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THE STRESSES EXERTED BY GRAIN
ON RETAINING STRUCTURES

By

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A thesis submitted for the degree of
Doctor of Philosophy
in Civil Engineering
in

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ABSTRACT

This thesis is concerned with the loads that agricultural materials impose on the walls of their storage structures with special emphasis on grain in the relatively shallow structures found on farms in the United Kingdom.

A review of past work covers the basis of traditional techniques used to calculate wall loads, the ways in which wall loads have been measured, and the results obtained. It is shown that, when compared with calculations using existing design formulae, the results are very scattered.

A new stress/strain model for bulk grain is developed which uses four stress dependent parameters to relate shear and normal stresses to shear and normal strains. The model represents the behaviour of the material up to and including failure and includes the effect of shear induced dilation. The parameters for the model can all be measured using suitable tests in a triaxial cell. This stress/strain model is incorporated into a computer program, based on the incremental finite element method, that calculates the stresses within the stored material and at its boundaries.

The design of a silo on which wall loads can be measured is described. Considerable attention is paid to the instrumentation for measuring wall stresses. The effect of size of the sensing area and deflection of the gauge on the measured stress is considered in a theoretical manner and investigated by experiment. It is shown that the measured stress is highly sensitive to gauge deflection and a measurement technique is devised to account for the deflection.

A comparison of loads measured in the test silo with those calculated with the finite element program shows

generally good agreement when the wall of the test silo is relatively rigid. The performance of the finite element model in cases where significant wall deflection occurs is however disappointing and further work needs to be done to ensure a more realistic representation of material failure in the computer program.

An appendix compares the void ratio and orientation of the material in the silo with that in a triaxial sample.

LIST OF SYMBOLS

All the symbols have been defined in the text when they first occur but, for convenience, the following list of symbols is also given. Where symbols have been used only locally they have been omitted from this list.

[A]	Matrix of terms in stress/strain model of Chapter 3.
$a_{11}, a_{12},$	Components of [A].
a_{21}, a_{22}	
a_k, b_k	k^{th} Fourier coefficients.
[B]	Element strain/displacement matrix
C_{hh}	Sensitivity of horizontal transducer channel to horizontal force at lower pin joint.
C_{hv}	Sensitivity of horizontal transducer channel to vertical force at lower pin joint.
C_{vh}	Sensitivity of vertical transducer channel to horizontal force at lower pin joint.
C_{vv}	Sensitivity of vertical transducer channel to vertical force at lower pin joint.
D	Total grain depth; bin diameter; diameter of face plate; diameter of disk for probability calculations.
d	Diameter of circles on disk for probability calculations.
[D]	Element stress/strain matrix.
E	Young's modulus.
E_t	Tangent (incremental) Young's modulus.
F_m	Fraction of shear strength mobilised.
F_{ml}	Maximum value of F_m .

F_{mo}	Value of F_m at zero stress.
F_x	Total horizontal force on wall per unit width.
F_y	Total vertical force on wall per unit width.
$[F]$	Global nodal force vector.
$[\dot{F}]$	Incremental form of $[F]$.
$[F]^e$	Element nodal force vector.
K	K value, i.e. ratio of total horizontal force on a wall to that produced by a liquid of the same depth and specific weight.
k	Ratio of horizontal to vertical stress.
k_a	Rankine active value of k .
k_c	Value of k based on Coulomb's method.
k_o	"At rest" value of k .
$[k]$	Global stiffness matrix.
$[k_t]$	Tangent (incremental) global stiffness matrix.
$[k]^e$	Element stiffness matrix.
M_b	Bulk modulus.
M_{bt}	Tangent (incremental) bulk modulus.
M_s	Shear modulus.
M_{st}	Tangent (incremental) shear modulus.
N	Number of grains per unit volume.
P	Resultant total force on a wall.
R	Hydraulic radius of a bin.
$[R]$	Octahedral strain increment/Cartesian strain increment matrix.
S	Non-dimensional horizontal stress on wall.
$[S]$	Octahedral stress increment/Cartesian stress increment matrix.
V	Sum of vertical heights of features in Quantimet sample.

W	Weight of wedge of granular material.
Y	Non-dimensional depth below grain surface.
y	Depth below grain surface.
$\bar{y}$	Depth of centre of pressure below surface.
Z_h	Output of horizontal transducer channel at lower pin joint for no applied force.
Z_v	Output of vertical transducer channel at lower pin joint for no applied force.
α	Slope angle of granular material.
γ	Specific weight of granular material; octahedral shear strain.
δ	Angle of wall friction; deflection of face plate.
$[\delta]$	Global displacement vector.
$[\dot{\delta}]$	Incremental form of δ .
$[\delta]^e$	Element displacement vector.
ϵ	Octahedral normal strain.
ϵ_v	Volumetric strain.
ϵ_1, ϵ_2	Principal strains.
ϵ_3	
$[\epsilon]$	Element strain vector.
θ	Angle of rupture plane to horizontal.
μ	Coefficient of internal friction of granular material.
μ_s	Side wall friction coefficient.
μ_w	Front (main) and back wall friction coefficient.
ν	Poisson's ratio.
ν_t	Tangent (incremental) Poisson's ratio.
ρ	Bulk density of granular material.
σ	Octahedral normal stress.
σ_w	Normal stress on a wall.

σ_x	Horizontal stress (normal stress on y,z vertical plane); Normal stress on a vertical wall.
σ_{xy}	Shear stress in x,y plane.
σ_y	Vertical stress (normal stress on z,x horizontal plane).
σ_{yz}	Shear stress in y,z plane.
σ_z	Normal stress on x,y vertical plane.
σ_{zx}	Shear stress in z,x plane.
$\sigma_1, \sigma_2,$	Principal stresses.
σ_3	
$[\sigma]$	Element stress vector.
τ	Octahedral shear stress.
Φ	Angle of preferred orientation.
ϕ	Angle of internal friction of granular material; angle of rotation of Quantimet sample.
Ω	Magnitude of preferred orientation.

CHAPTER 1

INTRODUCTION

1.1 Changes brought about by mechanization

In the last century or so cereal harvesting has progressed from a hard manual task to an almost completely mechanized one. The first step in this progress was the development of the mechanical reaper which cut the standing crop and replaced the sickles and scythes (and much of the labour) that had previously been used. The first successful reapers, drawn by horses, were introduced at the end of the nineteenth century and represented a significant step in farm mechanization. However, once cut the crop was left on the ground awaiting manual tying into sheaves and stacking of the sheaves into stooks. The development of the reaper was quickly followed by that of the binder. As well as cutting the crop this machine formed it into sheaves which were automatically tied and left in the field for the farm labourer to stack.

During this development thought was also being given to the automatic threshing of the crop. Traditionally this had been done by spreading the crop on a specially prepared part of the barn floor and hitting it with a flail. Early threshing machines were simply mechanized flails and because of their extremely violent action breakages were common. Reliable mechanical threshing was made possible by the advent of the threshing drum. This separated the grain by passing the ears through the small gap between a revolving drum and stationary bars so stripping the grain from the ears.

Mechanical reaping binding and threshing completely changed the way in which grain was harvested. However, the method of storage changed little. Although there was

an increase in the amount of grain stored, it was still put into sacks and stored in the barn.

Perhaps the biggest single factor that changed the way in which grain is stored was the advent of the combine harvester. It was an obvious step to develop a machine that could produce grain directly from the standing crop in one pass and combine harvesters began to appear in the U.K. in the period just before the second world war. The reduction in harvesting time made possible by these machines meant that the total grain product from a season's growing was available in a very short time. This opened up the possibility of bulk processing of the crop - now that cutting and threshing was done in one operation, it made sense to dry the grain and, when market forces dictated, to sell it in one operation. Because of the advantages of bulk handling, the early combine harvesters which discharged into sacks gave way to machines that stored the grain in tanks. In a modern combine harvester these tanks are discharged, sometimes transferring several tonnes at a time, periodically into a trailer.

Mechanized harvesting has therefore changed the way in which grain is stored on the farm. In addition there are other factors that have changed the amount of grain stored, among which are:-

a) Farms have increased in size and have specialized. It is not unusual to have a large farm given over entirely to the production of cereals.

b) Increasing use of fertilizers, advances in plant breeding, harvesting efficiency and crop protection, have given rise to greatly increased cereal yields.

c) Economic factors, such as the balance between achieving a good price and receiving the return on the previous season's investment as quickly as possible, dictate an optimum time to sell. This may mean storing all of the harvested grain for a considerable time.

1.2 Bulk storage structures

Although storing grain in bulk on the farm is a comparatively recent phenomenon, storing grain in bulk per se is not. Very large grain storage structures, designed by proper engineering methods, were found at such places as dockyards and railheads well before they had their place on the farm. These structures tended to be tall compared with their plan dimensions presumably because ground space was at a premium in the built-up areas in which they were constructed. Consequently most of the early interest in design methods for bulk grain stores concentrated on tall structures.

The constraints on the design of farm grain stores are rather different from those on commercial stores. Space is more freely available but, because a store will usually not be in constant use, its capital cost must be lower in order to be economically justified. Farm structures therefore tend to be shallower but take up more ground space than commercial ones. This saves on the depth of foundations and the required strength of the wall leading to a cheaper structure per unit amount of grain stored. The two main types of structure found on the farm are :-

a) Bins. These are usually circular in section and their diameters are roughly of the same size as their height.

b) On-floor stores. These are stores where grain

is stored on a flat floor and retained by walls. The walls are sometimes part of the building structure, usually specially strengthened, and sometimes temporary and erected inside an existing building.

In recent years bins have given way to on-floor stores. This may be due to a variety of causes such as:-

a) A tractor can be driven into an on-floor store and used with a front mounted bucket to handle the grain. As the tractor is part of the normal farm equipment, this represents a saving in capital and a more versatile method than the provision of conveyors and augers which have to be used with bins.

b) An on-floor store can be used for other purposes, such as storing implements, when empty or only partially full of grain.

In early on-floor stores the retaining walls were very low, only one or two metres high. In these cases the design of the wall structure was determined primarily by factors other than the load exerted by the grain, for example the strength required to withstand accidental damage or wear and tear. However, the increasing amounts of grain stored on the farm have tended to increase storage depths and this has meant that the loads imposed by the grain have become a more important part of the design considerations.

The types of grain being stored on the farm are changing. For instance the growing of oats, which once formed a considerable part of grain production, has reduced to a comparatively low level while production of oil seed rape has increased dramatically. As the production of cereals in the European Economic Community goes into surplus the

trend away from cereals is likely to continue and other types of oil seeds (such as linseed) and protein rich grain (such as soya beans if varieties suitable for conditions in the U.K. can be bred) are likely to be produced in quantity.

For these two reasons, increased storage depths and more diverse types of stored materials, the Joint Consultative Organization for Research and Development in Agriculture and Food (J.C.O.) thought it necessary to commission the National Institute of Agricultural Engineering (N.I.A.E.) to carry out a research programme to investigate the loads exerted by agricultural materials on storage structures. This thesis contains an account of part of the research carried out under this commission.

1.3 Definition of terms and scope of the work

In this thesis the word "silo" is used to denote any structure for containing bulk grain. The word "retaining wall" is used to denote a wall retaining the grain in an on-floor store. A "bin" is taken to be a storage structure with a closed cross section.

The contribution made by this work is seen by the author to be the application of a modern technique of stress analysis to the mechanics of stored granular materials. To do this, a new stress/strain model for bulk grain was developed (Chapter 3) which was incorporated into a computer program, written by the author, that calculated the stresses within and at the boundaries of a mass of granular material (Chapter 4). To test the method of calculation, a full scale test silo was designed and built capable of being filled in various configurations with up to about

80 tonnes of wheat. Considerable attention was paid to the fundamental problems of measuring the stresses imposed by agricultural grain on a retaining wall (Chapter 5). A large series of experiments was conducted and the loads on a wall of the test silo were measured (Chapter 6) and then compared with those calculated using the computer program (Chapter 7). Although the mathematics of the computer program has been fully described (Chapter 4) a listing of the code has not been given. The interested reader may obtain such a listing from the author.

Because of the importance attached by the J.C.O. to the study of on-floor stores, the experimental work has been concerned with measuring the loads on flat retaining walls up to 4.5m high. However, the stress/strain model, the subsequent method of stress analysis, and the measurement techniques developed in this thesis should be applicable to most situations where the static behaviour of the stresses in granular materials is of interest.

CHAPTER 2

REVIEW OF PAST WORK

2.1 Methods of calculating wall loads

2.1.1 Introduction

As with many branches of engineering practice the design of retaining structures for granular materials has sometimes been carried out by a combination of experience, intuition and guesswork (so called rules of thumb), and sometimes by methods of a more analytical nature. It is not the intention of the author to present an historical review of methods in the former category as this has already been done by Pearson-Kirk (1967). Instead, the more common types of analytical methods that are still in current use will be examined.

To be mathematically correct, the solution to a problem in the statics of granular materials must satisfy certain conditions which are:-

- a) Every part of the material must be in a state of equilibrium.
- b) The stresses and strains within the material must satisfy the boundary conditions.
- c) The stresses and strains at all points must satisfy the stress/strain law (which in general includes a failure law) for the material.

2.1.2 Trial wedge methods

These methods, developed from the original work of Coulomb (1776), do not consider the stress distribution in the material at all. Instead, a large volume of material is analysed as a rigid body with concentrated forces acting on it. Fig. 2.1 illustrates the method in the special case of a vertical wall with a level filling. It is assumed that a plane of rupture, OB, develops extending from the

bottom of the wall OA to the surface and inclined at an angle θ to the horizontal. It is also assumed that the full internal friction of the material is mobilised along OB and that the resultant thrust, P, of the wall on the wedge OBA is inclined at some angle, δ , which is usually taken to be the angle of friction of the material on the wall. The weight of the material in the wedge, W, can be determined for any value of θ if the specific weight of the material is known and so the polygon of forces (Fig. 2.1, bottom) can be used to determine P. The particular value of θ which applies is arrived at by the following reasoning. A perfectly rigid wall will exert a force on the wedge which is higher than that exerted if the wall is allowed to move slightly. If the wall moves the force drops until a plane of rupture develops. The angle of this plane of rupture must be that for which the wedge needs the maximum support and so for which P is highest. Other values of θ are inadmissible because the material would fail before P could drop any lower. This reasoning reveals another assumption which is often forgotten i.e. some wall movement must take place. The theory does not specify just how much wall movement is necessary before a rupture surface will develop. There is also no guarantee that the stress system in the material will obey both the boundary conditions and the stress/strain law and no certainty of equilibrium for the material outside the area OBA. For instance, the material to the right of OB must exert on OBA a force having a horizontal component equal to the horizontal component of P. If the material sloped down to the ground to the right of point B this horizontal component would have to be provided by friction

at the floor and a smooth floor might not have the necessary coefficient of friction.

A variation on the basic method assumes that the rupture surface is not plane but takes on some assumed curvature, for instance part of a logarithmic spiral (Lambe and Whitman, 1969), but this refinement is used more often where passive conditions (wall moving into the material) are being considered.

An advantage of the trial wedge technique is that it can deal with any shape of surface or wall and can also be used when two walls are sufficiently close together so that the rupture surface intersects the back wall rather than the top surface (Airy, 1897). However, even when the geometry is simple and formulae for P can be derived, they can be complex (Airy, 1897), and in the general case, where P has to be calculated for trial values of θ until a maximum is obtained, the method is tedious.

2.1.3 Analyses assuming failure throughout the material

Methods of this type were first proposed by Rankine (1857) who analysed the stability of a pile of earth using the concept of the ellipse of stress (Love, 1944). However, in the author's opinion, analysis by this method is extremely complex and a more easily understandable technique is provided by use of the Mohr circle (Crandall et al., 1972). The Mohr circle is drawn on a set of axes where the abscissa of a point represents the normal stress and the ordinate represents the shear stress on a reference plane in the material. If the reference plane is rotated then the point representing the normal and shear stress on the plane describes a circle with its centre on the normal stress axis. The

angle between the point and the centre of the circle is twice the angle of inclination of the plane to the principal axes.

Fig. 2.2 (top) shows the forces acting on a parallelogram of granular material of unit width (into the plane of the paper). The broken lines show an exactly similar parallelogram to the right of the original one and the two are assumed to be part of a mass of material of infinite horizontal and vertical extent and of slope α . As the parallelograms are similar the normal and shear forces, F_{x1} and F_{xy1} , must be equal to the forces F_{x2} and F_{xy2} but opposite in direction. These forces can therefore be left out of the stability analysis of the parallelogram ABCD. Considering the other forces acting on ABCD, from the equilibrium of vertical forces

$$W = y\Delta x\gamma = \tau_a \sin \alpha \Delta x \sec \alpha + \sigma_a \cos \alpha \Delta x \sec \alpha$$

and horizontal forces

$$\sigma_a \sin \alpha \Delta x \sec \alpha = \tau_a \cos \alpha \Delta x \sec \alpha$$

where W is the weight of ABCD and γ the specific weight of the material. From the above two equations the stresses on the plane inclined at angle α and at depth y are

$$\sigma_a = \gamma y \cos^2 \alpha \quad \dots (2.1)$$

$$\tau_a = \sigma_a \tan \alpha \quad \dots (2.2)$$

Fig. 2.2 (bottom) shows the Mohr circle for a point at depth y . Rankine assumed that the material was in a state of failure according to the Mohr-Coulomb failure criterion (Lambe and Whitman, 1969). For a cohesionless material this criterion is represented on the diagram by a pair of straight lines inclined at $\pm\phi$, the angle of internal

friction, to the horizontal axis. The Mohr circle must be tangential to these lines to be consistent with Rankine's assumption otherwise there would be planes where the shear stress was greater than the normal stress multiplied by $\tan\phi$. The locus of points satisfying Eqn 2.2 is represented on the diagram by the line OA and so the stresses on the plane inclined at α are represented either by point B or point A. If α were equal to zero then σ_α would be the vertical stress, σ_y , which would also be one of the principal stresses. If we confine ourselves to active failure conditions then, for $\alpha = 0$, the vertical stress would be the major principal stress, point C. As α was increased, the stresses on the plane inclined at α would move from C to A. The stresses on the plane inclined at angle α are therefore given by point A rather than point B. A useful point on the Mohr circle is known as the origin of planes (Lambe and Whitman, 1969). This point has the property that a line drawn from it at any angle intersects the circle at a point representing the stress conditions on a plane in the material inclined at that angle. Drawing a line at α to the horizontal from point A to intersect the circle shows that point B is the origin of planes. A horizontal line drawn from point B intersects the circle at point D which therefore represents the stresses on a horizontal plane and has coordinates σ_y, σ_{xy} . Similarly, point E represents the stresses on a vertical plane and has coordinates $\sigma_x, -\sigma_{xy}$. Using Eqn 2.1 and considering the geometry of the diagram, it can be shown that

$$\sigma_x = \gamma_y \cos^2 \alpha \left\{ \frac{1 - \cos \alpha \sqrt{\sin^2 \phi - \sin^2 \alpha} - \sin^2 \alpha}{1 + \cos \alpha \sqrt{\sin^2 \phi - \sin^2 \alpha} - \sin^2 \alpha} \right\} \quad \dots (2.3)$$

and

$$\sigma_y = \gamma_y \cos^2 \alpha \left\{ \frac{1 + \sqrt{\sin^2 \phi - \sin^2 \alpha} + \sin^2 \alpha}{1 + \sqrt{\sin^2 \phi - \sin^2 \alpha} - \sin^2 \alpha} \right\} \dots (2.4)$$

and the ratio of horizontal to vertical stress is given by

$$k = \frac{1 + \cos \alpha \sqrt{\sin^2 \phi - \sin^2 \alpha} + \sin^2 \alpha}{1 + \cos \alpha \sqrt{\sin^2 \phi - \sin^2 \alpha} - \sin^2 \alpha} \dots (2.5)$$

The maximum inclination of the slope is when $\alpha = \phi$ as greater values than this would require larger shear stresses than can be supported by the internal friction of the material. If the value of α for a level surface ($\alpha = 0$) is substituted into Eqn 2.3 then the more familiar expression for σ_x results i.e.

$$\sigma_x = \gamma_y \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)$$

Rankine assumed that using a wall and a floor to contain the material did not change the stress conditions from those existing in an infinite mass. The normal stress acting on a vertical wall will therefore be σ_x and the shear stress σ_{xy} . Again, from the geometry of the diagram,

$$\sigma_{xy} = \sigma_x \tan \alpha$$

which shows that the inclination of the resultant force on the wall is α , the slope angle. An expression could also be derived for the angle of the resultant force on the floor in terms of α and ϕ . However, there is no reason why such frictional forces should be mobilised at the boundaries, for instance the floor and wall might not be rough enough. Rankine's method therefore takes no account of the boundary conditions.

Rankine's assumption that the presence of a wall does not change the stress conditions from those existing in

an infinite mass leads to anomalous results when the surface of the material slopes or when the wall is inclined.

For the first case consider a vertical wall where the surface of the material slopes at its angle of repose (Fig. 2.3, top left). For this condition $\alpha = \phi$ which, when substituted into Eqn 2.3 gives

$$\sigma_x = \gamma y \cos^2 \phi$$

For a downward slope $\alpha = -\phi$ and exactly the same result is obtained for σ_x whereas it would seem more sensible if the stress were lower. This anomaly is explained by Fig. 2.3 (top right) which shows the situation implicitly assumed by Rankine's method. The stresses on the wall are assumed to be equal to the stresses on the section OA of a vertical plane in an infinite mass of material. If the region to the right of OA only is considered, this corresponds to an upward slope and the normal stress on the wall is σ_x . If the region to the left of OA only is considered, this corresponds to a downward slope and this time the normal stress is σ'_x . For equilibrium, $\sigma_x = \sigma'_x$ and so the stresses for both upward and downward slopes are equal.

For the second case consider a wall inclined at an angle ψ to the vertical. The stress normal to the wall is not now σ_x but may be obtained by re-drawing the Mohr circle with $\alpha = 0$ (Fig. 2.3, middle). As point B is the origin of planes, a line drawn from B inclined at ψ to the vertical intersects the circle at a point F corresponding to the conditions on the plane of the wall. From the geometry of the figure, the stress normal to the wall is

$$\sigma_w = \frac{1 - \sin \phi \cos 2\psi}{1 + \sin \phi}$$

Now consider a wall inclined inwards by the same amount. The stresses on the plane of the wall are given by the intersection of the broken line with the circle, F' , and the normal stress is the same as for the case when the wall is inclined outwards. Again this result is rather surprising. One would expect a gradual transition from a maximum value of σ_w when $\psi = 90^\circ$ (i.e. a horizontal wall or floor) to zero when the wall slopes inwards at the angle of repose of the material. Fig. 2.3 (bottom left) shows the wall as it might exist in practice while Fig. 2.3 (bottom right) shows the situation implicitly assumed by Rankine's method. The region to the right of OA corresponds to a wall inclined outwards and the stress is σ_w while the region to the left of OA corresponds to a wall inclined inwards for which the stress is σ'_w . Again, for equilibrium, $\sigma_w = \sigma'_w$ and so the stress on an outward sloping wall is the same as that on one that slopes inwards.

The stress conditions at a wall with a given roughness can also be analysed assuming active failure throughout the material by use of the Mohr circle. Fig. 2.4 shows the Mohr circle where the shear stress on the vertical plane is given by

$$\sigma_{xy} = -\sigma_x \tan \delta \quad \dots (2.6)$$

and δ is the angle of friction of the material on the wall. This condition is represented by the line OA in the diagram and point E represents the stresses on the vertical plane in the material. A vertical line from point E intersects the circle at B which is therefore the origin of planes and so the stresses on the horizontal plane are given by point D. The ratio of horizontal to vertical stress is

given by

$$k = \frac{1 - \cos\delta \sqrt{\sin^2\phi - \sin^2\delta} - \sin^2\delta}{1 + \cos\delta \sqrt{\sin^2\phi - \sin^2\delta} + \sin^2\delta} \dots (2.7)$$

It should be noted that an explicit expression for σ_x and σ_y can only be obtained if it is assumed that Eqn 2.6 applies for all vertical planes in addition to the one at the wall.

In this case and for a level surface

$$\sigma_y = \gamma y$$

(Lambe and Whitman, 1969) and

$$\sigma_x = k\sigma_y$$

In a real case, however, the shear stress on vertical planes will drop with distance away from the wall and so for a large distance away the stress conditions will approach those for no wall friction. A constant ratio between the shear stress and the normal stress on any vertical plane also implies that there is a constant ratio between those on any horizontal plane. This cannot be true at the surface as there is no frictional boundary condition and so the wall friction effect must also fall as the surface is approached.

The foregoing discussion shows that if the wall and surface boundary conditions are to be satisfied as well as the condition of active failure then the material must have different stress conditions at different points. This problem was considered by Sokolovskii (1965) who solved the two dimensional differential equations of equilibrium subject to the Mohr-Coulomb failure condition. He solved these equations by a finite difference technique and showed that only part of the total mass (the zone of failure) bounded by the surface of the material, the wall, and a

surface of rupture, needed to be considered. Although this method is usually used for cases where the rupture surface passes out of the surface of the material, it has also been extended to cases where the vertical walls are too close together for this to happen (Horne and Nedderman, 1976) as is the case in deep grain bins.

Sokolovskii's method has an advantage over Rankine's in that the boundary conditions and the failure criterion within the zone of failure are all satisfied. However, neither of these methods satisfies all of the requirements of Sect. 2.1.1. As with Coulomb's method, the boundary conditions need not be satisfied outside the zone of failure. Also, as it has been assumed that the material is always in a state of active failure, the material has been represented by a perfectly plastic stress/strain law (Crandall et al., 1972). In other words it has been assumed that the strength limit of the material will be reached for all strains, however small.

To examine the validity of this last assumption consider a rectangular section box with frictionless vertical walls filled in horizontal layers. The vertical stress at any depth, y , will be $\sigma_y = \gamma y$. If the walls of the box are assumed to be perfectly rigid there will be no lateral strain in the contained material. Because of this the horizontal stress will be higher than that occurring when active failure conditions apply and so the Mohr circle for this condition will not reach the failure line (Fig. 2.5). Terzaghi performed experiments with various types of soils to determine the wall movement necessary to reach active conditions. When using dry sand (Terzaghi, 1934)

which, like the agricultural grains to be used in this work, can be assumed to be cohesionless, he found that the necessary wall movement was small, especially when the sand was compacted. The total horizontal force on the wall decreased to a minimum after an outward movement of only 0.05% of the height of the wall. The results of these tests are often used to justify the assumption that the active state will be reached with all materials and all walls. However, the rate of reduction of the stress on a wall with movement, or strain, will be determined by the stress/strain properties of the stored material. If a compressible material is placed under strain then a relatively small stress change will occur for a given amount of strain when compared with an incompressible one. Thus, when storing compressible materials such as agricultural grain, the rate of reduction of wall stress with outward movement will be less than that occurring with relatively incompressible materials such as sand.

2.1.4 Analyses using thin horizontal layers

Formulae derived from analyses of this type are usually used for deep bins i.e. those where the depth is much greater than the diameter or distance between walls (Janssen, 1895; Jaky, 1948; Reimbert and Reimbert, 1961) but they have also been used for shallow silos (Kagan, 1960; De Loringhoven, 1966).

The analysis for a deep bin starts by considering the equilibrium of a thin layer extending across the bin (Fig. 2.6). The horizontal stress at depth y is expressed as

$$\sigma_x = k\sigma_y$$

The upward frictional force on the layer, assuming limiting

friction, is

$$\sigma_x \mu_w P \Delta y = k \sigma_y \mu_w P \Delta y$$

where μ_w is the coefficient of friction at the wall, and P is the perimeter of the cross section.

The equilibrium of vertical forces gives

$$\sigma_y A + \gamma A \Delta x = k \sigma_y \mu_w P \Delta y + (\sigma_y + \Delta \sigma_y) A$$

Where A is the area of the cross section, which reduces to the differential equation

$$\frac{d\sigma_y}{dy} = \gamma - \frac{k \mu_w}{R} \sigma \quad \dots (2.8)$$

with a boundary condition for a level surface $\sigma_y = 0$ at $y = 0$. R is the hydraulic radius which equals the area divided by the perimeter.

Janssen (1895) assumed that k was a constant for a given type of grain and bin and solved Eqn 2.8 which gave

$$\sigma_y = \frac{\gamma R}{k \mu_w} \left[1 - \exp\left(\frac{-k \mu_w y}{R}\right) \right]$$

and

$$\sigma_x = \frac{\gamma R}{\mu_w} \left[1 - \exp\left(\frac{-k \mu_w y}{R}\right) \right] \quad \dots (2.9)$$

The value of k which should be used for designing a given bin is not well defined. Pleissner (1906) showed experimentally that k depended on the dimensions of the bin and the material from which it was made as well as the material filling it. Jaky (1948) made certain assumptions about the variation of the wall friction coefficient with depth after analysing the experimental work of Pleissner (1906) and Jamieson (1903) and also assumed the spatial variation of the shear stress. He showed theoretically that k should depend on the depth, horizontal distance

away from the centre line, wall friction coefficient and the specific weight and shear strength of the material. As is to be expected, the solution of Eqn 2.8 with such a complex form of k is far from simple and this detracts from the usefulness of the resulting formulae.

The uncertain value of k is a disadvantage when using Janssen's formula and so some writers (e.g. Toltz, 1898) have suggested that k should be taken as the value for active failure derived by Rankine (1857) i.e.

$$k = k_a = \frac{1 - \sin\phi}{1 + \sin\phi} \quad \dots (2.10)$$

This gives $k \approx 0.38$ for wheat ($\phi \approx 27^\circ$) compared with the value of 0.67 measured in a model by Janssen. Even if active conditions could be assumed, the use of Eqn 2.10 is incompatible with the boundary conditions of wall friction (Sect. 2.1.3). Because of this, Homes (1972) has suggested the use of a value derived by assuming that the angle of friction at the wall is equal to the angle of internal friction of the material which, by putting $\delta = \phi$ in Eqn 2.7 gives

$$k = \frac{1 - \sin^2\phi}{1 + \sin^2\phi}$$

or $k \approx 0.66$ for wheat.

Some workers have assumed that k and γ are not constant and have related them to vertical stress (Clower et al., 1973; Loewer et al., 1977; Ross et al., 1979). This means that an analytical solution to Eqn 2.8 is no longer possible and so the authors used a numerical finite difference procedure. However, the dependence of k and γ on y was not very great and the difference between assuming constant

and variable properties was slight.

The method as discussed so far assumes a value for k and calculates the stress distribution on the wall. Reimbert and Reimbert (1961) used the same general method but assumed that k approached a value, k_a , as given by Eqn 2.10 when the depth became very large and that the total vertical load carried by the walls, F_{yT} , was of the general form

$$F_{yT} = \frac{ay^2 + by + c}{dy + e}$$

where a , b , c , d , and e were constants. From the known boundary conditions they deduced the values of these constants and derived a formula for the horizontal stress

$$\sigma_x = \frac{\gamma R}{\mu_w} \left[1 - \frac{1}{\left(\frac{y}{B} + 1 \right)^2} \right] \quad \dots (2.11)$$

where, for a level surface

$$B = \frac{R}{\mu_w k_a}$$

By manipulating Reimbert's equations it can be shown that

$$k = k_a + \frac{k_a}{1 + y/B}$$

and so k decreases from $2k_a$ at the surface to k_a at great depth. It is interesting that this is in direct contradiction to Cain (1904) who said that k should approximate to k_a at the surface and should increase with depth.

Jaky (1948) also suggested that k should increase with depth. It is possible to use the same basic technique when the sides of the bin are sloping. In this case the areas of the horizontal layers change with depth but the method is very similar (De Loringhoven, 1966). The technique

of varying the horizontal extent of the layers has also been used to extend the method to long retaining walls with comparatively small depths of material. Both Kagan (1960) and De Loringhoven (1966) considered horizontal layers within regions bounded by a vertical wall, the surface of the material, and an inclined plane extending from the foot of the wall to the surface. Kagan assumed that the plane was inclined so as to give the maximum thrust on the wall while De Loringhoven assumed the inclination to be equal to the angle of repose of the material. Obviously the method gives the same total wall force as that using trial wedges (Sect. 2.1.2) because the conditions at the external boundaries of the set of layers are the same as those of the wedge. To derive the stress distribution they further assumed that there was no shear stress between layers and the vertical stress was constant over the area of each layer. This led to a non-linear variation of horizontal stress with depth which had the peculiar property that the horizontal stress was zero at the foot of the wall. It is this fact which makes the theories untenable. For instance, consider a mass of material of total depth D . The horizontal stress is zero at the surface and, by these theories, zero at depth D . At a depth equal to $D/2$ the horizontal stress is relatively high. Suppose now that the floor is considered to consist of all the material below depth $D/2$ or, alternatively, that the material below depth $D/2$ is replaced by a floor having exactly the same properties as this material. The theories then say that the horizontal stress at depth $D/2$ is zero whereas it should be the same as before. Both these treatments are subject

to the same criticisms as Coulomb's theory but in addition to this, De Loringhoven assumed that the plane of rupture was not the one giving maximum force on the wall and so there must be steeper planes within the material on which the Mohr-Coulomb failure law is violated.

2.1.5 Finite element methods

These methods rely heavily on the use of a high speed electronic computer and, because of this, their use is a relatively recent occurrence. The technique regards the material as being divided into many small regions and the equilibrium and deformation of each region is determined by solving large sets of algebraic equations (Chapter 4). Because of this, the stress/strain law is taken into account, indeed knowledge of such a law is a requirement of the use of the method.

Most of the applications of finite element techniques for granular materials have been directed towards predicting settlements in soils (Chapter 3) but there have been a few studies of retaining wall behaviour for soils (Girijavallabhan and Reese, 1968; Morgenstern and Eisenstein, 1970; Clough and Duncan, 1971), and of agricultural materials (Jofriet et al., 1977) and powders (Reddy and Shook, 1976) in bins. The method has the advantage that all the requirements specified in Sect. 2.1.1 can be met but the development of stress/strain laws for granular materials that are suitable for inclusion in a finite element analysis has lagged behind the development of the techniques which can handle them. The application of the finite element method to the calculation of loads on agricultural storage structures waits, therefore, for the specification of a suitable stress/strain

law for the materials. It is with such a specification that Chapter 3 of this thesis is concerned.

2.1.6 Comparison of methods

Methods which assume that the full shear strength of the material is mobilised i.e. trial wedge and active failure methods, were developed for the design of walls retaining soils. They are often used in this context for gravity retaining walls, that is walls of a relatively massive nature where the tendencies for overturning and outward sliding are resisted mainly by the weight of the wall. This type of wall can remain structurally sound even for large amounts of movement by sliding outwards on the soil on which it rests. The large amount of allowable movement coupled with the small amount necessary to reach active conditions has prompted Lambe and Whitman (1969) to refer to this type of system as a "ductile" one. However, they also refer to "brittle" systems where the structure can only stand a small amount of movement before it collapses. Whether a system can be classed as ductile, and so justify active failure design, or brittle, when some other method is needed, depends on the relationship between the allowable structural movements and those necessary to mobilise the full shear strength of the stored material. It has already been stated that the movement necessary to mobilise active conditions is likely to be higher with agricultural materials than with soils. The allowable wall movements are also likely to be less. For instance, in a series of tests on five types of reinforced brickwork walls, each of just over 3m high, it was found that deflections of the top of the wall of between 0.76mm and 4.22mm were sufficient to cause

cracking and permanent strain of the wall (Maurenbrecher et al., 1976). As a further example, the radial movement in a steel bin of 6m diameter would only be about 4mm when the steel had reached its elastic limit. Where walls rely for their strength on members in compression, such as unreinforced brickwork or by using external struts, the allowable movement will be less as compression members generally collapse with very little movement. It seems therefore that methods for the calculation of wall loads for agricultural materials which assume active failure conditions should be used with caution or with large safety factors.

The major problem with methods derived from analyses using thin horizontal layers is deciding which value of k to use. The experiments of Janssen (1895) indicated a value of 0.67 for wheat in a small wooden bin whereas the value derived from Rankine's formula (Eqn 2.10) which is often recommended in codes of practice (National Research Council of Canada, 1970; American Concrete Institute, 1977; Ministry of Agriculture, 1977) gives a figure of about 0.38, roughly half that of Janssen's. Because of the wide choice of the value of k , the value that is actually used is not much more than a guess and as Ketchum (1929) stated, "inasmuch as something has to be guessed, why not guess at the design of the wall in the first place?"

The major advantage of these two types of method is that, once the so called constants (γ , k , ϕ etc.) are known, the method is very easy to apply. This is not the case with the finite element method which requires a computer to handle the many equations that need to be solved.

However, it is the only method to date that can satisfy all the requirements listed in Sect. 2.1.1. In addition, the finite element method is not restricted to any particular type of silo, tall bins are treated in the same way as shallow retaining walls except that the boundary conditions are different. It is unlikely that the finite element method will be used by the agricultural architect or designer in the very near future, but it might be possible to make use of it in a design, rather than just in a research context, by the preparation of charts or tables based on computer calculations. However as computers become cheaper and smaller and engineers become more used to using them for design purposes it is likely that such techniques will be used on a routine basis just as the more simple but less rigorous and less adaptable formulae are used today.

2.2 Methods of measuring wall loads

2.2.1 Total forces

Supporting the wall on load cells

The most direct method of measuring the total force on a retaining wall is to measure the forces acting at its supports. Fig. 2.7 shows a wall section supported on five load cells measuring forces F_1 to F_5 . The total horizontal force of the material on the wall per unit width is

$$F_x = (F_1 + F_2 + F_3)/W$$

and the total vertical force per unit width is

$$F_y = (F_4 + F_5)/W$$

where W is the width of the wall. The vertical position at which the resultant acts (the centre of pressure) is obtained by taking moments giving

$$\bar{y} = \frac{F_1 y_1 + (F_2 + F_3) y_2}{F_x}$$

Kagan and Perelman (1959) used the technique and attached strain gauges to horizontal and vertical rods supporting a wall section. A simpler technique, but one that yields less information was used by Dobie et al. (1966) who hinged the bottom of their wall and measured the horizontal supporting load at one point only. The supporting forces at the hinge were unknown and it was necessary to assume the position of the centre of pressure to calculate F_x . A stress distribution which increased linearly from the surface (triangular) was assumed giving $\bar{y} = 2D/3$ and so

$$F_x = \frac{3F_1(D - y_1)}{DW}$$

Measuring wall deflections

This method (Pleissner, 1906; McCalmont and Ashby, 1934; Pitman and Wright, 1973) depends on some assumption being made about the way the wall is supported. For instance, if the wall can be regarded as a beam, simply supported at each end with a uniform stress distribution, then the horizontal force per unit width is given by

$$F_x = \frac{76.8EIx}{l^3W}$$

(Roark, 1954) where E is the Young's modulus and I is the second moment of area of the wall material, x is the maximum outward deflection, and l is the length of the beam. The deflection depends on the stress distribution as well as the total load and so the distribution has to be assumed, but this effect is reasonably small. For instance, a triangular stress distribution gives a deflection equal to that for a uniform stress (although the height of the maximum deflection is slightly different) and a stress which

decreases linearly from a maximum in the middle to zero at each end gives a deflection only 28% greater (Roark, 1954).

The insensitivity of the maximum deflection to the stress distribution means that the method is unsuitable for measuring such distributions, which made McCalmont and Ashby (1934) abandon their attempt to measure the stress distribution of maize on a wooden bin using this technique. The need for accurate values of E and I presents further problems. Sometimes their values can be inferred by calibration, for instance by filling the structure with water, which also removes the need for an assumption about the means of support. However, calibration is difficult unless using small scale models and this, along with the fact that representing a real wall by some simple beam is usually an extreme idealization, makes the method inaccurate.

Measuring the force on the bottom of a bin

If the total weight of the material in a bin is known and the vertical force on the bottom measured, then the difference must be the vertical load carried by the walls. The total horizontal force on the walls can be obtained by multiplying by the coefficient of friction of the material on the wall. Janssen (1895) and Pleissner (1906) both used this method, Janssen using platform scales underneath a model bin and Pleissner suspending the bottom of a full scale bin on a steelyard. The coefficient of friction can be measured in a separate experiment but there is no guarantee that the friction will reach its limiting value in a real bin. The downward movement of the grain produced by the action of the weighing machine will tend to produce

limiting friction at the wall in the bin under test but it is not certain that the stress conditions in such a bin will be the same as those in a similar bin without the measuring equipment. The method does not, of course, give any indication of the distribution of the wall stress.

2.2.2 Stress distributions

Measuring strains in the structure

The previous methods have been used to investigate the validity of, or to derive constants for, stress distribution formulae. This can be done by calculating the total force from the formula and then comparing it with the measured value. However, this comparison is only a partial check because different stress distributions can lead to the same force and so a more direct method of measuring the stress distribution is usually required.

A simple way of estimating the horizontal stress in a cylindrical bin is to measure the hoop stress in the material. This has been done by measuring the circumferential strain with a tape measure (Gupta and Muir, 1971) and by using strain gauges (Collins, 1963; Pieper, 1968; Candura and Gusman, 1970). The horizontal stress is assumed to be

$$\sigma_x = \frac{2t\sigma}{D} \quad \dots (2.12)$$

where t is the thickness of the wall material, σ is the hoop stress in the wall and D is the bin diameter. It must be remembered that while this relationship is exact for an infinite cylinder when σ_x , t , and D do not vary with depth (Roark, 1954), it is only approximate for other cases. For instance, if a bin were cut horizontally into a number of thin cylinders they would all be free to expand independently and Eqn 2.12 could be applied to each one

to derive σ_x as a function of depth. However, in a real case the bottom boundary of one cylinder must expand by the same amount as the top boundary of the one beneath it and so they cannot act independently. Stresses at one level affect the circumferential strains not only at that level but also at the surrounding levels which leads to a stress averaging effect.

Another problem occurs near the bottom of the bin due to the restraining influence of the base which reduces the amount of circumferential expansion. It can also occur if the resistance to circumferential expansion changes abruptly at some levels as in the full scale bin used by Candura and Gusman (1970). Here the bin was made of three separate horizontal rings bolted together at flanged joints. The flanges had the effect of stiffening the circumference and the resulting hoop strain distribution was a series of three bulges being virtually zero at the joints. As the hoop stress was obtained by multiplying the strain by a constant, the calculated stress distribution was similar. Such a distribution can only be compared in a very rough way with an existing formula and is of no real use in the investigation of the stress distribution imposed on the wall.

Measuring the load transferred to a relatively simple part of the structure

Sometimes the design of a structure is such that nearly all of the imposed load is taken on a part of the structure for which the stress analysis is relatively simple. An example of this is in bins where reinforcing bands are used in conjunction with a relatively weak wall material. Amundsen (1945) used a cylindrical bin which was made of

a number of vertical hardboard panels. The panels were not connected along their vertical edges but were held together by horizontal steel bands around the bin. As there could have been no continuous hoop stress in the panels, the imposed horizontal stress was carried by the tension in the bands (and by friction where the panels met the floor, which was ignored). Amundsen derived the stress distribution by assuming that the panels transferred loads to each band from an area extending from points midway between that band and the next band above and below. This was an approximation and depended on the panels not acting as continuous beams between the bands. However, as the panels were made of a relatively flimsy material it was probably a reasonable assumption. One practical limitation of this method is that friction between the panels and the bands can cause strains that vary around the circumference. Consequently Amundsen had to use several strain gauges on each band.

It is important to note that even if a component is assumed to give no strength to a structure for design purposes, in practice it might well resist a significant part of the imposed load. For instance, Dinnie and Beard (1970) attempted to calculate the imposed load on a cylindrical brickwork bin by measuring the circumferential strain with gauged tapes. It was assumed that all the load would be taken by reinforcing hoops placed between the courses of brickwork as is the normal practice when designing such a structure. However the brickwork must have resisted a considerable part of the hoop stress because the strains were too small to measure which rendered the experiment a

complete failure.

Stewart (1972) transferred the loads imposed on a small scale experimental structure to precision made vertical beams. The wall was divided horizontally into twenty three independent sections which rested on the precision beams so transferring the load to the beams at twenty three discrete points. Calculating the imposed stress distribution on such a beam involves differentiating the bending moment with respect to distance along the beam. This means that enough measurements must be taken to give a reliable indication of the bending moment distribution and so Stewart took measurements at eleven places on each beam. A problem with the differentiation of a set of experimental points is that amplification of experimental errors which leads to scatter in the derivative can occur. This is because small errors in the data can lead to relatively large errors in the difference between consecutive data points. Stewart overcame this by fitting smooth curves to the bending moment distributions before differentiation. Although this technique reduces the scatter, it does so at the expense of decreasing the resolution of the measurement because some smoothing of the experimental data takes place as a result of curve fitting.

Replacing part of the wall by a stress measuring guage

All the methods of measuring stress distributions discussed so far are subject to a load transfer effect. That is a load imposed on the wall by the stored material does not produce a stress in the wall material in that area alone. This results in a smoothing of the imposed stress distribution. It would be better to measure stresses on

parts of the wall which are sensitive to loads applied to that part only. This obviously cannot be done without modifying the existing wall which poses practical problems illustrated by the following example.

Perhaps the simplest way of measuring stress is to fix a rubber bag filled with a liquid to the wall (e.g. Pleissner, 1906). The rubber bag (Fig. 2.8, top) is extremely flexible and is assumed not to carry any of the imposed load and so the pressure in the liquid is equal to the stress exerted by the stored material on the bag. The liquid pressure can be measured easily with a manometer. Although the method is simple and requires little in the way of specialized equipment it is subject to the following criticisms:-

a) The relatively flat wall is modified by fixing the rubber bag to it. The abrupt change in shape of the wall may affect the stresses on it especially in the most important region i.e. at the bag.

b) The surface of the bag may have different frictional properties to that of the part of the wall it has replaced which may also affect the stress.

c) Some liquid must be displaced out of the bag into the manometer. This results in the bag reducing in volume as the stress on it increases which produces a strain in the stored material and hence a change in the stress that is being measured.

d) The rubber bag can distort even in the absence of liquid displacements by bulging in one place in response to a depression in another. This would occur on loading the silo as the surface of the material passed over the

area of the cell and might further affect the stresses imposed on the cell.

A slight improvement on the rubber bag is to replace it with a rigid cell that has the face in contact with the grain closed by a thin rubber membrane. Kramer (1944) used a cell of this nature and also attempted to minimize the affect of a) above by using small ramps at the edges of the cell to produce a smoother transition between the cell face and the wall. The method can be further improved by mounting the cell so that the face is flush with the wall and covering most of the surface of the membrane with a rigid plate leaving a gap all round (Jamieson, 1903; Bovey, 1904). As well as minimizing d) above, this latter improvement means that the cover can be made of a similar material to the wall so minimizing the effect of b). Dale and Robinson (1954) further improved on this method by limiting the movement of the diaphragm cover into the stored material. When the cover was flush with the bin wall it was restrained from further inward movement by rigid stops. The liquid in the cell was held under pressure during the filling of the bin and when a reading was required the pressure was gradually reduced until the cover was pushed slightly backwards off the stops. This resulted in a much smaller movement of the diaphragm cover and so probably a more reliable stress reading.

A different type of gauge transfers the load on a face plate let into the wall to a separate force measuring device (Fig. 2.8, middle) and the imposed stress is calculated by dividing the force by the area of the face plate. By making the face plate from a material similar to that

of the wall and mounting the face plate flush with the wall the effects of a) and b) above can be minimized. The face plate can also be rigid, unlike the rubber diaphragm, which makes this type of gauge relatively immune from the effects of d). The device that measures the force can take several forms, for example a steelyard (Roberts, 1884), spring balance (Fordham, 1937), bars in bending where the strain is measured by a dial gauge (McCalmont, 1938) or by strain gauges (Saul, 1953; Stroppel, 1968), strain gauged tapes in tension (Caughey et al., 1951), strain gauged cantilevers (Yaeger and Pratt, 1974) and commercial load cells (Yaeger and Pratt, 1974; Powell et al., 1977).

A third type of gauge combines features from the previous types by making the diaphragm much less flexible than the first type and using it as a load sensing element (Fig. 2.8, bottom). There is no liquid present with this type of gauge, instead the diaphragm carries all the load and the strain in the diaphragm is measured and calibrated against a known stress. Williams and Ross (1968) used a strain gauge fixed to the back of a circular diaphragm while North (1954) and Barr (1962) used a similar but slightly different technique. North made a separate cell that was fixed to the wall of a bin, the sensitive member being a thin rectangular plate, while Barr cut rectangular sections from the wall of a cylindrical corrugated silo and welded them back in along the vertical edges only. Both these devices can be considered as edge clamped beams and both used strain gauges to measure the strains in them.

This type of gauge can be set flush with the wall and its greater rigidity compared with the rubber diaphragm

type makes it far less susceptible to d) above. However, it is nearly always impossible to make the face of the gauge of the same material as the wall (Except where the wall is made of thin material anyway, for example in Barr's technique). A further slight disadvantage with the method is that the strain in a diaphragm or beam depends on the load distribution on it (Roark, 1954) and so these gauges should be calibrated with an applied stress distribution close to that which will be encountered in practice.

All the gauges mentioned have the disadvantage that their deflection changes the stress they are trying to measure. This effect was first noticed by Roberts (1884) whose face plates were either 305mm square, 610mm square, or 915mm square. Levers were used to transmit the force on the face plates to a steelyard weighing machine which was counterbalanced so that the face plates were pressed against stops as the bin was loaded. The force was measured by removing weights from the counterbalance until movement of the steelyard occurred. Roberts expected the steelyard to move suddenly to its limit of travel as soon as the counterbalance weight was equal to the load imposed by the wheat but unfortunately as soon as the steelyard moved the load dropped off which stopped further movement. Roberts had to remove many counterbalance weights before the steelyard finally reached its limit and he recorded two stresses, one for the initial movement (the "dormant" stress) and one where the steelyard had reached its limit (the "active" stress). The active stress was about half the dormant value which Roberts attributed to "the elastic force of the grain expending itself before the mass of the

grain came into the state of mobility". Unfortunately Roberts did not record the movement of the face plate but the sensitivity of the measured stress to movement was obviously much greater than he expected.

This sensitivity was the subject of an investigation by the Waterways Experiment Station (1944) who were interested in the value of the sensitivity for sand. They placed a stress gauge in the bottom of a 711mm diameter tank and carefully placed sand over the gauge to a depth of 254mm. A rubber membrane covered the sand above which air pressure was applied. The deflection of the gauge could be changed by inserting different springs in its mounting and diameter of the cell to deflection ratios in the range 60 to 20,000 were investigated. Four cell diameters were used from 76 to 305mm and as the diameter to deflection ratio (D/δ) was increased, either by changing the cell diameter or the deflection, the indicated stress increased from about 50% of the air pressure at $D/\delta = 60$ to about 88% at $D/\delta = 1000$ after which it stayed reasonably constant. The 12% difference for $D/\delta \geq 1000$ was attributed to friction between the sand and the side of the tank and it was concluded that provided D/δ was greater than 1000 the indicated stress would not be affected by face plate deflection.

This method of investigating the sensitivity of measured stress to deflection does not quite reproduce the conditions in a real silo because the boundary conditions are different. For example, in the experiments a constant pressure was applied to the sand only 254mm away from the face plate. Because of this constant pressure, the normal stress on the sand at a distance of 254mm away from the gauge also

remained constant. In a larger mass of material the normal stress would probably have reduced as a result of the strain in the sand caused by the deflection of the face plate. It would seem therefore that this technique underestimates the sensitivity to movement but unfortunately by an unknown amount.

Fordham (1937) investigated the sensitivity of measured stress to face plate deflection by filling a container with sand and measuring the force on a face plate in one of the side walls as the face plate was moved. The face plate was 108mm square and at a depth of 406mm below the surface, the opposite side wall being 610mm away. In this test Fordham found that the measured stress halved with an outward movement of only 25 μ m which, taking D as the diameter of the inscribed circle, is equivalent to an error of 50% at a D/ δ of 4000 - a conclusion quite different to that of the Waterways Experiment Station. Once again the boundary conditions were different from those in a real silo as there was a fixed boundary at a distance of 610mm. A perfectly rigid boundary would lead to a sensitivity greater than that occurring with a larger mass of material. This is because a strain in the sand at the opposite wall in response to face plate deflection would lead to an immediate lowering of the stress there whereas replacing the rigid wall with more material of a finite compressibility would lead to a more gradual lessening of the stress.

There is another difference between the test employed by Fordham and that of the Waterways Experiment Station. In Fordham's test the face plate was moved after the load was applied whereas in the Waterways Experiment Station

test the face plate deflected continuously as the load was applied. The results of the former test would therefore be more appropriate to gauges that deflect only when a reading is taken (e.g. Roberts, 1884; Dale and Robinson, 1954) whereas those of the latter test would be more applicable to gauges that deflect continuously (e.g. Jamieson, 1903; Powell et al., 1977).

As well as being affected by the boundary conditions the sensitivity of measured stress to face plate deflection is also likely to be affected by the properties of the material being used. The situation is similar to that when a wall is allowed to deflect (Sect. 2.1.3) and again the sensitivity will be affected by the stress/strain properties of the stored material, which will not be the same as those for sand. It seems that at this stage all that can be said about the sensitivity of the measured stress to face plate deflection is that it can be high and it might depend on the boundary conditions and the stress/strain properties of the stored material. It would be prudent therefore to treat general recommendations for the allowable deflection with caution and gauges to measure stress should be designed to take account of and correct for any movement that occurs.

Chaplin (1976) corrected for the deflection of a gauge by measuring the sensitivity of the indicated stress to deflection in a separate experiment. This was done by removing shims which allowed the face plate to deflect by measured amounts. The stresses for various deflections were recorded and the results used to correct for the known deflections that occurred in subsequent experiments. A more sophisticated way of compensating for the deflection is to measure it

and use it as a feedback signal to control a compensating force. When the movement is zero the force applied to the gauge by the stored material equals the compensating force. Williams and Ross (1968) used the signal from the strain gauges attached to a diaphragm to control an air pressure applied to the reverse side and achieved a D/δ ratio of 5000. Stroppel (1968) transferred the load from a face plate to a strain gauged bar but instead of mounting the bar on the silo structure, a compensating force was applied to it so keeping the face plate in position. This force was applied by filling or emptying a water tank with valves controlled by signals from a set of contacts which detected the movement at the face plate. Stroppel was able to achieve a diameter to deflection ratio of 20000 with a 100mm diameter face plate.

2.3 Analysis of previous measurements of wall loads

2.3.1 Method of comparison

The measurements to be analysed here were carried out using a wide range of techniques and silo types (Table 2.1) storing different agricultural grains (Table 2.2). To compare the results of these experiments it is necessary to reduce the data to some form of common base. It seems sensible to transform the data in such a way that they can be compared with a relevant design formula. The experimental work has been divided into two main groups:-

a) Cases where formulae based on Coulomb's or Rankine's methods would be used. These are long flat retaining walls or bins with flat sides which can be considered as shallow, i.e. where the depth is not greater than four times the hydraulic radius (National Research Council of Canada, 1970).

b) Cases where Janssen's or Reimbert's formulae would be used. These are bins with flat or curved sides of any depth.

Obviously some silos can be in both groups.

Where the experimenter has used a deeper bin but taken measurements at several depths, data from shallow depths have been included in the first group. Each main group has been further divided on a point of experimental technique. Some experimenters used one gauge to measure the stress at one point on the silo wall (Table 2.1). The stress distribution with depth was then derived by filling the silo to different depths and recording the stress on the one gauge. In the absence of any "bottom effect" this would be equivalent to having an array of gauges at different depths and recording stress simultaneously. However, in reality the friction at the floor may tend to support some of the load that would otherwise be transmitted to the wall. There is evidence for the existence of this effect from the results of Saul (1953) and Kagan and Perelman (1959). The experimenters who used the variable depth technique placed their gauges close to the floor and so their measurements could have resulted in figures which were lower than the actual stress distributions.

To compare the experimental results with design formulae, the specific weight and angle of internal friction of the stored material and the coefficient of wall friction must be known. If these were measured by the experimenter, they have been used. Otherwise they have been taken from a standard source of design data (American Society of Agricultural Engineers, 1977). The figures used are listed

Table 2.1

Variations in experimental technique

Experimenter	Dimensions of cross section, m	Max. depth, m	Gauge deflection to diameter ratio	Number of replicates	Type of measurement
Roberts (1884)	2.06 x 1.83*	>4R	Not stated	2 - 6	Variable depth
Jamieson (1903)	3.66 x 4.11* 0.305 x 0.305*	>4R	12000	1	" "
Bovey (1904)	4.08 x 3.76	>4R	"	1	" "
Pleissner (1906)	1.5 x 1.5 1.57 x 1.57 3.15 x 2.80 2.90 x 2.51	>4R	Not stated	1	" "
Kramer (1944)	2.88 x 2.88	>4R	Not stated	2	" "
Amundsen (1945)	5.82 diam.	3.05	Part of structure	1	Fixed depth
Caughey et al. (1951)	0.457 diam.	>4R	3100	4	Variable depth
Saul (1953)	3.35 x 3.35	2.74	2400	1 - 2	Fixed depth
Dale and Robinson (1954)	0.457 diam.	>4R	4900	1	" "
Kagan and Perelman (1959)	Long retaining wall	4.2	Not stated	1	" "

Table 2.1 (continued)

Experimenter	Dimensions of cross section, m	Max. depth, m	Gauge deflection to diameter ratio	Number of replicates	Type of measurement
Pieper (1968)	0.7 diam.	>4R	Part of structure	8	Fixed depth
Stroppel (1968)	1.0 x 1.0	0.5	20000	2 - 5	" "
Gupta and Muir (1971)	0.25 diam.	0.63	Part of structure	5	" "
	0.47 diam.	0.88			
	0.67 diam.	1.05			
	0.97 diam.	1.47			
Stewart (1972)	1.22 x 2.44	1.22	Part of structure	3	" "

- Notes.
1. Where it is not stated on which side of a rectangular bin the stresses were measured, an asterisk appears in column 2.
 2. For rectangular gauges the diameter has been taken as four times the hydraulic radius of the gauge.
 3. If the number of replicates was not stated it has been assumed to be one.
 4. Variable depth refers to the type of measurement where one gauge was used (or two in the case of Caughey et al.) and the stress distribution derived by filling to different depths.

Table 2.2

Parameters used in design formulae

Experimenter	Grain	Wall material	γ kN/m ³	μ_w	μ
Roberts (1884)	Wheat	Rough wood	8.1	0.41*	0.47*
Jamieson (1903)	Wheat	Rough wood	7.9	0.44	0.53
	"	Steel trough plate	"	0.47	"
	"	Sheet steel	"	0.36	"
	"	Smooth wood	"	0.40	"
Bovey (1904)	Wheat	Rough wood	7.7	0.41*	0.47*
Pleissner (1906)	Wheat	Rough wood	8.2	0.41*	0.47*
	Rye	" "	7.7	0.41*	0.47*
	Rape	" "	7.0	0.47*	0.47*
	Flax	" "	6.7	0.41*	0.46*
	Wheat	Rough concrete	8.2	0.44*	0.47*
	Rye	" "	7.9	0.44*	0.47*
Kramer (1944)	Rice (var. Rexora)	Rough wood	5.7	0.66	0.68
	Rice (var. Blue rose)	" "	5.7	0.60	0.73
Amundsen (1945)	Wheat	Fibre board	7.2	0.40	0.47*

Table 2.2 (continued)

Experimenter	Grain	Wall material	γ kN/m ³	μ_w	μ
Caughey et al. (1951)	Wheat	Concrete	7.2	0.35	0.51
	Maize	"	7.4	0.25	0.51
	Soya beans	"	7.2	0.27	0.58
Saul (1953)	Maize	Rough wood	7.3	0.34*	0.52*
Kagan and Perelman (1959)	Wheat	Rough wood	7.7	0.47	0.49
Pieper (1968)	Barley	Aluminium	7.7	0.37	0.55
	"	"	"	0.55	"
Stroppel (1968)	Barley	Emery paper	7.2	0.58	0.58
	"	Card	"	0.29	"
Gupta and Muir (1971)	Wheat	Polyethylene	8.7	0.37	0.40
Stewart (1972)	Sorghum	Steel	7.1	0.32	0.49

Note. An asterisk indicates that the value was obtained from a standard source of design data.

in Table 2.2. When three or more replicates of the same experiment were carried out, the stresses at a given depth have been averaged and regarded as a single data point.

2.3.2 Comparison on the basis of Coulomb's method

Reynolds and Steedman (1976) give a set of formulae for the calculation of stress distributions based on Coulomb's method. These are all of the form

$$\sigma_w = k_c \gamma y \quad \dots (2.13)$$

Where k_c is a constant. Eqn 2.13 can be written in a non-dimensional form as

$$\frac{\sigma_w}{Y D k_c} = \frac{y}{D} \quad \dots (2.14)$$

The left hand side can be thought of as a non-dimensional stress, S , and the right hand side as a non-dimensional depth, Y . Dividing by the total grain depth, D , means that the range of Y is always 0 to 1.0 which makes plotting and comparisons easier. For experiments where the grain depth was greater than four times the hydraulic radius, D has been taken as $4R$.

If the experimental data agreed with the figures calculated using Coulomb's method then, from Eqn 2.14, they would all lie on a straight line of slope 1.0 when plotted on axes of S vs Y . However the data from experiments using the variable depth technique is considerably scattered (Fig. 2.9, top) with values of S ranging from 75% below the line in the case of Roberts to 196% above it in the case of Pleissner. The reasons for this scatter are not clear and it may be due to a variety of causes, for example:-

a) The instrumentation may not have been sufficiently accurate to give a reliable indication of the stress.

b) For all the data plotted in Fig. 2.9 (top) the width of the silo was of the same order as the maximum grain depth. The different combinations of width and wall friction coefficients for each experiment may have affected the measured stresses by different amounts.

c) Differences in silo designs and corresponding wall deflections may have affected the measured stress by different amounts.

d) There may have been a bottom effect (Sect. 2.3.1) which, due to differences in floor friction coefficient, may have been different in each experiment.

It is impossible to say whether any one of these reasons or a combination of all of them could have led to the scatter in the data. As mentioned in Sect. 2.2.2, the effect of gauge deflection on the measured stress is not known and so even if the deflection of each gauge for each measurement had been recorded there would be no means by which the data could be corrected. Also, there are no methods that can calculate the effect of friction at the floor or deflections of the silo walls. The effect of side wall friction can, however, be included by using Janssen's formula to reduce the data to a common form. This will be done in a later section.

Data from experiments using gauges at different depths to determine the stress distribution are shown in Fig. 2.9 (bottom). Although there is still considerable scatter in the data, there is less than in those of Fig. 2.9 (top). The reduction in scatter could be due to the floor friction only affecting data in the region $y/D = 1.0$ rather than for all values of y/D as in Fig. 2.9 (top). On the other

hand it could be due to three out of the four silos being wider (relative to the maximum depth) than those of Fig. 2.9 (top) so reducing side wall friction effects, or simply because the experiments of Fig. 2.9 (bottom) were carried out seventy years or so later during which time experimental techniques and equipment could have improved. A bottom effect is only apparent in the data of Saul and of Kagan and Perelman. It is significant that these experiments were carried out on large scale structures whereas those where a bottom effect is not apparent (Stroppel, Stewart) were carried out on models. It is likely that the surfaces of the models were smoother than those of the full scale structures which could have resulted in a lower floor friction. If the data that are modified by floor friction are discounted then those remaining are nearly all above the line corresponding to calculations by Coulomb's method. This suggests that the full shear strength of the material is not always mobilised as assumed in Coulomb's method.

2.3.3 Comparison on the basis of Rankine's method

The formulae derived from the Mohr circle in Sect. 2.1.3 can be reduced to a form similar to Eqn 2.13 i.e.

$$\sigma_w = k_a \gamma y$$

where k_a is a constant for any particular configuration. The equation can again be transformed to a non-dimensional form

$$\frac{\sigma_w}{\gamma D k_a} = \frac{y}{D}$$

and a plot of the non-dimensional stress against the non-dimensional depth will show how well Rankine's method predicts the experimental data. Fig. 2.10 (top) shows

the data from experiments using the variable depth technique. All the experiments were carried out with a flat grain surface and a vertical wall and for these cases Rankine's method gives stresses equal to those which would be obtained using Coulomb's method with no wall friction. As the wall friction coefficients are all similar (Table 2.2), this results in a virtually constant ratio between k_a and k_c . Fig. 2.10 (top) is therefore very similar to Fig. 2.9 (top) except that the non-dimensional stresses are all about 20% lower as a result of dividing by the larger value of k_a . The scatter in the data when using Rankine's method for comparison is just as large as when using Coulomb's method which shows that the former method accounts for changes in the experimental configuration in much the same way as the latter when the grain surface is level, the wall is vertical and the wall friction coefficient is confined to a narrow range.

Fig. 2.10 (bottom) shows data from experiments which used several gauges to measure the stress distribution. As well as including data from experiments with a flat grain surface and a vertical wall (Saul, Stewart), Fig. 2.10 (bottom) also includes data from sloping surfaces (Kagan and Perelman) and inclined walls (Stroppel). Comparing Fig. 2.10 (bottom) with Fig. 2.9 (bottom) it can be seen that the scatter in Saul's and Stewart's data has remained about the same while that in Kagan and Perelman's and Stroppel's has increased considerably. This is due to the anomalous results obtained using Rankine's method for downward sloping grain surfaces or walls that are inclined inwards (Sect. 2.1.3) and emphasises the unsuitability

of using this method for such cases.

2.3.4 Comparison on the basis of Janssen's and Reimbert's formulae

As mentioned in Sect. 2.1.3, modern codes of practice usually recommend the use of the Rankine active value of k in Janssen's formula. Using this modification, Janssen's formula (Eqn 2.9) can be written in the non-dimensional form

$$S = 1 - \exp(-Y)$$

where the non-dimensional stress is

$$S = \frac{\sigma_x \mu_w}{R}$$

and the non-dimensional depth is

$$Y = \frac{y \mu_w k_a}{R}$$

Reimbert's formula (Eqn 2.11) can also be written in terms of these quantities as

$$S = 1 - \frac{1}{(1 + Y)^2}$$

which means that both formulae can be represented by single lines on axes of S vs Y .

Data from both silos in group b) (Sect. 2.3.1) have been reduced to the non-dimensional form but in cases where very deep silos have been used, measurements have only been included for depths up to about twice the diameter. This means that the highest value of Y is about 1.3. The data from experiments using the variable depth technique (Fig. 2.11, left) and those from experiments using stress gauges at different depths (Fig. 2.11, right) are considerably scattered. Again, it is impossible to say whether this

is due to poor instrumentation or the inability of the formula to predict wall stresses for a range of different silo configurations. Certainly the scatter is no less in those data which are common to Fig. 2.11 and Figs 2.9 and 2.10 which means that the extra feature inherent in Janssen's and Reimbert's formulae (i.e. taking the friction around the complete circumference into account rather than just that at one of the walls) does not help in predicting the wall stresses.

The stresses plotted in Fig. 2.11 (right) are nearly all higher than the line corresponding to Janssen's formula while those on Fig. 2.11 (left) are spread approximately equally above and below the line. The lower stresses of Fig. 2.11 (left) could be caused by the bottom effect reducing the stresses measured using the variable depth technique. The stresses from the experiments of Gupta and Muir are all very much higher than those calculated using Janssen's formula. Their containers were made of polyethylene film which had no vertical rigidity and therefore deformed freely in a vertical direction with the grain settlement. In such a container no wall friction could act on the grain which would increase the vertical and hence the horizontal stresses and so explain some of the increase in stress measured in these experiments. However, even if a wall friction coefficient of zero is used the data are still above a line corresponding to Janssen's formula.

Reimbert's formula gives higher values of S than those given by Janssen's formula and so more of the data points are below the line corresponding to Reimbert's formula. This is because, at small depths the ratio between horizontal

and vertical stress is considerably higher than the Rankine active value (see Sect. 2.1.4). However, although use of this formula may be more appropriate if active failure of the material does not occur, it is unlikely that it would be used in the design of the relatively shallow silos with which this work is concerned.

2.4 Conclusion

From Sect. 2.3 it is clear that the data measured by a variety of experimenters are not predicted consistently by any of the standard design methods. It is impossible to say whether this is due to unsuitable instrumentation or formulae that are not applicable to the wide range of conditions covered in the experiments. It has been shown that existing methods for predicting stresses all make gross assumptions and some of these may be unreasonable for agricultural materials. On the other hand the measurement of stresses in granular materials is difficult and the errors produced by, for instance, the deflection of a stress gauge are sometimes not appreciated and always of an unknown magnitude. It is the general aim of this work to increase knowledge in these areas by investigating a new method of calculating stresses on silo walls and also by improving the way in which stresses can be measured.

