0 in a new plant
installed at time 0).

Other symbols used hereafter are

K_t - capital outlay per unit output capacity, in a new
plant installed at t (in particular, note K_0),

r - rate of discount,

T - the economic life of a plant; (also used for $t = T$).

(It is also assumed that salvage value of plant is 0).

The basic equation (identity) of the S-method is:

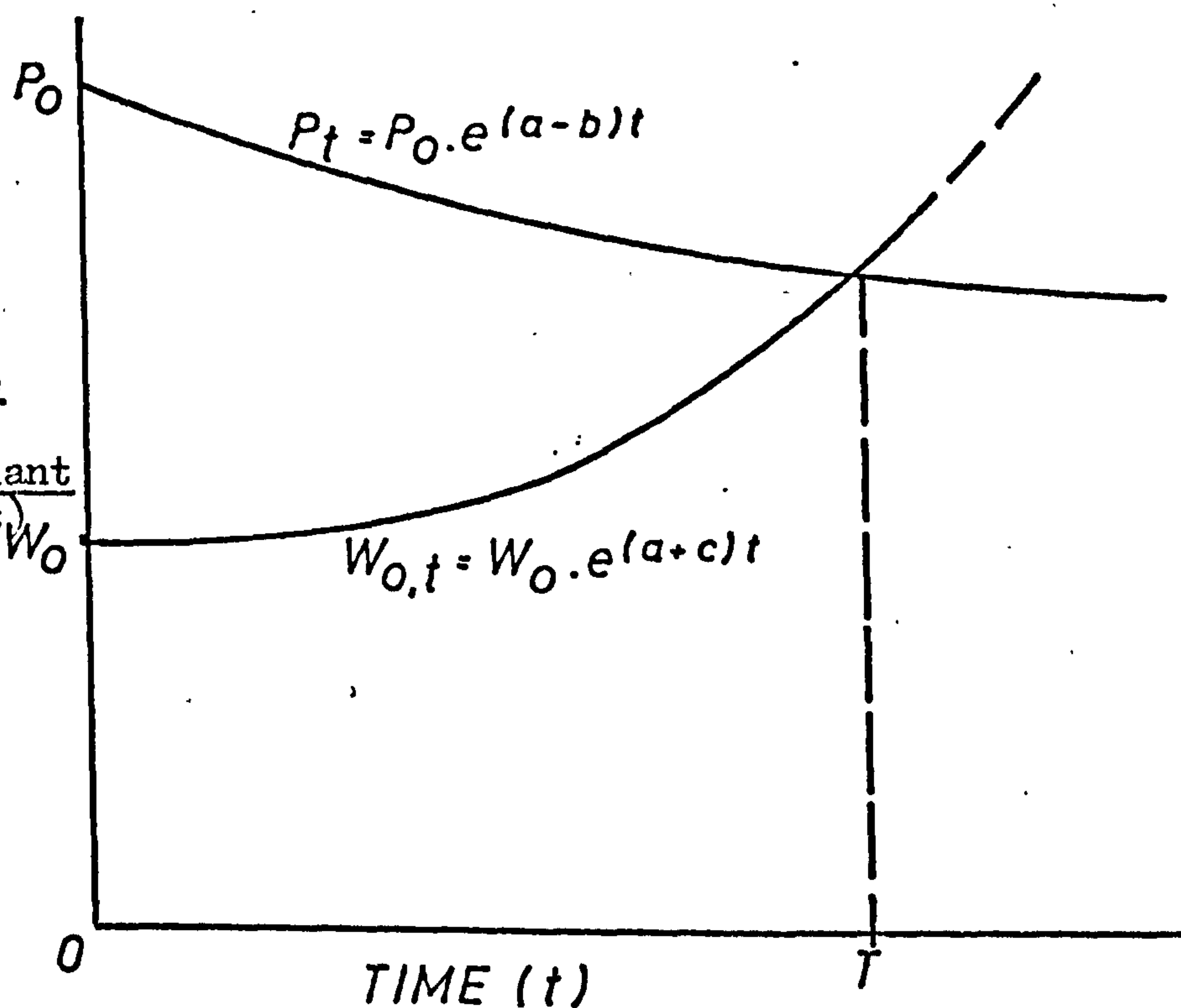
$$\underbrace{K_0}_{\text{Capital outlay}} = \underbrace{P_0 \int_0^T e^{-(r+b-a)t} dt}_{\text{Discounted gross earnings from time "0" to "T" ,}} - \underbrace{W_0 \int_0^T e^{-(r-a-c)t} dt}_{\text{Discounted operating cost from time "0" to "T" }} \quad (3)$$

or after integration,

$$K_0 = P_0 \cdot \frac{1 - e^{-(r+b-a)T}}{r + b - a} - W_0 \cdot \frac{1 - e^{-(r-a-c)T}}{r - a - c} \quad (4)$$

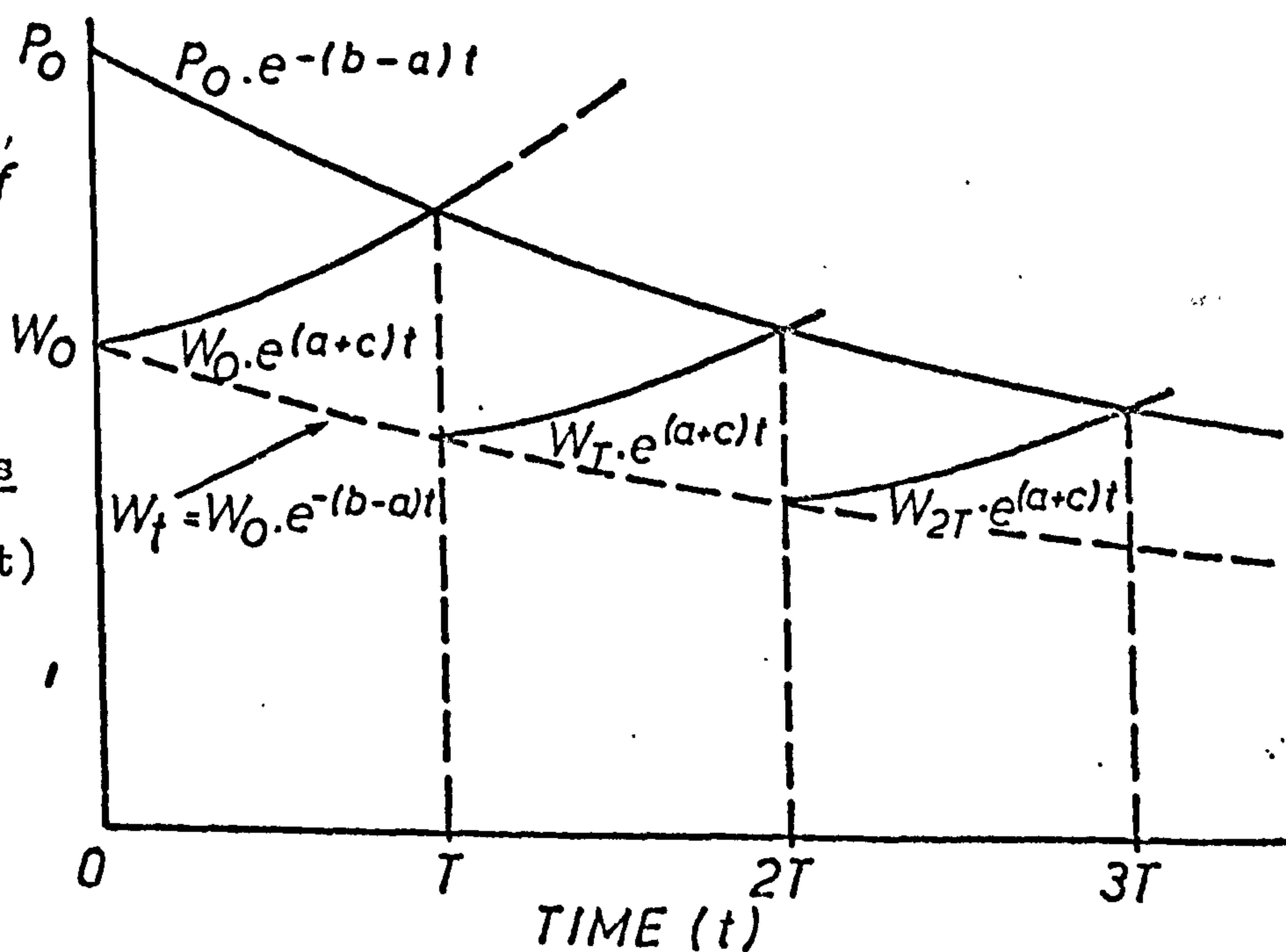
Price, cost,
per unit of
output.

Fig. 6.1 S-Model time
movement of price and
operating cost in a plant
(for notation see text)



Price, cost,
per unit of
output.

Fig. 6.2 S-Model time
movement of price and
operating cost in
successive generations
of plant
(for notation see text)



It is interesting to see how T , the economic life, is determined. As suggested by Figure 6.1 and Figure 6.2 it is simply that point of time where the operating cost curve intersects the price curve - a point beyond which the price does not cover the operating cost. Mathematically this is verified by differentiating (3) with respect to T :

$$\frac{\partial K_o}{\partial T} = P_o \cdot e^{-(r+b-a)T} - W_o \cdot e^{-(r-a-c)T} \quad (5)$$

on equating the derivative to 0 we obtain

$$P_o \cdot e^{-(r+b-a)T} - W_o \cdot e^{-(r-a-c)T} = 0 \quad (6)$$

or

$$P_o = W_o \cdot e^{(b+c)T}$$

(The second derivative $\frac{\partial^2 K_o}{\partial T^2}$ is negative, i.e. T gives a 'maximum' point)

T can then be expressed as

$$T = \frac{1}{b+c} \cdot \ln \frac{P_o}{W_o} \quad (7)$$

This leads us to an interesting conclusion. We can express the price P_o as a function of the initial operating cost W_o , the capital outlay K_o , the discounting factor - and the determinants of future price movement - a , b , and c ; that is, independently of the length of life.

$$K_o = \frac{P_o}{r+b-a} \left(1 - e^{-(r+b-a)T} \left[\frac{1}{b+c} \ln \frac{P_o}{W_o} \right] \right) - \frac{W_o}{r-a-c} \left(1 - e^{-(r-a-c)T} \left[\frac{1}{b+c} \ln \frac{P_o}{W_o} \right] \right) \quad (8)$$

(P_o is determined by approximation methods)

6.2.2. The Neoclassical Microeconomic Context.

An important feature of the S-system is that P_0 is the long term marginal cost price. (More strictly, P_0 is that part of the output price which covers labour and capital costs). To qualify as long term marginal cost price, P_0 has to produce (considering future operating cost associated with the new investment and future output prices) a normal return on investment. Future output prices are assumed such that they would also give exactly "normal return" on future investment made at any future date. In such circumstances an investor would be indifferent as to the date of incurring "expansion" investment. Not surprisingly therefore, R. Turvey uses this "indifference" in defining long-run-marginal-cost price.¹⁾

P_0 "gives" a normal return "r" on the investment K_0 . This is verified immediately from equations (3) and (4) above. It still needs showing how the future prices, which are assumed given in these equations, would "give" normal return on future investment. This is done very simply as a logical extension of (4) above.

By definition K_t and W_t , for any "t" are :

$$K_t = K_0 e^{-(b-a)t} \quad (9)$$

$$W_t = W_0 \cdot e^{-\gamma(b-a)t} \quad (10)$$

If in equation (4) we substitute K_t for K_0 and W_t for W_0 then in order to maintain equality we have to introduce P_t such that,

1) R. Turvey, "Marginal Cost" The Economic Journal Vol. 79 No. 2 (June 1969) pp. 282-299. See also discussion in Chap. 3.

$$P_t = P_o \cdot e^{-(b-a)t} \quad (11)$$

which is the "assumed" price in (1) above.

The economic importance of the conclusion given in (11) is that in the S-system "prices" are determined on long term costs consideration ("investment" costs and operating costs); therefore they are long term marginal cost prices.

Another important feature of the S-system is that the technological change in it is "neutral". The system assumes that the proportional expenditures on "capital" relative to expenditure on labour is not affected by the technological change (and also not by cost escalation). Using the terminology developed in Chapter 3 the S-system does not concern itself with the 'movement of production function' but with comparing the optimal combination of "labour" and "investment" in each period. Expressed in monetary rather than physical units the S-system is concerned with "neutral" change in the monetary sense i.e. proportional money expenditure.

An illustration of the movement of operating costs in successive replacements of plant is given in Figure 2. Note the constant length of life of successive generations of plant.

6.2.3. Derived Decision Models

The S-system forms three decision models

- a) a pricing model,
- b) an expansion investment (new investment) model,
- c) a replacement model.

Table 6.1 below summarises the meaning of the system's parameters in each model and specifies the "unknowns" inherent in each one.

Table 6.1 S-system: Parameters and "unknowns" in the three decision models.

	Pricing	Expansion	Replacement
K_o	capital outlay in best practice plant	capital required	capital required
P_o	minimum price required	market price (or revenue)	op. cost in old plant
W_o	op. in best practice plant	op. costs in new plant	op. costs in new plant
Unknowns:	P_o	r	$P_o^{1)}$
	T	T	T

1) In this case the meaning of the unknown is the operating cost in old plant that would justify replacement.

The decision in the three cases is as follows:

1. Pricing - set the price according to calculated P_o
2. Expansion investment - accept project if $r \geq r_o$
(r_o = the required rate of return)
3. Replacement - replace if operating costs in old plant are greater or equal to P_o

6.3 Minor Extensions of the Basic Model.

6.3.1. Treatment of Inflation

Inflation has become a central consideration in the analysis of projected future cash flows. It is therefore important to state how inflation can be catered for by the S-system. Obviously real life inflation is a complex phenomenon and only a limited aspect of it can enter a "model".

Here I shall show how the S-system can accommodate the overall expected rate of inflation. In a model of 'certainty' as the S-system is this would be equivalent to the assumption that inflation will increase all costs by the same percentage annually (Neutral inflation). We shall denote this rate of inflation - "f".

In the original model "r" and "a" are expressed with reference to constant-money costs and prices, thus "r" is the 'real' cost of capital (or 'real' discount factor) and "a" is the 'real' wage increase rate, now, we shall denote " r_m " as the money rate of cost of capital (or the current rate of cost of capital) and " a_m " as the money rate of wage increase. So

$$\left. \begin{aligned} r_m &= r + f \\ a_m &= a + f \end{aligned} \right\} \dots (12)$$

In particular we could write

$$r_m - a_m = (r_m - f) - (a_m - f) = r - a \dots (13)$$

Equation (13) leads us to an interesting result.

In the basic equation (equation (3)) and all the derived equations "r" and "a" always appear together as (r - a) which as stated here is equal to $r_m - a_m$. Since no other parameters in the basic equation are affected by the rate of inflation f we could rewrite the basic equation in current money terms:

$$K_0 = P_0 \int_0^T e^{-(r_m + b - a_m)t} dt' - W_0 \int_0^T e^{-(r_m - a_m - c)t} dt \dots (14)$$

So the S-system can be expressed in "money term" and include implicitly a rate of inflation. This does not require any additional structural parameter of "inflation"; the basic relations are not distorted by inflation. It must be added that no similar simple transformation can be operated on other investment-replacement models.

6.3.2. Taxation and the S-System

Taxation, i.e. corporate income tax is a major 'real life' consideration in investment decision making and should therefore be examined in this discussion of the S-system. Much like inflation, taxation is a complex phenomenon. To include taxation in the S-system would require some simplifying assumptions about the way tax is calculated and collected¹⁾. It would also require the fundamental assumption that the basic mathematical relations in the model hold true in a world-with-tax.

Tax can be incorporated in the S-model in a number of ways. The conventional way is that of modifying the income factors (in the S-system this would be the right hand side of equation (4)) and of leaving the capital investment factor unchanged. This is a straight-forward treatment as it considers cash flows when they occur. Mathematical manipulation of the U.K. tax parameters (as were in force in 1971-72) has revealed that in the particular case of the S-system all the tax parameters can be conveniently moved to left hand side of equation (4) thereby modifying the investment side rather than the income side. This means that the taxation "package" can be looked at as changing the 'effective' investment without altering the before-tax income. This approach enables charts based on the original S-model to be used in the solution of a world-with-tax problems. Such charts are presented in Chapter 7.

1) For example, income, for tax purpose, is recognized at the time of sale and not on 'production point' basis. Income in the S-system is calculated on 'production point' basis. (The same is also true of the Terborgh system and the Merrett & Sykes System). So, an assumption must be made either that output is sold the moment it is produced or that the "capital" includes some working capital element (to cover the financial cost of the time difference between production and selling). The latter alternative requires that for tax calculation there are two capital elements: one depreciable and one (working capital) not. For simplicity the former alternative is chosen; income and tax are calculated on 'production point' basis.

The basic equation modified for pre-April 1972 U.K. tax is given here as equation (15). It is presented as a fairly general treatment of taxation, not necessarily confined to U.K. tax (in practice of course, tax is calculated as discrete annual lump sum and not as flow. The analysis is done on a continuous mode to fit the original S-model).

$$K_0 \cdot \left[1 - \underbrace{\frac{1}{e^{rm}}(A \cdot X)}_{\text{First year allowance factor}} + \underbrace{(1-A) \cdot X \cdot \frac{d}{d+r}}_{\text{Total annual allowances factor}} \right] \cdot \underbrace{\left(1 - \frac{X}{e^{rn}} \right)^{-1}}_{\text{After tax coefficient}} =$$

$$= \frac{P_0}{r+b-a} \cdot (1-e^{-(r+b-a)T}) - \frac{W_0}{r-a-c} \cdot (1-e^{(r-a-c)T}) \dots (15)$$

The additional symbols are

X - corporation tax rate

A - first year capital allowance factor

d - reducing balance of capital for tax purposes

m - the time lag for realisation of first year and annual allowances

n - the time lag for corporation tax payment on earnings

therefore, $\frac{1}{e^{rm}}$ is the impact of the delay in the realisation of the capital allowances.

The tax allowance elements in the equation are: a 'first year' investment allowance, and a reducing balance depreciation allowance on the remainder.

Equation (15) represents a number of cases:

1. (General) pre April 1972 U.K. Corporation Tax on plant and machinery,
2. (A=1) post 1972 U.K. Corporation Tax on plant and machinery i.e. 100% first year allowance,

3. (A = 0) U.S. Corporate income tax - double-declining-balance depreciation,
4. (A = 0, d = 0) U.K. Corporation Tax on commercial buildings.

Thus for example the net-of-tax capital coefficient for plant and machinery (U.K. Post April 1972) is:

$$\frac{1 - X \cdot e^{-rm}}{1 - X \cdot e^{-rn}} \quad \dots (16)$$

The net-of-tax capital coefficient for commercial buildings is:

$$\frac{1}{1 - X \cdot e^{-rn}} \quad \dots (17)$$

Table 6.2 below illustrates how delays in recovery of the 100% first year allowance change the effective capital factor (note that an infinite delay of this allowance is equivalent to no allowance - as is the case of commercial buildings).

Table 6.2

Capital Net-of-Tax Co-efficients

Calculated for 100% first year allowance realised m years after investment (1½ year's lag in corporation tax payment at 50%)

Rate of Return (r)	Number of Years (m)						
	1	1.5	2	3	5	10	∞
5	.98	1.0	1.02	1.07	1.14	1.30	1.87
10	.96	1.0	1.03	1.10	1.22	1.41	1.75
15	.95	1.0	1.04	1.13	1.27	1.48	1.66
20	.94	1.0	1.05	1.15	1.29	1.48	1.59
25	.93	1.0	1.06	1.17	1.31	1.47	1.53

6.3.3. Combined Consideration of Taxation and Inflation

Equation (16) accommodates all the tax considerations on its left hand side, the capital side.

The right hand side of the equation, the income side, is in its original form as in equation (3). Following the discussion in 6.3.1, it is insensitive to inflation. The impact of inflation cancels out since in the expression " $r-a$ " both r and a contain the same inflation element.

The left hand side of the equation (15), on the contrary, is sensitive to inflation. Inflation is contained only in " r " the discounting factor. So the higher the rate of inflation, the higher the effective discounting. This is a source of theoretical concern. Inflation, in a world-with-tax, is not a neutral factor the way it is in "a-world-without-tax". Generally speaking, inflation reduces the benefit of delayed capital allowances thereby increasing the effective capital investment (reducing profitably).

In practice the problem is not as serious as it may seem. Take for example the current U.K. tax treatment of plant and machinery. Here, when time lag for tax payment and for first year allowance are the same the net-of-tax coefficient is unity and thus is not affected by the rate of inflation. (See Table 6.2). Plant and machinery, I may add, is the class of investment with which the S-system is most concerned.

Table 6.3 shows the impact of taxation and inflation in a slightly more complex case. It is based on first year allowance of 80% and annual reducing balance of 25% thereafter. Corporation tax rate is 40% and time lags are 15 years for both tax payment and first year allowance. Calculation is done for £1.00 investment.

The left hand side of equation (15) for this case would be:

$$\left[1 - e^{-1.5r} \cdot (0.8 \cdot 0.4 + \frac{0.2 \cdot 0.4 \cdot 0.25}{r + 0.25}) \right] \cdot (1 - 0.4 \cdot e^{-1.5r})^{-1}$$

Table 6.3

F(r) The Value of the Modified Capital Side in Equation (15) of an Initial £1.00 Capital Outlay

C A L C U L A T E D V A L U E S

r	F(r)
•1	1.03
•105	1.03071
•11	1.03136
•115	1.03198
•12	1.03255
•125	1.03308
•13	1.03357
•135	1.03402
•14	1.03445
•145	1.03484
•15	1.0352
•155	1.03553
•16	1.03583
•165	1.03611
•17	1.03636
•175	1.03659
•18	1.0368
•185	1.03699
•19	1.03716
•195	1.0373
•2	1.03743
•205	1.03754
•21	1.03764
•215	1.03772
•22	1.03779
•225	1.03784
•23	1.03788
•235	1.03791

Table 6.3 shows calculated values of the effective capital for the discounting factor values over the range 10% - 23.5%. It shows that over this range there are only minute differences in effective capital - less than 0.008. This implies that the effective capital - within the particular assumptions behind Table 6.3 - is insensitive to the rate of inflation.

The overall impact of taxation in modifying the capital factor in this particular case is 3% - 4%.

6.4 Summary

The S-model has been developed as an integrative dynamic pricing-investment-replacement model. Thus the same mathematical structure can be interpreted as a long-term marginal cost pricing method, an expansion investment model, or a replacement model. When used in different applications some of the model's parameters have different meanings.

The model, a specified neoclassical 'vintage model' includes parameters specifying the impact of technological change, plant deterioration and wage increase on operating costs and on capital costs, in new vintages of investment starting with today's investment. The basic model lends itself to include additional factors. Thus the inclusion of inflation and taxation is demonstrated.

The basic S-model in its original continuous mode is convenient for mathematical analysis. Such analysis shows the determination of the optimal length of life of investment and the calculation of another 'unknown', according to application. They are: in a pricing model - the long term marginal cost price to charge; in a new investment model - the IRR of an investment proposal, and in a replacement model - the initial operating cost saving that would justify replacement.

An investigation of the numerical behaviour of the model over a wide range of parameter values both in the continuous mode and in an equivalent discrete mode is given in a separate Chapter - Chapter 7. The results there are also used in demonstrating the S-model's solution of simple investment and replacement problems.

CHAPTER 7

S-METHOD: COMPUTER SOLUTIONS AND INVESTIGATION

7.1 General

Much of my work on the S-system is the development of computer packages, tables and charts. Their possible use is as an aid to managers in price and investment decision making¹⁾ and as an aid for students of management economics in studying problems of price and investment. In addition the packages, tables and charts are a way of analysing the S-system itself. The numerical and visual examples show, rather conveniently, the relations among the underlying parameters of the system. Formal mathematical analysis of the behaviour of the S-system over a similar range of parameters to that covered by the computer - if at all possible - would be extremely tedious.

As the first step I wrote the FORTRAN program INVS1 to mount the basic S-model on a computer (ICL 1905 of the City University). The program generates 'solutions' (i.e. S-system equilibrium points) for a wide range of parameter values. The printout of INVS1 and of similar programs are presented and discussed in this Chapter.

In Section 7.2 I present the conversion of the basic continuous model into a discrete mode. It was felt that the discrete mode employing periodical cost and revenue information would be more realistic²⁾ than a continuous mode using densities of costs and

1) See Sir R. Shone, "Price and Investment Relationships" Paul Elek, London, 1975. p. XV. Note that Shone's comment on his book applies in particular to the packages the tables, and the charts. - "It is not a vade mecum giving handy answers to particular problems, but a general survey covering the principles involved in the related issues of pricing and investment."

2) See Chapter 4, Passim.

revenues. Later in the Chapter the computer program INVS1 and its Flow Chart are presented. Section 7.3 discusses and presents the Tables and the Charts. Section 7.4 gives two examples for the use of the Tables in replacement and new investment decisions.

Analyses of the model and of its presentation in this Chapter are given in the Appendices to this Chapter. Appendix 7.1 contains the computer program, and tables for the continuous mode of the model. Reading through the results one verifies the similarity and differences between the two modes of the S-system. Figure 7.7 compares the results of the continuous and the discrete modes for the "same" set of parameters.

Appendix 7.2 presents some of the difficulties of efficient graphical presentation by offering a number of alternatives to the presentation used in Section 7.3. Alternative presentations give some insight into the behaviour of the system with different sets of parameters.

Appendix 7.3 refers to formula (8) in Chapter 6 which presented the length of life of investment as a function of only $\left(\frac{P_o}{W_o}\right)$ and $(b + c)$. Tables of "life" are presented as a function of these two parameters.

Appendix 7.4 unlike the rest of the Chapter, deals with points outside the model's equilibrium. In particular it shows the reduction of profitability when a "wrong" "length of life" is employed in investment appraisal. Inevitably, such an analysis using either NPV or IRR criterion deals only with selected instances but it reveals that the profitability is insensitive to changes or errors in the 'length of life'. Some of the implications of this are discussed in Chapter 8.

