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MATRIX PENCIL REPRESENTATION OF STRUCTURAL TRANSFORMATIONS OF PASSIVE ELECTRICAL NETWORKS

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ABSTRACT

The paper examines the problem of systems redesign within the context of passive electrical networks by considering the problem of multi-parameter and topology changes, and their representation. This representation may be used to investigate the impact of such changes on properties such as characteristic frequencies. The general problem area is the modelling of systems, whose structure is not fixed but evolves during the system life-cycle. The specific problem we are addressing is the study of effect of changing the topology of an electrical network that is changing individual elements of the network into elements of different type and value, augmenting / or eliminating parts of the network and developing a framework that allows the study of the effect of such transformations on the natural frequencies. This problem is a special case of the more general network redesign problem. We use the Impedance-Admittance models and we establish a representation of the different types of transformations on such models. The representation of the structural transformations is given in terms of the companion pencil that preserves the natural topologies of the RLC network.

Index Terms— Systems Theory, Networks Theory

1. INTRODUCTION

The problem of redesigning autonomous (no inputs or outputs) passive electric networks [1], [2] aims to change the network (natural frequencies) by modification of the types of elements, possibly their values, interconnection topology and possibly addition, or elimination of parts of the network. As such, this is a problem that differs considerably from a standard control problem, since it involves changing the system itself without control and aims to achieve the desirable system properties, as these may be expressed by the natural frequencies by system re-engineering. In fact, this problem involves the selection of alternative values for dynamic elements (inductances, capacitances) and non-dynamic elements (resistances) within a fixed interconnection topology and/or alteration of the network interconnection topology and possible evolution of the (increase of elements, branches). The aim

of the paper is to define an appropriate representation framework that allows the deployment of control theoretic tools for the re-engineering of properties of a given network when there are multi-parameter variations within a fixed, or variable cardinality network topology. We use impedance and admittance modelling [2], [3] for passive electrical networks and consider here systems with no sources (autonomous descriptions), since our current interest is on the shaping of natural frequencies. The emphasis here is on the study of the different representations of the passive network that enable the investigation of the transformations on such models as structural transformations. The problem considered here is:

- Define the representation of changes of a many dynamic, or non-dynamic elements with preservation, or alteration of existing topologies without changes in the overall nodal or loop cardinality of the network and define a framework for studying natural frequency assignment.

The overall aim is to explore the structure and representations of the Impedance-Admittance model $W(s)$ and introduce appropriate representation of the above transformations which enable the study of the shaping of natural frequencies. Matrix representations of the above transformations are introduced as additions of structural transformations on the $W(s)$ model. For RL (resistor-inductor) or RC (resistor-capacitor) networks the corresponding impedance or admittance models become matrix pencils and it has been shown that the single parameter variation problem is equivalent to Root Locus problems [4]. The general case of RLC networks is considered and we introduce the notion of companion pencil, $sF + G$, that has the same non-zero structure with $W(s)$ and preserves the natural topological properties of the network. We establish the representation of cardinality preserving transformations as additive transformations on $sF + G$ [5] and show that natural frequency assignment may be studied within the exterior algebra framework of the Determinantal Assignment Problem [6], [7], [8].

2. PASSIVE NETWORK MODELS AND TOPOLOGIES

2.1. Impedance and Admittance Models

In the network loop analysis method, the variables are selected such that the vertex law is automatically satisfied. Here, we consider only planar graphs with b branches and n vertices. We then consider the variables associated with each of the meshes and we define them as loop variables. The overall system is reduced to a number of meshes, which are $[q = (b - n + 1)]$ [3],[9], referred to as *loop cardinality of the network*. If we denote by $(f_1, f_2, \dots, f_q)$ the set of the Laplace transforms of the loop currents and by $(u_{s1}, \dots, u_{sq})$ the set of Laplace transforms of equivalent voltage sources, then the loop or impedance model is defined by [10]:

$$Z(s)f(s) = u_s(s) \quad (1)$$

where $Z(s)$ has elements $z_{ii}(s)$ expressing the sum of impedances in loop i and $z_{ij}(s)$ is the sum of impedances common between loops i and j . This is known as the loop or impedance model and it is an integral-differential symmetric matrix and $Z(s)$ is the *network impedance matrix*.

Alternative modeling is the method using the across variables from each vertex to some reference vertex. The number of vertex equations is in general $p = (n - 1)$ and will be referred to as *nodal cardinality of the network*. If we denote by $(u_1, u_2, \dots, u_n)$ the Laplace transforms of the node voltages and by $(i_{s1}, \dots, i_{sn})$ the set of Laplace transforms of equivalent current sources, then the node or admittance model is defined by [2]:

$$Y(s)u(s) = i_s(s) \quad (2)$$

where: $y_{ii}(s)$ is the sum of admittances in node i ; $y_{ij}(s)$ is the sum of admittances common between nodes i and j . This is referred to as the node or admittance model and it is an integral-differential symmetric matrix and $Y(s)$ is referred to as the *network admittance matrix*.