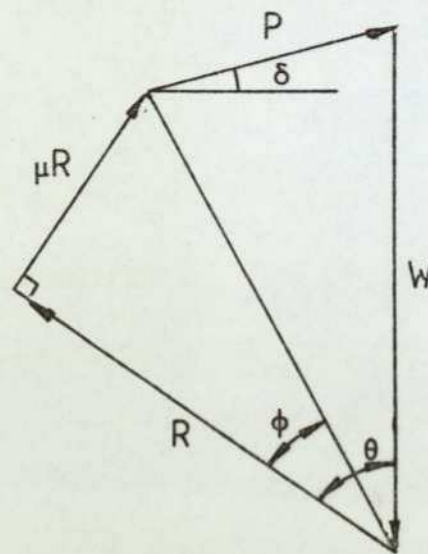
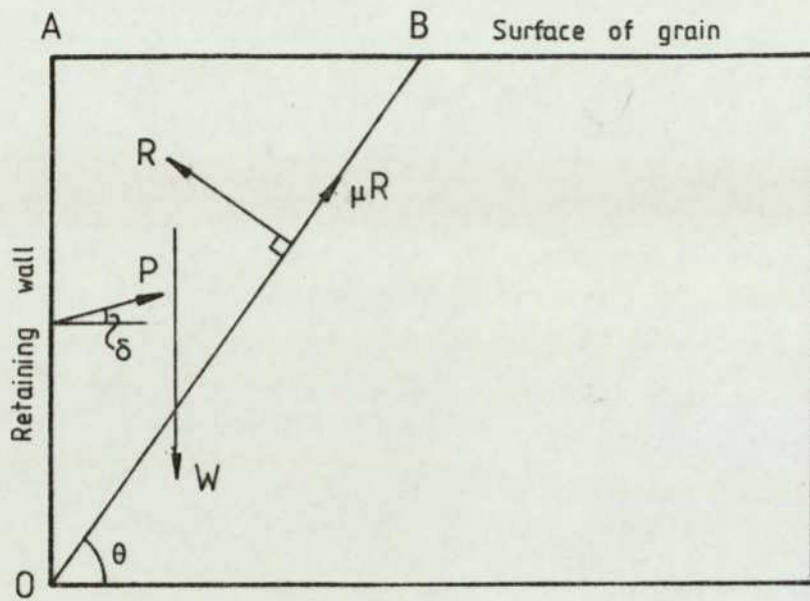


Fig. 2-1. Trial wedge method. Top, forces assumed to act on wedge of material; Bottom, polygon of forces.

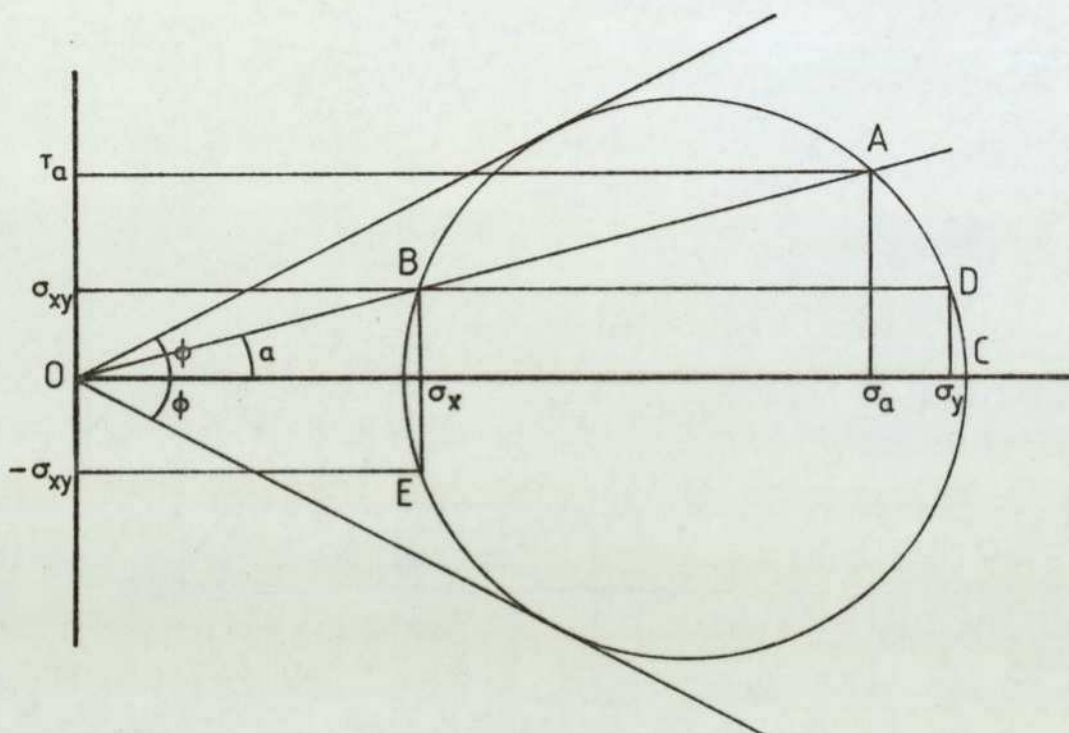
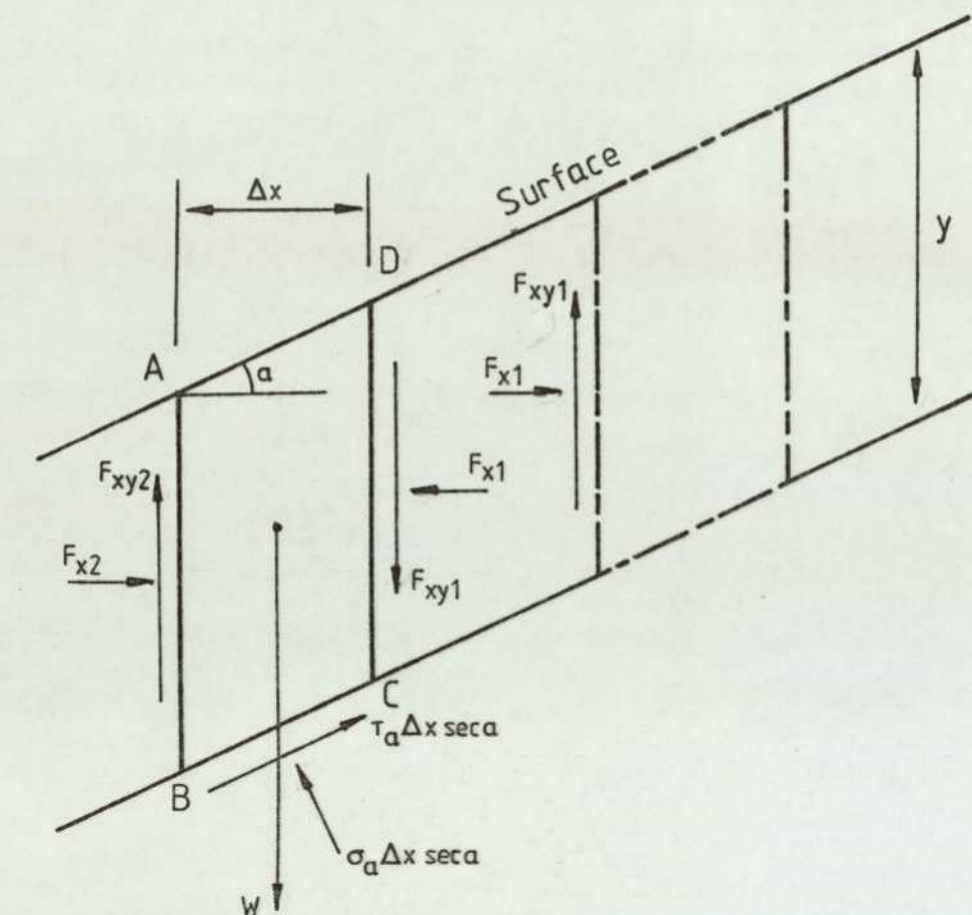


Fig. 2.2. Infinite sloping material. Top, forces acting on a parallelogram of material; Bottom, Mohr circle for this stress condition.

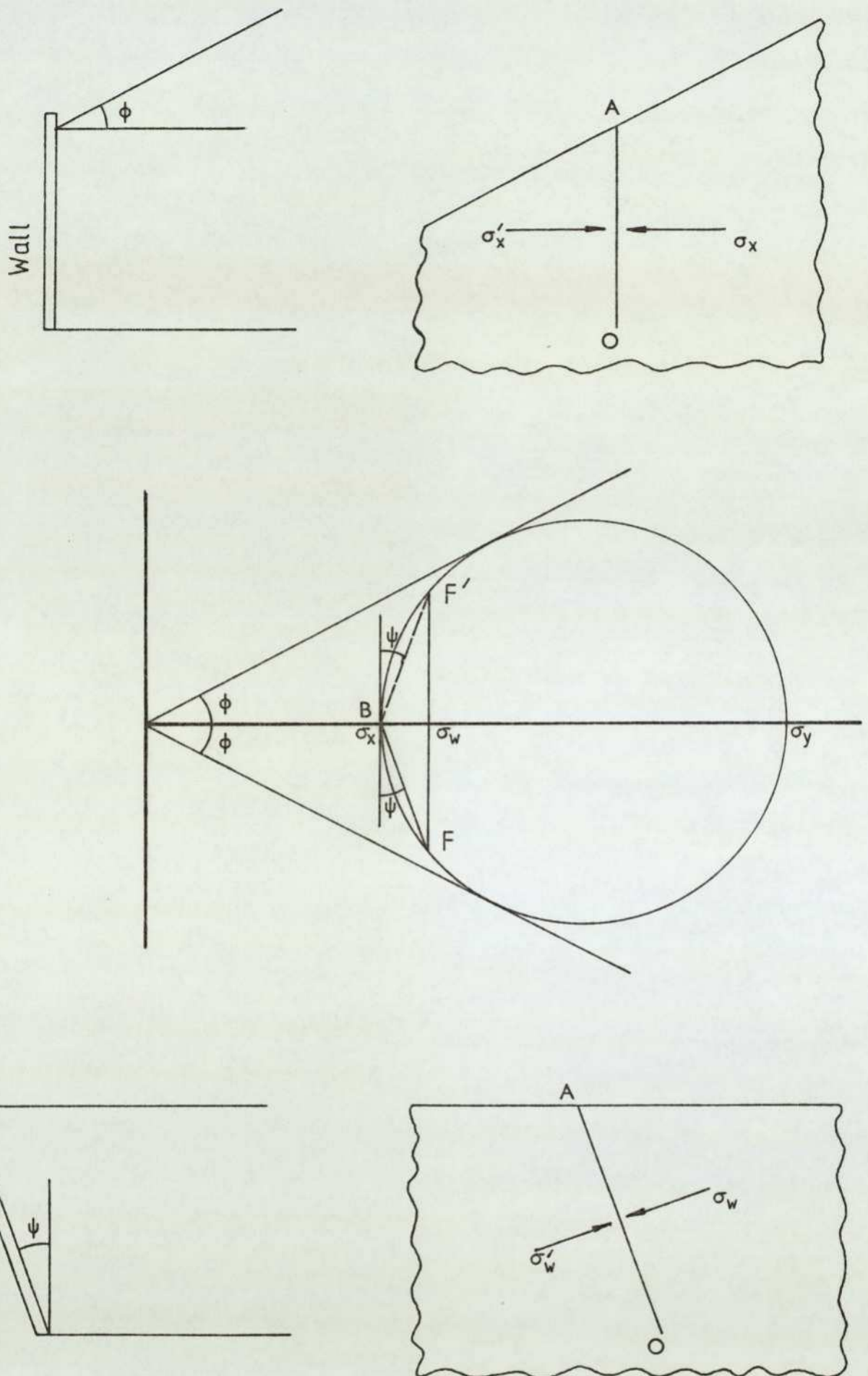


Fig. 2.3. Conditions giving anomalous results when using Rankine's method. Top left, wall with upward sloping material; Top right, infinite slope assumed by Rankine's method; Middle, Mohr circle for inclined wall. Bottom left, inclined wall. Bottom right, infinite mass of material assumed by Rankine's method.

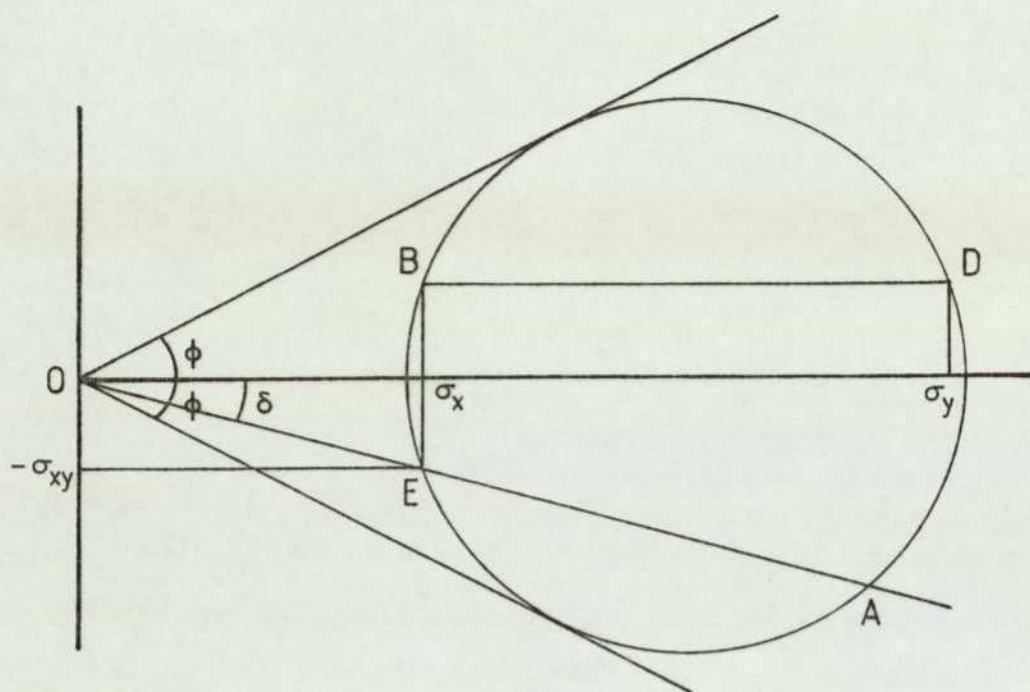


Fig. 2.4. Mohr circle for stress conditions where $\sigma_{xy} = -\sigma_x \tan \delta$

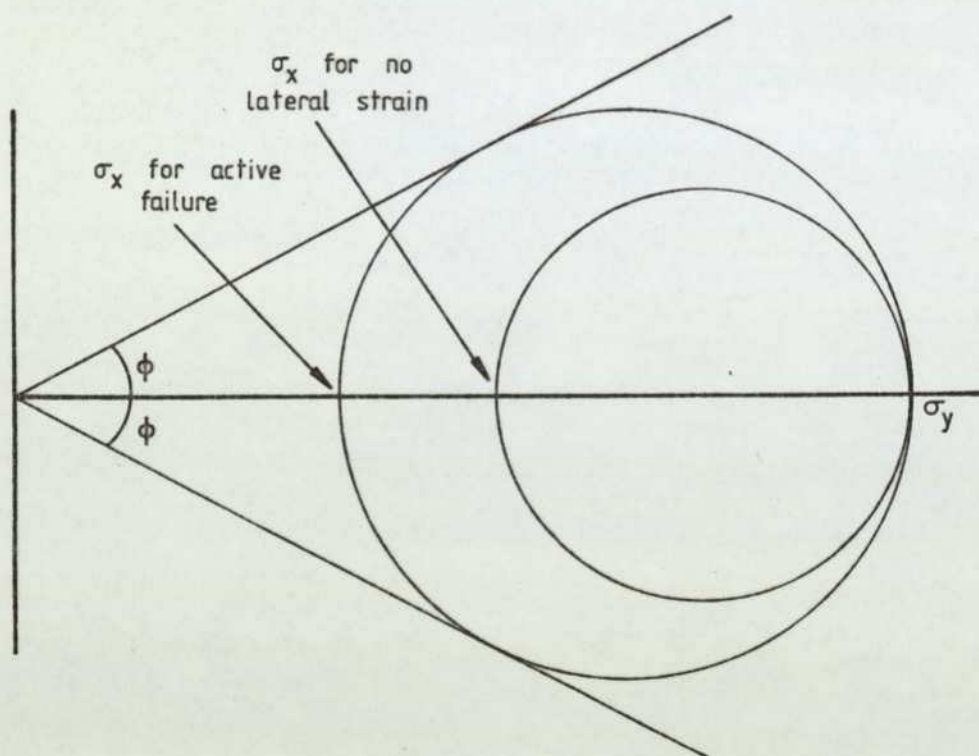


Fig. 2.5 Mohr circles for material with a horizontal surface in a frictionless box. Circles drawn for no lateral strain and with sufficient strain to cause active failure.

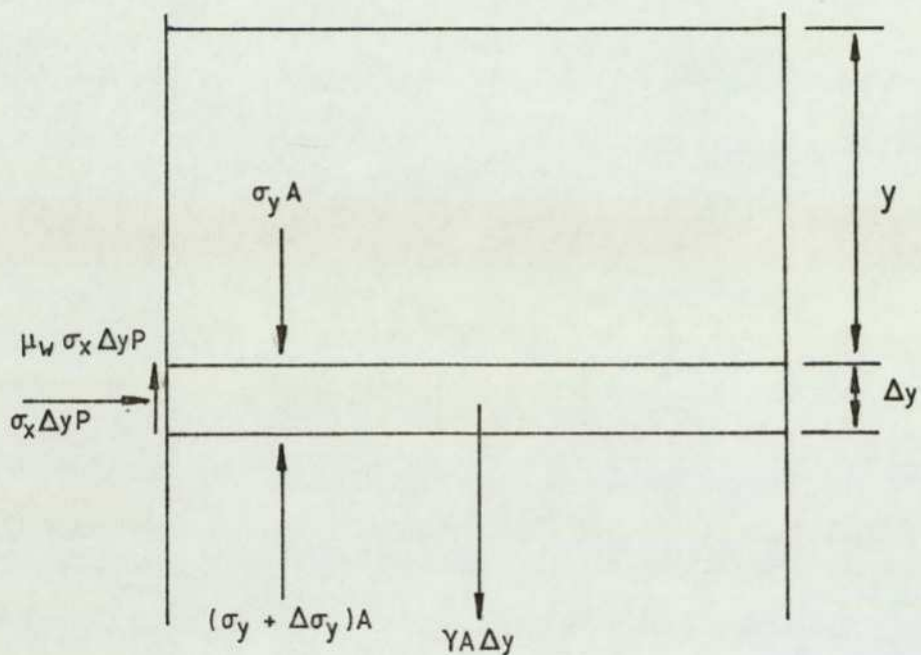


Fig. 2.6. Forces on a thin horizontal layer of material.

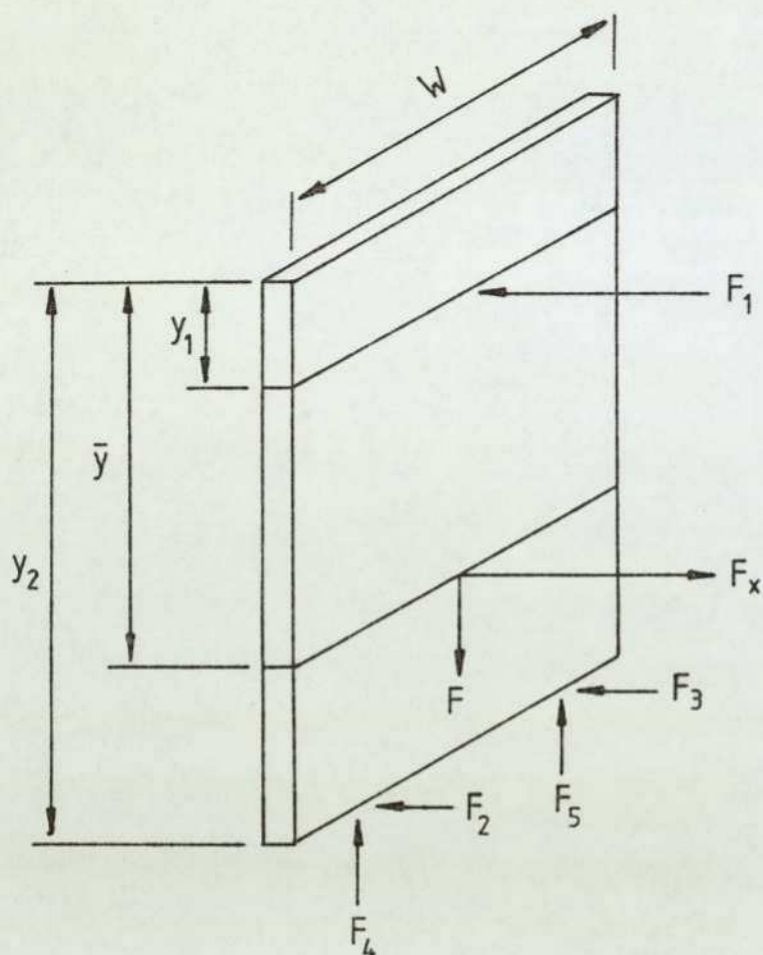


Fig. 2.7. Imposed and measured forces on a plane wall.

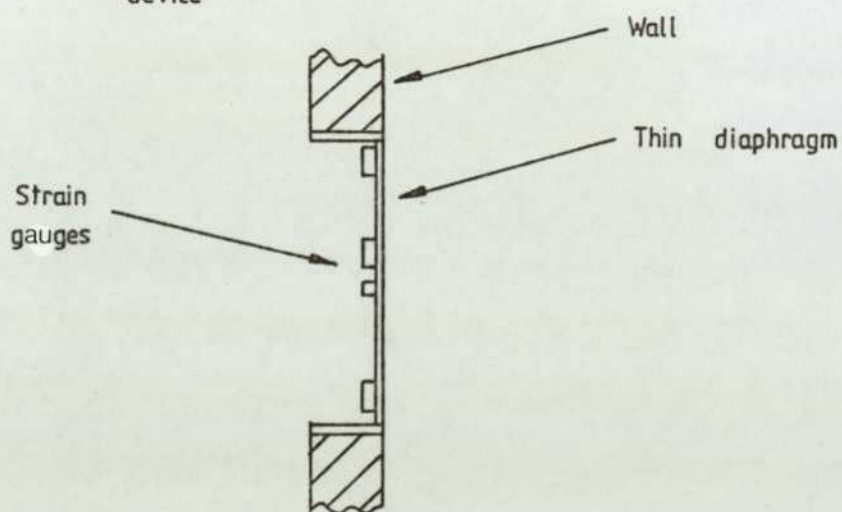
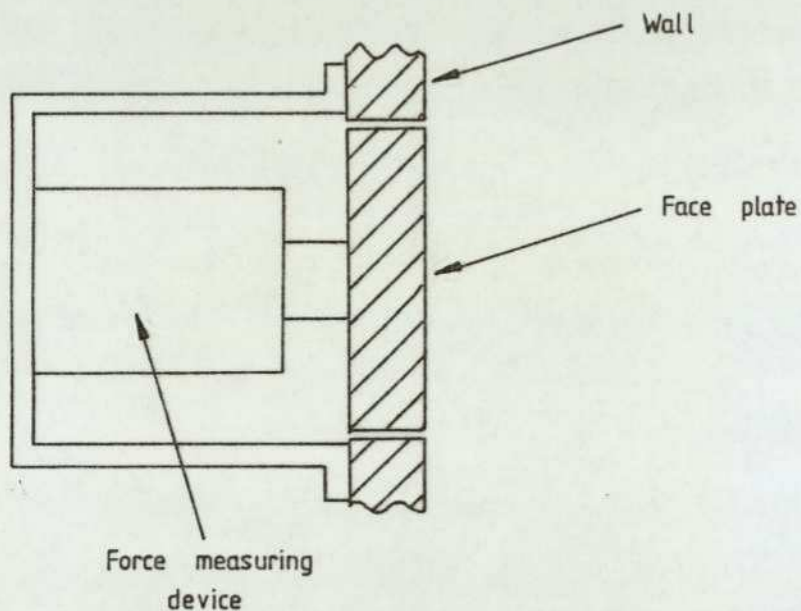
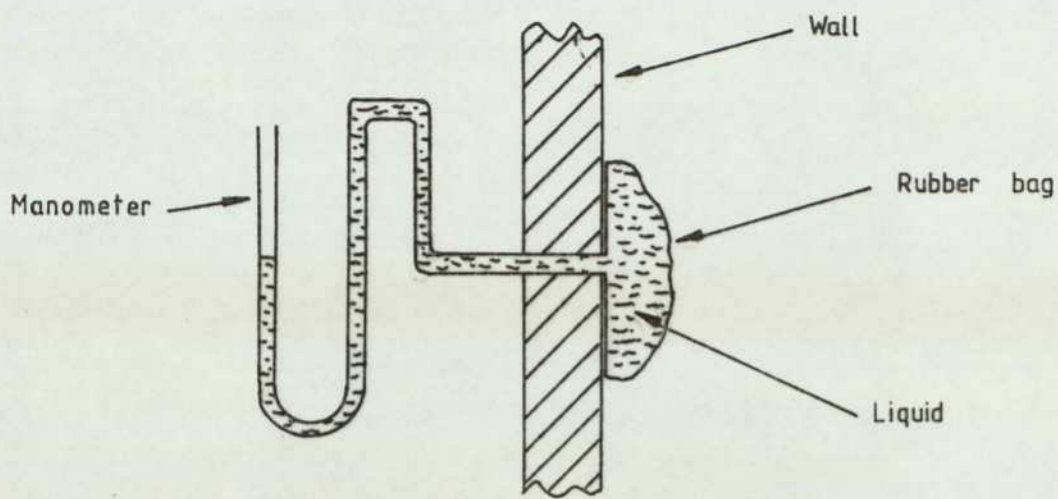


Fig. 2.8. Types of gauge for measuring wall stress. Top, rubber bag displacing liquid into a manometer; Middle, face plate transferring load to a force measuring device; Bottom, strain gauges measuring deflection of a diaphragm.

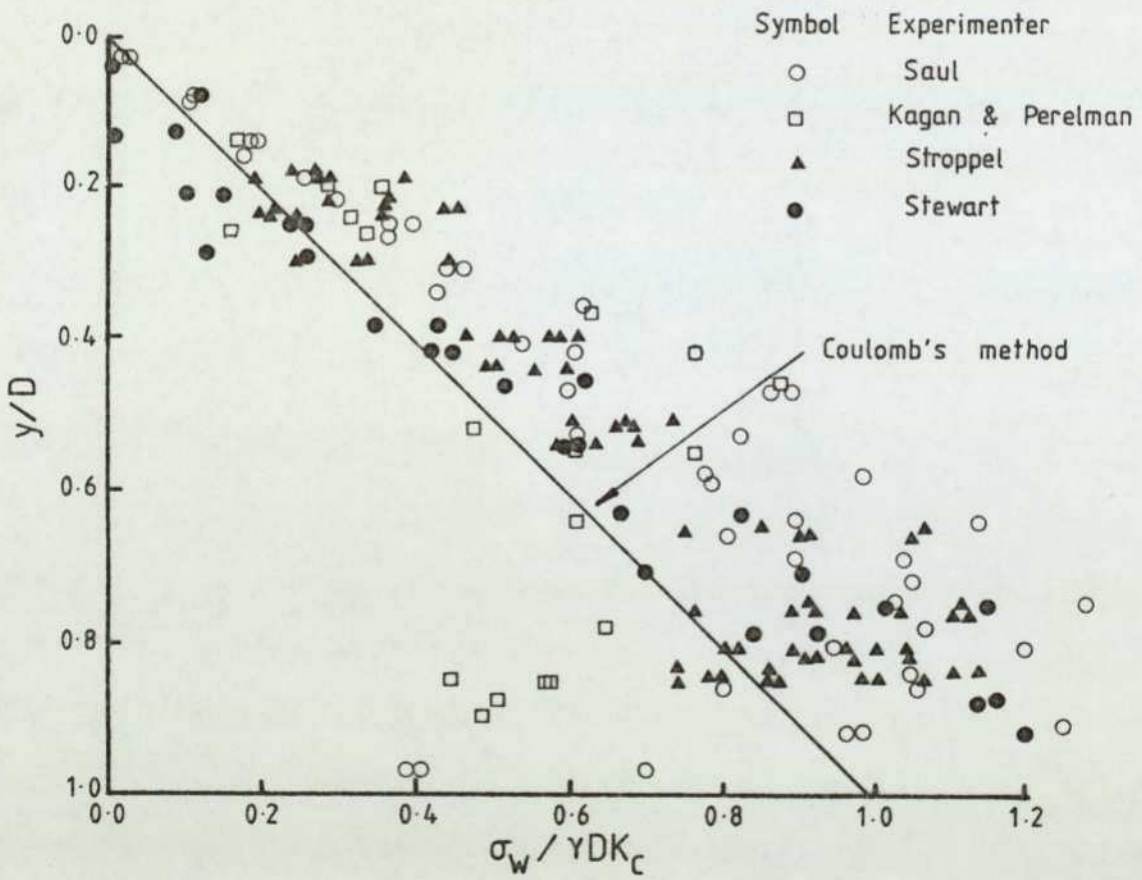
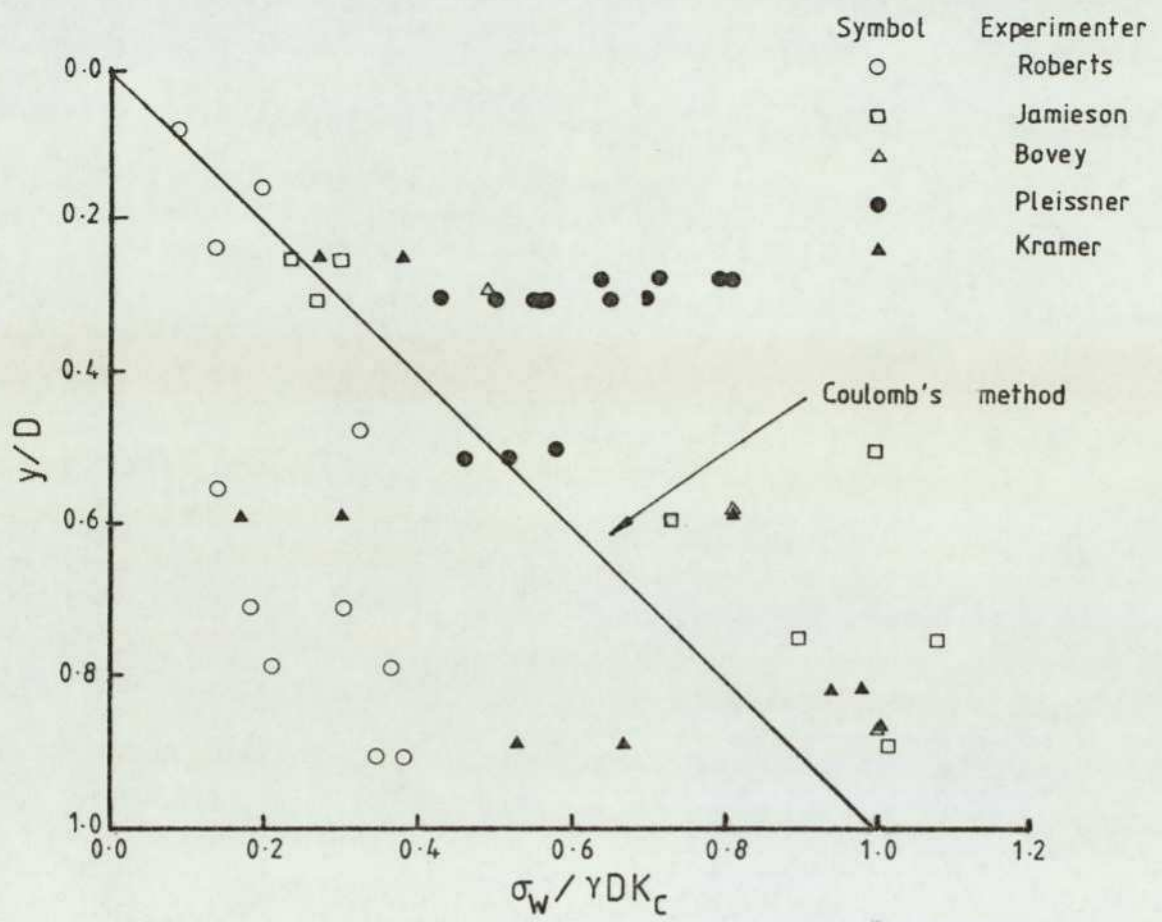


Fig. 2.9. Data of non-dimensional stress vs non-dimensional depth compared with Coulomb's method. Top, variable depth technique; Bottom, fixed depth technique.

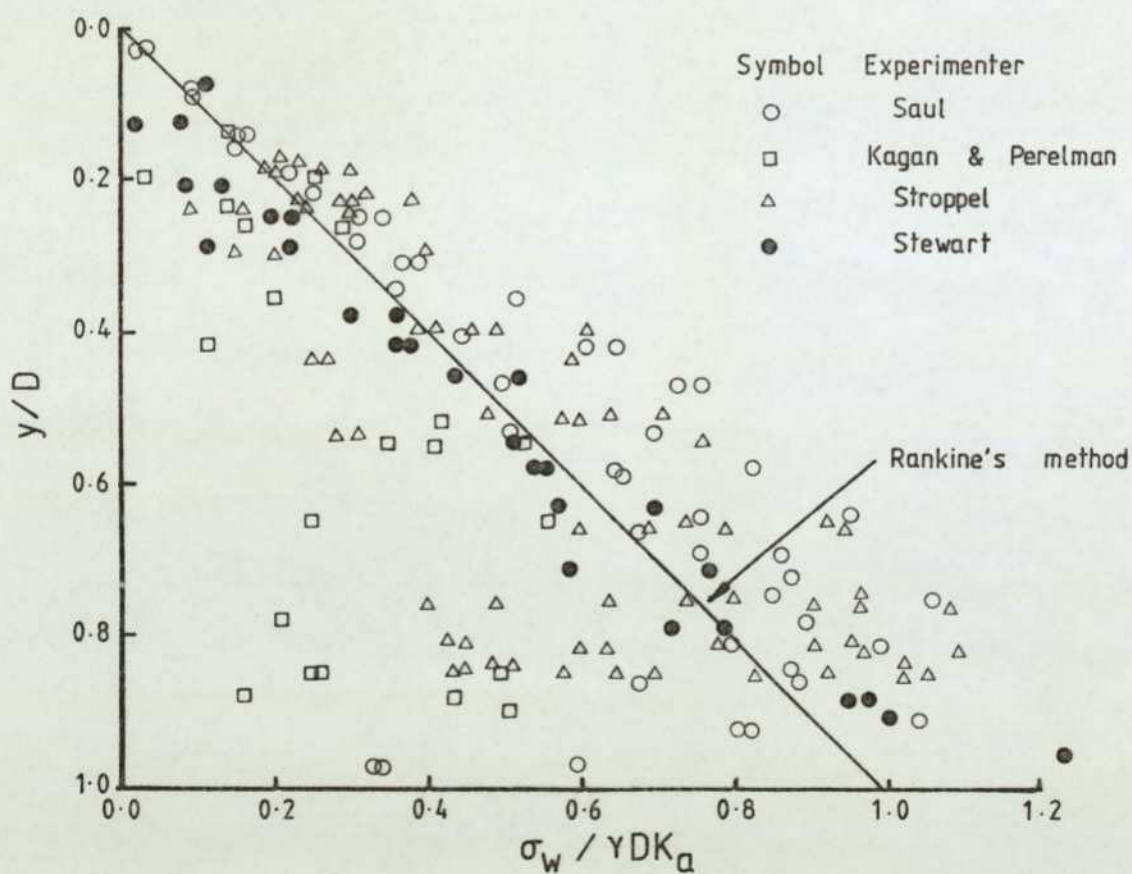
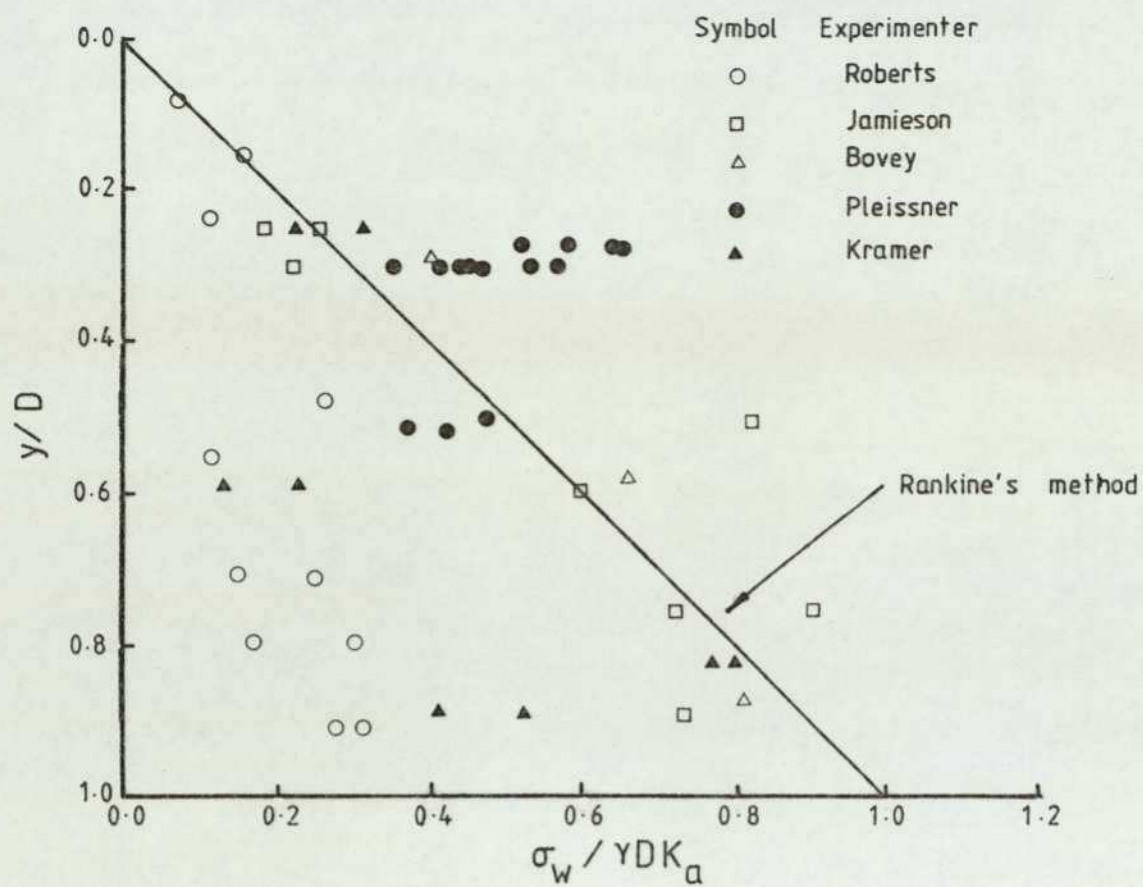


Fig. 2.10. Data of non-dimensional stress vs non-dimensional depth compared with Rankine's method. Top, variable depth technique. Bottom, fixed depth technique.

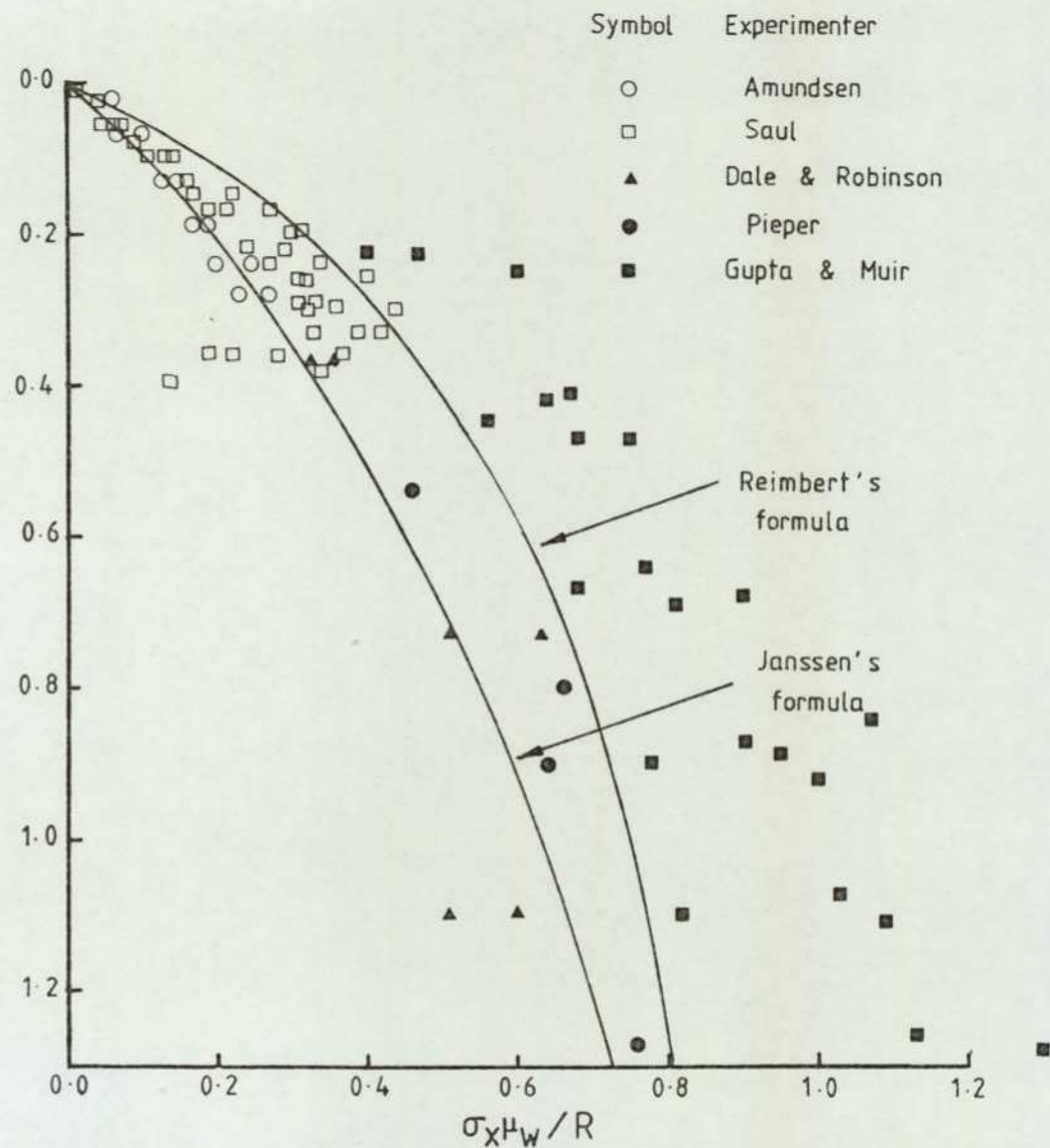
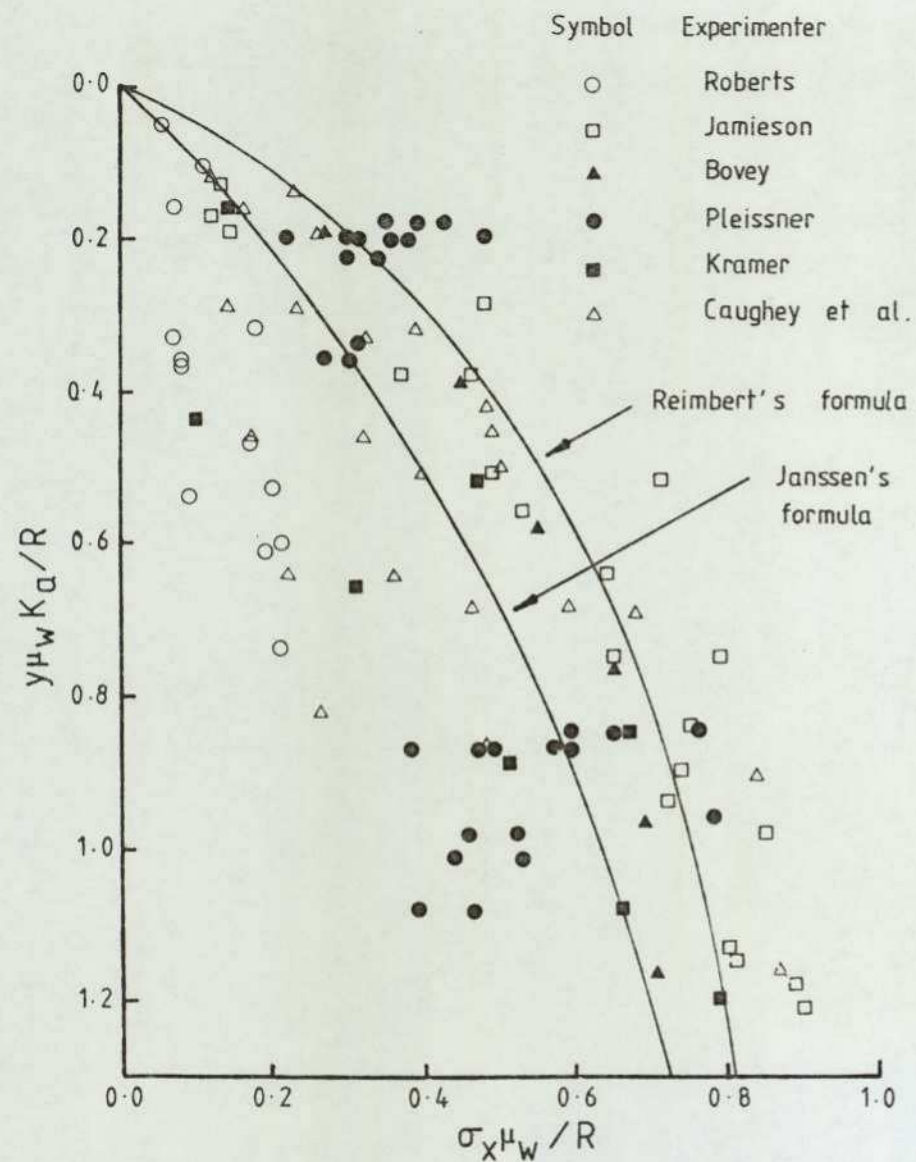


Fig. 2.11. Data of non - dimensional stress vs non - dimensional depth compared with Janssen's and Reimbert's formulae. Left, variable depth technique. Right, fixed depth technique.

CHAPTER 3

A STRESS/STRAIN MODEL FOR BULK GRAIN

3.1 Introduction

The stress/strain behaviour of granular materials is usually thought of in terms of changes in shape (distortional or shear strains) and changes in volume (normal strains). In materials that are linearly elastic, isotropic, and homogeneous, the two types of strain are caused by their corresponding stresses only. That is shear stress (proportional to differences in the principal stresses) causes shear strain and normal stress (proportional to the average of the principal stresses) causes normal strain. There is no cross-coupling between the two types of quantity. To illustrate this, Fig. 3.1 (left) shows a two dimensional object of isotropic, homogeneous, and linearly elastic material that is initially circular and loaded so that the two principal stresses are equal. If the load is increased keeping the two principal stresses equal (Fig. 3.1, middle), there will be no distortion (the object will remain circular) but the area will change. If this isotropic stress system is changed again so that the principal stresses are unequal, giving rise to shear stress, but their average value remains constant, the circle will distort into an elliptical shape but the area will stay unchanged (Fig. 3.1, right). In a general loading condition where the normal and shear stresses both change, the total strain (assumed to be small) can be obtained by superposing the shear and normal strain components.

The characteristics of the behaviour of granular soils in terms of distortional and volumetric effects are (Lambe and Whitman, 1969):-

- a) If the mean normal stress is held constant, the

material becomes progressively easier to distort as the shear stress increases and above a certain critical shear stress the material will distort without limit and so be in a state of failure. The shear stress needed to cause failure increases as the mean normal stress increases.

b) Distortion is accompanied by a volume change which can be an increase and is known as dilation.

c) If the material is subjected to a stress which is then removed, not all of the strain that occurs during loading will be recovered on unloading. This unrecovered or "plastic" strain is especially apparent when the material is subjected to shear stresses.

It can be seen that, as well as being non-linear and exhibiting plastic behaviour, soils are different from linear elastic materials in that there is cross-coupling between shear and normal quantities. As both soils and grain are granular materials, their stress/strain properties should have similar forms and so a literature review was carried out, taking note especially of the work already done in the field of soil mechanics, to see if any of the material models used in the finite element analyses of soil mechanics situations would be suitable for the present problem.

In Chapter 4 it will be shown that the finite element method can be used to analyse the stresses in a granular material by representing the loading by steps within each of which the stress/strain properties of the material are assumed to be linear. In the opinion of the author this "incremental elasticity" technique (Desai and Abel, 1972) is an ideal one with which to model the filling of a storage structure for the following reasons. Firstly, a structure

is filled incrementally, although in the real process the increments are infinitesimally small; secondly, the representation of the process as a series of elastic steps does not alter the engineer's original conception of the finite element method and so makes it easier to understand; and thirdly, for the same reason as the second point, the computer program to implement the incremental elasticity method can be derived from a standard one with only minor alterations. Because of these three points it was decided to review only the literature where the incremental elasticity method has been used and to avoid material models based on, for example, the minimum energy principle of thermodynamics (Rowe, 1962) or on the laws of plasticity (Lade and Duncan, 1975).

3.2 Previous work

3.2.1 Hooke's law in terms of octahedral stresses and strains

For materials that are linearly elastic, homogeneous, and isotropic, the relationships between the principal stresses and strains are described by the set of equations (Love, 1944)

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} \quad \dots\dots(3.1)$$

which is known as Hooke's law. Two constant parameters only are needed to describe this type of material, the Young's modulus, E , and the Poisson's ratio, ν . Previous authors have used these equations to describe soils and other granular materials sometimes representing E and ν by non-linear stress dependent functions. Before reviewing

the work of these authors in more detail, it will be shown that this set of equations cannot describe one of the important characteristics of granular materials mentioned in the previous section, that is the volume change of the material under the action of shear stresses.

Firstly it is necessary to define some measure of distortional and volumetric strains in a three dimensional sample. It can be shown (Love, 1944) that the maximum shear strain in each of the principal strain planes is proportional to the difference in the principal strains which lie in that plane. One measure of the average distortion is therefore the octahedral shear strain defined as

$$\gamma = \frac{2}{3} \left[(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2 \right]^{1/2} \dots\dots(3.2)$$

which shows that γ is proportional to the root-mean-square value of the three maximum shear strains. The volumetric strain is equal to the sum of the three principal strains and so their mean or the octahedral normal strain

$$\epsilon = \frac{1}{3} (\epsilon_1 + \epsilon_2 + \epsilon_3) \dots\dots(3.3)$$

is a measure of the volumetric strain. The corresponding stressess are defined as the octahedral shear stress

$$\tau = \frac{1}{3} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2} \dots\dots(3.4)$$

and the octahedral normal stress

$$\sigma = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) \dots\dots(3.5)$$

By manipulating Eqns 3.1 to 3.5 it can be shown that

$$\begin{bmatrix} \gamma \\ \epsilon \end{bmatrix} = \begin{bmatrix} 1/M_s & 0 \\ 0 & 1/3M_b \end{bmatrix} \begin{bmatrix} \tau \\ \sigma \end{bmatrix} \dots\dots(3.6)$$

where the shear modulus is

$$M_s = \frac{E}{2(1+\nu)}$$

and the bulk modulus is

$$M_b = \frac{E}{3(1-2\nu)}$$

Eqn 3.6 clearly shows that, if Hooke's law is used to describe a material, volumetric strains can only be ascribed to normal stresses. These equations cannot therefore predict any volume change with shear because of the lack of off-diagonal terms in the coefficient matrix.

3.2.2 Simple models where E and ν are independent of stress difference

As well as using Hooke's law to describe granular materials, some authors have simplified their stress/strain relationships even further by assuming that either the Young's modulus and Poisson's ratio are constant (Christian, 1968; Duncan and Dunlop, 1969; Jofriet et al., 1977; Smith, 1970) or are simple functions of minor principal stress (Duncan et al., 1968) or octahedral normal stress (Huang, 1968).

These simple descriptions may suffice where small stress changes are superposed, on an existing, much higher, stress state such as the loading of footings (Christian, 1968; Smith, 1970) or roadways (Duncan et al., 1968; Huang, 1968) on over-consolidated soils. In these cases, because the soils were saturated clays the authors generally regarded them as nearly incompressible, that is volume changes due to shear and normal stresses were small. If this is accepted, then the diagonal form of Eqn 3.6 and Hooke's law, with Poisson's ratio close to 0.5, may suffice for the description of the material. Also, because of the relatively small stress changes compared with the existing stress, a constant value of E, or one which varies only with σ_3 or σ (to represent the increased resistance to distortion of the material with greater confinement) may be used.

These simple forms cannot however be used in any

realistic way to represent cases where large stress changes occur or, for example, in dry granular materials, where dilational volume changes may be significant. Despite this, Duncan and Dunlop (1969) used constant values of E and ν to represent a clay soil in which an excavation was made. Here the initial high value of lateral stress was reduced to zero by applying an equally large but opposite stress change. Jofriet et al. (1977) also used constant values to calculate the lateral stresses on the wall of a grain bin. In this latter case and also in the present work, the grain was taken from an initial low state of stress when it was first deposited, to a higher stress state as more grain was placed on top of it, which according to classical retaining wall analysis would have approached a failure condition. Because this stress path encompassed a large part of the shear stress/strain characteristic of the material and also because in this dry granular material, dilational volume changes may have been significant, representation of the material by constant values of Young's modulus and Poisson's ratio was clearly inappropriate.

3.2.3 Models where E and ν depend on stress difference

As mentioned in Sect. 3.1, granular materials become progressively easier to distort as the shear stress is increased. To include this behaviour, some authors (Chang and Duncan, 1970; Clough and Duncan, 1971; Clough and Woodward, 1967; D'Appolonia and Lambe, 1970; Desai and Reese, 1970; Duncan and Chang, 1970; Dunlop and Duncan, 1970; Girijavallabhan and Reece, 1968; Kulhawy et al., 1969; Lee and Idriss, 1975; Yong and Hanna, 1977) have used Hooke's law but made the Young's modulus a decreasing

function of principal stress difference. A typical method is that of Kulhawy et al., (1969) who simulated the construction of an embankment using an incremental loading technique. In the simulation, successive layers of soil were considered to be placed one on top of the other, the placing of a new layer adding to the load on the soil mass below. Young's modulus and Poisson's ratio were assumed to depend on $(\sigma_1 - \sigma_3)$ and were calculated for the stress conditions at the start of each load increment. These incremental values, E_t and v_t , were then used to calculate the stresses and strains for the small change in load, after which new values of E_t and v_t were calculated for the new stress state. The incremental procedure was continued until the end of the simulation.

The variation of E_t and v_t with stress was obtained from triaxial compression tests at constant confining stress. For a linear elastic material under this test condition ($\sigma_2 = \sigma_3 = \text{constant}$), Eqn 3.1 may be used to show that

$$E_t = E = (\sigma_1 - \sigma_2) / \dot{\epsilon}_1$$

and

$$v_t = v = -\dot{\epsilon}_3 / \dot{\epsilon}_1$$

where a dot above a quantity indicates a small change in that quantity. Hyperbolic curves were fitted to data of $(\sigma_1 - \sigma_3)$ and ϵ_3 vs ϵ_1 following the method of Kondner and Zelasko (1963) and the failure of the material was represented by ϵ_1 becoming very large as the stress difference approached its asymptotic value $(\sigma_1 - \sigma_3)_{ult}$. The increased shear strength of the material with increased confinement was represented by making $(\sigma_1 - \sigma_3)_{ult}$ a function of σ_3 and the incremental values of Young's modulus and

Poisson's ratio and their dependence on the stresses were obtained by differentiation. The difference in behaviour of the material between loading and unloading and hence the plastic nature of the shear strain was represented by making the Young's modulus much higher when $(\sigma_1 - \sigma_3)$ was reducing. This value of E_t , which was obtained from unloading tests, was then used until $(\sigma_1 - \sigma_3)$ was equal to or greater than the previous maximum.

As Kulhawy et al. used Hooke's law with stress dependent moduli, their soil model can be described in terms of shear and normal quantities by the set of equations

$$\begin{bmatrix} \dot{\gamma} \\ \dot{\epsilon} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \dot{\tau} \\ \dot{\sigma} \end{bmatrix} \quad \dots\dots(3.7)$$

which are incremental forms of Eqn 3.6 in which the matrix elements a_{11} (the reciprocal of the shear modulus) and a_{22} (one third of the reciprocal of the bulk modulus) are functions of stress. As stated previously, because of the diagonal form of these equations, no normal strains can be ascribed to shear stresses. The way in which this diagonal form affects the measured values of E_t and ν_t is best explained by regarding Eqn 3.7 as having an element missing in the coefficient matrix. This term, a_{21} , would appear as the lower off-diagonal element and, if it were present, would relate the change in octahedral normal strain to the change in octahedral shear stress. In the test used by Kulhawy et al. to measure E_t and ν_t (the triaxial compression test at constant confining stress), τ and σ both change and so the normal strain increment $\dot{\epsilon}$, contains a component, $a_{21}\dot{\tau}$ arising from the change in shear stress and another, $a_{22}\dot{\sigma}$, arising from the change in normal stress. However,

because Kulhawy et al. implicitly assumed that $a_{21} = 0$, the whole of $\dot{\epsilon}$ was ascribed to a_{22} , thus adding a term $a_{21}\dot{\epsilon}/\dot{\sigma}$ to the true value of a_{22} . This modification to a_{22} depends therefore on the relative values of $\dot{\epsilon}$ and $\dot{\sigma}$ applied to the sample when tested and so the resulting values of a_{22} will depend on the type of test used to measure them, whereas they should be a property of the material alone. The extent to which this modification affects the calculated values of E_t and v_t also depends on the test conditions. In the method of Kulhawy et al., the calculation of E_t did not involve normal strains and so the modification to a_{22} was provided entirely by v_t . In some cases the rate of shear induced volume increase was greater than the rate of volume decrease due to the normal stress and this net volume increase resulted in a calculated value of v_t in excess of 0.5. Such a material would have a negative bulk modulus and a volume increase would be predicted with increasing stress under isotropic compression - a prediction clearly at variance with observation. The material would also be mechanically unstable (Love, 1944) and so, when implementing the finite element method, v_t had to be limited to a value just below 0.5.

3.2.4 Models using independent measurements of bulk and shear modulus

Some authors (Domaschuk and Wade, 1969; Domaschuk and Valliapan, 1975; Drnevitch, 1975; Battelino and Majes, 1977) have stated that Young's modulus and Poisson's ratio are not suitable parameters with which to describe granular materials and that bulk and shear moduli, or constrained and shear moduli, measured in separate tests would be more

appropriate. Drnevitch (1975) has illustrated this point by the use of "uncertainty multipliers" which are the sensitivities of the terms in the stress strain matrix of Hooke's law to errors in the determination of the Poisson's ratio. When ν approaches 0.5 these sensitivities become very large, due to a term $(1-2\nu)$ in the denominators of some of the elements, and, for example, a change in ν from 0.499 to 0.4999 (0.18%) changes $\partial \dot{\sigma}_x / \partial \dot{\epsilon}_x$ by a factor of 10. This shows that the limitation of ν_t to a value just below 0.5 (e.g. Kulhawy et al., 1969) can have large effects on the terms of the stress/strain matrix depending on the precise value of ν_t that is arbitrarily chosen.

Domaschuk and Wade (1969) measured the bulk modulus of dry sand by performing an isotropic compression test in a triaxial cell. In this test, the sample was compressed by water pressure with no other loads applied, thus varying σ with τ equal to zero. A curve was fitted to data of σ vs 3ϵ and the tangent bulk modulus, M_{st} (equivalent to $1/3a_{22}$ in Eqn 3.7), was calculated by use of the relationship

$$M_{bt} = \left. \frac{d\sigma}{d(3\epsilon)} \right|_{\tau=\text{const}}$$

giving M_{bt} as a function of σ . The test was only carried out with a single value of τ (zero) and so any possible variation of M_{bt} with τ was not investigated. The shear modulus was measured, again in a triaxial cell, by performing several tests in which τ was varied keeping σ constant for each test. The incremental shear modulus, M_{st} (equivalent to $1/a_{11}$), was calculated by fitting a curve to data of $\sqrt{3}\tau$ vs $\sqrt{3}\gamma$ at each value of σ and differentiating, that is

$$M_{st} = \left. \frac{d(\sqrt{3}\tau)}{d(\sqrt{3}\gamma)} \right|_{\sigma=\text{const}}$$

In this way, the variation of M_{st} with τ and σ was determined. Domaschuk and Valliappan (1975) later used the same method to investigate the bulk and shear moduli of clay while Battelino and Majes (1977) used it for a sand and a clay sample. This technique has an advantage over the triaxial compression test at constant confining stress in that the parameter a_{22} is not modified by that part of the volumetric strain caused by shear being wrongly ascribed to normal stress. This is because in the test to determine a_{22} the shear stress is held constant and so the term $a_{21} \dot{\tau} / \dot{\sigma}$ mentioned in Sect. 3.4 is zero. However this method, while measuring values of a_{11} and a_{22} , still neglects the off-diagonal term a_{21} and also a possible term a_{12} .

3.3 A new incremental model

3.3.1 Coupling between normal and shear terms

The previous section has shown that all the models reviewed are deficient in that they cannot allow for cross coupling between normal and shear quantities. However, once the equations are written in the form which exposes this fact (Eqn 3.7), it becomes a relatively simple matter to rectify it. The cross-coupling can be included by replacing the zeros in Eqn 3.7 by terms a_{12} and a_{21} which, on multiplying the matrices, gives

$$\begin{aligned}\dot{\gamma} &= a_{11}\dot{\tau} + a_{12}\dot{\sigma} \\ \dot{\epsilon} &= a_{21}\dot{\tau} + a_{22}\dot{\sigma}\end{aligned}\quad \text{.....(3.8)}$$

The material is now described by four independent parameters, instead of the two used in Hooke's law, which can all be measured in suitable tests.

3.3.2 Tests to evaluate parameters

Eqns 3.8 show that if σ is constant (that is $\dot{\sigma} = 0$) then a_{11} is equal to the change in $\dot{\gamma}$ divided by the corresponding change in $\dot{\tau}$ and the other three parameters can be defined in a similar way. The four parameters can therefore be measured using two types of test, in each of which one of the octahedral stresses is held constant while the other is varied and the resulting octahedral strains measured. The four parameters are obtained from the expressions

$$\left[a_{11} = \frac{\dot{\gamma}}{\dot{\tau}} \text{ and } a_{21} = \frac{\dot{\epsilon}}{\dot{\tau}} \right]_{\sigma = \text{const.}} \dots\dots(3.9)$$

and

$$\left[a_{12} = \frac{\dot{\gamma}}{\dot{\sigma}} \text{ and } a_{22} = \frac{\dot{\epsilon}}{\dot{\sigma}} \right]_{\tau = \text{const.}} \dots\dots(3.10)$$

The two types of test are best illustrated in principal stress space (Fig. 3.2) where the coordinates of a point are the three principal stresses acting on the sample under test. Also shown are the hydrostatic axis, along which all three principal stresses are equal, and an octahedral plane, which is a plane equally inclined to all three axes. A pictorial representation of the octahedral stresses for any point, P, in principal stress space may be obtained by manipulating Eqns 3.4 and 3.5. Thus the distance of the point from the hydrostatic axis measured along the octahedral plane containing the point equals $\sqrt{3}\tau$ and the distance of that octahedral plane from the origin measured along the hydrostatic axis equals $\sqrt{3}\sigma$. Lines parallel to the hydrostatic axis, therefore, represent stress paths for which τ is constant and lines lying in octahedral planes represent stress paths for which σ is

constant.

Any test in which the principal stresses are varied may be represented by a path through principal stress space but, with the standard triaxial equipment to be used in the tests here, two of the principal stresses, conventionally σ_2 and σ_3 are equal and so possible stress paths are confined to the plane that contains the σ_1 axis and which bisects the angle between the σ_2 and σ_3 axes, i.e. the triaxial plane.

The validity of a general stress/strain law should be investigated over the whole principal stress space and also when the axes of this space are rotated during the test. However, equipment for performing such tests is not generally available and the great amount of test work necessary would be prohibitive. The justification for deriving the model parameters from measurements on the triaxial plane only for this particular application is as follows:-

a) It is expected that the intermediate principal stress, σ_2 , will be of a similar value to σ_3 . For example, the angle of internal friction of dry cereal grains is about 26° (Lawton and Marchant, 1980) which gives a coefficient of active pressure, k_a , of 0.39. The coefficient of pressure for no lateral strain for granular materials has been approximated by the formula (Jaky, 1944)

$$k_0 = 1 - \sin \phi$$

and so k_0 will be about 0.56. For a long retaining wall where plane strain conditions apply, the maximum ratio between σ_2 and σ_3 (i.e. that assuming active failure conditions) is k_0/k_a or only about 1.4. This ratio is felt to be within the range over which the model can be reason-

ably extrapolated.

b) The angle of friction between the grain and the walls of the silo will be about half the angle of internal friction of the grain (Lawton, 1980; Lawton and Marchant, 1980). The maximum rotation of the principal stress axes away from the horizontal and vertical directions will be at the wall (assumed to be vertical) where

$$\sigma_{xy} = \sigma_x \mu_w$$

The Mohr circle can be used to show that the rotation of the principal axes under these conditions is only about 9° which is also felt to be within the range of extrapolation of the model.

The stress path for the test in which σ is constant may be determined by combining Eqn 3.5 with the triaxial test condition $\sigma_2 = \sigma_3$ to give

$$\sigma_1 = 3\sigma - 2\sigma_3 \quad \text{.....(3.11)}$$

and so this set of tests can be represented by a family of straight lines of slope -2 and intercept 3σ in the σ_3, σ_1 plane. Using Eqn 3.4 in a similar manner, the stress paths for the tests in which τ is kept constant are given by

$$\sigma_1 = \frac{3\tau}{\sqrt{2}} + \sigma_3 \quad \text{.....(3.12)}$$

and can be represented by another family of straight lines in the σ_1, σ_3 plane of slope 1 and intercept $3\tau/\sqrt{2}$.

The immediate use of the model produced by this work will be confined to loading situations where both the shear and the normal stresses will always be increasing. It is to be expected that the stress/strain behaviour will be different for decreasing stresses but, at present, tests have been confined to one direction only along each stress

path. If at a later date it becomes necessary to consider unloading situations, tests could be carried out in each direction and different model parameters determined for each one.

3.4 Experimental technique

3.4.1 Apparatus

Samples were tested in a standard triaxial cell suitable for a sample size of 100mm diameter and 200mm long. The axial loads on the piston were applied with weights and the movement of the piston was measured with a dial guage. A constant pressure apparatus is usually incorporated into triaxial equipment to prevent the confining stress from being affected by fluid displacements resulting from sample volume changes. In a common form (Bishop and Henkel, 1962), the pressure is provided by the difference between the levels of mercury in two pots. The upper pot is suspended from a spring with an appropriate spring rate, and any displacement of mercury between the pots caused by changes in the sample volume causes the spring to extend so lowering the pot that it is supporting and preventing any change in mercury head. A typical commercially available constant pressure apparatus (Engineering Laboratory Equipment type EL27-300) is designed to operate in the range 0-690kPa, whereas the maximum confining pressure envisaged in this work is about 16kPa. Obviously, a small pressure error in the commercial apparatus, caused by say spring non-linearity or hysteresis, which would be quite acceptable in the range for which it was designed could give a large error in the range considered here. To give acceptable performance at low confining stresses, a new constant pressure apparatus

has been designed. It also has an extra feature in that it provides a means of conveniently and accurately measuring the sample volume change.

The principle of operation is the same as the common form of constant pressure apparatus. The confining stress is provided by the difference between the levels of mercury in the two pots B and C (Fig. 3.3) but the spring has been replaced by an electric balance (Sauter type SD2COCT/0.1) which has a very accurate and constant spring rate. Further, the change in mass reading (Δm) of the balance is proportional to the volume of liquid displaced (ΔV) i.e.

$$\Delta V = \frac{\Delta m}{\rho_m - \rho_w}$$

where ρ_m and ρ_w are the densities of mercury and water. Providing there are no volume changes elsewhere in the system between pot B and the cell, ΔV equals the change in sample volume.

A further aspect of working at very low pressures is that the variation of water pressure over the sample length is not negligible. For a 200mm long sample this variation is about 2kPa. In this work, all tests results have been interpreted on the basis that the confining stress is that which exists at a point halfway up the sample. Although the system in this work used weights to apply the load to the piston of the cell, in many triaxial apparatuses the mid point of the sample moves upwards during the test because the top platen is held stationary by the piston bearing upon it while the cell body and the bottom platen is driven upwards against the load frame. Thus the mid point of the sample moves at half the rate

of the cell body. An ideal way of overcoming this problem would be to have a third pot (A) containing water that moved and remained level with the mid point of the sample. Any pressure change in the cell due to movement of the cell body would then be offset by an equal pressure change on the other side of pots B and C. The pressure at the mid point of the sample could be measured with a transducer fixed to pot A. However, it was felt that this would add unnecessary complication to a reasonably simple system so when using the apparatus in a standard load frame pot A is attached to the cell body itself with the pressure transducer (Ether type UP) level with the mid point of the unstrained sample. When, as in this work, weights are used to apply the load, pot A is placed on the bench alongside the triaxial cell. The set pressure can be adjusted by pressurizing pot A with air from a precision low pressure regulator (Norgren type 11-919-999).