The Appendices form a part of Chapter 7. They relate to various computer operations done on the S-system. They have been separated from the main body of the Chapter only as a matter of convenience as each deals with some isolated aspect of the model or the presentation.

7.2 S-Model in the Discrete Mode

7.2.1 The Discrete Mode and the Computer Solution Process

The basic equations:-

$$K_o = P_o \int_0^T e^{-(r+b-a)t} dt - W_o \int_0^T e^{-(r-a-c)t} dt \quad (1)$$

and

$$P_o \cdot e^{(a-b)T} = W_o \cdot e^{(a+c)T} \quad (2)$$

have discrete equivalents:-¹⁾

$$K_o = P_o \sum_{i=1}^T \frac{1}{(1+r+b+a)^i} - W_o \sum_{i=1}^T \frac{1}{(1+r-a-c)^i} \quad (3)$$

(where $r, a, b, c, \geq 0$; $b+c > 0$)

and

$$P_o \cdot \frac{1}{(1+b-a)^Y} = W_o \cdot \frac{1}{(1-a-c)^Y} \quad ; \quad T = [Y] \quad 2) \quad (4)$$

respectively.

1) It is difficult to define "exact" equivalent. For example $e^{-(r+b-a)t}$ can be written as $e^{-rt} \cdot e^{-bt} \cdot e^{at}$. This may have the equivalent of $\frac{1}{(1+r)^t} \cdot \frac{1}{(1+b)^t} \cdot \frac{1}{(1-a)^t}$ or of $\frac{1}{(1+r)^t} \cdot \frac{1}{(1+b)^t} \cdot (1+a)^t$;

there are many other "equivalents". The type of discrete equivalent used in equations (3) and (4) clearly is very convenient mathematically; this is why I chose it, but it is arguable that it is somewhat less accurate than other equivalents.

2) The expression $[]$ means the whole part of a number, thus if $Y = 11.7$ then $T = 11$

The following changes were made for the computer enumeration process.

- 1) K_0 value was assumed $K_0 = 1$,
- 2) $(r + b - a)$ was replaced by a single variable G (in the program INVS1 it is labelled $K1$); $(r - a - c)$ was replaced by a single variable H (in the program it is labelled $K2$).
- 3) Equation (3) was re-written as an explicit function of P_0 .
- 4) Equation (4) was enlarged by $(1 + r)^{-y}$.

The operative equations are, therefore,

$$P_0 = \frac{1}{\sum_{i=1}^T \frac{1}{(1+G)^i}} + \frac{W_0 \cdot \sum_{i=1}^T \frac{1}{(1+H)^i}}{\sum_{i=1}^T \frac{1}{(1+G)^i}} \quad (5)$$

$$P_0 \frac{1}{(1+G)^y} = W_0 \frac{1}{(1+H)^y} \quad (6)$$

The actual computation process is given in the following Flow Chart and printout of the computer program INVS1.

The discrete presentation used here is mathematically analogous to the continuous presentation of Chapter 6. But, the annual jumps that are implied by it require new strict interpretation of the parameters.

The most convenient interpretation for our purpose is:

K_0 - Investment outlay for a unit annual capacity assumed to concentrate at mid year 0 (i.e. - "now").

P_0 - The 'current price' of output (or in replacement problems - operating cost of the old plant) - "now" - at mid year 0.

W_0 - The wages or operating cost of the new plants per unit of output, "now", at mid year 0.

The actual 'first year price' is P_0 modified by technological change and wage increase as well as - for investment assessment - by the rate of discount, namely it is modified by the parameter "G". The actual 'first year wages' is W_0 modified by one year wage increase and by one year deterioration (as from the time of investment).¹⁾

Alternative discontinuous reconstruction can be made with other assumptions. Here is one:

P_0 and W_0 refer to year 1. (Therefore the notation P_1 and W_1 is preferable there). This way they represent directly first year operating revenue and costs.

The basic equation then becomes:

$$K_0 = P_1 \cdot \frac{1}{1+r} \sum_{i=1}^T \frac{1}{(1+r+b-a)^{i-1}} - W_1 \cdot \frac{1}{1+r} \sum_{i=1}^T \frac{1}{(1+r-a-c)^{i-1}} \quad (7)$$

This approach, though intuitively very appealing, is very difficult to apply. It is inconvenient for charts and tables because it includes an additional explicit parameter 'r' outside the instrumental parameters "G" (=r+b-a), and "H" (=r-a-c) (see Appendix 7.2 on charting problems).

It should be stated though without quantification, that in most of the practical problems the difference between the two 'methods' is very small.²⁾

1) The conceptual difficulty of including a "notional" deterioration in year 1 - first year of operation - is discussed in Shone (1975) p.188. Shone offers some pragmatic justifications for this approach.

2) Quantifying this statement is tedious as it should refer separately to various $\frac{W_0}{K_0}$ combinations. In general, the longer the life of the investment (or the smaller the sum "b+c") the smaller is the difference between the 'methods'.

7.2.2 Inflation and Taxation in the Discrete Mode.

The use of the Tables and Charts of the discrete mode for a "world-with-tax" requires a modification of the relevant coefficients.

a. Inflation

Here the analogy is straightforward. As in the continuous mode we have

$$r_m = r_r + f \quad (\text{The current discount factor equals the sum of the 'real' discount factor and the rate of inflation. See Chap. 6}).$$

b. Taxation

Here the analogy is based on substituting continuous discounting by discrete discounting. The net-of-tax capital coefficient for plant and machinery (where 100% first year allowance is given) in the theoretical, continuous case is

$$\frac{1 - X \cdot e^{-rm}}{1 - X \cdot e^{-rn}}$$

(For notation and discussion see 6.3.2.).

In the discrete case this net-of-tax capital coefficient for plant and machinery (100% first year allowance) becomes

$$\frac{1 - X (1+r)^{-m}}{1 - X (1+r)^{-n}}$$

Shone, with particular interest in the development in British taxation, studies in detail the introduction of discrete coefficients in a number of Tax categories.¹⁾

7.3 Tables and Charts

The 5 tables given in this Chapter are completed for a representative value of capital outlay of £1.00. Each of the tables is given for a specific value of W_0 or, more strictly for any $\frac{W_0}{K_0}$. The values are -

1) Shone (1975) pp. 63-69. The subjects covered are: a) plant and machinery, b) commercial buildings, c) cars for business use, d) industrial buildings, and e) development grants.

0.10, 0.15, 0.20, 0.25, 0.50.¹⁾

The iterative 'solution' process uses G (denoting $r + b - a$) and H (denoting $r - a - c$) as the other computation parameters. The ranges covered are:

$$0 \leq G \leq 36 \text{ and } -0.20 \leq H \leq 0.20$$

thus precluding negative r or excessively high r as a basis for long run assessment.

Parameter values used are:

For G : 0.00, 0.02, 0.04 ... , 0.34, 0.36

For H : -0.20, -0.18, -0.16, ... , 0.18, 0.20.

The "solution" itself is two-valued, it includes the remaining two parameters of the S-system P_0 and T . For tabulation purpose the parameter P_0 (initial revenue) was substituted by F (initial earnings) such that $F = P_0 - W_0$.

The tables themselves (e.g. Tables 7.1 - 7.5) though capable of giving the desired answer still are fairly inconvenient. A convenient manual and visual tool for solution are Charts based on the Tables.

The number of possible ways of presenting the results graphically is very large. It was decided that the main mode of presentation will be based on a separate Chart for each value of W_0 , (for other modes see Appendix 7.2). Each such chart shows how G and H relate to F and to T .

A further graphical and practical improvement was the replacement of H by a new parameter M . $M = G - H$ and as such it means "the annual 'degradation' of existing plant" or the combined impact of obsolescence and deterioration.²⁾

1) Tables were computed and printed for the additional $\frac{W_0}{K_0}$ values: 0.01, 0.02, 0.03, 0.05, 0.08, 0.30, 0.40, 0.75, 1.00. More Tables can be calculated by changing the DATA card in the computer program.

2) Note, $G = r + b - a$ and $H = r - a - c$, therefore $M = G - H = r + b - a - r + a + c = b + c$. The derivation of M is, therefore, immediate and does not impose any difficulty in constructing the charts.

The area of a chart for a given value of W_0 is thus used for two separate "maps". The "maps" are:

1. The relation between the initial earnings-F and the parameter G for given values of the parameter M (i.e. isoquants of M on (F,G)plane).
2. The relation between plant length of life T and the parameter G for given values of the parameter M (i.e. isoquant of M on (T,G)plane).

Charts are given for $W_0 = 0.10, 0.15, 0.20, 0.25$ and 0.50 in Figures 7.2 - 7.6.

Other methods of obtaining charts from the Tables are discussed in Appendix 7.2 to this Chapter.

7.4 Examples for the use of the Charts

a. Example 1: Replacement analysis (world-without-tax)

Company A considers replacing a 16-year-old workshop by a new one. The new one will not require new space, the old building with a few minor changes will be used. The cost of the new equipment is estimated at £130.000 and the shop will require 10 employees at the total labour cost of £26.500 p.a. At the moment the workshop employs 25 workers at an annual cost of £57.000 a year. It is assumed that no change in other cost elements will take place, no additional output will be obtained from the installation of new plant. The wage increase rate is assumed to be 3% p.a. in real terms while average annual technological progress, or obsolescence, is assumed to be 4% of the operating cost a year.¹⁾ Deterioration,

¹⁾For strict meaning of a rate of embodied technical progress used here see Sir R. Shone "Price and Investment Relationship". Paul Elek, London, 1975, pp 33-36.

though mainly in the form of parts substitution, may be assumed here to raise operating cost at a rate of 2% a year (of the operating costs at the beginning of that year). The company's real cost of capital is regarded to be about 9%. Is immediate replacement justified?

Answer:

$$W_o = \frac{\pounds 26.500}{\pounds 130.000} \simeq 20\% \text{ Therefore, we shall use the Chart for } W_o = 20\%$$

$$G = r + b - a = 9\% + 4\% - 3\% = 10\%$$

$$M = b + c = 4\% + 2\% = 6\%$$

From the Chart we get $F \simeq 22\%$,

or in money terms $\pounds 130.000 \times 0.22 = \pounds 28.600$

The company has initial savings of $\pounds 57.000 - \pounds 26.500 = \pounds 30.500$ therefore, immediate replacement is justified.

As a by-product, one gets the "length of life" of the new workshop = 22 years.

b. Example 2: New investment (world-with-tax)

Assume corporation tax rate of 50%, no capital allowances(as for commercial buildings) and no time lag in tax payment.

$$b = a = 4\% \quad c = 0\%$$

$$\frac{W_o}{K_o} = 20\% ; \quad \frac{P_o}{K_o} = 48\% ; \quad \text{and } \therefore F = 28\%$$

What is the before-tax and after-tax DCF rate of return and the before-tax and after-tax life of the investment? (Use the Charts in order to find the answer. Notice that: $G = r + b - a$; $M = b + c$;
 $F = \frac{P_o}{K_o} - \frac{W_o}{K_o}$)

A convenient way of solving the problem is through steps (A), (B) and (C).

- (A) Calculate the length of the life of the investment from the Chart for $\frac{W_o}{K_o} = 0.20$ and $F = 28\%$.

Answer: $G = 24\%$ and $T = 22$ years.

- (B) Choose the Chart for after-tax calculation by computing

$\frac{W_o}{K_o} = \frac{W_o}{K_o} \cdot \frac{K_o}{K}$; K is the modified, after-tax value of K_o (see the left hand side of the equation (15) in Chapter 6).

In this case $\frac{K_o}{K} = 0.5$ so $\frac{W_o}{K_o} = 10\%$;

Answer: use the Chart $\frac{W_o}{K_o} = 10\%$.

- (C) Find G and T (in the Chart for $\frac{W_o}{K_o} = 10\%$) for $F(\text{after tax}) = \frac{28}{200} = 14\%$

Answer: $G = 8\%$, and $T = 22$ years.

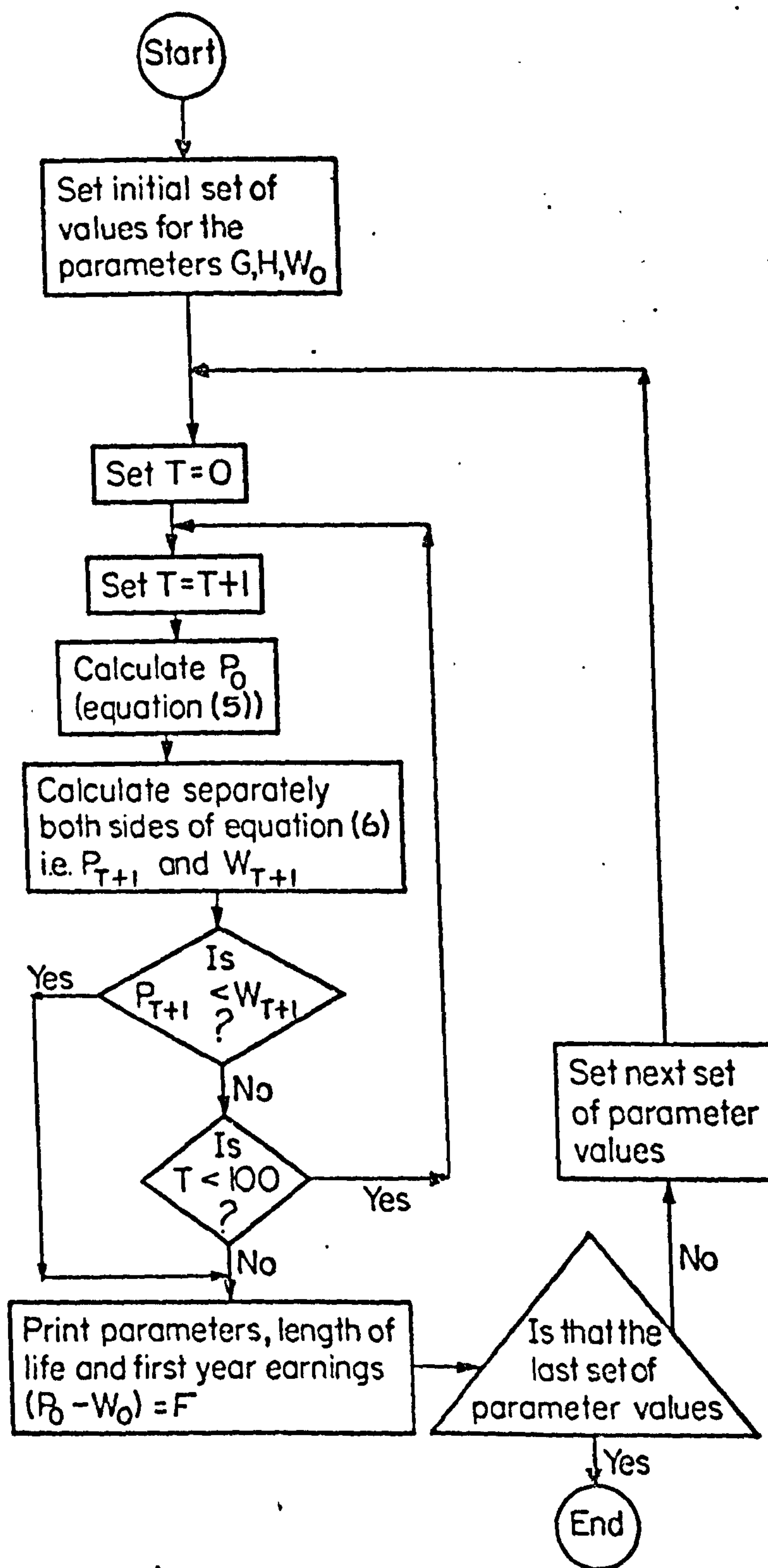
The DCF rate of return ($=G - (b-a)$) is 24% before tax and 8% after tax.

The life of the investment is 22 years and is not affected by tax (in some cases the life before and after tax may differ by 1 year as the curves are drawn to the nearest whole year)¹⁾.

Example 2 as given here is a very simple example. It serves mainly to show how a modified K_o is used in a-world-with-tax problems. More interesting examples for a-world-with-tax problems are given by Shone.²⁾

1) From equation (7) in Chapter 6 we know that $T = \frac{1}{(b+c)} \times \ln \frac{P_o}{W_o}$. Therefore any modification of K_o that does not affect the parameters in this equation will not affect the length of life T .

2) Shone (1975). Prof. Shone has extended the tax modification of K_o (as developed in Chapter 6) to the discrete model and he deals with a number of U.K. Tax arrangements which include a) 100% first year allowance, b) straight line depreciation allowance, c) reducing balance depreciation allowance, d) no allowance. He also incorporates time lags in tax payments.



1)
Fig. 7.1 A Schematic Flow Chart of Calculation-S Model

- 1) Rather than using my original drawings in Figures 7.1-7.10 I have reproduced the neater draughtsman's copy as printed in Sir Robert Shone's "Price and Investment Relationships" (1975). I acknowledge Sir Robert's permission to do so.

FORTTRAN PROGRAM - INVS1

```

MASTER INVS1
DIMENSION W0(7),PR(19),IT(19),OK1(19),OK2(21)
DATA W0/ .01,.02,.05,.10,.25,.50,1.00 /
WRITE (2,5)
5  FORMAT(1H1,30X,41HINVESTMENT AND TECHNOLOGICAL CHANGE MODEL/
C 1H0,30X,54HCALCULATION OF FIRST YEAR EARNINGS AND INVESTMENT LIFE
C/31X,66HFOR VARIOUS PROPORTIONS OF INITIAL YEARLY WAGES TO CAPITAL
C OUTLAY / 1H0,30X,21HCAPITAL OUTLAY = 1.0 )
DO 3 I = 1,19
3  OK1(I) = (I-1)*2 /100.
DO 4 L = 1,21
4  OK2(L) = (L+2 -22) / 100.
DO 7 IA= 1,7
WRITE (2,6) IA, W0(IA),(OK1(I),I=1,19 )
6  FORMAT ( 1H1,10X,7HTABLE ,12,26H  INITIAL WAGE PROPORTION ,F4.2/
C 1H0,3X24K1,19(2X,F4.2)/4H0 K2)
DO 8 K2 = 1,21
DO 9 K1 = 1,19
IF(K1+10-K2) 140,140,10
10  IF(K2-11)12,120,12
12  IF(K1-1)13,130,110
140 PR(K1) = .0
IT(K1) = 0
GO TO 9
110 A = ( 1.0 + W0(IA) / OK2(K2))*OK1(K1)
B = W0(IA)*OK1(K1) / OK2(K2)
C = 1.0 / (1.0 + OK2(K2))
CT= C
D = 1.0 / (1.0 + OK1(K1))
DT= D
ITT=1
111 ITT= ITT +1
P0 = ( A-B*CT)/ (1.0 -DT )
IF ( ITT -101) 112,19,19
112 CT =CT + C
DT =DT + D
IF(P0/(W0(IA)*(1.-OK1(K1)- OK2(K2))* ITT)-1.) 19,19,111
120 D = 1.0 / (1.0 + OK1(K1))
DT = D
ITT=1
121 P0 = (1.0 +W0(IA) * ITT)* OK1(K1)/(1.0-DT)
ITT = ITT+1
IF ( ITT -101) 122,19,19
122 DT = DT +D
IF(P0/(W0(IA)*(1.+OK1(K1)- OK2(K2))* ITT)-1.) 19,19,121
130 C = 1.0/ ( 1.0 + OK2(K2) )
CT= C
ITT=1
131 P0 = (1.0 + W0(IA)*(1.0 -CT)/ OK2(K2))/ITT
ITT = ITT +1
IF(ITT-101) 132,19,19
132 CT=CT+C
IF(P0/(W0(IA)*(1.-OK1(K1)- OK2(K2))* ITT)-1.) 19,19,131
19 PR (K1) = P0-W0(IA)
IT (K1) = ITT -1
9 CONTINUE
WRITE (2,17) OK2(K2) ,(PR(N),N=1,19)
WRITE (2,18) (IT(M),M=1,19)
17 FORMAT(1H0,F4.2,2H F,19(1X,F5.3) )
18 FORMAT (6X,1HT,19(1X,I3,2X))
8 CONTINUE
7 CONTINUE
13 STOP
END

```

END OF SEGMENT, LENGTH 526, NAME INVS1

Table 7.1. S-Model, $W_0 = 0.10$

G		0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36
H																				
-0.20	F	0.312	0.346	0.381	0.390	0.421	0.453	0.486	0.510	0.519	0.518	0.577	0.607	0.638	0.669	0.700	0.732	0.764	0.748	0.775
	T	7	7	7	6	6	6	6	6	5	5	5	5	5	5	5	5	5	4	4
-0.18	F	0.295	0.315	0.345	0.379	0.391	0.422	0.453	0.484	0.517	0.521	0.550	0.578	0.608	0.637	0.667	0.698	0.728	0.760	0.751
	T	8	7	7	7	6	6	6	6	5	5	5	5	5	5	5	5	5	5	4
-0.16	F	0.262	0.295	0.315	0.347	0.379	0.395	0.424	0.454	0.485	0.516	0.525	0.553	0.581	0.609	0.638	0.668	0.697	0.727	0.757
	T	8	8	7	7	7	6	6	6	6	6	5	5	5	5	5	5	5	5	5
-0.14	F	0.240	0.265	0.297	0.314	0.340	0.380	0.399	0.427	0.457	0.486	0.517	0.530	0.557	0.585	0.613	0.641	0.669	0.698	0.727
	T	9	8	8	7	7	7	6	6	6	6	6	5	5	5	5	5	5	5	5
-0.12	F	0.216	0.245	0.269	0.300	0.324	0.353	0.383	0.414	0.432	0.461	0.490	0.519	0.549	0.563	0.589	0.617	0.644	0.672	0.701
	T	10	9	8	8	7	7	7	7	6	6	6	6	6	5	5	5	5	5	5
-0.10	F	0.190	0.219	0.247	0.274	0.304	0.329	0.358	0.388	0.418	0.438	0.466	0.494	0.523	0.552	0.569	0.595	0.622	0.649	0.677
	T	11	10	9	8	8	7	7	7	7	6	6	6	6	6	5	5	5	5	5
-0.08	F	0.162	0.194	0.224	0.252	0.284	0.310	0.340	0.364	0.393	0.422	0.445	0.472	0.500	0.528	0.556	0.576	0.602	0.628	0.655
	T	12	11	10	9	8	8	8	7	7	7	6	6	6	6	6	5	5	5	5
-0.06	F	0.139	0.168	0.200	0.230	0.259	0.289	0.316	0.348	0.371	0.399	0.428	0.452	0.479	0.506	0.534	0.562	0.584	0.610	0.636
	T	14	12	11	10	9	9	8	8	7	7	7	6	6	6	6	6	5	5	5
-0.04	F	0.106	0.142	0.175	0.206	0.237	0.266	0.296	0.324	0.353	0.379	0.407	0.435	0.461	0.487	0.514	0.541	0.568	0.596	0.618
	T	18	15	13	11	10	9	9	8	8	7	7	7	6	6	6	6	6	6	5
-0.02	F	0.071	0.113	0.149	0.183	0.215	0.245	0.275	0.304	0.332	0.360	0.388	0.415	0.443	0.471	0.496	0.522	0.549	0.576	0.603
	T	27	19	15	13	12	10	10	9	8	8	7	7	7	7	6	6	6	6	6
0.00	F	0.000	0.079	0.121	0.157	0.191	0.223	0.254	0.283	0.312	0.341	0.369	0.397	0.424	0.451	0.479	0.505	0.531	0.558	0.584
	T	0	29	20	16	13	12	11	10	9	8	8	8	7	7	7	6	6	6	6
0.02	F	0.000	0.000	0.087	0.150	0.168	0.200	0.232	0.265	0.293	0.322	0.351	0.379	0.406	0.434	0.461	0.488	0.516	0.541	0.567
	T	0	0	31	21	16	14	12	11	10	9	9	8	8	7	7	7	6	6	6
0.04	F	0.000	0.000	0.000	0.098	0.148	0.176	0.210	0.242	0.273	0.303	0.333	0.360	0.389	0.416	0.445	0.471	0.498	0.526	0.552
	T	0	0	0	34	22	17	14	12	11	10	9	9	8	8	7	7	7	6	6
0.06	F	0.000	0.000	0.000	0.000	0.110	0.151	0.187	0.221	0.253	0.284	0.314	0.342	0.371	0.400	0.427	0.456	0.482	0.508	0.535
	T	0	0	0	0	37	23	18	15	13	11	10	10	9	8	8	7	7	7	7
0.08	F	0.000	0.000	0.000	0.000	0.124	0.164	0.200	0.233	0.265	0.295	0.325	0.354	0.383	0.412	0.438	0.465	0.494	0.519	0.549
	T	0	0	0	0	40	24	18	15	13	12	11	10	9	8	8	8	7	7	7
0.10	F	0.000	0.000	0.000	0.000	0.140	0.177	0.212	0.245	0.277	0.307	0.337	0.366	0.395	0.422	0.450	0.477	0.506	0.536	0.566
	T	0	0	0	0	44	26	19	16	13	12	11	10	9	9	8	8	7	7	7
0.12	F	0.000	0.000	0.000	0.000	0.157	0.192	0.226	0.259	0.290	0.320	0.350	0.379	0.408	0.434	0.463	0.489	0.519	0.549	0.579
	T	0	0	0	0	47	27	20	16	14	12	11	10	9	9	8	8	7	7	7
0.14	F	0.000	0.000	0.000	0.000	0.174	0.208	0.241	0.273	0.304	0.334	0.363	0.392	0.421	0.447	0.476	0.506	0.536	0.566	0.596
	T	0	0	0	0	50	28	21	17	14	12	11	10	9	9	8	8	7	7	7
0.16	F	0.000	0.000	0.000	0.000	0.195	0.225	0.257	0.288	0.318	0.348	0.378	0.406	0.433	0.461	0.489	0.519	0.549	0.579	0.609
	T	0	0	0	0	54	30	23	17	15	13	11	10	9	9	8	8	7	7	7
0.18	F	0.000	0.000	0.000	0.000	0.211	0.242	0.273	0.303	0.333	0.363	0.393	0.423	0.453	0.483	0.513	0.543	0.573	0.603	0.633
	T	0	0	0	0	57	31	22	18	15	13	11	10	9	9	8	8	7	7	7
0.20	F	0.000	0.000	0.000	0.000	0.230	0.260	0.290	0.320	0.350	0.380	0.410	0.440	0.470	0.500	0.530	0.560	0.590	0.620	0.650
	T	0	0	0	0	60	33	23	19	16	14	12	11	10	9	9	8	8	7	7