2.2. The Autonomous Natural Impedance-Admittance Model and Topologies

When we consider networks with no inputs (no current, or voltage sources) the resulting admittance, or impedance network models may be described in a unifying way as:

$$\{pB + p^{-1}C + D\}\underline{x}(t) = \underline{0} \quad (3)$$

where p , $p - 1$ are respectively the differential, integral operators respectively and $\underline{x}(t)$ is the vector of nodal voltages, or loop currents. Such a description may be referred to as the natural autonomous network description and the operator $W(s) = sB + s^{-1}C + D$ will be called the natural network operator. Note that for the case of admittance we have that B is a matrix of A-type elements (i.e. mass, inductance, capacitance), C is the matrix of T-type elements (i.e. spring,

inductance) and D is a matrix of D type elements (i.e. resistance). For the case of impedance the reverse holds true. Hence, B is the matrix of T-type elements, C is the matrix of A-type elements and D is the matrix of D-type elements. The symmetric operator $W(s)$ is thus a common description of $Y(s)$ and $Z(s)$ matrices. The operator $W(s)$ describes the dynamics of the network and of special interest are the properties of its zeroes. Network modeling uses the system graph, which is the basic topological structure that generates the system equations. We may introduce some additional topologies, which are linked to the specifics of the Node and Loop analysis. The detailed topological structures that emerge depend on the nature of the elements in the network. The mass, inertia and capacitance store energy by virtue of their across- variables (velocity, voltage) and they are referred to as A-type energy storage units [2]. Springs and inductances store energy by virtue of their through- variables and are called T-type energy- storage devices. The dampers and resistances dissipate energy and will be called D-type elements.

2.2.1. The Vertex Topology

Every network may be represented in terms of a set of vertices, or nodes and all branches between two vertices may be represented by an admittance function. The nature of the elements in the branches of the natural vertex graph defines an element dependent topology, which is characterized by adjacency type matrices. If we set the external sources to zero, the reduced graph will be referred to as the kernel vertex graph. For a given kernel vertex graph we define A-vertex sub-graph by eliminating from the kernel vertex graph all T- and D-type edges. Similarly, we define the T-vertex sub-graph by eliminating all A- and D-type edges and the D-vertex sub-graph by eliminating all A- and T-type edges. The sub-graph of the natural vertex graph obtained by eliminating all T-, D-, A- type elements represents the location of the through variable sources and will be called the source-vertex sub-graph, or simply S-vertex sub-graph.

2.2.2. The loop topology

The loop topology is a notion dual to that of the vertex topology and it is based on the following principle: Every network of n vertices and b edges (branches) may be represented by $q = (b - n + 1)$ loops leading to independent equations. All branches common between two loops may be represented by an impedance function. Specification of the values of through variables for the loops defines the values of all across variables in the network. In a similar way to the case of nodal analysis, we may define the loop topology based on the kernel loop graph and its sub-graphs the A-loop sub-graph, the T-loop sub-graph, the D-loop sub-graph and the source-loop sub-graph [2].

3. THE LINEARISATION OF THE AUTONOMOUS NATURAL IMPEDANCE-ADMITTANCE MODEL

Starting from the integral-differential model of (3), described by the operator $W(s)$ the natural question that arises is how we can transform it to an equivalent first-order, matrix pencil description, which preserves the topology of the network. We introduce a new set of variables, $\hat{x} = [x, \tilde{x}]^t$, $p^{-1}\tilde{x} = \hat{x}$ which reduces (3) to a first order description given by equation (5) which has an associated matrix pencil $sF + G$ defined by (6) and referred to as the network matrix pencil which is defined:

$$\begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} p\tilde{x} \\ \tilde{x} \end{bmatrix} = \begin{bmatrix} -D & -C \\ I & 0 \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix} \quad (4)$$

$$sF + G = \begin{bmatrix} sB+D & C \\ -I & sI \end{bmatrix} \quad (5)$$

Note that the above autonomous differential description preserves the topological properties of the network as these are represented by the B, C, D matrices, but its dimensionality is not necessarily minimal (dimensionality of $sF + G$). The pencil derived is structured, but not symmetric in the general case and it will be referred to as the *companion pencil* of the network. The zeroes of $W(s)$ define the natural frequencies of the network. Note that:

$$\left| \begin{bmatrix} sB+D & C \\ -I & sI \end{bmatrix} \right| = s^k |sB + s^{-1}C + D|$$

or

$$|sF + G| = s^k \cdot |W(s)|, W(s) \in k \times k(s) \quad (6)$$

Remark (1): The non-zero natural frequencies of the network are given by the zeroes of the pencil and thus this pencil may be used for the study of assignment of natural frequencies under different types of transformations. For the special cases where the network is characterized only by A- and D- type elements or T- and D- type elements

$$\widetilde{W}(s) = sB + D, \widehat{W}(s) = \widehat{s}C + D, \widehat{s} = s^{-1} \quad (7)$$

which are symmetric matrix pencils [5]. These pencils are derived from passive networks and thus inherit the passivity properties [4], [2].

4. NETWORK TRANSFORMATIONS

The general modeling for passive electrical networks provides a description of networks in terms of symmetric, integral, differential operator, $W(s) = sB + s^{-1}C + D$. It is clear that the network may be represented by the triple of matrices structural transformations $\{C, B, D\}$. The study of the structural changes on the network may be expressed as transformations on the matrices $\{C, B, D\}$. The general classes of structural transformations which may preserve, or alter the cardinality of the network, and may also change its different types of topology are defined below.

4.1. Classification of Structural Transformations

Type 1: Changing the values of the components of the system without changing the topology as this is described by $\{C, B, D\}$ tuple.

Type 2: Altering the nature of components by transformations on tuple without changing the element cardinality of the network.

Type 3: Modifying the networks topology and changing the cardinality of elements by removing components / subsystems.

Type 4: Augmenting the networks topology and changing the cardinality of elements of the system by adding subsystems to the existing topology of the network. In the following we focus on Cases 1, 2 preserving the loop, or nodal cardinality and thus the dimensionality of $\{C, B, D\}$. These transformations are then expressed as:

Definition 1: Given the triple of matrices $\{C, B, D\}$ we consider transformations on the network matrices of the type

$$C' = C \pm c(x, b), B' = B \pm l(x, b), D' = D \pm r(x, b) \quad (8)$$

which preserve the physical elements cardinality (loop, or nodal cardinality) and depend on the real parameter $x \in \mathbb{R}$ and the position vector $b \in \mathbb{R}^k$. In fact, consider the changes $c(x, b)$, $l(x, b)$, $r(x, b)$ which have the general form $f(x, b)$ [4] where:

$$f(x, b) = xbb^T \text{ for } b = e_i, \text{ or } b = e_i - e_j, i \neq j \quad (9)$$

4.2. Examples

Consider the electrical network of Figure (1). The network variables are the loop currents I_1, I_2, I_3 . The impedance model expresses the impedances in the three loops and thus has the form of (9). We now assume that in this network we change the corresponding topology by adding the elements L_4, R_5, C_3 as shown in Figure (2).

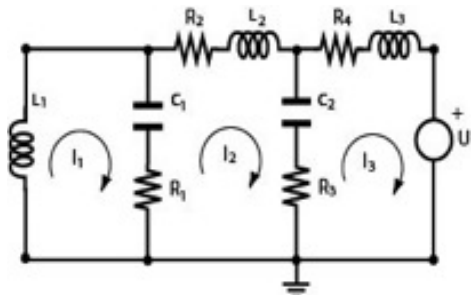


Figure 1: Initial RLC network

Specifically the transformations are:

- Add a resistor to loop 1
- Add an inductance common to loops 1 and 2

- Add a capacitor to loop 2

$$\begin{aligned}
Z(s) &= \begin{pmatrix} C_1^{-1} & -C_1^{-1} & 0 \\ -C_1^{-1} & C_1^{-1} + C_2^{-1} & -C_2^{-1} \\ 0 & -C_2^{-1} & C_2^{-1} \end{pmatrix} s^{-1} \\
&+ \begin{pmatrix} R_1 & -R_1 & 0 \\ -R_1 & R_1 + R_2 + R_3 & -R_3 \\ 0 & -R_3 & R_3 + R_4 \end{pmatrix} \\
&+ \begin{pmatrix} 0 & 0 & 0 \\ 0 & L_2 & 0 \\ 0 & 0 & L_3 \end{pmatrix} s = \\
&= s^{-1}C + D + sB
\end{aligned} \tag{10}$$

Using the formulation (11) the above transformations can be expressed formally with modification to the corresponding matrices as shown below:

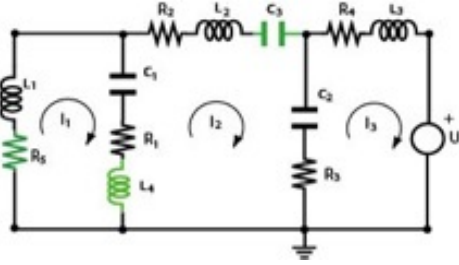


Figure 2: Augmented RLC network

- (i) For the A-type elements:

$$C' = C + \frac{1}{C_3} b_3 b_3^t, \text{ where } b_2 = e_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^t \tag{11}$$

The above expresses the addition of capacitor C_3 to loop 2. Hence, we have:

$$\begin{aligned}
C' &= \begin{bmatrix} C_1^{-1} & -C_1^{-1} & 0 \\ -C_1^{-1} & C_1^{-1} + C_2^{-1} & 0 \\ 0 & 0 & C_3^{-1} \end{bmatrix} + \\
&+ C_3^{-1} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \\
&= \begin{bmatrix} C_1^{-1} & -C_1^{-1} & 0 \\ -C_1^{-1} & C_1^{-1} + C_2^{-1} + C_3^{-1} & 0 \\ 0 & 0 & C_3^{-1} \end{bmatrix} \tag{12}
\end{aligned}$$

- (ii) For the D-type elements:

$$D' = D + R_3 b_1 b_1^t, \text{ where } b_1 = e_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^t \tag{13}$$

The above expresses the addition of resistor R_3 to loop 1. Hence, we have:

$$D' = \begin{bmatrix} R_1 & -R_1 & 0 \\ -R_1 & R_1 + R_2 + R_3 & -R_3 \\ 0 & -R_3 & R_3 + R_4 \end{bmatrix} +$$

$$+ \begin{bmatrix} R_5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} R_1 + R_5 & -R_1 & 0 \\ -R_1 & R_1 + R_2 + R_3 & -R_3 \\ 0 & -R_3 & R_3 + R_4 \end{bmatrix} \tag{14}$$

- (iii) For the T-type elements:

$$B' = B + L_4 b_{12} b_{12}^t, \text{ where } b_{12} = e_1 - e_2 \tag{15}$$

The above expresses the addition of inductance to the branch common to loops 1 and 2. Hence, we have:

$$\begin{aligned}
B' &= B + L_4 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} L_1 & 0 & 0 \\ 0 & L_2 & 0 \\ 0 & 0 & L_3 \end{bmatrix} + \\
&+ \begin{bmatrix} L_4 & -L_4 & 0 \\ -L_4 & L_4 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} L_1 + L_4 & -L_4 & 0 \\ -L_4 & L_2 + L_4 & 0 \\ 0 & 0 & L_3 \end{bmatrix} \tag{16}
\end{aligned}$$

Summarizing, the transformed network is described by the corresponding matrices B' , C' , D' , which lead to the new impedance matrix $W(s)$ describes the above transformations, is given by: $\tilde{W}(s) = s^{-1}C' + sB' + D'$.

Remark (2): The presence of an element of A-, T-, D-type is expressed by an entry in the corresponding matrix C , B , T respectively. In specific:

- If an element is present in the i th loop (node), then its value is added in the i th position of the respective matrix.
- If an element is common to the i th and j th loop then its value is added to the i th and j th loop diagonal entries, as well as subtracted from the (i, j) and (j, i) position of the corresponding matrix.

Theorem (1): Consider a network described by the triple C, B, D with natural operator $W(s) = sB + s^{-1}C + D$ and corresponding companion pencil $sF + G$. Any network preserving cardinality transformation (combination of Type 1 and 2) may be represented by a triple $\{C^*, B^*, D^*\}$ and it results in a companion pencil $sF' + G'$ defined by:

$$\begin{aligned}
sF' + G' &= sF + G + \begin{bmatrix} sB^* + D^* & C^* \\ 0 & 0 \end{bmatrix} \\
&= (sF + G) + (sH + K) \tag{17}
\end{aligned}$$

Thus, structural transformations that preserve network cardinality are expressed as structured additive perturbations on the companion matrix pencil. This allows for the development of a framework for determinantal assignment of natural frequencies discussed next. A version of this problem was recently considered in [5].

5. CONCLUSIONS

The paper has examined the problem of redesign of passive electric networks as a problem describing the structure evolution of systems linked to changes in the nature of topology, and values of the physical elements. Four different types of structural transformations have been defined and for the two which preserve the network cardinality it has been shown that these transformations are expressed as additive structured transformations on the companion pencil. The assignment of natural frequencies of the network may then be formulated as a spectrum assignment of matrix pencils under additive transformations and may be studied within the framework of Determinantal Assignment introduced in [6], [7]. Amongst the problems under investigation is the study of spectrum assignment under special families of (H,K) transformations, the characterization of the fixed frequencies (if any) and the derivation of conditions for arbitrary assignment of such frequencies.

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