It can easily be shown that the required relationship between the cross sectional area of the pots and the spring rate (change in mass reading per unit displacement) of the balance is

$$\frac{P_w}{a_A} = (P_m - P_w) \left[\frac{P_m - P_w}{K_s} - \frac{1}{a_B} - \frac{1}{a_C} \right]$$

where K_s is the spring rate, and a_A , a_B , a_C are the cross sectional areas. The apparatus (Fig. 3.4) was made from standard sized perspex tube and the areas of the pots were adjusted to the required values by glueing small pieces of perspex inside them. Stiffeners were glued to the tops and bottoms of the pots to minimise their deflections and hence volume changes during tests. In preliminary tests

in which sample volume changes were simulated by running water out of the cell, it was found that the apparatus could easily control the confining stress to within 50Pa. The resolution of the balance was 1g giving a resolution in sample volume change of 80mm^3 or 0.005% of a 100mm diameter by 200mm long sample.

3.4.2 Preliminary tests

Tests to evaluate the friction on the piston

The major principal stress on the sample in the triaxial cell is given by

$$\sigma_1 = \frac{W_p + F_t}{A_s} + \sigma_3 \quad \dots\dots(3.13)$$

where W_p is the weight of the top platen, F_t is the total force exerted by the piston on the top platen, and A_s is the cross sectional area of the sample. Although σ_3 varies over the length of the sample due to the different depth in the water of each part, this has been neglected and an average value, measured at the mid point of the unstrained sample has been used. With this approximation, F_t can be expressed as

$$F_t = W_a + W_{pi} - F_b - F_u \pm F_p \quad \dots\dots(3.14)$$

where W_a is the weight applied to the top of the piston, W_{pi} is the weight of the piston, F_b is the upward bouyancy force on the piston equal to the weight of the water it displaces, and F_u is the upthrust on the piston due to the superimposed water pressure. F_p is the frictional force on the piston which is positive when the piston moves upwards and negative when it moves downwards. F_b and F_u are given by

$$F_b = \ell A_p \gamma_w$$

and

$$F_u = A_p \sigma_3$$

where l is the distance that the piston extends into the water, A_p is the cross sectional area of the piston, and γ_w the specific weight of water.

The expression $W_{pi} - F_b \pm F_p$ was evaluated in a separate test as follows. The cell was filled with water with the sample absent and the piston extended into the water by the amount usual at the start of a test. The inlet valve to the cell was then closed and the piston pressed downwards slightly and allowed to return noting the water pressure, σ_{3d} , when the piston had stopped moving. The total downward force on the piston was then

$$W_{pi} - F_b - A_p \sigma_{3d} + F_p = 0 \quad \text{.....(3.15)}$$

The piston was raised slightly and allowed to fall back noting again the water pressure, σ_{3u} , when the piston had stopped moving. In this case the total downward force was

$$W_{pi} - F_b - A_p \sigma_{3u} - F_p = 0 \quad \text{.....(3.16)}$$

From Eqns 3.15 and 3.16

$$W_{pi} - F_b + F_p = A_p \sigma_{3d}$$

and

$$W_{pi} - F_b - F_p = A_p \sigma_{3u}$$

Substituting into Eqns 3.14 and 3.13 gives the expressions for the major principal stress

$$\sigma_1 = \frac{1}{A_s} \left[W_p + W_a + A_p (\sigma_{3u} - \sigma_3) \right] + \sigma_3 \quad \text{.....(3.17)}$$

for downward movement of the piston and

$$\sigma_1 = \frac{1}{A_s} \left[W_p + W_a + A_p (\sigma_{3d} - \sigma_3) \right] + \sigma_3 \quad \text{.....(3.18)}$$

for upward movement. These expressions can now be combined

with either Eqn 3.11 or Eqn 3.12, depending on the type of test (constant σ or constant τ) to be carried out, so giving the required relationship between σ_3 and W_a . In these calculations, the approximation was made that A_s was equal to the initial cross sectional area rather than that at any instant during the test. This made it possible to produce tables giving the value of σ_3 corresponding to each value of W_a beforehand. The stresses to be kept constant were calculated after each test using the actual cross sectional area of the sample and were all within 1% of their nominal values.

Errors due to the rubber membrane

Because of the large sizes of the grains used (maximum diameter about 8mm), an extra term was included in the measurement of volume change, caused by the shape of the rubber membrane between adjacent grains on the outside surface of the sample changing when the confining stress was changed. To measure this term, separate experiments were conducted using dummy samples. These samples, which were the same size as and had similar external textures to the real ones, were made by glueing grains to the outside of wooden cylinders and so, as the dummy samples were very rigid, it was assumed that the volume change measured was caused by the membrane effect mentioned above. The measured volume change was found to be proportional to the change in confining stress but depended on whether it was increasing or decreasing. A different constant of proportionality was therefore used in correcting for the membrane effect depending on the type of grain used and on the sign of the change in confining stress.

As the confining stresses were low, it was felt that the extra stress due to possible tensions in the rubber membrane might be significant. The need to apply a correction for this was avoided by deliberately making the membrane slightly oversize so that it was never in tension during the course of a test. This was achieved by stretching the membrane on a 150mm diameter tube and soaking it in hot water prior to use.

3.4.3 Test method

Sample preparation

Girijavallabhan and Reese (1968) found that the parameters in their Hooke's law model for sand varied with the initial void ratio of the sample. The same is likely to be true for the four parameters to be determined here but this investigation has been confined to a standard method of sample preparation which was shown in preparatory tests to give a virtually constant bulk density and hence initial void ratio. The extent to which the initial void ratio of a triaxial sample differs from that of the grain deposited in a much larger silo will be investigated in the appendix.

The samples were prepared using a split mould placed around the lower platen by the method outlined by Bishop and Henkel (1962) the lower platen being covered with a PTFE disk to reduce friction. The standard method of filling was to pour the grain into a tube of the same diameter as the mould and fixed above it. A screen with perforations large enough to allow the grains to drop slowly through was placed inside the tube, 250mm above the centre of the mould. After the mould was filled and the top platen, also covered with a PTFE disk, was sealed to the rubber membrane,

a small confining stress was applied by partially evacuating the inside of the membrane so supporting the sample when the mould was removed. The magnitude of the stress applied during sample preparation was measured by using the transducer on the constant pressure apparatus to measure the pressure difference between the outside of the membrane and the air on the inside. This pressure difference was not allowed to rise above 2kPa.

Slight differences in stress history were unavoidable although care was taken to keep their magnitudes to a minimum. To reduce the effect of this variability, all tests were started at a confining stress of at least 4kPa, the increase from the maximum of 2kPa used in sample preparation swamping any slight differences in stress history.

Stress paths used

The methods of testing with constant σ and constant τ were similar in that the two principal stresses were varied simultaneously to satisfy either Eqn 3.11 or Eqn 3.12. For constant σ , σ_1 was increased while σ_3 was reduced and for constant τ , σ_1 and σ_3 were both increased. Tests were carried out along six stress paths for constant values of σ between 8 and 18kPa and seven paths for constant values of τ between 0.94kPa and 6.60kPa. The sample was loaded by first putting a weight on the piston and then adjusting the confining stress to the corresponding value read from the previously prepared table (Sect. 3.4.2). Thus the continuous stress paths were approximated by a series of steps, between ten and twenty steps being used for each test.

3.4.4 Calculation of stresses and strains

As the volume strain is (Lambe and Whitman, 1969)

$$\epsilon_v = \epsilon_1 + \epsilon_2 + \epsilon_3$$

and, because of symmetry, ϵ_2 can be assumed to equal ϵ_3 , then the lateral strain is given by

$$\epsilon_3 = (\epsilon_v - \epsilon_1)/2$$

ϵ_v was obtained from the reading on the electric balance of the constant pressure apparatus (Sect. 3.4.1) and ϵ_1 by dividing the movement of the piston by the initial sample length. The average cross sectional area of the sample used in Eqns 3.17 and 3.18 was calculated from (Bishop and Henkel, 1962)

$$A_s = A_{so} \frac{1 - \epsilon_v}{1 - \epsilon_1}$$

The octahedral stresses were then calculated using Eqns 3.4 and 3.5 with the condition $\sigma_2 = \sigma_3$ and the octahedral strains using Eqns 3.2 and 3.3 with $\epsilon_2 = \epsilon_3$.

3.5 Results

3.5.1 Tests results for wheat

Results from tests on a wheat sample in which σ was held constant (Figs 3.5, top and 3.6, top) show that the relationships between the strains and the shear stress are far from linear. Because of this, the a terms will be functions of the stresses. These functions are determined by fitting curves to the experimental data and differentiating them. Sometimes the differentiation of experimental data can be hazardous because small errors in the data lead to relatively large errors in the difference between consecutive data points. Fitting curves and differentiating, rather than using differences in the experimental data,

helps to overcome this because the data is smoothed by the fitted curved.

The choice of the zero level for strain will not affect derivatives and so all strains have been plotted with respect to the strain at the start of the test. In both Fig. 3.5 (top), and Fig. 3.6 (top) the curves are asymptotic to vertical lines at shear stresses approximately proportional to the normal stress. In Fig. 3.5 (top) the slopes of the curves at low values of τ decrease with increasing σ while those in Fig. 3.6 (top) increase as σ is increased. The normal strains (Fig. 3.6, top) show peak values at about 75% of the asymptotic shear stress.

Curves which share these properties are the hyperbola

$$\gamma = \frac{K_1 \tau}{\sigma(1 - \tau/K_2 \sigma)} + C_1 \quad \text{.....(3.19)}$$

for the shear strain, and the sum of a hyperbola and a straight line

$$\epsilon = K_3 \tau \sigma - \frac{0.0625 K_3 K_2' \sigma^2}{(1 - \tau/K_2' \sigma)} + C_2 \quad \text{.....(3.20)}$$

for the normal strain. It is worthwhile pointing out that K_2 defines one of the more important properties of the material. When the shear stress is K_2 times the normal stress the shear strain becomes infinite and so K_2 is analogous to the coefficient of internal friction of the Mohr-Coulomb failure criterion.

When fitting these curves to the data, the values of the constants were chosen so that the fitted curves passed through the first and last experimental points and the one closest to a value of τ which was 75% of the distance from the first to the last value. After fitting a curve

for each value of σ , K_1 and K_3 were averaged for the set of six curves. The asymptotic values of stress defined by K_2 and K_2' were regarded as arising from the same cause, that is the failure of the sample and so a value averaged from the results of both sets of tests was taken to represent both K_2 and K_2' . The values of the constants K_1 to K_3 , which characterise the responses to shear stress over the whole of the experimental range, are given in Table 3.1 along with some other physical properties of the materials.

The parameters a_{11} and a_{21} can be determined by partial differentiation of Eqns 3.19 and 3.20 with respect to τ to give

$$a_{11} = \frac{K_1}{\sigma(1 - \tau/K_2\sigma)^2} \quad \dots\dots(3.21)$$

$$a_{21} = K_3\sigma - \frac{0.0625K_3}{(1 - \tau/K_2\sigma)^2} \quad \dots\dots(3.22)$$

The initial slope of the relationship between shear strain and normal stress (Fig. 3.7, top) was negative for low values of τ becoming positive as τ was increased. Although the relationships were not linear, the form of this non-linearity was not apparent and so a straight line was fitted to each set of experimental points by the method of least squares of the form

$$Y = S\sigma + C_3 \quad \dots\dots(3.23)$$

where the slope of the line, S , was given by

$$S = K_4\tau + K_5$$

The relationships between normal strain and normal stress (Fig. 3.8, top) were fitted by parabolic curves

$$\epsilon = K_6\sigma + K_7\sigma^2 + C_4 \quad \dots\dots(3.24)$$

using three points as before. It was found that the terms K_6 and K_7 were approximately constant for all values of τ and so mean values were taken to represent the responses to normal stress over the whole of the experimental range. The values of the constants K_5 to K_7 are also given in Table 3.1.

Eqns 3.23 and 3.24 can be differentiated with respect to σ to give the remaining a terms as

$$a_{12} = K_4\tau + K_5 \quad \dots\dots(3.25)$$

$$a_{22} = K_6 + 2K_7\sigma \quad \dots\dots(3.26)$$

3.5.2 Results for other types of grain

In addition to the wheat sample just described, samples of rapeseed, tick beans, and barley of two different moisture contents, were tested. The wheat and barley grains were shaped approximately like prolate spheroids with the ratio between major and minor axes about 2.2 and 2.6 respectively, while the rapeseed and tick beans were approximately spherical. Even so, the stress/strain relationships were all of the same form as those of the wheat sample and so similar curves were fitted to the data. Table 3.1 shows the values of the K terms for all the samples tested. The equivalent diameters of the grains are the diameters of the spheres in a dense hexagonal packing having the same number per unit volume as the grains. The number per unit volume was derived from the bulk density and 1000 grain mass.

3.6 Use of the model

3.6.1 Method

The trapezoidal rule of numerical integration (Korn and Korn, 1968) was used to predict the strains

Table 3.1

Parameters defining stress/strain characteristics along with other physical properties for all the materials tested. K_1 and K_2 are non-dimensional but for K_3 to K_7 the units are consistent for stresses measured in kPa and strains in per thousand.

Type of grain	Moisture content, % wet basis	Bulk density, kg/m ³	Equivalent diameter, mm	K_1	K_2	K_3	K_4	K_5	K_6	K_7
Wheat	14.8	678	4.8	4.77	.635	.0140	.060	-.331	.616	-.0107
Barley	13.6	635	4.5	3.07	.594	.0276	.075	-.223	.625	-.0116
Barley	20.1	625	4.6	7.27	.716	.0472	.063	.076	1.134	-.0186
Rapeseed	8.5	607	2.1	2.13	.572	.0179	.047	.017	.280	-.0046
Tick beans	13.8	740	8.4	3.11	.524	.0215	.062	-.034	.679	-.0191

occurring in a sample under the action of applied stresses. The loading sequence to be simulated was first divided into small steps. To calculate the strain change occurring as a result of a stress change, the values of the parameters a_{11} , a_{12} , a_{21} , and a_{22} were calculated at the mean values of the stresses applied during the step. For instance, if the stresses at the start of the first and second steps were τ_0 , σ_0 and τ_1 , σ_1 then a_{11} was calculated using Eqn 3.21 where

$$\tau = (\tau_0 + \tau_1)/2$$

and

$$\sigma = (\sigma_0 + \sigma_1)/2$$

After calculating the a terms, Eqns 3.8 were used to calculate the strain increments, $\dot{\gamma}$ and $\dot{\epsilon}$, where the stress increments were given by

$$\dot{\tau} = \tau_1 - \tau_0$$

$$\dot{\sigma} = \sigma_1 - \sigma_0$$

The strain levels at the end of a step were calculated by adding the strain increments to the strains existing at the start of a step. The strains existing at the start of the first step were chosen to be zero.

3.6.2 Simulations of tests used to derive parameters

The use of the model involves a numerical integration procedure and relies on parameters (the a terms) which are obtained from differentiating reasonably simple functions derived from curves fitted to the experimental data. Thus if the loading sequences used in the tests to derive the a terms are simulated, the measured and predicted strains will not agree exactly and the comparison between

the two will be an indication of how well the simple functions fit the whole ranges of experimental data and also of the errors due to the numerical integration.

The predicted strains for the wheat sample are shown in Figs 3.5 (bottom) and 3.6 (bottom) for the constant σ tests and Figs 3.7 (bottom) and 3.8 (bottom) for the constant τ tests. In Fig 3.5 (bottom) the shapes of the curves and their asymptotic stresses are well represented but the strains on the initial parts of the curves are generally overestimated, especially at the lower values of σ . In Fig. 3.6 (bottom) the shapes of the curves and the asymptotic levels of stresses are again well represented, but, as is to be expected, the predicted curves are a much more idealised version of the experimental ones. As a linear function was used to represent the shear strain/normal stress characteristic, the predicted relationships cannot be expected to represent the shape of the experimental ones. Despite this, the predictions do show the increase in the slopes as τ is increased and the strain levels are reasonably well represented. The measured normal strains for constant τ are well represented, Fig. 3.8 (bottom) showing good agreement both in the shape of the curves and in the values of the strains.

3.6.3 Use of model where τ and σ change simultaneously

The success of the model depends on its ability to predict strains over a range of possible stress paths. It has been shown that the model exhibits similar characteristics to actual samples in cases where either the shear or the normal stress is held constant. It can be used when the stresses change simultaneously by superpos-

ing the effect of the shear stress change on that of the normal stress change. Although the material is non-linear in terms of stresses and strains, by Eqns 3.8 it is linear in terms of small change in these quantities. Superposing the effects of shear and normal stresses is therefore valid if the changes are small.

To determine the performance when both stresses change simultaneously, the shear and normal strains were measured in three standard triaxial compression tests with the confining stress at 10kPa and are compared with the values calculated using the new model in Fig. 3.9. The shapes of the characteristics are predicted well, both the failure of the sample and the initial volume decrease followed by an increase (dilation) being shown. The predicted normal strain (Fig. 3.9, top) is however generally higher than the measured value although the peak of the curve occurs at the correct value of shear stress. Considering the scatter in the experimental results, the shear strain (Fig. 3.9, bottom) is predicted well although the predicted strains are slightly high at low values of shear stress.

Standard triaxial compression tests were also simulated for the other types of grains using the parameters in Table 3.1. The calculated strains along with those from practical tests are shown in Figs 3.10 to 3.13. As with the wheat sample, the shapes of the strain curves are predicted well but the predicted peak value of the normal strain is larger than the experimental one. The relative magnitudes of normal strain and shear stress at failure are, however, well predicted. The peak normal

strains are predicted about equal for tick beans and rapeseed, about 1.75 times higher for wheat and dry barley and about 4.7 times higher for wet barley, these ratios being very close to those observed experimentally. The shear stress at failure is predicted as being lowest for tick beans, followed in ascending order by rapeseed, dry barley, wheat, and wet barley, which agrees with the experimental order.

3.7 Comparisons with other models

3.7.1 The model of Kulhawy et al. (1969)

In this model the results of standard triaxial compression tests are used to determine values for the tangent Young's modulus and Poisson's ratio. Seven such tests were carried out with confining stresses in the range 4-16kPa on the wheat sample already described. Hyperbolic curves of the form

$$(\sigma_1 - \sigma_3) = \frac{\epsilon_1}{1/E_i + \epsilon_1/(\sigma_1 - \sigma_3)_{ult}} \quad \text{.....(3.27)}$$

were fitted to the data where E_i is the initial slope of the hyperbola and $(\sigma_1 - \sigma_3)_{ult}$ is the asymptotic value of the deviator stress. As with the results of Kulhawy et al. for sand, it was found that E_i increased for increasing confining stress (although very slightly) and their relationship could be represented by

$$E_i = 4.54 \sigma_3^{0.06} \quad \text{.....(3.28)}$$

σ_3 being measured in kPa and strains in per thousand. $(\sigma_1 - \sigma_3)_{ult}$ was found to be proportional to σ_3 and represented by

$$(\sigma_1 - \sigma_3)_{ult} = 2.25 \sigma_3 \quad \text{.....(3.29)}$$

Combining Eqns 3.28 and 3.29 with Eqn 3.27 and differentiating gives the tangent Young's modulus as

$$E_t = \left[1 - (\sigma_1 - \sigma_3)/2.25\sigma_3 \right]^2 4.54\sigma_3^{0.06}$$

To determine the tangent Poisson's ratio, curves were fitted to data of ϵ_3 vs ϵ_1 giving

$$-\epsilon_3 = K_8\epsilon_1 + K_9(1 - \exp(-K_{10}\epsilon_1)) \quad \dots\dots(3.30)$$

where K_8 , K_9 and K_{10} varied with the confining stress and were given by

$$K_8 = 0.63 - 0.0052\sigma_3$$

$$K_9 = -1.25 - 0.242\sigma_3$$

$$K_{10} = 0.525 - 0.0261\sigma_3$$

The tangent Poisson's ratio was obtained as a function of $(\sigma_1 - \sigma_3)$ and σ_3 by differentiating Eqn 3.24 with respect to ϵ_1 and substituting for ϵ_1 using Eqn 3.21 giving

$$\nu_t = K_8 + K_9 K_{10} \exp \left[\frac{-K_{10}(\sigma_1 - \sigma_3)}{4.54\sigma_3^{0.06}(1 - (\sigma_1 - \sigma_3)/2.25\sigma_3)} \right]$$

with the provision that ν_t was limited to be less than or equal to 0.5 (Sect. 3.2.3).

The predicted variation of γ with τ using the model of Kulhawy et al. for the tests where σ was held constant (Fig. 3.14, top) shows that the experimental shear strains (Fig. 3.5, top) are well represented. However this model also predicts that the normal strain for these tests is always zero whereas Fig. 3.6 (top) shows this not to be the case. Fig. 3.14 (bottom) shows that the variation of ϵ with σ for the tests in which τ was held constant (Fig. 3.8, top) is reasonably well predicted when τ is low but over-estimates ϵ for the higher values of τ . The model

also predicts that γ will be zero for all these tests whereas some shear strain was measured (Fig. 3.7, top). The shear strain in the triaxial compression test is predicted well (Fig. 3.15) as is to be expected because the model parameters are derived from these tests. The normal strain is also reasonably well represented up to the peak of the curve although the peak value is rather low but, because the tangent Poisson's ratio must not be allowed to rise above 0.5, no dilation at all can be predicted and the normal strain remains flat at the peak value.

3.7.2 The model of Domaschuk and Wade (1969)

The two parameters for this model were derived from the same tests as those of the new model, and can be related to the terms of Eons 3.8. In this model the octahedral strain increments are related to the octahedral stress increments by the equations

$$\dot{\gamma} = \dot{\tau} / M_{st}$$

$$\dot{\epsilon} = \dot{\sigma} / M_{bt}$$

Their tangent shear modulus is thus equal to $1/a_{11}$ and their tangent bulk modulus is equal to $1/3a_{22}$. The model is similar to the new one except that the cross-coupling terms, a_{12} and a_{21} , in Eqns 3.8 are equal to zero. The predicted shear strain/shear stress and the normal strain/normal stress relationships with σ and τ constant will therefore be the same as those predicted by the new model as they depend only on a_{11} and a_{22} . The strains which depend on the cross-coupling terms however (shear strain/normal stress and normal strain/shear stress) will always

be predicted as zero. Fig. 3.15 shows the simulation of the triaxial compression test using the model of Domaschuk and Wade. The predicted shear strain is very close to that of the new model. This is because the shear and normal stress increments are of the same order whereas a_{11} is generally much larger than a_{12} and so nearly all the shear strain in this type of test arises from the change in shear stress and a zero value of a_{12} has little effect. The normal strain is shown as increasing throughout the test, the dilation not being predicted at all. This is because the model assumes that normal strain increments are caused only by changes in normal stress, and, because the parameter a_{22} is positive for all values of stress over the range considered, the normal strain must always increase.

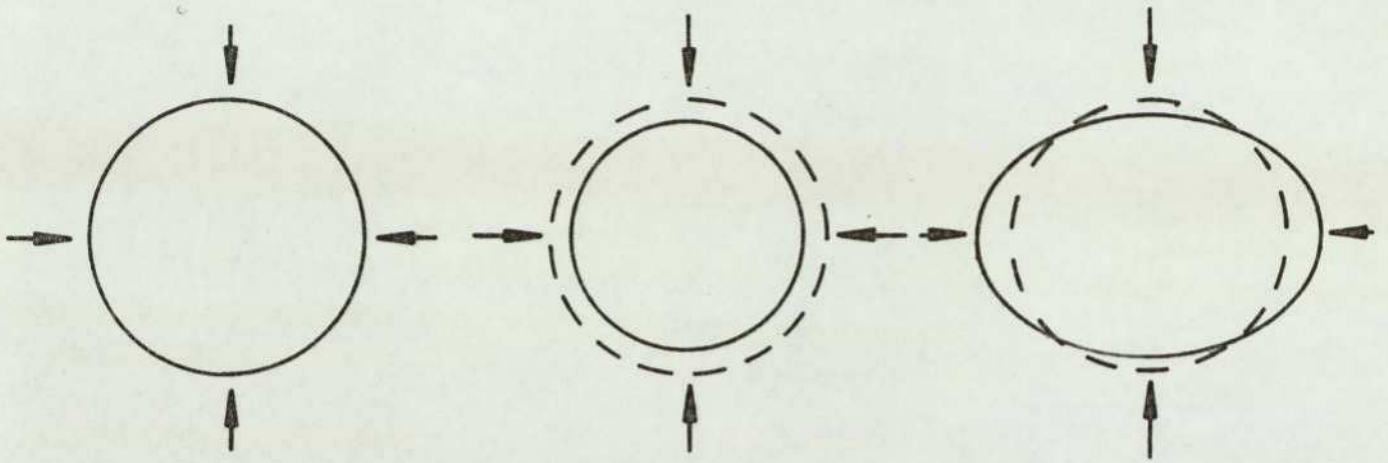


Fig. 3-1 Two dimensional object under various stress conditions. Left, initial isotropic stress. Middle, isotropic stress change. Right, anisotropic stress change keeping mean stress constant. Original shape shown thus :- - - - -.

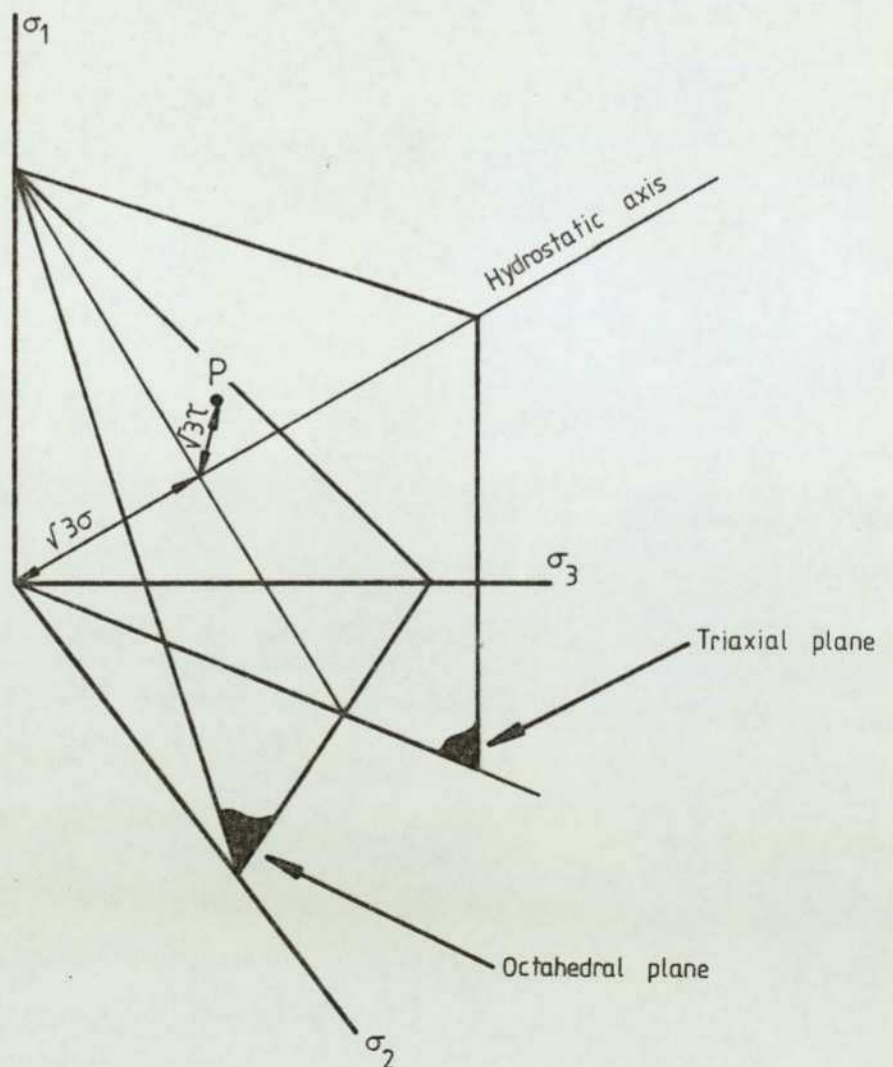


Fig. 3.2 Principal stress space.

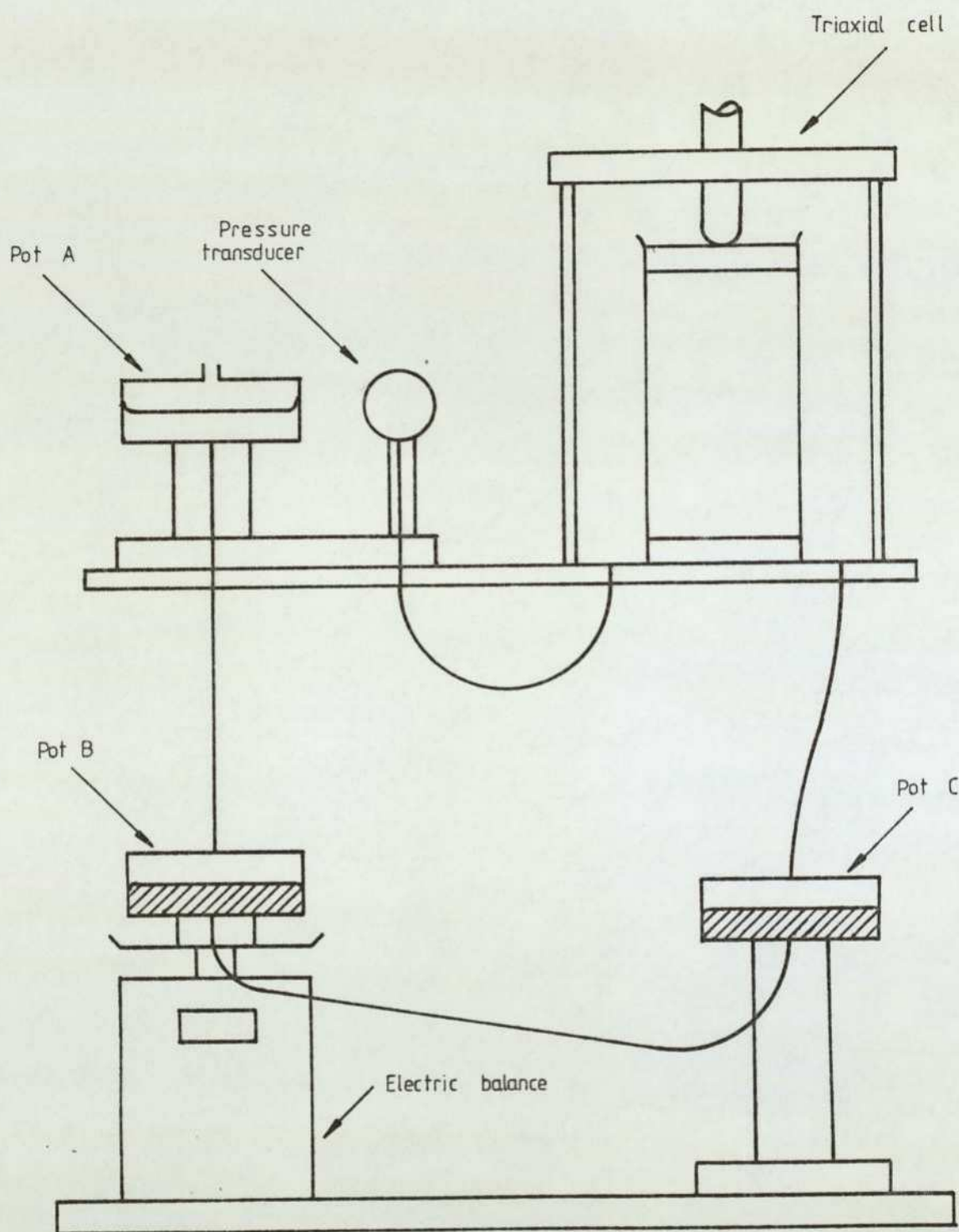


Fig. 3.3 Schematic diagram of the constant pressure apparatus.

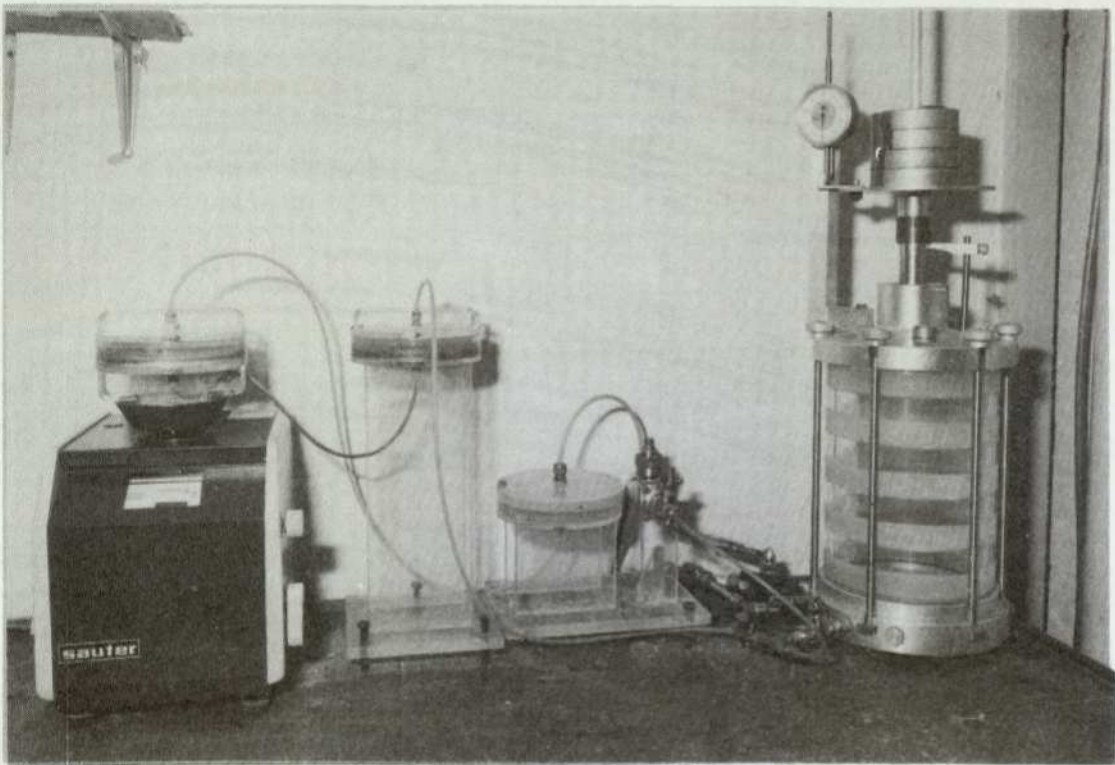


Fig. 3.4. Triaxial equipment with constant pressure apparatus.

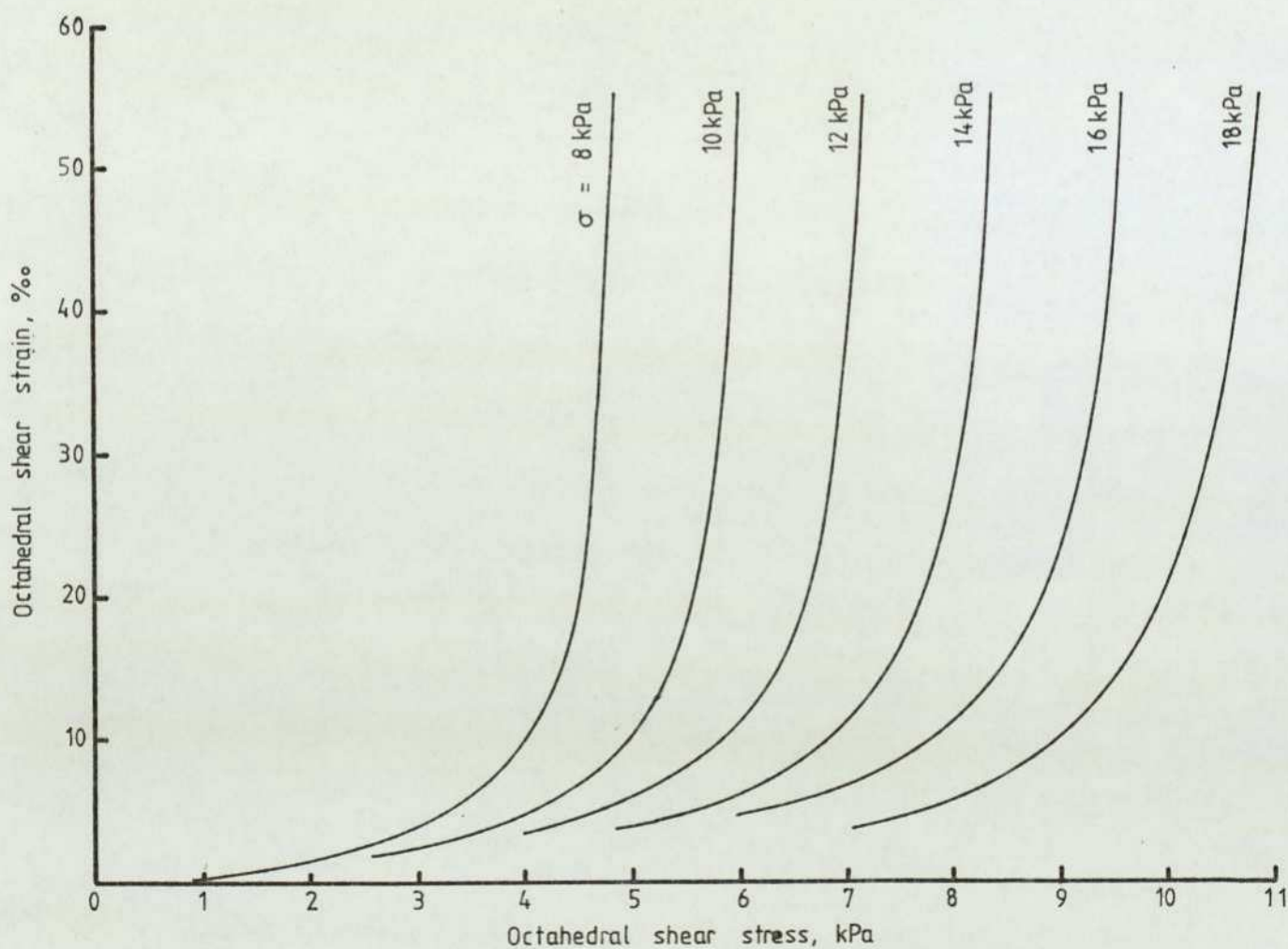
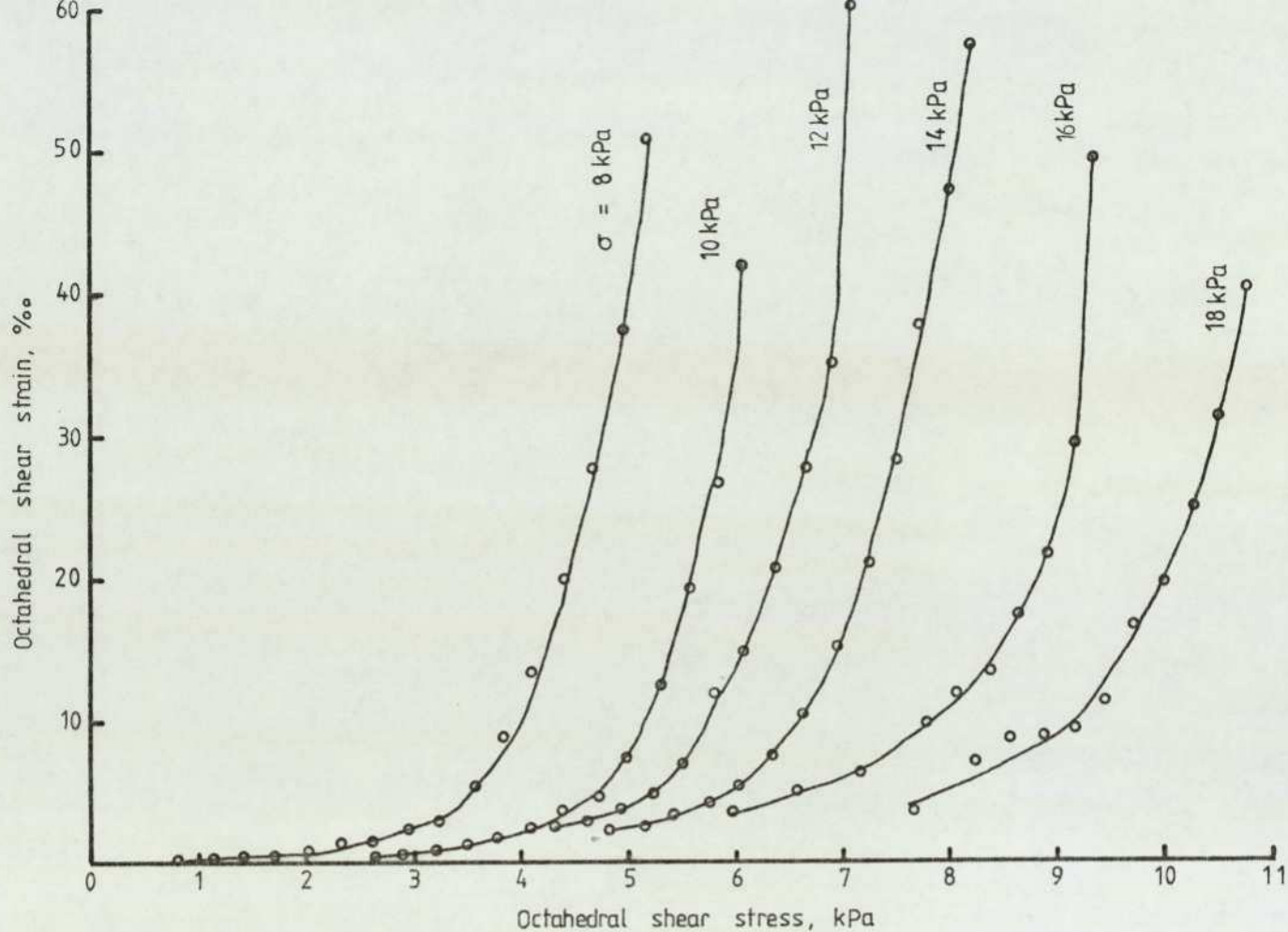


Fig. 3.5 Octahedral shear strain vs. octahedral shear stress at different values of octahedral normal stress for wheat. Top, experimental. Bottom, predicted using new model.

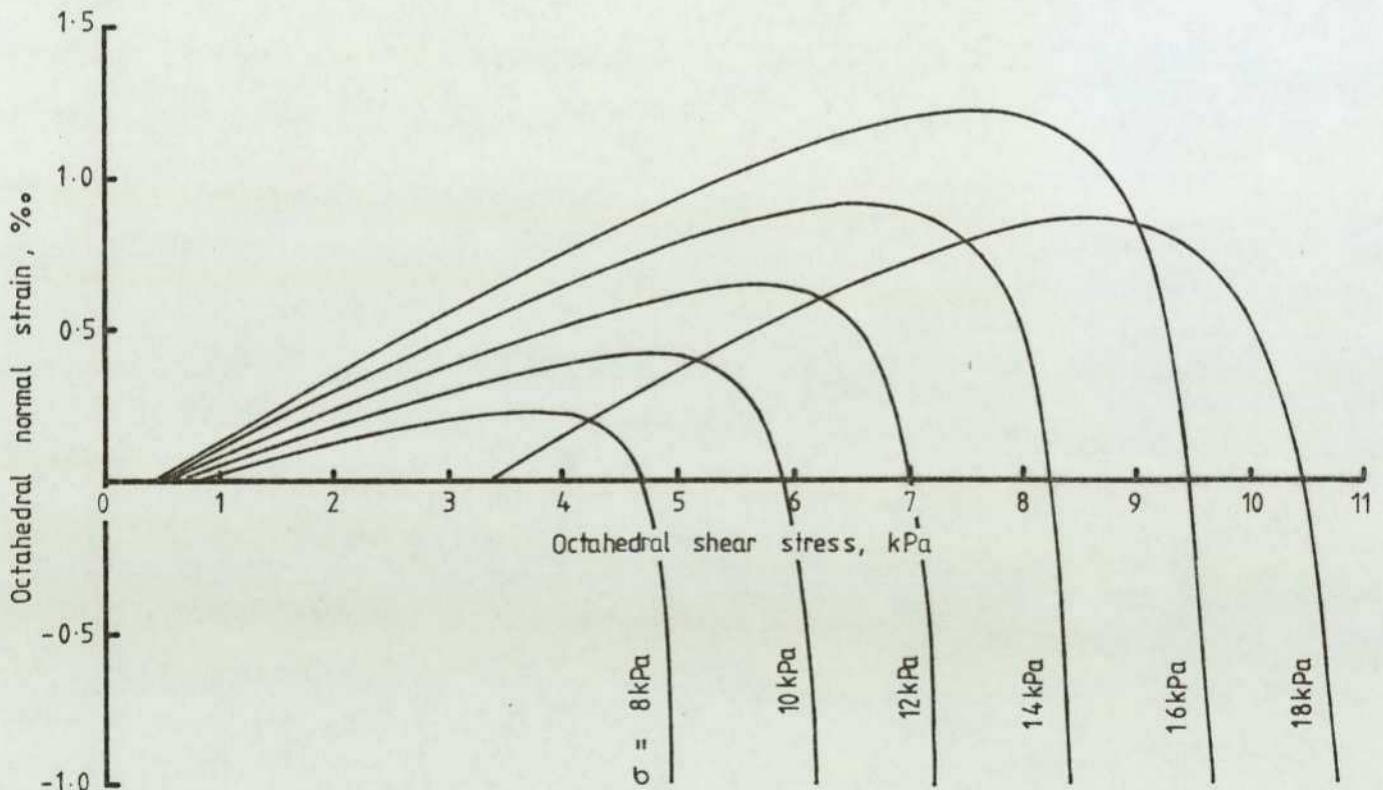
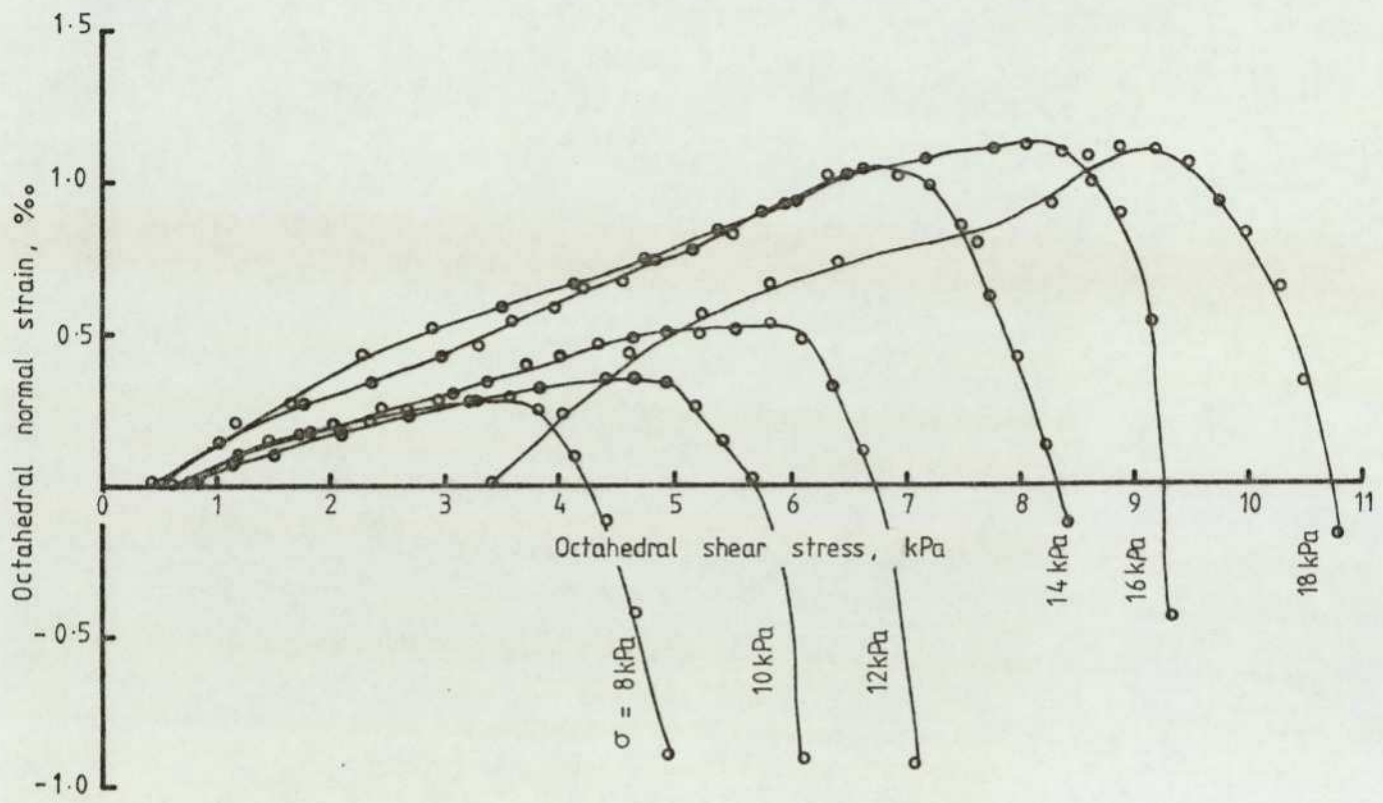


Fig. 3.6 Octahedral normal strain vs. octahedral shear stress at different values of octahedral normal stress for wheat. Top, experimental. Bottom, predicted using new model

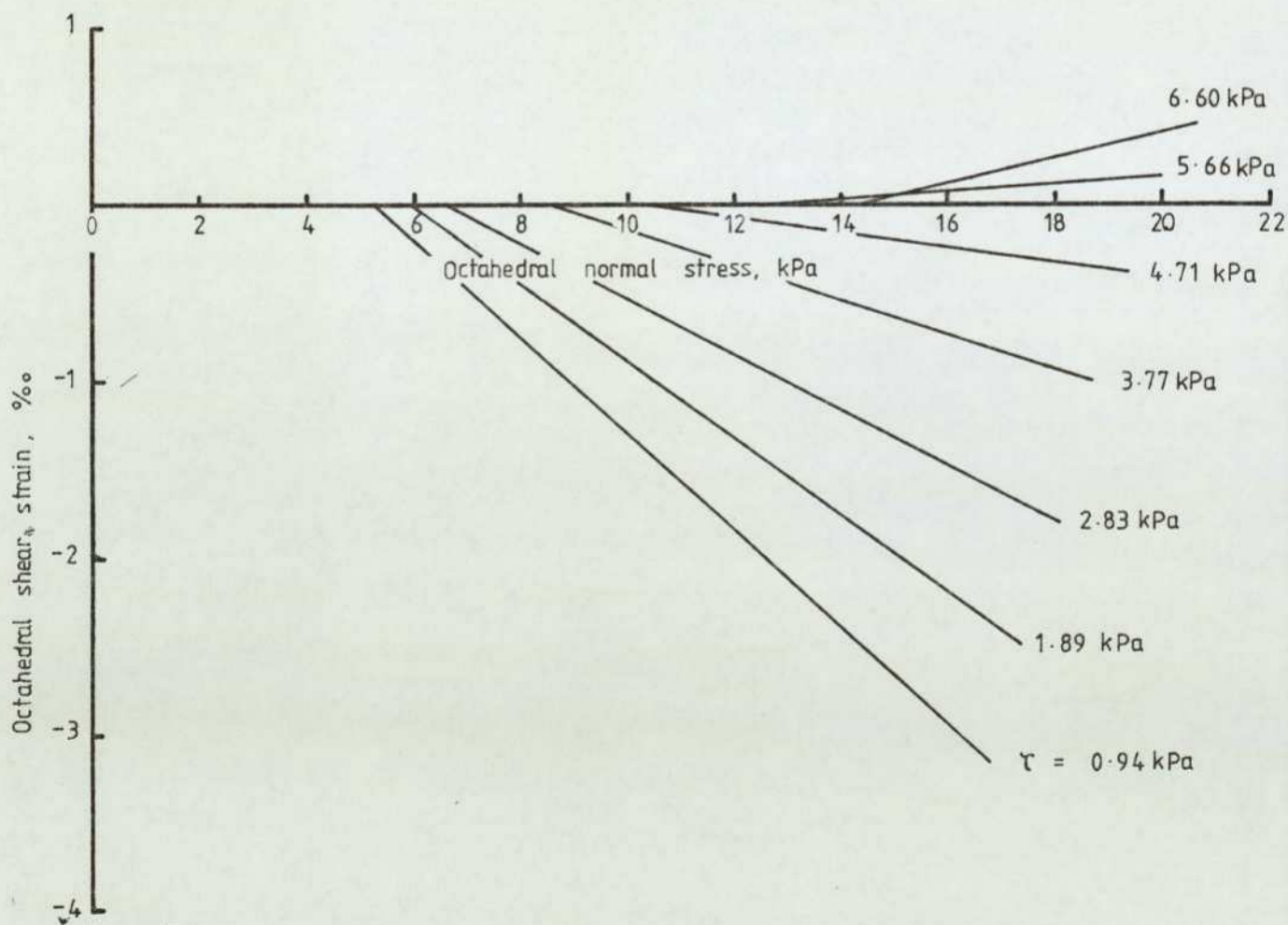
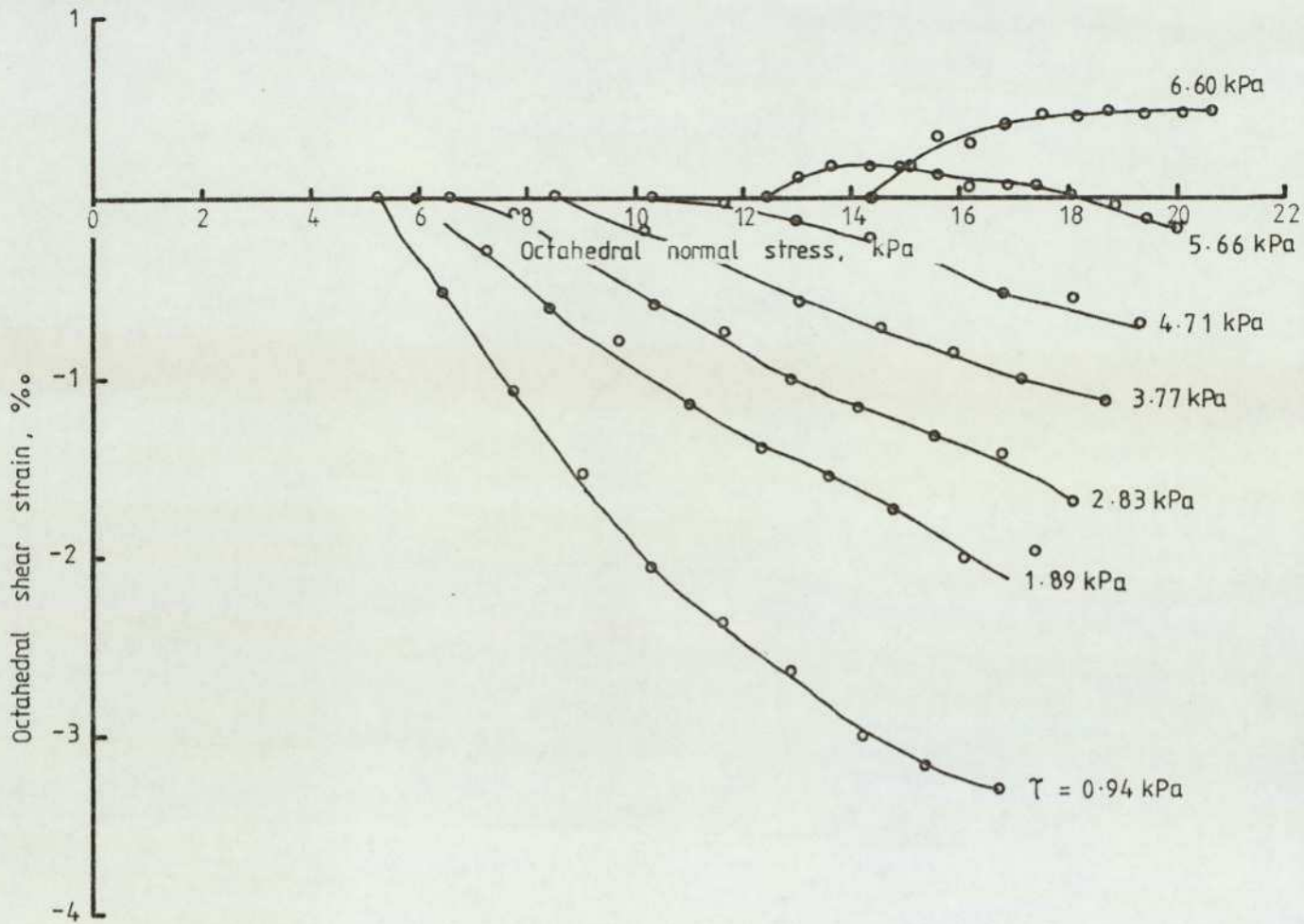


Fig. 3.7 Octahedral shear strain vs. octahedral normal stress at different values of octahedral shear stress for wheat. Top, experimental. Bottom, predicted using new model.

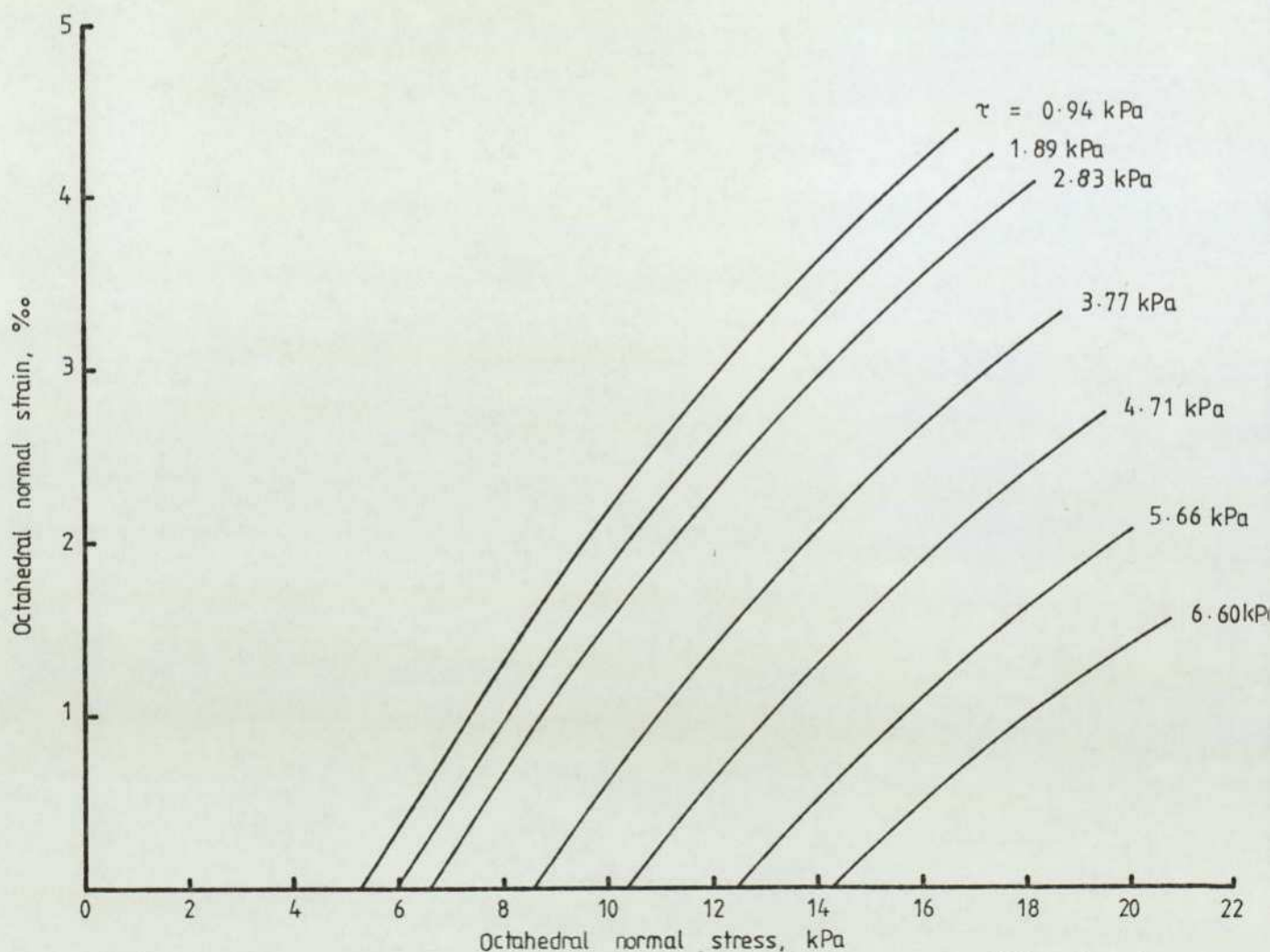
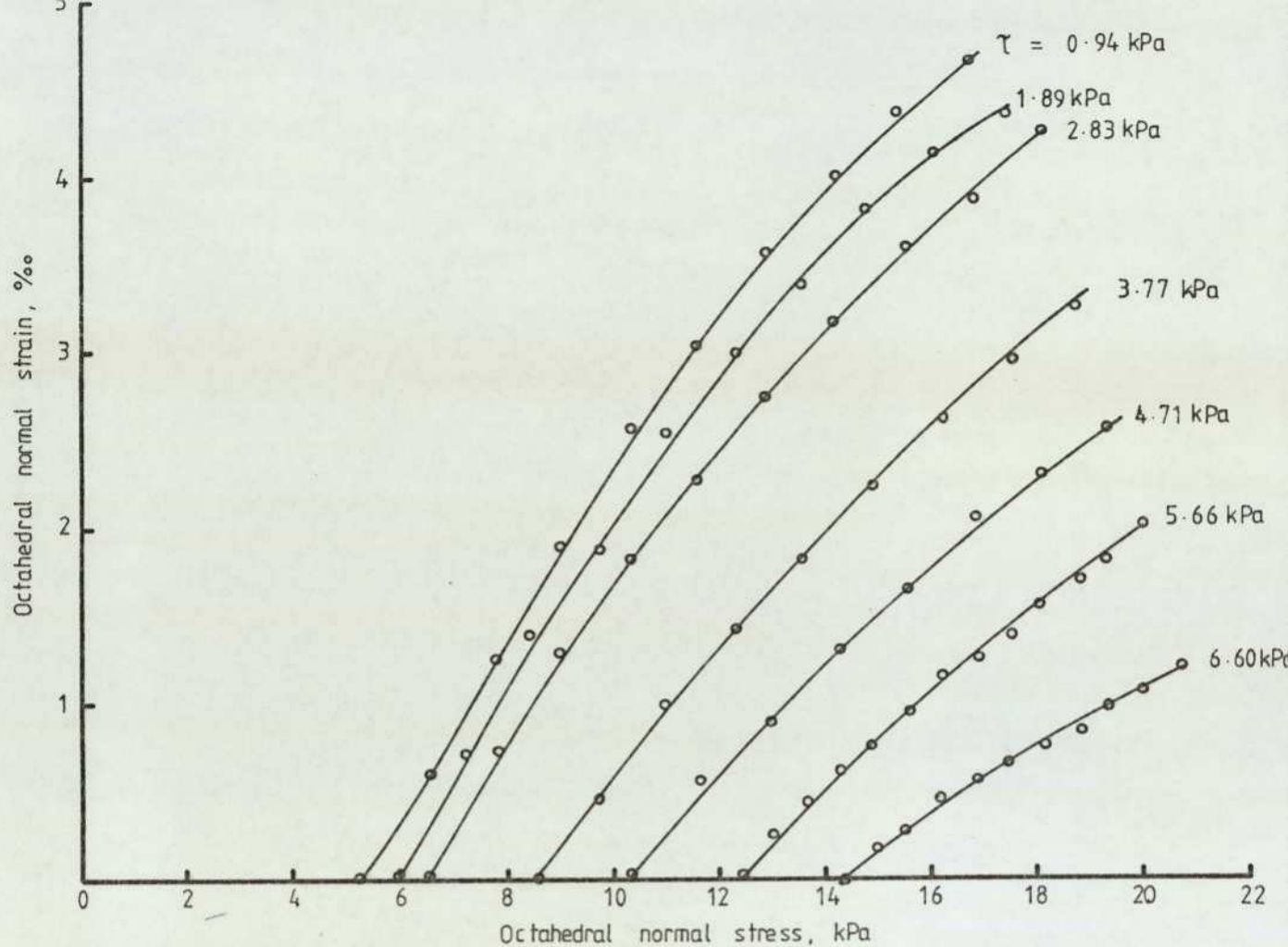


Fig. 3.8 Octahedral normal strain vs. octahedral normal stress at different values of octahedral shear stress for wheat. Top, experimental. Bottom, predicted using new model.

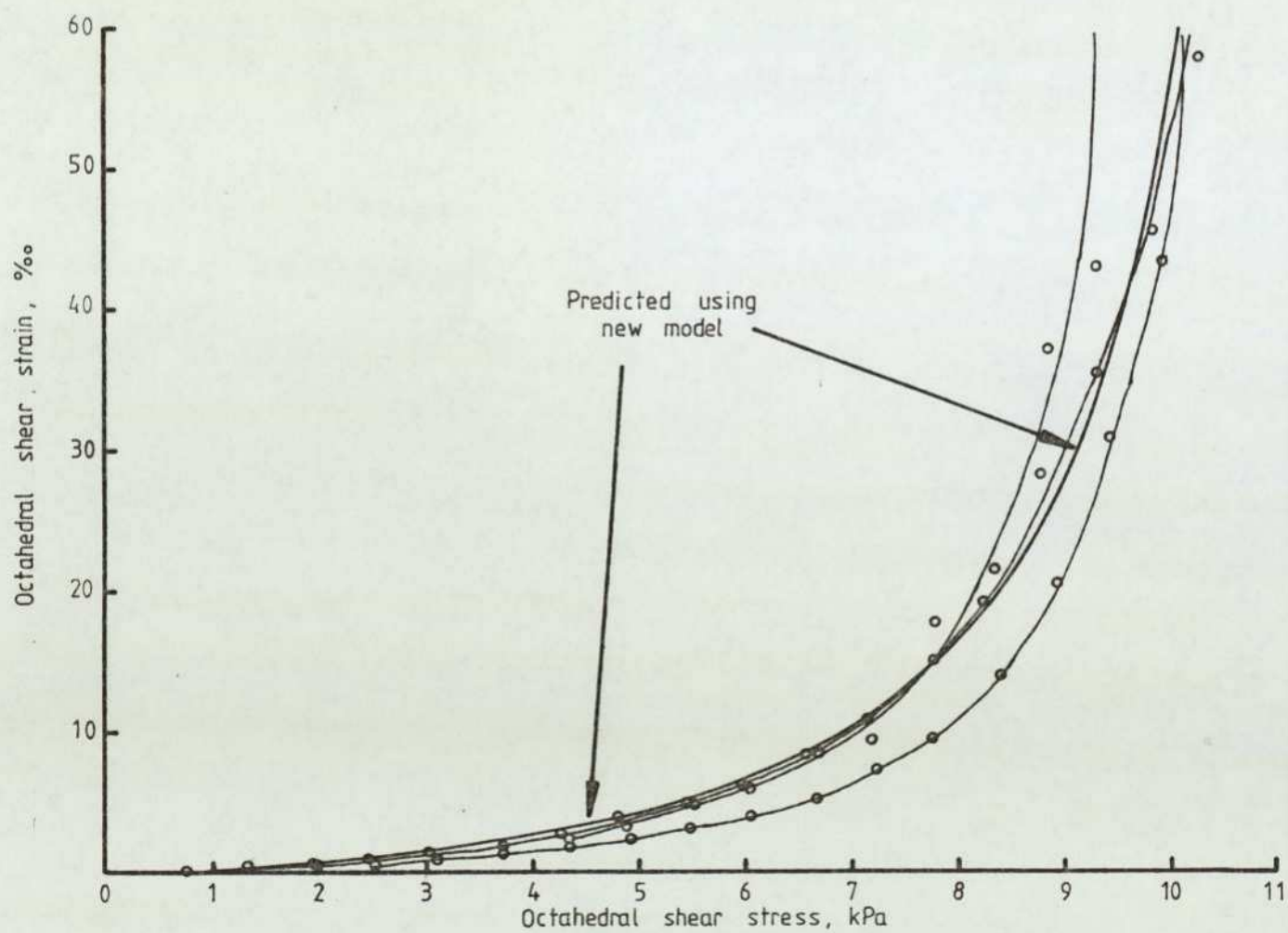
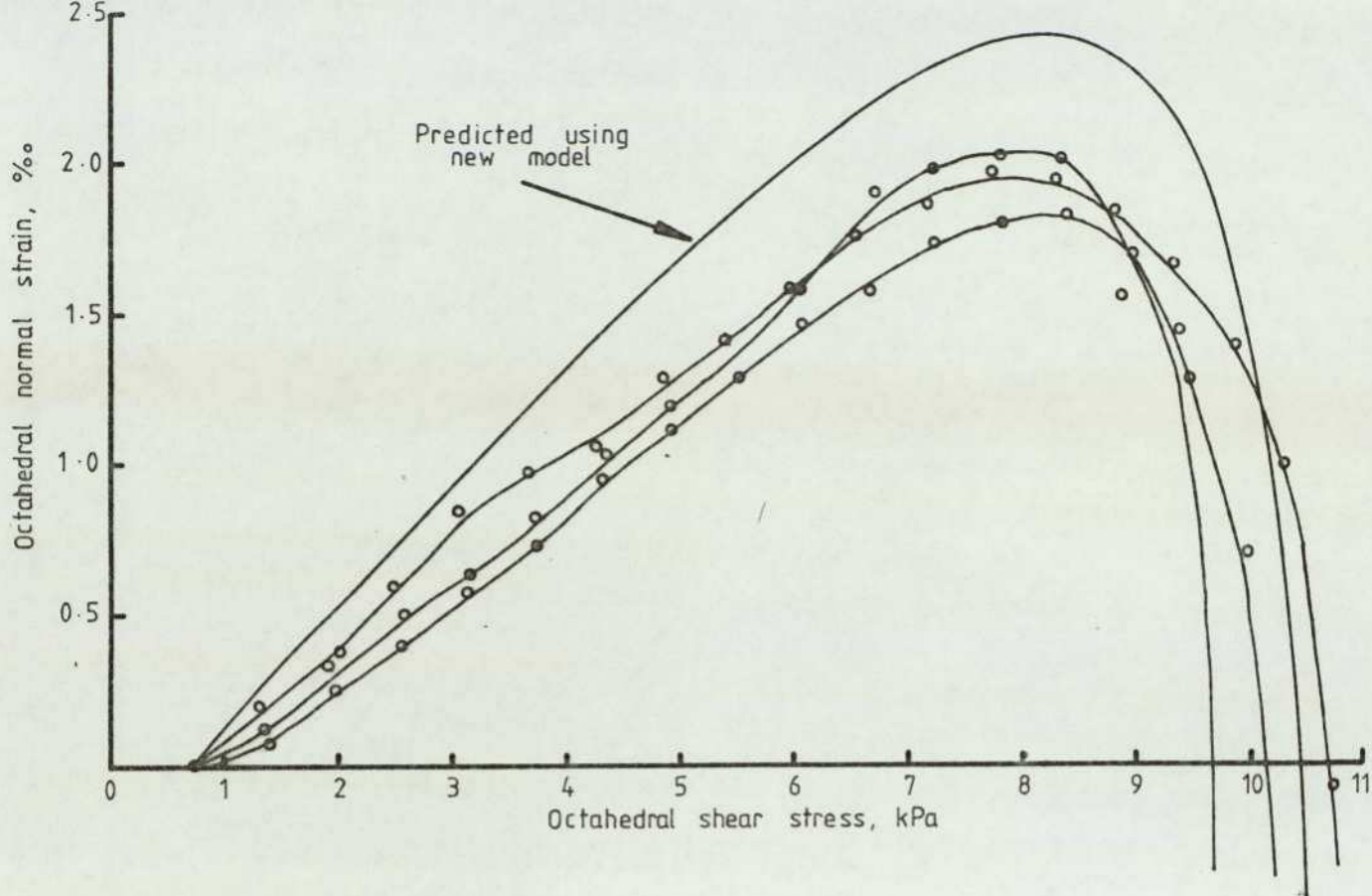


Fig. 3.9 Measured ($\circ-\circ$) and predicted (—) strains vs. octahedral shear stress for wheat in triaxial compression with $\sigma_3 = 10$ kPa. Top, octahedral normal strain. Bottom, octahedral shear strain.