Table 7.2. S-Model, $W_0 = 0.15$

C	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36
H																			
-0.20	F	0.369	0.403	0.443	0.483	0.520	0.554	0.590	0.625	0.662	0.694	0.716	0.747	0.779	0.811	0.843	0.876	0.909	
	T	6	6	6	5	5	5	5	5	5	4	4	4	4	4	4	4	4	4
-0.18	F	0.351	0.369	0.405	0.441	0.479	0.487	0.520	0.553	0.587	0.622	0.657	0.686	0.716	0.747	0.778	0.809	0.841	0.872
	T	7	6	6	6	5	5	5	5	5	5	4	4	4	4	4	4	4	4
-0.16	F	0.313	0.351	0.371	0.405	0.441	0.477	0.489	0.521	0.554	0.587	0.620	0.655	0.689	0.718	0.748	0.778	0.809	0.840
	T	7	6	6	6	6	5	5	5	5	5	5	4	4	4	4	4	4	4
-0.14	F	0.289	0.315	0.351	0.374	0.407	0.442	0.477	0.495	0.524	0.555	0.588	0.620	0.654	0.687	0.721	0.751	0.780	0.810
	T	8	7	7	6	6	6	6	5	5	5	5	5	5	4	4	4	4	4
-0.12	F	0.253	0.290	0.318	0.353	0.378	0.411	0.444	0.478	0.497	0.527	0.558	0.590	0.622	0.654	0.687	0.725	0.754	0.783
	T	8	8	7	7	6	6	6	5	5	5	5	5	5	5	4	4	4	4
-0.10	F	0.225	0.257	0.293	0.322	0.356	0.385	0.415	0.447	0.480	0.502	0.532	0.562	0.593	0.624	0.656	0.688	0.702	0.730
	T	9	8	8	7	7	6	6	6	6	5	5	5	5	5	5	4	4	4
-0.08	F	0.194	0.229	0.265	0.297	0.328	0.361	0.395	0.420	0.452	0.484	0.509	0.538	0.568	0.598	0.628	0.659	0.690	0.709
	T	10	9	8	8	7	7	7	6	6	6	5	5	5	5	5	5	4	4
-0.06	F	0.163	0.200	0.235	0.271	0.303	0.334	0.366	0.399	0.424	0.457	0.489	0.521	0.544	0.573	0.603	0.633	0.663	0.716
	T	12	10	9	9	8	7	7	7	6	6	6	5	5	5	5	5	5	4
-0.04	F	0.128	0.169	0.207	0.245	0.277	0.309	0.341	0.372	0.405	0.433	0.464	0.494	0.526	0.552	0.580	0.609	0.638	0.698
	T	15	12	11	10	9	8	7	7	7	6	6	6	6	5	5	5	5	5
-0.02	F	0.086	0.134	0.174	0.213	0.249	0.283	0.316	0.349	0.380	0.411	0.441	0.471	0.501	0.532	0.560	0.588	0.616	0.674
	T	22	14	13	11	10	9	8	8	7	7	6	6	6	5	5	5	5	5
0.00	F	0.000	0.095	0.142	0.183	0.221	0.257	0.291	0.324	0.356	0.388	0.419	0.450	0.479	0.509	0.539	0.569	0.624	0.652
	T	0	24	16	13	11	10	9	8	8	7	7	6	6	6	5	5	5	5
0.02	F	0.000	0.000	0.101	0.150	0.192	0.230	0.265	0.300	0.333	0.365	0.397	0.427	0.458	0.488	0.517	0.546	0.605	0.633
	T	0	0	24	17	14	12	10	9	8	8	7	7	6	6	6	6	5	5
0.04	F	0.000	0.000	0.000	0.111	0.159	0.201	0.239	0.275	0.309	0.342	0.374	0.406	0.436	0.466	0.497	0.526	0.584	0.615
	T	0	0	0	27	18	14	12	10	9	8	7	7	6	6	6	6	6	5
0.06	F	0.000	0.000	0.000	0.122	0.170	0.211	0.249	0.284	0.318	0.351	0.384	0.415	0.446	0.476	0.507	0.536	0.564	0.593
	T	0	0	0	30	19	15	12	11	10	9	8	8	7	7	6	6	6	6
0.08	F	0.000	0.000	0.000	0.000	0.135	0.181	0.222	0.259	0.295	0.329	0.362	0.394	0.425	0.456	0.486	0.518	0.546	0.574
	T	0	0	0	0	32	20	15	15	11	10	9	8	8	7	7	6	6	6
0.10	F	0.000	0.000	0.000	0.000	0.000	0.149	0.194	0.233	0.271	0.306	0.340	0.375	0.405	0.435	0.467	0.496	0.525	0.557
	T	0	0	0	0	0	34	21	16	13	11	10	9	8	8	7	7	7	6
0.12	F	0.000	0.000	0.000	0.000	0.000	0.165	0.207	0.246	0.283	0.319	0.352	0.385	0.417	0.447	0.479	0.508	0.536	0.566
	T	0	0	0	0	0	37	22	16	13	11	10	9	8	8	7	7	7	7
0.14	F	0.000	0.000	0.000	0.000	0.000	0.181	0.222	0.260	0.296	0.330	0.364	0.397	0.427	0.459	0.491	0.519	0.549	0.579
	T	0	0	0	0	0	40	25	17	14	12	10	9	9	8	8	7	7	7
0.16	F	0.000	0.000	0.000	0.000	0.000	0.199	0.237	0.274	0.309	0.344	0.376	0.408	0.440	0.471	0.500	0.528	0.558	0.588
	T	0	0	0	0	0	42	24	17	14	12	11	10	9	9	8	8	8	8
0.18	F	0.000	0.000	0.000	0.000	0.000	0.217	0.253	0.289	0.324	0.358	0.390	0.421	0.453	0.484	0.513	0.543	0.574	0.604
	T	0	0	0	0	0	45	25	18	14	12	11	10	9	9	8	8	8	8
0.20	F	0.000	0.000	0.000	0.000	0.000	0.235	0.270	0.305	0.338	0.371	0.404	0.435	0.466	0.496	0.525	0.554	0.584	0.614
	T	0	0	0	0	0	47	26	19	14	12	11	10	9	9	8	8	8	8

G	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36
H																			
-0.20	F	0.436	0.447	0.486	0.524	0.546	0.605	0.647	0.638	0.672	0.708	0.743	0.779	0.815	0.852	0.889	0.927	0.965	1.003
T		6	5	5	5	5	5	5	6	6	6	6	6	6	6	6	6	6	6
-0.18	F	0.391	0.433	0.468	0.485	0.523	0.561	0.601	0.611	0.639	0.672	0.706	0.741	0.776	0.811	0.847	0.883	0.920	0.956
T		6	4	5	5	5	5	5	5	6	6	6	6	6	6	6	6	6	6
-0.16	F	0.351	0.391	0.411	0.450	0.486	0.523	0.560	0.598	0.636	0.660	0.673	0.707	0.740	0.774	0.809	0.844	0.879	0.914
T		6	6	6	5	5	5	5	5	5	5	6	6	6	6	6	6	6	6
-0.14	F	0.325	0.354	0.392	0.431	0.453	0.488	0.526	0.560	0.597	0.636	0.665	0.675	0.708	0.741	0.774	0.808	0.842	0.876
T		7	6	6	6	5	5	5	5	5	5	6	6	6	6	6	6	6	6
-0.12	F	0.287	0.327	0.357	0.394	0.432	0.457	0.491	0.526	0.561	0.597	0.633	0.647	0.679	0.710	0.743	0.775	0.808	0.841
T		7	7	6	6	6	5	5	5	5	5	5	6	6	6	6	6	6	6
-0.10	F	0.256	0.292	0.310	0.362	0.398	0.434	0.472	0.493	0.520	0.563	0.598	0.636	0.681	0.714	0.746	0.777	0.810	0.842
T		8	7	7	6	6	6	6	5	5	5	5	5	6	6	6	6	6	6
-0.08	F	0.222	0.260	0.301	0.334	0.373	0.402	0.438	0.475	0.500	0.533	0.567	0.601	0.635	0.658	0.688	0.719	0.749	0.781
T		9	8	8	7	7	6	6	6	5	5	5	5	5	6	6	6	6	6
-0.06	F	0.186	0.227	0.266	0.305	0.339	0.377	0.408	0.443	0.478	0.506	0.539	0.571	0.604	0.638	0.664	0.694	0.724	0.756
T		11	9	8	8	7	7	6	6	6	5	5	5	5	6	6	6	6	6
-0.04	F	0.146	0.192	0.234	0.273	0.310	0.345	0.382	0.414	0.448	0.483	0.513	0.545	0.577	0.609	0.642	0.675	0.700	0.730
T		13	11	10	9	8	7	7	6	6	6	5	5	5	5	5	5	6	6
-0.02	F	0.099	0.153	0.198	0.240	0.279	0.316	0.352	0.387	0.421	0.454	0.488	0.520	0.551	0.583	0.615	0.647	0.680	0.708
T		20	14	11	10	9	8	7	7	6	6	6	5	5	5	5	5	6	6
0.00	F	0.000	0.106	0.160	0.206	0.247	0.286	0.323	0.360	0.394	0.429	0.462	0.495	0.528	0.559	0.590	0.621	0.653	0.685
T		0	21	14	12	10	9	8	7	7	6	6	6	6	5	5	5	5	6
0.02	F	0.000	0.000	0.114	0.168	0.213	0.255	0.294	0.331	0.368	0.402	0.437	0.470	0.502	0.535	0.567	0.598	0.628	0.659
T		0	0	22	15	12	10	9	8	7	7	7	6	6	6	5	5	5	5
0.04	F	0.000	0.000	0.000	0.123	0.176	0.222	0.264	0.303	0.340	0.375	0.410	0.444	0.478	0.510	0.542	0.576	0.606	0.636
T		0	0	0	24	16	12	10	9	8	8	7	7	6	6	6	5	5	5
0.06	F	0.000	0.000	0.000	0.113	0.186	0.238	0.275	0.312	0.350	0.384	0.420	0.453	0.488	0.519	0.551	0.586	0.615	0.645
T		0	0	0	25	16	13	11	9	8	8	7	7	6	6	6	5	5	5
0.08	F	0.000	0.000	0.000	0.000	0.145	0.197	0.242	0.283	0.321	0.358	0.394	0.430	0.462	0.497	0.528	0.559	0.596	0.625
T		0	0	0	0	27	17	13	11	10	9	8	7	7	6	6	6	5	5
0.10	F	0.000	0.000	0.000	0.000	0.000	0.158	0.208	0.252	0.294	0.332	0.368	0.404	0.440	0.472	0.508	0.538	0.569	0.600
T		0	0	0	0	0	29	18	16	14	11	10	9	8	7	7	6	6	6
0.12	F	0.000	0.000	0.000	0.000	0.000	0.000	0.173	0.221	0.264	0.304	0.343	0.379	0.415	0.451	0.483	0.514	0.549	0.579
T		0	0	0	0	0	0	31	18	16	14	12	10	9	8	7	7	6	6
0.14	F	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.168	0.235	0.277	0.317	0.355	0.391	0.427	0.459	0.494	0.525	0.560
T		0	0	0	0	0	0	0	33	19	16	14	12	10	9	8	7	7	6
0.16	F	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.205	0.249	0.290	0.330	0.368	0.405	0.439	0.471	0.506	0.536
T		0	0	0	0	0	0	0	0	35	20	15	12	10	9	8	8	7	7
0.18	F	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.222	0.264	0.304	0.343	0.379	0.416	0.449	0.483	0.518
T		0	0	0	0	0	0	0	0	0	37	21	15	12	11	9	9	8	7
0.20	F	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.240	0.280	0.319	0.356	0.393	0.427	0.462	0.496	0.530
T		0	0	0	0	0	0	0	0	0	39	22	16	13	11	10	9	8	7

Table 7.4. S-Model, $W_0 = 0.25$

	0	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36
-20 F	0.463	0.506	0.551	0.596	0.596	0.634	0.672	0.712	0.751	0.792	0.832	0.874	0.913	0.958	0.923	0.957	0.991	1.023	1.059	1.039
T	5	5	5	5	5	4	4	4	4	4	4	4	4	4	3	3	3	3	3	3
-18 F	0.421	0.462	0.504	0.547	0.591	0.596	0.631	0.671	0.709	0.747	0.786	0.826	0.866	0.906	0.947	0.923	0.956	0.989	1.023	1.023
T	5	5	5	5	5	5	4	4	4	4	4	4	4	4	4	3	3	3	3	3
-16 F	0.398	0.423	0.463	0.503	0.545	0.587	0.598	0.634	0.671	0.708	0.745	0.783	0.821	0.860	0.899	0.939	0.924	0.956	0.989	0.989
T	6	5	5	5	5	5	5	4	4	4	4	4	4	4	4	4	3	3	3	3
-14 F	0.355	0.398	0.442	0.485	0.506	0.544	0.583	0.601	0.636	0.671	0.708	0.744	0.781	0.818	0.856	0.894	0.933	0.926	0.958	0.958
T	6	6	6	5	5	5	5	4	4	4	4	4	4	4	4	4	4	4	3	3
-12 F	0.323	0.357	0.399	0.442	0.467	0.506	0.545	0.584	0.604	0.639	0.673	0.709	0.744	0.780	0.817	0.854	0.891	0.928	0.929	0.929
T	7	6	6	6	5	5	5	5	5	4	4	4	4	4	4	4	4	4	3	3
-10 F	0.282	0.326	0.361	0.402	0.443	0.471	0.508	0.546	0.583	0.609	0.642	0.677	0.711	0.746	0.781	0.817	0.852	0.889	0.925	0.925
T	7	7	6	6	6	5	5	5	5	4	4	4	4	4	4	4	4	4	4	4
-08 F	0.246	0.291	0.329	0.366	0.405	0.443	0.476	0.512	0.549	0.587	0.625	0.663	0.701	0.748	0.783	0.818	0.853	0.889	0.925	0.925
T	8	7	7	6	6	6	5	5	5	4	4	4	4	4	4	4	4	4	4	4
-06 F	0.207	0.253	0.295	0.334	0.372	0.410	0.449	0.481	0.517	0.553	0.590	0.627	0.663	0.701	0.752	0.786	0.820	0.854	0.894	0.894
T	10	9	8	7	7	6	6	5	5	5	5	5	4	4	4	4	4	4	4	4
-04 F	0.163	0.212	0.258	0.300	0.339	0.380	0.415	0.453	0.488	0.522	0.558	0.593	0.630	0.669	0.691	0.723	0.756	0.789	0.823	0.823
T	12	10	9	8	7	7	6	6	5	5	5	5	4	4	4	4	4	4	4	4
-02 F	0.110	0.169	0.218	0.263	0.306	0.343	0.385	0.421	0.459	0.495	0.529	0.563	0.598	0.634	0.666	0.697	0.729	0.761	0.794	0.794
T	18	13	10	9	8	7	7	6	6	5	5	5	4	4	4	4	4	4	4	4
0.00 F	0.000	0.117	0.178	0.225	0.270	0.312	0.353	0.391	0.428	0.465	0.502	0.536	0.570	0.604	0.639	0.673	0.704	0.736	0.767	0.767
T	0	19	13	11	9	8	7	7	6	6	5	5	4	4	4	4	4	4	4	4
0.02 F	0.000	0.124	0.183	0.233	0.278	0.320	0.360	0.398	0.436	0.472	0.508	0.543	0.577	0.610	0.644	0.679	0.712	0.743	0.743	0.743
T	0	20	14	11	9	8	7	7	6	6	5	5	4	4	4	4	4	4	4	4
0.04 F	0.000	0.000	0.000	0.133	0.192	0.241	0.287	0.328	0.368	0.406	0.443	0.480	0.515	0.552	0.584	0.618	0.651	0.685	0.720	0.720
T	0	0	0	21	14	11	9	8	8	7	6	6	5	5	5	5	5	5	4	4
0.06 F	0.000	0.000	0.000	0.143	0.201	0.250	0.294	0.338	0.376	0.415	0.454	0.488	0.523	0.561	0.593	0.625	0.658	0.691	0.720	0.720
T	0	0	0	22	15	11	10	10	10	9	8	7	6	6	5	5	5	5	5	5
0.08 F	0.000	0.000	0.000	0.000	0.154	0.211	0.260	0.304	0.345	0.385	0.424	0.460	0.497	0.531	0.570	0.602	0.634	0.666	0.696	0.696
T	0	0	0	0	24	15	12	10	9	8	7	7	6	6	5	5	5	5	5	5
0.10 F	0.000	0.000	0.000	0.000	0.167	0.222	0.270	0.314	0.355	0.395	0.434	0.469	0.507	0.540	0.574	0.612	0.643	0.673	0.703	0.703
T	0	0	0	0	25	16	12	10	9	8	7	7	6	6	5	5	5	5	5	5
0.12 F	0.000	0.000	0.000	0.000	0.181	0.234	0.282	0.325	0.366	0.405	0.444	0.479	0.517	0.550	0.583	0.622	0.652	0.682	0.712	0.712
T	0	0	0	0	27	16	12	10	9	8	7	7	6	6	5	5	5	5	5	5
0.14 F	0.000	0.000	0.000	0.000	0.195	0.247	0.293	0.336	0.377	0.416	0.456	0.489	0.528	0.561	0.593	0.626	0.659	0.692	0.725	0.725
T	0	0	0	0	29	17	13	11	9	8	7	7	6	6	5	5	5	5	5	5
0.16 F	0.000	0.000	0.000	0.000	0.211	0.260	0.306	0.348	0.389	0.428	0.468	0.501	0.540	0.574	0.607	0.640	0.673	0.706	0.739	0.739
T	0	0	0	0	30	18	13	11	9	8	7	7	6	6	5	5	5	5	5	5
0.18 F	0.000	0.000	0.000	0.000	0.228	0.275	0.318	0.361	0.400	0.440	0.475	0.512	0.545	0.578	0.611	0.644	0.677	0.710	0.743	0.743
T	0	0	0	0	32	18	14	11	9	8	7	7	6	6	5	5	5	5	5	5
0.20 F	0.000	0.000	0.000	0.000	0.245	0.290	0.333	0.374	0.412	0.450	0.487	0.524	0.561	0.598	0.635	0.672	0.709	0.746	0.783	0.783
T	0	0	0	0	34	19	14	11	9	8	7	7	6	6	5	5	5	5	5	5

Table 7.5. S-Model, $W_0 = 0.50$

		G	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36
H.	-0.20	F	0.651	0.709	0.768	0.829	0.812	0.860	0.908	0.957	1.006	1.056	1.106	1.156	1.207	1.259	1.311	1.363	1.415	1.468	1.536
		T	4	4	4	4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2
	-0.18	F	0.592	0.647	0.703	0.760	0.818	0.811	0.857	0.904	0.952	0.999	1.048	1.096	1.145	1.195	1.245	1.295	1.346	1.397	1.448
		T	4	4	4	4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	-0.16	F	0.538	0.590	0.646	0.698	0.753	0.810	0.810	0.850	0.901	0.948	0.994	1.041	1.089	1.136	1.185	1.233	1.282	1.331	1.381
		T	4	4	4	4	4	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	-0.14	F	0.504	0.539	0.590	0.642	0.695	0.749	0.803	0.811	0.855	0.900	0.945	0.990	1.036	1.082	1.129	1.176	1.223	1.271	1.319
		T	5	4	4	4	4	4	4	3	3	3	3	3	3	3	3	3	3	3	3
	-0.12	F	0.444	0.503	0.542	0.591	0.642	0.693	0.745	0.798	0.812	0.856	0.899	0.943	0.988	1.032	1.078	1.123	1.169	1.215	1.261
		T	5	5	4	4	4	4	4	4	3	3	3	3	3	3	3	3	3	3	3
	-0.10	F	0.401	0.448	0.504	0.545	0.593	0.642	0.692	0.743	0.794	0.815	0.857	0.900	0.943	0.986	1.030	1.074	1.118	1.163	1.208
		T	6	5	5	4	4	4	4	4	4	3	3	3	3	3	3	3	3	3	3
	-0.08	F	0.343	0.403	0.451	0.505	0.549	0.596	0.644	0.692	0.742	0.792	0.818	0.859	0.901	0.943	0.986	1.029	1.072	1.115	1.159
		T	6	6	5	5	4	4	4	4	4	4	3	3	3	3	3	3	3	3	3
	-0.06	F	0.288	0.347	0.403	0.455	0.507	0.561	0.600	0.646	0.694	0.742	0.790	0.822	0.863	0.904	0.945	0.987	1.029	1.071	1.113
		T	7	6	6	5	5	5	4	4	4	4	4	3	3	3	3	3	3	3	3
	-0.04	F	0.228	0.293	0.353	0.409	0.459	0.510	0.563	0.604	0.650	0.696	0.743	0.790	0.827	0.867	0.907	0.947	0.988	1.029	1.071
		T	9	7	6	6	5	5	5	4	4	4	4	4	3	3	3	3	3	3	3
	-0.02	F	0.155	0.233	0.300	0.359	0.413	0.465	0.513	0.565	0.609	0.654	0.699	0.745	0.791	0.832	0.871	0.911	0.951	0.991	1.031
		T	15	9	8	7	6	5	5	5	4	4	4	4	4	3	3	3	3	3	3
	0.00	F	0.000	0.161	0.240	0.305	0.344	0.418	0.471	0.519	0.569	0.615	0.659	0.703	0.748	0.793	0.838	0.877	0.915	0.955	0.994
		T	0	14	9	8	7	6	5	5	5	4	4	4	4	4	3	3	3	3	3
	0.02	F	0.000	0.000	0.168	0.246	0.311	0.370	0.424	0.478	0.525	0.573	0.622	0.665	0.708	0.752	0.796	0.841	0.883	0.921	0.959
		T	0	0	14	10	8	7	6	5	5	5	4	4	4	4	4	4	3	3	3
	0.04	F	0.000	0.000	0.000	0.175	0.253	0.316	0.377	0.431	0.485	0.532	0.579	0.629	0.671	0.713	0.755	0.799	0.843	0.889	0.927
		T	0	0	0	15	10	8	7	6	5	5	5	4	4	4	4	4	4	3	3
	0.06	F	0.000	0.000	0.000	0.000	0.184	0.262	0.326	0.384	0.439	0.489	0.539	0.585	0.637	0.678	0.719	0.761	0.804	0.847	0.896
		T	0	0	0	0	15	10	8	7	6	6	5	5	4	4	4	4	4	4	3
	0.08	F	0.000	0.000	0.000	0.000	0.000	0.193	0.270	0.335	0.392	0.447	0.494	0.546	0.591	0.637	0.683	0.726	0.767	0.809	0.851
		T	0	0	0	0	0	16	11	8	7	6	6	5	5	4	4	4	4	4	4
	0.10	F	0.000	0.000	0.000	0.000	0.000	0.000	0.204	0.279	0.344	0.401	0.454	0.503	0.555	0.599	0.644	0.693	0.733	0.774	0.815
		T	0	0	0	0	0	0	17	11	8	7	6	6	5	5	5	4	4	4	4
	0.12	F	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.215	0.289	0.352	0.410	0.465	0.512	0.563	0.607	0.651	0.702	0.741	0.781
		T	0	0	0	0	0	0	0	18	11	9	7	6	6	5	5	5	4	4	4
	0.14	F	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.228	0.300	0.362	0.421	0.470	0.521	0.573	0.615	0.658	0.711	0.750
		T	0	0	0	0	0	0	0	0	19	11	9	7	7	6	5	5	5	4	4
	0.16	F	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.241	0.311	0.372	0.427	0.479	0.530	0.583	0.624	0.667	0.720
		T	0	0	0	0	0	0	0	0	0	19	12	9	8	7	6	5	5	5	4
	0.18	F	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.255	0.323	0.384	0.438	0.489	0.540	0.593	0.634	0.674	0.714
		T	0	0	0	0	0	0	0	0	0	20	12	9	8	7	6	5	5	5	5
	0.20	F	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.270	0.335	0.394	0.449	0.500	0.551	0.594	0.644	0.684	0.724
		T	0	0	0	0	0	0	0	0	0	21	13	10	8	7	6	5	5	5	5

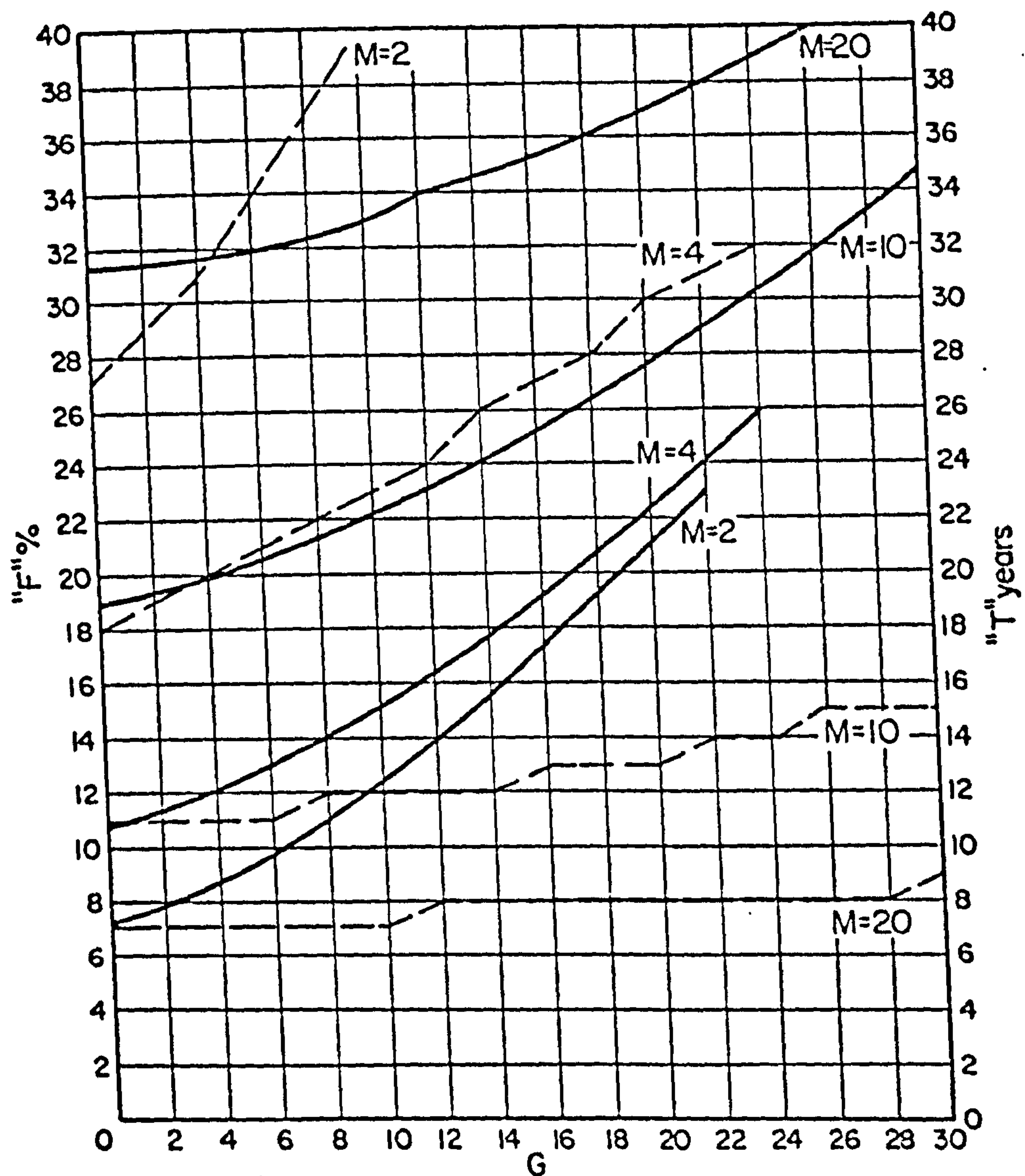


Fig. 7.2 S-Model.

$$\text{Initial} \frac{\text{Operating cost}}{\text{Capital outlay}} = 10 \text{ per cent}$$

————— F = Initial year per cent earnings on capital
 ----- T = Life of investment

G per cent = Sum of DCF return rate plus technological change rate minus wages increase rate

M per cent = Sum of technological change rate plus operating deterioration rate

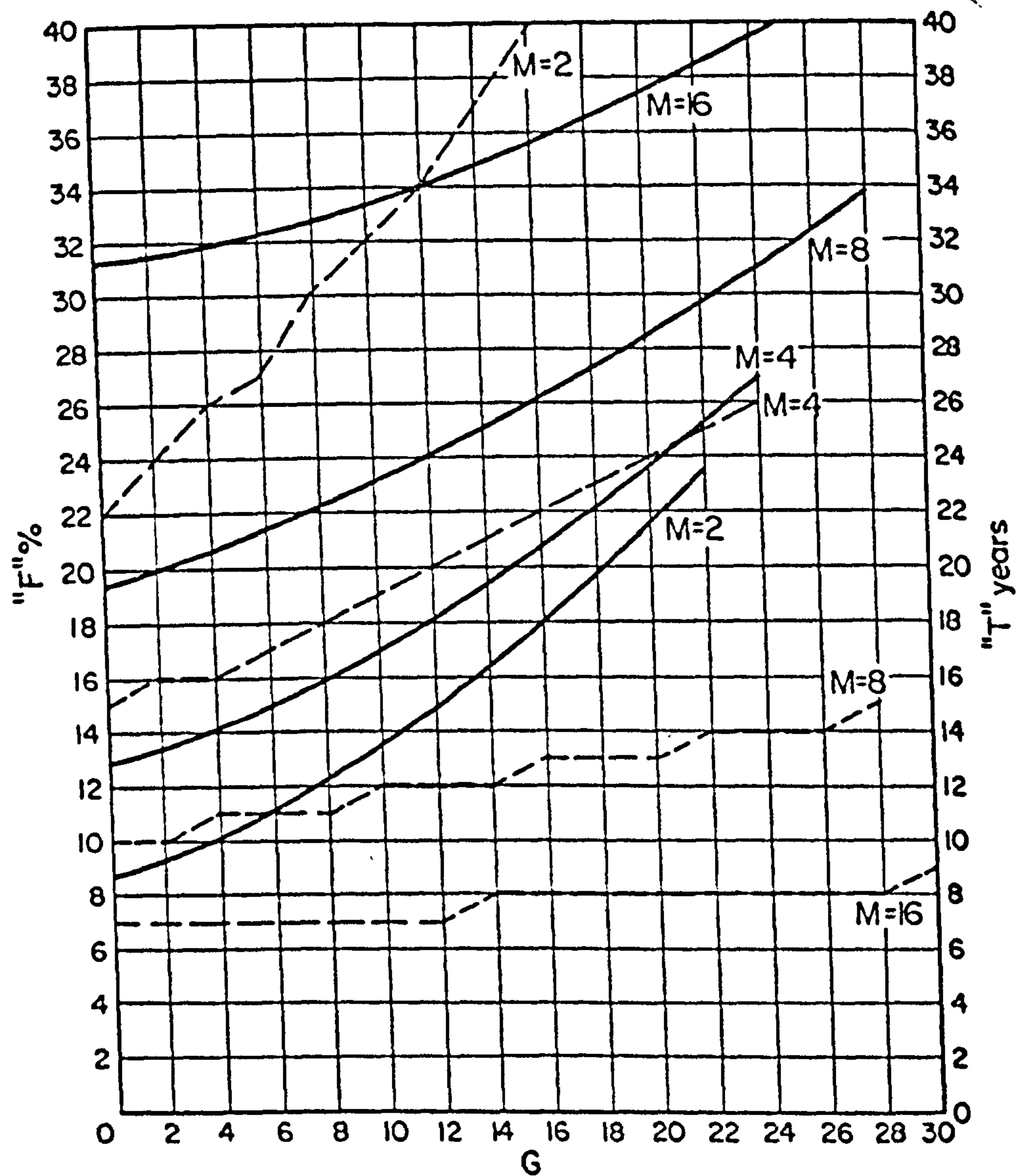


Fig. 7.3 S-Model

Initial $\frac{\text{Operating cost}}{\text{Capital outlay}} = 15$ per cent

————— F = Initial year per cent earnings on capital
 ----- T = Life of investment

G per cent = Sum of DCF return rate plus technological change rate minus wages increase rate

M per cent = Sum of technological change rate plus operating deterioration rate

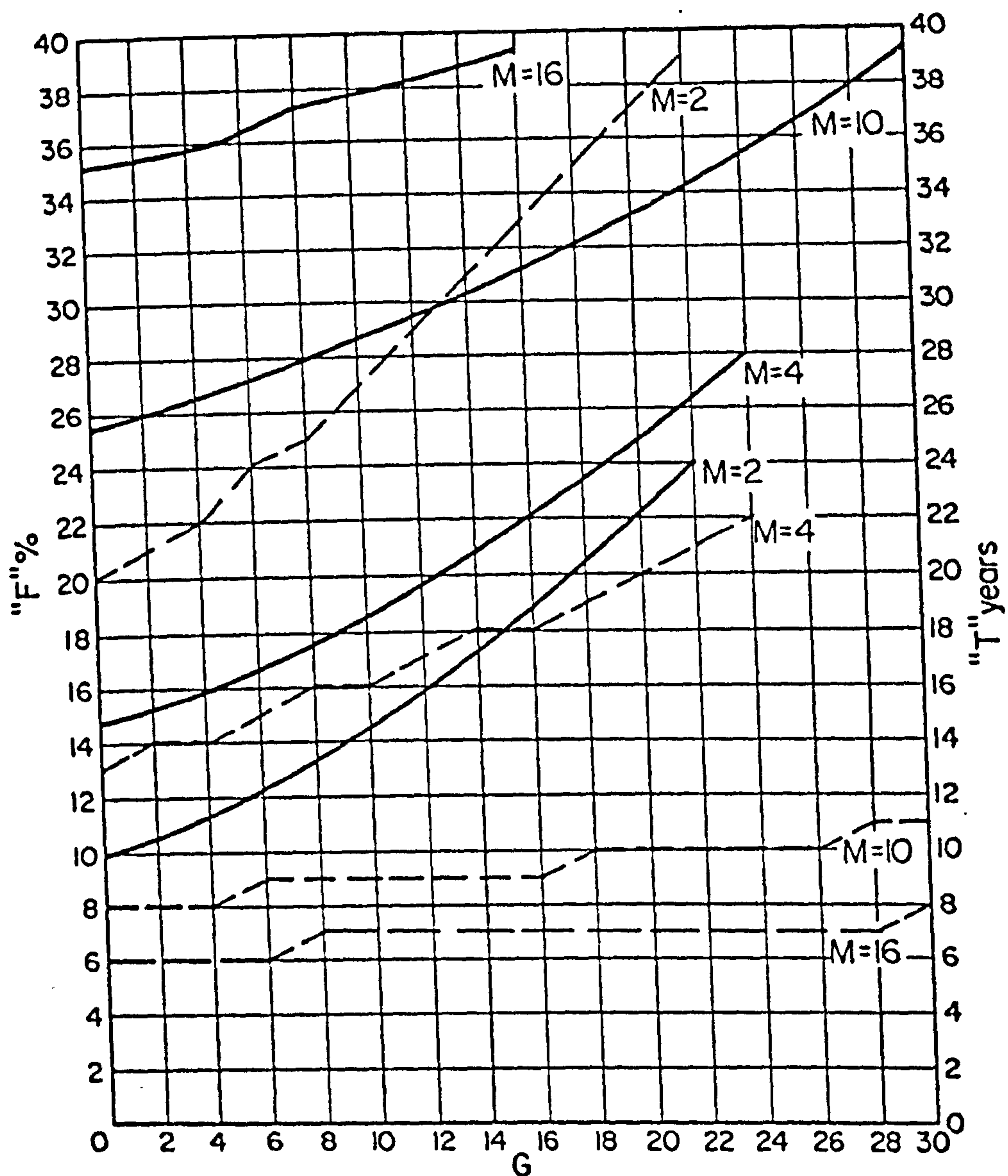


Fig. 7.4 S-Model

$$\text{Initial } \frac{\text{Operating cost}}{\text{Capital outlay}} = 20 \text{ per cent}$$

- F = Initial year per cent earnings on capital
- T = Life of investment
- G per cent = Sum of DCF return rate plus technological change rate minus wages increase rate
- M per cent = Sum of technological change rate plus operating deterioration rate

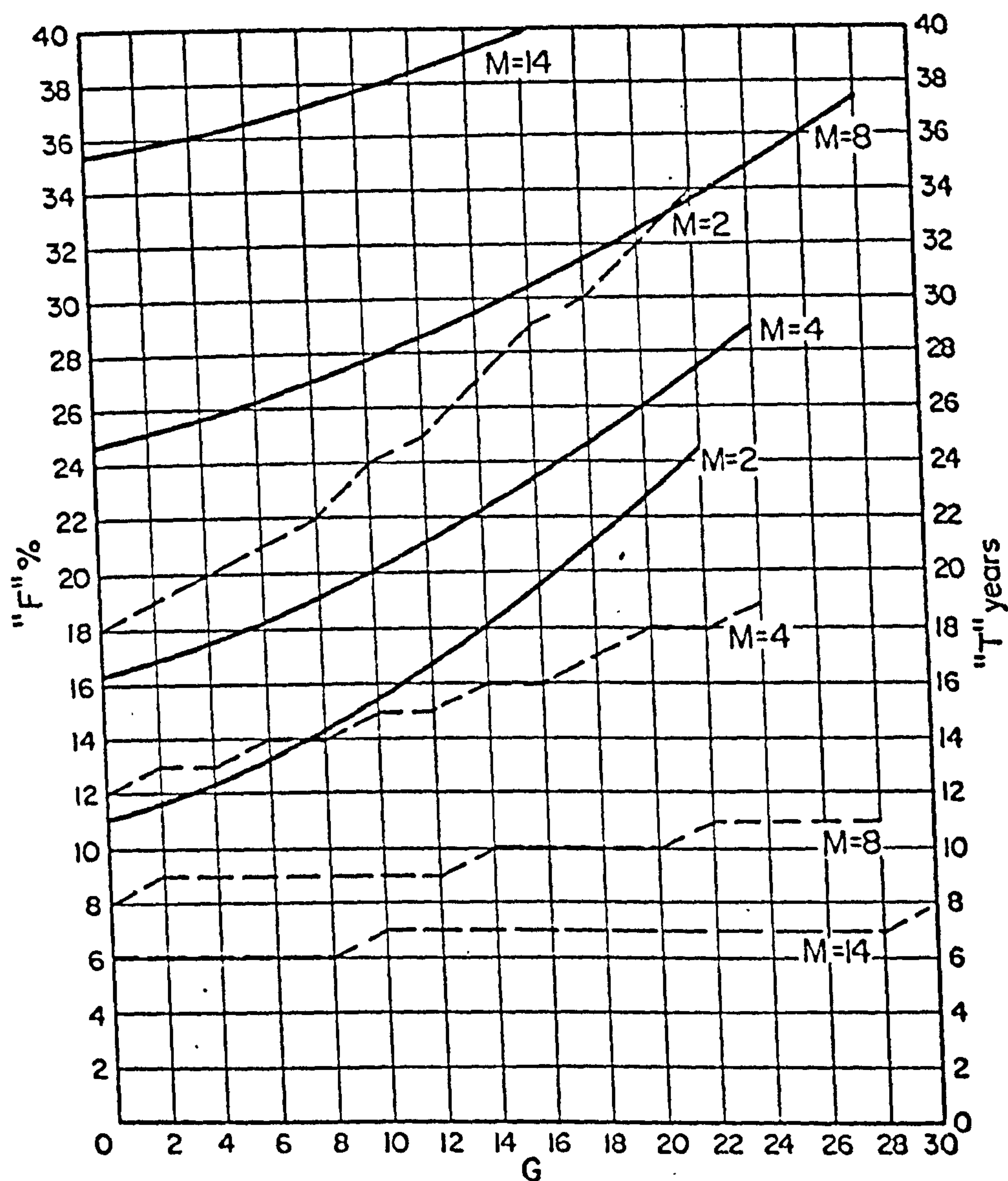


Fig. 7.5 S-Model

Initial $\frac{\text{Operating cost}}{\text{Capital outlay}} = 25$ per cent

- F = Initial year per cent earnings on capital
- T = Life of investment
- G per cent = Sum of DCF return rate plus technological change rate minus wages increase rate
- M per cent = Sum of technological change rate plus operating deterioration rate

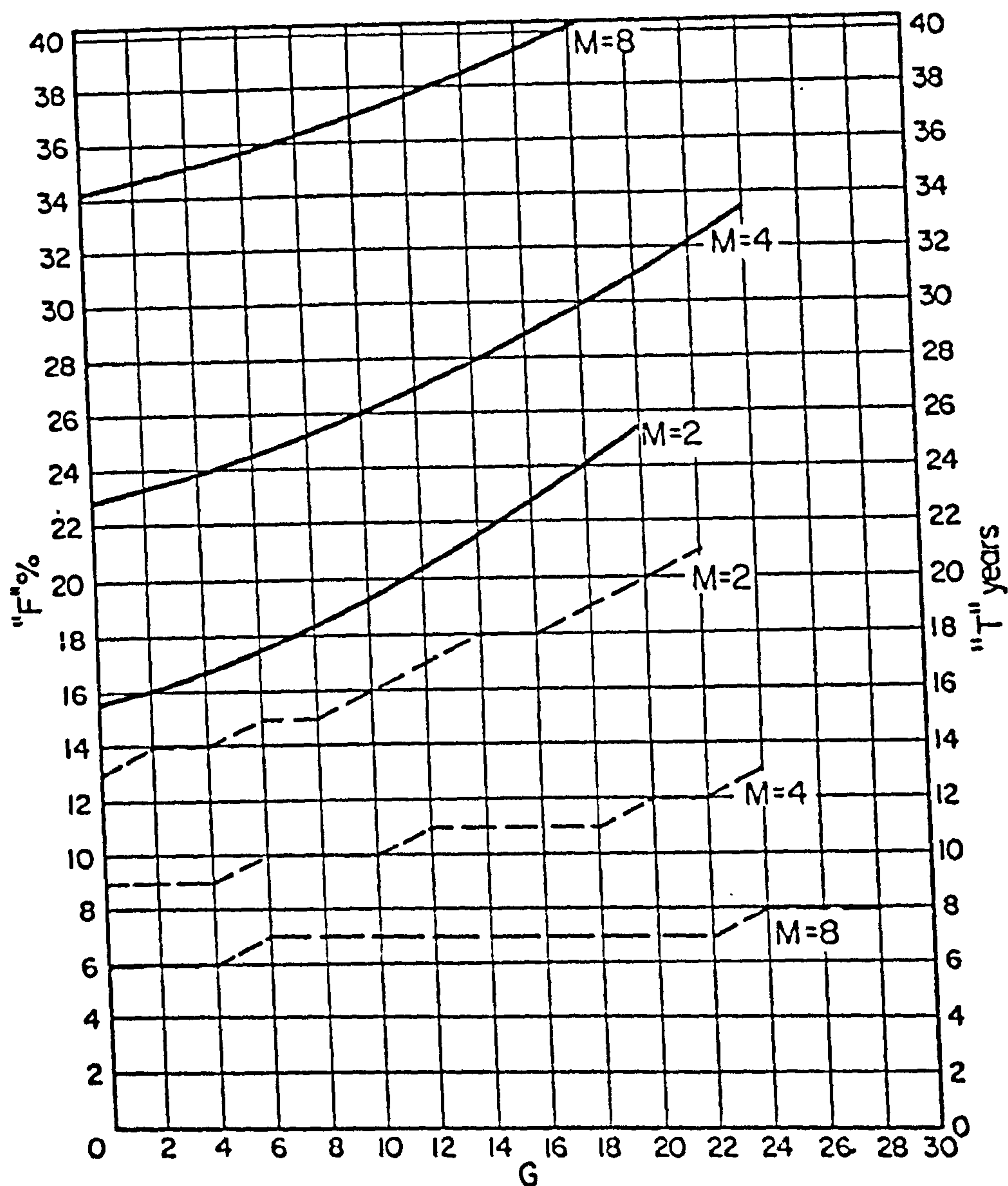


Fig. 7.6 S-Model.

Initial $\frac{\text{Operating cost}}{\text{Capital outlay}} = 50$ per cent

- F = Initial year per cent earnings on capital
- T = Life of investment
- G per cent = Sum of DCF return rate plus technological change rate minus wages increase rate
- M per cent = Sum of technological change rate plus operating deterioration rate

Appendix 7.1: Computer Solution for the Original Model
(Continuous Mode)

The Computer Program INVS2 that I wrote for the solution of the original model is given here. Also in this Appendix are Tables for W_0 values of 0.10, 0.25 and 0.50 (Tables 7.6 - 7.8) and a Chart comparing the results of the continuous and discrete modes for $W_0 = 0.10$. (Figure 7.7).

FORTTRAN PROGRAM - INVS2

```

MASTER INVS2
DIMENSION W0(7),PR(19),I1(19),OK1(19),OK2(21)
DATA W0/.01,.02,.05,.10,.25,.50,1.00 /
WRITE (2,5)
5  FORMAT(1H1,30X,41HINVESTMENT AND TECHNOLOGICAL CHANGE MODEL/
C 1H0,30X,54HCALCULATION OF FIRST YEAR EARNINGS AND INVESTMENT LIFE
C/31X,60HFOR VARIOUS PROPORTIONS OF INITIAL YEARLY WAGES TO CAPITAL
C OUTLAY / 1H0,30X,21HCAPITAL OUTLAY = 1.0 )
DO 3 I = 1,19
3  OK1(I) = (I-1)*2 / 100.
DO 4 L = 1,21
4  OK2(L) = (L+2 -22) / 100.
DO 7 IA = 1,7
WRITE (2,6) IA, W0(IA),(OK1(I),I=1,19 )
6  FORMAT ( 1H1,10X,7HTABLE ,12,26H INITIAL WAGE PROPORTION ,F4.2/
C 1H0,3X24K1,14(2X,F4.2)/4H0 K2)
DO 8 K2 = 1,21
DO 9 K1 = 1,19
IF(K1+10-K2) 140,140,10
10 IF(K2-11)12,120,12
12 IF(K1-1)13,130,110
140 PR(K1) = .0
IT(K1) = 0
GO TO 9
110 ITT=1
X1=EXP(-OK1(K1))
X2=EXP(-OK2(K2))
111 P0=OK1(K1)*(1.+W0(IA)+(1.-X2)/OK2(K2))/(1.-X1)
ITT=ITT+1
IF(ITT-100)112,112,19
112 X1=EXP(-ITT*OK1(K1))
X2=EXP(- ITT*OK2(K2))
IF(P0*X1-W0(IA)*X2)19,19,111
120 ITT=1
X1=EXP(-OK1(K1))
121 P0=OK1(K1)*(1.+W0(IA)+ITT)/(1.-X1)
ITT=ITT+1
IF(ITT-100)122,122,19
122 X1=EXP(-ITT*OK1(K1))
IF(P0*X1-W0(IA) )19,19,121
130 ITT=1
X2=EXP(-OK2(K2))
131 P0= (1.+W0(IA)+(1.-X2)/OK2(K2))/ITT
ITT=ITT+1
IF(ITT-100)132,132,19
132 X2=EXP(-ITT*OK2(K2))
IF(P0 -W0(IA)*X2)19,19,131
19 PR (K1) = P0-W0(IA)
IT(K1)=ITT-1
9 CONTINUE
WRITE (2,17) OK2(K2) ,(PR(N),N=1,19)
WRITE (2,18) (IT(N),N=1,19)
17 FORMAT(1H0,F4.2,2H I,19(1X,F5.3) )
18 FORMAT (5X,1HT,19(1X,15,2X))
8 CONTINUE
7 CONTINUE
13 STOP
END

```

END OF SEG-IF IT. LENGTH 500, NAME INVS2

Table 7.7. S-Model, Continuous Mode, $W_0 = 0.25$

	0	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36
-0.24	F	0.328	0.439	0.435	0.462	0.519	0.532	0.555	0.578	0.601	0.625	0.649	0.673	0.698	0.723	0.748	0.773	0.843	0.863
-0.28	F	0.355	0.461	0.457	0.484	0.541	0.554	0.577	0.600	0.623	0.646	0.669	0.692	0.715	0.738	0.761	0.784	0.854	0.874
-0.16	F	0.433	0.539	0.535	0.562	0.619	0.632	0.655	0.678	0.701	0.724	0.747	0.770	0.793	0.816	0.839	0.862	0.932	0.952
-0.14	F	0.512	0.618	0.614	0.641	0.698	0.711	0.734	0.757	0.780	0.803	0.826	0.849	0.872	0.895	0.918	0.941	1.011	1.031
-0.12	F	0.591	0.697	0.693	0.720	0.777	0.790	0.813	0.836	0.859	0.882	0.905	0.928	0.951	0.974	0.997	1.020	1.090	1.110
-0.10	F	0.670	0.776	0.772	0.800	0.857	0.870	0.893	0.916	0.939	0.962	0.985	1.008	1.031	1.054	1.077	1.100	1.170	1.190
-0.08	F	0.749	0.855	0.851	0.879	0.936	0.949	0.972	0.995	1.018	1.041	1.064	1.087	1.110	1.133	1.156	1.179	1.249	1.269
-0.06	F	0.828	0.934	0.930	0.958	1.015	1.028	1.051	1.074	1.097	1.120	1.143	1.166	1.189	1.212	1.235	1.258	1.328	1.348
-0.04	F	0.907	1.013	1.009	1.037	1.094	1.107	1.130	1.153	1.176	1.199	1.222	1.245	1.268	1.291	1.314	1.337	1.407	1.427
-0.02	F	0.986	1.092	1.088	1.116	1.173	1.186	1.209	1.232	1.255	1.278	1.301	1.324	1.347	1.370	1.393	1.416	1.486	1.506
0.00	F	1.065	1.171	1.167	1.195	1.252	1.265	1.288	1.311	1.334	1.357	1.380	1.403	1.426	1.449	1.472	1.495	1.565	1.585
0.02	F	1.144	1.250	1.246	1.274	1.331	1.344	1.367	1.390	1.413	1.436	1.459	1.482	1.505	1.528	1.551	1.574	1.644	1.664
0.04	F	1.223	1.329	1.325	1.353	1.410	1.423	1.446	1.469	1.492	1.515	1.538	1.561	1.584	1.607	1.630	1.653	1.723	1.743
0.06	F	1.302	1.408	1.404	1.432	1.489	1.502	1.525	1.548	1.571	1.594	1.617	1.640	1.663	1.686	1.709	1.732	1.802	1.822
0.08	F	1.381	1.487	1.483	1.511	1.568	1.581	1.604	1.627	1.650	1.673	1.696	1.719	1.742	1.765	1.788	1.811	1.881	1.901
0.10	F	1.460	1.566	1.562	1.590	1.647	1.660	1.683	1.706	1.729	1.752	1.775	1.798	1.821	1.844	1.867	1.890	1.960	1.980
0.12	F	1.539	1.645	1.641	1.669	1.726	1.739	1.762	1.785	1.808	1.831	1.854	1.877	1.900	1.923	1.946	1.969	2.039	2.059
0.14	F	1.618	1.724	1.720	1.748	1.805	1.818	1.841	1.864	1.887	1.910	1.933	1.956	1.979	2.002	2.025	2.048	2.118	2.138
0.16	F	1.697	1.803	1.799	1.827	1.884	1.897	1.920	1.943	1.966	1.989	2.012	2.035	2.058	2.081	2.104	2.127	2.197	2.217
0.18	F	1.776	1.882	1.878	1.906	1.963	1.976	1.999	2.022	2.045	2.068	2.091	2.114	2.137	2.160	2.183	2.206	2.276	2.296
0.20	F	1.855	1.961	1.957	1.985	2.042	2.055	2.078	2.101	2.124	2.147	2.170	2.193	2.216	2.239	2.262	2.285	2.355	2.375