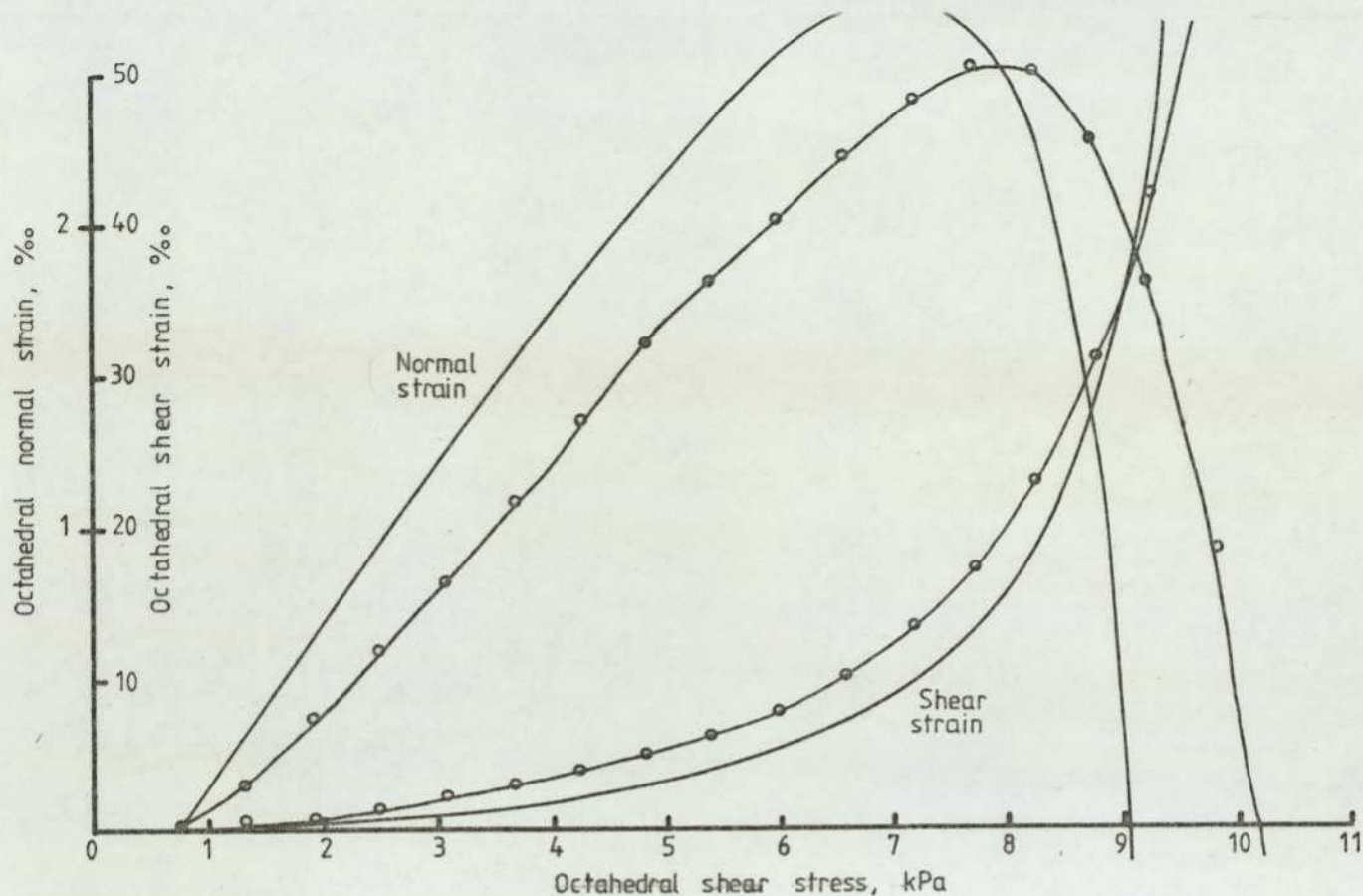


Fig. 3.10 Measured and predicted strains for barley (13.6% m.c.) in triaxial compression with $\sigma_3 = 10$ kPa .

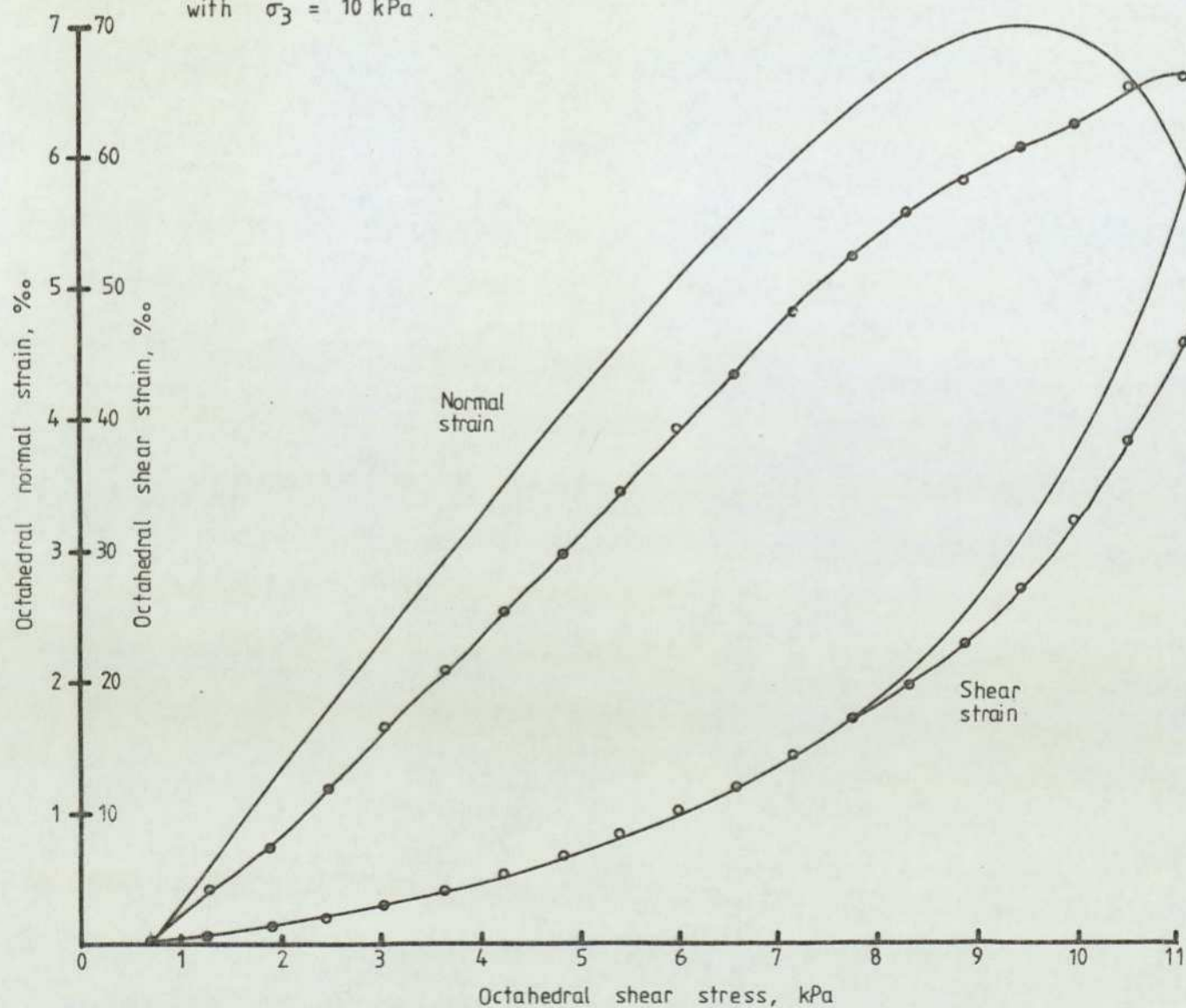


Fig. 3.11 Measured and predicted strains for barley (20.1% m.c.) in triaxial compression with $\sigma_3 = 10$ kPa .

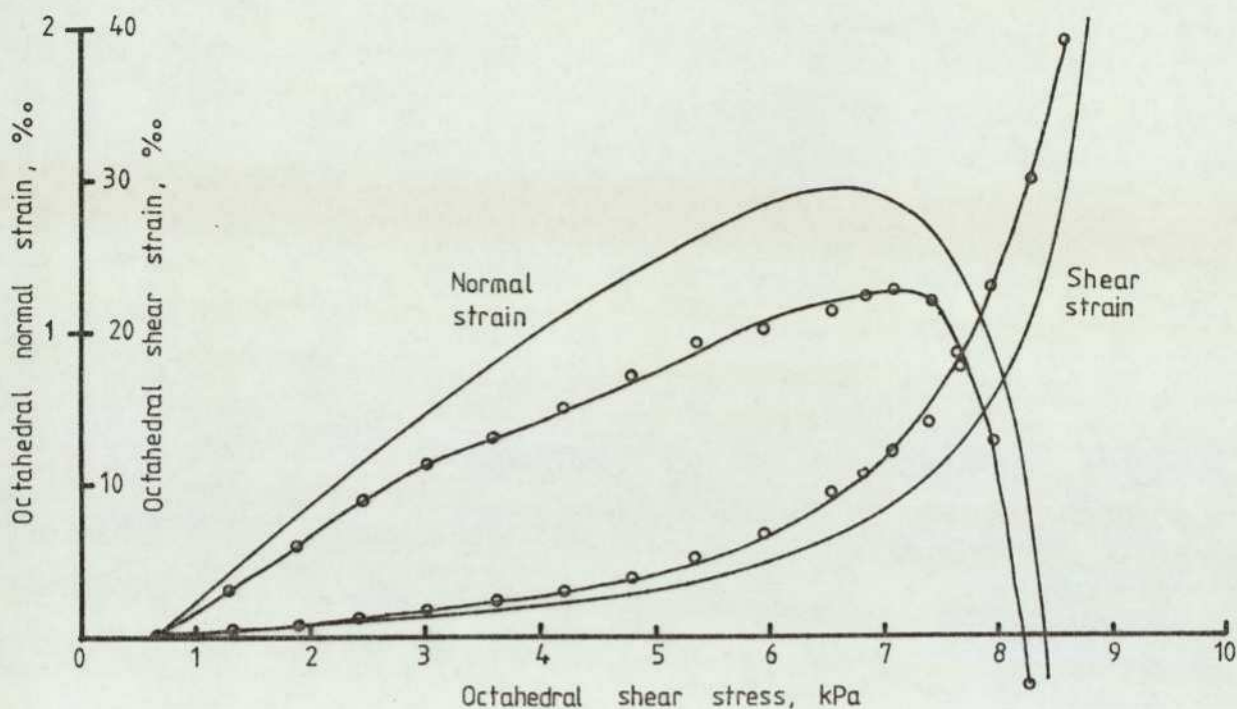


Fig. 3.12 Measured and predicted strains for rapeseed in triaxial compression with $\sigma_3 = 10$ kPa.

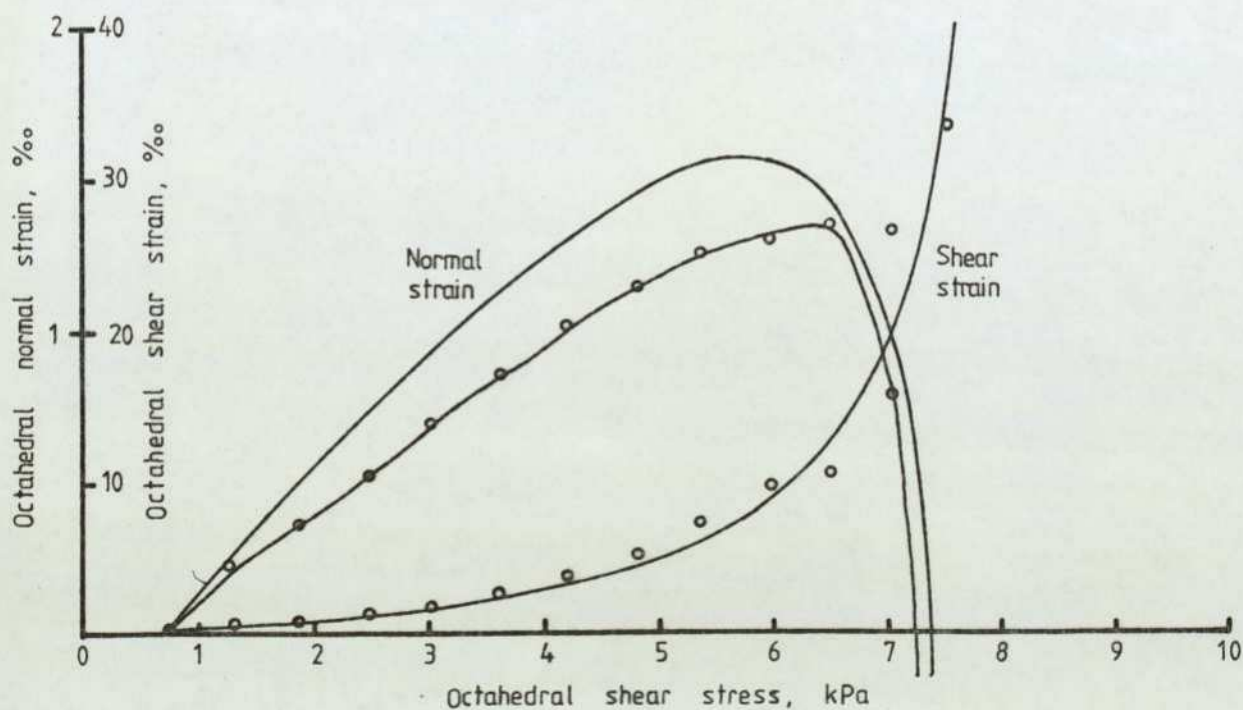


Fig. 3.13 Measured and predicted strains for tick beans in triaxial compression with $\sigma_3 = 10$ kPa.

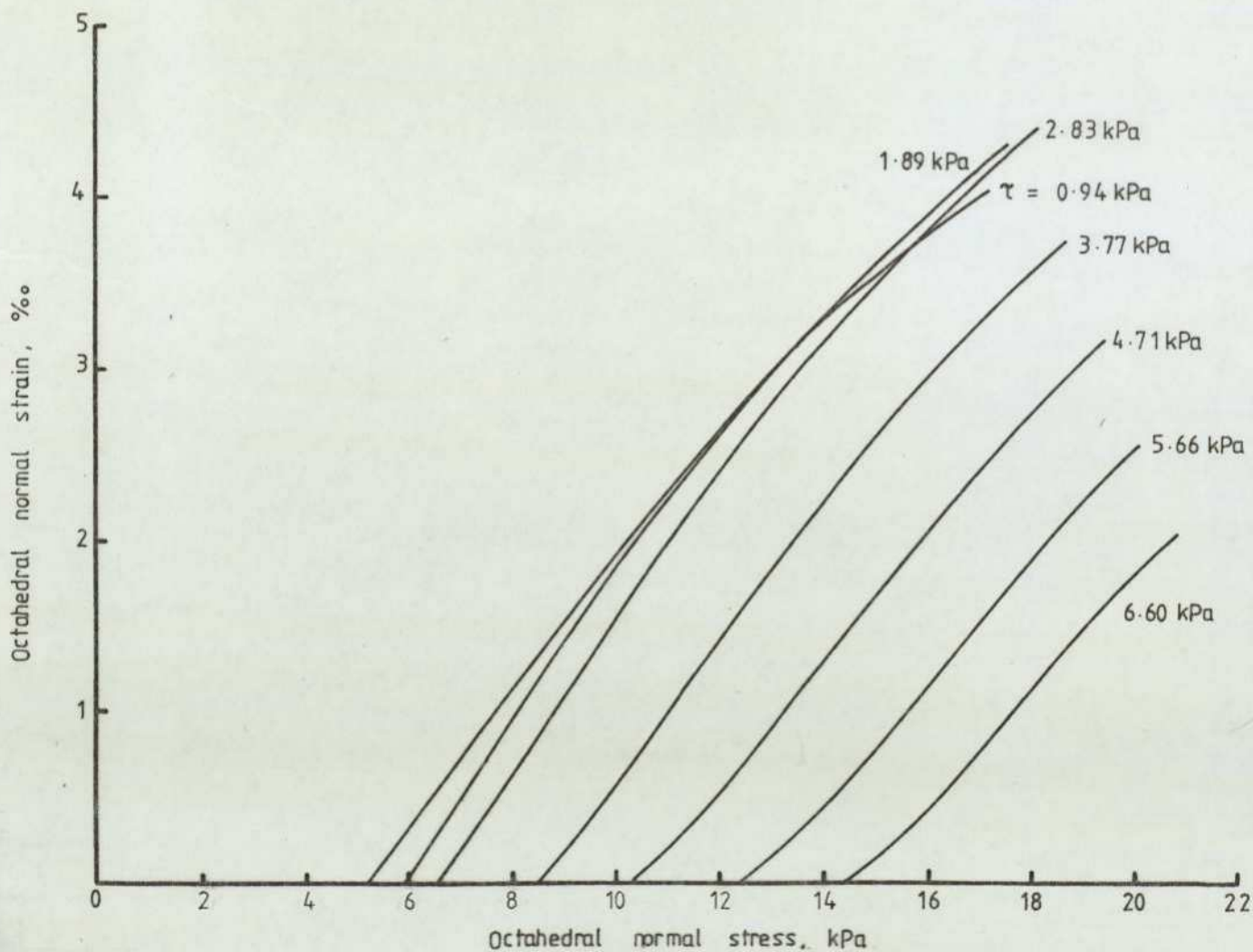
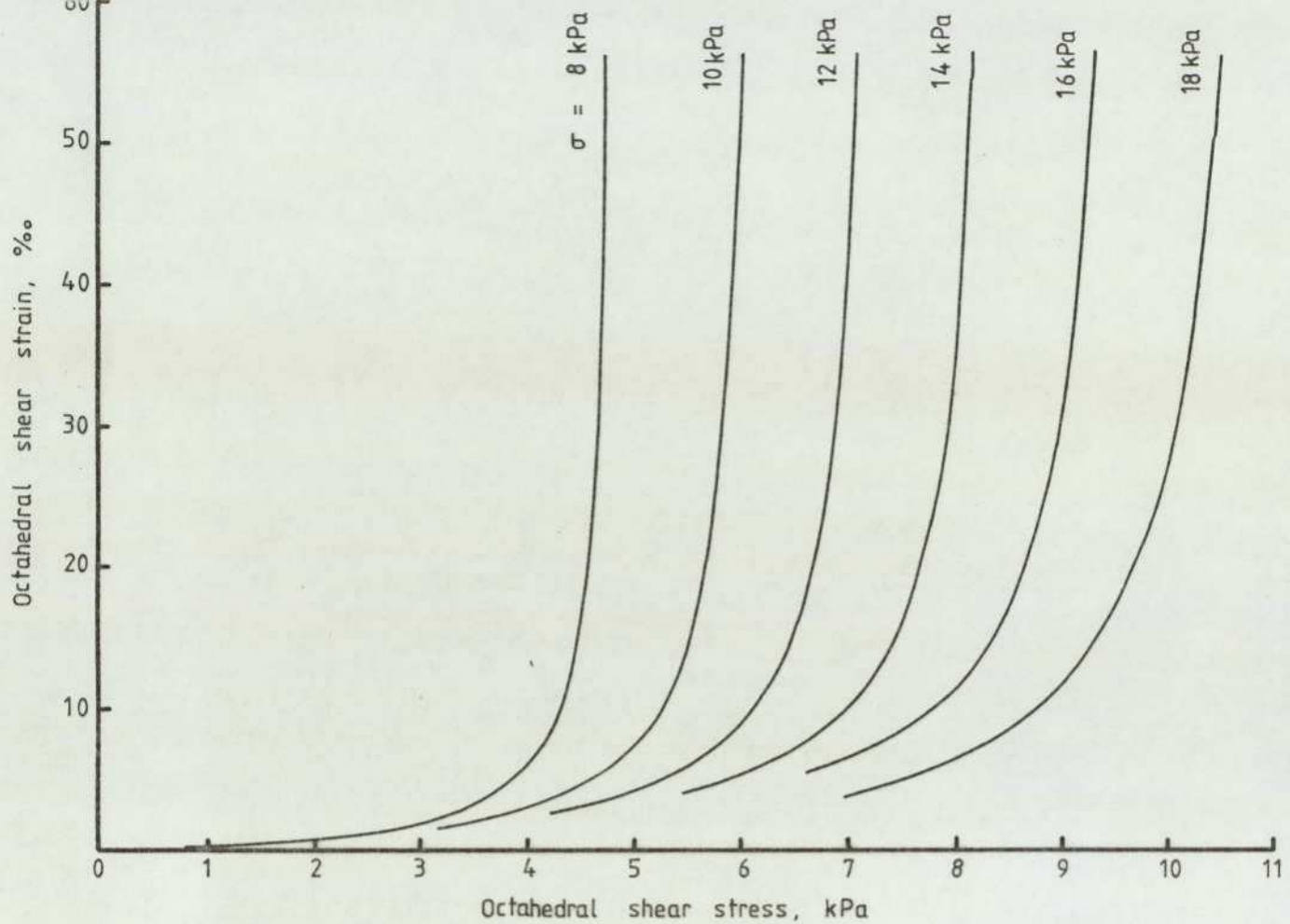


Fig. 3.14 Predicted strains for wheat using model of Kulhawy et al. (1969).

Top, octahedral shear strain vs. octahedral shear stress.

Bottom, octahedral normal strain vs. octahedral normal stress.

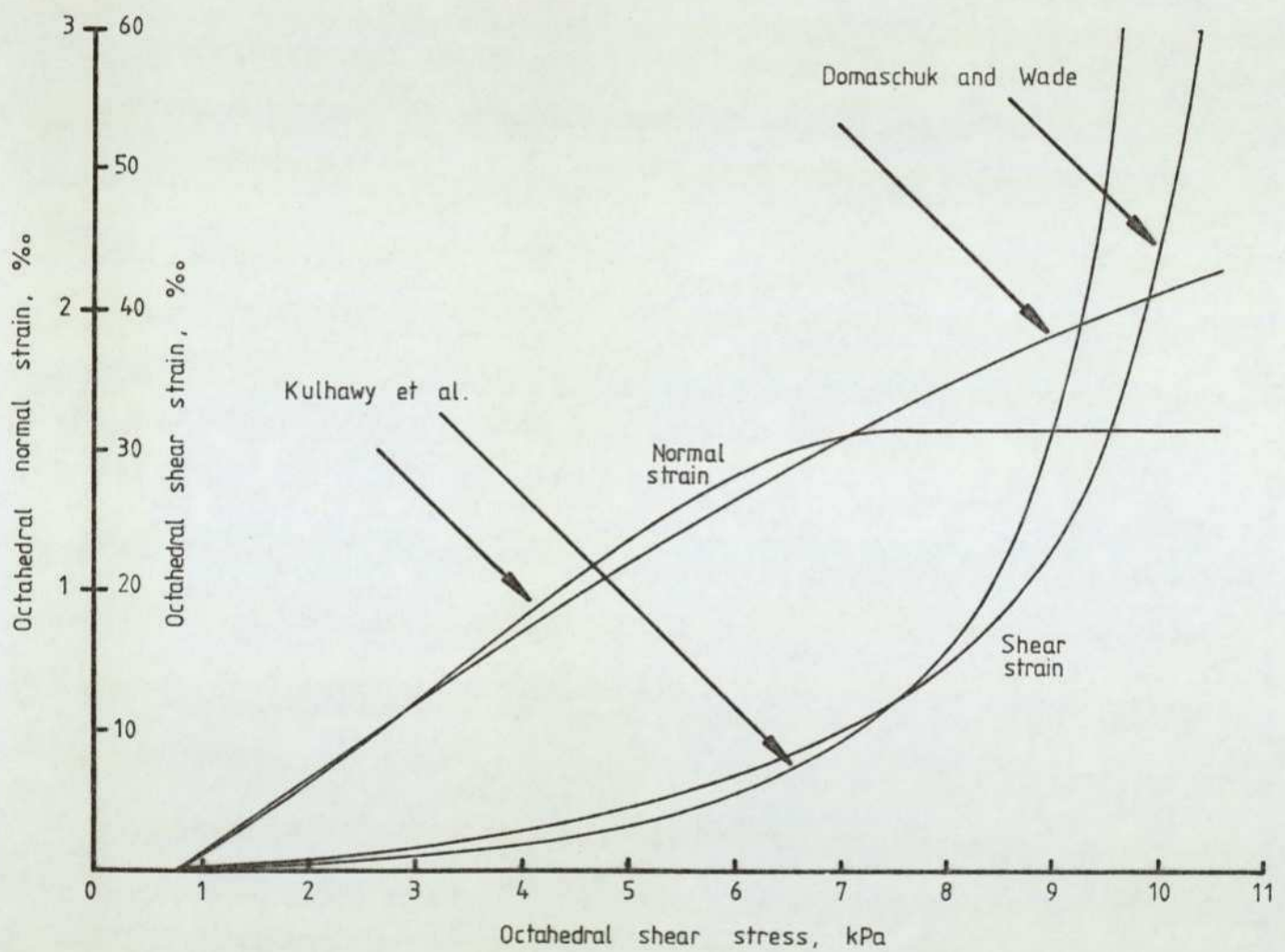


Fig. 3.15 Predicted strains for wheat in triaxial compression with $\sigma_3 = 10$ kPa using models of Kulhawy et al. (1969) and Domaschuk and Wade (1969).

CHAPTER 4

THE FINITE ELEMENT MODEL

4.1 Description of the finite element method

4.1.1 The basic form

The finite element method was proposed over twenty years ago (Turner et al., 1956) for the analysis of stresses in aircraft structures and since that time its use has become widespread for problems in many branches of engineering science. The application of the method as reported here is concerned only with a special class of problems in stress analysis i.e. those in which plane strain (straight retaining walls that are infinitely long or have frictionless side walls) or axi-symmetric (bins of circular cross section) conditions apply and so the brief description to be given here will be restricted to these cases. For a full treatment of the method the reader is referred to a standard text (e.g. Zienkiewicz, 1971; Desai and Abel, 1972).

The method consists of dividing a continuous body into many small regions (the elements) and regarding these elements as being connected together at discrete points (the nodes) along their common boundaries. For plane strain the element is a right prism while for axi-symmetric conditions the element is a toroidal solid of revolution. Many types of element cross section are possible but the simplest one is triangular with a node at each corner (Fig. 4.1). It is the simple triangular element which will be used throughout this work.

In general, the displacement of any point within an element is assumed to be an interpolation of the displacements at the nodes and with the triangular element a linear interpolation is assumed. This assumption leads to

a constant strain within the element but the strain may, of course, vary from element to element. It is possible to relate the strain in the element to the nodal displacements by a matrix equation

$$[\epsilon] = [B][\delta]^e \quad \dots (4.1)$$

The components of $[B]$ are functions of the geometry of the element and whether plane strain or axis-symmetric conditions apply (Zienkiewicz, 1971). The general strain vector which can cover both plane strain and axis-symmetric conditions has four components i.e.

$$[\epsilon] = \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \\ \epsilon_z \end{bmatrix}$$

although, for plane strain, $\epsilon_z = 0$. This vector will be called the Cartesian strain vector to distinguish it from the octahedral strain vector of Chapter 3. $[\delta]^e$ is the vector of nodal displacements for the element where

$$[\delta]^e = \begin{bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{bmatrix}$$

The components u_i and v_i , for example, are the displacements of node i in the x and y directions respectively.

The stresses within an element are related to the strains by a matrix $[D]$ i.e.

$$\sigma = [D][\epsilon] \quad \dots (4.2)$$

where $[\sigma]$ is the Cartesian stress vector

$$[\sigma] = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_z \end{bmatrix}$$

and $[D]$ is derived from the constitutive law for the material. For instance, for a linear elastic material the components of $[D]$ will be functions of the Young's modulus and Poisson's ratio and may be derived from Hooke's law.

The principle of virtual work can be used to determine the nodal displacements when the element is acted upon by external forces. The work done by the external forces must equal the strain energy stored in the element and Zienkiewicz (1971) uses this principle to show that the relationship between the nodal forces and the displacements for one element is

$$[F]^e = [K]^e [\delta]^e$$

where the nodal force vector, made up of the x and y components of the force at each node is

$$[F]^e = \begin{bmatrix} F_{xi} \\ F_{yi} \\ F_{xj} \\ F_{yj} \\ F_{xk} \\ F_{yk} \end{bmatrix}$$

and $[k]^e$, the element stiffness matrix is

$$[k]^e = \int [B]^T [D] [B] d(\text{volume}) \quad \dots (4.3)$$

where the integral is taken over the volume of the element and $[B]^T$ is the transpose of $[B]$.

Once the stiffness matrices for the individual elements have been derived, an overall matrix which relates the displacement of each node in the complete body to the external force applied to that node can be determined. This process, known as "assembly", produces the so called "global stiffness matrix", $[k]$, by adding in the contri-

butions of each element stiffness matrix, $[k]^e$, to the correct locations in the global matrix (Zienkiewicz, 1971).

The displacement of each node in the finite element representation of the body can be calculated by solving the set of equations produced by the assembly process i.e.

$$[F] = [k][\delta] \quad \dots (4.4)$$

where the components of $[F]$ are the forces applied to each node in the whole assembly and the components of $[\delta]$ are the displacements. Once the displacements are known, the strain and stress in each element can be calculated using Eqns 4.1 and 4.2.

The actual strain and stress distributions are approximated by a number of small areas (the elements) of constant stress and strain. This approximation will approach the actual continuous distribution as the elements are made smaller. Making the elements smaller does however generate more equations and so a balance has to be struck between accuracy and time available for solution.

An advantage of the method is that the elements can be made any size or shape and so they can be made smaller in areas where the stress is changing rapidly. They can also be made to follow the boundaries of the real body, although with the triangular element a curved boundary is approximated by a series of straight lines. A further advantage is that each element can have different stress/strain properties which makes it possible to analyse stresses in bodies which are not homogeneous. Also the method is formulated in matrix terms which makes it ideally suited to handling by electronic computer.

4.1.2 Nonlinear materials - Incremental methods

The simple assembly and solution process just described is ideally suited to the analysis of stress in linear elastic materials but it is clear from Chapter 3 that the material used in this work, bulk agricultural grain, is far from linear. However, the finite element method can be extended to cover non-linear materials in a variety of ways and the technique to be discussed here is the incremental loading method.

Fig. 4.2, which shows the compression/load characteristic of a non-linear spring, is used to illustrate the technique. Suppose the initial compressive load on the spring L_0 is increased to L_T resulting in an increase in compression of the spring from Y_0 to Y_T . To simulate this loading, the total load is divided into a number of smaller steps $\dot{L}$ and Fig. 4.2 (right) shows the loading situation just before the first step is applied. A tangential compliance can be defined by differentiating the compression/load characteristic to give

$$C_t(L) = \frac{dY}{dL}$$

and as the spring is non-linear C_t will depend on L . The compression at the end of the first step is calculated as

$$Y_1 = Y_0 + \dot{L} C_t(L_0)$$

and the load is

$$L_1 = L_0 + \dot{L}$$

The compression at the end of the second step would be

$$Y_2 = Y_1 + \dot{L} C_t(L_1)$$

and so on until the loading sequence has been completed. The real compression/load characteristic has in fact been approximated by a series of linear pieces and hence the term "piecewise linear" is often applied to this method.

In this case, i.e where the slope of the compression/load characteristic is always decreasing, the method suffers from the fact that the calculated compression gradually becomes greater than the true one. This effect can be reduced by making the load increments smaller or by using an estimated value of C_t which more closely represents the average value over the load step.

The incremental loading method is used in non-linear finite element analysis in a similar way (Desai and Abel, 1972) and, in general terms, a set of equations

$$[\dot{F}] = [k_t][\dot{\delta}] \quad \dots (4.5)$$

analogous to Eqn 4.4 is solved, where $[\dot{F}]$ is the load increment vector $[k_t]$ is the tangent stiffness matrix and $[\dot{\delta}]$ is the vector of displacement increments.

Because the material has a non-linear stress/strain law, the components of $[k_t]$, like the compliance of the non-linear spring, are dependent on the stress level. The basic process is illustrated by the flow chart of Fig. 4.3.

The incremental loading method is very suitable for the problem in hand for two reasons:-

a) The process of filling a silo is an incremental one, although in a real case the increments are infinitesimally small, and so the filling can be simulated real-

istically.

b) The stress/strain law of the material (Chapter 3) although formulated in terms of octahedral rather than Cartesian stresses and strains is defined in incremental terms.

4.2 Derivation of the element stress/strain matrix

4.2.1 Relationships between octahedral and Cartesian quantities

The tangent global stiffness matrix, $[k_t]$, is assembled from the individual element stiffness matrices using Eqn 4.3. These matrices relate the four Cartesian stress components to the four Cartesian strain components and so may in general contain sixteen terms. The material model developed in Chapter 3 relates the two octahedral strains to the two octahedral stresses and contains only four terms. Clearly some method is needed which derives the tangent stress/strain matrix, $[D]$, from the relationships defined by Eqn 3.8.

Love (1944) states that the quantities

$$I_1 = \sigma_x + \sigma_y + \sigma_z \quad \dots (4.6)$$

and

$$I_2 = \sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x - \sigma_{xy}^2 - \sigma_{yz}^2 - \sigma_{zx}^2 \quad \dots (4.7)$$

are invariant with respect to rotation of the x, y, and z axes, and so by rotating the axes so that they are coincident with the principal axes

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 \quad \dots (4.8)$$

and

$$I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \quad \dots (4.9)$$

Using Eqns 4.8, 4.9, 3.4, and 3.5, the octahedral shear stress is given by

$$\tau = \frac{\sqrt{2}}{3} [I_1^2 - 3I_2]^{1/2} \quad \dots (4.10)$$

and the octahedral normal stress by

$$\sigma = \frac{I_1}{3} \quad \dots (4.11)$$

For the stress conditions dealt with here, σ_{yz} and σ_{zx} are zero and so, substituting from Eqns 4.6 and 4.7 into Eqns 4.10 and 4.11

$$\tau = \frac{\sqrt{2}}{3} [\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x\sigma_y - \sigma_y\sigma_z - \sigma_z\sigma_x + 3\sigma_{xy}^2]^{1/2} \dots (4.12)$$

and

$$\sigma = \frac{1}{3} [\sigma_x + \sigma_y + \sigma_z] \quad \dots (4.13)$$

Eqns 4.12 and 4.13 define the relationships between the Cartesian stresses and the octahedral stresses but the requirement is for relationships between the increments of stress. The chain rule of partial differentiation may be used here to give

$$\dot{\tau} = \dot{\sigma}_x \frac{\partial \tau}{\partial \sigma_x} + \dot{\sigma}_y \frac{\partial \tau}{\partial \sigma_y} + \dot{\sigma}_z \frac{\partial \tau}{\partial \sigma_z} + \dot{\sigma}_{xy} \frac{\partial \tau}{\partial \sigma_{xy}}$$

and

$$\dot{\sigma} = \dot{\sigma}_x \frac{\partial \sigma}{\partial \sigma_x} + \dot{\sigma}_y \frac{\partial \sigma}{\partial \sigma_y} + \dot{\sigma}_z \frac{\partial \sigma}{\partial \sigma_z}$$

or in matrix form

$$\begin{bmatrix} \dot{\tau} \\ \dot{\sigma} \end{bmatrix} = [S] \begin{bmatrix} \dot{\sigma}_x \\ \dot{\sigma}_y \\ \dot{\sigma}_{xy} \\ \dot{\sigma}_z \end{bmatrix} \quad \dots (4.14)$$

where [S] is a 2 x 4 matrix whose elements are the partial derivatives of the octahedral stresses with respect to the Cartesian stresses. The differentiations can be performed on Eqns 4.12 and 4.13 to give

$$[S] = \frac{1}{3} \begin{bmatrix} (2\sigma_x - \sigma_y - \sigma_z)/3\tau & (2\sigma_y - \sigma_z - \sigma_x)/3\tau & 2\sigma_{xy} & (2\sigma_z - \sigma_x - \sigma_y)/3\tau \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

The relationships between the strain increments are derived in a similar way by noting that (Love, 1944)

$$J_1 = \epsilon_x + \epsilon_y + \epsilon_z$$

and

$$J_2 = \epsilon_x \epsilon_y + \epsilon_y \epsilon_z + \epsilon_z \epsilon_x - \frac{1}{4} (\epsilon_{xy}^2 + \epsilon_{yz}^2 + \epsilon_{zx}^2)$$

are invariants of strain and so

$$\begin{aligned} \gamma &= \frac{2\sqrt{2}}{3} [J_1^2 - 3J_2] \\ &= \frac{2\sqrt{2}}{3} \left[\epsilon_x^2 + \epsilon_y^2 + \epsilon_z^2 - \epsilon_x \epsilon_y - \epsilon_y \epsilon_z - \epsilon_z \epsilon_x + \frac{3}{4} (\epsilon_{xy}^2 + \epsilon_{yz}^2 + \epsilon_{zx}^2) \right]^{1/2} \dots (4.15) \end{aligned}$$

and

$$\epsilon = \frac{J_1}{3} = \frac{1}{3} [\epsilon_x + \epsilon_y + \epsilon_z] \quad \dots (4.16)$$

By differentiating Eqns 4.15 and 4.16, the relationship

$$\begin{bmatrix} \dot{\gamma} \\ \dot{\epsilon} \end{bmatrix} = [R] \begin{bmatrix} \dot{\epsilon}_x \\ \dot{\epsilon}_y \\ \dot{\epsilon}_{xy} \\ \dot{\epsilon}_z \end{bmatrix} \quad \dots (4.17)$$

may be derived where [R] is a 2 x 4 matrix given by

$$[R] = \frac{1}{3} \begin{bmatrix} 4(2\epsilon_x - \epsilon_y - \epsilon_z)/3Y & 4(2\epsilon_y - \epsilon_z - \epsilon_x)/3Y & 2\epsilon_{xy} & 4(2\epsilon_z - \epsilon_x - \epsilon_y)/3Y \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

Combining Eqn 4.14 with the incremental form of Eqn 4.2 gives

$$\begin{bmatrix} \dot{\tau} \\ \dot{\sigma} \end{bmatrix} = [S][D] \begin{bmatrix} \dot{\epsilon}_x \\ \dot{\epsilon}_y \\ \dot{\epsilon}_{xy} \\ \dot{\epsilon}_z \end{bmatrix} \quad \dots (4.18)$$

and combining Eqn 3.8 with Eqn 4.17 gives

$$\begin{bmatrix} \dot{\tau} \\ \dot{\sigma} \end{bmatrix} = [C][R] \begin{bmatrix} \dot{\epsilon}_x \\ \dot{\epsilon}_y \\ \dot{\epsilon}_{xy} \\ \dot{\epsilon}_z \end{bmatrix} \quad \dots (4.19)$$

where

$$[C] = [A]^{-1}$$

and

$$[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Comparing Eqns 4.18 and 4.19, the relationship

$$[S][D] = [C][R] \quad \dots (4.20)$$

is obtained.

The left and right hand sides of Eqn 4.20 are matrices with dimensions (2 x 4) and, as all the components of the left hand matrix must equal all those of the right hand one for equality, Eqn 4.20 provides eight individual equations for the components of the matrix [D].

4.2.2 Extra assumptions

The material model developed in Chapter 3 cannot define the sixteen components of [D] explicitly. In fact a model with four independent parameters is the most sophisticated that can be derived from measurements using a standard triaxial cell because there are only two independent stresses and two independent strains operating in such a device. More complex models are the result of

making additional assumptions. For instance, eight equations have been derived above because although the material model has been derived from stress states on the triaxial plane, it has been assumed to apply for states away from it.

[D] contains a total of sixteen components and so a further eight independent equations must be found. The additional assumptions used to determine these extra equations are as follows:-

a) A normal strain in the direction of one coordinate axis affects the normal stresses in the direction of the other two axes equally. This assumption is illustrated in Fig. 4.4 (left) which shows a rigid cubic box of unit dimensions containing the granular material. If one face (in this case the top face) of the cube is moved inwards giving a strain change $\dot{\epsilon}_y$, then it is assumed that the stress changes on the faces of the box, $\dot{\sigma}_x$ and $\dot{\sigma}_z$ are equal. As the faces of the box can be moved in any of three directions, this assumption gives three equations

$$\frac{\partial \sigma_x}{\partial \epsilon_y} = \frac{\partial \sigma_z}{\partial \epsilon_y} \quad \text{or} \quad d_{12} = d_{42}$$

$$\frac{\partial \sigma_y}{\partial \epsilon_x} = \frac{\partial \sigma_z}{\partial \epsilon_x} \quad \text{or} \quad d_{21} = d_{41}$$

$$\frac{\partial \sigma_y}{\partial \epsilon_z} = \frac{\partial \sigma_x}{\partial \epsilon_z} \quad \text{or} \quad d_{24} = d_{14}$$

b) A shear strain in one plane does not affect the stress normal to that plane. Fig. 4.4 (right, shows the cubic box subjected to a shear strain $\dot{\epsilon}_{xy}$. As relative motion occurs between adjacent grains there is a tendency

for the area of the material in the plane of the motion to increase to overcome interlocking (Lambe and Whitman, 1969). However, the area is fixed due to the rigid nature of the box and so this expansion is resisted by stress changes $\dot{\sigma}_x$ and $\dot{\sigma}_y$ at the faces of the box. As there is no relative movement in the other two planes then $\dot{\sigma}_z$ will be zero. Because there is only one non-zero shear strain to be considered, this assumption gives rise to one equation which is

$$\frac{\partial \sigma_z}{\partial \epsilon_{xy}} = 0 \quad \text{or} \quad d_{43} = 0$$

c) The shear stress in the x,y plane is affected only by changes in the corresponding shear strain and the sensitivity of this shear stress to changes in the shear strain is the same as the sensitivity of the octahedral shear stress to changes in the octahedral shear strain. This gives rise to four equations

$$\frac{\partial \sigma_{xy}}{\partial \epsilon_x} = 0 \quad \text{or} \quad d_{31} = 0$$

$$\frac{\partial \sigma_{xy}}{\partial \epsilon_y} = 0 \quad \text{or} \quad d_{32} = 0$$

$$\frac{\partial \sigma_{xy}}{\partial \epsilon_{xy}} = \frac{\partial \tau}{\partial \gamma} \quad \text{or} \quad d_{33} = c_{11}$$

$$\frac{\partial \sigma_{xy}}{\partial \epsilon_z} = 0 \quad \text{or} \quad d_{34} = 0$$

where c_{11} is the first element of $[C]$. All the above assumptions are valid for the case of linear, homogeneous,

isotropic, elastic materials.

It may seem to the reader that rather a large number of assumptions have been made but these assumptions must be set in their proper context. Firstly, the level of assumption is less than that made by previous workers (Chapter 3) who derived the matrix $[D]$ by measuring two independent parameters on the material sample and assumed that the relationships between the components of $[D]$ were the same as for a linear elastic material obeying Hooke's law. Thus they produced $[D]$ from two measured parameters whereas the author has produced $[D]$ from four measured parameters. Secondly, even if the sixteen components of $[D]$ could be measured directly, this would need a vast amount of controlled experimentation to determine the sensitivity of each stress component to changes in each strain and also the way in which these sensitivities depend on the stress levels. As well as the large volume of experimental work, a device would need to be designed that could apply three independent normal stresses and one shear stress to a sample and measure the associated strains. The author feels that both these tasks are impracticable.

The eight individual equations defined by Eqn 4.20 together with the eight from the extra assumptions are now sufficient to define $[D]$ which, on solution of the sixteen equations is given overleaf where

$$[N] = [C][R]$$

$$[D] = \begin{bmatrix} n_{21} + \frac{2n_{11}}{3s_{11}} & n_{22} - \frac{n_{12}}{3s_{12}} & \frac{3n_{23}s_{12} - n_{13} + s_{13}c_{11}}{s_{12} - s_{11}} & n_{24} - \frac{n_{14}}{3s_{14}} \\ n_{21} - \frac{n_{11}}{3s_{11}} & n_{22} + \frac{2n_{12}}{3s_{12}} & \frac{n_{13} - s_{13}c_{11} - 3s_{11}n_{23}}{s_{12} - s_{11}} & n_{24} - \frac{n_{14}}{3s_{14}} \\ 0 & 0 & c_{11} & 0 \\ n_{21} - \frac{n_{11}}{3s_{11}} & n_{22} - \frac{n_{12}}{3s_{12}} & 0 & n_{24} + \frac{2n_{14}}{3s_{14}} \end{bmatrix}$$

4.3 Description of the computer program

4.3.1 The basis of the program

The loading of grain into a silo is usually a continuous process in that new grain is continually being added to the grain mass already in place. At the same time the boundaries of the grain mass, for example the walls of the silo, may be deflecting. The computer program simulates this continuous process by one in which discrete layers of grain are successively added to the mass and where each additional layer may be accompanied by a discrete deflection of the structure (Fig. 4.5).

The simulation starts with a layer, which is usually divided into several finite elements, being deposited onto the floor of the silo. The load on the layer arises from the weight of the grain itself and may be represented by applying downward forces to the nodes (Zienkiewicz, 1971). The weight of each element is divided into three equal parts and each part is assumed to act at one of the nodes of the element. These forces appear in the load vector $\dot{F}$ of Eqn 4.5. The global stiffness matrix $[k_t]$ is then

assembled by calculating $[D]$ for each element, forming the element stiffness matrix (Eqn 4.3), and then adding in the contribution of each element to the appropriate location of $[k_t]$. Once Eqn 4.5 is set up it is solved (Sect. 4.3.3) for the unknown displacement vector $[\dot{\delta}]$, any known boundary displacements being eliminated from the set of equations beforehand. The resulting strains and stresses are then calculated for each element by application of Eqns 4.1 and 4.2. After the new layer has been placed, a new stiffness matrix is set up and the equations solved for the displacement increments which occur during the load step. The strain and stress increments are calculated as before and the strain and stress levels after placing the second layer are calculated by adding the increments to the values calculated at the previous step. This process is continued until the full loading history has been simulated.

It is important to remember that $[\dot{F}]$ is an incremental load vector and that when the new layer is placed $[\dot{F}]$ contains the incremental loads only i.e. those that arise from the weight of the material in the new layer.

4.3.2 The two passes

As the stress/strain properties of bulk grain depend on the stress levels, some representative value of stress must be chosen at which to evaluate them. One method is to use the stresses at the start of the load step and to evaluate the stresses at the end of the step by

$$[\sigma_{n+1}] = [\sigma_n] + [\dot{\sigma}(n)]$$

where $[\sigma_n]$ is the stress vector at the end of the n^{th} step and $\dot{\sigma}(n)$ indicates that the stress increment has been

calculated using the material properties at the stress levels at the end of the n^{th} step. This particular method of numerical integration is known as Euler's method. Although simple, Euler's method can lead to significant departures from the stresses of the real continuous system (as in Fig. 4.2) and so some refinement is called for. The method used in this work is one of the general category of Runge-Kutta methods (Smith, 1977). It is a two pass method in that it requires the set of equations to be set up and solved twice for each load step. The first pass uses Euler's method and produces a trial set of stresses

$$[\sigma_{n+1}'] = [\sigma_n] + [\dot{\sigma}(n)]$$

The second pass repeats the first one but evaluates the material properties with the stresses at the end of the first pass and produces a second trial set of stresses

$$[\sigma_{n+1}''] = [\sigma_n] + [\dot{\sigma}(n+1)']$$

The final values for the stresses at the end of the $n+1^{\text{th}}$ step are taken as the average of the two trial values.

This version of the Runge-Kutta technique is known as Heun's method (Smith, 1977)

4.3.3 Solution of equations

A characteristic of the global stiffness matrix is that it is sparse, i.e. many of its components are zero. This sparseness suggests that an iterative method for solving the equations would be appropriate (Korn and Korn, 1968) as does the fact that a reasonable starting value for the iterative solution is available from the solution at the previous pass (for the second pass) or from the previous step (for the first pass). In another applic-

ation of the finite element method to a non-linear material (Marchant, 1976), the author found that the method of successive over-relaxation (Forsythe and Wasow, 1960) gave a good balance between computer time and storage requirement and so this method has been used here. The method takes each row of the matrix equation 4.5 in turn and rearranges it so that the corresponding unknown displacement is the subject of the equation. For instance, row i , which is

$$\dot{F}_i = \sum_{\ell=1}^m (k_t)_{i\ell} \dot{\delta}_\ell$$

where m is the number of equations, gives

$$\dot{\delta}_i = \left[F_i - \sum_{\ell=1, \ell \neq i}^m (k_t)_{i\ell} \dot{\delta}_\ell \right] / (k_t)_{ii} \quad \dots (4.21)$$

A residual, r , is defined as being the difference between the value of $\dot{\delta}_i$ at the stage of the iteration reached and the value given by the right hand side of Eqn 4.21. This residual is then multiplied by a constant, Ω , (the over-relaxation constant) and added to $\dot{\delta}_i$ thus producing an improved estimate. After each row of Eqn 4.5 has been dealt with in this way, the iteration cycle is repeated from row 1 until all the residuals are less than a pre-set figure. If $\Omega = 1$ the method is identical to the Gauss-Seidel method (Korn and Korn, 1968) but a faster rate of convergence can be achieved by setting Ω to a figure between 1 and 2. Unfortunately no general rule exists for the optimum value of Ω but a value of about 1.3 has been found to be satisfactory in this work.

4.3.4 Elements with special stress conditions

The calculation of the parameter a_{11} of Chapter 3 within the finite element program presents a problem when the normal stress, σ , is zero. In this case a_{11} becomes

infinite because it contains σ in its denominator. The problem occurs when the stresses are being calculated in an element for the first time (newly placed elements) and to overcome it a term σ' is used in place of σ to calculate a_{11} and a_{21} . σ' is set equal to σ except that it is not allowed to fall below the minimum value used in the tests to determine a_{11} and a_{21} (i.e. 4kPa, Chapter 3).

Another problem occurring with newly placed elements is that the term

$$F_m = \frac{\tau}{K_2 \sigma}$$

is indeterminate because both τ and σ are zero. With a finite value of σ and with increasing τ , this term increases from zero and approaches a value of 1.0 as the material approaches failure. F_m may therefore be thought of as the mobilized fraction of the shear strength. The choice of a suitable initial value for F_m (F_{m0}) will be returned to in Sect. 4.4.3.

As the finite element method gives only an approximation to the true stresses, conditions of stress may occur in some elements which cannot occur in the real grain. Two cases are identified here:-

a) Elements in which the strength limit of the grain is exceeded. The bulk grain becomes progressively easier to distort as it approaches failure and so the changes in shear stress become smaller for a given change in shear strain. In the extreme case, when $F_m = 1$, a change in the shear strain causes no change in the shear stress at all and so F_m cannot increase any further. In the finite element representation however the stress changes are

discrete and F_m may change rapidly, especially at low values of stress, and may overshoot the value of 1.0. This problem is dealt with by limiting the value of F_m used to calculate a_{11} and a_{21} to a figure (F_{m1}) that is just below 1.0. The choice of a suitable value will also be dealt with in Sect. 4.4.3.

b) Elements in which the principal stresses are not all compressive. As the bulk grain is cohesionless, the normal stress on all planes must be compressive. In the simulation however, again due to its discrete nature, one or more tensile principal stresses may occur. When this arises a procedure similar to the initial stress method described by Zienkiewicz (1971) is used.

This method enables stresses that are outside a permitted boundary to be reduced to that boundary value. This is done by first calculating the amount by which the stresses in the element exceed the boundary i.e.

$$[\sigma_O] = [\sigma] - [\sigma_B]$$

where $[\sigma_B]$ is the boundary stress vector and $[\sigma_O]$ is the overshoot. A set of nodal forces equivalent to the overshoot is calculated from the relationship

$$[F_O] = [B]^T [\sigma_O]$$

(Zienkiewicz, 1971) and the set of equations (Eqn 4.4) solved again with $[F_O]$ added to the load vector $[F]$ and an initial stress vector $-[\sigma_O]$ added to the element stresses. In this way the amount by which the element stresses overshoot the boundary is shed from the element and redistributed on to its neighbours. The procedure as described by Zienkiewicz is an iterative one and the process of calculating the overshoot and redistributing

it is repeated until the changes from one iteration to the next are not significant. This method can be very efficient in terms of computing time because, although the whole set of equations must be solved on each iteration cycle, the matrix $[k]$ of Eqn 4.4 does not change. If the equations are solved by inverting $[k]$ the inversion need only be carried out once and subsequent solutions are obtained by matrix multiplication i.e.

$$[\delta] = [k]^{-1}([F] + [F_0])$$

The method is applied here in a slightly different form by calculating the principal stresses in each element after the second pass at each load step. If any of the principal stresses are negative (i.e. tensile), the program attempts to eliminate all the stress components in that element by setting $[\sigma_0]$ equal to $[\sigma]$. At this stage, the method used here differs from that described by Zienkiewicz. Because of the way that the equations are solved (Sect. 4.3.3) the inverse of $[k_t]$ is not available and so repeated solution of the equations would be very time consuming. To overcome this problem, $-[\sigma_0]$ is added to $[\sigma]$ at the end of the second pass of each load step but the load redistribution is not carried out until the next load step. By using this procedure, tensile stresses can be eliminated without the need for extra solutions of the equations.

4.3.5 Boundary friction

The program allows limiting friction to be simulated at any of the horizontal or vertical boundaries using the same method as Jofriet et al. (1977). If, for instance, a horizontal boundary has a coefficient of friction μ_b

then the horizontal forces at the nodes along this boundary will be given by

$$F_{xi} = \mu_b F_{yi}$$

where the direction of the horizontal force is in opposition to the horizontal displacement at the boundary.

Each node where limiting friction applies will give rise to one such equation (although in the author's program the forces are incremental forces) and these extra equations are incorporated into the main set (Eqn 4.5) as follows. The rows from Eqn 4.5 arising from the equilibrium of node i are

$$\dot{F}_{xi} = R_{xi} \quad \dots (4.22)$$

$$\dot{F}_{yi} = R_{yi} \quad \dots (4.23)$$

where $\dot{F}_{xi}$ and $\dot{F}_{yi}$ are the incremental horizontal and vertical forces at node i and R_{xi} and R_{yi} are the right hand sides of the relevant rows of Eqn 4.5. To incorporate the frictional boundary condition Eqn 4.23 is multiplied by μ_b and subtracted from Eqn 4.22 which gives

$$0 = R_{xi} - R_{yi}$$

and eliminates Eqn 4.23 from the set. This procedure and the corresponding one for vertical boundaries is repeated for each frictional node.

After their equations had been solved, Jofriet et al. examined each frictional node to see if the assumed sign of the friction was consistent with the relative directions of the force and the displacement. If the force was not in opposition to the displacement the sign of the friction was changed and the equations solved again. The author has found that this refinement is unnecessary and, with

the author's program, the sign of the friction is specified in the data.

4.3.6 Side wall friction

Plane strain conditions apply only to infinitely long straight retaining walls or walls of a finite length where the side walls are frictionless. In any practical case neither of these conditions are fulfilled and so some means must be found to assess the effect of side wall friction on the loads on the main walls. The ideal way of doing this would be to treat the stress analysis as a completely three dimensional one. However, experience with the two dimensional analysis has shown that, although possible in principle, this would be impracticable. As well as needing many more nodes, extending the method into three dimensions would require the solution of three equations for each node at each step instead of two and so would lead to solution times which would be unacceptably long.

Bransby and Smith (1975) have shown that the three dimensional problem can be reduced to one in two dimensions by assuming that the shear stress mobilized at the side walls changes at a uniform rate across the width i.e. in the z direction. They have shown that the three dimensional differential equations describing the equilibrium of the granular mass reduce to a form similar to those applying to the plane strain case except that an extra body force term must be included. This body force term is analogous to the gravitational force but its direction is opposite to that of the displacement. If the width of the silo is W then the magnitude of this body force is

$$\rho = \frac{2\mu_s\sigma_z}{W}$$

and so the actual force on a prism with a small cross sectional area ΔA extending across the width of the silo is $2\mu_s \sigma_z \Delta A$.

Bransby and Smith used this technique in conjunction with the method of characteristics (Sokolovskii, 1965) to investigate the effect of side wall friction on model tests but it is also suited for use with the incremental finite element method. In the author's program the body force and the normal stress in the z direction are incremental quantities. The body force for any particular step is calculated using the z stress increments and the displacement directions which occurred at the last step. This is felt to be reasonable as the increments in stress are fairly constant if each new layer is of a constant thickness but it does slightly underestimate the value of the body force because the stress increment at the top layer is assumed to be zero. This extra body force is incorporated in the same way as the gravitational body force (specific weight) that is by taking the area ΔA as the area of an element and assigning one third of the force to each node. As mentioned by Bransby and Smith, this method will underestimate the effect of friction near the side walls and overestimate it near the middle but should lead to a figure representative of the average effect across the width of the silo.

4.4 General characteristics of the finite element model

4.4.1 Non-linear relationships between octahedral and Cartesian quantities

Initial trials of the computer program using non-linear relationships between the octahedral stresses and

strains and the Cartesian stresses and strains (the [S] and [R] matrices of Sect. 4.2.2) were disappointing. The large strains which occurred when the material was close to failure gave rise to components of the [R] matrix which varied widely from element to element and between load increments. This led to extremely uneven and unrealistic stress distributions and sometimes to the setting up of sets of equations for which the solution routine was unstable.

To overcome this problem the components of [S] and [R] had to be kept constant and were set at the values which apply for stress conditions on the triaxial plane. The justification for this simplification is the same as that given in Sect. 3.3.2, i.e. for the applications dealt with here the intermediate principal stress will be of a similar value to the minor principal stress.

4.4.2 Effect of changing the element size

When using the finite element method a balance must be made between the time taken for the preparation of data and running the program, and accuracy. Smaller elements will lead to increased accuracy but at the expense of an increased time taken to solve the problem. Experiments were therefore carried out with the computer program to determine the effect of changing the element size on the calculated values of wall stress and the time taken for solution.

A plane strain situation was chosen for this investigation which simulated the loading of wheat in horizontal layers into a silo where the distance between the front and back walls was 4m and the final depth of grain was

also 4m. The stress/strain properties of the wheat were as given in Table 3.1. The walls of the silo were assumed to be perfectly rigid and the front and back walls to have a coefficient of friction of 0.3. There was assumed to be no slip between the grain and the floor of the silo. This assumption was justified because the maximum ratio between the shear and normal stresses calculated by the program at the nodes along the floor did not exceed 0.05, a value less than any practical coefficient of friction. The side walls of the silo were frictionless and the values of the parameters F_{m0} and F_{m1} (Sect. 4.3.4) were taken to be 0.5 and 0.968 respectively. The seemingly strange choice for F_{m1} is that which gives a maximum value for a_{11} (that at failure) equal to 1000 times its minimum value (that under hydrostatic stress conditions). The standard element configuration is shown in Fig. 4.6. The elements were all isosceles triangles obtained by dividing a rectangle into four and the loading was simulated by dividing the filling into equal horizontal layers. As the cross section was symmetrical it was only necessary to perform calculations on one half, the centre line being represented by a frictionless vertical boundary. The horizontal stress on the wall was calculated at each boundary node by averaging the stresses in all elements connected to that node.

To determine the effect of changing the vertical size of the elements, the program was run using six, eight, ten, and twelve horizontal layers. The stress distributions calculated using different numbers of layers (Fig. 4.7) were almost indistinguishable but the time taken for the solution roughly doubled for each additional two layers

and ranged from 90 seconds (6 layers) to 567 seconds (12 layers).

To determine the effect of changing the horizontal size of the elements, the half cross section was divided into two, three, and four columns of rectangles using eight layers (Fig. 4.8). Again the stress distributions (Fig. 4.9) were little affected by the size of the elements but the solution time roughly doubled for each extra column and varied between 105 seconds (2 columns) and 486 seconds (4 columns).

For general use it would be inadvisable to use the least number of columns along with the least number of layers even though the accuracy would probably be acceptable with the silo configuration of Fig. 4.6. The effect of element size on the calculated stress distribution may depend on the silo geometry and so to provide a "factor of safety" it is recommended that at least three columns of elements and eight layers be used as a general rule. Unfortunately, if the cross section is not symmetrical the whole of it must be analysed. To keep the same element size as in the symmetrical case this would require six columns which, because the solution time roughly doubles for each extra column, would take about 2000 seconds to solve. This is felt to be unreasonably long and so for unsymmetrical cross sections five columns are recommended. If necessary, three columns could be used in one half and two in the other where accuracy on one particular side is more important.

4.4.3 Effect of changing the parameters F_{m0} and F_{m1}

The parameter F_{m0} defines the fraction of the strength

of the material mobilized when the stresses are zero, that is on the first pass of the program for elements in newly placed layers (Sect. 4.3.4). Because F_{mo} should represent some average value during an element's initial load increment, it was felt that the effect of changing this parameter might depend on the stress conditions likely to be found during that load increment. Because of this the computer program was run to simulate two different cases leading to different stress systems. In the first, both the front and the back silo walls were allowed to deflect by outward rotation about their bases during the loading. The final wall movement, measured at the top of the wall, was 0.1% of the depth. Eight layers were used with three columns of rectangles for half the cross section as in Fig. 4.8. In the second case only one wall was allowed to rotate outwards to a final figure of 1% of the depth of grain and, since the system was no longer symmetrical the whole cross section had to be analysed and five vertical columns of rectangles were used (Fig. 4.10). In both cases the amount of rotation was equal for each layer applied and F_{mo} was varied between 0.2 and 0.8 with F_{m1} set at 0.968. The solution times for F_{mo} equal to 0.2, 0.4, 0.6, and 0.8 were 402, 360, 168, and 183 seconds for 0.1% rotation and 657, 627, 294, and 378 seconds for 1% rotation.

The effect of changing F_{mo} is, to some extent, self compensating. For example, setting F_{mo} to a value close to 1.0 means that newly placed elements will have a high value of the parameter a_{11} (Sect.3.3.1) because a_{11} contains a term $(1 - F_{mo})^2$ in its denominator. This results in a large octahedral shear strain being necessary to give

any appreciable change in the octahedral shear stress. However, the parameter a_{22} is independent of F_m and so the octahedral normal strain needed for a given change in the octahedral normal stress is unaffected. Because of the presence of surrounding material and the boundaries of the silo, there is some constraint on the strain in any particular element and so this large value of F_{mo} leads to an octahedral shear stress change that is small compared with the change in octahedral normal stress. This results in the material being stronger i.e. F_m is lower, at the end of the first pass and so the element starts the second pass with a low value of F_m as a result of a high value at the beginning of the first pass. Conversely if a low value of F_{mo} is chosen the element starts the second pass with a higher value and so, as the mean of the stress changes over the two passes is taken, the stress conditions at the end of the first step are reasonably independent of the choice of F_{mo} . This compensation also occurs from step to step as well as between separate passes on the same step and as more layers are placed on top of an element the choice of F_{mo} becomes increasingly less significant. This effect can be seen in the stress distributions at the walls of the silo (Figs 4.11 and 4.12) for the two different conditions of wall rotation where the spread of calculated stresses is larger nearer the surface. For both the 0.1% rotation and the 1% rotation (where the stresses on each wall are different) a value of F_{mo} of 0.6 gives stresses which are about halfway between the extreme readings. As this value also requires a significantly smaller computing time, it will be used for the rest of the work.

The parameter F_{m1} is the maximum allowable value of F_m (Sect. 4.3.4). From Eqn 3.21, a_{11} changes from its value under hydrostatic stress conditions (when $F_m = 0$) to a maximum (when $F_m = F_{m1}$) in the ratio $(1 - F_{m1})^2:1$ and so F_{m1} also determines the allowable range of a_{11} . With $F_{m0} = 0.6$ and using the same two conditions of wall rotation as in the previous case, F_{m1} was changed in the range 0.683 to 0.990 ($a_{11}(\text{max})/a_{11}(\text{min}) = 10, 100, 1000, \text{ and } 10000$). The stresses at the walls were virtually indistinguishable for all values of F_{m1} and, as they were so close to those of Fig. 4.12 with $F_{m0} = 0.6$, they were not plotted. The solution times were also virtually the same for all values of F_{m1} . The lack of dependence of the stresses on F_{m1} was due to the fact that very few elements reached a stress state such that F_m was greater than 0.683 and so the choice of F_{m1} was unimportant. Kulhawy et al. (1969) used Hooke's law with a non-linear form of Young's modulus (E_t) to describe their material when simulating the construction of an earth bank dam (Sect. 3.2.3). They found that the calculated stresses were unaffected by the choice of the minimum value of E_t provided it was less than 1/1000 times the value under hydrostatic conditions. As a_{11} is proportional to $1/E_t$ in a material that obeys Hooke's law, the ratio $a_{11}(\text{max})/a_{11}(\text{min})$ in the author's material model is analogous to $E_t(\text{min})/E_t(\text{max})$ in that of Kulhawy et al. and so a value of 1000 as used by Kulhawy et al. will be used for the rest of the work.

4.5 Effect of parameter changes on wall stresses

4.5.1 Effect of wall rotation

The computer program was run to calculate the horiz-

ontal wall stresses for the same silo as used in previous runs i.e. depth of grain, 4m; distance between front and back walls, 4m; front and back wall friction coefficient, 0.3. The filling was assumed to be in eight horizontal layers and the side walls were assumed to be frictionless. The computer program was run to simulate the front wall rotating about its base so that the top of the wall moved outwards by 0, 0.1%, 0.2%, 0.5%, and 1% of the final depth of grain (D). The final rotations were assumed to be achieved in equal steps at each load increment. The stresses (Fig. 4.13) became progressively smaller as the maximum wall rotation was increased and those for the run with 1% wall rotation were about 10% lower than those with zero rotation. The ratio of the horizontal force of the grain on the wall to that of a liquid having the same density (the K value) was also calculated for each run by integrating the stress distributions using the trapezoidal rule. The K values varied from 0.424 to 0.380 and their relationship to the rotation was represented extremely well by a straight line (Fig. 4.14).

The stresses calculated with the computer program can be compared with the practical tests of Terzaghi (1934). Terzaghi measured the force on a wall 2.14m high by 4.28m wide which formed one face of a bin measuring 4.28m from the front to the back wall. In one series of tests the bin was filled by lightly pouring sand into it after which the wall was rotated outwards about its base. Terzaghi found that the K value decreased from 0.405 for no rotation to a minimum of 0.241 at a rotation of 0.42% of the height. This represents an average rate of 96% reduction of K for

each 1% of wall rotation whereas the rate calculated for wheat is only an 11% reduction of K for each per cent of wall rotation.

The difference between Terzaghi's experimental results and those from the finite element program are due to the following two causes:-

a) In Terzaghi's case the wall was rotated after filling the bin. Rotation after filling leads to higher strains in the material behind the wall because material at a given level is subjected to the whole of the wall movement at that level. Conversely, if the wall is allowed to rotate during filling, material at a given level is subjected only to the movement that occurs after that level has been placed. Hence rotation after filling is likely to reduce the stresses more quickly.

b) Terzaghi used sand (void ratio 0.46) whereas the author's material was wheat. It has already been pointed out that more compressible materials are likely to require more wall movement to reduce the stresses upon them (Sect. 2.1.3) and wheat is more compressible than sand. For example, the ratio of volumetric strain change to octahedral normal stress change ($3\sigma_{22}$) is about 1.2MPa^{-1} for wheat (Sect. 3.5.1) whereas the equivalent value for loosely poured sand (void ratio 0.50) is about 0.24MPa^{-1} (Domaschuk and Wade, 1969).

The classical formulae for calculating wall loads (Sect. 2.1) nearly always assume that the shear strength of the stored material is fully mobilized whereas the finite element method does not. Comparison of the results from the finite element program with those obtained from a class-

ical formula will therefore determine how much this assumption affects the calculated stresses. The stress distributions plotted in Fig. 4.13 were obtained by assuming a finite value of front wall friction but zero side wall friction. The results are therefore directly comparable with those calculated by Coulomb's trial wedge method. Airy (1897) gave a formula for the horizontal force per unit width, F_x , derived from the trial wedge method:-

$$F_x = \frac{\gamma D^2}{2A} \left(\frac{A - \mu}{1 - \mu\mu_w + (\mu + \mu_w)A} \right) \dots (4.24)$$

where

$$A = \mu + \sqrt{\frac{\mu(1 + \mu^2)}{\mu + \mu_w}}$$

The stress on the wall at distance y below the surface can be derived by differentiating this expression with respect to depth hence

$$\sigma_x = \frac{\gamma y}{A} \left(\frac{A - \mu}{1 - \mu\mu_w + (\mu + \mu_w)A} \right) \dots (4.25)$$

A problem when comparing the results given by Eqn 4.25 with those given by the finite element program is the choice of a suitable value for the coefficient of internal friction, μ , as this quantity is not directly related to any of the parameters required for the stress/strain model developed in Chapter 3. Because of this a series of tests was conducted on the same wheat sample as used in Chapter 3, using the large shear box developed by Lawton and Marchant (1980). The coefficient of internal friction was measured as 0.51 with negligible cohesion. This measured value of μ and the same values of μ_w (0.3) and γ (6.65 kN/m³) as used in the computer runs were substituted into

Eqns 4.24 and 4.25. The resulting stress distribution (Fig. 4.13) and the K value (0.32) were lower than those from the finite element program by between 16% (1% rotation) and 24% (zero rotation).

It is useful at this point to consider the possible K values which can occur with the finite element program. From Sect 4.3.4

$$\tau = F_m(0.635\sigma)$$

where the figure of 0.635 is the value of K_2 for wheat (Sect. 3.5.1). Substituting for τ and σ from Eqns 3.4 and 3.5

$$\left[(\sigma_1 - \sigma_2) + (\sigma_2 - \sigma_3) + (\sigma_3 - \sigma_1)^2 \right]^{1/2} = 0.635 F_m (\sigma_1 + \sigma_2 + \sigma_3) \quad \dots (4.26)$$

If the analysis is confined to the simplified case of no boundary friction and a flat grain surface then the principal stresses are the normal stresses in each of the three coordinate directions and $K = \sigma_3 / \sigma_1$. Unfortunately σ_2 , the stress normal to the x,y plane, is not known but for the present let $\sigma_2 = C\sigma_3$. The minimum value of K will be that at failure i.e. when $F_m = 1$. In this case, dividing Eqn 4.26 by σ_3 and manipulating gives

$$\frac{1}{K^2}(1.60) - \frac{1}{K}(2.81C + 2.81) + 1.60(C^2 + 1) - 2.81C = 0 \quad \dots (4.27)$$

which is a quadratic equation in $1/K$. Sect. 3.3.2 showed that C is unlikely to be outside the range 1.0 to 1.4 and the solutions of Eqn 4.27 for the extreme values of C and where $\sigma_1 > \sigma_3$ are $K = 0.29$ ($C = 1.0$) and $K = 0.24$ ($C = 1.4$). The K value derived using Eqn 4.24 with $\mu_w = 0$ and $\mu = 0.51$ is 0.38 which shows that the finite element program could predict stresses which were lower than those using Eqn 4.25 if enough wall movement were allowed so that the material

failed completely.

Eqn 4.26 can be re-solved to give the value of F_m required to produce a K value equal to that derived using Eqn 4.24. This results in $F_m = 0.78$ for $C = 1.0$ and $F_m = 0.65$ for $C = 1.4$. In the computer runs the average value of C was about 1.1 which shows that F_m had to reach about 0.75 before the wall stresses would have been reduced to those given by Eqn 4.25 whereas in most of the elements F_m was less than 0.65 even at 1% wall rotation (Fig. 4.15).

The positions of the centres of pressure (Fig. 4.14) were also calculated by integrating the stress distributions using the formula

$$\frac{\bar{y}}{D} = \frac{1}{D} \frac{\int \sigma_x y dy}{\int \sigma_x dy}$$

where $\bar{y}$ is the height of the centre of pressure above the floor. Once again the trapezoidal rule was used. Three points are apparent from this calculation:-

a) The centre of pressure is located at about one third the height up the wall, which is the position derived using Eqn 4.25.

b) The change in height of the centre of pressure as the wall is rotated is small.

c) This small change is in a downwards direction with increasing rotation.

These conclusions are in qualitative agreement with Terzaghi's practical results for loose sand (Terzaghi, 1934) in which the centre of pressure decreased from 0.34 to 0.30 of the depth of the sand for a rotation of 0 to 0.45% of the depth.

4.5.2 Effect of front and back wall friction

The silo configuration of Sect. 4.4.2 was again used in the computer runs to investigate the effect of changing the friction at the front and back walls. The walls were both assumed to be completely rigid and their coefficients of friction were assumed to be equal and were varied in the range 0 to 0.4.

The stresses calculated using the finite element program (Fig. 4.16) were integrated to give the K values. Once again the K values calculated using the finite element program were higher than those calculated from Eqn 4.24. However, the amount by which K was reduced for a given change in μ_w (Fig. 4.17) was very similar for both methods.

When Eqn 4.25 is used the centre of pressure is always one third of the height above the floor and Fig. 4.17 shows that those calculated using the finite element program were close to this especially for zero friction. However, as the friction was increased the stresses at the floor fell at a greater rate than the stresses at the higher levels which shifted the centre of pressure slightly upwards.

4.5.3 Effect of side wall friction

Once again the same silo configuration was used but this time with a constant front and back wall friction coefficient of 0.3. The side wall spacing was set at 4m and the coefficient of friction at the side walls was varied in the range 0 to 0.4.

The trial wedge method does not take side wall friction into account but Janssen's formula (Eqn 2.9) does and so can be used for a comparison with the finite element program.

Janssen's formula assumes that the wall friction is constant around the perimeter of the bin whereas in the silo configurations used with the finite element program μ_s is usually different from μ_w . However, an examination of the derivation of this formula shows that an equivalent value can be used which is a weighted average of the frictions along the perimeter, the weighting factors being the lengths of the perimeter over which the respective coefficients apply. If the rectangular section silo used has a side wall spacing of W and a distance from front to back walls of L then the coefficient of friction that should be used in Eqn 2.9 is

$$\mu_w = \frac{\mu_w W + \mu_s L}{w + L}$$

The ratio between the horizontal and vertical stresses, k , used in Eqn 2.9 was that assuming total failure throughout the material (Eqn 2.10) i.e. the value normally used for design purposes. Again, the stresses calculated with the classical formula were lower than those calculated using the finite element program (Fig. 4.18). However, the reduction of the K value with side wall friction (Fig. 4.19) was virtually the same for both methods of calculation despite the actual magnitude of K being lower when using the classical formula. (Note that k which is the ratio of horizontal to vertical stress is not equal to K).

The centres of pressure calculated using each method were very nearly equal (Fig. 4.19) and both methods predicted a slight increase in the height of the centre of pressure as the side wall friction increased.

The effect of side wall friction was also investigated at different widths with the depth at 4m and different

depths with the width at 2m and the results are shown in Figs 4.20 and 4.21. It can be seen that the stresses decreased as the width became smaller but the differences between the stresses with and without side wall friction were smaller as the total depth decreased. This suggests that there might be a unique relationship between the dimensionless parameters ΔK (the difference between the K value with side wall friction and that without) and $D\mu_s/W$. These parameters have been plotted (Fig. 4.22) for all the computer runs that included side wall friction and it can be seen that a single straight line describes the relationship well. This line must pass through the origin as the side wall friction effect must be zero when $\mu_s = 0$ or when W is infinite and so its slope can be calculated from the formula

$$C_s = \frac{(\text{sum of } y \text{ values})}{(\text{sum of } x \text{ values})}$$

which gives $C_s = 0.082$.

4.5.4 A possible simple formula for the K value

The changes in K can be related to the values of rotation and front wall friction acting separately by fitting straight lines to the data points of Fig. 4.14 and Fig. 4.17. This gives parameters $C_r (=0.045)$ and $C_w (=0.153)$ which are analogous to the parameter C_s . It is suggested that these parameters can be combined in a linear manner to give a simple formula for calculating the K values for rectangular section silos with a level filling of wheat where the wall is rotating outwards about its base. This formula has the form

$$K = K_0 - C_w\mu_w - C_r r - C_s \frac{D\mu_s}{W}$$

where K_0 is the K value with zero front wall friction, zero rotation, and zero side wall friction. From Fig. 4.17 K_0 is 0.470 which gives the formula

$$K = 0.470 - 0.153\mu_w - 0.045r - 0.082 \frac{D\mu_s}{W} \quad \dots (4.28)$$

Table 4.1 shows all the values calculated using the finite element program along with those using Eqn 4.28 and it can be seen that the agreement is excellent over a wide range of conditions except when the depth is not equal to 4m. The relatively poor agreement for shallower depths could be due to K_0 being dependent on the depth and so K_0 was re-calculated by integrating the stress distribution with $\mu_w = 0$ of Fig. 4. but this time down to depths of 2m and 3m only. This gave values for K_0 of 0.491 and 0.479 respectively which, when used instead of 0.470 in Eqn 4.28 resulted in calculated values of K which were much closer to the ones from the finite element program. These are shown in parentheses in Table 4.1.

It is felt that this approach could lead to the production of simple design formulae from results obtained from the finite element program. However, more work would need to be done in the following areas:-

- a) An investigation of the K values over a wider range of the variables.
- b) The determination of equivalent values for r when the wall movement is not a simple rotation.
- c) The determination of equivalent values for D when the surface is not flat.
- d) The determination of the variation of K_0 , C_w , C_r , and C_s with the mechanical properties of the grain, probably as functions of dimensionless groups formed with the a para-

meters of Chapter 3, the specific weight, and the depth, e.g. $C_r = f(a_{22}/\gamma D)$.

These points are left as suggestions for further work.

4.5.5 Silos with circular cross sections

Although the practical measurements of wall stresses in this work are confined to a silo with a rectangular cross section (Chapter 6), the finite element program has been written to handle cases of silos with circular cross sections. The horizontal wall stresses on a silo with perfectly rigid walls filled with wheat to a depth of 4m have been calculated with the finite element program. In the first set of runs the diameter was 2m and the wall friction was varied from zero to 0.4 while in the second set the wall friction was held constant at 0.3 and the diameter was changed from 2m to 4m. In both sets the stresses (Figs 4.23 and 4.24) were higher than those calculated using Janssen's formula but, as with the rectangular section silos, the reduction of the K value with wall friction and the heights of the centres of pressure were similar (Fig. 4.25).

Table 4.1

K values calculated using the finite element program and also using the proposed simple formula.

μ_w	r %	D m	W m	μ_s	K F.E.M	K Eqn 4.28
0.3	0	4	4	0	0.424	0.424
0.3	0.1	4	4	0	0.419	0.420
0.3	0.2	4	4	0	0.415	0.415
0.3	0.5	4	4	0	0.401	0.402
0.3	1.0	4	4	0	0.380	0.379
0	0	4	4	0	0.470	0.470
0.1	0	4	4	0	0.453	0.455
0.2	0	4	4	0	0.438	0.439
0.3	0	4	4	0	0.424	0.424
0.4	0	4	4	0	0.412	0.409
0.3	0	4	4	0.1	0.416	0.416
0.3	0	4	4	0.2	0.407	0.408
0.3	0	4	4	0.3	0.399	0.400
0.3	0	4	4	0.4	0.391	0.391
0.3	0	4	1	0.3	0.332	0.326
0.3	0	4	2	0.3	0.375	0.375
0.3	0	4	6	0.3	0.407	0.408
0.3	0	2	2	0	0.452	0.424 (0.445)
0.3	0	2	2	0.3	0.423	0.400 (0.420)
0.3	0	3	2	0	0.437	0.424 (0.433)
0.3	0	3	2	0.3	0.398	0.387 (0.396)

Note:- The figures in parentheses were obtained by making K_0 a function of depth.

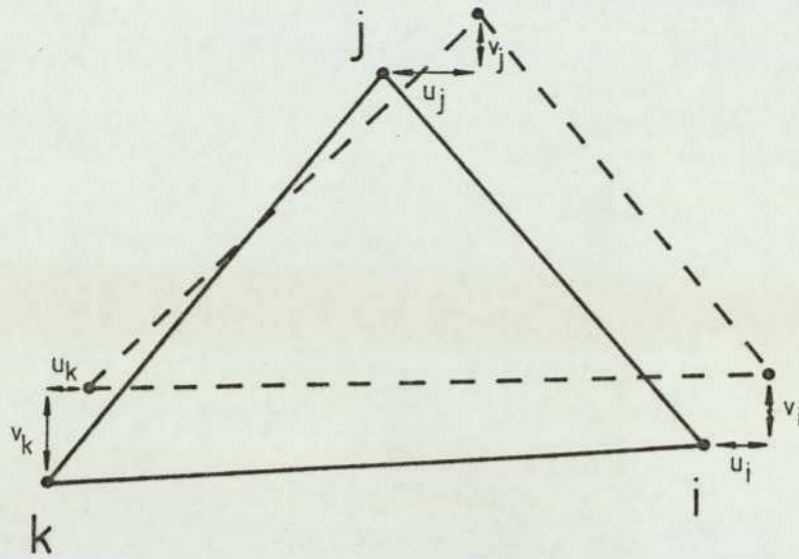


Fig. 4.1 Simple triangular cross section finite element shown before (solid lines) and after (broken lines) distortion. Nodes shown thus :- • .

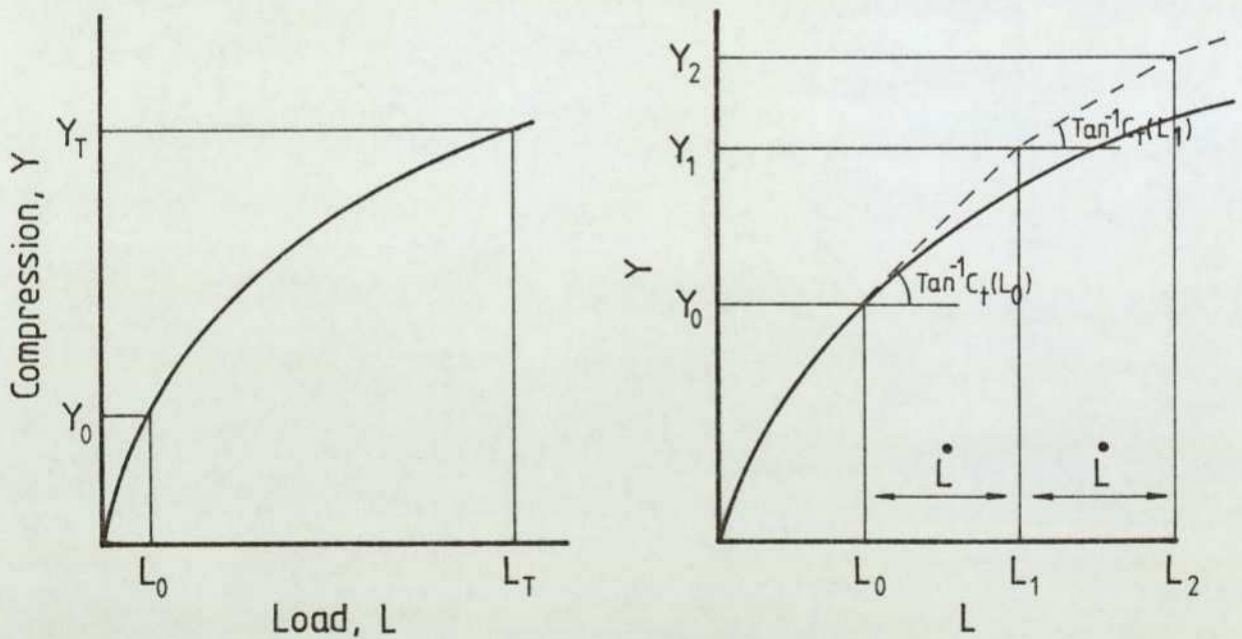


Fig. 4.2 Left, compression/load characteristic of a non-linear spring. Right, detail of first part of simulation showing true curve (solid curve) and simulated curve (broken lines).

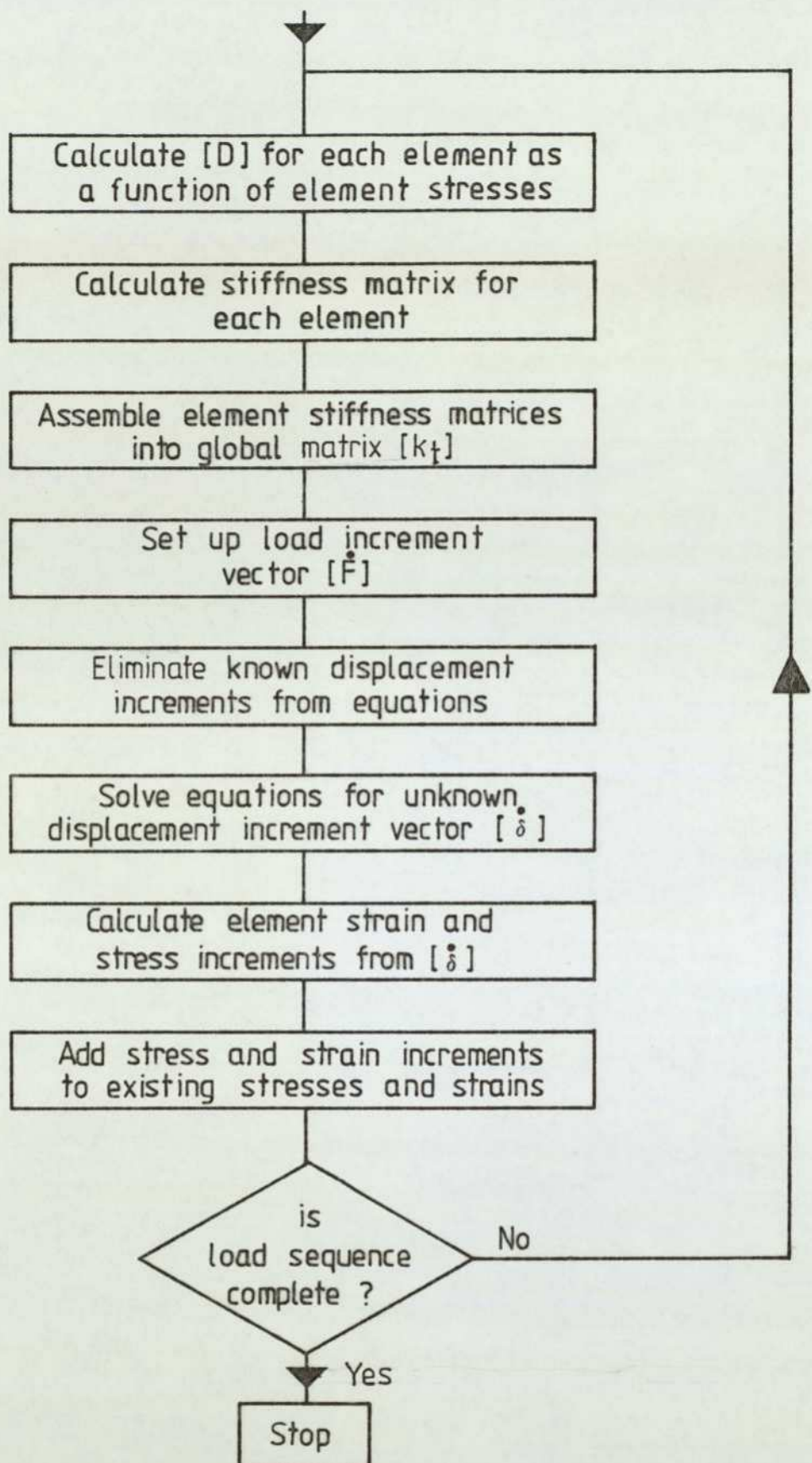


Fig. 4.3 Flow chart illustrating incremental finite element method.

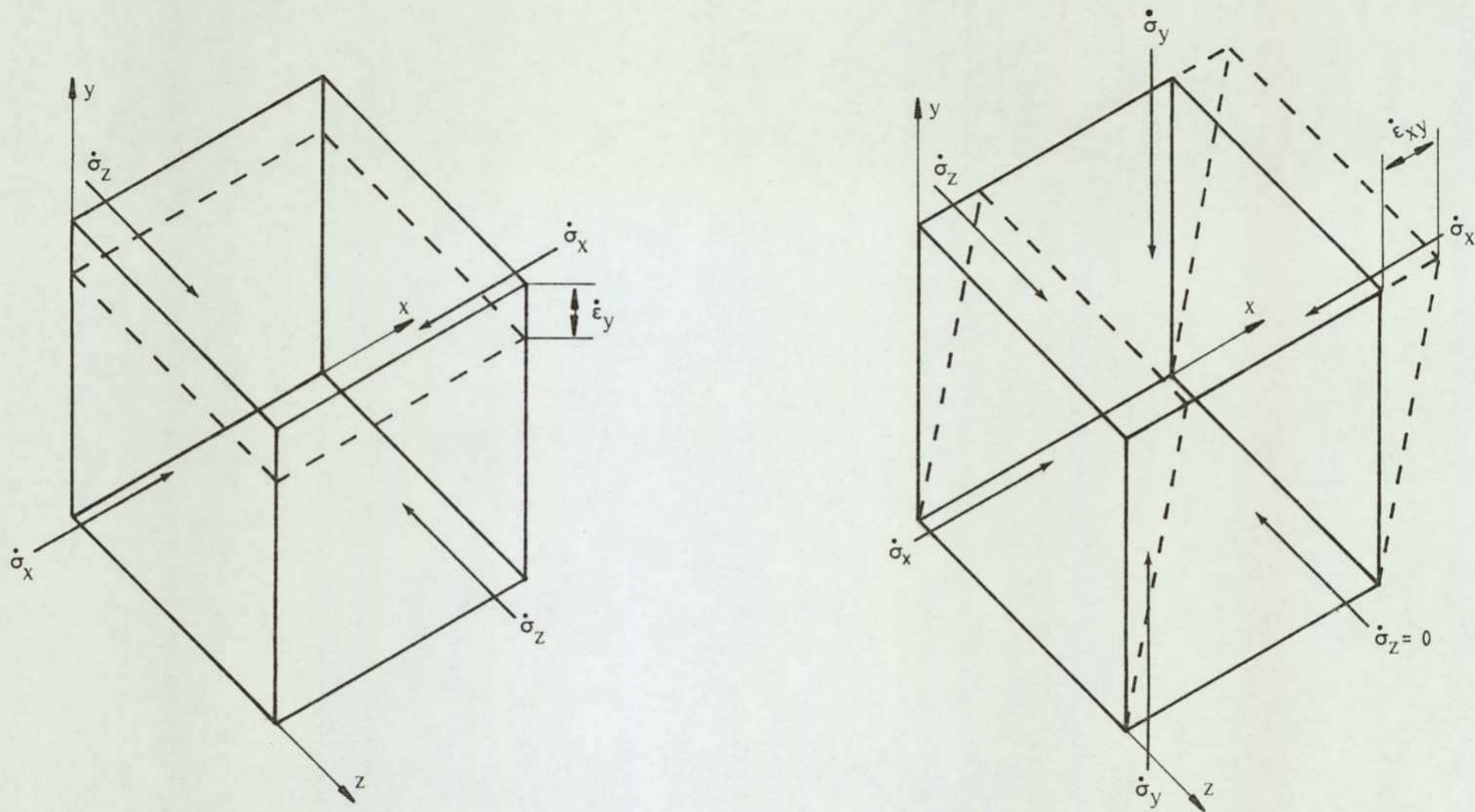


Fig. 4-4 Cubic box of unit dimensions before (solid lines) and after (broken lines) distortion. Left, normal strain change $\dot{\epsilon}_y$. Right, shear strain change $\dot{\epsilon}_{xy}$.

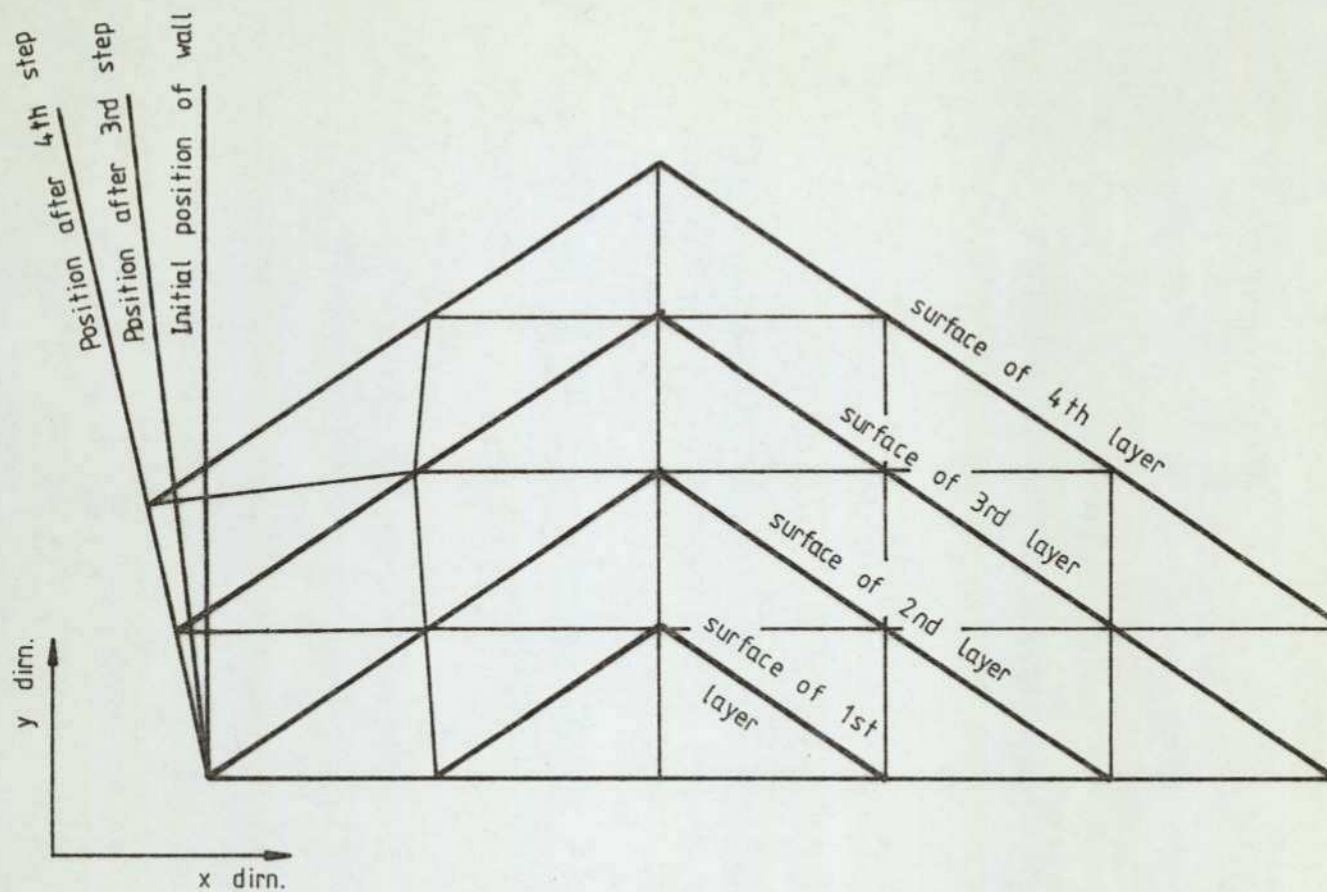


Fig. 4.5 Incremental simulation of the loading of a silo. Example of an element arrangement at the end of the 4th step.

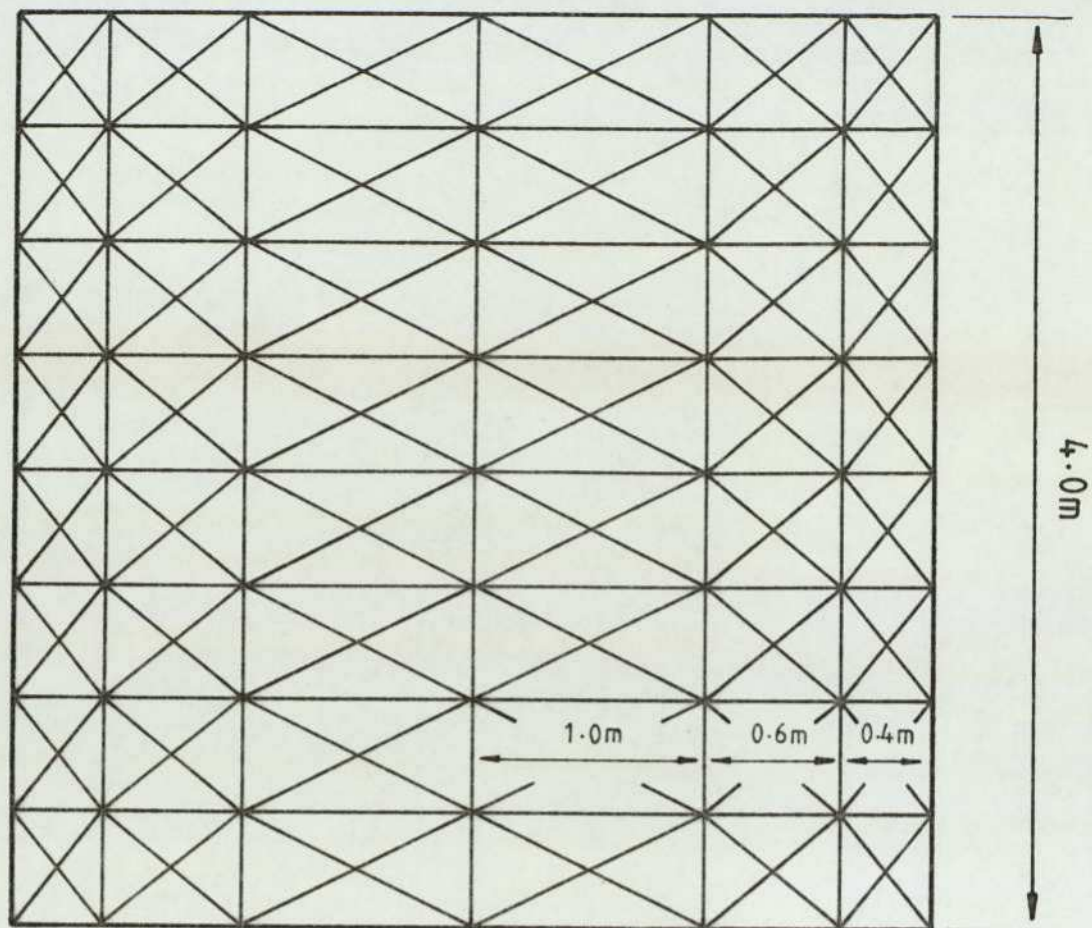


Fig. 4.6 Standard element configuration.

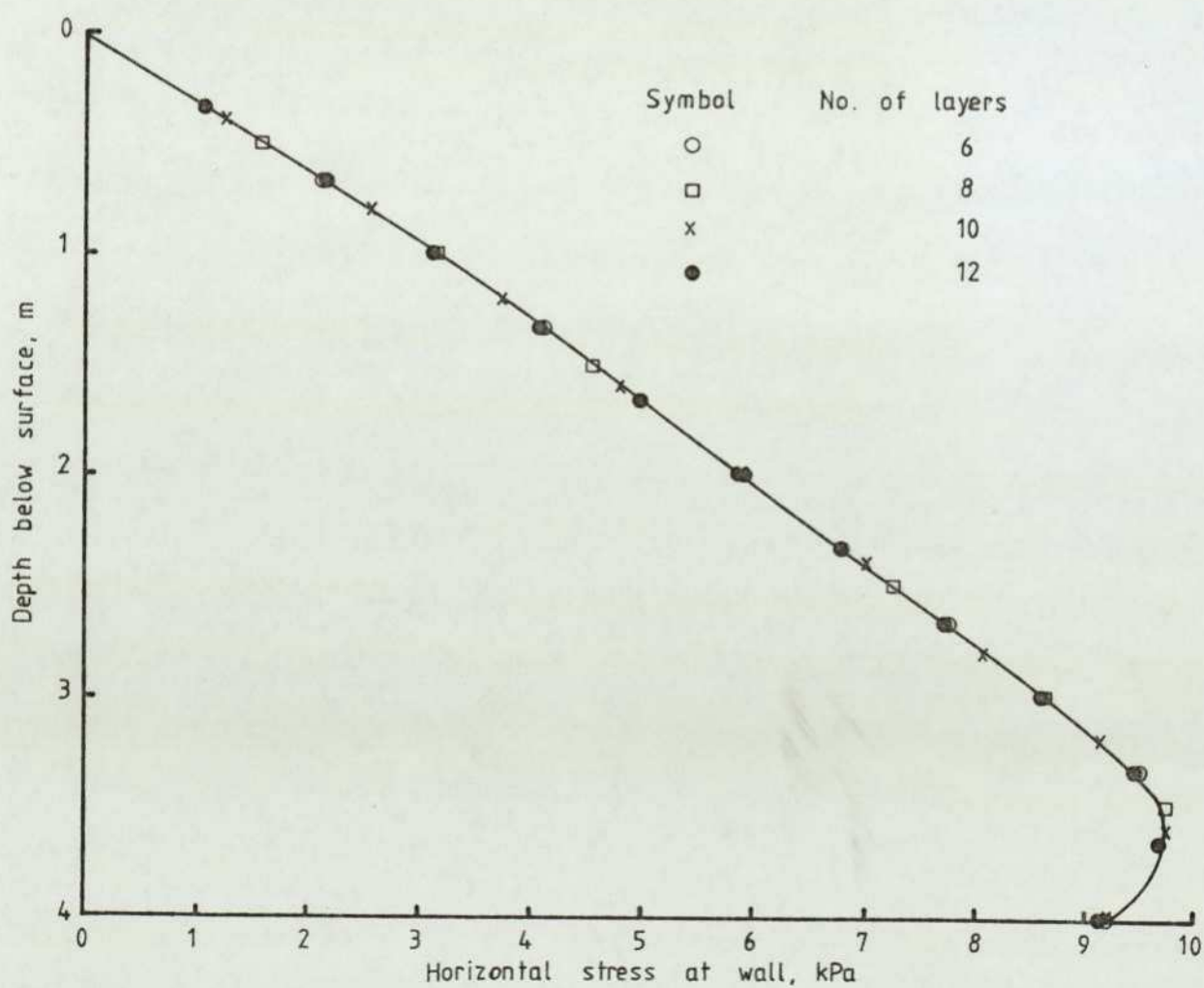


Fig. 4.7 Stress distributions at the wall for various numbers of layers.

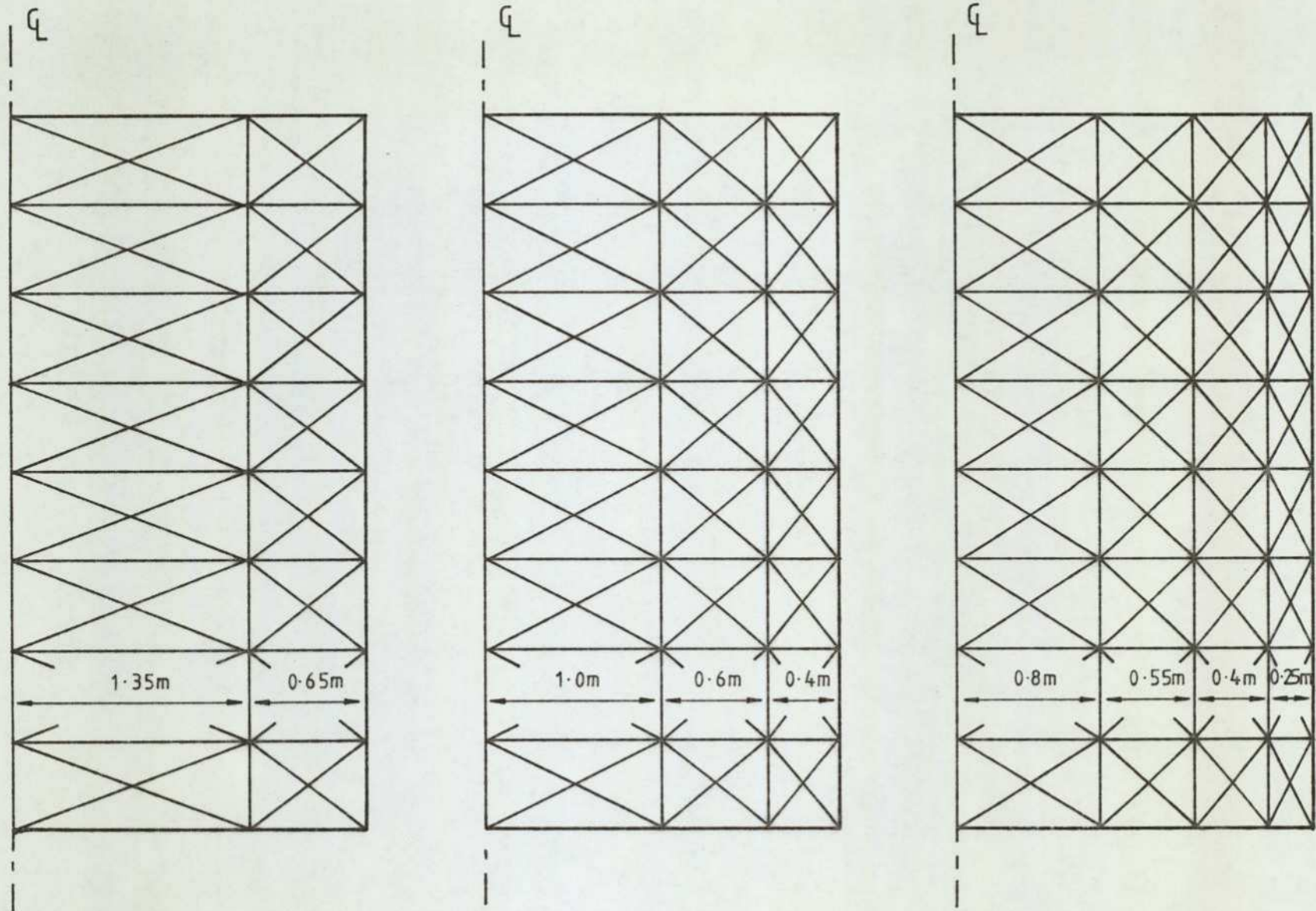


Fig. 4.8 Element configurations for various numbers of vertical columns. Half of symmetrical system only shown.

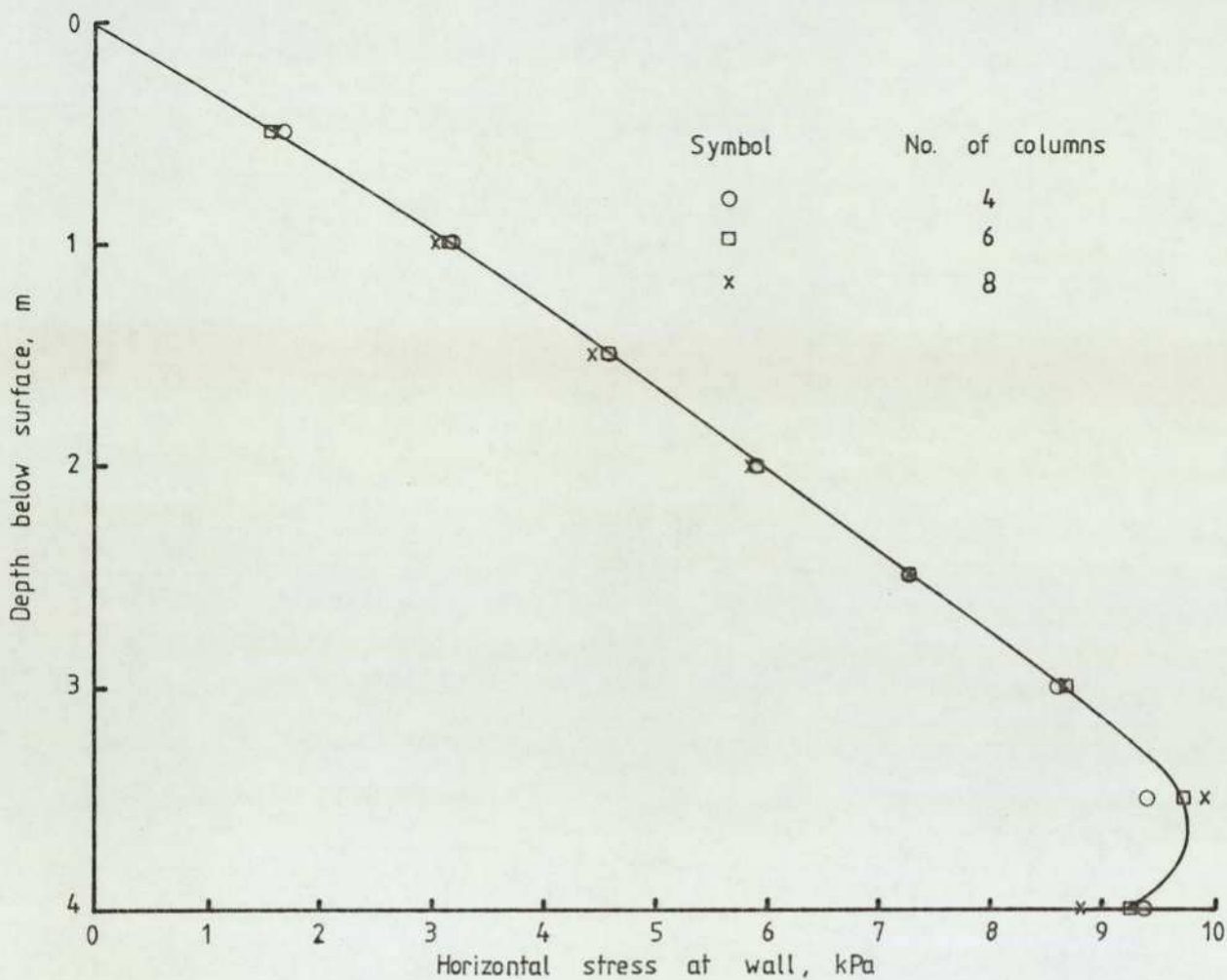


Fig. 4.9 Stress distributions at the wall for various numbers of vertical columns. Line drawn for six columns.

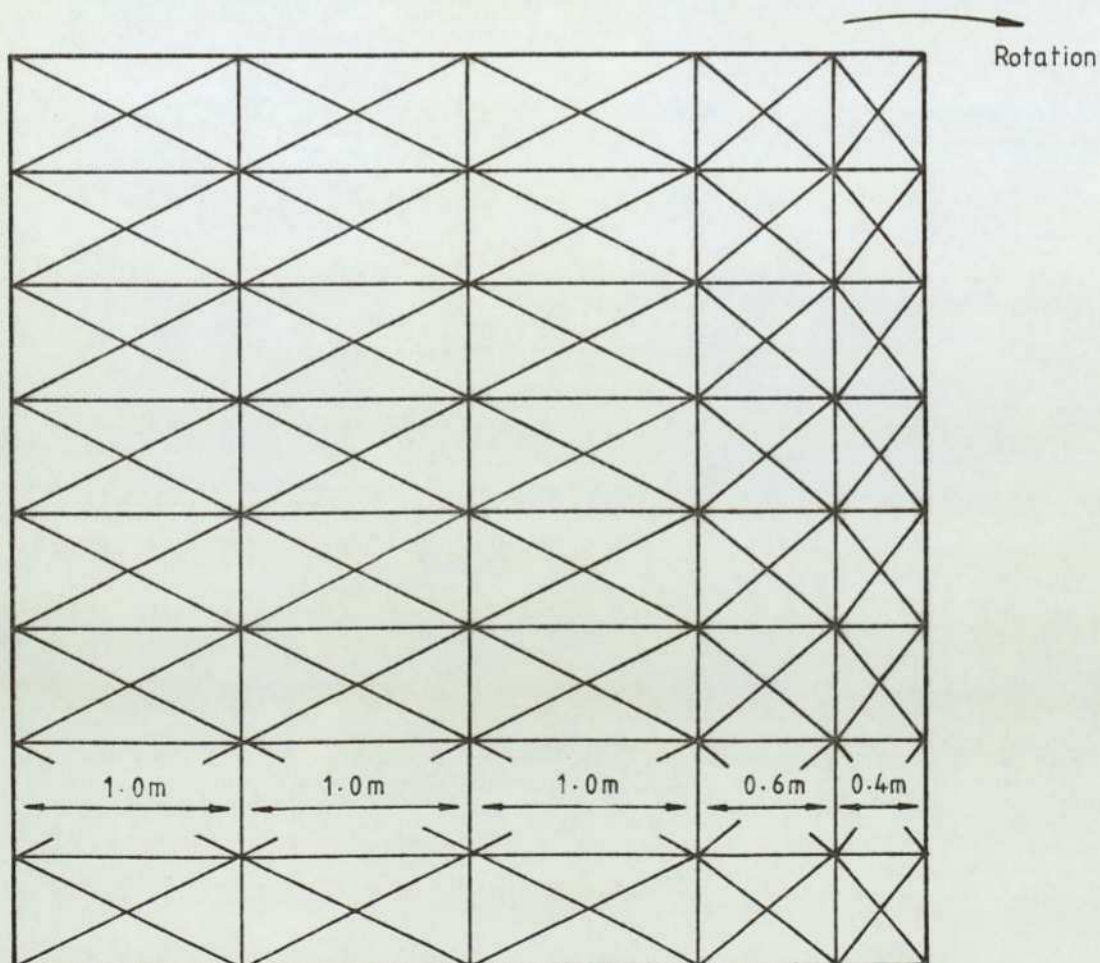


Fig. 4.10 Element configuration for runs in which one wall was rotated.

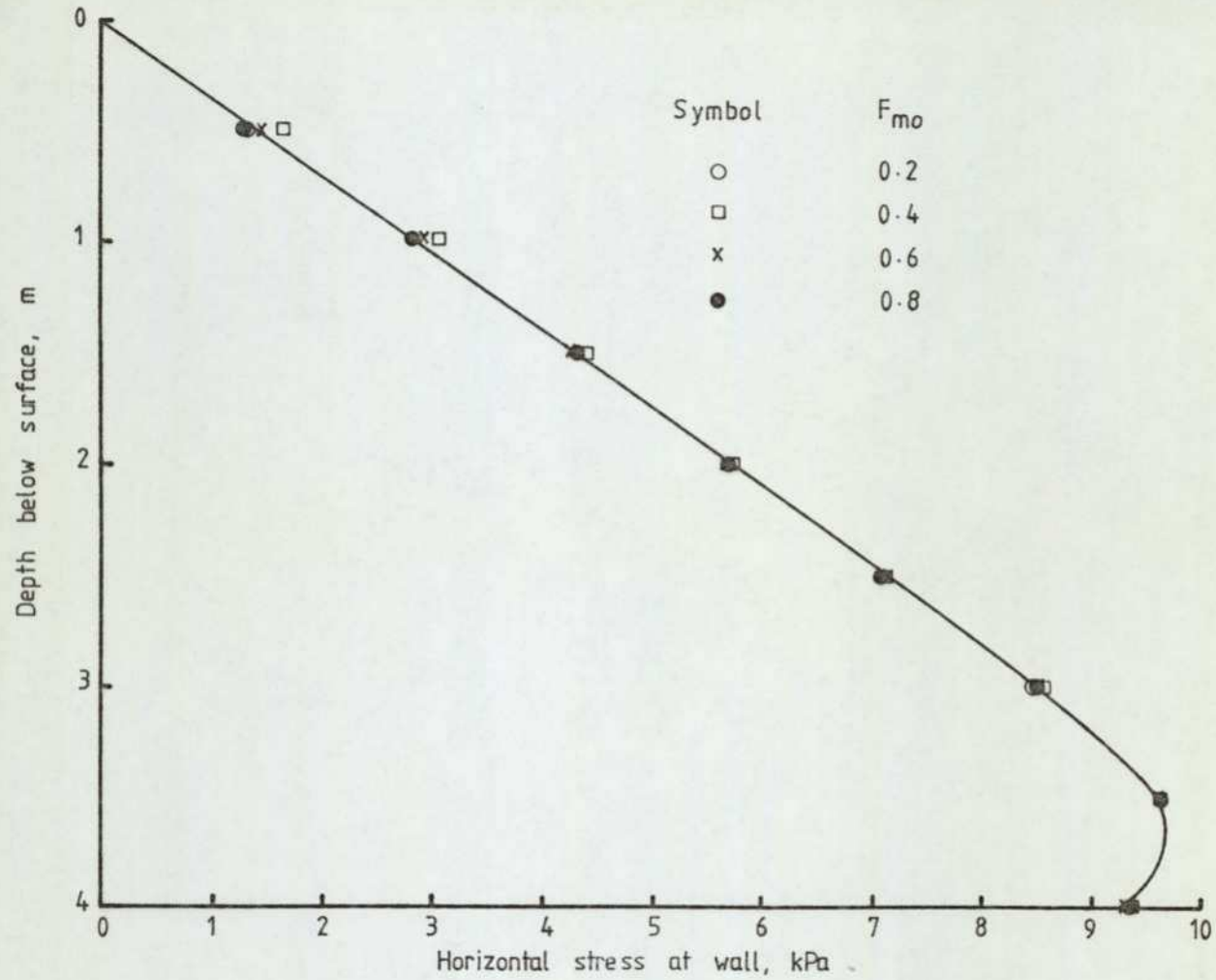


Fig. 4.11 Stress distributions at the walls for various values of F_{m0} with a 0.1% rotation of the front and back walls. Line drawn for $F_{m0} = 0.6$.

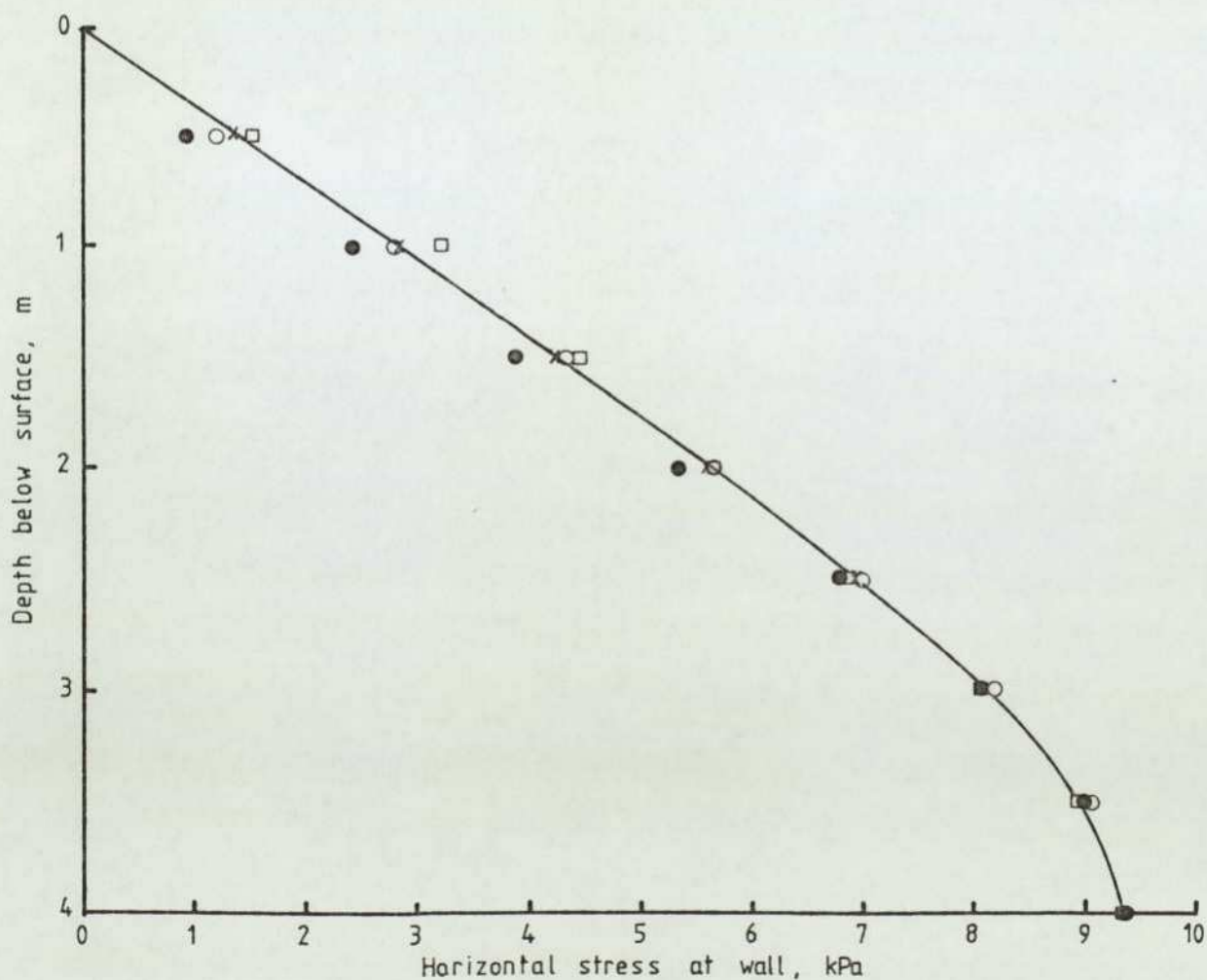
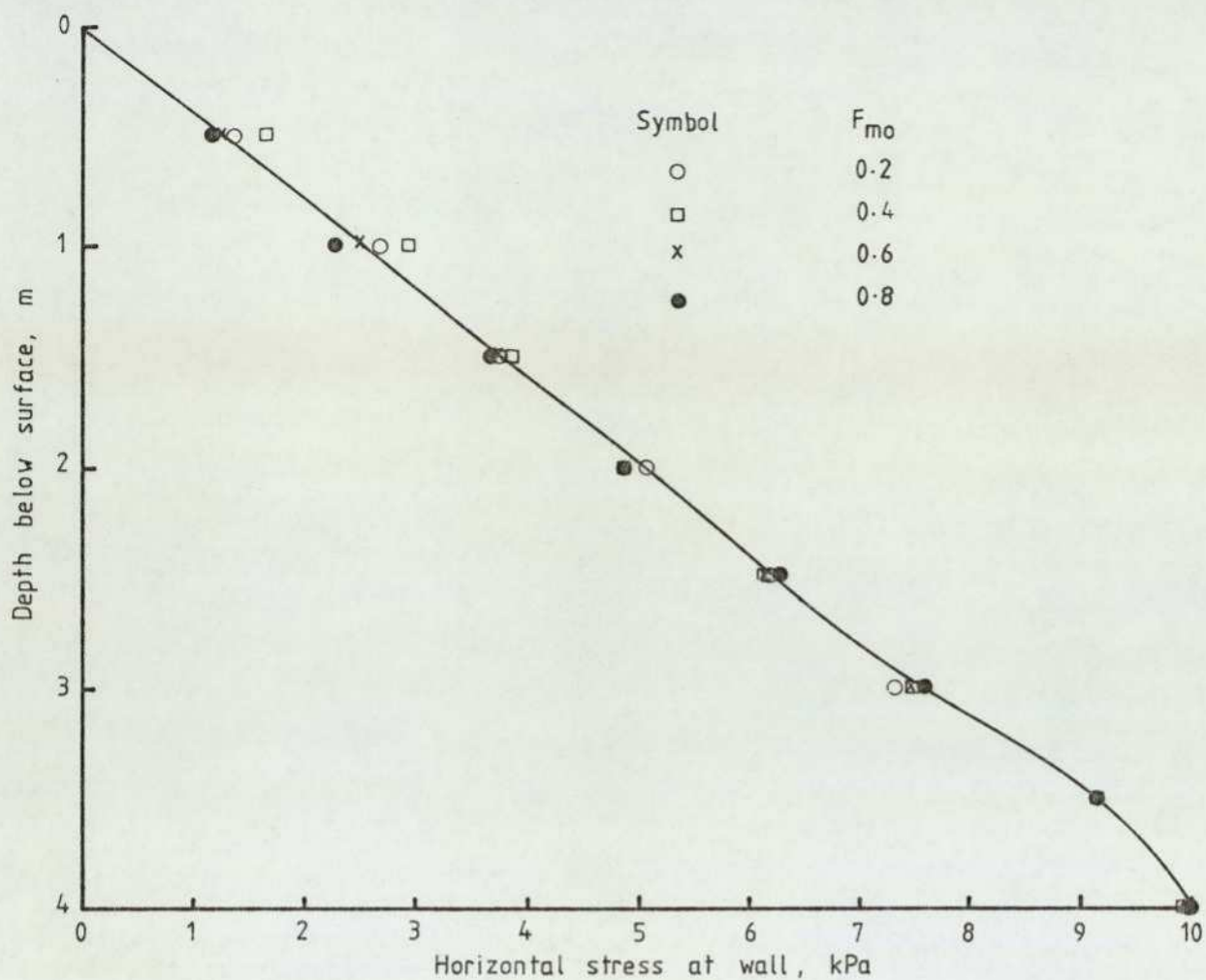


Fig. 4.12 Stresses at the front (top) and back (bottom) walls for various values of F_{mo} at 1% front wall rotation. Lines drawn for $F_{mo} = 0.6$.

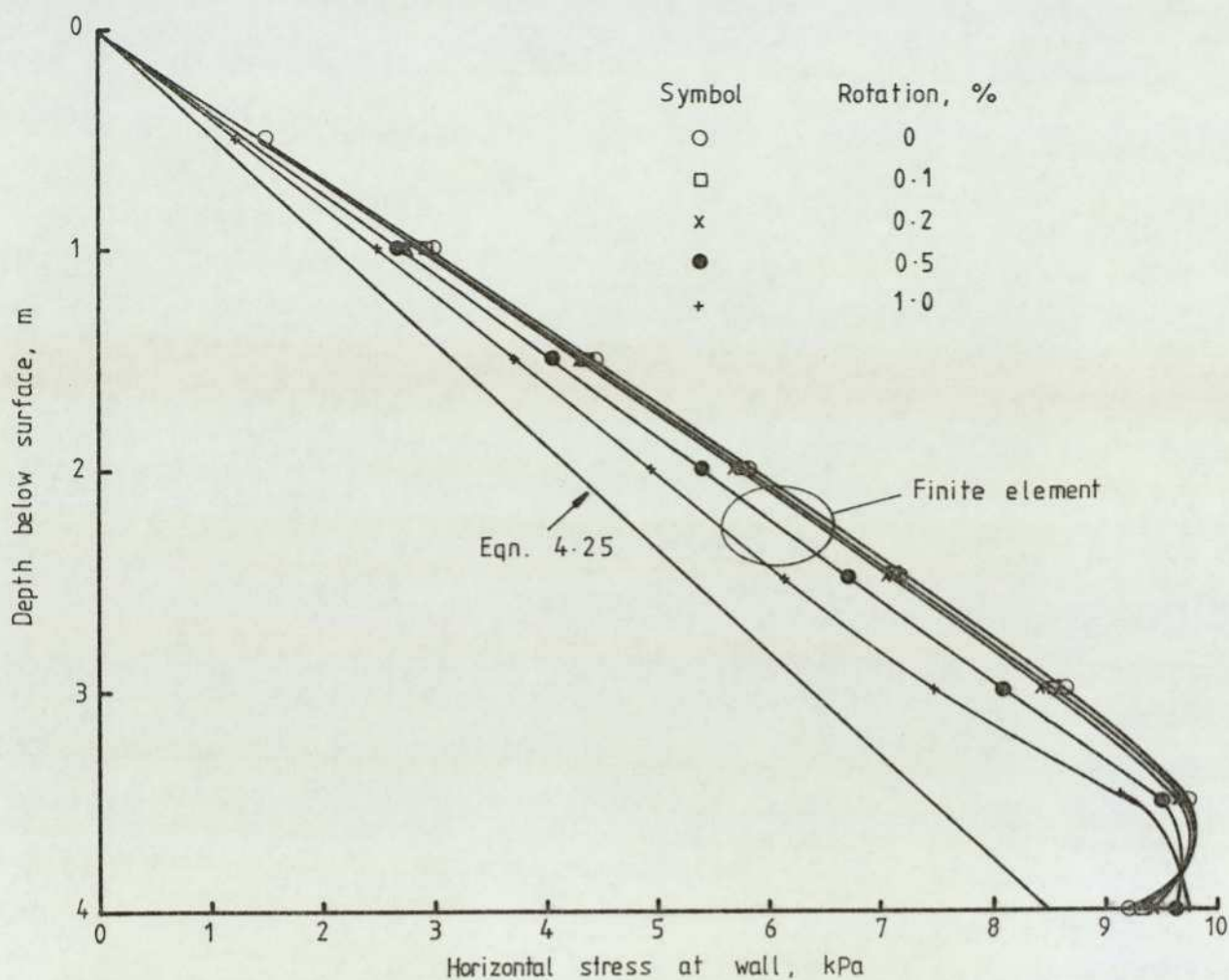


Fig. 4.13 Stresses at the front wall for various wall rotations calculated with the finite element program along with the distribution calculated using Eqn. 4.25.

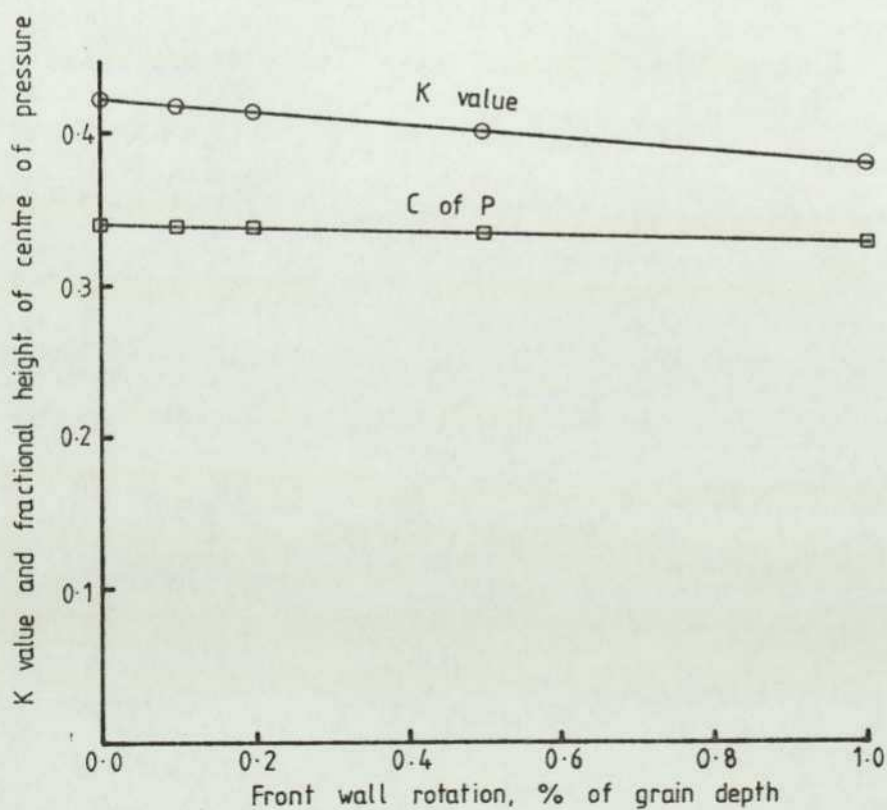


Fig. 4.14 K value and centre of pressure at various wall rotations calculated with the finite element program with $\mu_w = 0.3$ and $\mu_s = 0$.

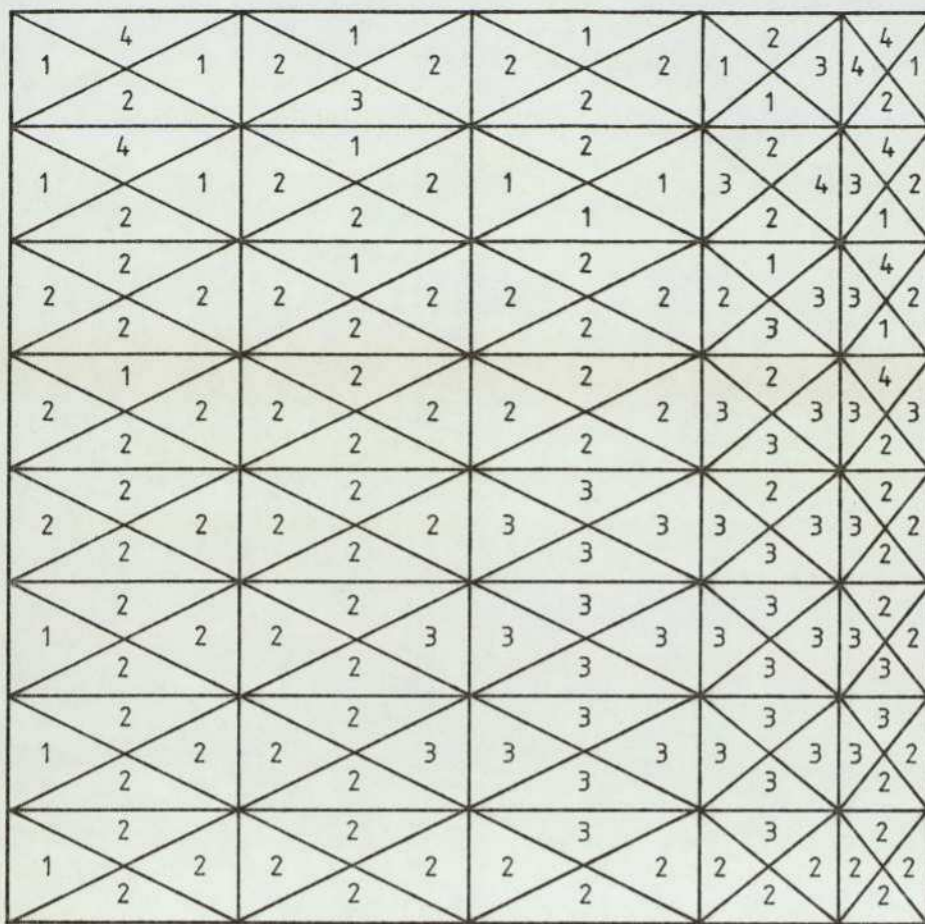


Fig. 4.15 Variation of the fraction of strength mobilized (F_m) throughout the silo with 1% rotation of front wall. 1, $F_m \leq 0.6$; 2, $0.6 \leq F_m \leq 0.65$; 3, $0.65 \leq F_m \leq 0.70$; 4, $F_m \geq 0.70$.