Table 7.8. S-Model, Continuous Mode, $W_0 = 0.50$

M																									
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.026	0.028	0.030	0.032	0.034	0.036							
-0.040	0.002	0.004	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020	0.022	0.024	0.0												

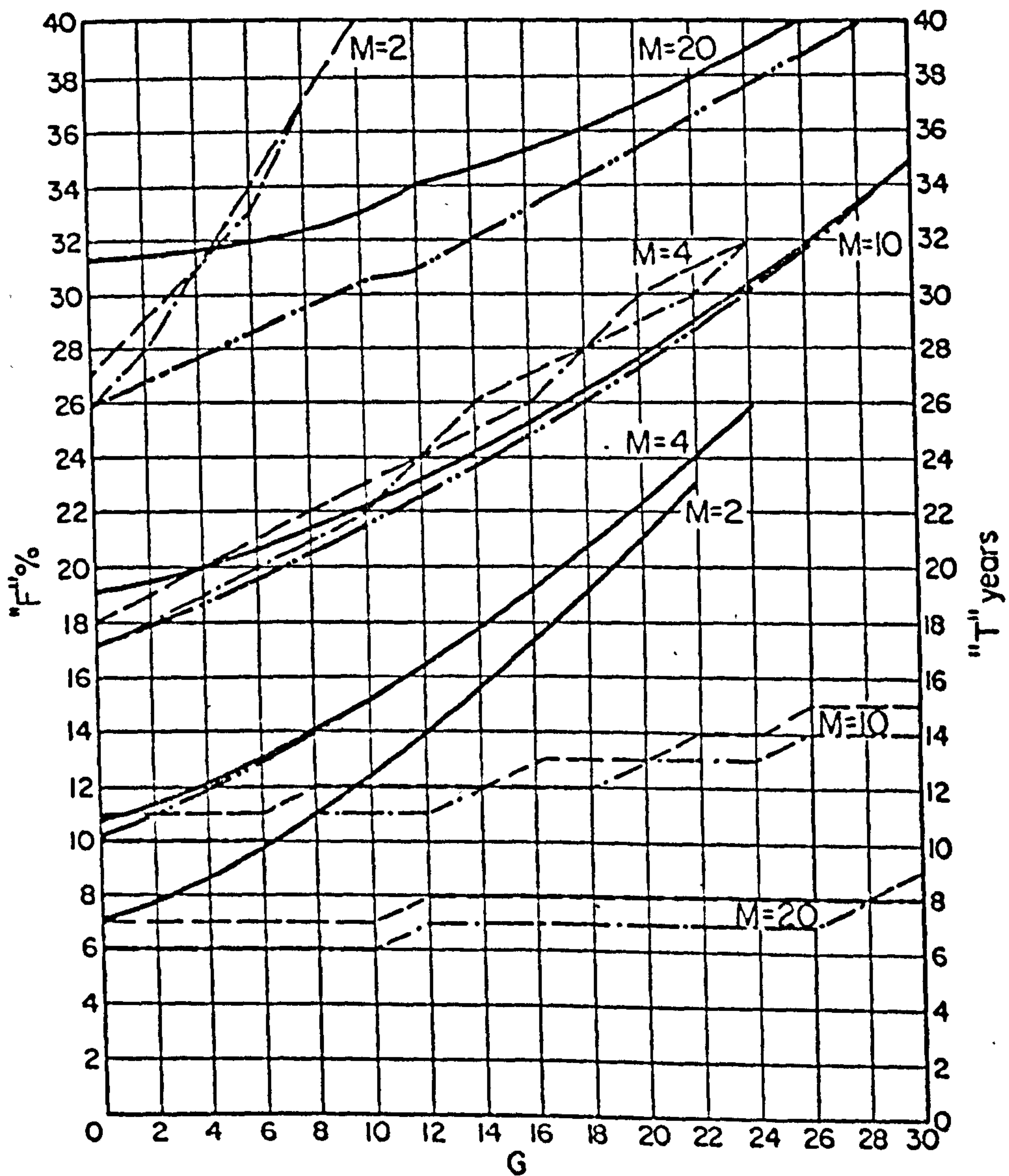


Fig. 7.7 S-Model, Comparison of Results: Continuous and Discrete Modes
Initial $\frac{\text{Operating cost}}{\text{Capital outlay}} = 10$ per cent

- F = Initial year per cent earnings on capital (discrete)
- T = Life of investment (discrete)
- .. — F = Initial per cent earnings on capital (continuous)
- . — T = Life of investment (continuous)
- G per cent = Sum of DCF return rate plus technological change rate minus wages increase rate
- M per cent = Sum of technological change rate plus operating deterioration rate

Appendix 7.2: Alternatives to the Graphical Presentation

There are many ways of giving the results a graphical-visual presentation. Here we present a few additional ways to the one used in the main body of this chapter. Each one of these alternatives has a few obvious advantages, but also, and that is not less obvious, some disadvantages as well.

The inability of any single two dimensional presentation to cover all the information in an easy way stems from the number of parameters that the S-method covers. Even if we ignored T - "the length of life" as being a by-product, we would be left with 4 explicit variables - F, W_0 , G, M (related to £1.00 initial capital outlay). In fact, we try to reduce a 4 dimensional space into 2 dimensions. With limited graphical sophistication (no use of colour or semi-transparent paper) we had to "divide" the values of one parameter into a number of separate charts. Each chart is plotted for one single value of that parameter. This division is, therefore, the "Charts dimension". In the main presentation this parameter is W_0 . In each chart we have one parameter on the horizontal "X axis" and one on the vertical "Y axis" (in the main presentation these are the parameters "G" and "F", respectively). The last parameter, the "free-axis" is "imposed" on each chart in the form of isoquants. Each graph represents a single value of that parameter ("M" in the main presentation). These lines are comparable, e.g. to barometric pressure lines (isobars) drawn on a geographical map. The variable T in the main presentation is used in an analogous way to the variable F and the plane of the graph in fact covers that dimension as well.

The charts given in this Appendix are:-

	<u>X Axis</u>	<u>Y Axis</u>	<u>Free Axis</u>	<u>Charts Axis</u>
Chart 1: (Figure 7.8)	G	F, T	W_o	M (for M = 4)
Chart 2: (Figure 7.9)	W_o	F, T	G	M (for M = 4)
Chart 3: (Figure 7.10)	W_o	F, T	M	G (for G = 10)

More complications arise if we try to divide the parameter G into "r" and "b - a". Though the division has great intuitive attraction it is proved impractical because it complicates the process of finding the 'answer'. It seems clear that an early calculation of G as a sum of "r" and "b - a" before using the charts is much more practical and, therefore, recommended.

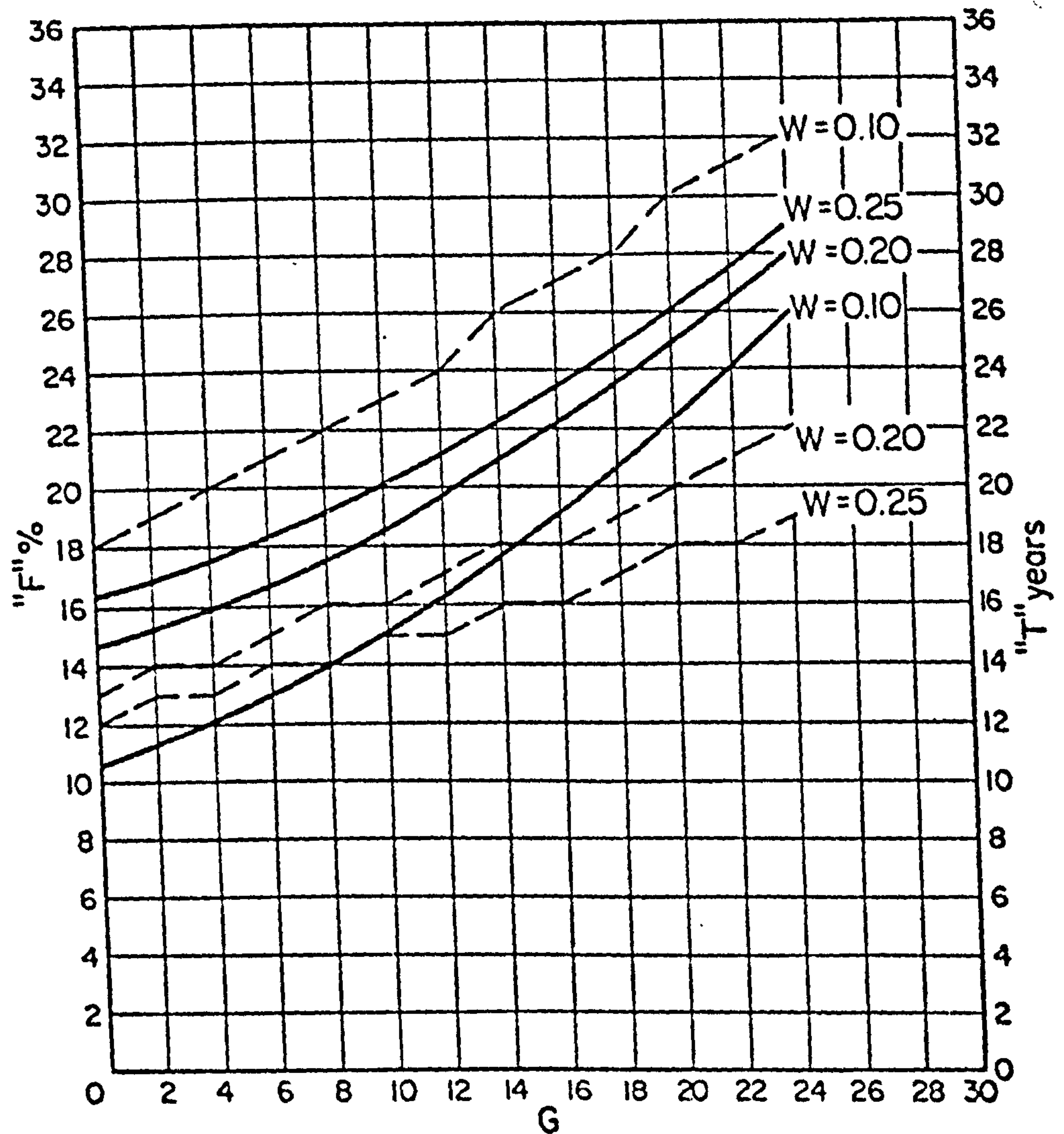


Fig. 7.8 S-Model

M (technical change rate + deterioration rate) = 4 per cent.

———— F = Initial year per cent earnings on capital

----- T = Life of investment

G per cent = Sum of DCF return rate plus technological change rate minus wages increase rate

W = Initial $\frac{\text{Operating cost}}{\text{Capital outlay}}$

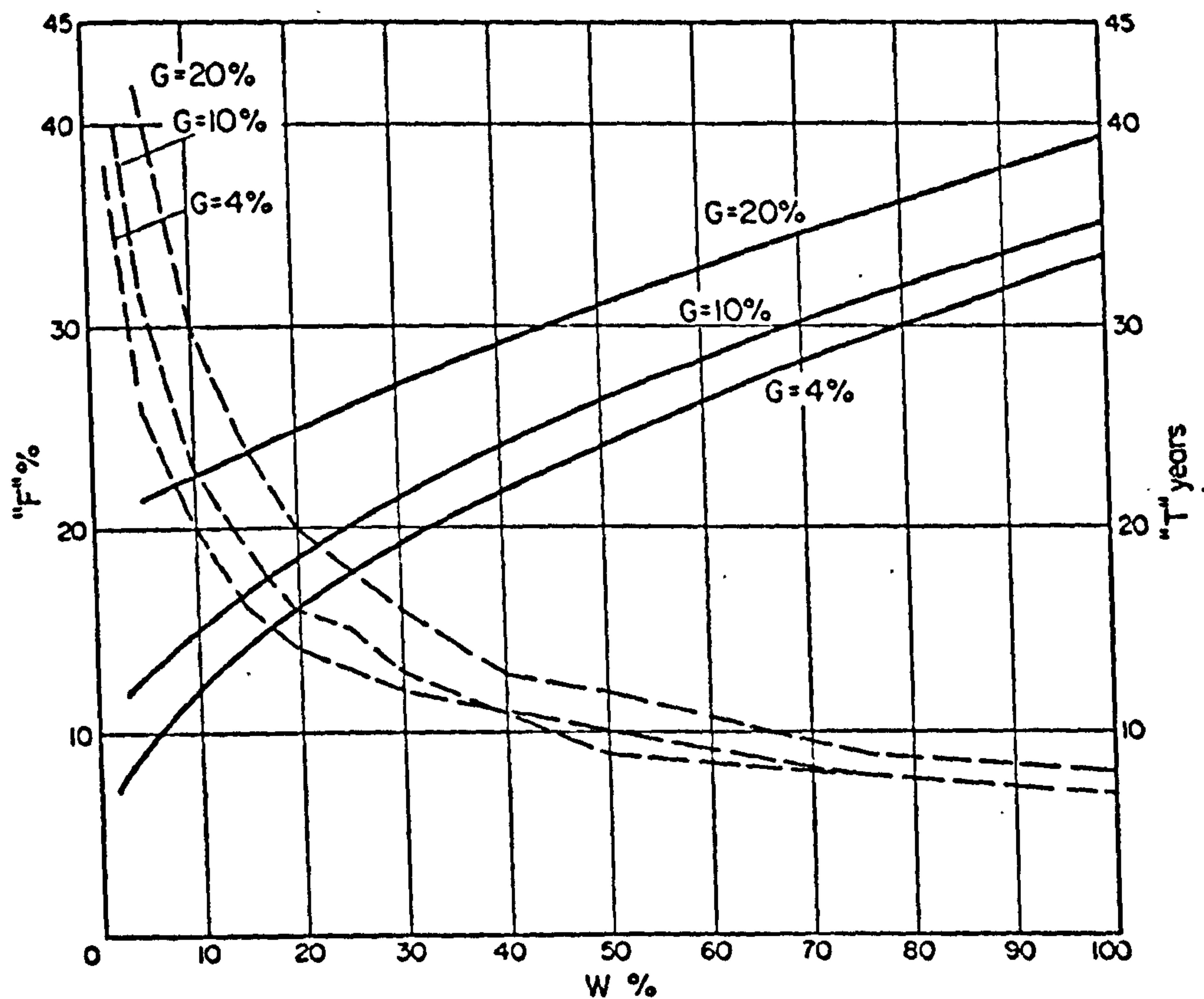


Fig 7.9 S-Model

M (technical change rate + deterioration rate) = 4 per cent

————— F = Initial year per cent earnings on capital

----- T = Life of investment

G per cent = Sum of DCF return rate plus technological change rate minus wages increase rate

$$\therefore W = \text{Initial} \frac{\text{Operating cost}}{\text{Capital outlay}}$$

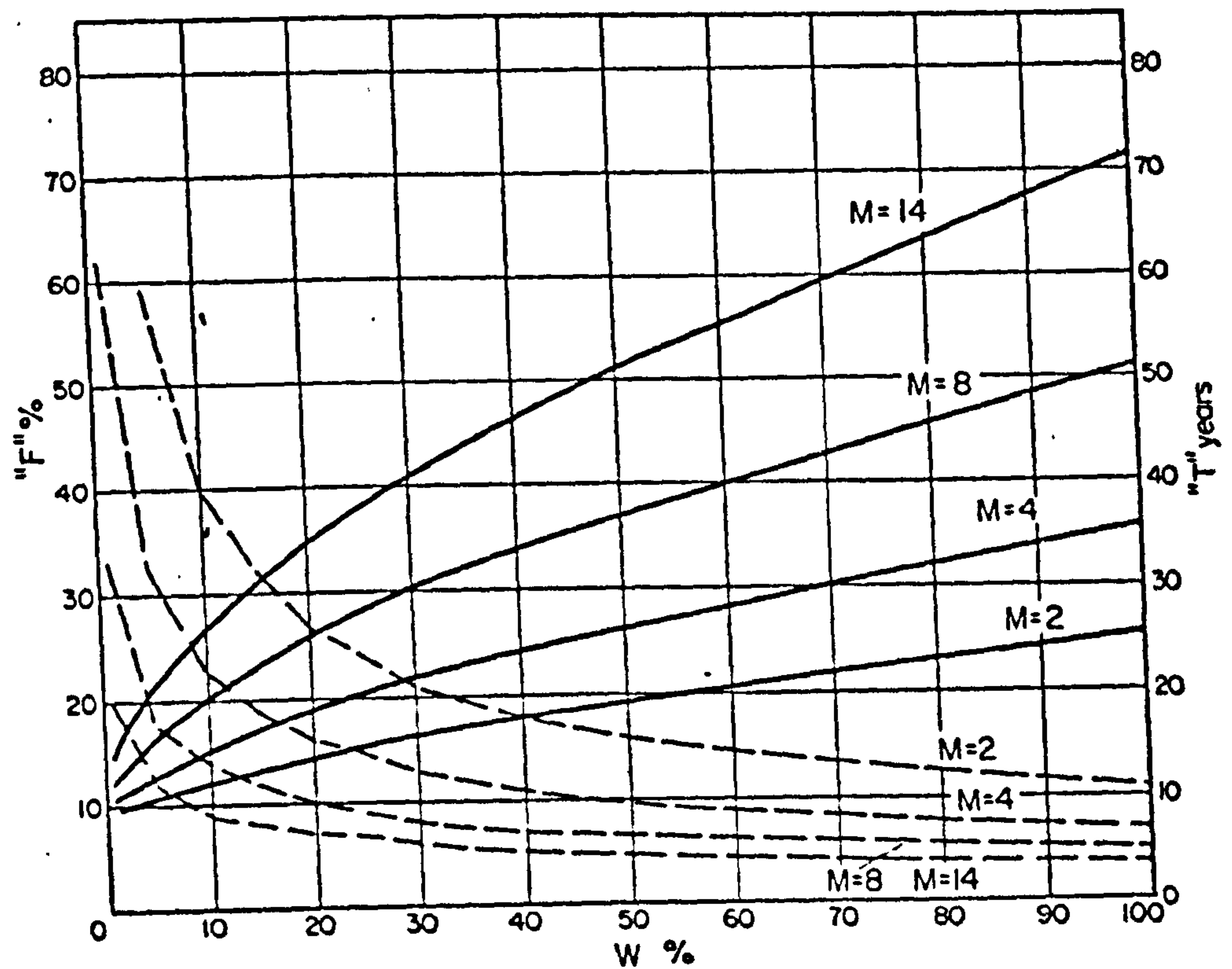


Fig. 7.10 S-Model

For $G = 10$ per cent

———— F = Initial year per cent earnings on capital

----- T = Life of investment

$W = \text{Initial } \frac{\text{Operating cost}}{\text{Capital outlay}}$

$M = \text{Sum of technological change rate plus operating deterioration rate}$
 $(M = b + c)$

Appendix 7.3: The Length of Life as a Function of M (=b+c), and $\frac{P_o}{W_o}$

Though the calculation of the lengths of life is not a target in itself in the applications of the model, it is possible to calculate the length of life simply as a function of "b + c" and $\frac{P_o}{W_o}$.

This derives from Equation(6) Chapter 6:-

$$P_o \cdot e^{(a-b)T} = W_o \cdot e^{(a+c)T},$$

from where we obtain:-

$$T = \frac{1}{b+c} \cdot \ln \frac{P_o}{W_o}$$

(see equation (7) in Chapter 6)

Table 7.9 below gives the length of life as a function of "b+c" and $\frac{P_o}{W_o}$. Some insight into the application of the S-method can be gained from looking at the figures. (Note that lengths of life of 100 years or more are given the value 99.9. This has been done to simplify the tabulation).

FORTTRAN PROGRAM - LIFE

```

MASTER LIFE
DIMENSION A1(20),B1(20),SHOP(20)
DO 14 JK=0,20,20
DO 7 I=1,20
  A1(I) = (I+JK)/100.
  B1(I) = 1./A1(I)
7 CONTINUE
DO 13 L=1,5,4
  WRITE (2,8) (A1(I),I=1,20)
8 FORMAT (40H1      CALCULATION OF LENGTH OF LIFE /1H0,6X,
  C 4HB+C ,20(1X,F4.2)/7H0 PD/W0,105X,8H PD/W0/1X )
  AL=L/10.
  XL= L
  DO 12 J1=1,40
  XL= XL+AL
  Y= ALOG(XL)
  DO 11 K=1,20
  SHP = Y*31(K)
  IF(SHP - 99.9) 10,10,9
9 SHP = 99.9
10 SHOP(K) = SHP
11 CONTINUE
  WRITE (2,6) XL,(SHOP(K),K=1,20),XL
  FORMAT (2H ,F5.2,4X,20(1X ,F4.1),3X,F5.2 )
12 CONTINUE
13 CONTINUE
14 CONTINUE
  STOP
  END

```

END OF SEGMENT, LENGTH 188, NAME LIFE

Table 9. CALCULATION OF LENGTH OF LIFE

B+C		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
P ₀ , %		1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90	3.00	3.10
1.10	9.5	4.8	3.2	2.4	1.9	1.6	1.4	1.2	1.1	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0	0.0	0.0
1.20	13.2	9.1	6.1	4.6	3.6	3.0	2.6	2.3	2.0	1.8	1.7	1.5	1.4	1.3	1.2	1.1	1.0	0.9	0.8	0.7	0.6	0.5
1.30	26.2	13.1	8.7	6.6	5.2	4.4	3.7	3.3	2.9	2.6	2.4	2.2	2.0	1.9	1.7	1.6	1.5	1.4	1.3	1.2	1.1	1.0
1.40	33.2	16.8	11.2	8.4	6.7	5.6	4.8	4.2	3.7	3.4	3.1	2.8	2.6	2.4	2.2	2.1	2.0	1.9	1.8	1.7	1.6	1.5
1.50	40.5	20.3	13.5	10.1	8.1	6.8	5.8	5.1	4.5	4.1	3.7	3.4	3.1	2.9	2.7	2.5	2.4	2.3	2.2	2.1	2.0	1.9
1.60	47.1	23.5	15.7	11.8	9.4	7.8	6.7	5.9	5.2	4.7	4.3	3.9	3.6	3.4	3.1	2.9	2.8	2.6	2.4	2.3	2.2	2.1
1.70	53.1	26.5	17.7	13.3	10.6	8.8	7.6	6.6	5.9	5.3	4.8	4.4	4.1	3.8	3.5	3.3	3.1	2.9	2.8	2.7	2.6	2.5
1.80	58.8	29.4	19.6	14.7	11.8	9.8	8.4	7.3	6.5	5.9	5.3	4.9	4.5	4.2	3.9	3.7	3.5	3.3	3.1	3.0	2.9	2.8
1.90	64.2	32.1	21.4	16.0	12.8	10.7	9.2	8.0	7.1	6.4	5.8	5.3	4.9	4.6	4.3	4.0	3.8	3.6	3.4	3.3	3.2	3.1
2.00	69.3	34.7	23.1	17.3	13.9	11.6	9.9	8.7	7.7	6.9	6.3	5.8	5.3	5.0	4.6	4.3	4.1	3.9	3.7	3.6	3.5	3.4
2.10	74.2	37.1	24.7	18.5	14.8	12.4	10.6	9.3	8.2	7.4	6.7	6.2	5.7	5.3	4.9	4.6	4.4	4.1	4.0	3.9	3.8	3.7
2.20	78.8	39.4	26.3	19.7	15.8	13.1	11.3	9.9	8.8	7.9	7.2	6.6	6.1	5.6	5.3	4.9	4.6	4.4	4.2	4.1	4.0	3.9
2.30	83.3	41.6	27.8	20.8	16.7	13.9	11.9	10.4	9.3	8.3	7.6	6.9	6.4	5.9	5.6	5.2	4.9	4.6	4.4	4.3	4.2	4.1
2.40	87.5	43.8	29.2	21.9	17.5	14.6	12.5	10.9	9.7	8.8	8.0	7.3	6.7	6.3	5.8	5.5	5.1	4.9	4.6	4.5	4.4	4.3
2.50	91.5	45.8	30.5	22.9	18.3	15.3	13.1	11.5	10.2	9.2	8.3	7.6	7.0	6.5	6.1	5.7	5.4	5.1	4.8	4.7	4.6	4.5
2.60	95.5	47.8	31.9	23.9	19.1	15.9	13.7	11.9	10.6	9.6	8.7	8.0	7.4	6.8	6.4	6.0	5.6	5.3	5.0	4.8	4.7	4.6
2.70	99.3	49.7	33.1	24.8	19.9	16.6	14.2	12.4	11.0	9.9	9.0	8.3	7.6	7.1	6.6	6.2	5.8	5.5	5.2	5.0	4.8	4.7
2.80	99.9	51.5	34.3	25.7	20.6	17.2	14.7	12.9	11.4	10.3	9.4	8.6	7.9	7.4	6.9	6.4	6.1	5.7	5.4	5.1	4.9	4.8
2.90	99.9	53.2	35.5	26.6	21.3	17.7	15.2	13.3	11.8	10.6	9.7	8.9	8.2	7.6	7.1	6.7	6.3	5.9	5.6	5.3	5.1	5.0
3.00	99.9	54.9	36.6	27.5	22.0	18.3	15.7	13.7	12.2	11.0	10.0	9.2	8.5	7.8	7.3	6.9	6.5	6.1	5.8	5.5	5.3	5.2
3.10	99.9	56.6	37.7	28.3	22.6	18.9	16.2	14.1	12.6	11.3	10.3	9.4	8.7	8.1	7.5	7.1	6.7	6.3	6.0	5.7	5.5	5.4
3.20	99.9	58.2	38.8	29.1	23.3	19.4	16.6	14.5	12.9	11.6	10.6	9.7	8.9	8.3	7.8	7.3	6.8	6.5	6.1	5.8	5.6	5.5
3.30	99.9	59.7	39.8	29.8	23.9	19.9	17.1	14.9	13.3	11.9	10.9	9.9	9.2	8.5	8.0	7.5	7.0	6.6	6.3	6.0	5.8	5.7
3.40	99.9	61.2	40.8	30.6	24.5	20.4	17.5	15.3	13.6	12.2	11.1	10.2	9.4	8.7	8.2	7.6	7.2	6.8	6.4	6.1	5.9	5.8
3.50	99.9	62.6	41.6	31.3	25.1	20.9	17.9	15.7	13.9	12.5	11.4	10.4	9.6	8.9	8.4	7.8	7.4	7.0	6.6	6.3	6.1	6.0
3.60	99.9	64.0	42.7	32.0	25.6	21.3	18.3	16.0	14.2	12.8	11.6	10.7	9.9	9.1	8.5	8.0	7.5	7.1	6.7	6.4	6.2	6.1
3.70	99.9	65.4	43.6	32.7	26.2	21.8	18.7	16.4	14.5	13.1	11.9	10.9	10.1	9.3	8.7	8.2	7.7	7.3	6.9	6.5	6.3	6.2
3.80	99.9	66.8	44.5	33.4	26.7	22.3	19.1	16.7	14.6	13.4	12.1	11.1	10.3	9.5	8.9	8.3	7.8	7.4	7.0	6.7	6.5	6.4
3.90	99.9	68.0	45.4	34.0	27.2	22.7	19.4	17.0	15.1	13.6	12.4	11.3	10.5	9.7	9.1	8.5	8.0	7.6	7.2	6.8	6.6	6.5
4.00	99.9	69.3	46.2	34.7	27.7	23.1	19.8	17.3	15.4	13.9	12.6	11.6	10.7	9.9	9.2	8.7	8.2	7.7	7.3	6.9	6.7	6.6
4.10	99.9	70.5	47.0	35.3	28.2	23.5	20.2	17.6	15.7	14.1	12.8	11.8	10.9	10.1	9.4	8.8	8.3	7.8	7.4	7.0	6.8	6.7
4.20	99.9	71.6	47.8	35.9	28.7	23.9	20.5	17.9	15.9	14.4	13.0	12.0	11.0	10.3	9.6	9.0	8.4	8.0	7.6	7.2	6.9	6.8
4.30	99.9	72.9	48.6	36.5	29.2	24.3	20.8	18.2	16.2	14.6	13.3	12.2	11.2	10.4	9.7	9.1	8.5	8.1	7.7	7.3	7.0	6.9
4.40	99.9	74.1	49.4	37.0	29.6	24.7	21.2	18.5	16.5	14.8	13.5	12.3	11.4	10.6	9.9	9.3	8.7	8.2	7.8	7.4	7.1	7.0
4.50	99.9	75.2	50.1	37.6	30.1	25.1	21.5	18.8	16.7	15.0	13.7	12.5	11.6	10.7	10.0	9.4	8.8	8.4	7.9	7.5	7.2	7.1
4.60	99.9	76.3	50.9	38.2	30.5	25.4	21.8	19.1	17.0	15.3	13.9	12.7	11.7	10.9	10.2	9.5	8.9	8.5	8.0	7.6	7.3	7.2
4.70	99.9	77.4	51.6	38.7	31.0	25.8	22.1	19.3	17.2	15.5	14.1	12.9	11.9	11.1	10.3	9.7	9.1	8.6	8.2	7.7	7.4	7.3
4.80	99.9	78.4	52.3	39.2	31.4	26.1	22.4	19.6	17.4	15.7	14.3	13.1	12.1	11.2	10.5	9.8	9.2	8.7	8.3	7.8	7.5	7.4
4.90	99.9	79.5	53.0	39.7	31.8	26.5	22.7	19.9	17.7	15.9	14.4	13.2	12.2	11.3	10.6	10.0	9.4	8.9	8.4	8.0	7.6	7.5
5.00	99.9	80.5	53.6	40.2	32.2	26.8	23.0	20.1	17.9	16.1	14.6	13.4	12.4	11.5	10.7	10.1	9.5	9.0	8.5	8.1	7.7	7.6