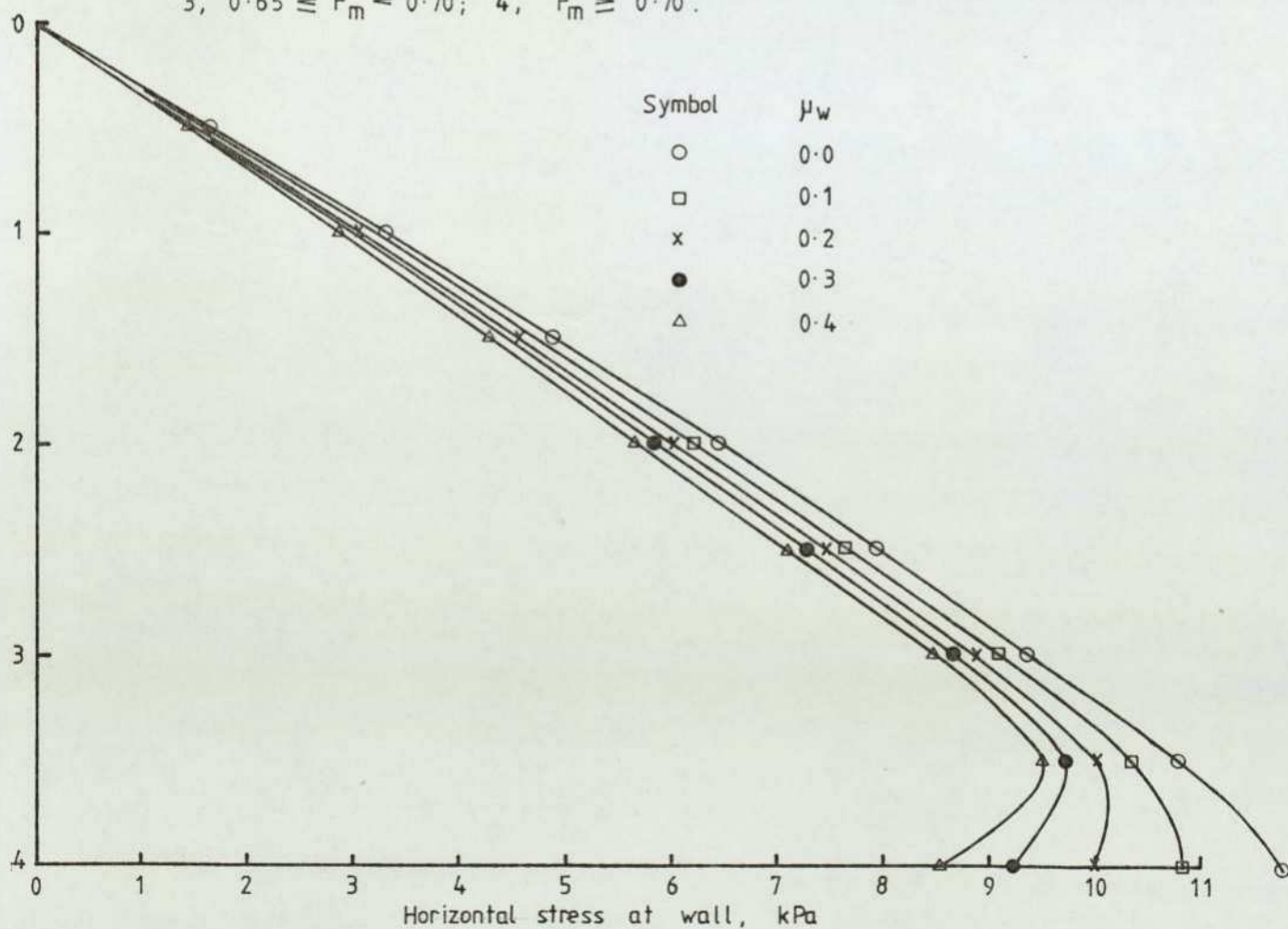


Fig. 4.16 Stresses at the front wall for various values of front and back wall friction calculated with the finite element program.

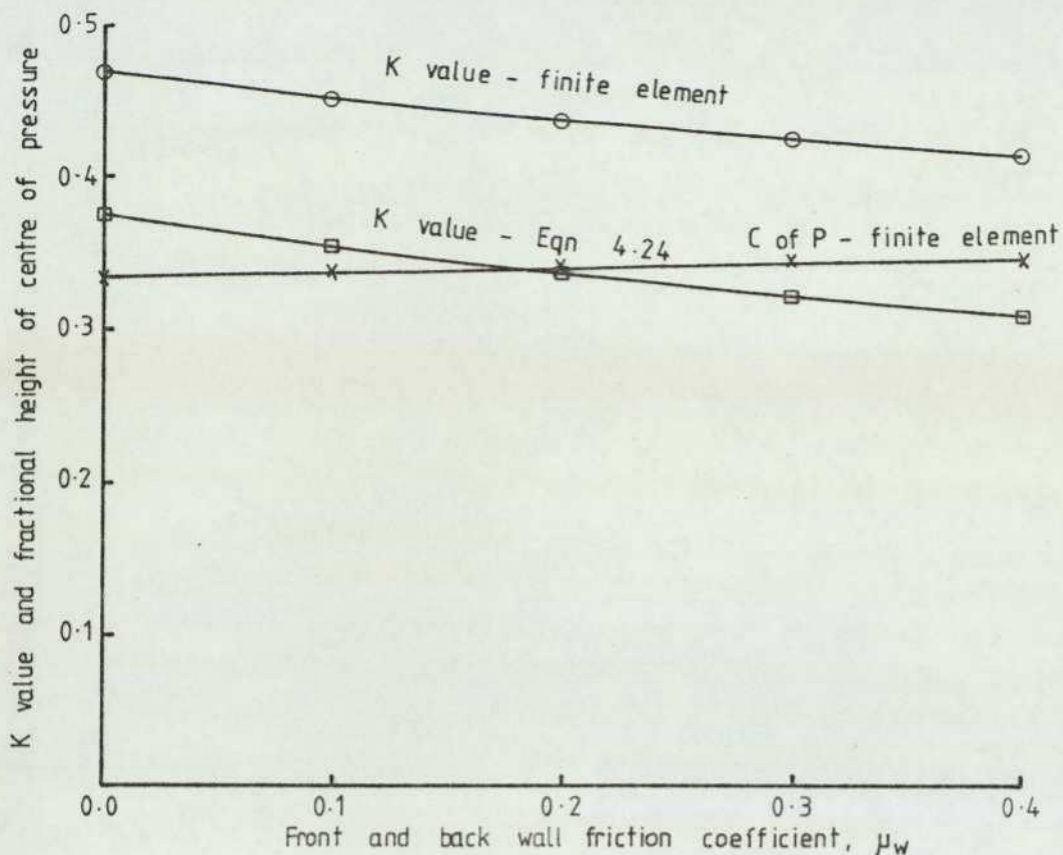


Fig. 4.17 K value and centre of pressure at various values of μ_w with $\mu_s = 0$ and no wall rotation.

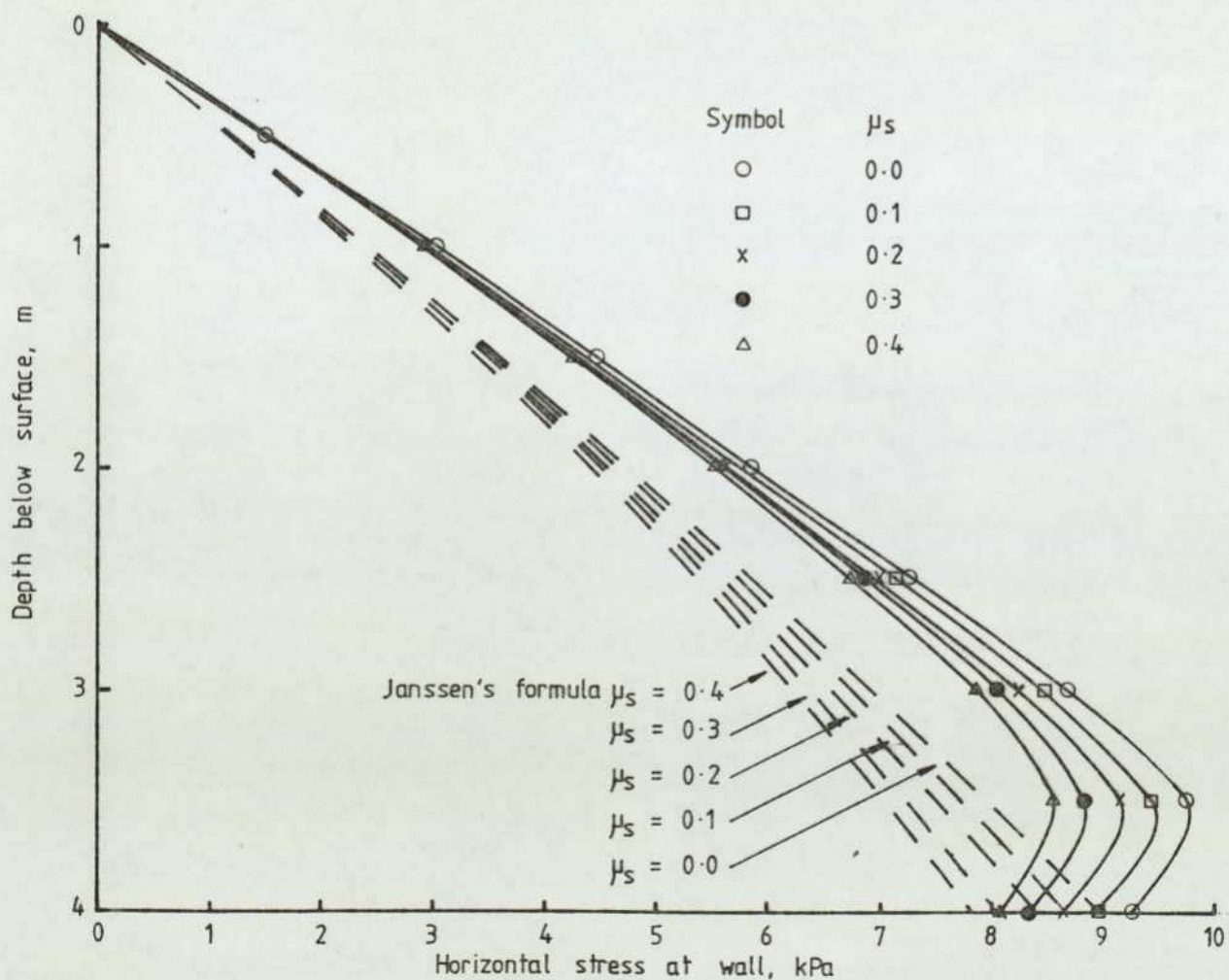


Fig. 4.18 Stress distributions for various values of μ_s calculated using the finite element program (full lines) and Janssen's formula (broken lines).

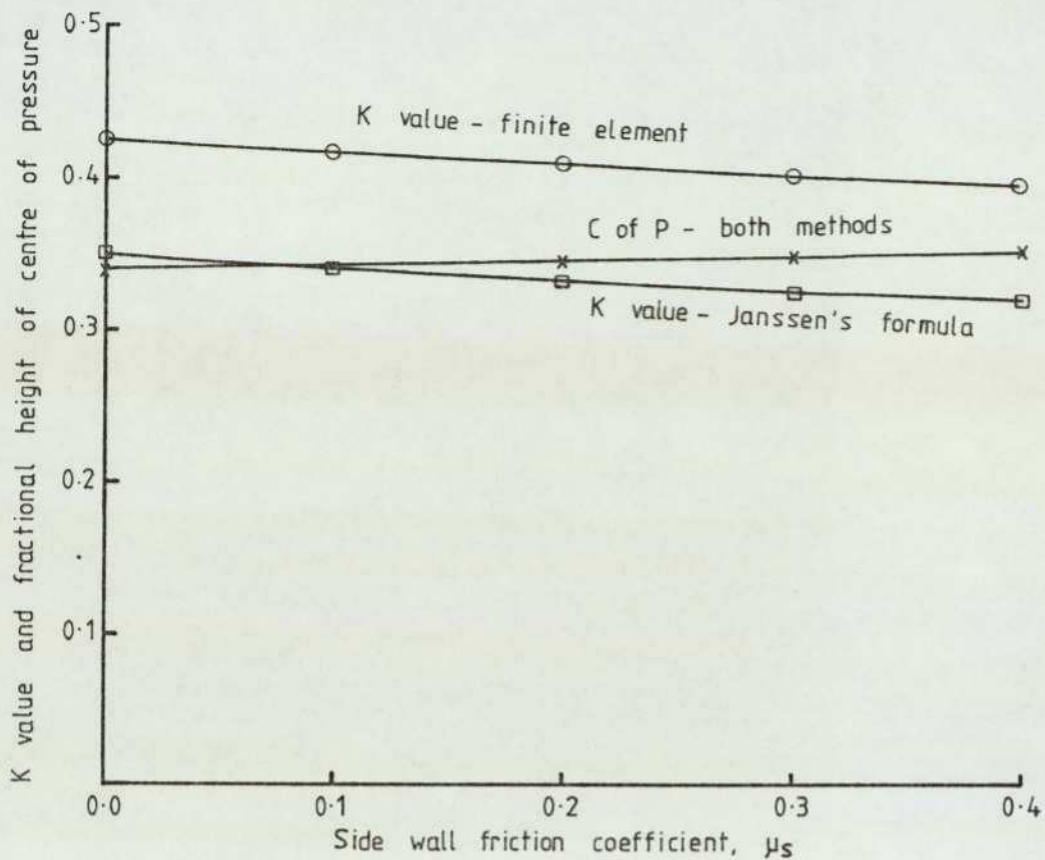


Fig. 4.19 K value and centre of pressure at various values of μ_s with $\mu_w = 0.3$ and no wall rotation.

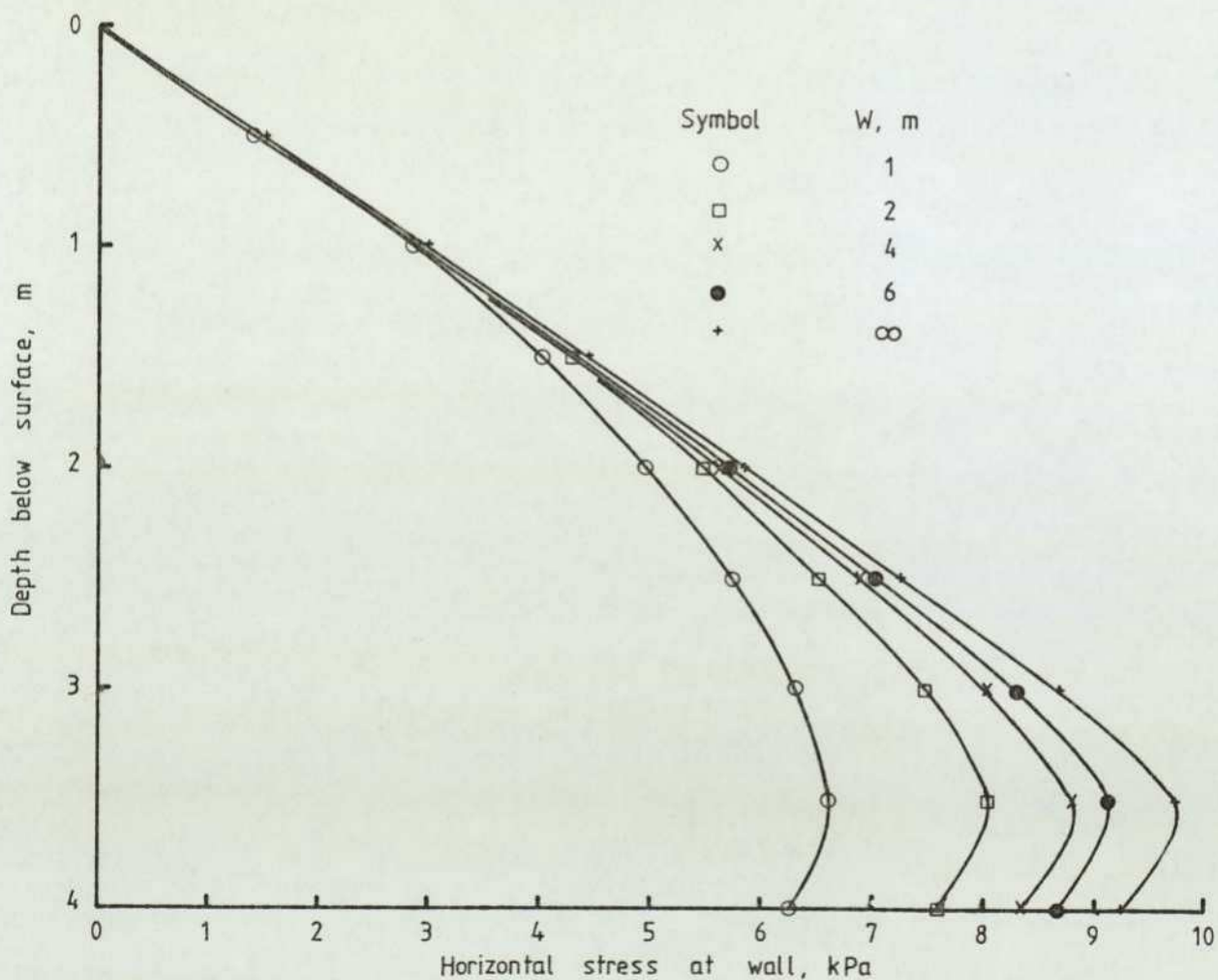


Fig. 4.20 Stress distributions for various widths with $\mu_w = \mu_s = 0.3$ and no rotation.

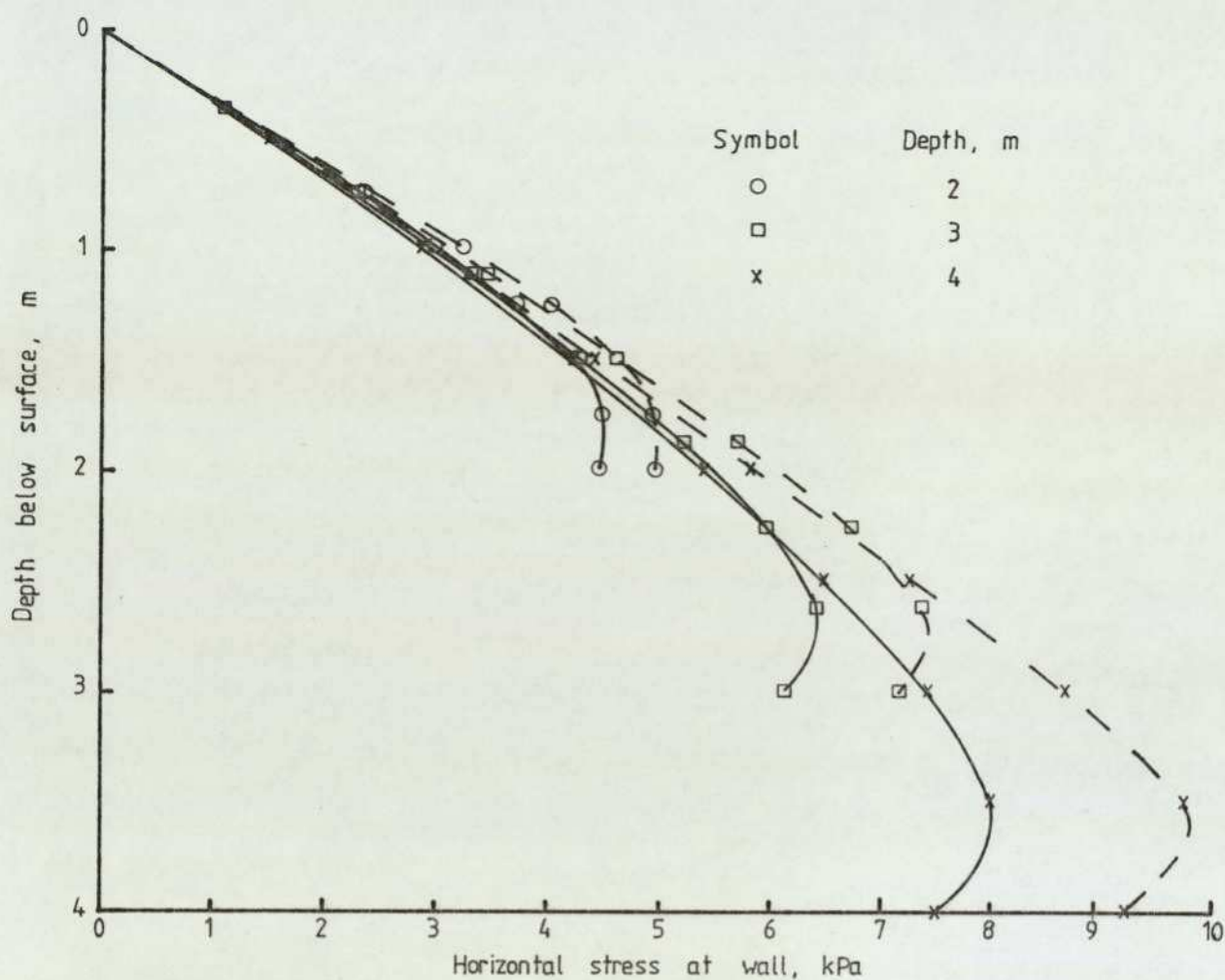


Fig. 4.21 Stress distributions for various depths with $W = 2\text{m}$ and $\mu_W = 0.3$. $\mu_s = 0.3$ (full lines), $\mu_s = 0.0$ (broken lines).

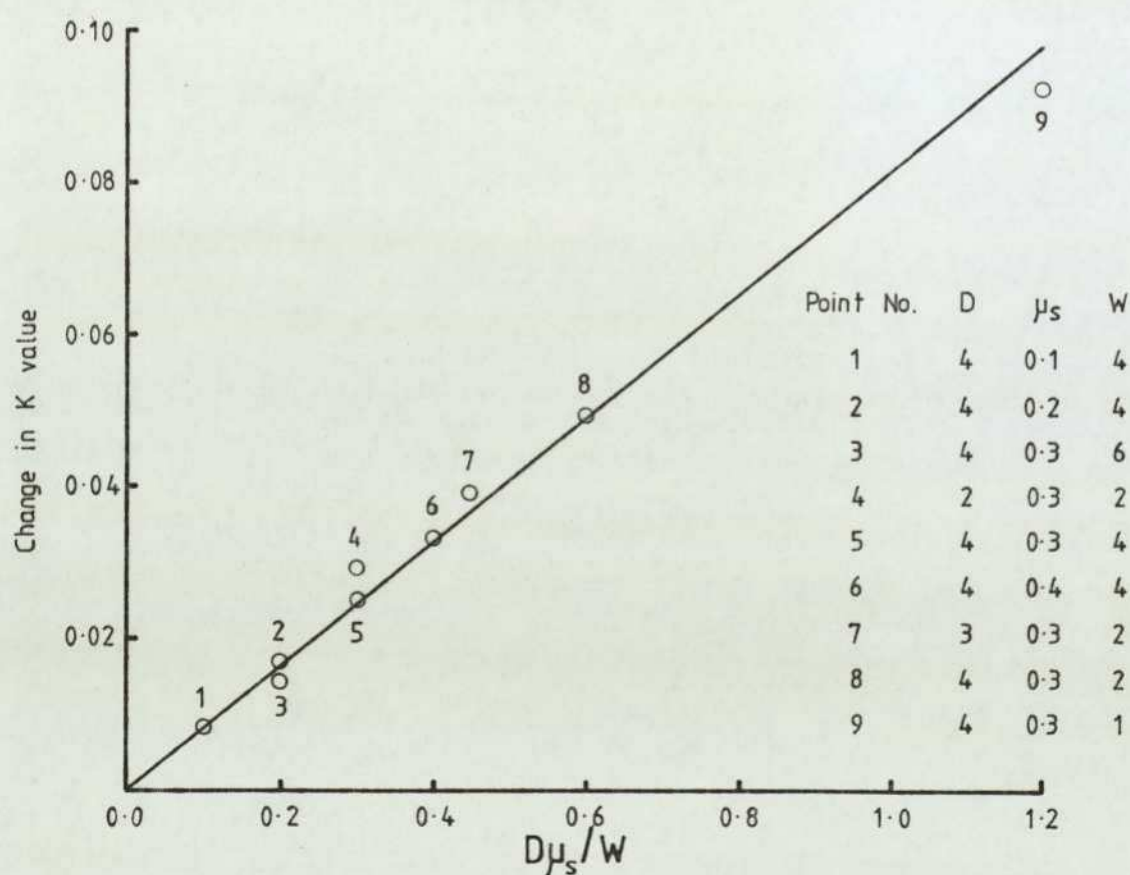


Fig. 4.22 Change in the K value vs. the side wall friction parameter.

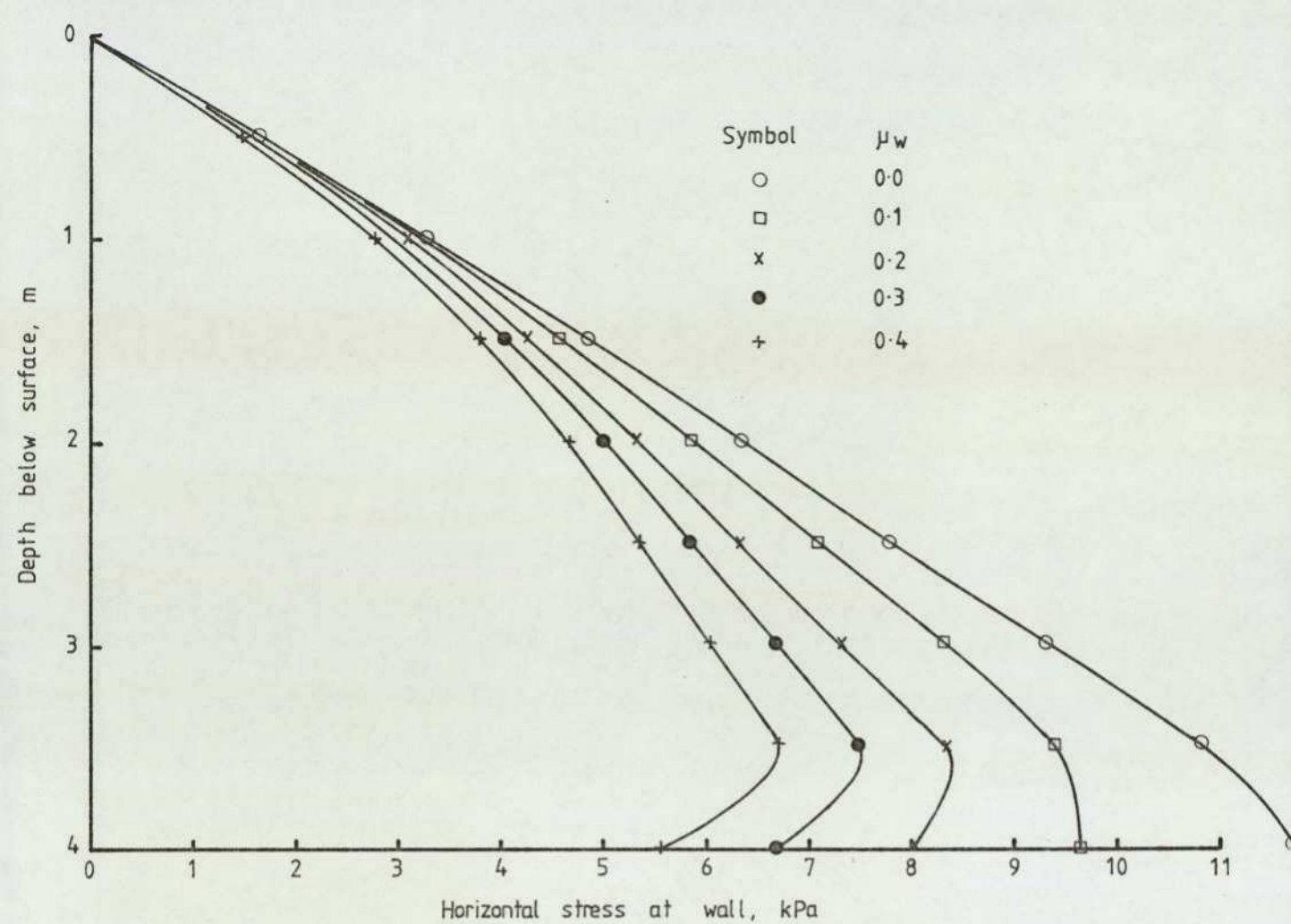


Fig. 4.23 Stress distribution at the wall of a circular section silo for various values of wall friction with diameter = 2m.

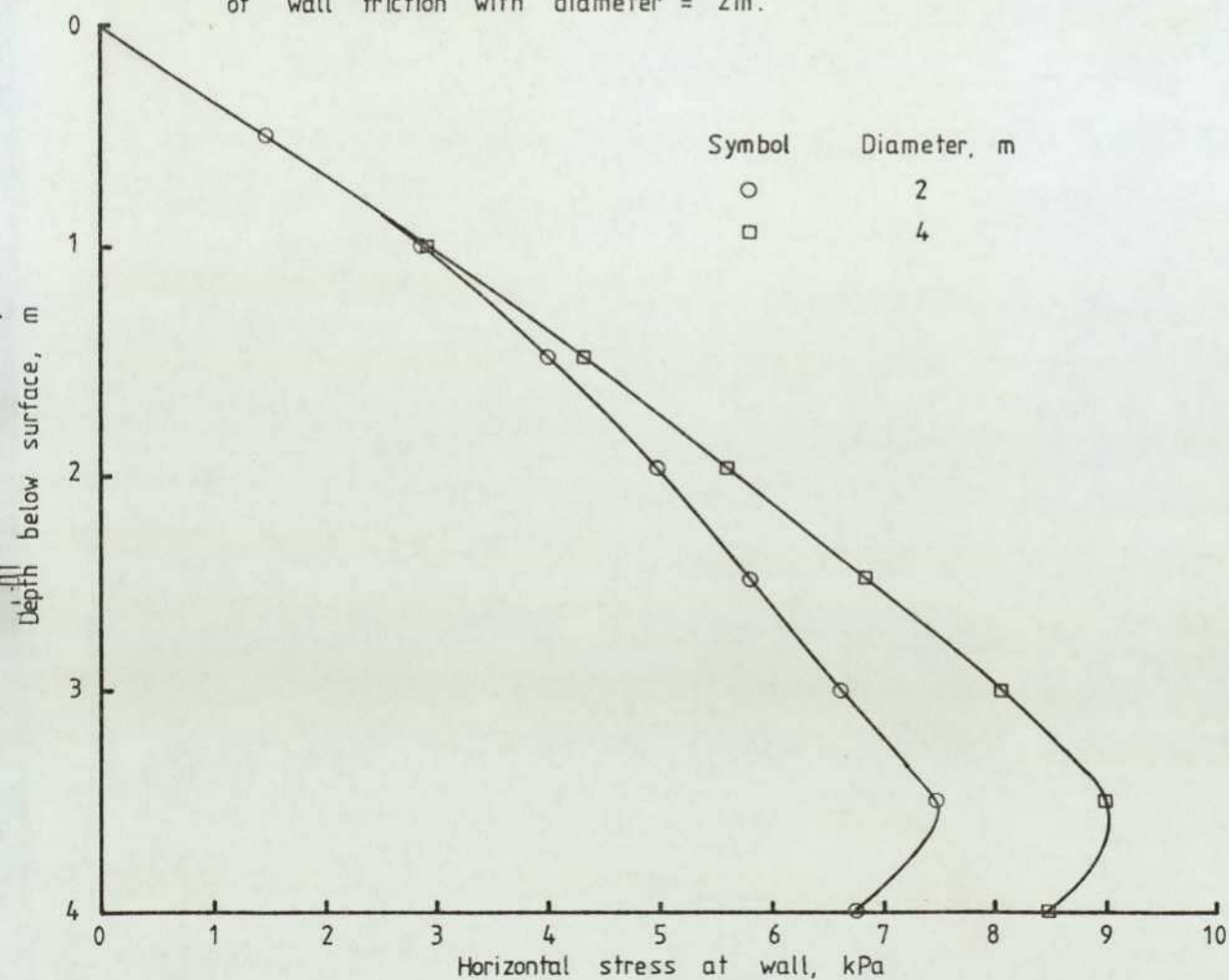


Fig. 4.24 Stress distribution at the wall of a circular section silo for two diameters with $\mu_w = 0.2$

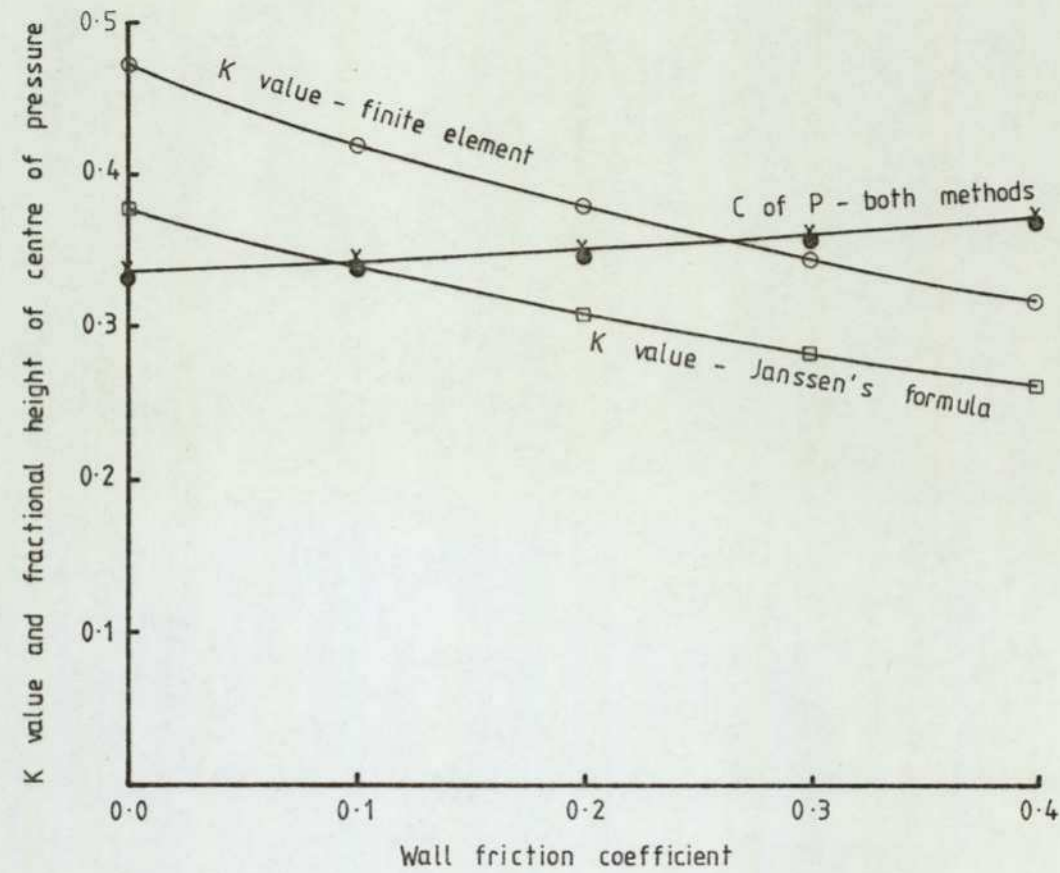


Fig. 4.25 K value and centre of pressure at various values of μ_w for a circular section silo of depth 4m and diameter 2m.

CHAPTER 5

EXPERIMENTAL EQUIPMENT FOR THE MEASUREMENT OF WALL LOADS

5.1 Design of the test silo

5.1.1 Design philosophy

The loads imposed on a silo by stored grain are expected to depend on a number of variables, among which are:-

a) The mechanical properties of the grain and the silo, e.g. the bulk density and stress/strain properties of the grain, degree and mode of flexibility of the structure, friction between the structural material and the grain.

b) The geometry of the silo, e.g. whether the retaining walls are curved or flat, the proximity of the side walls.

c) The way in which the silo is filled, e.g. the shape of the deposited heap of grain, the depth of fill.

Even if a silo could be designed so that the above quantities could be varied in any required manner, its cost would be prohibitive. However, it is felt that the design to be presented here enables the effects of the variables to be studied in sufficient detail to validate the finite element model yet still falls within the financial constraints of the project (National Institute of Agricultural Engineering, 1977). It is also likely that the structure will be used to study the loads resulting from materials having much larger particle sizes (e.g. potatoes) than the ones of immediate interest and so, while some of the measuring and handling equipment has been designed specifically for small grains, the basic design could be adapted for other materials at a later date.

5.1.2 Main wall framework

The structures laboratory of the Farm Buildings Division at N.I.A.E. has a specially designed floor that is provided with tapped sockets spaced in a 1m square grid pattern.

To suit this spacing the main retaining wall, on which force and stress measurements were taken, was made up of six identical steel framed modules each 4.5m high and 1m wide. By using different numbers of modules, the wall could be varied in width from 1m to 6m in steps of 1m (Fig. 5.1). Each module was made of heavy gauge (150 x 100 x 10mm) rectangular hollow steel members welded together and connected by two pin joints on to two posts which were welded to T-shaped sections (feet) bolted to the floor. External struts, each one serving two adjacent modules, supported the main retaining wall and they were pin jointed to sections bolted to the floor and also to the modules (Fig. 5.2). The wall could be rotated about its lower pin joint either after the silo had been filled or during filling. In the first case the struts were shortened by activating the hydraulic cylinders which formed their lower ends, and in the second case by inserting steel disk springs between the ends of the struts and the body of the hydraulic cylinder (Fig. 5.3). By replacing some or all of the springs by rigid stops, various wall compliances could be achieved. A set of stops in each of five different sizes was made which allowed for wall rotations of up to about 80mm away from the vertical when measured at the top of the wall.

5.1.3 Facing panels and stress measuring points

Each main wall module carried four 1m square timber reinforced plywood panels which were bolted to the steelwork. The 19mm birch faced plywood was reinforced on the outside (i.e. the side not in contact with the grain) by 100 x 50mm fully prepared timber sections glued and screwed to the plywood. Smaller panels were bolted below the

lowest square panel, filling in the spaces between the feet, and also above the highest square panel so completing the plywood facing of the modules.

In twelve of the square panels, two round holes were cut which enabled instruments to measure wall stress to be fitted (Sect. 5.3). Fig. 5.1 shows the centres of the holes on a vertical line at a symmetrical spacing of 500mm and so with the facing panels in this configuration the complete wall had measuring points equally spaced at 0.5m intervals vertically and 1m horizontally. As the panels were square, they could be removed from the module, rotated through 90° and replaced. In this alternative configuration the measuring points were at 0.5m intervals horizontally and 1m intervals vertically which was more suitable for the study of horizontal stress distributions. Three of the smaller bottom panels were provided with measuring points in the spaces between the feet and these points, which were below the level of the lower pin joints (Fig. 5.2, right), enabled stresses to be measured close to the floor.

5.1.4 Side walls and back wall

The side and back walls were 4.5m high and 4.5m and 6m long respectively and were bolted to the fixing points in the laboratory floor. The internal distance between the side walls could be varied between 5.7m and 0.7m in steps of 1m by moving the west side wall with a hoist which travelled along a mobile overhead gantry. There was insufficient space in the laboratory to support the walls externally and so the side walls were tied together at five points whilst the back wall was supported by internal steel ties bolted to the floor. Two of the side wall ties

were fastened at the top of the walls and so were not buried in the grain. To minimise any effect that the other three ties might have had on the stresses exerted on the main wall they were all positioned below the Rankine active slip zone (Lambe and Whitman, 1969) and chains were used instead of rigid members so that they could move to some extent with the grain. Furthermore, by using chains it was easier to change the width of the silo as, with suitable end couplings, the length of the chains could be varied easily. A plan view of the test silo is shown in Fig. 5.4.

To prevent grain spilling from the side walls when the main wall was rotated, triangular plywood fillet pieces (Fig. 5.5) were bolted to the main wall, one on each side. A 50mm wide plastic strip was screwed to the edge of each fillet piece so forming a lip type seal which was closed by the pressure of the grain acting on it.

5.1.5 Grain handling and storage

Between fillings of the test silo, the grain was stored in an adjacent holding bin and transferred between the two with augers. When the silo was less than 3m wide it was unloaded by allowing the grain to escape into a hopper via a hatch at the base of the west wall, the auger then transferring it to the holding bin. When the silo was 3m wide or more, the unloading auger was buried in the test silo during loading but, again to minimise interference with stresses on the main wall, its position was below the Rankine active slip zone.

A second auger was used to fill the test silo and was placed with its intake in the holding bin. This auger delivered grain to a horizontal conveyor that was suspended

from the overhead gantry above the test silo with its belt travelling parallel to the main wall (Fig. 5.6). Angled brushes were fitted to the conveyor which swept the grain from the belt into the silo. With this arrangement, the grain was deposited into a heap, the apex of which was a reasonably straight and horizontal ridge parallel to the main wall.

5.2 Measurement of the total forces on the main wall

5.2.1 Measuring system

The experimental measurements in this work were carried out using up to three modules on the east half of the main wall only. The compressions in the four struts supporting the east half of the main wall were measured using commercial load cells (Novatech type F204), which formed the top end of each strut. The forces at the points where each module was pin jointed to the floor were measured by strain gauging the posts on each foot section. Two sets of strain gauges (Micromasurements type EA06-250 VF-350) were applied to each post in the positions shown in Fig. 5.7 and wired in Wheatstone bridge configuration. This enabled the horizontal and vertical component of the force at each pin joint to be measured (Perry and Lissner, 1962).

Zero readings of the transducer outputs were taken before the silo was loaded and the forces at the pin joints calculated. After loading, the forces were again calculated from a further set of readings and the load due to the presence of the grain was derived by subtraction. The horizontal and vertical components of the total force on the main wall and the height of the centre of pressure were calculated by resolution of forces and taking moments.

5.2.2 Calibration technique

The load cells in the struts were supplied with calibration certificates by the manufacturers. To calibrate the transducers on the foot sections, standard load cells (Mayes type MTLC), which had previously been calibrated on the universal testing machine (W.H. Mayes Ltd) in the Engineering Research Department at N.I.A.E., were used in the device shown in Fig. 5.9. The standard load cells were set with their axes in horizontal and vertical positions and forces in the range 0-10kN were applied to the posts by rotation of the nuts on the threaded shafts.

It can be shown (Perry and Lissner, 1962) that the output voltages of the Wheatstone bridge circuits are

$$V_h = K_h [2Y_{ah} - 2Y_{ch} + 2Y_{av} - 2Y_{cv}] \quad \dots (5.1)$$

for the horizontal channel and

$$V_v = K_v [Y_{av} - Y_{bv} + Y_{cv} - Y_{dv} + Y_{ah} - Y_{bh} + Y_{ch} - Y_{dh}] \quad \dots (5.2)$$

for the vertical channel, where K_h and K_v are the sensitivities of the horizontal and vertical channels. The suffices on the strains (Y) refer to the position and the force causing the strain, i.e. Y_{ah} is the strain at position a (Fig. 5.7) caused by the horizontal force. If the expected values of the strains measured in microstrains are substituted into Eqns 5.1 and 5.2 then

$$V_h = K_h [240 + 240 + 40 - 40]$$

$$V_v = K_v [20 + 6 + 20 + 6 + 120 - 0 - 120 - 0]$$

The detector should therefore isolate each force component by adding strains caused by one component and subtracting strains caused by the other. However, during initial calibrations it was found that loads applied in a precisely

horizontal direction resulted in an output from the vertical channel as well as the expected output from the horizontal channel and vice-versa. This cross-coupling could have arisen from a variety of causes, for instance manufacturing errors in the gauges or the posts and slight misplacing of the gauges. It was especially apparent in the vertical channel because the strains caused by the horizontal force were much larger than those caused by the vertical force. For instance, if a 3% difference in nominally equal detected strains occurred due to the previously mentioned errors, the outputs would be

$$V_h = K_h[247.2 + 240 + 41.2 - 40] = 487.2 \times K_h + 1.2 \times K_h$$

$$V_v = K_v[20.6 + 6.2 + 20 + 6 + 123.6 - 0 - 120 - 0]$$

$$= 52.8 \times K_v + 3.6 \times K_v$$

In this example the vertical force caused $1.2K_h$ (0.3%) of the total horizontal output and the horizontal force caused $3.6K_v$ (6.4%) of the total vertical output. It can be seen that the cross-coupling on to the horizontal channel would be negligible while that on to the vertical channel would not. The calibration technique must therefore allow for cross-coupling.

In an ideal situation the outputs of the horizontal and vertical channels would be proportional to the horizontal and vertical forces (F_h and F_v) only, and could be represented by the matrix equation

$$\begin{bmatrix} V_h \\ V_v \end{bmatrix} = \begin{bmatrix} C_{hh} & 0 \\ 0 & C_{vv} \end{bmatrix} \begin{bmatrix} F_h \\ F_v \end{bmatrix} \quad \dots (5.3)$$

However, in practice there will be cross-coupling, a zero

offset, and possibly temperature drift. To admit the possibility of these effects, Eqn 5.3 must be modified as follows

$$\begin{bmatrix} V_h \\ V_v \end{bmatrix} = \begin{bmatrix} C_{hh} & C_{hv} \\ C_{vh} & C_{vv} \end{bmatrix} \begin{bmatrix} F_h \\ F_v \end{bmatrix} + \begin{bmatrix} Z_h \\ Z_v \end{bmatrix} + T \begin{bmatrix} t_{ch} \\ t_{cv} \end{bmatrix} \dots (5.4)$$

To investigate the effect of temperature changes the following procedure was adopted. Before the foot sections were bolted to the laboratory floor, one of them was placed in a temperature controlled cabinet. The cabinet temperature was raised to 30°C and the output voltage of each strain gauge channel was measured after allowing the foot section to reach thermal equilibrium. The cabinet temperature was then reduced to 2°C, thermal equilibrium re-established, and the outputs measured again. The change in output voltage over this temperature range did not exceed 0.1mV. As the variations of temperature in the laboratory would be much less than this between taking zero readings and readings on load (usually 5 hours maximum) and the expected full load outputs were about 10mV (horizontal channel) and 2mV (vertical channel) the temperature drift was expected to be small. This was confirmed when the foot sections were bolted to the laboratory floor and their outputs measured during the course of several days. The effect of temperature changes was therefore assumed to be negligible and the terms t_{ch} and t_{cv} in Eqn 5.4 were set to zero.

To calibrate each transducer, the following procedure was adopted. Firstly three loads 0, 4, and 10kN were applied in the horizontal direction only and the outputs of the horizontal channel V_{h0h} , V_{h4h} , V_{h10h} and the vertical channel V_{v0h} , V_{v4h} , V_{v10h} noted. The procedure was then repeated but this time applying the loads in the vertical direction

only giving outputs V_{h0v} , V_{h4v} , V_{h10v} and V_{v0v} , V_{v4v} , V_{v10v} . The Z terms of Eqn 5.4 were calculated by averaging the two sets of no load readings i.e.

$$Z_h = (V_{h0h} + V_{h0v})/2$$

$$Z_v = (V_{v0h} + V_{v0v})/2$$

The C terms were calculated by fitting least squares straight lines to the appropriate items of data. For instance, C_{hv} (the sensitivity of the horizontal channel to vertical loads) was obtained by fitting a straight line to data of the horizontal output versus the vertical load and was calculated as

$$C_{hv} = (V_{h4v} + V_{h10v} - 2V_{h0v})/-14$$

Once the matrix components in Eqn 5.4 had been determined it was possible to calculate the horizontal and vertical components of any loads applied to a post by simple matrix manipulation so that

$$\begin{bmatrix} F \\ F \end{bmatrix} = [C]^{-1} \begin{bmatrix} V_h & Z_h \\ V_v & Z_v \end{bmatrix}$$

where [C] is the matrix of C terms.

5.2.3 Results of calibrations

The outputs of each channel (channel numbering convention, Fig. 5.8) for the calibration loads are shown in Table 5.1 and the components of the C and Z matrices are shown in Table 5.2. The sign convention for the loads is shown in Fig. 5.7.

5.3 Measurement of stresses on the main wall

5.3.1 Theory

In Sect. 2.2.2 various methods of measuring the stresses imposed on a wall by stored grain were discussed. It was

Table 5.1

Outputs of transducer channels in mV at calibration loads

Channel	Horizontal loads, kN			Vertical loads, kN		
No.	0	4	10	0	-4	-10
7	0.01	-3.95	-10.07	0.01	0.10	0.10
1	0.04	0.05	0.05	0.04	0.77	1.87
8	-0.06	-4.17	-10.60	-0.06	0.01	0.16
2	-0.03	-0.11	-0.28	-0.03	0.69	1.72
9	0.01	-3.91	-9.93	0.01	0.13	0.27
3	-0.02	-0.08	-0.31	-0.02	0.77	1.79
10	-0.02	-3.69	-9.36	-0.02	0.03	0.12
4	-0.11	-0.03	-0.02	-0.11	0.73	1.82
11	0.03	-3.98	-10.32	0.03	0.09	0.12
5	-0.10	-0.02	0.01	-0.10	0.64	1.68
12	0.02	-3.79	-9.73	0.02	-0.05	-0.14
6	0.00	0.02	0.00	0.01	0.80	1.97

Table 5.2

Components of calibration matrices for each transducer

Channel	Z_h	Z_v	C_{hh}	C_{hv}	C_{vh}	C_{vv}
Nos	mV	mV	mV/kN	mV/kN	mV/kN	mV/kN
7,1	0.01	0.04	-1.003	-0.013	0.001	-0.183
8,2	-0.06	-0.03	-1.046	-0.021	-0.024	-0.176
9,3	0.01	-0.02	-0.990	-0.027	-0.025	-0.186
10,4	-0.02	-0.11	-0.929	-0.014	0.012	-0.198
11,5	0.03	-0.10	-1.026	-0.011	0.014	-0.180
12,6	0.02	0.00	-0.969	0.009	0.001	-0.196

concluded that measurements of strains in the wall structure would only give a rough indication of the stress distribution, and a better method was to use separate stress measuring gauges (sensing heads) at various positions on the wall. This section considers the theoretical aspects of the design of such devices

Required size of face plate

The area over which the wall stress is sensed will be termed the face plate. The stress acting over the area of a face plate is not continuous as it is made up of many smaller forces which are the contact forces between each grain and the face plate. Even if the stress in the region of the face plate is uniform, there will be some uncertainty in the total force on the face plate (equal to the stress times the area) due to the different possible numbers of grain contacts and the possible uneven distribution of forces between them.

To give some idea of the magnitude of this uncertainty and so to quantify the likely errors due to this cause, Kallstenius and Bergau (1956) assumed that the grains (in their case soil particles) were all spheres of equal size transmitting equal forces. They also assumed that the grains were arranged on the face plate so that those on the periphery were touching each other, forming a circle. This peripheral circle was considered to be either just on or just off the face plate and so the total range of possible forces acting on the face plate was equal to the sum of the contact forces of the grains on the circle. This error range was shown to be a function of the ratio of the diameter of the face plate to that of the grains. For an error of less than $\pm 3\%$,

Kallstenius and Bergau calculated that the ratio needed to be greater than 50 which would mean a face plate diameter of the order of 300mm for cereal grains. These calculations only put an upper bound, consistent with the assumptions of equal regular grain size and packing, on the possible error and give no indication of the probability of a given error occurring. Errors approaching the upper bound, for instance, may have little probability of occurring and it may be possible to reduce the diameter of the face plate while still keeping the likely error within reasonable bounds. A more realistic analysis, taking into account error probabilities, has been developed by the author and is given below.

If it is assumed that the grains are of equal size and, because of a regular packing, transmit equal forces to the face plate under the action of a uniform stress (as with the method of Kallstenius and Bergau) then the problem reduces to one of finding the probability of a given number of grains being in contact with the face plate. To do this, a computer program was written to simulate the placing of a disk on two types of regular array of circles, cubic and hexagonal. A "cell" was scanned by fixing the centre of the disk at regularly spaced points within the cell. A cell was defined as being the area which formed the basis of the array, that is the cell for the cubic packing was a square and that for the hexagonal packing was a 60° rhombus (Fig. 5.10), the sides of the two types of figure being equal to the diameter of the circles. The number of circles falling within the area of the disk was counted for each scanning point and, when all the points

had been scanned, the frequency of occurrence of each number of circles lying within the area was noted. In this way a probability distribution of the number of circles on the disk for each ratio of disk to circle diameter was determined. There was obviously no need to scan more than one cell as the probability distribution repeated itself from cell to cell. The mean number of circles on the disk, $\bar{n}$, was calculated from the probability distribution using the formula

$$\bar{n} = \sum_{n=0}^{\infty} np(n)$$

where $p(n)$ is the probability of n circles being on the disk. As expected, $\bar{n}$ was always equal to the disk area divided by the cell area. A measure of the fractional spread of possible numbers of circles on the disk (and so of possible forces) was the coefficient of variation

$$v = \frac{1}{\bar{n}} \left[\sum_{n=0}^{\infty} (n - \bar{n})^2 p(n) \right]^{1/2}$$

which was determined from the probability distribution as a function of D/d .

In practice the grains will not be spherical and so some equivalent grain diameter must be defined. In the method of Matthies and Petersen (1974) measurements were taken of the bulk density, ρ , and the thousand grain mass, m , to derive the number of grains per unit volume, N , as

$$N = \frac{1000\rho}{m} \quad \dots (5.5)$$

An equivalent diameter, d_{eq} , was then calculated equal to the diameter of the spheres packed in a regular way which had the same number in a unit volume as the grains. For example, a cubic packing has a solid fraction of 0.524 and so the number of spheres per unit volume is

$$N = 0.524 \div (\pi d^3/6)$$

The equivalent diameter is therefore given by

$$d_{eq} = N^{-1/3} \quad \dots (5.6)$$

If a hexagonal packing is assumed (solid fraction 0.740)

$$d_{eq} = 1.123N^{-1/3} \quad \dots (5.7)$$

At first sight it would appear that V depends on the way in which the spheres are assumed to be packed as the probability distribution and the equivalent grain diameter both depend on the packing. This dependence is however very small as the methods of determining the probability and the equivalent diameter are mutually compensating. That is, if the packing is assumed to be more dense then the cell area becomes smaller for the same diameter of sphere so giving a narrower probability distribution. However, the number of grains per unit volume must stay constant, as it is determined from measurements of m and ρ and does not depend on the assumed packing. d_{eq} must therefore be larger with the more dense packing to keep the same number of grains in a unit volume. Thus the potentially smaller variation is offset by the increase in d_{eq} leaving V substantially constant. Fig. 5.11 illustrates this by plotting V against the dimensionless quantity $N^{1/3}D$ which is a function only of the measurable quantities m , ρ , and D . It can be seen that the one curve closely fits the data from the two types of packing. This shows that if the bulk grain is represented by a regular packing of spheres, V depends only on the measurable properties of the bulk and not on the way it is assumed to be packed.

Fig. 5.11 also shows that more variation in the possible forces on the face plate is likely as $N^{1/3}D$ (which, substituting for $N^{1/3}$ from Eqn 5.6 or Eqn 5.7, is proportional to D/d)

decreases as predicted by Kallstenius and Bergau. Several measures of likely error are possible, retaining the $\pm 3\%$ figure from Kallstenius and Bergau, V is equal to this when $N^{1/3}D = 8$ or using another criterion, assuming the measurements are normally distributed, 99% of them will be between $\pm 3\%$ of the mean value when $N^{1/3}D = 14$. It was decided that a convenient size for the face plate was about 100mm diameter and the final size as designed was 106mm. This should satisfy the latter criterion for values of N as low as 2.3×10^6 per cubic metre (equivalent diameters up to 7.6mm for cubic packing).

Allowable deflection of the face plate

If the face plate of a sensing head retreats even slightly under the action of the stress it is attempting to measure, then the local strains caused by the deflection will reduce the stress in the region of the face plate and a stress lower than the true one will be measured. Kallstenius and Bergau (1956) attempted to quantify the sensitivity of the measured stress to face plate deflection. They assumed that the retained material was linearly elastic and derived the following equation for the error in measured stress

$$\frac{\Delta\sigma}{\sigma_t} = \frac{4\delta E v^2}{\pi D \sigma_t (1 - v^2)}$$

Where $\Delta\sigma$ is the error in stress, σ_t is the true stress, δ is the deflection and D the diameter of the face plate, E is the Young's modulus and v the Poisson's ratio of the elastic material. Although their analysis was an extremely idealized one, especially as the material was assumed to be linearly elastic, two important points are apparent:-

- a) The fractional error in measured stress depends

on the actual value of the stress.

b) The error also depends on the stress/strain properties of the retained material.

These points reinforce the arguments in Sect. 2.2.2 that recommendations for the allowable deflections of a face plate in soils should be treated with caution. Not only are the stress/strain properties of the bulk grain different from soils (Chapter 3), but the stress levels are also likely to be very different. For instance, a maximum wall stress of about 16kPa is envisaged in this work whereas the Waterways Experiment Station cells (W.E.S., 1944) were calibrated in the range 140 - 690kPa. Obviously the sensitivity of the measured stress to face plate deflection will need to be re-examined for agricultural grain but, for the present, the design of the stress measuring equipment will be based on the assumption that the deflection should be at least as small as that specified by the W.E.S. With the present face plate diameter of 106mm, the specified diameter to deflection ratio (1000) gives a maximum deflection of 106 μ m.

5.3.2 Design of stress measuring equipment

Sensing heads

A schematic diagram of a sensing head fitted into a wall facing panel is shown in Fig. 5.12. A plywood faced steel plate fits into a hole cut into the timber panel which is reinforced with a steel ring. This face plate is supported on a shaft suspended from a carrier by two beryllium copper suspensions. The rear end of the shaft terminates in a stud which passes through a hole in a locking bridge which, when locked, prevents rearwards movement of the face plate. The horizontal stress exerted by the grain

is measured by first screwing a force transducer onto the rear of the carrier, its sensitive point just touching the stud. The bridge is then unlocked and is pushed backwards by the internal springs so transferring the load from the bridge on to the sensitive point of the transducer. The exact technique and the transducer will be described in later sections. Fig. 5.13 shows a front (grain side) view of the main wall where the face plates can be seen.

Force transducer framework

The position of the stud with respect to the carrier will not be precisely the same for each sensing head and so a means of adjustment is provided by carrying the force transducer in a framework on runners (Fig. 5.14) the ends of which are threaded so that they can be joined with nuts to the sensing head. The other ends of the runners are fixed to the back plate of the framework into which is fitted a micrometer head. When the micrometer head is rotated the force transducer is pushed along the runners against the resistance of two springs which minimise any backlash in the system.

Force transducer

The requirements of the force transducer are listed below:-

- a) The transducer must be simple and robust as it may be necessary in the future to use the device on a farm where sophisticated test or repair equipment is not available.
- b) The transducer must be accurate, free from hysteresis, and must maintain its calibration and accuracy over long periods.
- c) The effect of environmental changes, especially

temperature, on calibration and zero offset must be small.

d) The output of the device must be high enough so that effects due to "noise" are negligible. The supply and output devices must also be as simple as possible for the same reason as given in a).

e) The deflection of the transducer under load must be very small.

To satisfy these requirements, a completely new type of transducer has been designed that is powered by compressed air. The new transducer, shown diagrammatically in Fig. 5.15, is circular in cross section and symmetrical about the centre line. The movable part A is held in position by two thin beryllium copper suspensions, B, screwed into the fixed part C. Compressed air is supplied from a regulator (Norgren type 11-913-110) into two flow control valves (Air Automation type FC1) each of which supplies one annular air chamber, D. The air escapes radially, outwards and inwards, between the ground faces of the fixed and movable parts and the difference in pressure between the two annular chambers is measured at the tapings E. As most of the pressure drop in the system occurs at the flow control valves, small changes in the width of the gaps between the fixed and movable parts have little effect on the rate of airflow. A horizontal force applied as shown in Fig. 5.15 will increase the size of the left-hand gap and reduce the size of the right-hand one. Since the flow rate to each side is essentially constant, the pressure at the left-hand tapping will decrease while that at the right-hand one will increase. The pressure difference is a measure of the applied force and can be calibrated against a known load.

The pressure distribution for the radial airflow system of one side of the transducer is shown in Fig. 5.16. The applied force, F , is balanced by the air pressure in the gap between the two parts and is given by

$$F = \pi P_0 (R_3^2 - R_2^2) + 2\pi \left(\int_{R_1}^{R_2} p_1 r dr + \int_{R_3}^{R_4} p_2 r dr \right) \dots (5.8)$$

where P_0 is the pressure above atmospheric in the annular chamber. To determine the pressure distributions p_1 and p_2 (Fig. 5.16) it will be assumed that the air is incompressible, its physical properties are independent of pressure, and velocity pressures are negligible compared with static pressures. If the volume rate of flow is Q , then the mean velocity at any radius, r , is

$$V_m = \frac{Q}{2\pi r h}$$

The flow rates between the gaps of the transducer are such that the flow is laminar (the highest Reynolds number is about 300 at no load), giving a parabolic variation of velocity in a direction normal to the flow. The velocity gradient at the walls and normal to them is

$$\frac{dv}{dn} = \frac{6V_m}{h} = \frac{3Q}{\pi r h^2}$$

giving the shear stress at the wall as

$$\tau = \frac{3Q\mu}{\pi r h^2}$$

where μ is the absolute viscosity of the air. Considering the balance of forces on an annular element of radius r , width dr , and thickness h , the following differential equation is obtained

$$p + r \frac{dp}{dr} = \frac{-6\mu Q}{\pi h^3}$$

Solving and substituting the boundary conditions $p = P_0$ at $r = R_2$ and $p = 0$ at $r = R_1$ (for p_1) and $p = P_0$ at $r = R_3$ and $p = 0$ at $r = R_4$ (for p_2) leads to expressions for the

pressure distributions

$$p_1 = \frac{P_0 R_2}{R_2 - R_1} \left(1 - \frac{R_1}{r} \right) \quad \dots (5.9)$$

$$p_2 = \frac{P_0 R_3}{R_4 - R_3} \left(\frac{R_4}{r} - 1 \right) \quad \dots (5.10)$$

and for the flow rates

$$Q_1 = \frac{\pi P_0 R_2 h^3}{6\mu(R_2 - R_1)} \quad \dots (5.11)$$

$$Q_2 = \frac{\pi P_0 R_3 h^3}{6\mu(R_4 - R_3)} \quad \dots (5.12)$$

where Q_1 is the flow rate via the inner gap and Q_2 is the flow rate via the outer gap.

Substituting for p_1 and p_2 from Eqns 5.9 and 5.10 into Eqn 5.8 gives

$$F = \pi P_0 (R_3 R_4 - R_2 R_1) \quad \dots (5.13)$$

The transducer of Fig. 5.15 is equivalent to two of the systems of Fig. 5.16 connected together. The applied force F_t is balanced by the difference between the forces at the two gaps. Denoting each side by the suffices a and b and substituting into Eqn 5.13 gives

$$F_t = \pi(P_{0a} - P_{0b})(R_3 R_4 - R_2 R_1) \quad \dots (5.14)$$

This shows that the relationship between the applied force and the differential pressure depends only on four of the physical dimensions of the device and so the calibration should be independent of environmental changes (neglecting the change in radii with thermal expansion) and of the supply pressure and flow rate.

The relationship between force and gap size for the

single gap system of Fig. 5.16 is obtained by combining Eqns 5.11 and 5.13 to give

$$F = \frac{6\mu Q_1(R_2 - R_1)(R_3R_4 - R_2R_1)}{R_2h^3}$$

In the transducer, a small increase in size, Δh , of one gap is accompanied by an equal decrease in the size of the other and so the applied force is given by

$$F_t = \frac{6\mu Q_1(R_2 - R_1)(R_3R_4 - R_2R_1)\{(h_0 - \Delta h)^{-3} - (h_0 + \Delta h)^{-3}\}}{R_2}$$

where h_0 is the gap size at no load. Because Δh is small compared with h_0 , Taylor's theorem can be used to derive the spring rate as

$$\frac{F_t}{\Delta h} = \frac{36\mu Q (R_2 - R_1)(R_3R_4 - R_2R_1)}{R_2h_0^4} \dots (5.15)$$

The critical dimensions in the final design of the transducer were:-

$$\begin{aligned} R_1 &= 6\text{mm} \\ R_2 &= 9\text{mm} \\ R_3 &= 19\text{mm} \\ R_4 &= 25\text{mm} \\ h_0 &= 37\mu\text{m} \end{aligned}$$

and the flow control valves were adjusted so that P_0 was equal to 40kPa with a supply pressure (upstream of the flow control valves) of 200kPa. These dimensions resulted in a theoretical air consumption at no load of $0.72 \times 10^{-3} \text{ m}^3/\text{s}$, a sensitivity of 0.76kPa/N, and a spring rate at no load of 8.6N/ μm .

A transducer has been designed therefore which is simple and robust and is also capable of using equally simple supply and output devices (e.g. a compressed air

cylinder can be used as a supply and a mercury manometer as the output device). In theory the calibration of the device is not sensitive at all to environmental changes, or even changes in supply pressure, and its spring rate is very high and should result in a deflection of only about $11\mu\text{m}$ at the full load of 100N. Hence the transducer should satisfy the requirements listed at the start of this section.

5.3.3 Tests on stress measuring equipment

Calibration of force transducer

The transducer was calibrated with its axis vertical and its sensitive point uppermost using the arrangement shown in Fig. 5.17. In this position the weight of the movable part of the transducer contributes to the total load and so the movable part and the beryllium copper suspensions were weighed separately before the transducer was assembled. The weight of the suspensions was considered to be shared between the fixed and the movable parts and so the extra load was calculated as the sum of the weight of the movable part and half the weight of the suspensions. The weight of the hanger was adjusted by adding lead wire so that the total load on the transducer in the vertical position without any added weights was exactly 1kg. The transducer was placed inside a temperature controlled cabinet at 15.8°C and, after allowing about 2 hours for equilibrium to be established, the device was zeroed with its axis horizontal. It was then placed in a vertical position and loading started. The load was increased in steps of 1kg up to a total of 14kg and then reduced, again in steps of 1kg. The transducer was also loaded at two other

temperatures, 28.5°C and 2.7°C , after allowing about 2 hours for equilibrium to be established at each temperature.

Separate calibrations were carried out to determine the effect of changing the supply pressure. Three sets of loadings were performed in the range 1-10kg, all at 16.5°C , with supply pressures of 150, 200, and 250kPa. In these tests the flow control valves, which were merely needle valves, were left in their standard positions and so the air flow rate was considerably different for each test. The results of these calibrations are given in Tables 5.3 and 5.4.

Table 5.3 shows that the hysteresis was negligible - only once was there any difference between the outputs for increasing loads and those for decreasing loads. This difference was only 0.1kPa or about 0.1% of the maximum output. The effect of changing the temperatures was also very small, the slope of a straight line fitted to the data changed by only 0.14% for a temperature change of 26°C and the intercept of the line changed by 0.6% of the maximum output. The close correspondence of the outputs with the fitted straight line shows that the calibration was extremely linear, the maximum deviation from the calculated values being 0.4% of the maximum output.

It can be seen from Table 5.4 that the calibration was also little affected by the supply pressure and hence the airflow rate - a change of $\pm 25\%$ of the nominal supply pressure caused a change in the slope of the fitted line of 0.7% and a change in the intercept of 0.9% of the maximum output.

The insensitivity of the calibration to temperature

Table 5.3

Output of force transducer (kPa) for calibrations at three temperatures, T, with supply pressure 200kPa. Key to columns:- I, output for increasing load; D, output for decreasing load; I_c and D_c , calculated output from a least squares straight line fit for increasing and decreasing loads respectively.

Total load kg	T = 15.8°C				T = 28.5°C		T = 2.7°C	
	I	D	I_c	D_c	I	I_c	I	I_c
1	7.2	7.2	7.4	7.4	7.2	7.5	7.7	8.0
2	13.9	14.0	14.2	14.2	14.0	14.2	14.5	14.8
3	20.9	20.9	21.0	21.0	20.8	21.0	21.4	21.6
4	27.8	27.8	27.8	27.8	27.8	27.8	28.4	28.4
5	34.6	34.6	34.6	34.6	34.6	34.6	35.3	35.2
6	41.6	41.6	41.4	41.4	41.6	41.3	42.3	42.0
7	48.4	48.4	48.1	48.1	48.4	48.1	49.1	48.8
8	55.2	55.2	54.9	54.9	55.2	54.9	55.9	55.6
9	62.0	62.0	61.7	61.7	61.9	61.7	62.7	62.3
10	68.7	68.7	68.5	68.5	68.6	68.4	69.4	69.1
11	75.3	75.3	75.3	75.3	75.3	75.2	76.1	75.9
12	82.0	82.0	82.1	82.1	81.9	82.0	82.7	82.7
13	88.6	88.6	88.8	88.8	88.6	88.8	89.2	89.5
14	95.2	95.2	95.6	95.6	95.1	95.5	95.6	96.3
Slope of least squares fit, kPa/N	0.692 0.692				0.692		0.692	
Intercept, kPa	0.7 0.7				0.7		1.3	
Correlation coefficient	0.999964				0.999965		0.999933	

Table 5.4

Output of force transducer (kPa) for calibrations at three supply pressures, P, at a temperature of 16.5°C. The columns are labelled as in Table 5.3.