Table 9. CALCULATION OF LENGTH OF LIFE (Cont'd)

B+C		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
P ₀ /H ₀																					
5.50	99.9	85.2	56.8	42.6	34.1	28.4	24.4	21.3	18.9	17.0	15.5	14.2	13.1	12.2	11.4	10.7	10.0	9.5	9.0	8.5	
6.00	99.9	89.6	59.7	44.8	35.8	29.9	25.6	22.4	19.9	17.9	16.3	14.9	13.8	12.8	11.9	11.2	10.5	10.0	9.4	8.6	
6.50	99.9	93.6	62.4	46.8	37.4	31.2	26.7	23.4	20.8	18.7	17.0	15.0	14.4	13.4	12.5	11.7	11.0	10.4	9.9	9.4	
7.00	99.9	97.3	64.9	48.6	38.9	32.4	27.9	24.3	21.6	19.5	17.7	15.2	15.0	13.9	13.0	12.2	11.4	10.8	10.2	9.7	
7.50	99.9	99.9	67.2	50.4	40.3	33.6	28.8	25.2	22.4	20.1	18.3	15.8	15.5	14.4	13.4	12.6	11.9	11.2	10.6	10.1	
8.00	99.9	99.9	69.3	52.0	41.6	34.7	29.7	26.0	23.1	20.8	18.9	17.3	16.0	14.9	13.9	13.0	12.2	11.6	10.9	10.4	
8.50	99.9	99.9	71.3	53.5	42.8	35.7	30.6	26.8	23.8	21.4	19.5	17.8	16.5	15.3	14.3	13.4	12.6	11.9	11.3	10.7	
9.00	99.9	99.9	73.2	54.9	43.9	36.6	31.4	27.5	24.4	22.0	20.0	18.3	16.9	15.7	14.6	13.7	12.9	12.2	11.6	11.0	
9.50	99.9	99.9	75.3	56.3	45.0	37.5	32.2	28.1	25.0	22.5	20.5	18.6	17.3	16.1	15.0	14.1	13.2	12.5	11.8	11.3	
10.00	99.9	99.9	76.8	57.6	46.1	38.4	32.9	28.8	25.6	23.0	20.9	19.2	17.7	16.4	15.4	14.4	13.5	12.8	12.1	11.5	
10.50	99.9	99.9	78.4	58.8	47.0	39.2	33.6	29.4	26.1	23.5	21.4	19.6	18.1	16.8	15.7	14.7	13.8	13.1	12.4	11.8	
11.00	99.9	99.9	79.9	59.9	48.0	40.0	34.3	30.0	26.6	24.0	21.8	20.0	18.4	17.1	16.0	15.0	14.1	13.3	12.6	12.0	
11.50	99.9	99.9	81.4	61.1	48.8	40.7	34.9	30.5	27.1	24.4	22.2	20.4	18.8	17.4	16.3	15.3	14.4	13.6	12.9	12.2	
12.00	99.9	99.9	82.8	62.1	49.7	41.4	35.5	31.1	27.6	24.8	22.6	20.7	19.1	17.7	16.6	15.5	14.6	13.8	13.1	12.4	
12.50	99.9	99.9	84.2	63.1	50.5	42.1	36.1	31.6	28.1	25.3	23.0	21.0	19.4	18.0	16.8	15.8	14.9	14.0	13.3	12.6	
13.00	99.9	99.9	85.5	64.1	51.3	42.7	36.6	32.1	28.5	25.6	23.3	21.4	19.7	18.3	17.1	16.0	15.1	14.2	13.5	12.8	
13.50	99.9	99.9	86.8	65.1	52.1	43.4	37.2	32.5	28.9	26.0	23.7	21.7	20.0	18.6	17.4	16.3	15.3	14.5	13.7	13.0	
14.00	99.9	99.9	88.0	66.0	52.8	44.0	37.7	33.0	29.3	26.4	24.0	22.0	20.3	18.9	17.6	16.5	15.5	14.7	13.9	13.2	
14.50	99.9	99.9	89.1	66.9	53.5	44.6	38.2	33.4	29.7	26.7	24.3	22.3	20.6	19.1	17.8	16.7	15.7	14.9	14.1	13.4	
15.00	99.9	99.9	90.3	67.7	54.2	45.1	38.7	33.9	30.1	27.1	24.6	22.6	20.8	19.3	18.1	16.9	15.9	15.0	14.3	13.5	
15.50	99.9	99.9	91.4	68.5	54.8	45.7	39.2	34.3	30.5	27.4	24.9	22.8	21.1	19.6	18.3	17.1	16.1	15.2	14.4	13.7	
16.00	99.9	99.9	92.4	69.3	55.5	46.2	39.6	34.7	31.8	27.7	25.2	23.1	21.3	19.8	18.5	17.3	16.3	15.4	14.6	13.9	
16.50	99.9	99.9	93.4	70.1	56.1	46.7	40.0	35.0	31.1	28.0	25.5	23.4	21.6	20.0	18.7	17.5	16.5	15.6	14.8	14.0	
17.00	99.9	99.9	94.4	70.8	56.7	47.2	40.5	35.4	31.5	28.3	25.8	23.6	21.8	20.2	18.9	17.7	16.7	15.7	14.9	14.2	
17.50	99.9	99.9	95.4	71.6	57.2	47.7	40.9	35.8	31.8	28.6	26.0	23.9	22.0	20.4	19.1	17.9	16.8	15.9	15.1	14.3	
18.00	99.9	99.9	96.3	72.3	57.8	48.2	41.3	36.1	32.1	28.9	26.3	24.1	22.2	20.6	19.3	18.1	17.0	16.1	15.2	14.5	
18.50	99.9	99.9	97.3	72.9	58.4	48.6	41.7	36.5	32.4	29.2	26.5	24.3	22.4	20.8	19.5	18.2	17.2	16.2	15.4	14.6	
19.00	99.9	99.9	98.1	73.6	58.9	49.1	42.1	36.8	32.7	29.4	26.8	24.5	22.6	21.0	19.6	18.4	17.3	16.4	15.5	14.7	
19.50	99.9	99.9	99.0	74.3	59.4	49.5	42.4	37.1	33.0	29.7	27.0	24.8	22.8	21.2	19.8	18.6	17.5	16.5	15.6	14.9	
20.00	99.9	99.9	99.9	74.9	59.9	49.9	42.8	37.4	33.3	30.0	27.2	25.0	23.0	21.4	20.0	18.7	17.6	16.6	15.8	15.0	
20.50	99.9	99.9	99.9	75.5	60.4	50.3	43.1	37.8	33.6	30.2	27.5	25.2	23.2	21.6	20.1	18.9	17.8	16.8	15.9	15.1	
21.00	99.9	99.9	99.9	76.1	60.9	50.7	43.5	38.1	33.8	30.4	27.7	25.4	23.4	21.7	20.3	19.0	17.9	16.9	16.0	15.2	
21.50	99.9	99.9	99.9	76.7	61.4	51.1	43.8	38.4	34.1	30.7	27.9	25.6	23.6	21.9	20.5	19.2	18.1	17.0	16.1	15.3	
22.00	99.9	99.9	99.9	77.3	61.8	51.5	44.2	38.6	34.3	30.9	28.1	25.8	23.8	22.1	20.6	19.3	18.2	17.2	16.3	15.5	
22.50	99.9	99.9	99.9	77.8	62.3	51.9	44.5	38.9	34.6	31.1	28.3	26.0	24.0	22.2	20.8	19.5	18.3	17.3	16.4	15.6	
23.00	99.9	99.9	99.9	78.4	62.7	52.3	44.8	39.2	34.8	31.4	28.5	26.1	24.1	22.4	20.9	19.6	18.4	17.4	16.5	15.7	
23.50	99.9	99.9	99.9	78.9	63.1	52.6	45.1	39.5	35.1	31.6	28.7	26.3	24.3	22.6	21.0	19.7	18.6	17.5	16.6	15.8	
24.00	99.9	99.9	99.9	79.5	63.6	53.0	45.4	39.7	35.3	31.8	28.9	26.5	24.4	22.7	21.2	19.9	18.7	17.7	16.7	15.9	
24.50	99.9	99.9	99.9	80.0	64.0	53.3	45.7	40.0	35.5	32.0	29.1	26.7	24.6	22.8	21.3	20.0	18.8	17.8	16.8	16.0	
25.00	99.9	99.9	99.9	80.5	64.4	53.6	46.0	40.2	35.8	32.2	29.3	26.8	24.8	23.0	21.5	20.1	18.9	17.9	16.9	16.1	

Table 9. CALCULATION OF LENGTH OF LIFE (Cont'd.) :

R+C		0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.40
Pd/MO																					
5.50	3.1	7.7	7.4	7.1	6.8	6.6	6.3	6.1	5.9	5.7	5.5	5.3	5.2	5.0	4.9	4.7	4.6	4.5	4.4	4.3	
6.00	3.5	8.1	7.8	7.5	7.2	6.9	6.6	6.4	6.2	6.0	5.8	5.6	5.4	5.3	5.1	5.0	4.8	4.7	4.6	4.5	
6.50	3.9	8.5	8.1	7.8	7.5	7.2	6.9	6.7	6.5	6.2	6.0	5.8	5.7	5.5	5.3	5.2	5.1	4.9	4.8	4.7	
7.00	4.3	8.8	8.5	8.1	7.8	7.5	7.2	6.9	6.7	6.5	6.3	6.1	5.9	5.7	5.6	5.4	5.3	5.1	5.0	4.9	
7.50	4.6	9.2	8.8	8.4	8.1	7.7	7.5	7.2	6.9	6.7	6.5	6.3	6.1	5.9	5.8	5.6	5.4	5.3	5.2	5.0	
8.00	4.9	9.5	9.2	8.7	8.3	8.0	7.7	7.4	7.2	6.9	6.7	6.5	6.3	6.1	5.9	5.8	5.6	5.5	5.3	5.2	
8.50	5.2	9.7	9.3	8.9	8.6	8.2	7.9	7.6	7.4	7.1	6.9	6.7	6.5	6.3	6.1	5.9	5.8	5.6	5.5	5.4	
9.00	5.5	10.0	9.6	9.2	8.8	8.5	8.1	7.8	7.6	7.3	7.1	6.9	6.7	6.5	6.3	6.1	6.0	5.8	5.6	5.5	
9.50	5.7	10.2	9.8	9.4	9.0	8.7	8.3	8.0	7.7	7.4	7.2	7.0	6.8	6.6	6.4	6.3	6.1	5.9	5.8	5.6	
10.00	5.9	10.5	10.0	9.6	9.2	8.9	8.5	8.2	7.9	7.7	7.4	7.2	7.0	6.8	6.6	6.4	6.3	6.1	5.9	5.8	
10.50	6.1	10.7	10.2	9.8	9.4	9.0	8.7	8.4	8.1	7.8	7.6	7.3	7.1	6.9	6.7	6.5	6.4	6.2	6.0	5.9	
11.00	6.3	10.9	10.4	10.0	9.6	9.2	8.9	8.6	8.3	8.0	7.7	7.5	7.3	7.1	6.9	6.7	6.5	6.3	6.1	6.0	
11.50	6.5	11.1	10.6	10.2	9.8	9.4	9.0	8.7	8.4	8.1	7.9	7.6	7.4	7.2	7.0	6.8	6.6	6.4	6.3	6.1	
12.00	6.7	11.3	10.8	10.4	9.9	9.6	9.2	8.9	8.6	8.3	8.0	7.8	7.5	7.3	7.1	6.9	6.7	6.5	6.4	6.2	
12.50	6.9	11.5	11.0	10.5	10.1	9.7	9.4	9.0	8.7	8.4	8.1	7.9	7.7	7.4	7.2	7.0	6.8	6.6	6.5	6.3	
13.00	7.1	11.7	11.2	10.7	10.3	9.9	9.5	9.2	8.8	8.5	8.3	8.0	7.8	7.5	7.3	7.1	6.9	6.7	6.6	6.4	
13.50	7.3	11.9	11.4	10.9	10.4	10.0	9.6	9.3	9.0	8.7	8.4	8.1	7.9	7.7	7.4	7.2	7.0	6.8	6.7	6.5	
14.00	7.5	12.1	11.6	11.1	10.6	10.2	9.8	9.4	9.1	8.8	8.5	8.2	8.0	7.6	7.5	7.3	7.1	6.9	6.8	6.6	
14.50	7.7	12.3	11.8	11.3	10.8	10.4	10.0	9.6	9.3	9.0	8.6	8.3	8.1	7.9	7.6	7.4	7.2	7.0	6.9	6.7	
15.00	7.9	12.5	12.0	11.5	11.0	10.5	10.2	9.8	9.5	9.1	8.8	8.5	8.3	8.1	7.8	7.6	7.4	7.2	7.0	6.9	
15.50	8.1	12.7	12.2	11.7	11.2	10.7	10.3	9.9	9.6	9.2	8.9	8.7	8.4	8.2	7.9	7.7	7.5	7.3	7.1	6.9	
16.00	8.3	12.9	12.4	11.9	11.4	10.9	10.4	10.0	9.7	9.3	9.0	8.8	8.5	8.3	8.0	7.8	7.6	7.4	7.2	7.0	
16.50	8.5	13.1	12.6	12.1	11.6	11.1	10.6	10.2	9.9	9.5	9.2	8.9	8.7	8.4	8.2	7.9	7.7	7.5	7.3	7.1	
17.00	8.7	13.3	12.8	12.3	11.8	11.3	10.8	10.4	10.1	9.7	9.4	9.1	8.9	8.6	8.3	8.1	7.9	7.7	7.5	7.3	
17.50	8.9	13.5	13.0	12.5	12.0	11.5	11.0	10.5	10.2	9.8	9.5	9.2	9.0	8.7	8.4	8.2	8.0	7.7	7.5	7.3	
18.00	9.1	13.7	13.2	12.7	12.2	11.7	11.2	10.7	10.4	10.1	9.7	9.4	9.1	8.8	8.5	8.3	8.1	7.8	7.6	7.4	
18.50	9.3	13.9	13.4	12.9	12.4	11.9	11.4	10.9	10.5	10.2	9.8	9.5	9.2	8.9	8.7	8.4	8.2	7.9	7.7	7.5	
19.00	9.5	14.1	13.6	13.1	12.6	12.1	11.6	11.1	10.6	10.3	9.9	9.6	9.3	9.0	8.7	8.5	8.3	8.0	7.7	7.5	
19.50	9.7	14.3	13.8	13.3	12.8	12.3	11.8	11.3	10.8	10.4	10.1	9.7	9.4	9.1	8.8	8.5	8.3	8.0	7.8	7.6	
20.00	9.9	14.5	14.0	13.5	13.0	12.5	12.0	11.5	11.0	10.5	10.2	9.8	9.5	9.2	8.9	8.7	8.4	8.2	7.9	7.7	
20.50	10.1	14.7	14.2	13.7	13.2	12.7	12.2	11.7	11.2	10.7	10.3	9.9	9.6	9.3	9.0	8.7	8.5	8.3	8.0	7.8	
21.00	10.3	14.9	14.4	13.9	13.4	12.9	12.4	11.9	11.4	10.9	10.5	10.1	9.7	9.4	9.1	8.8	8.5	8.3	8.0	7.7	
21.50	10.5	15.1	14.6	14.1	13.6	13.1	12.6	12.1	11.6	11.1	10.6	10.2	9.8	9.5	9.2	8.9	8.7	8.4	8.1	7.9	
22.00	10.7	15.3	14.8	14.3	13.8	13.3	12.8	12.3	11.8	11.3	10.8	10.4	9.9	9.6	9.3	9.0	8.7	8.5	8.2	7.9	
22.50	10.9	15.5	15.0	14.5	14.0	13.5	13.0	12.5	12.0	11.5	11.0	10.5	10.1	9.7	9.4	9.1	8.8	8.5	8.2	7.9	
23.00	11.1	15.7	15.2	14.7	14.2	13.7	13.2	12.7	12.2	11.7	11.2	10.7	10.3	9.9	9.6	9.3	9.0	8.7	8.4	8.1	
23.50	11.3	15.9	15.4	14.9	14.4	13.9	13.4	12.9	12.4	11.9	11.4	10.9	10.5	10.1	9.7	9.4	9.1	8.8	8.5	8.2	
24.00	11.5	16.1	15.6	15.1	14.6	14.1	13.6	13.1	12.6	12.1	11.6	11.1	10.6	10.2	9.8	9.5	9.2	8.9	8.6	8.3	
24.50	11.7	16.3	15.8	15.3	14.8	14.3	13.8	13.3	12.8	12.3	11.8	11.3	10.8	10.4	9.9	9.6	9.3	9.0	8.7	8.4	
25.00	11.9	16.5	16.0	15.5	15.0	14.5	14.0	13.5	13.0	12.5	12.0	11.5	11.0	10.5	10.1	9.7	9.4	9.1	8.8	8.5	

Appendix 7.4: The Relation of Profitability to Investment Length of Life (Sensitivity Analysis)

The discussion so far has dealt only with optimal points, i.e. optimal parameter combinations. It is important to investigate what happens "outside" this optimum. Thus, for example, it is important to know how profitability of an investment project is being reduced if a project's life is shortened or extended - away from the optimum. In this Appendix I show the relation between profitability and length of life. Section 1 in this Appendix presents a computer program for DCFROR sensitivity analysis of the continuous mode and gives examples for it. Section 2 gives an analysis of the present value of costs for an example in the discrete mode. (Shone gives a similar P.V. analysis for the continuous mode)¹⁾. All these results show that profitability is very insensitive to the shortening or the extension of the life of the project.

1) Shone (1975) op. cit. p.43

Section 1 DCF Rate of Return - Sensitivity Analysis

FORTTRAN PROGRAM-SENS

```

MASTER SENS
C SENSITIVITY ANALYSIS OF THE INTERNAL RATE OF RETURN IN SHONE S MODEL
C TO THE LENGTH OF LIFE PARAMETER
DIMENSION R(10),P(10),I(10),DP(8),RDN(8),TON(8)
DATA R / .04,.06,.08,.10,.12,.14,.16,.18,.20,.22 / ,
C      P / .253,.263,.273,.285,.297,.311,.325,.340,.356,.373 / ,
C      T/15.,16.,16.,17.,18.,18.,19.,20.,21.,21. / ,
C      DR / -.02,-.015,-.01,-.005,-.005,-.01,-.015,-.02 / ,
WD = 0.10
DO 36 10 = 1, 10
DO 35 11 = 0,2,2
DO 34 12 = 1,4
J = 11*2 + 12
TT3 = T(10)
TT1 = T(10) * 11
TT2 = (TT3 + TT1) / 2.
RJ = R(10) + DR (J)
DO 33 13 = 1,10
RJJ=RJ-.04
IF (ABS(RJJ) .LT. .00001 ) GO TO 60
BONB = P(10)*(1-EXP(-(RJ+.02)*TT2)) / (RJ +.02 )
C      -WD* ( 1.-EXP(-(RJ -.04)*TT2 )) / (RJ -.04 ) -1.
29 IF ( BONB ) 30,32,31
30 TT1 = TT2
GO TO 32
31 TT3 = TT2
32 TT2 = (TT3 + TT1) / 2.
33 CONTINUE
RDN(J) = RJ
TON(J) = TT2
34 CONTINUE
35 CONTINUE
WRITE (2,40) P(10),WD ,R(10),T(10),(TON(K),K=1,8),
C      (RDN(L), L =1, 8 )
40 FORMAT (1H0,8HPICE = ,F5.3,15H DP, COSTS = ,F4.2,
C 20H MAX DCF RETURN R=,F3.2 ,10H AT T = ,F2.0, 8H YEARS/
C 1X,1HT, 8 (2X,F4.1 ) / 1X,1HR, 8 ( 2X,F4.3 ) )
36 CONTINUE
STOP
60 BONB = P(10)*(1-EXP(-(RJ+.02)*TT2)) / (RJ +.02 )
C      -WD * TT2 -1.
GO TO 29
END

```

END OF SEGMENT, LENGTH 279, NAME SENS

Table 7.10 S-Model, Continuous Mode - Sensitivity Analysis by Program
SENS - showing how DCFROR is related to length of life.

Worked Examples:

Parameters

$$W_0 = 0.10, \quad b = 4\%, \quad a = 2\%, \quad c = 2\%$$

DOCUMENT SENS

121/08/72 AT 22.20

PRICE = 0.253 UP. CUSIS = 0.10 MAX LCF RETURN R=.04 AT T = *15. YEARS

T 10.5 11.0 11.7 12.6 16.4 19.4 20.1 20.6
R .020 .025 .030 .035 .035 .030 .025 .020

PRICE = 0.263 UP. CUSIS = 0.10 MAX LCF RETURN R=.06 AT T = *16. YEARS

T 10.5 11.1 11.8 12.8 19.8 21.0 21.8 22.4
R .040 .045 .050 .055 .055 .050 .045 .040

PRICE = 0.273 UP. CUSIS = 0.10 MAX LCF RETURN R=.08 AT T = *16. YEARS

T 10.7 11.4 12.2 13.4 20.6 22.3 23.4 24.3
R .060 .065 .070 .075 .075 .070 .065 .060

PRICE = 0.285 UP. CUSIS = 0.10 MAX LCF RETURN R=.10 AT T = *17. YEARS

T 10.6 11.3 12.2 13.4 22.7 24.8 26.4 27.5
R .080 .085 .090 .095 .095 .090 .085 .080

PRICE = 0.297 UP. CUSIS = 0.10 MAX LCF RETURN R=.12 AT T = *18. YEARS

T 10.7 11.4 12.4 13.9 24.2 27.6 30.1 31.9
R .100 .105 .110 .115 .115 .110 .105 .100

PRICE = 0.311 UP. CUSIS = 0.10 MAX LCF RETURN R=.14 AT T = *18. YEARS

T 10.5 11.2 12.2 13.7 28.4 34.6 36.0 36.0
R .120 .125 .130 .135 .135 .130 .125 .120

PRICE = 0.325 UP. CUSIS = 0.10 MAX LCF RETURN R=.16 AT T = *19. YEARS

T 10.4 11.2 12.2 13.9 34.1 38.0 38.0 38.0
R .140 .145 .150 .155 .155 .150 .145 .140

PRICE = 0.340 UP. CUSIS = 0.10 MAX LCF RETURN R=.18 AT T = *20. YEARS

T 10.2 11.0 12.1 13.9 40.0 40.0 40.0 40.0
R .160 .165 .170 .175 .175 .170 .165 .160

PRICE = 0.356 UP. CUSIS = 0.10 MAX LCF RETURN R=.20 AT T = *21. YEARS

T 10.0 10.7 11.8 13.6 42.0 42.0 42.0 42.0
R .180 .185 .190 .195 .195 .190 .185 .180

PRICE = 0.373 UP. CUSIS = 0.10 MAX LCF RETURN R=.22 AT T = *21. YEARS

T 9.6 10.3 11.4 13.0 42.0 42.0 42.0 42.0
R .200 .205 .210 .215 .215 .210 .205 .200

Note: When required length of life 'T' for a given rate of return 'R' is more than twice the required length of life (starred number) for maximum DCF return then the program SENS prints 'T' value that is twice that starred optimal-length-of-life.

Section 2. PV Sensitivity Analysis.

Table 7.11 S-Model, Discrete Mode: PV of operating profit for various lengths of life.

Parameters: $K_o = 1.00$; $P_o = .50$; $W_o = .25$.

T	(1) $P_o \cdot \sum_{i=1}^T \frac{1}{(1+G)^i}$	(2) $W_o = \sum_{i=1}^T \frac{1}{(1+H)^i}$	(3) (1) - (2)
1	0.4464	0.2358	0.2106
2	0.8450	0.4583	0.3867
3	1.2009	0.6682	0.5327
4	1.5187	0.8663	0.6524
5	1.8024	1.0531	0.7493
6	2.0557	1.2293	0.8264
7	2.2819	1.3951	0.8868
8	2.4838	1.5524	0.9314
9	2.6641	1.7004	0.9637
10	2.8251	1.8400	0.9851
11	2.9688	1.9717	0.9971
12	3.0972	2.0960	1.0012
13	3.2118	2.2132	0.9986
14	3.3141	2.3237	0.9904
15	3.4054	2.4281	0.9773
16	3.4870	2.5265	0.9605
17	3.5598	2.6193	0.9405
18	3.6248	2.7069	0.9179
19	3.6829	2.7895	0.8934
20	3.7377	2.8675	0.8672

Note that the P.V. over the range of 9-16 years is within 5% of the optimal P.V. at 12 years. The optimal P.V. is not exactly 1 because of rounding errors.

CHAPTER 8

S - METHOD REVIEWED AND COMPARED WITH OTHER METHODS

The bulk of this Chapter, in Section 8.1, is a methodological comparison of the S - system and the 'practical' replacement systems discussed in Chapter 5. The parameters of the S system are interpreted in a way that facilitates comparison and emphasis is inevitably put on the S system as replacement model.

Section 8.2 comments on the technical change element in the S system which is a key factor behind its development. Section 8.3 lists a large number of considerations that enter real life investment decision making and which are somewhat outside the assumptions (and for that matter outside the ORM and MAPI models).

8.1 Methodological Comparison with the 'Practical' Replacement Methods

The 'practical' methods of assessing replacement proposals discussed in Chapter 5 are: the Optimal Replacement Method of Merrett and Sykes; the various MAPI models developed by George Terborgh; and the adaptation of Terborgh's method by Connor and Evans. The method presented in Chapter 6 and 7 is wider in context than the others. It aims at a closer tie with established and developing economic theory and deals with problems that are not covered by the others. In particular it provides a unified method of dealing with problems of pricing as well as with new investment and replacement decisions. This type of advantage is crucial to the S - system but it will not be examined here¹⁾. Here I shall examine only how it compares with other methods or "systems" in their special field - assessment of replacement projects.

1) See Sir Robert Shone, "Price and Investment Relationships", Elek 1975; Passim where these relations are discussed.

There is a great similarity between the S - model and the other methods. This can be demonstrated by comparing Figures 6.1 and 5.2. Shone himself shows how Terborgh's model can be used in a pricing decision¹⁾. There are nevertheless subtle differences some of them will be discussed here. I have already analysed mathematically both Terborgh's method (Chapter 5) and Shone's method (Chapter 6), so the comparison will be mainly in way of discussion.

I would like to add at the outset that there is no evidence of a large scale acceptance of the MAPI model in the USA or of the other methods in the UK. If anything, there is strong evidence to the contrary²⁾. Therefore, the proper way to assess the value of the S - method is as something novel rather than as replacing existing methods.

8.1.1. Two Theoretical Aspects

It is of great interest to make a formal structural comparison between the S - model and the other models. Here I shall consider two aspects. One aspect will be to express the S - model in terms of "standard assumptions" - comparable to Terborgh's two standard assumptions. The second aspect will be to look at the S - model from the next-year-comparison point of view.

1) ibid p. 99.

2) See Chapter 4 above.

I find both aspects very valuable in understanding similarity and difference between the S - model and the others.

i) "Standard Assumptions"

Using Terborgh's terminology (5.4.) the S - model can be expressed, and in fact enlarged by the following two standard assumptions (for notation see Chapter 6):

- 1) A Future Challenger at time t will have the adverse minimum of the Present Challenger multiplied by $e^{(a-b)t}$.
- 2) The Present Challenger will accumulate deterioration at the rate of

$$W_0 \left[e^{(a+c)t} - e^{(a-b)t} \right]$$

Terborgh's highly comparable standard assumptions are:

- 1) Future Challengers will repeat the adverse minimum of the present one.
- 2) The Present Challenger will accumulate deterioration at a constant rate.

The adverse minimum of the Present Challenger is basically the 'F' parameter, in Chapter 7.

ii) Next Year Comparison

As emphasised in Chapter 5, Merrett and Sykes use Terborgh's cost model to arrive at a replacement decision by what seems to be

a different approach. They compare immediate replacement with a postponed replacement, in effect they compare two infinite streams of costs. The result was, as stressed in Chapter 5, that for the same cost model one would get exactly the same replacement rule.

It will be shown that the same approach can be applied to the cost pattern of the S - model. There are a number of ways of showing that. I shall use the discrete S - model (See Chapter 7) to compare immediate investment and a one year postponed investment.

Costs associated with immediate investment are:

$$\begin{array}{ccccccc}
 0 & 1 & & T & T+1 & & 2T & \dots \\
 \hline
 K_0 & & W_0 \cdot \frac{1}{(1-a-c)^i} & K_T & & W_t \cdot \frac{1}{(1-a-c)^{i-T}} & K_{2T} & \text{Year}=i
 \end{array}$$

Costs associated with a one-year-postponed investment are:

$$\begin{array}{ccccccc}
 0 & 1 & 2 & & T+1 & T+2 & & 2T+1 & \dots \\
 \hline
 & K_1 & & W_1 \cdot \frac{1}{(1-a-c)^{i-1}} & & K_{T+1} & & W_{t+1} \cdot \frac{1}{(1-a-c)^{i-T-1}} & & K_{2t+1} & \text{Year}=i
 \end{array}$$

The incremental cash flow to consider is

$$\begin{aligned}
 & K_0 + W_0 \sum_{i=1}^T \frac{1}{(1+r-a-c)^i} + \frac{1}{(1+r)^T} (K_T + W_t \sum_{i=1}^T \frac{1}{(1+r-a-c)^i}) + \dots \\
 & - \frac{1}{1+r} (K_1 + W_1 \sum_{i=1}^T \frac{1}{(1+r-a-c)^i}) - \frac{1}{(1+r)^{T+1}} (K_{T+1} + W_{t+1} \sum_{i=1}^T \frac{1}{(1+r-a-c)^i}) - \dots (1)
 \end{aligned}$$

Which is an inconvenient expression, fortunately we can simplify this expression ⁽¹⁾ with the aid of equation (3) in Chapter 7, which can be restated as:

$$K_t = P_t \sum_{i=1}^T \frac{1}{(1+r+b-a)^i} - W_t \sum_{i=1}^T \frac{1}{(1+r-a-c)^i} \quad (2)$$

$$\text{or } K_t + W_t \sum_{i=1}^T \frac{1}{(1+r-a-c)^i} = P_t \sum_{i=1}^T \frac{1}{(1+r+b-a)^i} \quad (3)$$

The expression on the right hand side appears in (1), separately for each generation of replacement in both replacement chains.

Also we know (from (4) in Chapter 7) that

$$P_t = P_0 \cdot \frac{1}{(1+b-a)^t} \quad (4)$$

Therefore expression (1) can be rewritten in the convenient form of:

$$P_0 \sum_{i=1}^{\infty} \frac{1}{(1+r+b-a)^i} - \frac{1}{(1+r)} P_1 \sum_{i=2}^{\infty} \frac{1}{(1+r+b-a)^i} \quad (5)$$

$$\text{or } P_0 \cdot \frac{1}{(1+r+b-a)} \quad (6)$$

Expression (6) is the discounted difference between the cost associated with a chain of replacement starting by an immediate investment and a chain of replacement starting by a new investment next year.

So, regarding replacement problems where a chain of replacement starting next year is preceded by a one year retention of an existing asset we can say that immediate replacement is preferable to replacement

next year (and therefore justified) if next year (undiscounted) operating costs associated with the existing plant exceed

$$P_0 \cdot \frac{1}{(1+b-a)}.$$

This is virtually the same criterion used in the direct approach that is derived from information on the immediate replacement only (and from assumptions on the behaviour of future cost).

It is very important to be aware of this possible interpretation of the decision rule in the S - model.

8.1.2. Exponential Change vs Linear Change - Discussion

The ORM system and to a great extent the MAPI system discussed in Chapter 5 assume that technological change is constant over time in absolute, not percentage, terms. This leads to the conclusion that the operating costs in new plant decrease by constant absolute amounts. This means that, if technological change reduces costs at 5 percent a year, then operating costs would be negative after twenty years. Perhaps it would be claimed that it is not the technological change element alone that is included in the linear parameter, but a combination of technological change and deterioration in performance at existing plant which sets a linear pace of advantage of the new over the old with the passage of time. Such an argument has limited significance since deterioration is in many cases, a minor factor. Indeed this argument is not advanced by Terborgh or Merrett and Sykes.

Another argument can be that the 'negative' costs affect only future generations of investments and that because of discounting

the effect is significant in relation to what really matters - the current replacement decision. One cannot ignore this argument, but still it seems that assumptions of linearity in the face of the dominant effect of technological change rather than deterioration might lead to wrong answers through the effect on the first replacement. (The profitability of the new investment will be underestimated because the effect of future technological change is exaggerated). The S - method is not caught in the pitfalls of linear relative decline of incumbent plant. It explicitly assumes the separate impact of technological change and deterioration, both with an exponential time pattern (therefore, the possibility of negative costs is precluded).

Further, in practice firms experience the impact of technical progress embodied in new plants commonly in the form of their own wage and other operating costs rising without their being able to pass on such rising costs into higher prices. They are competing with new and more efficient plants which can pay the higher costs without corresponding increases in prices. This means that technical progress is felt by existing plant as a rise in wages and other costs, and these are normally measured as a percentage increase and not an absolute rise. It should be noted that neither the ORM nor the MAPI method make a distinction between deterioration proper and the impact of wage increases in existing plants. The S - system does make the distinction.

Again, when dealing with problems of inflation, the issue is one of an expected percentage rise in prices and costs and not of an absolute or simple interest rise. The treatment of inflation in the linear models is unsatisfactory¹⁾.

Hence, it seems more natural and to accord more closely with experience to use a model that is based on percentage parameters, as in the S - system, rather than to use one with linear parameters as is the case in the other models discussed.

8.1.3. The Complications in the Determination of W_0/K_0

Nevertheless the use of a percentage and not an absolute measure of change introduces an additional link between the elements of change and the initial operating costs (in themselves linked to the capital outlay). This necessitates additional information and could perhaps counter some of the appeal of the S - method. But the extra information required is more apparent than real.

Certainly the S - system requires clear information about the total relevant or responsive operating costs at new plant, since it is a percentage change in these responsive costs which is a key parameter. It also needs a clear measure of the difference in costs between new and old plant. The other methods require only the information about the difference in operating costs between the old and the new, and a measure of future absolute changes in cost at existing

1) See Sir Robert Shone, (1975) p.98.

and still newer plant. In all cases, however, whether absolute or percentage changes are sought, these have to be determined from a wide profile of well-documented or estimated values of operating-costs items, otherwise the figures have no quantitative value and are nothing more than a guess¹⁾. True, there can be operating-cost elements that are not affected by the replacement proposal; therefore they need not be quantified in the assessment process (provided that continuation with the activity at all is viable). But none of the systems require these additional details. The S - method specifically includes in the initial operating-cost parameter only those elements that are affected by deterioration and are responsive to improvement through new investment.

The additional complication, if any, that the method entails, is a complication of judgement. It is a matter of deciding on the value of the initial operating cost that will best reflect the impact of both deterioration and obsolescence. This is rather a 'goodness of fit' problem within the set of assumptions (exponential as against linear rates of change) and with the independently available information.

8.1.4. Scrap Value

The literature on investment and replacement devotes considerable attention to the question of scrap value. Yet the S - system does not include any scrap-value parameters. The reason is that it is difficult to build a simple model that includes a number of important economic factors, as this system does, and still cater for a scrap variable.

1) See Exhibits 4.1, 4.2 and 5.1.

In practice, over a considerable industrial range, the complication does not exist, mainly because much industrial equipment commonly has little, if any salvage value. In addition, time discount makes this value, at the time of the investment decision, insignificant. Therefore, no damage will be incurred if any ultimate scrap value of the new investment is disregarded when assessing its merits. Salvage value of existing plant creates some problems. A way to overcome them is by charging the (notional) interest on the realizable value of existing plant and its capital consumption (i.e. annual reduction in salvage value), if any, to the operating cost of the existing plant.

In the cases where the ultimate scrap value of the new equipment is expected to be significantly high, a simple approximation can be used. It is to subtract the discounted estimated scrap value of the new plant at the estimated replacement time from its purchase value when installed. The calculation then uses a modified capital value in assessing the required operating cost savings. The solution using the graphs of Chapter 7 also includes a calculated 'length of life'. For any practical purposes and where the scrap value is not expected to vary significantly with age at 'about the time of replacement' the given answer is satisfactory¹⁾.

1) In theory it is possible to include various patterns of salvage in the S - system. I attempted this during an early stage of my work. But since this inclusion significantly complicates the basic equation of Chapter 6 and in effect precludes a design of multiple cut-off rate charts, I left this consideration out.

Finally, the type of scrap value that is included in the MAPI and ORM systems is not, as one might perhaps expect, a sophisticated element that has an independent time pattern and thus determines, and in turn is determined by, the length of life of the investment in an explicit calculation. It is just a guess - the eventual scrap value at the time of replacement of the new equipment. And this time of replacement is itself supposed to be known. It can in practical terms be said that the S - system is capable of dealing with the scrap value broadly to the same extent as the other systems.

8.1.5. A Single Cut-Off Rate v. A Multiple Cut-Off Rate

Terborgh uses a single cut-off rate of 8.25% for assessing replacement projects. Though he states that his method still gives fair results when this parameter deviates within limits from the prescribed value, no measure for this impact are given. The diligent user of Terborgh's approach has to produce his own tables to get the correct answer for other values of the cost of capital. Merrett and Sykes's method adopts a similar basic approach. Their cut-off rate is 8%; they claim that the results do not vary significantly with rates of discount over the range 8% to 12%. Many users may find themselves outside the range. Connor and Evans use a discount rate of 10% as the basis of their tables. These tables show the gross cost saving which is required to justify replacement on the basis that a 10 per cent return on capital is needed.

In contrast to the other methods, the S - method permits easy change from one cut-off rate to another. The user chooses

his own rate and can make a sensitivity analysis to see whether the project is viable at other costs of capital in case he is wrong in his first estimation of the cost of capital.

8.1.6. Apparent Replacement Criteria

The literature on replacement (see Chapter 5) is concerned with deciding on the optimal replacement date through the minimization of costs streams or through short cuts to answering this question. With the establishment of DCF methods in the late 1950's there was a great temptation to convert every investment decision rule to some form of DCFROR. That is why both Terborgh and Merrett and Sykes introduced the use of the artificial and elusive concept of 'relative returns'. The concept has been dealt with in Chapter 5 in considering both the MAPI and ORM systems. It can be summarised again as follows. Replacement is possible either immediately or at a given time T in the future. The time series of costs for each alternative is known. Now, examine the additional advantage (or disadvantage) accruing from immediate replacement over replacement at T , (T is usually 1 year). The difficulty is that the additional advantage is calculated as a certain rate of return on the investment carried out now - the 'relative return'. If the 'return' exceeds the cut-off rate, then immediate replacement is indicated; if not, it simply means do not replace.

This decision process, albeit leading in many cases to correct results (provided, of course, that the assumptions on which the decision is made are valid) is awkward and foreign to the way the businessman thinks. He looks for a direct answer to the question, 'Does it pay me to replace or not?' He does not

want to be involved in the technical details of 'relative return' over T years or in any other 'indirect' answers. He looks for a straight forward answer. And this the S - system aims to provide.

The criterion employed by the S - system (and in fact by Terborgh's earlier work and by Connor and Evans's) is simply to indicate the level of operating cost saving that a new asset needs in order to justify its introduction. Yet it should be again stressed that the S - system inherently takes account of the impact of future action whether the firm itself or its competitors on the profitability of immediate replacement through a view that is taken about the likely future pattern of cost and price changes.

8.1.7. Technical Progress

The Technical change 'obsalescence' in the Terborgh and Merrett and Sykes models is expressed as a linear percentage per annum of the capital investment. But there is no attempt there to arrive at any logical connection between this figure and the realities of technical change. These models essentially are concerned with financial principles and methodology rather than with the underlying economic processes. Nevertheless, it can be deduced that , in these models technical progress affects operating costs but not capital costs. So while the S - system assumes that technological change is neutral as between the saving of capital and labour - a form of change which coincides with long-term observation over a considerable range of industries and which

many economists would hold as a norm - Merrett and Sykes state that capital costs of 'successive generations of assets are assumed to remain constant in real terms', while the same successive generations of new plant are assumed to have lower operating costs as compared with earlier plant. In effect they, as well as Terborgh assume a biased technical advance with operational saving but no capital saving as successive new vintages of new plant are developed.

8.1.8. The Length of Life

One apparent difference between the S - method and other methods in particular Terborgh's (1967) and Connor and Evans is the approach to the 'length of life'. The S - method computes a length of life from information about technological change. The others, it may be said, compute technological change from information about the life of new equipment'.

It may be argued that mathematically, in all the systems discussed, there is little or no difference whether one uses the life as an external parameter or whether one used the technological change (and deterioration) as the external parameter. In all systems, for given cost parameters there is (nearly) a one-to-one correspondence between these two¹⁾. If the correct value of the external parameter is used then the correct decision is arrived at either way. But such an argument clearly ignores consideration of causality. It is the technological change that

1) Only 'nearly' so because the life parameter is discrete, see discussion in 5.4.

determines the length of life equipment and not vice versa.

In Chapter 5 I made the supposition that Terborgh converted the MAPI method in 1950 to use 'N' - the length of life as an external parameter, mainly for considerations of computational simplicity. Computational simplicity of this sort is hardly a justification today, with easy availability of computers.

There is another argument not methodological but pragmatic that appears to favour the use of a length of life as an external parameter. This is the simple fact that profitability of a characteristic investment and replacement project is highly insensitive to the length of life of the project. This is true and evidence for that is given in Chapters 5 and 7. But as Shone points out, gross errors in estimation of life can still be made and are made¹⁾.

The use of correct length of life is particularly important in new investment decisions and in pricing decisions. Both these decisions are fairly clearly subjected to the test of the market place. Companies might price themselves out of the market because of an excessive expectation for a fast technological change or they might miss an opportunity to capture a new market by a new investment. As Shone points out, there are wide implications to that²⁾

8.1.9. Comparison of Numerical Results: Exponential vs. Linear Models

Systematic comparison of results between the systems is extremely difficult. This is due to differences in parameters

1) Shone (1975) op. cit. Chapter 11.

2) Shone (1975) loc. cit.

used (use of length of life or obsolescence-deterioration as the external parameter), to difference in time patterns (whether first year income is concentrated a year after investment or only half a year, whether the incidence of deterioration starts a year after investment or two years after it, etc.), to differences in actual decision criteria, etc. Also there is the fundamental question of parameter estimation, which may not be independent of the decision model used¹⁾.

So, in order to produce some meaningful comparison I have reduced the S - model to its original continuous mode and reduced the basic Terborgh's model to its continuous equivalent - the one I developed in Appendix 5.2.

The result of a few numerical comparisons are given in Table 8.1. I used the same capital cost and discounting factor for both models - which is unambiguous. I also used Terborgh's constant gradient as the corresponding initial 'gradient' for the S - system. In the S - system this gradient is not a single parameter but a more sensitive combination of a number of parameters (W_0 , a , b , and c , to be precise).

Table 8.1. shows a range of results in the exponential model that corresponds to a single result in the linear model. The trends are clear to see. Differences are significant in capital intensive projects ($W_0 = 10\%$). Also of interest is the distinction that the exponential model makes between technical change and deterioration. The linear model is 'blind' to this distinction.

1) The discussion on the purpose, comparability and consistency of models in microeconomics in Chapter 3 is relevant here, too.

Table 8.1

The Linear Continuous Model and the Exponential Continuous Model (S Model)

Numerical Comparison for Corresponding Parameters

r	linear continuous model		Exponential continuous model							
			$W_o=10\%$		$W_o=25\%$		$W_o=50\%$		$W_o=100\%$	
	G=20		b=0.20 c=0		b=0.08 c=0		b=0.04 c=0		b=0.02 c=0	
	T	F	T	F	T	F	T	F	T	F
6	11	223	7	388	9	285	10	253	10	238
10	12	240	8	412	10	306	10	273	11	256
14	13	260	8	439	10	330	11	295	12	277
18	14	283	8	469	11	356	12	319	13	301

	G=20		b=0 c=0.20		b=0 c=0.08		b=0 c=0.04		b=0 c=0.02	
	T	F	T	F	T	F	T	F	T	F
6	11	223	6	285	8	249	9	236	10	229
10	12	240	6	304	9	265	10	253	11	246
14	13	260	7	319	9	285	10	273	11	267
18	14	283	7	340	10	306	11	295	12	289

Capital investment - £1000

r = cost of capital (%)

W_o = Initial operating costs, expressed as a rate of capital costs

G = the annual Gradient of the linear model £20 p.a.

b = technical progress rate in S - model

- c = deterioration rate in S model
- a = wage increase factor 'assumed 0 in the examples above'
- T = length of life (figure rounded to a whole years)
- F = "First year earning" or "initial adverse minimum" expressed in £ s.

The calculations for the linear model derive from equation (28) in Chapter 5 and for the exponential model from equation (4) in Chapter 6. The given values for the exponential model are from the Printout of FORTRAN program INVS2, given in Appendix 7.1.

8.2 The Technical Progress As a Key Element in the S - Model¹⁾

8.2.1. General Points

Technological change has numerous meanings in Technical and Economic papers. The S-model is concerned with a more limited range of concepts. As a microeconomic model it deals with technological change as affecting the production side only. That is to say, the questions of improved quality of output and introduction of new products are not examined. Furthermore, in this model the interest is not in the process itself, but in how it affects and rather how it should affect these types of decision:

1. New investment decision
2. Pricing decision for the output
3. Plant and equipment replacement decision

I would mention that for the purpose of the S - model improved design of equipment and the application of economies of scale in the production process are considered technological change exactly in the same way as major innovations in production. Being aware of the complications that are involved in this 'simplification' the model uses the criterion of the reduction of production costs over time at fixed factor prices of a unit of output as the 'technological change'.

Another assumption is that the technological change considered is a 'single tier' phenomenon. This means that e.g. an asset replacement decision today is 'somehow' related to future successive replacements of the asset in question. The case where a 'higher tier'

1) The content of this section is related to observations made in Chapter 2 and 3.

technological change would require the abandonment in the foreseeable future of the whole system that includes this asset is not dealt with. In such case the assessment of replacement of a given asset should be, for any practical purpose, confined to the end of life of the 'system' and not be based on an infinite chain of replacements of that asset.

These assumptions whether specified or not are the same as those that underlie most of the work in the area, notably work on equipment replacement.

An all important feature of the S - model is that it unifies replacement and expansion investment decisions, and these two again with long term marginal cost pricing into one single mathematical framework. This, to my best knowledge has not been done by any other normative models. "Unification" means that the same mathematical structure is used for either of the three.

In constructing the specified nature of technological change that is incorporated in the model the following requirements were observed.

1. The technological change element is introduced in a manner that would facilitate a unique solution to:-
 - a) What price to charge for the output?
 - b) Is new investment proposal viable?
 - c) Is it the right time to replace existing plant?
2. The nature of technological change as specified follows

Economic argumentation and past observations. It should be sufficient to cover a wide range of real cases.

3. The model is reasonably simple to allow its use without much effort by students of Managerial Economics.
4. The technological change element in the model is particularly suitable for the realities of regulated industries, price-leader-industries, etc: the Steel Industry in Britain and elsewhere being a case of special interest.

The chosen profile of technological change answers all four requirements. In particular it gives a clear solution to the three questions in 1. It is a 'neutral technological change'; it reduces capital costs at the same rate it reduces operating costs. These are both reduced (or increased) exponentially with time. Beside being in line with past development in industry in general it gives the user of the model a fairly good answer even in cases where the change is biased either to capital saving or to labour saving.

The assumption that technological change is exponential and not linear follows economic logic and as it turns out does not complicate the use of the model. The model can cater for three types of technological change.