Total load kg	P = 200kPa		P = 150kPa		P = 250kPa	
	I	I _c	I	I _c	I	I _c
1	7.2	7.3	7.0	7.1	7.6	7.7
2	14.0	14.1	13.9	14.0	14.4	14.4
3	20.9	20.9	20.8	20.8	21.2	21.2
4	27.9	27.8	27.7	27.6	28.1	28.0
5	34.7	34.6	34.5	34.5	34.8	34.8
6	41.6	41.4	41.5	41.3	41.7	41.6
7	48.2	48.3	48.3	48.2	48.5	48.4
8	55.2	55.1	55.0	55.0	55.2	55.2
9	61.9	61.9	61.8	61.8	62.0	62.0
10	68.6	68.8	68.5	68.7	68.6	68.8
Slope of least squares fit, kPa/N	0.696		0.697		0.692	
Intercept, kPa	0.4		0.3		0.9	
Correlation coefficient	0.999985		0.999986		0.999992	

and supply pressure as predicted in Sect. 5.3.2 has therefore been confirmed. It is also interesting to compare the measured sensitivity, about 0.70 kPa/N, with the predicted value, 0.76kPa/N. The correspondence is reasonably close despite the assumption of Sect. 5.3.2 that the air was incompressible.

Deflection of the force transducer and framework

The deflection of the transducer plus that of its framework was measured by mounting the framework on a heavy gauge (20mm) steel plate instead of on a sensing head. The transducer was moved along its runners by rotating the micrometer head until contact was established with the plate when the differential pressure was noted. The micrometer head was then screwed further in and readings of the differential pressure taken at various positions of the micrometer head. As the steel plate was assumed to be perfectly rigid, the deflection of the transducer and its framework was equal to the change in micrometer reading. The deflection/differential pressure characteristic (Fig. 5.18) was found to be slightly curved with a maximum deflection of 25 μ m at a differential pressure (70kPa) corresponding to the nominal 100N full load, giving a diameter to deflection ratio of 4000 at this load. This is only a quarter of the maximum deflection recommended by W.E.S. for soils and so well within the limit set out in Sect. 5.3.1 for the initial studies.

Size of the face plate

The spread of measured forces will in practice be greater than those calculated in Sect. 5.1.1 because the grains are not spherical and are packed in a random way.

Although the size and shape of each grain is usually very similar, there will be some variation that will tend not only to vary the number of grains in contact with the face plate but also to vary the individual contact forces.

The extent of the spread of forces on a face plate was determined by fitting a sensing head of the same design as that used on the main wall into a tank (Fig. 5.19) which consisted of two chambers each 12.7mm wide and separated by a 450 μ m thick latex membrane. With the tank in a horizontal position, seeds were loaded into the lower chamber and then the two chambers were screwed together. Air pressure was then applied to the upper chamber, thus exerting a pressure on the seeds via the latex membrane, and the complete tank rotated into a vertical position. The pressure of the seeds on the wall of the tank, which was assumed to be equal to the air pressure on the other side of the membrane was measured using the force transducer.

Three types of seeds were used, rapeseed, wheat, and tick beans. Runs were carried out for each type of seed starting with the air pressure at 10kPa, increasing to 16kPa, reducing to 4kPa, and then increasing again to 10kPa. Force measurements were taken at each pressure step and a straight line, passing through the origin, was fitted to the data of air pressure in the upper chamber vs force acting on the face plate. Twelve runs were carried out for each type of seed and the seeds were re-packed between runs. As a control, twelve runs were carried out using table salt for which it was assumed that errors due to particle size could be neglected. The bulk densities of the three types of seed were measured in a standard bushel weight apparatus

(W.T. Avery Ltd) and the one thousand seed masses were measured by weighing several lots of one hundred seeds and averaging. Eqn 5.5 was used to calculate the numbers of seeds per unit volume, N , and the value of the function $N^{1/3}D$ was determined for each type of seed. The random errors due to effects other than the seed size were removed using the data from the runs with table salt.

The experimentally measured coefficients of variation are shown in Fig. 5.11. Although the observed variations are in the correct relation to each other, they are about 2.5 times the theoretical ones which were derived assuming regular packings of spheres. This shows that the non-spherical seed shape and irregular packing have a significant effect on the spread of measured forces. However, if it is assumed again that the variations are normally distributed, then for rapeseed more than 99% of all readings will be within $\pm 3\%$ of the mean while the corresponding values for wheat and tick beans are 95% and 67% lying within this range. The performance was therefore felt to be satisfactory for seeds up to about the size of tick beans.

Movements in the sensing head

Initial calculations showed that elastic strains in the components of the sensing head would lead to a negligible displacement of the face plate at the likely maximum loads. However, the locking bridge is unlocked and re-locked several times when pressure readings are taken during the course of an experiment and so the face plate may not return precisely to its original position. To measure the possible variations in position, a dial guage with a resolution of $2\mu\text{m}$ was clamped on to the rear flange of the carrier of the sensing

head on the tank of Fig. 5.19 to measure the movement of the stud. The zero reading of the dial guage was noted and the air pressure increased to 10kPa after which the bridge was repeatedly locked and unlocked. The readings of the dial guage after locking did not change by more than $3\mu\text{m}$ from the zero reading.

Effect of face plate deflection on measured stress

It was mentioned in Sect. 5.3.1 that, until more information was available, the stress measuring equipment would be designed with a lower limit of 1000 on the ratio of the face plate diameter to the deflection. The equipment reported in Sect. 5.3.2 achieved a diameter to deflection ratio of four times this figure. However, the amount by which the face plate deflection affects the measured stress is still not known and so tests were carried out to quantify this effect.

The test silo was filled with barley to a depth of 4.5m. The force transducer and framework were screwed on to a sensing head and the transducer advanced, by rotating the micrometer head, so that it was just in contact with the stud. This point was easily determined by a sudden increase in the output of the transducer. The bridge was then unlocked, the output noted, and the bridge re-locked. A second measurement was then taken by advancing the transducer with the bridge locked until the output was equal to the first reading. The bridge was then unlocked. This pre-stressing of the transducer resulted in a much smaller deflection than would otherwise have occurred because the change in applied force when the bridge was unlocked for the second time was much smaller. With the load now taken on

the transducer, the micrometer was used to move the transducer outwards in $2\mu\text{m}$ steps, then back in past the start position and finally out to the start position again. The output of the transducer was noted at each step. The maximum outward movement was set approximately equal to that which would have occurred when the bridge was unlocked for the first time, while the maximum inward movement was set at about 30% of this figure. This procedure was repeated with the force transducer screwed on to the other sensing heads in turn. To calculate the movement of the face plate with respect to the start position, the micrometer movement was corrected by adding to it the slight deflection of the transducer itself. This deflection was caused by the change in force from the start position to the position under consideration and was obtained from the known differential pressure/deflection relationship (Fig. 5.18).

A representative sample of the relationships between face plate deflection and the measured stress is shown in Fig. 5.20. It can be seen that the relationships are all well represented by straight lines and show little hysteresis, and that the slopes of the lines increase as the measured stress at the start position increases. The slope of each line is approximately proportional to this measured stress which means that the error in measured stress due to movement can be expressed as a given percentage of the actual stress, per unit of movement. The figure is about 1.5% error per micron of movement and if a deflection is taken equal to the upper limit of the W.E.S. criterion (i.e. $106\mu\text{m}$ for this diameter of face plate), the resulting error will be 159%. Of course, an error of greater than 100% would

not be possible and it is to be expected that the rate of fall in measured stress with deflection would gradually become less but even so, this emphasises the point that a criterion for soils cannot be used for materials that have different mechanical properties and are under different stress conditions. The high value of error per unit deflection also means that the stresses measured with the device described here would also be in error if some means were not devised to correct for the deflection.

5.3.4 Measurement technique

Fortunately, because the relationships plotted in Fig. 5.20 are linear, it is possible to devise a technique by which the sensitivity of the stress to face plate deflection is measured at the same time as the stress itself. Because the deflection for any force is known (Fig. 5.18), this makes it possible to correct the measured stress for the deflection. As each characteristic is a straight line, it can be determined uniquely by the coordinates of any two points on it. Once these have been identified, the force at zero deflection can be calculated by coordinate geometry. Referring to Fig. 5.21 this is done as follows:-

a) The micrometer head is rotated until the transducer is just in contact with the stud so giving a small output, F_0 (about 3kPa).

b) The bridge is unlocked which changes the load on the transducer so that the output becomes F_1 . This force change results in a deflection of the transducer and hence a deflection of the face plate of $\delta(F_1) - \delta(F_0)$. $\delta(F_1)$ and $\delta(F_0)$ are the deflections at the respective forces which are obtained from Fig. 5.18. Point 1 has therefore

coordinates of $(\delta(F_1) - \delta(F_0), F_1)$.

c) The micrometer head is used to advance the transducer by a set amount, ℓ , ($10\mu\text{m}$) and the new force reading, F_2 , noted. The face plate is thus advanced by a distance equal to ℓ minus the deflection of the transducer which is due to the change in force from F_1 to F_2 , that is $\ell - \delta(F_2) - \delta(F_1)$. The coordinates of point 2 are thus $(\delta(F_2) - \delta(F_0) - \ell, F_2)$.

d) The bridge is locked and the transducer withdrawn. The transducer and framework can now be unscrewed from the sensing head and, if necessary, moved to the next one.

e) The force which would exist if no movement had occurred (point 3) is calculated from the coordinates of the known points as

$$F_t = F_1 + \frac{(F_2 - F_1)(\delta(F_1) - \delta(F_0))}{\ell - \delta(F_2) + \delta(F_1)}$$

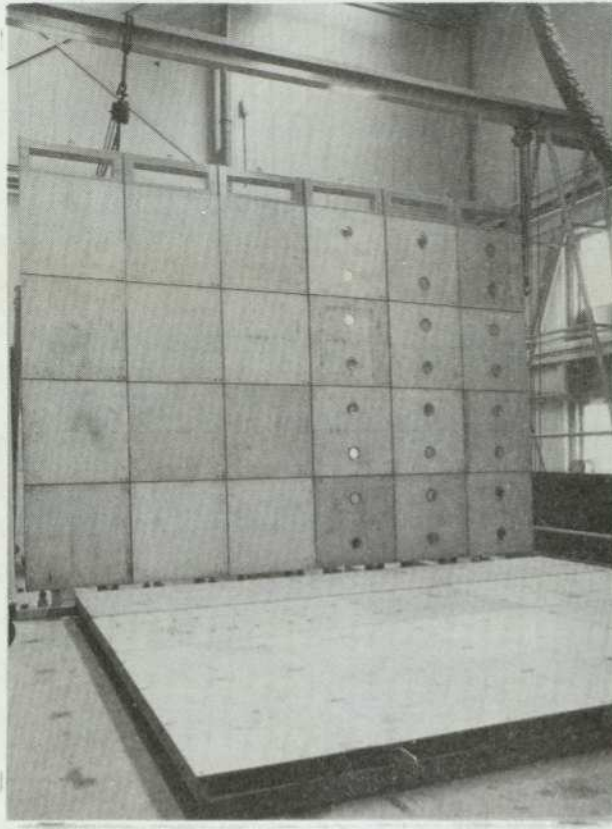


Fig. 5.1. Main wall of test silo.

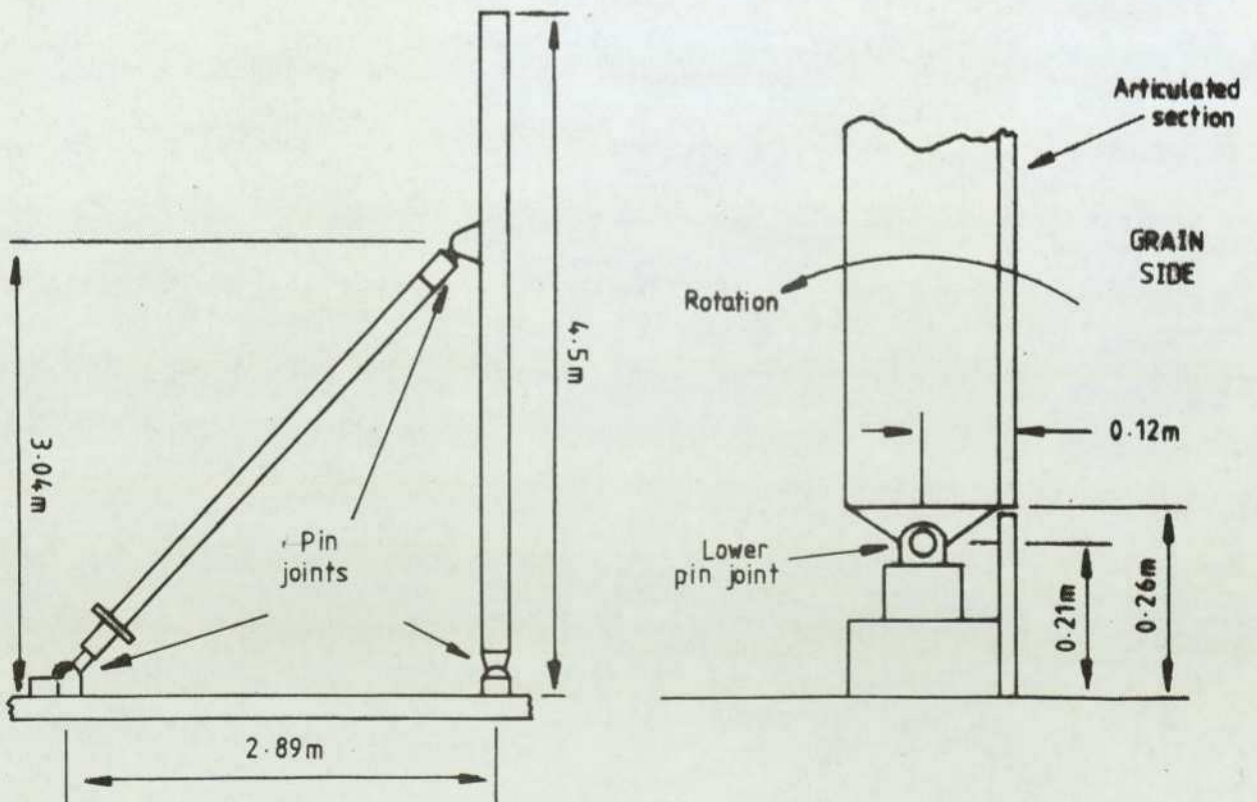


Fig. 5.2. Left, arrangement of struts and positions of pin joints. Right, detail of bottom of main wall.

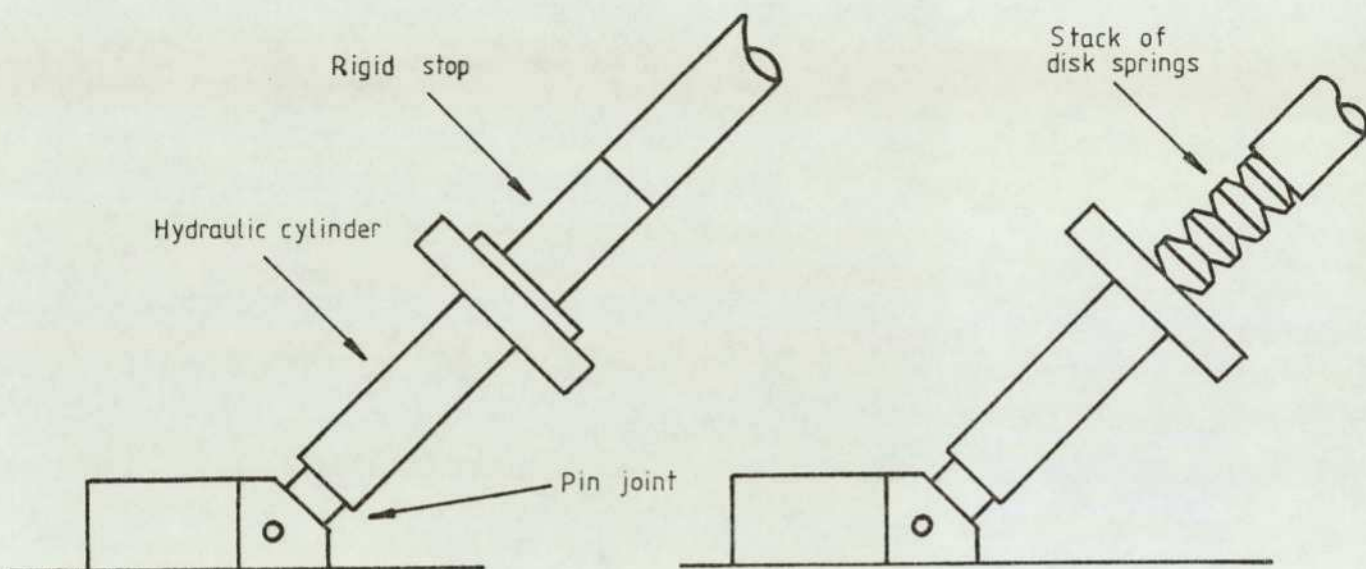


Fig. 5.3. Lower ends of struts showing alternative methods of rotating the main wall. Left, rigid stops which are removed to rotate the wall after filling. Right, springs which allow the wall to rotate during filling.

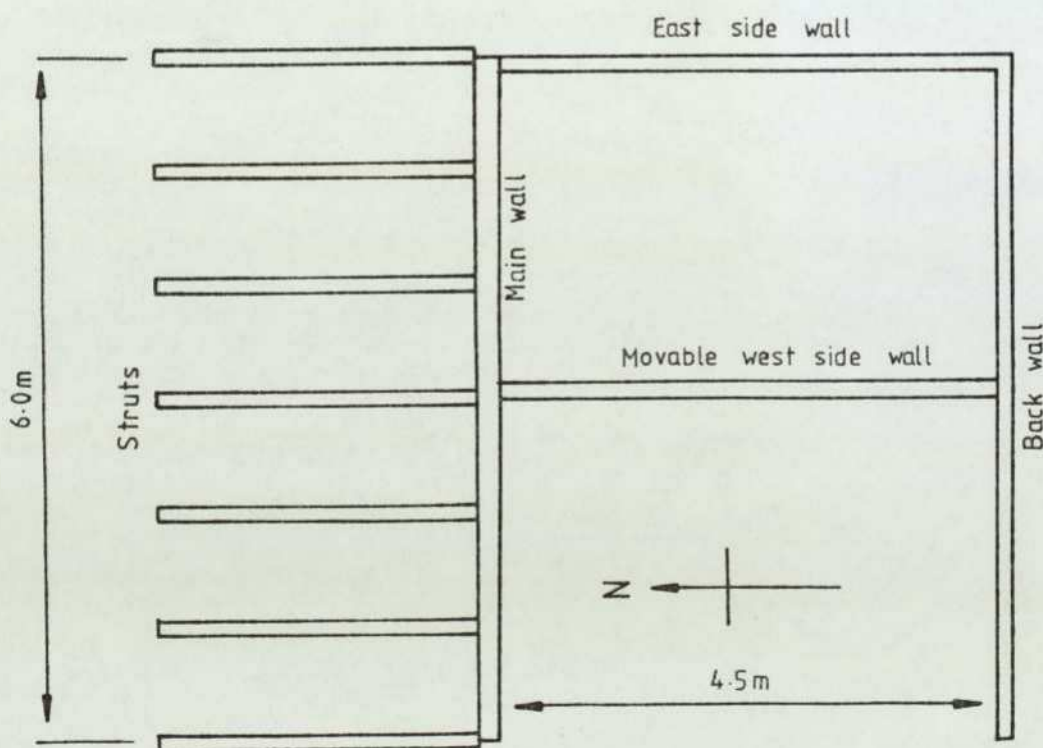


Fig. 5.4. Plan view of test silo.

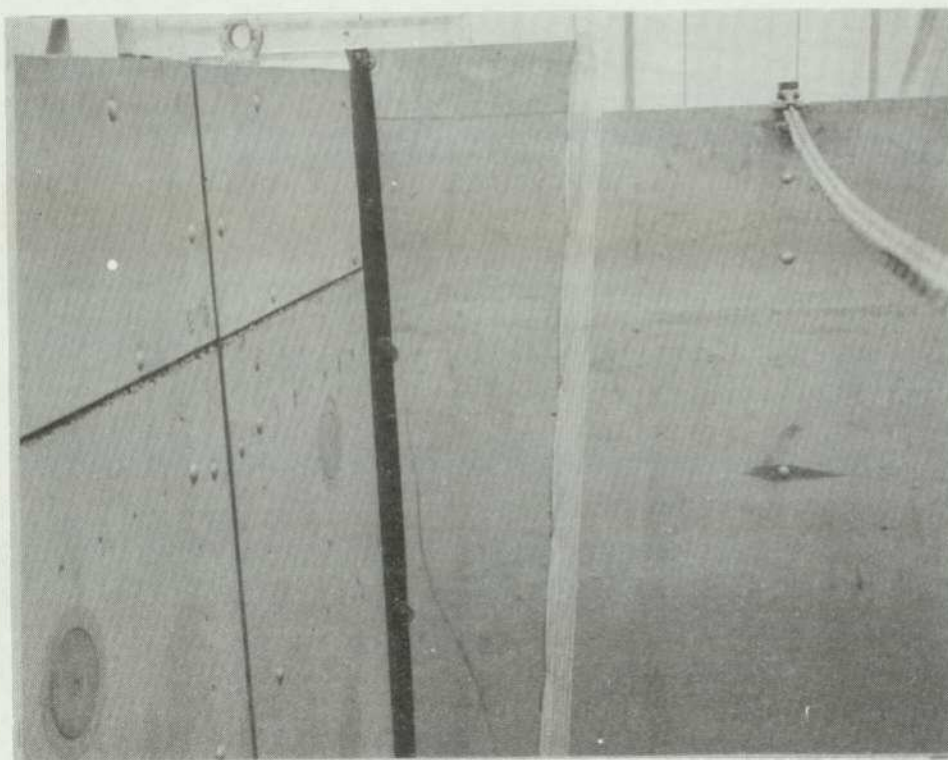


Fig. 5.5. Plywood fillet piece.

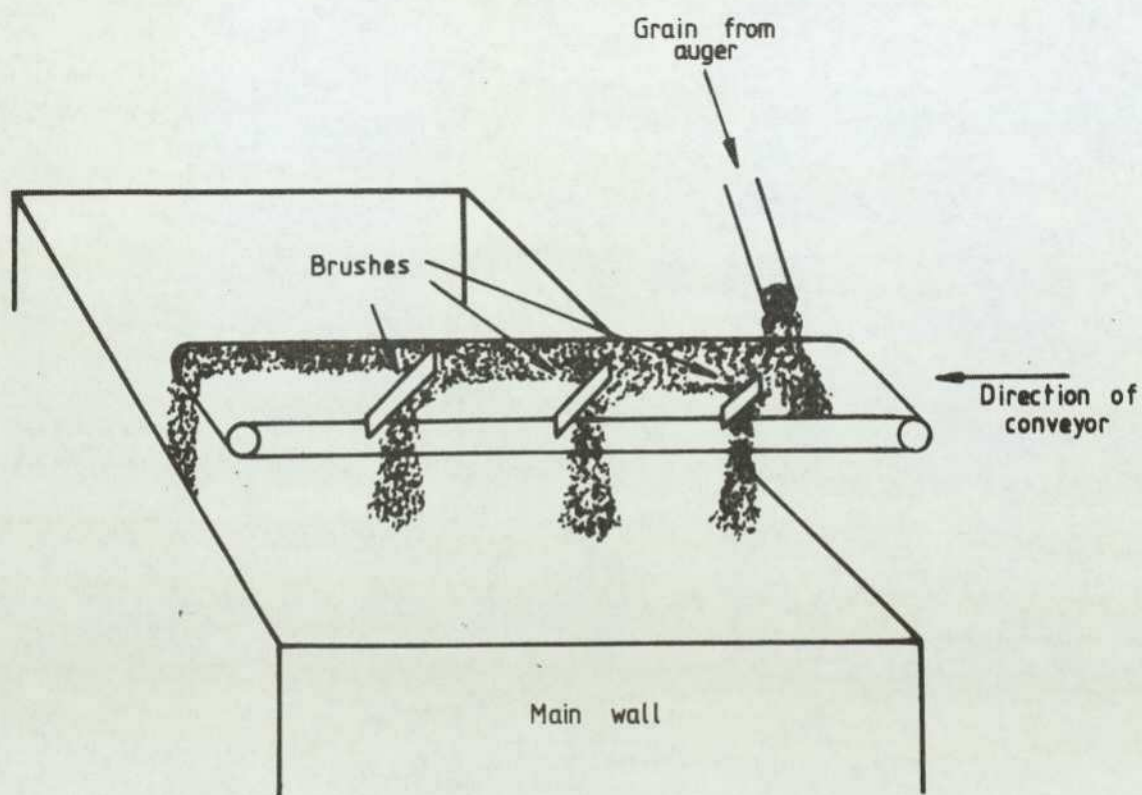


Fig. 5.6. Schematic diagram of grain handling system.

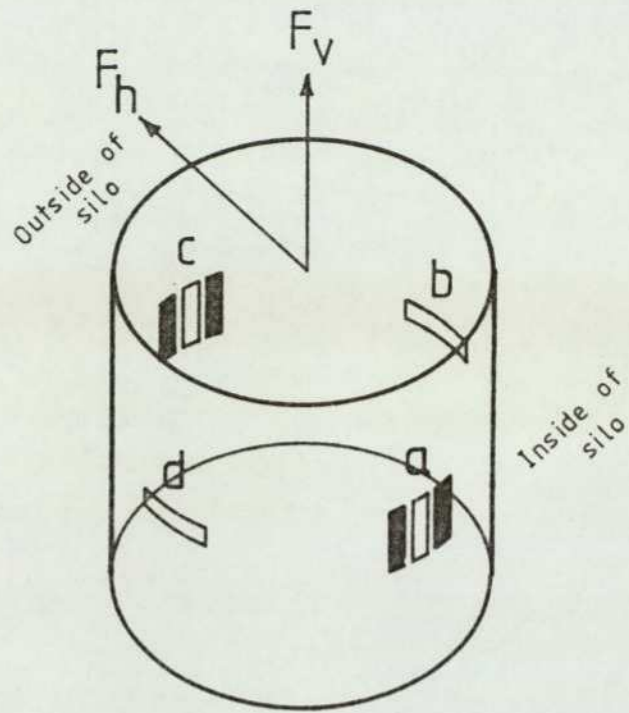


Fig. 5.7 . Positions of strain gauges on a post. □ Vertical channel.
 ■ Horizontal channel.

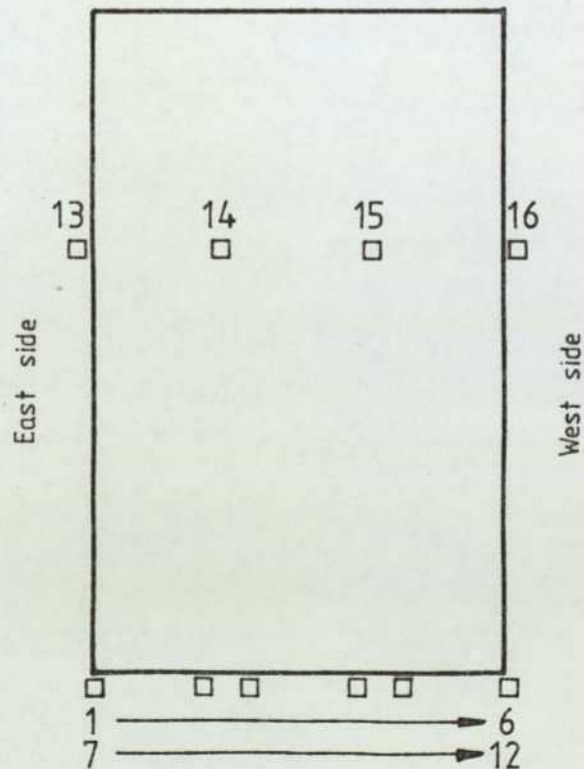


Fig. 5.8 . Rear view of main wall showing channel numbers of overall force transducers. (Channels 1 to 6, vertical forces; Channels 7 to 12, horizontal forces.)

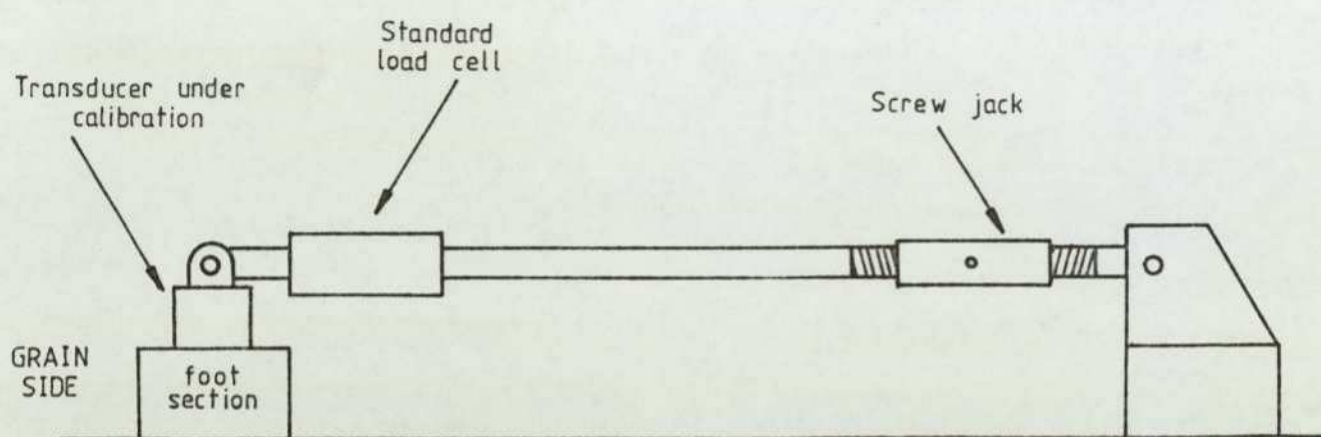
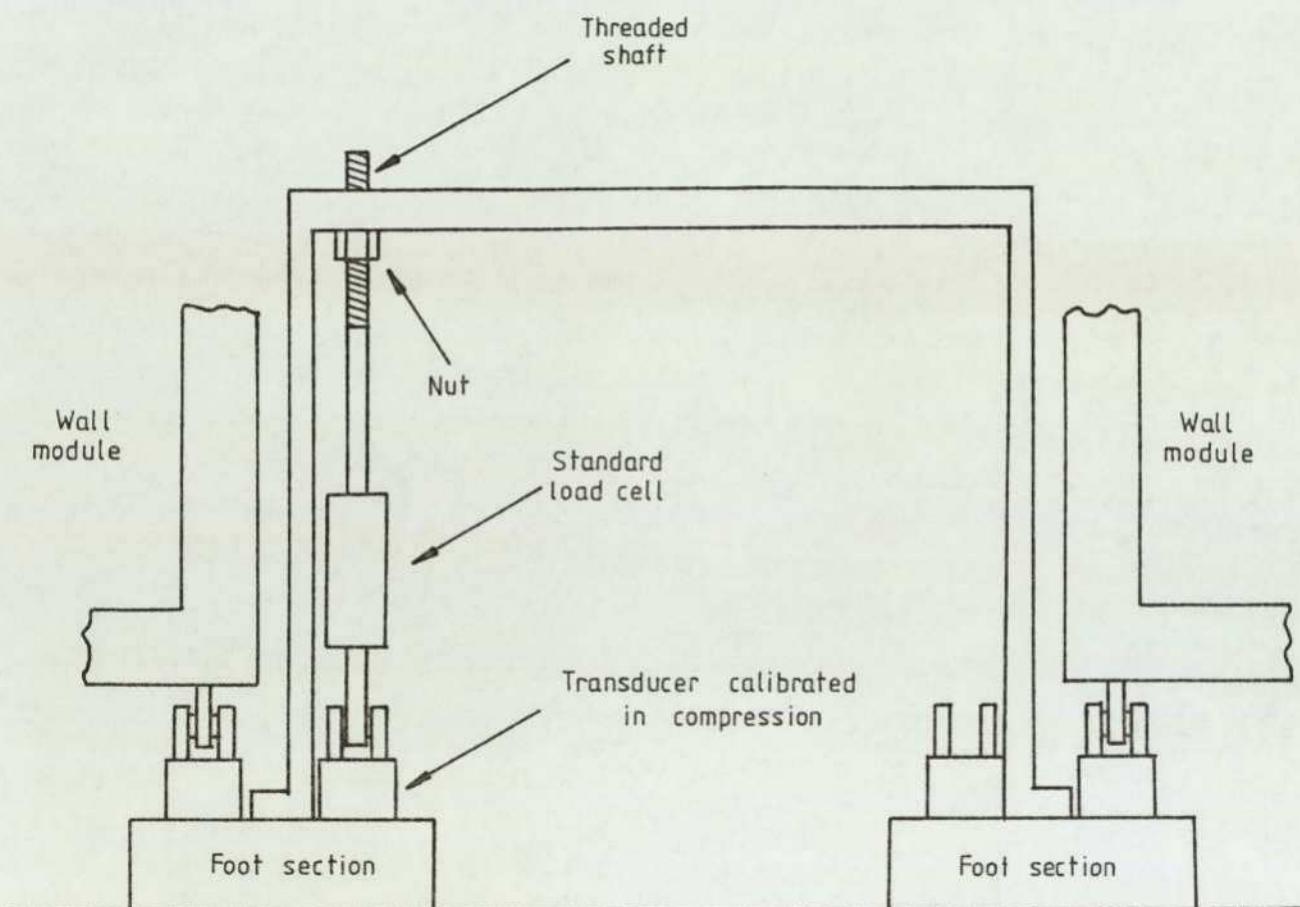


Fig. 5-9. Equipment for calibrating transducers on foot sections. Top, vertical calibration. Bottom, horizontal calibration.

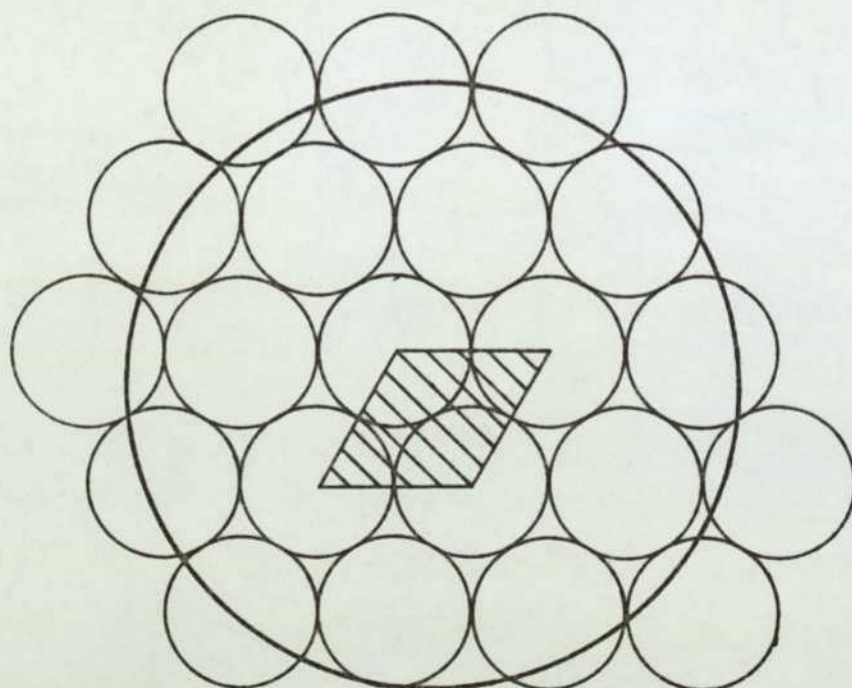
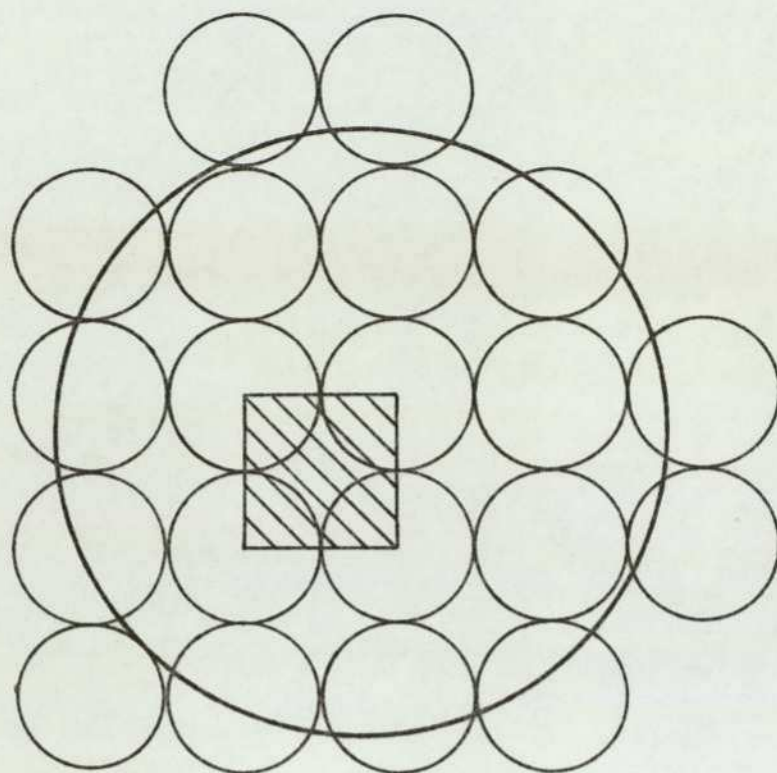


Fig. 5-10. Disk placed on two types of regular array of circles. Top, cubic array; Bottom, hexagonal array. Unit cell area shown hatched.

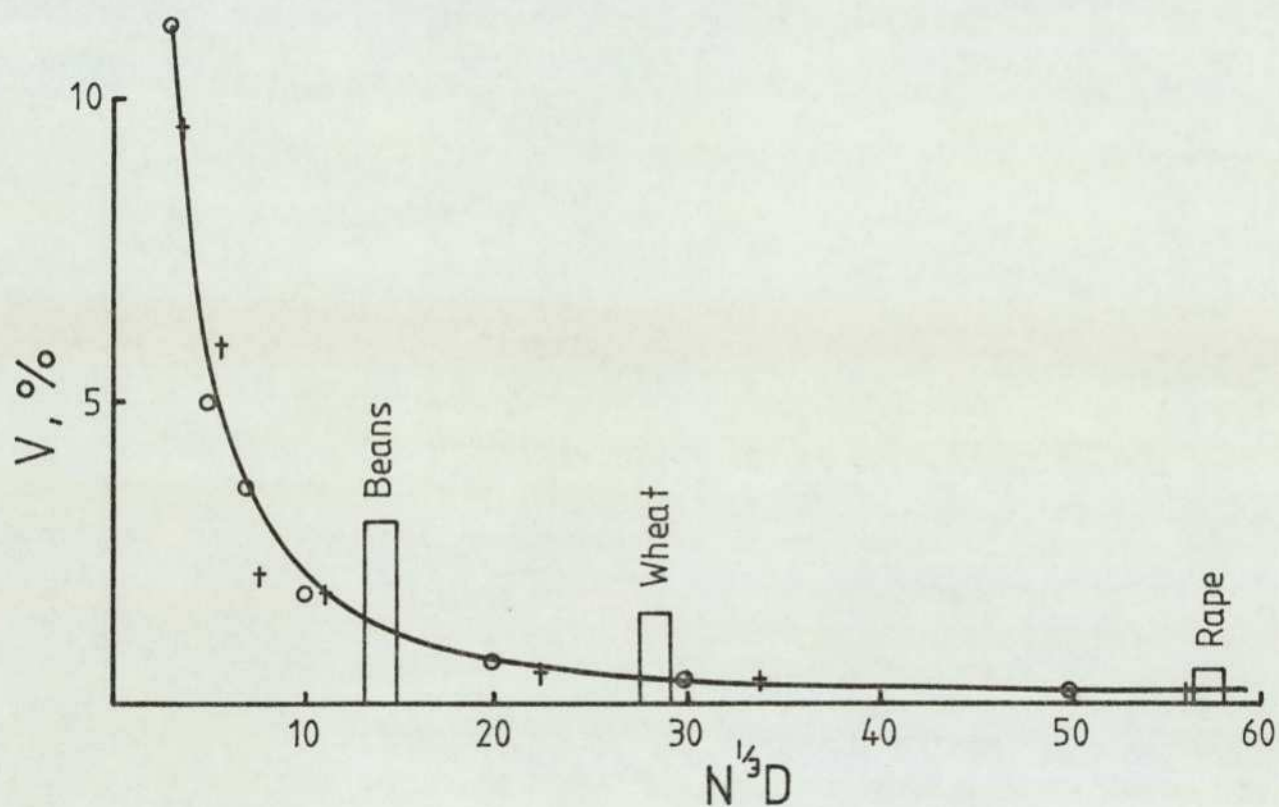


Fig.5.11. Theoretical coefficient of variation, V , vs. $N^{1/3}D$. o, Cubic array; +, Hexagonal array. Vertical bars show practical results.

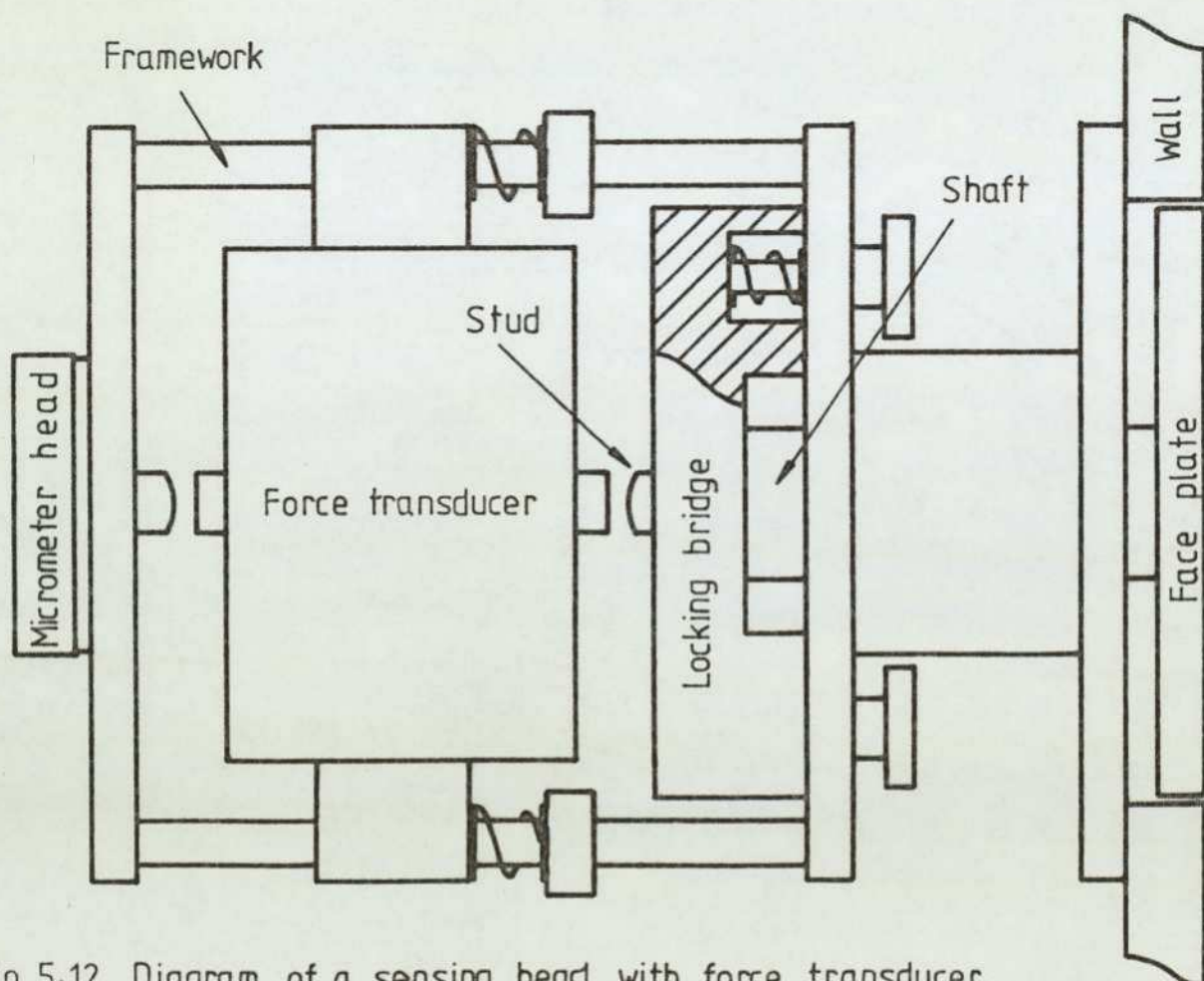


Fig.5.12. Diagram of a sensing head with force transducer attached.

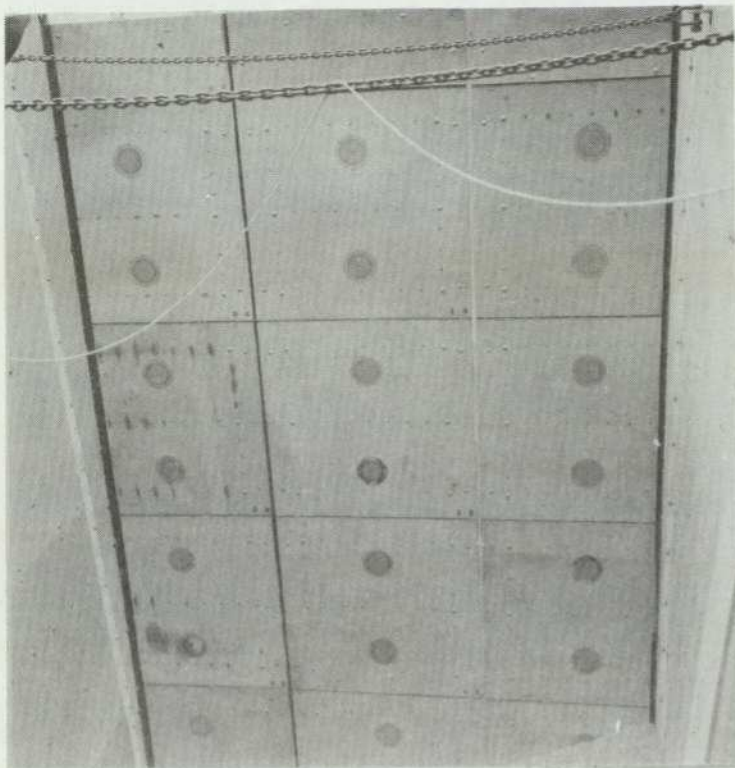


Fig. 5.13. Front view of main wall showing face plates.

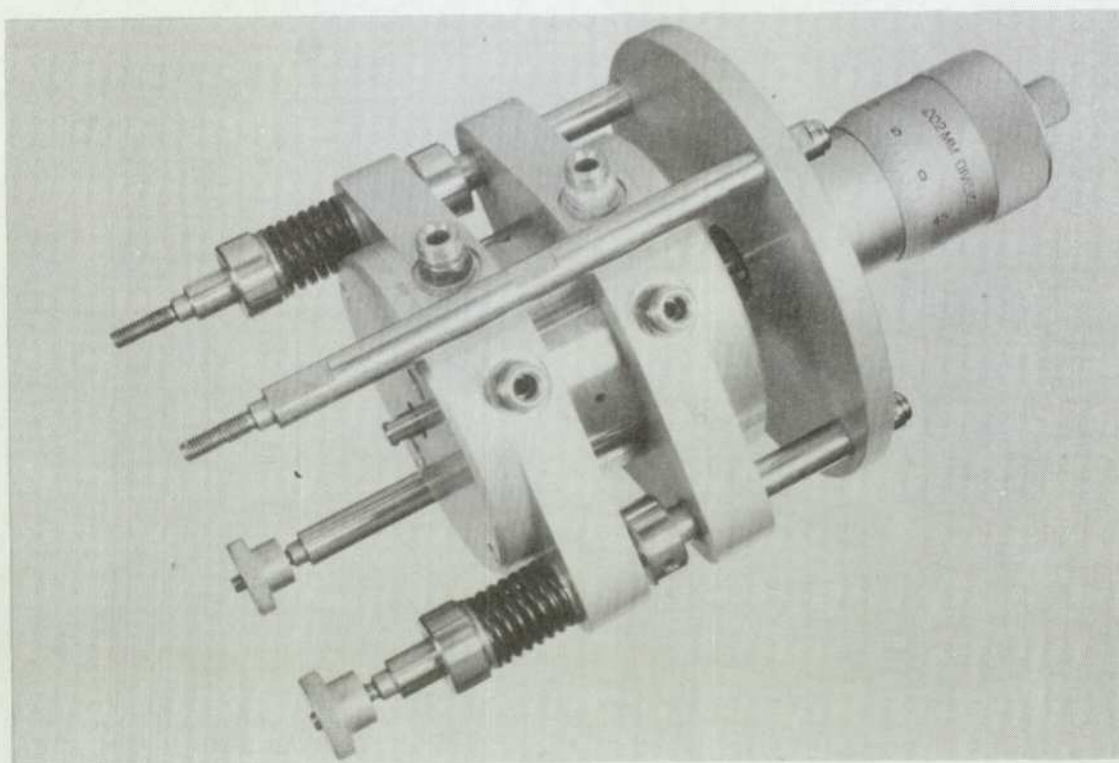


Fig. 5.14. Force transducer on framework.

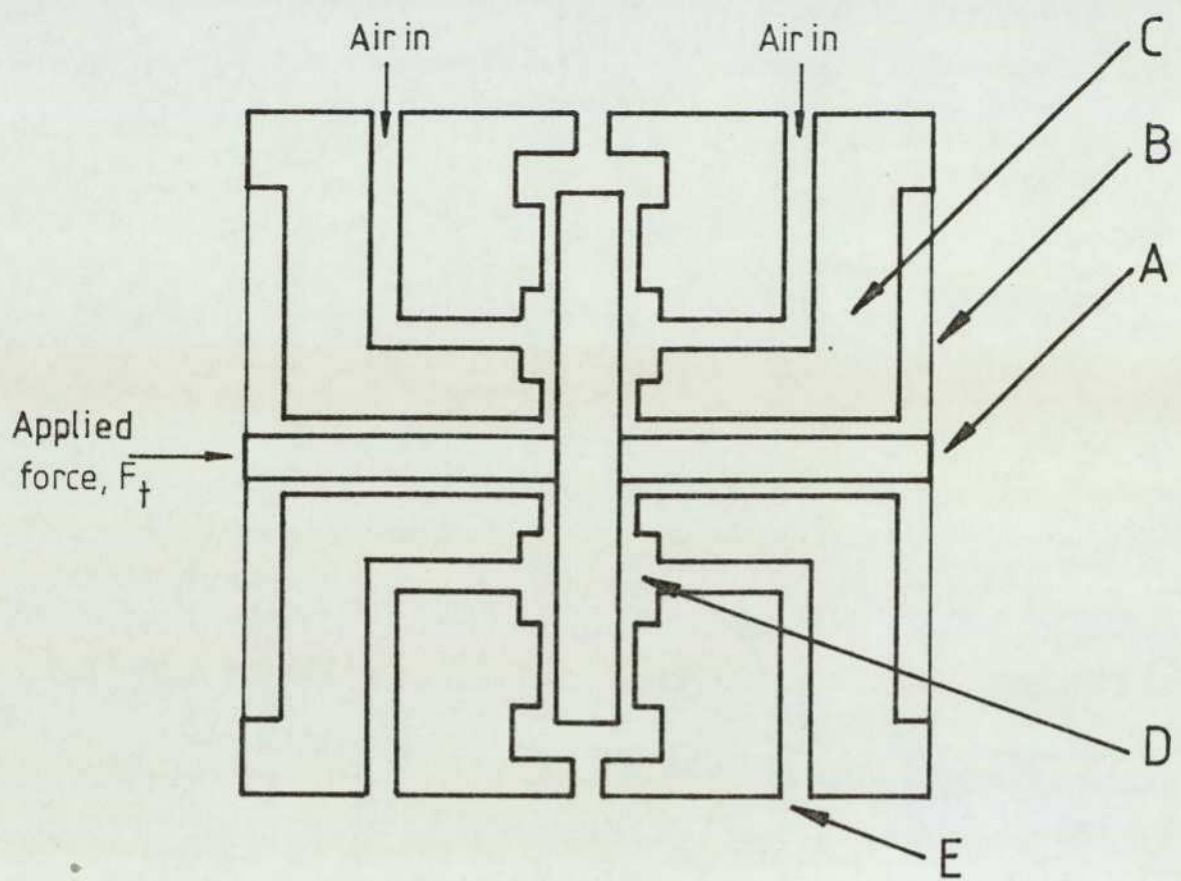


Fig. 5.15. Diagram of a cross section through the force transducer.

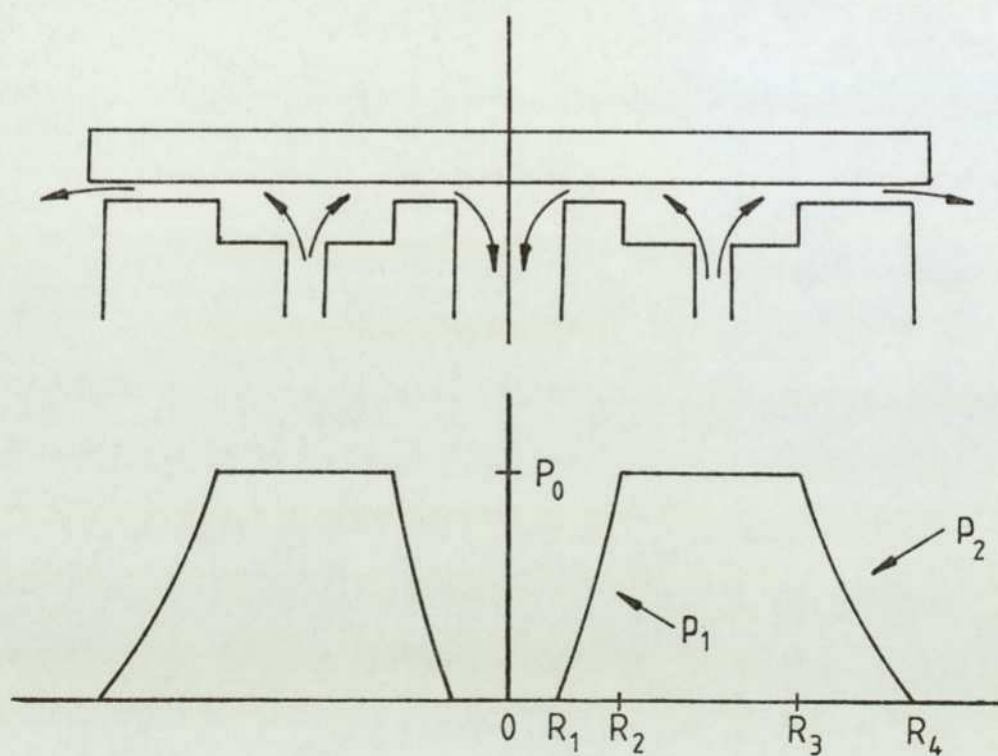


Fig. 5.16. Radial air flow system and pressure distribution (above atmospheric) for one side of the transducer.

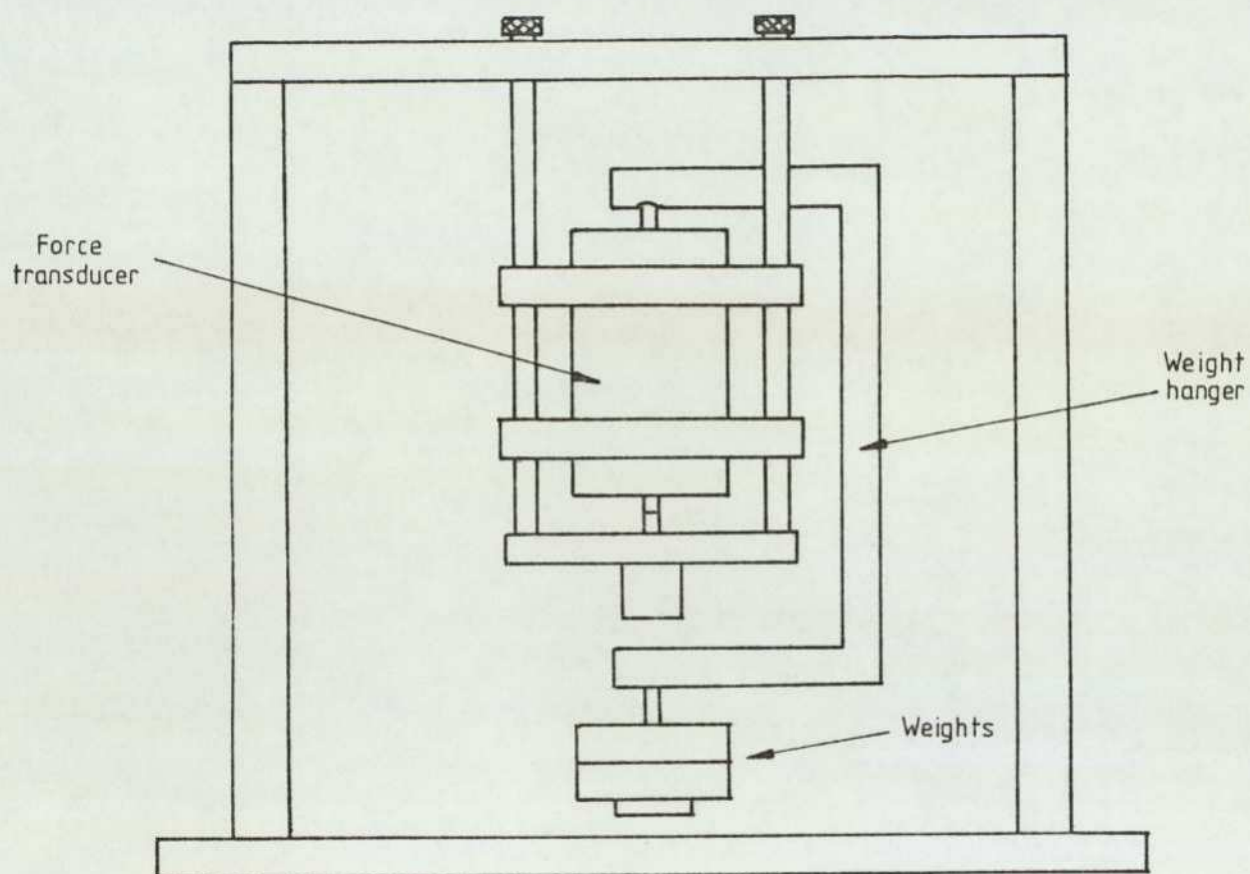


Fig. 5-17. Method of calibrating the force transducer.

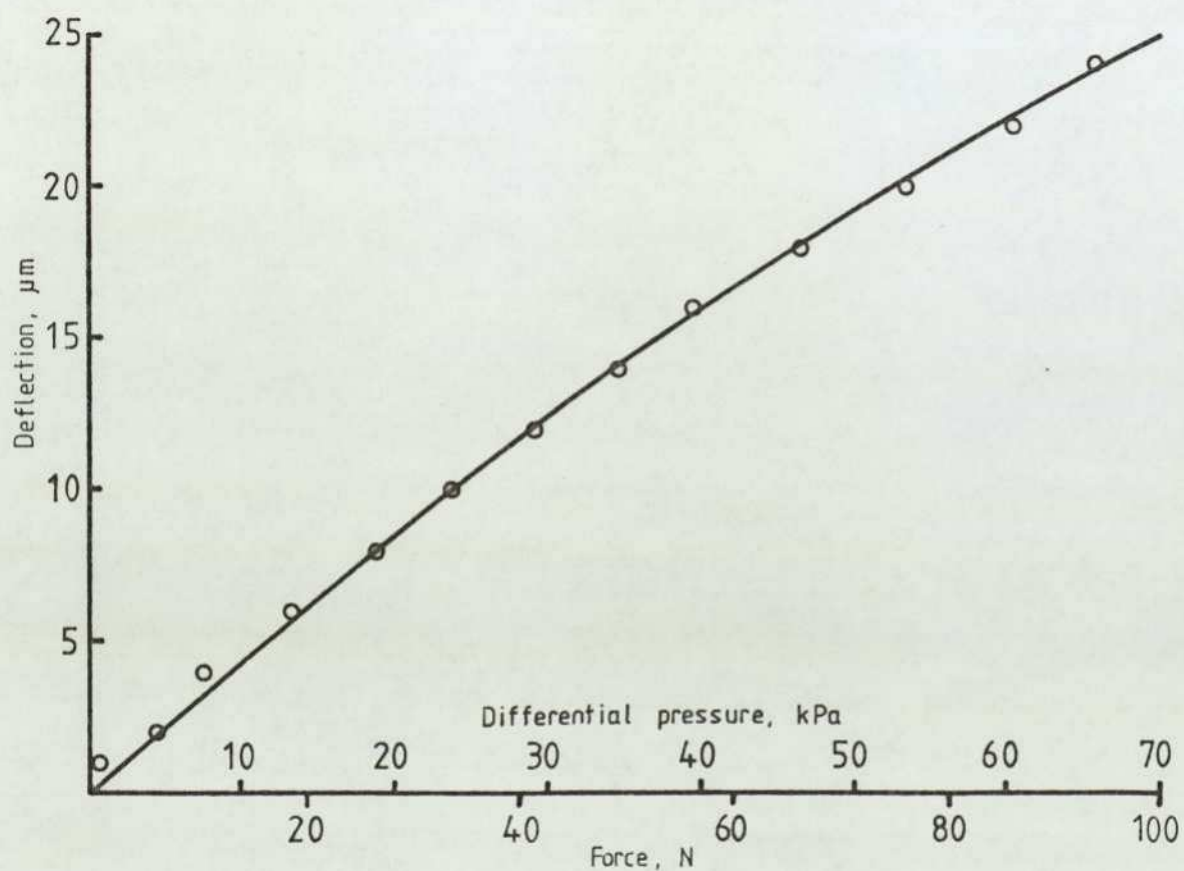


Fig. 5-18 Deflection vs. force and output for the transducer.

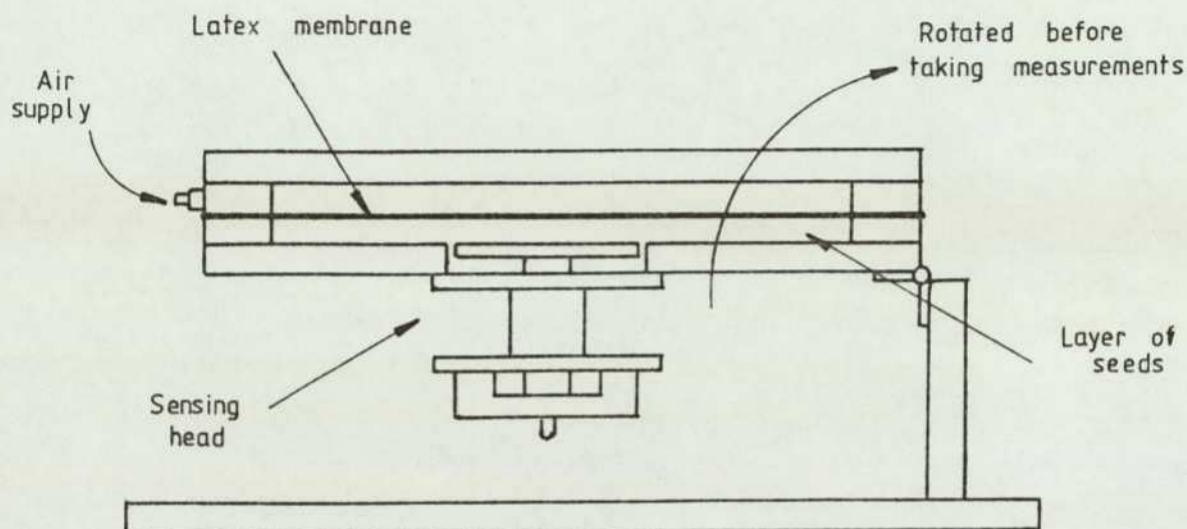


Fig. 5.19. Cross section of tank for face plate diameter measurements shown in horizontal (loading) position.

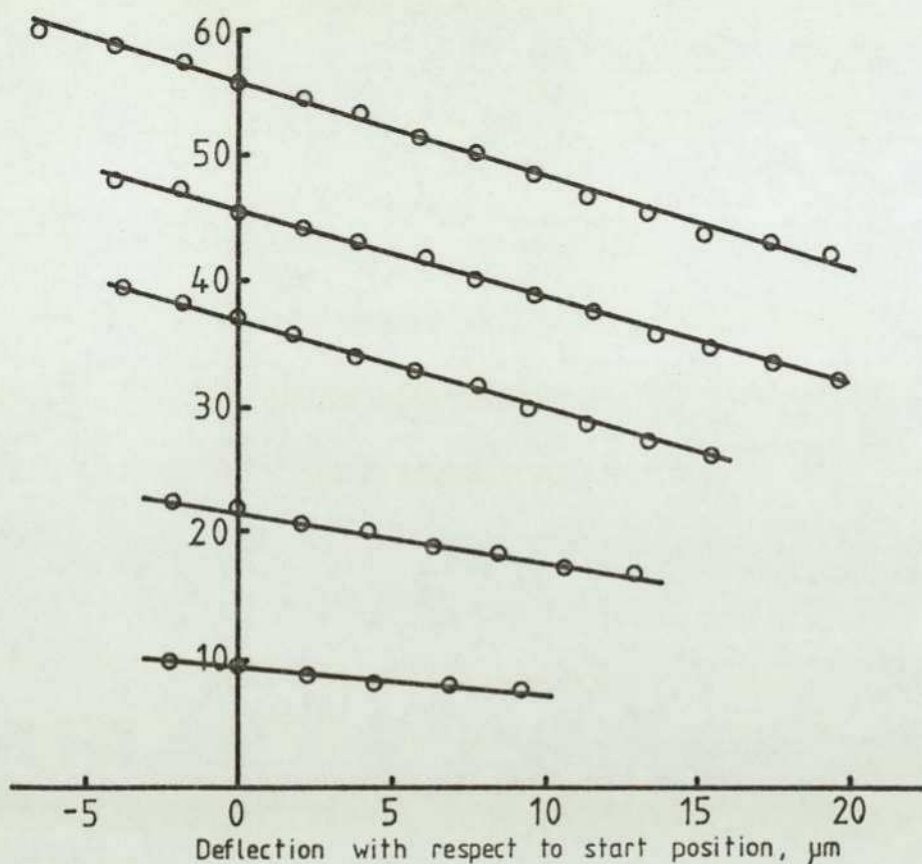


Fig. 5.20. Output of transducer vs. deflection of face plate at five different sensing heads. (The hysteresis band is too small to show on this graph.)

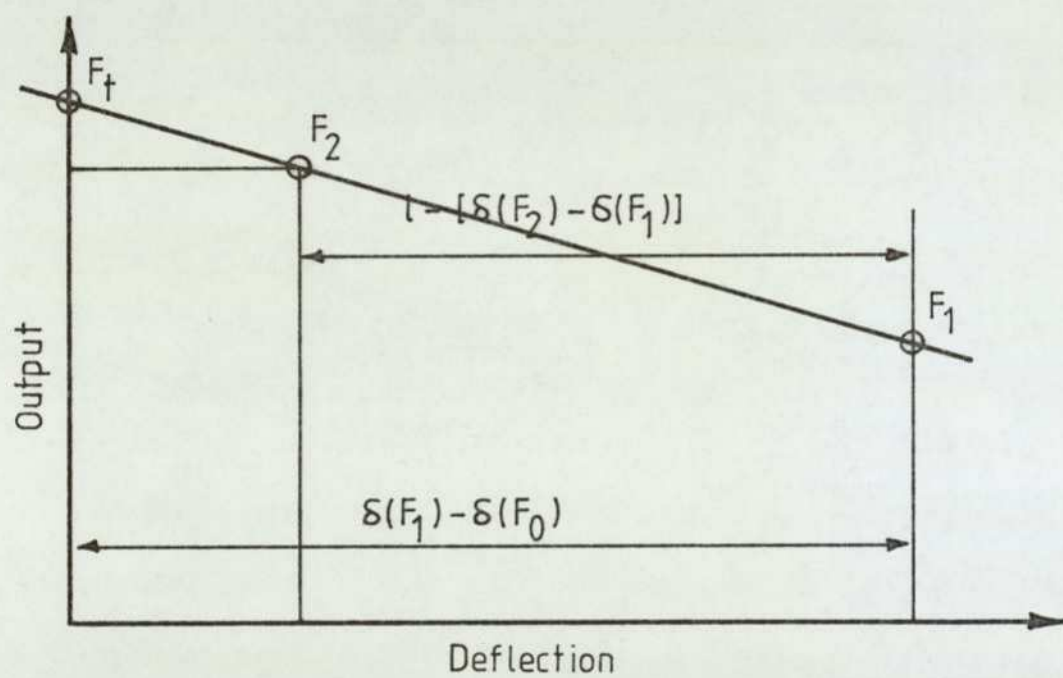


Fig. 5-21. Output of transducer vs. deflection of face plate illustrating measurement technique.