1. Embodied technological change - technological change that affects costs in new plant only. (This is the standard interpretation of technological change in the model - the parameter 'b')
2. Disembodied technological change that affects both new and incumbent plant. (This factor is like a negative

wage increase, i.e. negative 'a')

3. Residual technological change that affects only existing plant, mainly through improved usage. (This is a negative deterioration, i.e. negative 'c')

8.2.2. Uses of the S - Model

The model has been used for a number of years with considerable success by students of Managerial Economics at the Graduate Business Centre of the City University. The model has been used in a special research project on steel prices at Nottingham University¹⁾.

The model specifies technological change as a separate parameter. This parameter is of theoretical value and is useful in class work. In practical applications a simpler concept can be used instead - (Within the S - model). It is cost reduction (or increase) in successive new plants which is the impact of the above mentioned pure technological change offset by the concurrent wage increase²⁾. This pair of forces has the same effect on operating costs and capital costs (the wage increase affects capital costs through its impact on the capital-goods-industry).

On balance it seems that the approach to technological change in this model is valuable in solving practical problems, in applied economic research and as a teaching tool. There are no absolute ways of assessing its incremental value to existing theories and

1) The results of that work constitute Part III of Sir R. Shone (1975) op. cit.

2) Using the notation of Chapter 6 it is (b-a).

methods. Perhaps the main value of the model and the innovative content of it is to produce reasonably good solution to each one of the problems it deals with - pricing, new investment and replacement and to demonstrate the inherent relationships of all three.

8.3 S - Model Assumption and 'Real Life' Considerations ¹⁾

The S - model like any other specified decision model uses some simplifying assumptions about what parameters determine optimal decision. In this section I shall discuss elements that are not covered by the model. Some of these elements can be supplemented to it without difficulties, others require different interpretations of the structural parameters and yet others cannot be incorporated in it at all. Of the elements not included some are of minor importance because they do not affect the calculated values much, others because the cost of wrong decisions, due to their omission is small. Some elements are not included because they apply only to rare cases. The importance of elements like scrap value or risk cannot be easily denied and their omission from the model was a matter of judgement during the process of the model construction. The neatness of the model and the clarity of exposition were weighed against their inclusion and were found preferable to loading the model by the additional variable. As a matter of fact all deterministic models are based on preference of clarity and neatness to the inclusion of risk elements. The following discussion does not give an exhaustive list of all the factors that govern relevant decisions, its aim is to cover what I regard as the important factors and to demonstrate why it is impossible to include some of them in a workable system.

8.3.1. Different Time Patterns for Elements that are Included in the Model

- i. Technological change - (cost reduction) is assumed embodied in new investment, neutral in the sense that

1) No strictly comparable discussion, "MAPI model assumptions and 'real life' considerations" is given in this thesis. But much of such discussion should be implied from this Chapter and other places in this thesis.

it affects operating costs and capital costs to the same extent. It is assumed that cost reduction is gradual and is at an exponential rate. In reality cost reduction of a given production process moves along these lines only by chance. Usually it is different, cost reduction is a mixture of embodied and disembodied changes it is not neutral and it occurs in 'jumps', furthermore, these jumps are in many cases foreseeable. On the other hand, it would be most cumbersome to introduce a 'jump' parameter in a general decision model. If the S - model is to be judged not on what it excludes but on its incremental merits to existing models certainly an important point in its favour is the way it deals with technological change. The other 'practical' models Terborgh's and Merrett and Sykes's ignore the impact of technological change on capital costs, they do not segregate a 'pure' technological change factor and a wage increase factor, they rather look at the mixed effect. Their assumption of linear cost reduction can create serious decision errors.

- ii. Deterioration - is assumed to start immediately after the installation of new plant and to be an exponential function of operating cost, thus not affecting output at all. Again, in reality deterioration does not necessarily start immediately. More often than not, it is a time function of maintenance cost and not other operating costs. In some cases an important element of deter-

ioration is the reduction of output. It is impossible within the strict assumptions of the model of a constant number of units of output per time, to cater for this type of deterioration. Looser assumptions are required for that. Here is the place to say that the other models bypass the problem by dealing with money proceeds only and by avoiding the question of whether reduction in income is due to increased costs or reduced output or both. The general view behind this model is that in most relevant cases the impact of deterioration is secondary to obsolescence in importance and that a constant rate over the life is a convenient and useful 'average'.

- iii. Capital Investment. If strictly interpreted the model assumes capital investment to occur at a point of time rather than over a number of years. Moreover, this interpretation does not allow for a running-in period and full production is assumed to start immediately after investment. The problem it creates is more likely to arise in major new projects than in replacement projects. The difficulty in using the model may nevertheless be mitigated by capitalising all capital costs into the last year before full production starts.

8.3.2. Other Elements

- i. Scrap value - is omitted from this model. (See discussion in 8.1.4., above). The case where this omission might lead to expensively wrong decisions is where there is a great reduction in discounted scrap value at about the time signalled by the model for the 'length of life' of the investment, or where the discounted scrap value is very high. In both cases the DCF return might be

significantly different from the one calculated. The reason for the exclusion is doublefold. It facilitates very easy and still meaningful solutions within the theoretical background underlying the model. Secondly, in most industrial applications - whole plants, heavy machines and complex installations - there is no much scrap value as such because the costs of transfer and reinstallation of old equipment would be prohibitive.

ii. Risk - is an important element that is not included explicitly. That is to say, the model does not deal with more than a single line of outcome. Technical change, deterioration and wage increase rate are all predetermined. The model does not cater for the possibility of other values of the parameters than those explicitly used. The deficiency is mitigated by a careful choice of the parameters. (e.g. in expansion investment decisions, by raising the required return rate in the calculation.) The use of charts broadens the scope for risk or sensitivity analysis in the way of showing the impact of changed parameters on the results. The charts nevertheless are not a sufficient sensitivity analysis tool as they cannot show the impact on profitability of errors in the estimation of parameters.

iii. Taxation - is not inherently included but it can be incorporated in the model, as shown in Chapter 6 (therefore, it will not be discussed here).

- iv. Financial considerations - arise when various financial alternatives are attached to the projects. Examples are: hire purchase schemes, payment agreement, buy or lease considerations. The whole range of different financial circumstances in fact, revolves around the question of a multiple choice of equipment a question avoided by other models as well as this one. There is no ready solution to the problem. The rate of gearing is a most important financial consideration even when it is not related directly to a single project. It affects the firm's cost of capital and interacts with taxation.
- v. More action alternatives. The number of alternatives that a firm has, especially in replacement problems is undoubtedly greater than just buying new equipment or retaining old equipment. First, there can be a purchase of secondhand machines of various ages, second, there can be an overhaul and modernisation of old plant and third, there can be the choice of time; introduction of new plant does not have to be done simultaneously with closure of old plant. There is the argument of keeping old plant as standby to cope with over-capacity or the argument to use plant, e. g. vehicles, on more limited assignments than prior to their displacement.
- vi. Change of scale. The model does not deal with the case where there is a change of scale of production in successive vintages of equipment. It does not deal with the

problem of partly combined replacement and expansion.

The reason for that is that the model in its more restrictive interpretation is centred on the concept of unit of output (i.e. fixed effective capacity) per unit of time. Therefore, all the values: capital costs, operating costs and, revenue are determined for a fixed output. A different scale of output would require a different set of parameters. But it must be added that even a looser interpretation of the parameters as in Terborgh's work does not overcome the fundamental difficulty of answering simultaneously a cost minimization problem (replacement) and profit maximization (expansion).

- vii. Technical change concept . Technological change here is simply cost reduction per unit of output. The model is based on the assumption that technological change affects all users in exactly the same way and at the same time. This is a part of perfect competition situation or its hypothetical imposition (long range marginal cost pricing in the Nationalised Industries) that serve as a key assumption behind this multiple purpose model. In reality of course, diffusion of new methods takes time. Some users are quicker than others in using new technologies undoubtedly there is the element of "learning by doing" - the act of investment in new technologies enhances technical advancement. This sort of relation where the rate of technological change and the frequency of replacement are mutually determined is not sought in the model

and probably is not of much importance in actual decisions: technological change is, therefore, given as an external parameter. The model can cope with both embodied and disembodied types of technical change¹⁾.

- viii. Multiple process, multiple output, multiple market. These are vast bodies of complications. They all cause serious difficulties in pricing problems and in expansion investment problems. The S - model as an economic model is a single process, single output, single market model (and so are in fact the MAPI model and the ORM model - if one attempts to attribute to them an economic meaning rather than a financial meaning). The difficulties are not so great perhaps in applying the model to replacement problems. Multiple output and multiple market are considerations in income maximisation not in cost minimisation. Replacement of a multiple process requires more details but is not different in principle from replacement of a single process.
- ix. Business cycles. This is another problem of change in volume of output, it is mainly in decision on timing of replacement. Its explicit introduction would make the model complicated but there are cases when real decisions sought ought to take account of market conditions. Availability of funds too, enters this line of consideration.

1) Disembodied change in the form of negative 'c' or 'a' parameters in the basic S - model in Chapter 6.

- x. Limited Service Life Problems are met in many expansion and replacement problems. The service given by an asset in question is known to discontinue as from a certain point in the future. This is because a contract ends because of the advent of a substitute to the service given by the asset, or because production is due to start in a new factory so that the machines in the old one will be scrapped (therefore there is no reason to link a machine in the old plant with a similar machine that will be used in the new factory sometime in the future). Again this type of problems requires a separate treatment from the one given by the model.

8.3.3. Final Remarks

By listing a great number of complications in the use of the model I only tried to throw some light on the genuine difficulties involved in taming the general problem. It was not suggested that other models do the job better. On the contrary, other models only evade certain problems. For example, by dealing with monetary yearly income they simply avoid the question of whether deterioration is due to cost increase or due to reduction in output. If they at all seem applicable to a greater number of problems than the S - model it is because they give looser interpretation to highly comparable parameters.

8.4 Summary

The S-model and the underlying model of the MAPI are structurally very similar though there are some obvious structural differences between them. Greater differences appear when the models are used as decision methods, i.e. as replacement decision methods. Thus the MAPI method culminates as a manual that uses charts drawn for a very restrictive range of the value of the main parameters. The S-method covers a wider range of these parameters but at the same time does not directly include considerations of tax and future scrap values.

Similarity between the S-model and basic linear model is illustrated. Firstly the S-model is defined by way of two standard assumptions in analogy to the way the original MAPI model is defined and secondly the decision rule of the discrete S-model is constructed in the form of explicit next-year-comparison, the way the Merrett & Sykes's model is constructed.

Differences between the S-method and the other methods appear in a number of areas. First the S-model is an exponential model while the others are linear. The future movements of costs in the linear models are expressed independently of the initial costs while in the S-model they are not. There are differences in the treatment of scrap value and in the apparent investment criteria. The linear models use the length of life as an external variable unlike the S-model which prefers an explicit use of the rate of technological change. The S-model is drawn for a multiple cut-off-rate while all the others employ a single rate. A numerical comparison highlights basic differences between the models.

Technological change is a key factor behind the construction of the S-model. The model is capable of including various types of technological change in the three problems, pricing investment and replacement it caters for.

Many considerations that enter real life investment or replacement decisions are left out of decision models, the S-model is no exception. The ability of the S-model to cope with some of these considerations is discussed.

CHAPTER 9

CONCLUDING OBSERVATIONS

Most of the observations and conclusions in the various subjects discussed in this study are given in earlier Chapters and are condensed in Chapter summaries. They will not be repeated here. There nevertheless are some more general observations, these will be given here.

9.1 Robustness of One-Year-Comparison in Replacement Problems.

The intuitive approach to the replacement problem, the one advanced by Smith¹⁾ and practised to a certain extent by Merrett & Sykes²⁾ is to find the optimal date of the first replacement. Once this date is found then one knows whether to replace immediately or not.

The methods advanced in this study, the S-method (in its discrete form)³⁾ and the MAPI method⁴⁾, one way or another are based on comparing immediate replacement with a one year retention of existing asset followed then by replacement. These methods do not directly examine how immediate replacement compares with a 2-year postponement, a 3-year postponement, etc. Terborgh in his first book⁵⁾ tried to do such a multiple comparison by computing an 'adverse minimum' of the defender, but later abandoned this additional comparison.

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- 1) V.L. Smith, "Investment and Production", Harvard University Press Cambridge, Mass., 1966.
 - 2) A.J. Merrett & A. Sykes, "Capital Budgeting and Company Finance", Longmans, London, 1966.
 - 3) Sir Robert Shone, "Price and Investment Relationships", Elek, London, 1975.
 - 4) Latest Version: G. Terborgh, "Business Investment Management", MAPI, Washington D.C., 1967.
 - 5) G. Terborgh, "Dynamic Equipment Policy", MAPI, Washington D.C. 1949.

The decision rule employed therefore is: "accept a replacement proposal if immediate replacement is preferable to a one year postponement, reject a replacement proposal if otherwise". Let us now investigate where, within the behavioural cost assumptions this rule will lead to the universally correct answer and where it will not. The possibilities are:

- (1) Immediate replacement is preferable to all later replacement dates. The rule indicates 'replace now' and it is correct.
- (2) Replacement next year (and perhaps replacement at later dates as well) is preferable to immediate replacement. The rule indicates 'do not replace', and it is correct.
- (3) Immediate replacement is preferable to replacement next year but a first replacement at some date later than next year is preferable to immediate replacement. The rule indicates 'replace now' and it is incorrect.

It may be said that, within the assumptions, the decision rule satisfies sufficient conditions for 'rejection' and only necessary (but not sufficient) conditions for acceptance of replacement proposals. The rule therefore never wrongly indicates rejection. It may though (possibility (3) above) commit the error of indicating 'action', i.e. replacement, while 'no-action' is the right answer.

The decision rule thus is 'myopic'; and this has caused concern to some writers¹⁾. I shall briefly show that this 'myopia' of the S-model and of the MAPI model is an extremely robust and powerful simplification.

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- 1) e.g. a) B. Shore, "Replacement Decisions Under Capital Budgeting Constraints", The Engineering Economist, Summer 1975, pp. 243-256.
 - b) D.W. Gentry and T.B. Johnson, "An Investment Policy for Equipment Replacement", Mineral Industry Bulletin Vol. 17 (Jan. 1974) pp. 1-13.

Myopic investment decision rules are mentioned in other contexts in economics sometimes they are rejected (on theoretical grounds)¹⁾ and sometimes totally accepted²⁾. The justification here will be based on mathematical grounds as well as on institutional grounds. The justification will follow the argument that cases where the rule is wrong are rare and if it is wrong the costs of being wrong are relatively small.

Table 9.1 demonstrates the analysis required in order to investigate how sufficient is the basic rule of comparing immediate replacement with a one year postponement of replacement. For simplicity I have included in the Table only one additional option, that of replacing in two years time. Using a MAPI model and following the discussion in Chapter 5 we may say that once the first replacement has taken place then all relevant costs associated with the succession of 'challengers' to follow are equal to a constant annuity series of the 'adverse minimum'. Thus, within this model, the only costs that may upset the 'myopic' decision rule are those related to the 'defender'. So what we are looking for in Table 9.1 are cases where immediate replacement is preferable to replacement in year 1 and where replacement in year 2 is preferable to replacement now - two requirements that will be quantified below.

Assuming a discounting factor ' r ' let us look at the simple case where the defender has no salvage value. This may turn out a rather artificial case but it will conveniently illustrate the analysis and the type of conclusions to be deduced from it.

1) S.A. Marglin, "Approaches to Dynamic Investment Planning", North-Holland, Amsterdam, 1963.

2) K.J. Arrow, "Optimal Capital Policy, The Cost of Capital and Myopic Decision Rules", Annals of the Institute of Statistical Mathematics, Vol. XVI, Nos. 1-2, 1964, pp. 21-30.

Table 9.1 Costs and Cost Equivalents Associated with Different Dates of First Replacement (Model used - basic MAPI model)

	Year	0	1	2	3	4	...
(a)	Replace 'now'	-	A	A	A	A	...
		(-S ₀)					
(b)	Replace next year	-	OI ₁	A	A	A	...
			(-S ₁)				
(c)	Replace in year 2	-	OI ₁	OI ₂	A	A	...
				(-S ₂)			

Notation:

- A - Adverse minimum - annuity equivalent of the relevant costs of the new asset and its successors, computed as from the year following replacement.
- S_i - Salvage value (if any) of old asset at the time of replacement (i = 0, 1, 2, ...)
- OI_i - 'Operating inferiority' of old asset (the defender) i.e. the difference in operating cost in year i between the old asset and the best available new asset in the year (i = 1, 2 ...)

Note: A similar table can be constructed on the basis of S-model. The MAPI model is slightly simpler here as it employs the same adverse minimum 'A' every year. The S-model would use a different adverse minimum value every year (though it would be the same for all options in a year). See 8.11.

First requirement is:

PV ("Immediate replacement" less "Replacement next year") < 0

or

$$PV = \frac{(A - OI_1)}{1 + r} = \frac{X}{1+r} < 0 \quad (\text{Using } A - OI_1 = X)$$

Second requirement is:

PV ("Immediate replacement" less "Replacement in two years") > 0

or

$$PV = \frac{A - OI_1}{1 + r} + \frac{A - OI_2}{(1 + r)^2} > 0$$

$$\text{or } (A - OI_2) > -X(1+r)$$

So one may deduce that in order for the decision rule to be incorrect it is not enough that the "operating inferiority" in year 2 be lower than the adverse minimum it has to be substantially lower to counter the impact of X in year 1 and that of the interest on X. In effect what is sought is reduction of $2X + Xr$ in the operating costs of the defender between years 1 and 2. The higher X is, the more unlikely is the possibility that replacement in year 2 will be preferable to replacement now.

As said above, the example proved to be artificial. This in itself stresses how unlikely such situations are. Normally, one would expect that:

$$OI_2 \geq OI_1$$

It is possible, of course, that the defender may have a positive salvage value. I shall not outline this mathematically, the procedure for doing it is the same as above. Cases where difficulties arise are just as artificial as in the example 'without salvage value' above, though, perhaps, less objectionable¹⁾.

The S-model and the MAPI model give the detailed cost behaviour of the first challenger and they assume certain relations between successive vintages of equipment. There are nevertheless realistic situations outside these assumptions. If there is expected to be a technological break-through that will result in a substantial cost reduction to be started in two years time and if this is not reflected in costs of earlier vintages of equipment then the standard 'myopic' rule used will not suffice.

A judicious use of the decision rule of comparing immediate replacement with a postponed one is still, in most cases, an extremely useful way of reducing information about a replacement problem, even if not directly used in the decision making. So, a very clear indication to

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- 1) If we assume constant salvage value or even a constant annual decrease in salvage value the situation will be the same as above, where only an 'absurd' value of OI_2 will render the decision rule incorrect.

A more subtle situation arises if one assumes a large drop in salvage value over the first year such that would make immediate replacement marginally preferable to replacement in a year's time. If this is followed by very low or no further reduction in salvage value in the second year, coupled with low or no increase in operating inferiority then there will be a situation where the myopic decision rule will be incorrect. But even then, over two years, the cost of being incorrect does not exceed the discounted value of the drop in salvage value over the first year (this conclusion is developed from Table 9.1).

replace or a very clear indication to avoid replacement are particularly valuable. There are few other approaches that would do that much.

The use of a one year comparison or any implied decision tool derives its strength also from the institutional framework in which decisions are made.

If budgeting and tentative authorisation of projects is an annual exercise then the natural period of possible postponement of a replacement is one year which therefore should be regarded as the immediate alternative to replacement 'now'.

A justification for limiting the comparison to a one-year-comparison is the fact that detailed budgeting is done for a period of only one year. There is little or no value in determining today an exact date for a future replacement. This is so because the question of replacement could and should be re-examined in the future taking into account relevant information that will be available then. See Figure 4.1.

9.2 Budgeting, Rationing and the Use of Replacement Models.

Here and there in Chapter 4 above I pointed out that companies do not always employ a purely financial budget. Such a budget would select any project put forward that would satisfy a given profitability test showing that the project will generate sufficient income to pay for the funds it requires. With varied frequencies companies may add and employ more constraints in the process of projects selection - a situation generally described as 'rationing'. Thus, a company may delimit the volume of funds allocated for replacement investment in a given year. Alternatively it may require that replacement projects should satisfy a cut-off rate that is higher than what funds actually cost.

There are a number of reasons for this practice, some of them associated with business cycles. In periods of boom companies may have special preference for expansion projects. In periods of severe recession companies may scrap old plants without matching this with any explicit replacement. Such cyclical phenomena usually are the product of rationing budgets. Sometimes cycles are catered for without the need for rationing, so when funds are abundant and the cost of capital low more replacement projects than the perennial average will pass the profitability test and vice versa in periods of shortage of funds.

The practical replacement models can conveniently fit in within a financial budgeting system (provided they cover the particular cost of capital used by the system). A question to ask would be: how useful can such stylised models be within a rationing budget system?. My answer is: they are still useful tools of reducing information; but one has to treat them with caution.

The S-model of course has not been developed in a manual-form, so one cannot give a very specific answer as to how a rationing budget can accommodate it; some points of interest and difficulty will however be mentioned. If the rationing is in the form of a cut-off rate higher than the company's cost capital either for replacement proposals alone or for a mixed-bag of replacement and expansion proposals then this higher cut-off rate may be used. If the rationing is in volume of replacement budget then some additional measure of profitability like Terborgh's 'urgency rating' can be employed. If the rationing is in the form of a fixed overall investment budget, so that replacement and expansion proposals compete for the same pool of funds then there is no simple single criterion for accepting projects. Perhaps the proper solution in this case is to produce a set of information - one that can be judged by a number of criteria. As far as replacement proposals go such a set would include: the amount of capital required, the amount by which the saving in operating cost exceeds 'F' the first year earning,¹⁾ the required rate of return, etc.

The MAPI and the ORM schemes give answers that are linked to particular tax and cost of capital situations. Following a fashionable trend in the 1960s these answers are already in the form of a discounted rate of return. Their solutions may be useful if there is a rationing budget for replacement projects: projects can be ranked according to their relative return ('urgency rating' or 'DCF return on extended yield'). If the rationing is determined by a higher cut-off rate hurdle then their solution is of very limited use. (Charts based on these methods are constructed for a given single cut-off rate). This is so particularly

1) See definition of 'F' in 7.3; also refer to Table 6.1.

if expansion projects compete with replacement projects for the same funds. The most deceiving and therefore undesirable use of these models is where both expansion and replacement projects compete for funds allocated from the same fixed investment budget. Here the relative return concept can be a very misleading tool for ranking. The practical solution as mentioned above in the context of S-model is to produce information that will be judged by multiple criteria.

9.3 An Interpretation of the Technological-Change Factor in Replacement Models.

Loosely defined, the 'technological-change' factor may be regarded as the same economic force in all replacement models. One may detect, though, a number of possible ways to interpret this force.

The term technological change or obsolescence as used by V.L. Smith¹⁾ and by Merrett & Sykes²⁾ has one, particular, sense. It describes expected (operating) cost reduction in future vintages of plant and equipment as they become available. The immediate use of such technological progress is in calculating the 'opportunity cost' of keeping an incumbent asset for a prescribed time - as against replacing it; The 'opportunity cost' is the 'benefit forgone', i.e. the benefit from replacement. The moment to replace is when this opportunity cost exceeds the benefit from keeping the incumbent asset for the prescribed length of time.

1) V.L. Smith, Op. Cit.

2) A.J. Merrett & A. Sykes, Op. Cit.

The second approach to technological change is more conveniently associated with the S-model¹⁾. It is not a separate approach but rather a complementary approach. If the previous approach regards technological change as an improvement of future alternatives, then the second approach regards technological change as a direct constraint on today's investment one which shortens the (expected) life of investment carried out today, as compared with a situation where such constraint does not exist. As a production (i.e. investment) constraint technological change has its cost or 'shadow price'. Replacement, or for that matter any investment, will be justified only if the benefit from it will cover the cost of that constraint. In the S-model the cost of the technological change constraint forms a part of the 'F' factor.²⁾

This interpretation of technological change as a direct, nearly explicit, production constraint associated with investment today has an important insight value, in particular in the face of technological uncertainty. It enables us to relate the magnitude of a required 'F' to the 'severity' of a technological change 'constraint'. We know that the higher the technological change rate the sooner will equipment installed today be replaced and the faster will be the implementation of yet further future improvement, embodied in the next generation of investment. But this also means that new equipment installed today will have 'to pay for itself' the required cost of capital in a shorter period, or, in other words, the faster the technical change the higher is 'F'. The Tables and Charts in Chapter 7 illustrate the sensitivity of F (and the life of investment) to the rate of technological change.

1) See Sir R. Shone, Op. Cit.

2) See definition of 'F' in 7.3; also refer to Table 6.1.

In 3.3.4 above I discussed changes in the expectation of technological change showing e.g. how an increase in the expected rate of technological change would raise the required output price.¹⁾ Such an argument can be quantified with the aid of the Tables and the Charts. These may show the required increase in 'F' to compensate for a marginal increase in the expected rate of technological change.

1) See discussion in 3.3.4 above. Detailed investigation of these changes is given by:
H.R. Fisher, "Obsolescence and Optimum Replacement Timing",
The Chemical Engineer, April 1963, pp. 86-96.

9.4 Summary

The theoretical formulation of the replacement problem looks at the replacement problem as the selection of the optimal date of replacement. The solution procedure of the S-model and the MAPI model which in effect compares replacement "now" and "next year" give only a 'local optimization', confined to these two possible dates. Ignoring possible later dates for the first replacement the decision rule these models employ may therefore be considered a 'myopic rule'. It is not the object of 'practical' models to forecast the optimal replacement date but simply to answer whether to replace this year or not. As a 'proxy' to answering that question the decision rule appears to be extremely robust. It is powerful mathematically and justified institutionally.

Though models like the S-model and the MAPI model are devised for financial budgeting purpose they are still of some use in a rationing budget.

It is expedient for the purpose of investment appraisal to consider future technological change simply as an investment constraint: a factor that reduces the attractiveness of investing in replacement of existing assets. The Tables and Charts in Chapter 7 are useful in quantifying the effects and changes in expected technological progress have on pricing, investment and replacement.

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