CHAPTER 6

MEASUREMENTS OF WALL LOADS

6.1 Experimental method

6.1.1 The material

The material used to fill the silo was a good quality feed wheat (variety Maris Huntsman) from the 1977 harvest. Properties of the wheat were measured when first delivered and they were as follows:- Bulk density (Avery bushel weight apparatus) 678kg/m^3 ; moisture content (oven dried for 16hrs at 130°C) 14.8% w.b.; 1000 grain mass 53.1g. The angle of internal friction of the sample was measured using the N.I.A.E. shear box (Lawton and Marchant, 1980) and was found to be 27.0° with a negligible cohesion.

The wheat was transferred between the holding bin and the test silo over fifty times during the course of the experiments and this resulted in some deterioration of the grain. A considerable number of broken grains were present in the final sample and a large amount of dust accumulated in the laboratory. The above properties of the wheat were measured again at the end of the experimentation to see if they had been affected by the repeated handling, and the results were:- Bulk density 662kg/m^3 ; moisture content 15.7% w.b.; 1000 grain mass 35.9g; angle of internal friction 26.3° . The percentage of broken grains by weight had risen from virtually zero to 41%.

6.1.2 Configuration of the silo

The test silo was filled in two basic configurations. In one, the 'A' configuration, the overhead conveyor (Sect. 5.1.5) was positioned so that the ridge of the heap of grain was mid-way between the main and back walls and the grain sloped down at its angle of repose to the main and back walls. In the other, the 'F' configuration, the grain

was deposited at the main wall and sloped down towards the back wall. In these basic configurations the ridge was just below the top of the side walls but experiments were also carried out at a lesser depth for each configuration. When the test silo was unloaded, a considerable amount of grain was left on the floor after the auger intake was exposed and could not be removed without a large amount of manual work. To minimise the amount of shovelling, the test silo was not completely emptied after each experimental run. Instead, enough grain was shovelled into the auger intake so that the floor was clear of grain for about 300mm away from the main wall. From there the grain sloped at its natural angle of repose up to the back wall. Thus each run was started with a triangular prism of grain on the floor with a base about 3.9m long and height about 1.7m.

To investigate the effect of different silo widths, the spacing of the side walls was changed so that either one, two, or three main wall modules were in use. The internal widths of the silo corresponding to these cases were 0.7m, 1.7m, and 2.7m respectively. The effect of rotation of the main wall about its lower pin joint was studied by using the disk springs in the main wall struts (Sect. 5.1.2).

A coding convention was adopted so that, for example, 2F referred to configuration F with the greater depth using two main wall modules and 1A $\frac{1}{2}$ referred to configuration A with the lesser depth (the so-called "half depth") using one main wall module. When the springs were used in the struts, an S was added to the code. The dimensions of the cross sections of the heaps is shown in Fig. 6.1.

A run number was allocated each time the silo was filled (Table 6.1). Where two configurations are shown with the same run number, this indicates that the filling was stopped to enable the "half depth" readings to be taken and then continued to full depth. It was not always possible to do this in one working day and in these cases the silo was emptied after the half depth loads were read and the full depth loads were measured after filling on another day. In each case the forces and stresses on the main wall were measured after allowing sufficient time for the dust in the laboratory to settle, i.e. about one hour.

6.2 Results

6.2.1 Deflections of the main wall

The deflections of the steel framework of the main wall were measured with respect to two vertical beams spaced at approximately one third and two thirds of the distance across the main wall. Deflections were measured at five heights on each beam as the silo was filled in the 3A and 3F configurations. Fig. 6.2 shows the horizontal deflections calculated by averaging the two readings from the beams at each of the five heights. The deflection of the centre of a wooden panel with respect to the steelwork was also measured and was found to increase linearly with the mean of the stresses measured at its two sensing heads at a rate of $52\mu\text{m/kPa}$. The maximum deflection of any part of the wall did not exceed 1.9mm in the 3A configuration or 2.1mm in the 3F configuration.

When the disk springs were used, the change in length of each strut was measured as the silo was filled. The loads in the two struts at the sides of the main wall were

Table 6.1

Run numbers and configurations for the experimental
runs and the dates on which they were carried out

Run	Con.	Date	Run	Con.	Date	Run	Con.	Date	Run	Con.	Date
32	3A	2.7.79	52	3F	22.8.79	68	2A	10.10.79	80	1F $\frac{1}{2}$	16.11.79
33	3A	4.7.79	53	3F	24.8.79	69	2A $\frac{1}{2}$	11.10.79	"	1F	"
34	3A	5.7.79	54	3FS	4.9.79	"	2A	"	81	1F $\frac{1}{2}$	19.11.79
35	3AS	9.7.79	55	3FS	5.9.79	70	1A $\frac{1}{2}$	29.10.79	"	1F	"
36	3AS	11.7.79	56	3FS	6.9.79	"	1A	"	82	1F $\frac{1}{2}$	20.11.79
37	3AS	13.7.79	57	3A $\frac{1}{2}$	18.9.79	71	1A $\frac{1}{2}$	30.10.79	"	1F	"
38	3A	17.7.79	58	3A $\frac{1}{2}$	19.9.79	72	1A	31.10.79	83	1F $\frac{1}{2}$	21.11.79
39	3A	19.7.79	59	3A $\frac{1}{2}$	20.9.79	73	1A $\frac{1}{2}$	1.11.79	"	1F	"
40	3A	23.7.79	60	3F $\frac{1}{2}$	24.9.79	"	1A	"	84	2F $\frac{1}{2}$	26.11.79
41	3AS	25.7.79	61	3F $\frac{1}{2}$	25.9.79	74	1A $\frac{1}{2}$	5.11.79	"	2F	"
42	3AS	26.7.79	62	3F $\frac{1}{2}$	26.9.79	"	1A	"	85	2F $\frac{1}{2}$	27.11.79
43	3AS	27.7.79	63	2F $\frac{1}{2}$	3.10.79	75	1A	6.11.79	"	2F	"
44	3A	31.7.79	64	2F	"	76	1A $\frac{1}{2}$	7.11.79	86	2F $\frac{1}{2}$	28.11.79
45	3F	1.8.79	65	2F $\frac{1}{2}$	4.10.79	"	1A	"	"	2F	"
46	3F	6.8.79	"	2F	"	77	1A $\frac{1}{2}$	9.11.79	87	2A $\frac{1}{2}$	4.12.79
47	3F	7.8.79	66	2F $\frac{1}{2}$	5.10.79	"	1A	"	"	2A	"
48	3FS	9.8.79	"	2F	"	78	1F $\frac{1}{2}$	14.11.79	88	2A $\frac{1}{2}$	5.12.79
49	3FS	13.8.79	67	2A $\frac{1}{2}$	9.10.79	"	1F	"	"	2A	"
50	3FS	14.8.79	"	2A	"	79	1F $\frac{1}{2}$	15.11.79	89	2A $\frac{1}{2}$	6.12.79
51	3F	21.8.79	68	2A $\frac{1}{2}$	10.10.79	"	1F	"	"	2A	"

less than those in the middle two struts but, as the springs in each strut were graded to suit the likely loads, the change in length of each strut was very nearly equal and the main wall deflected virtually as a plane. Fig. 6.3 shows the change in length of the struts (average of the four struts) as the fillings progressed.

6.2.2 Stress distributions

The horizontal stresses on the main wall were measured using the technique described in Sect. 5.3. The plywood panels were positioned so that the sensing heads were in vertical columns with a horizontal spacing of 1.0m. For each run, an average value of the stress measured at each depth was derived (Figs 6.4 to 6.14) using the following technique:-

When three main wall modules were used there were three sensing heads in a horizontal line at various levels below the surface (the depths are given in Fig. 6.1). One of the sensing heads was situated on the vertical centre line and the other two were at a distance of 1m either side. The side walls were spaced at a distance of 1.35m either side of the centre line. A second order polynomial was fitted to the horizontal stress distribution defined at these three points, the average of this distribution being given by

$$\bar{\sigma}_x = 0.304\sigma_{xe} + 0.392\sigma_{xm} + 0.304\sigma_{xw}$$

where σ_{xe} , σ_{xm} , and σ_{xw} are the stresses at the east, middle, and west sensing heads respectively. When two wall modules were used, a first order polynomial (straight line) was fitted which had an average of

$$\bar{\sigma}_x = 0.5\sigma_{xe} + 0.5\sigma_{xw}$$

In the cases where one wall module was used, there was only one sensing head at each depth and so the stress at this single head was plotted. In Figs 6.4 to 6.14, where several fillings gave nearly equal stresses, the abscissa of each symbol is correct but some ordinates have been displaced to avoid overlapping. The correct levels of the sensing heads are given in Fig. 6.1.

6.2.3 Overall forces

The overall horizontal and vertical forces per unit width of the main wall for each experimental run are plotted in Figs 6.15 to 6.17. It is possible to derive the overall horizontal force from the stress measurements by integration. This has been done by applying the trapezoidal rule to the experimental points of Figs 6.4 to 6.17. Because the overall force transducers measure forces on the articulated section of the main wall (Sect. 5.1.2), the integration was carried out from the grain surface down to the bottom of the articulated section only. The overall horizontal forces calculated in this way are also shown in Figs 6.15 to 6.17.

6.2.4 Centres of pressure

The centres of pressure on the articulated section of the main wall are shown in Figs 6.18 to 6.19. As with the horizontal forces, it is possible to calculate the centres of pressure by integrating the experimental points of Figs 6.4 to 6.17 over the articulated section. These calculated values are also shown in Figs 6.18 and 6.19.

6.3 Discussion

6.3.1 Comparison between overall force measurements and stress distributions

The area over which the wall stress was actually

measured (i.e. the area of the face plates of the sensing heads) represented only a small proportion, about 2%, of the total main wall area. Thus there would have been some random sampling errors between the horizontal wall force measured by the overall force transducers and that calculated by integrating the stress distributions. However, Figs 6.15 to 6.17 show that the forces calculated by integration were nearly always greater than those from the overall force transducers by an average of about 6%. A t-test (Davies and Goldsmith, 1977) showed that this difference would have been caused by random errors in less than 0.1% of cases (i.e. the difference was significant at the 0.1% level).

Part of this difference could have been caused by friction between the fillet pieces (Sect. 5.1.4) and the side walls of the silo but calculations based on Janssen's formula and using a coefficient of friction of 0.3 between the fillet pieces and the side walls suggest that this would not account for more than about 2% difference in the 3A configuration. This explanation would also have resulted in a larger percentage error as the width was reduced which, from Figs 6.15 to 6.17, did not occur. It seems likely therefore that errors in the instrumentation accounted for a difference of about 4%.

Because it was impossible to apportion the error between its possible causes, the true values of the forces per unit width were taken as the average of those measured by the two methods and the stress distributions were multiplied by a factor that made them consistent with the corresponding forces. The stress and overall force readings

for each run in a configuration were combined to form an average stress distribution for that configuration. The average distributions were factored as mentioned above and plotted in Figs 6.20 to 6.23. The corresponding average forces per unit width for each configuration are shown in Fig. 6.24.

6.3.2 Effect of wall rotation

In the 3AS configuration, the horizontal movement of the main wall at the point where the grain surface intersected it had an average value of 51mm or 1.5% of the depth of the grain at the main wall. The corresponding figure for the 3FS configuration was 2.1%. Despite this Figs 6.18 and 6.19 show that the difference between the stress distributions on the flexible wall and those on the relatively rigid wall were small. The force per unit width in the 3AS configuration (Fig. 6.24) was 7.0% less than that in the 3A configuration (significant at the 1% level) and the force per unit width in the 3FS configuration even showed a slight increase (2.4%), but a t-test showed this only just to be significant at the 5% level. Three possible reasons for these small differences are:-

a) The loads on the main wall were reduced to their minimum values by the slight wall deflection which occurred when the springs were absent and so the extra compliance provided by the springs had no further effect.

b) The deflection needed to reduce the stresses appreciably was greater than that achieved, even with the springs present.

c) The value to which rotation would reduce the wall load was not very different from that exerted on a

perfectly rigid wall.

The maximum deflection of the wall framework in the runs with no springs occurred in the 3F configuration and was about 1.9mm at 2.26m above the floor (Fig. 6.2). However, this represented the total deflection from the position at the start of the loading but when the grain reached this height the framework had already deflected by 0.6mm and so the grain experienced a movement of about 1.3mm after it had been deposited. The deflection of the plywood facing panels at that depth (taking the average as half the centre deflection and the rate of deflection as $52\mu\text{m/kPa}$) would have added a further 0.1mm giving a total deflection of 1.4mm or 0.03% of the grain depth.

Calculations with the finite element program suggest that, for a silo with a flat grain surface, the horizontal force per unit width decreases by only about 0.3% with a rotation about the base of 0.03% of the grain depth but is still decreasing steadily at 1% rotation (Fig. 4.14). Although the configurations used here do not have flat surfaces and the deflections are not rotations it is expected that the calculations would give the sensitivity to movement to at least the correct order of magnitude and so it seems unlikely that a) above is valid.

The rate of change of wall force with rotation calculated using the finite element program for a silo with a flat surface is 10.6% of the force on a rigid wall for each per cent of rotation (Fig. 4.14). If this rate applied to a silo in the 3AS configuration, the wall force would decrease by 16%, i.e. over twice that observed. In the 3FS configuration the corresponding decrease would be 22%

whereas a slight increase (although hardly significant) was actually observed. Clearly the measured sensitivity to wall movement for a sloping surface was lower than that predicted for a flat surface and the likelihood of b) above being applicable cannot be assessed with the finite element simulations carried out so far.

Sect. 2.1.3 shows that the stress system in a material sloping at its angle of repose and of infinite extent is such that all the material is in a state of active failure. Although the author's material is necessarily finite, it is expected that it will be nearer to a state of failure than if the surface were flat, even if the wall were perfectly rigid. If this is the case then c) above is supported because the loads on a wall that is allowed to rotate would be nearer to those on a rigid wall. A further piece of evidence to support this view is that in the 3FS configuration, when the slope was of a greater extent than that in the 3AS configuration, the sensitivity to wall rotation was much less (in fact measured as negative).

6.3.3 Effect of side wall spacing

Experiments were carried out with the ratio of the width to the depth of grain (defined for this purpose as the depth from where the grain surface met the main wall) varying from 1.23 to 0.20 in the A configurations and from 0.98 to 0.16 in the F configurations. The author could find no other systematic study which sought to determine the effect of side wall spacing on main wall loads on a large scale but it is generally believed that the force per unit width on the main wall will start to decrease as the width drops below twice the depth (Terzaghi, 1934). However,

the practical work to support this view has been confined to small scale models where the relative effects of side wall friction could be much larger. For instance, when the grain surface was observed on loading the silo it was seen that the grains over the greater proportion of the width were all moving at virtually the same velocity and only those closer than about 150mm from the wall were retarded by the drag at the side walls. If this represented the extent to which the shear stress at the walls penetrated into the grain mass then in a silo of width 2.7m it would have had little effect but in a model, which may be only 500mm wide, its effect would be much larger. This could explain why, in this large scale silo, side wall effects were small (Fig. 6.24) even when the width to depth ratio was as low as 0.16.

6.3.4 Vertical forces

The ratio between the horizontal and vertical forces on the main wall, i.e. the coefficient of friction mobilised at the wall, was reasonably constant for the runs using two and three main wall modules, the average being 0.27 (Figs 6.15 to 6.17). Also, the friction was little affected by rotation of the main wall which agreed with Terzaghi's tests (Terzaghi, 1934) where it was found that the wall friction was not affected by movement for loosely packed sand. The wall friction was in reasonable agreement with limiting values of between 0.18 and 0.24 recorded by Lawton (1980) for a similar wheat on plywood using various types of annular shear apparatus. It may therefore be concluded that the limiting value of wall friction was always mobilised during these tests.

The average wall friction for the runs using one main wall module was 0.42 which was significantly greater than that using two and three modules. This was probably due to the vertical friction force on the fillet pieces being measured along with that on the main wall which would have had a noticeable effect at the very narrow single module width.

6.3.5 Centres of pressure

The positions of the centres of pressure were, like the horizontal forces, slightly different depending on whether they were measured by the overall force transducers or calculated by integrating the stress distributions (Figs 6.18 and 6.19). However, the amount of the error was small, the average being 70mm lower, less than 2% of the grain depth, when integrating the stresses, and so the true values were taken as the average of the values calculated by each method. Averages for each configuration, calculated by combining data from all runs in that configuration, are shown in Fig. 6.25.

There are three general points to note from Figs 6.20 to 6.23 and Fig. 6.25:-

a) The stress distributions were reasonably linear with depth but usually the rate of increase of stress dropped nearer the floor. The centres of pressure should therefore have been at or above one third of the distance up the main wall. This is confirmed by Fig. 6.25 which shows that the heights of the centres of pressure were between 0.33 and 0.43 of the height of the articulated section above the bottom of that section.

b) Comparing the results with and without rotation

(Fig. 6.25) shows that the positions of the centres of pressure were little affected by allowing the wall to rotate by up to 2.1% of the grain depth.

c) Although the change in the stress distribution was small as the width was reduced, Figs 6.20 to 6.23 show that the reductions in stress occurred mainly nearer the bottom of the silo. This was reflected by an increase in the height of the centre of pressure as the silo became narrower (Fig. 6.25).

This characteristic behaviour of the centre of pressure is in agreement both with Terzaghi's tests (Terzaghi, 1934) on loose sand and also with the calculations using the finite element program with a flat grain surface (Sect. 4.5).

6.3.6 Comparison with calculations using classical formulae

Unfortunately none of the classical formulae strictly apply to the configurations used here. Rankine's formula cannot cope with side wall or main wall friction, Janssen's formula assumes a flat surface, and Coulomb's method ignores side wall friction. Neither of these formulae can predict the effect of wall deflections. However, they can be used to determine the sensitivities of the main wall loads to most of the variables. For instance, Coulomb's method can be used to determine the horizontal wall force for each configuration assuming that there is no side wall friction. Janssen's formula (using a weighted wall friction coefficient as in Sect. 4.5.3) can then be used to calculate the reduction in force due to the side wall friction effect in some equivalent configuration having a flat surface. In fact this approach is very similar to that leading to the simple formula for a flat surface developed in Sect. 4.5.4, Coulomb's

method being used to provide the K value with main wall friction, $K_o = C_w \mu_w$, and Janssen's formula being used to provide the sensitivity to side wall friction, C_s .

This approach was adopted to calculate the horizontal forces per unit width for the runs without springs in the main wall struts. The calculated forces are shown in Fig. 6.26. The equivalent configurations used when applying Janssen's formula were those having the same horizontal cross section and containing the same volume of grain but with a flat surface. The angle of internal friction and bulk density used were those determined on the grain in the condition existing at the start of the experimentation (27.0° and 678kg/m^3). The ratio of horizontal to vertical stress was that calculated from the Rankine expression (0.376) and the main wall and side wall friction was assumed to be the average figure mobilised in the tests using two and three main wall modules (0.27). The changes in angle of internal friction and bulk density between the start and the end of the experimentation were small (Sect. 6.1.1). Because they both reduced, the stresses calculated using Coulomb's and Janssen's methods were increased due to the change in the angle of friction and reduced again due to the change in bulk density. A few trial calculations showed the differences between the calculated stresses using the two sets of material properties to be negligible.

A comparison of the calculated values of the forces per unit width (Fig. 6.26) and the measured values (Fig. 6.24) shows that the values calculated with the classical formulae are lower than the measured values, even those in which the main wall was allowed to rotate. This again reinforces the proposal (Sect. 2.2) that stored agricultural materials

are not likely to be in the state of active failure assumed by classical formulae.

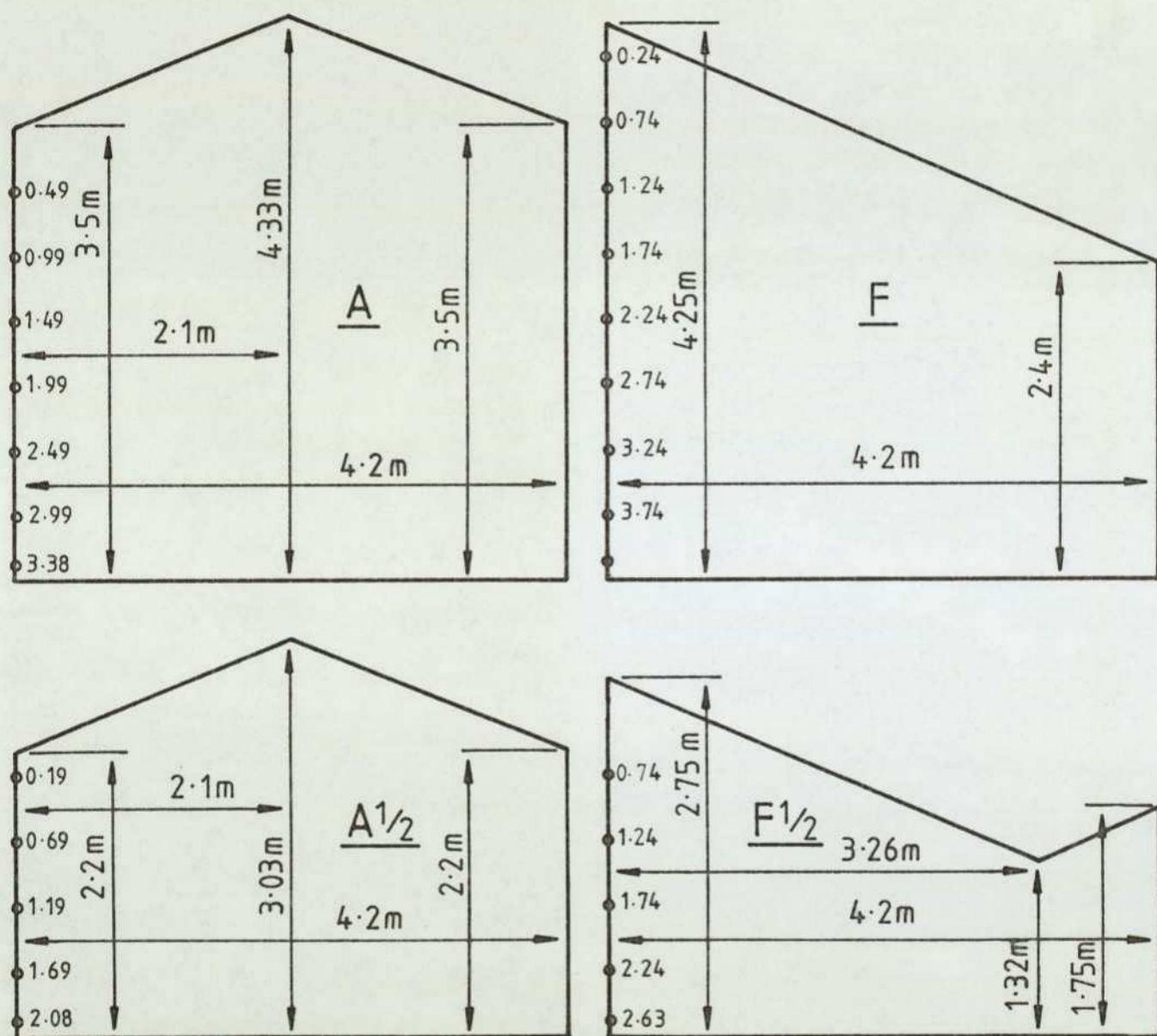


Fig. 6.1. Standard cross sectional dimensions for each configuration. Positions of sensing heads with distances below grain surface (m) shown thus :- o .

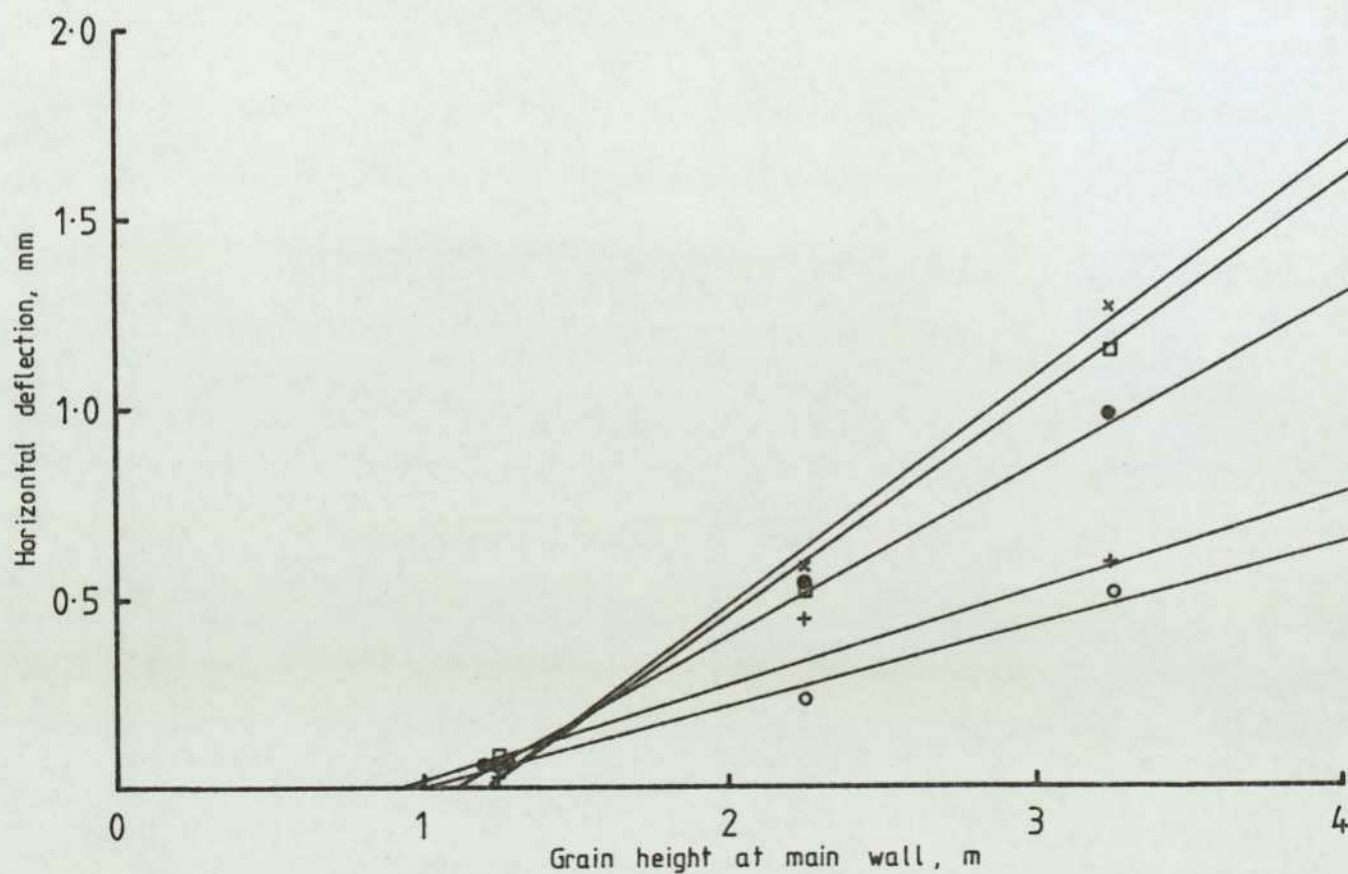
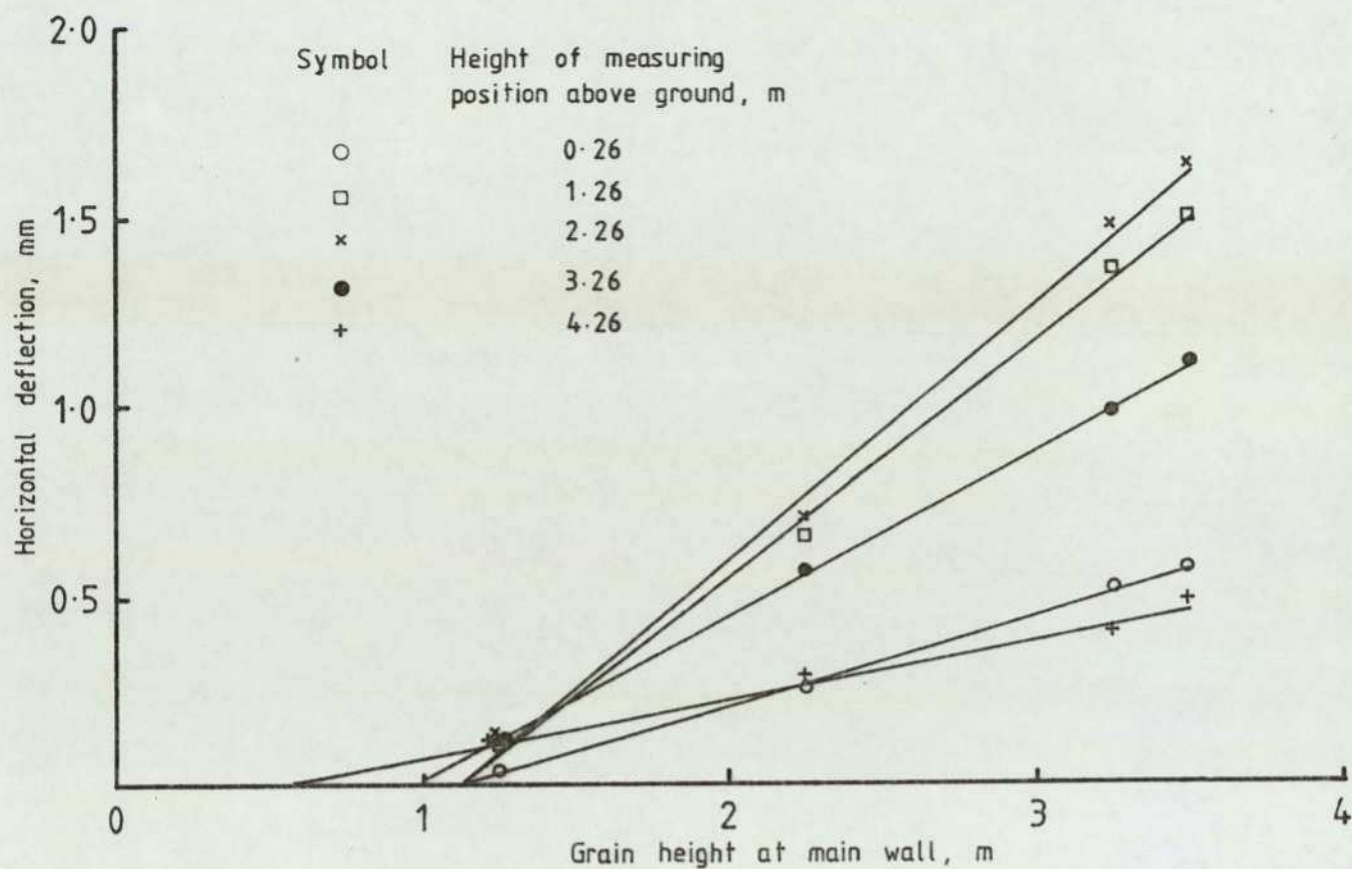


Fig. 6.2. Horizontal deflections at various points on the main wall framework measured as the silo was filled. Top, 3A configuration. Bottom, 3F configuration.

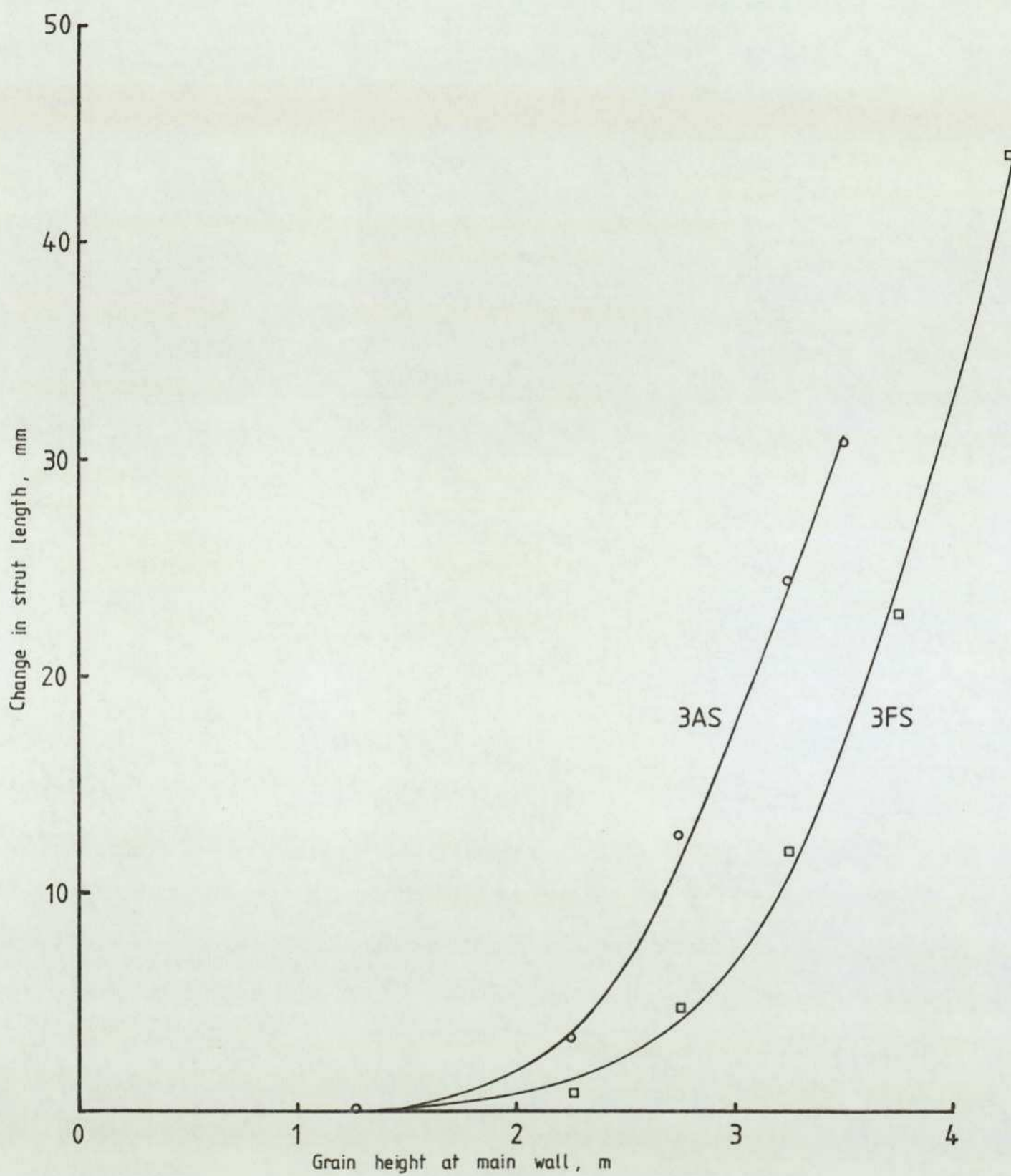


Fig. 6.3. Change in strut length measured as the silo was filled.

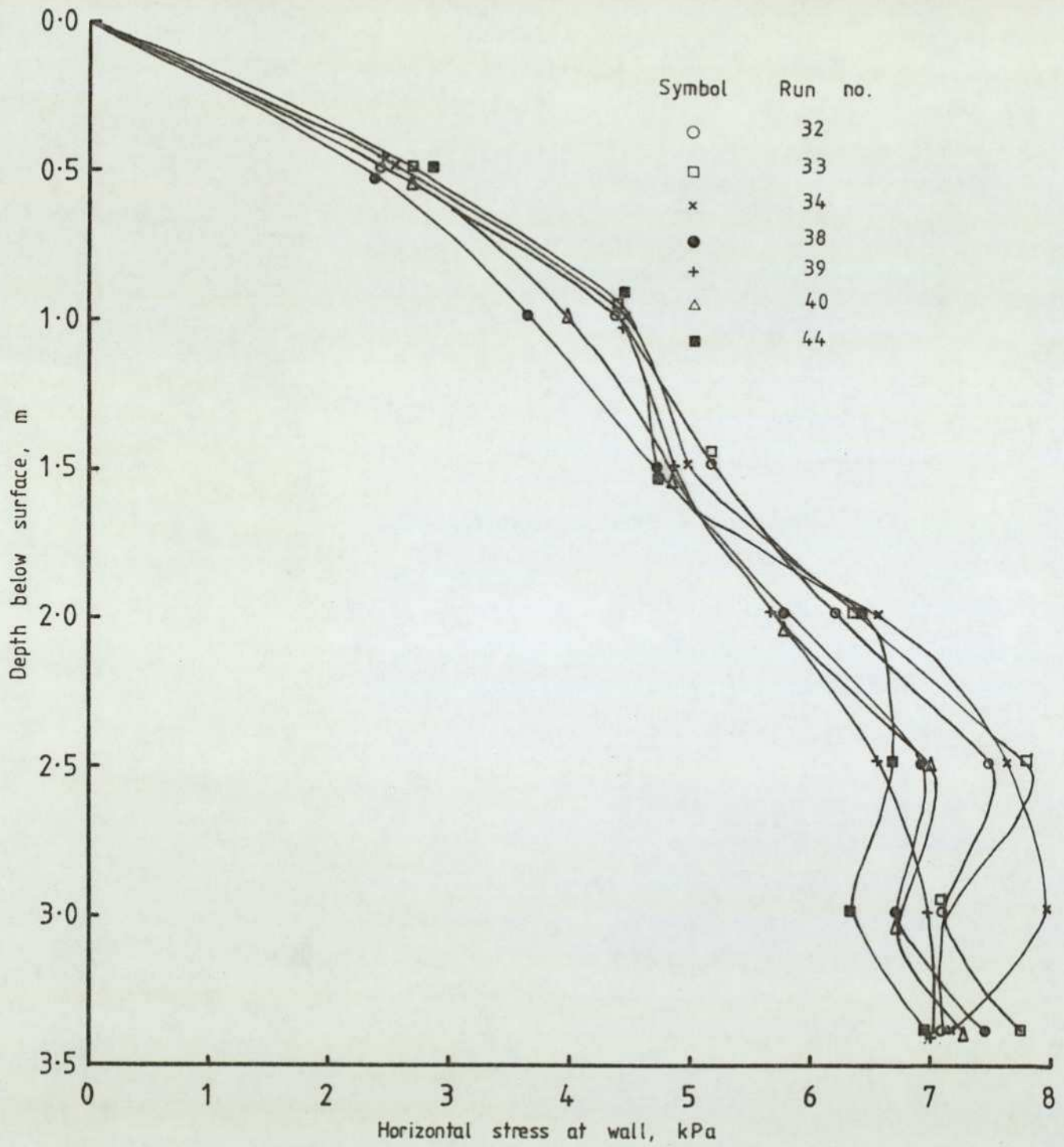


Fig. 6.4. Stress distributions on the main wall in the 3A configuration.

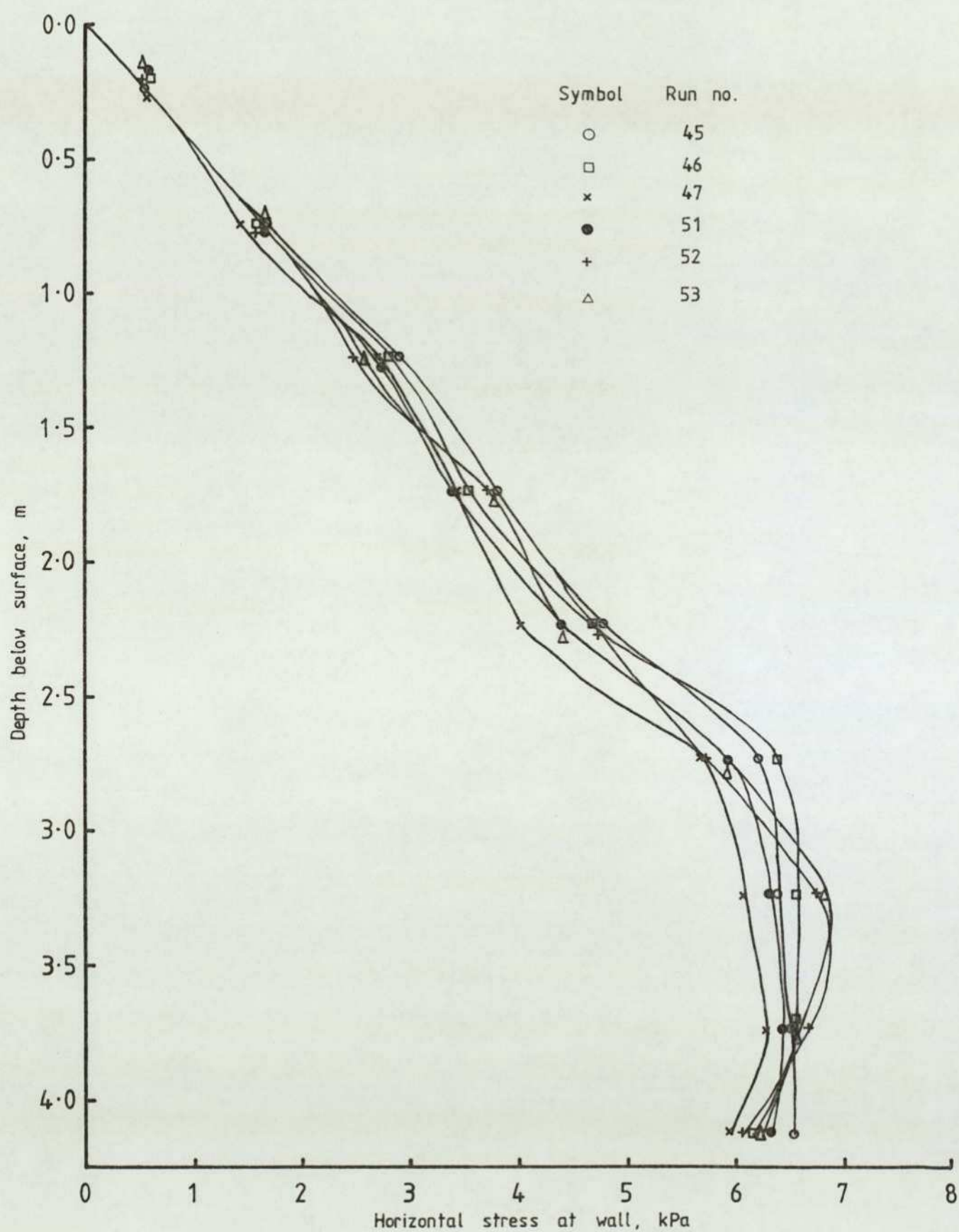


Fig. 6.5. Stress distributions on the main wall in the 3F configuration.

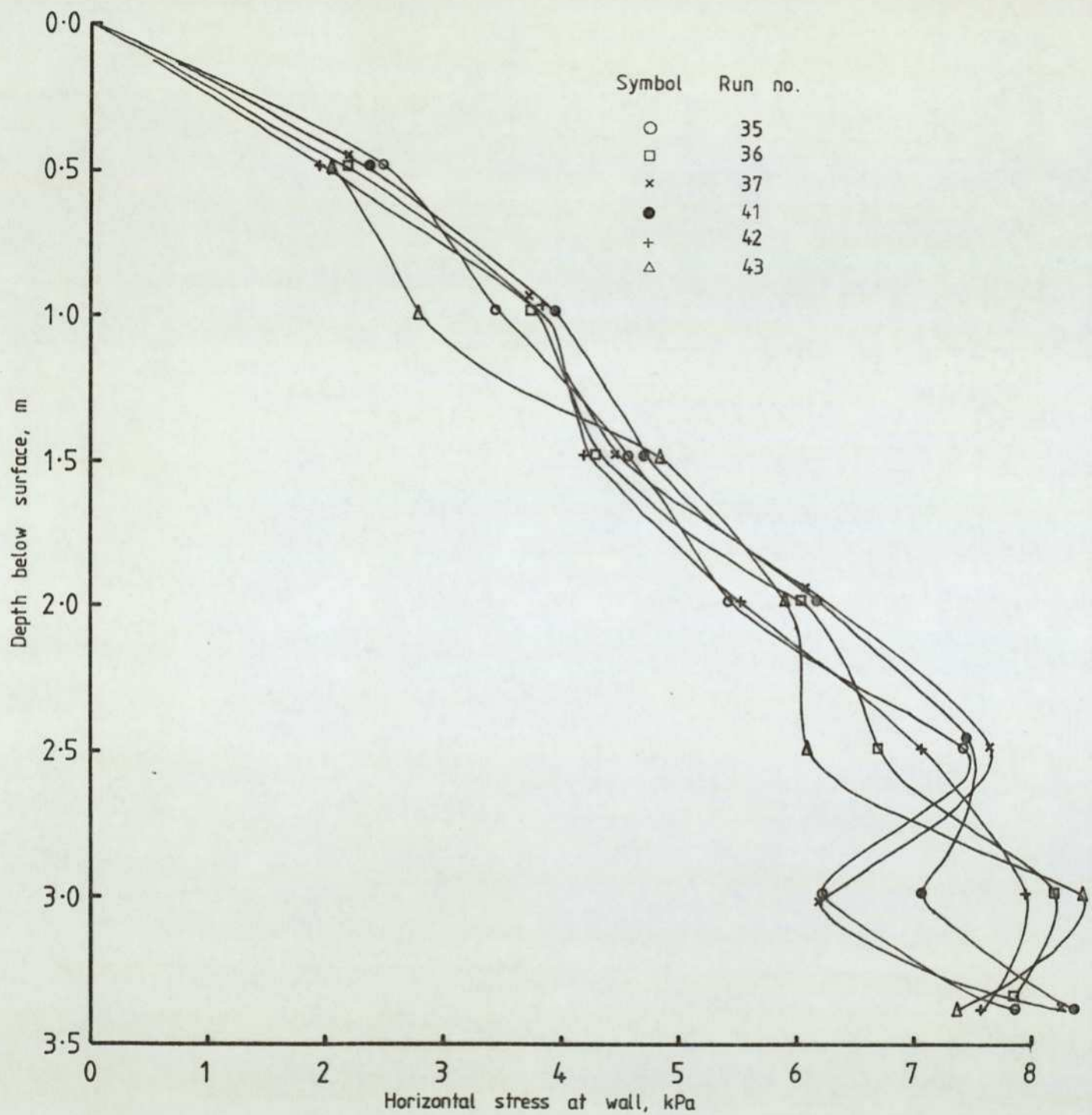


Fig. 6.6. Stress distributions on the main wall in the 3AS configuration.

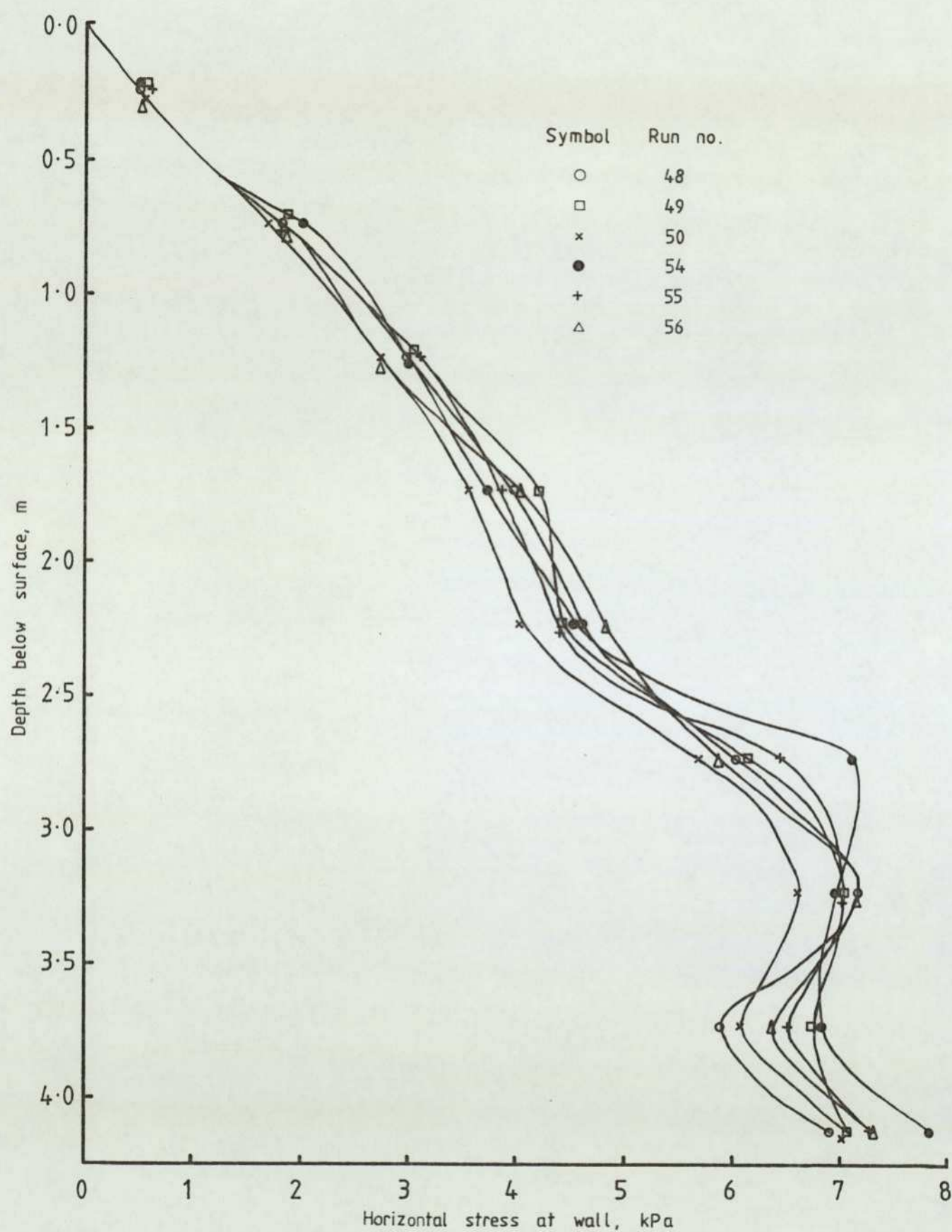


Fig. 6.7. Stress distributions on the main wall in the 3FS configuration.

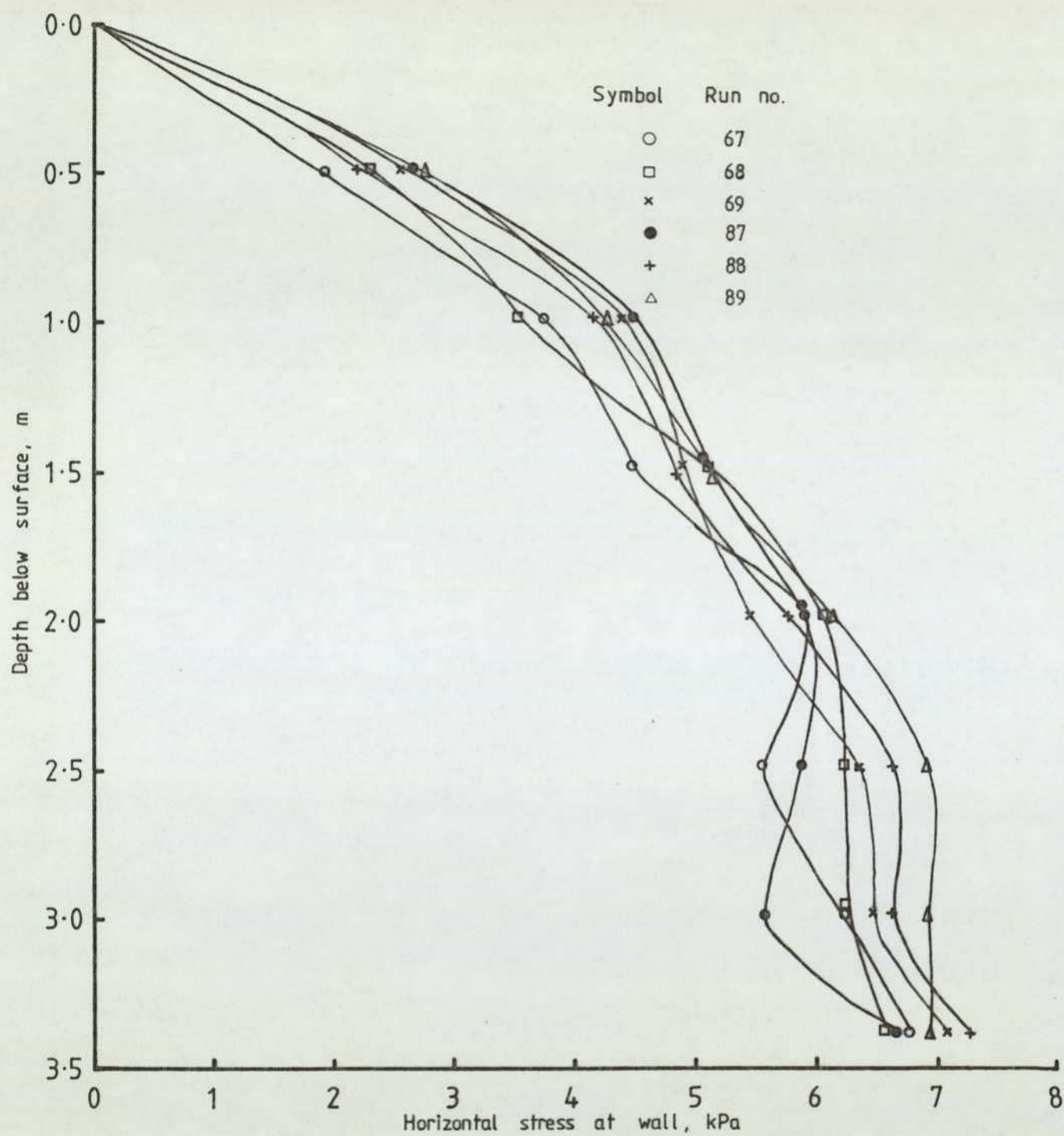


Fig. 6.8. Stress distributions on the main wall in the 2A configuration.

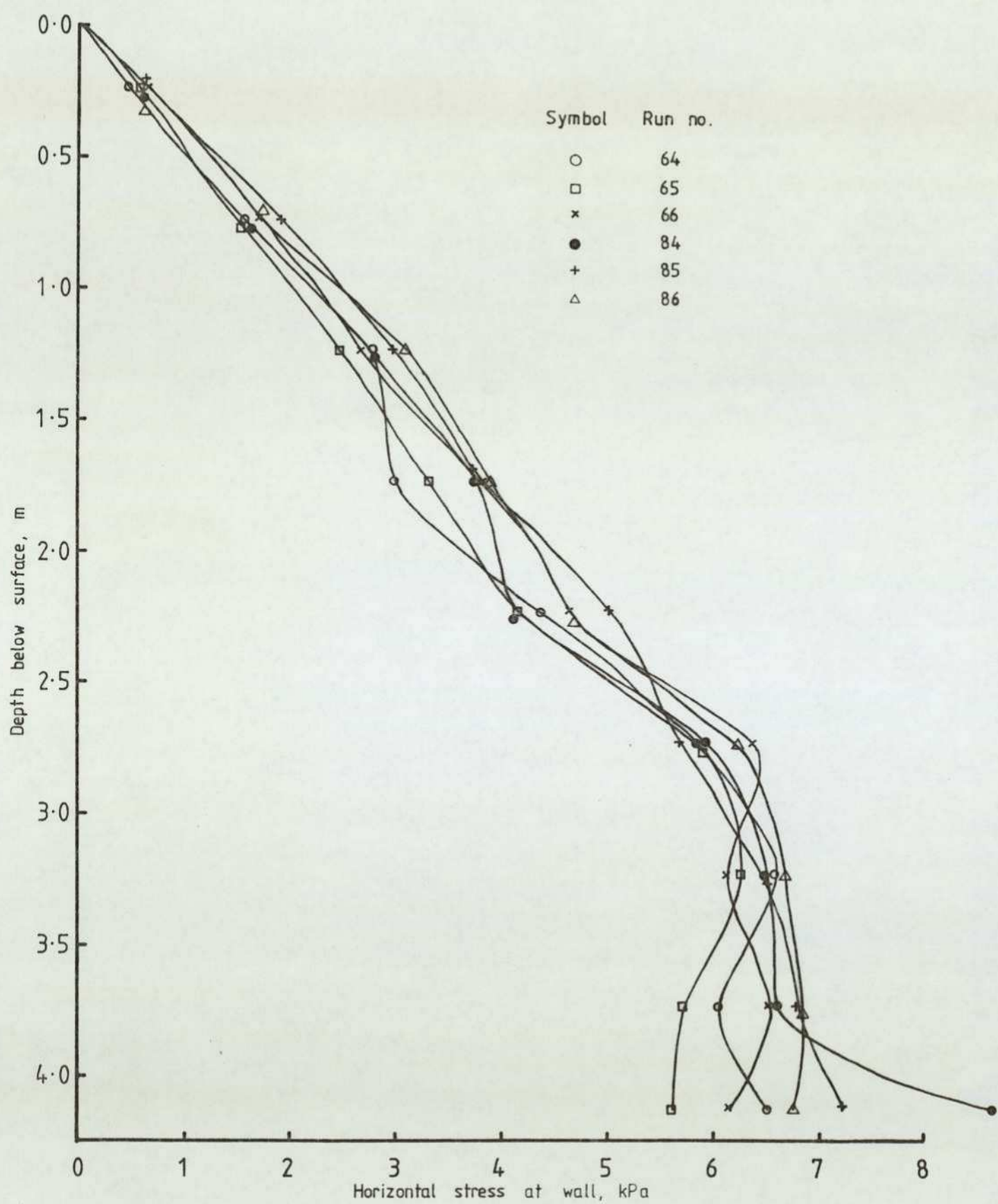


Fig. 6.9. Stress distributions on the main wall in the 2F configuration.

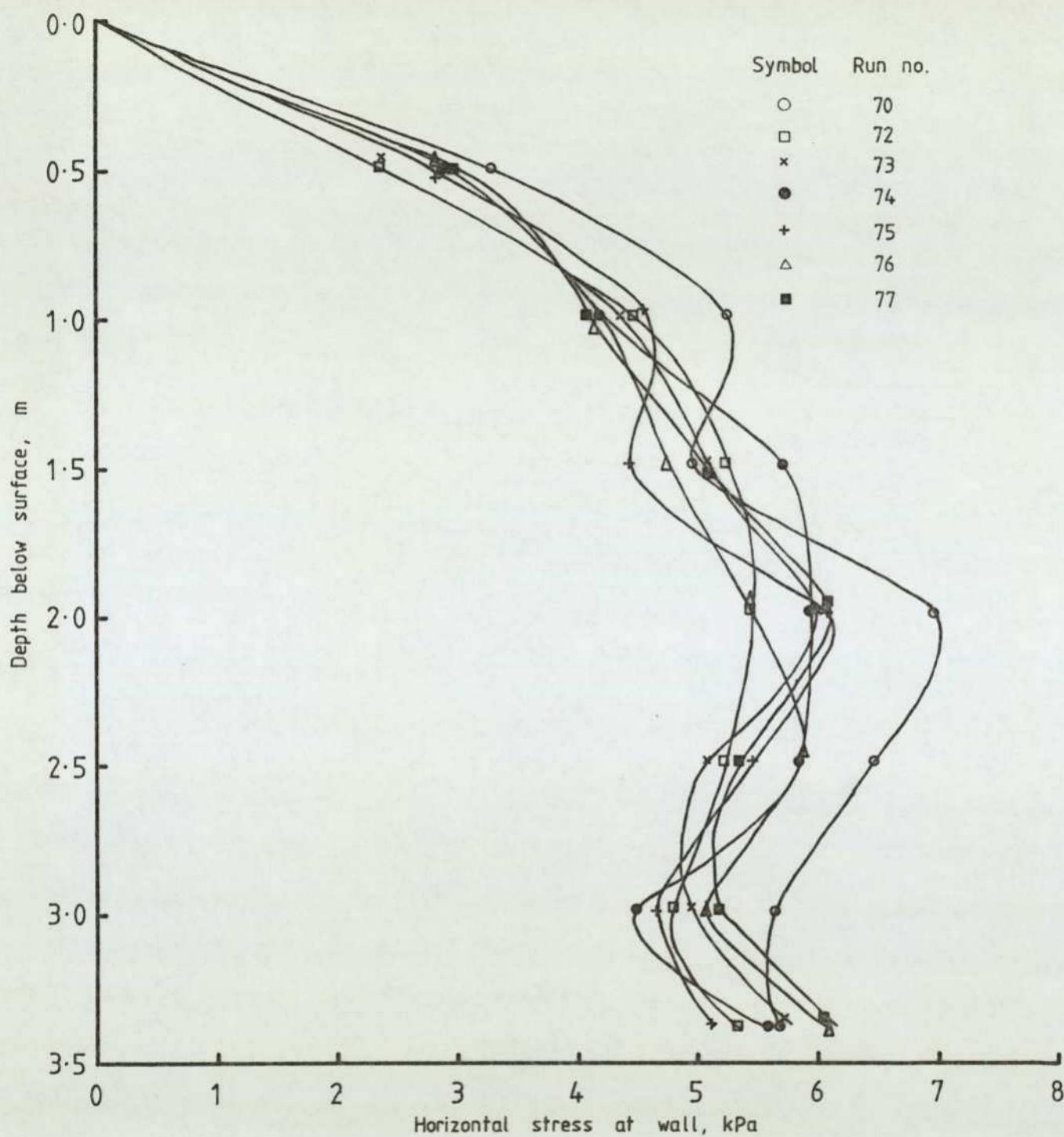


Fig. 6.10. Stress distributions on the main wall in the 1A configuration.

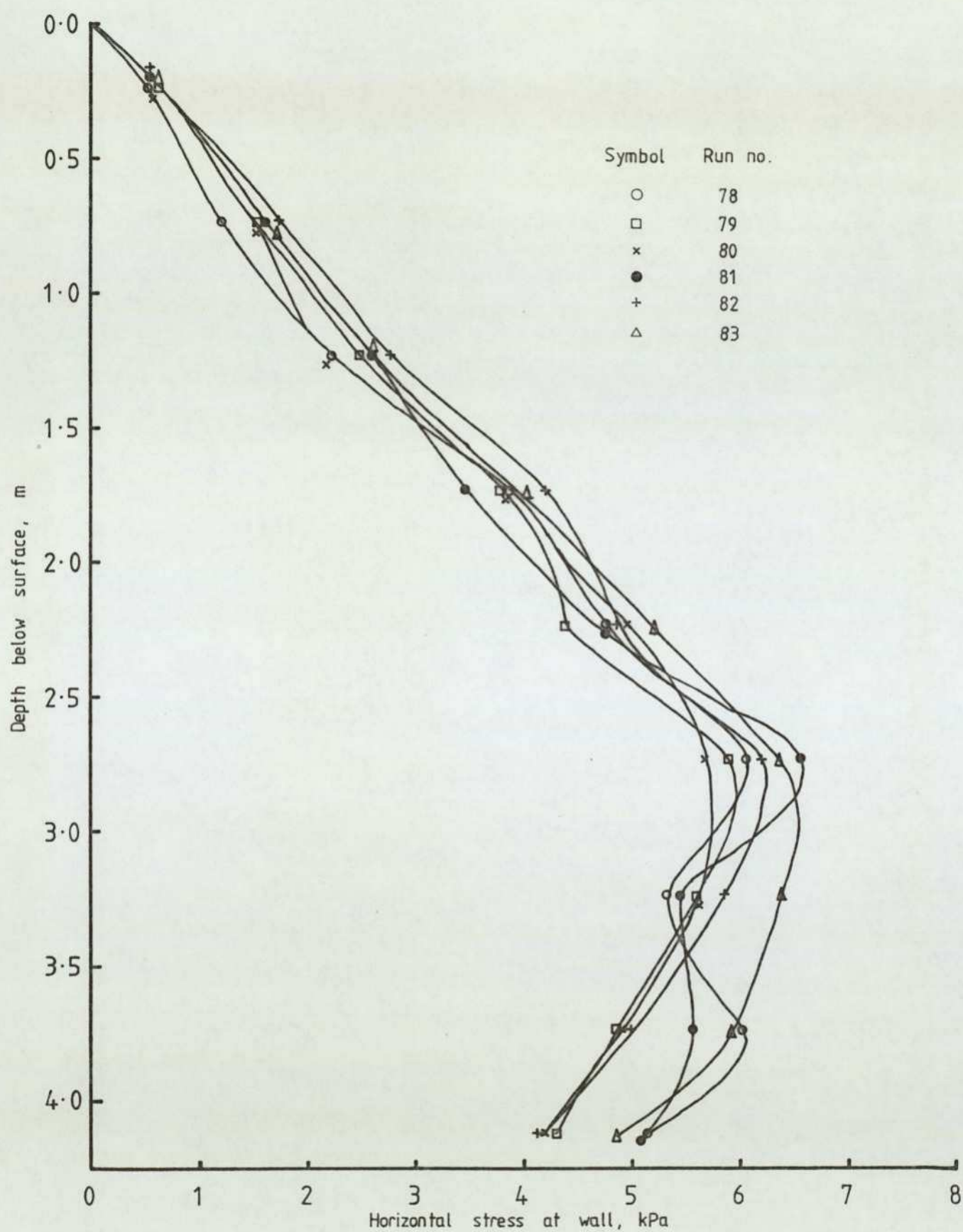


Fig. 6.11. Stress distributions on the main wall in the 1F configuration.

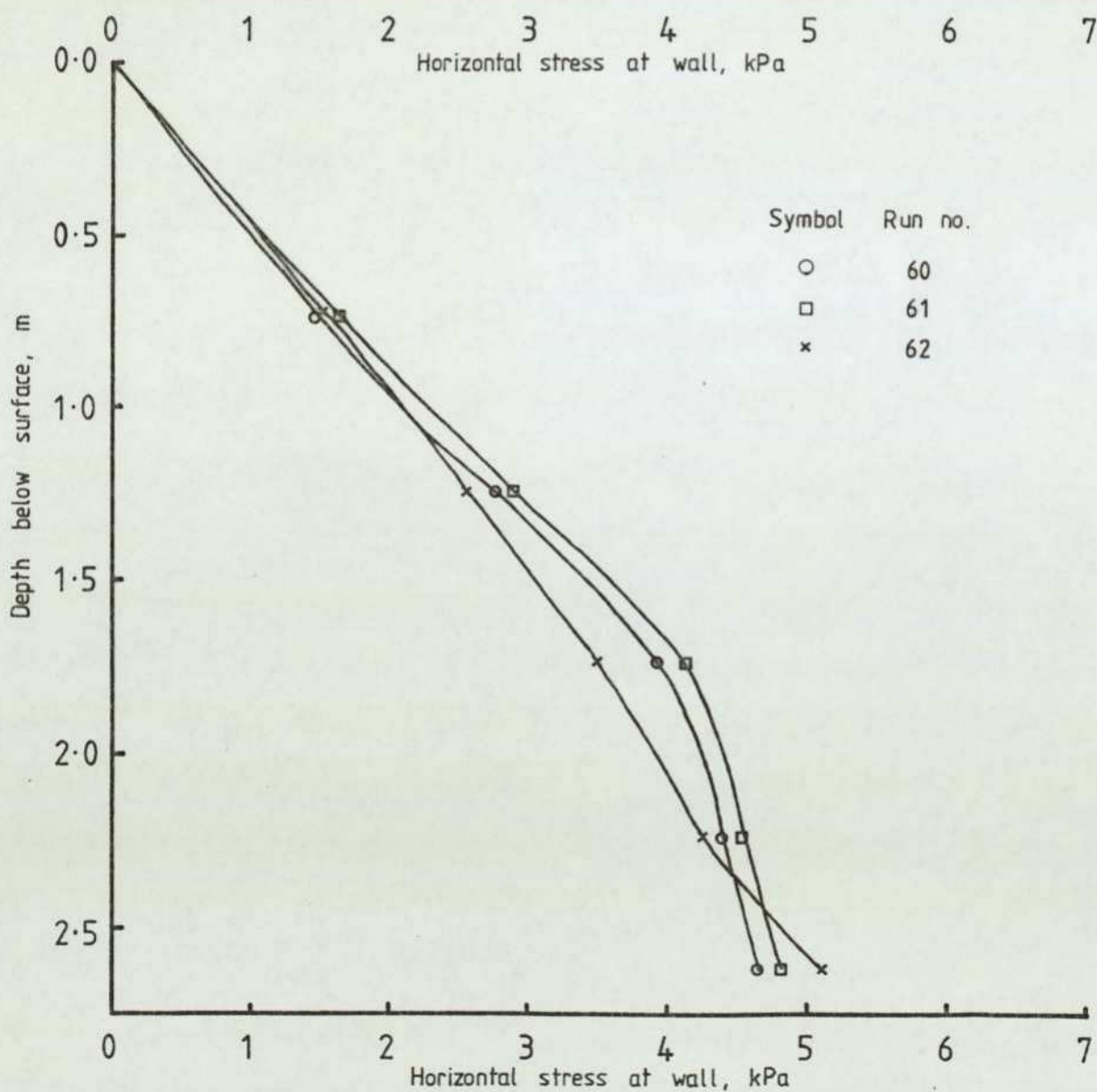
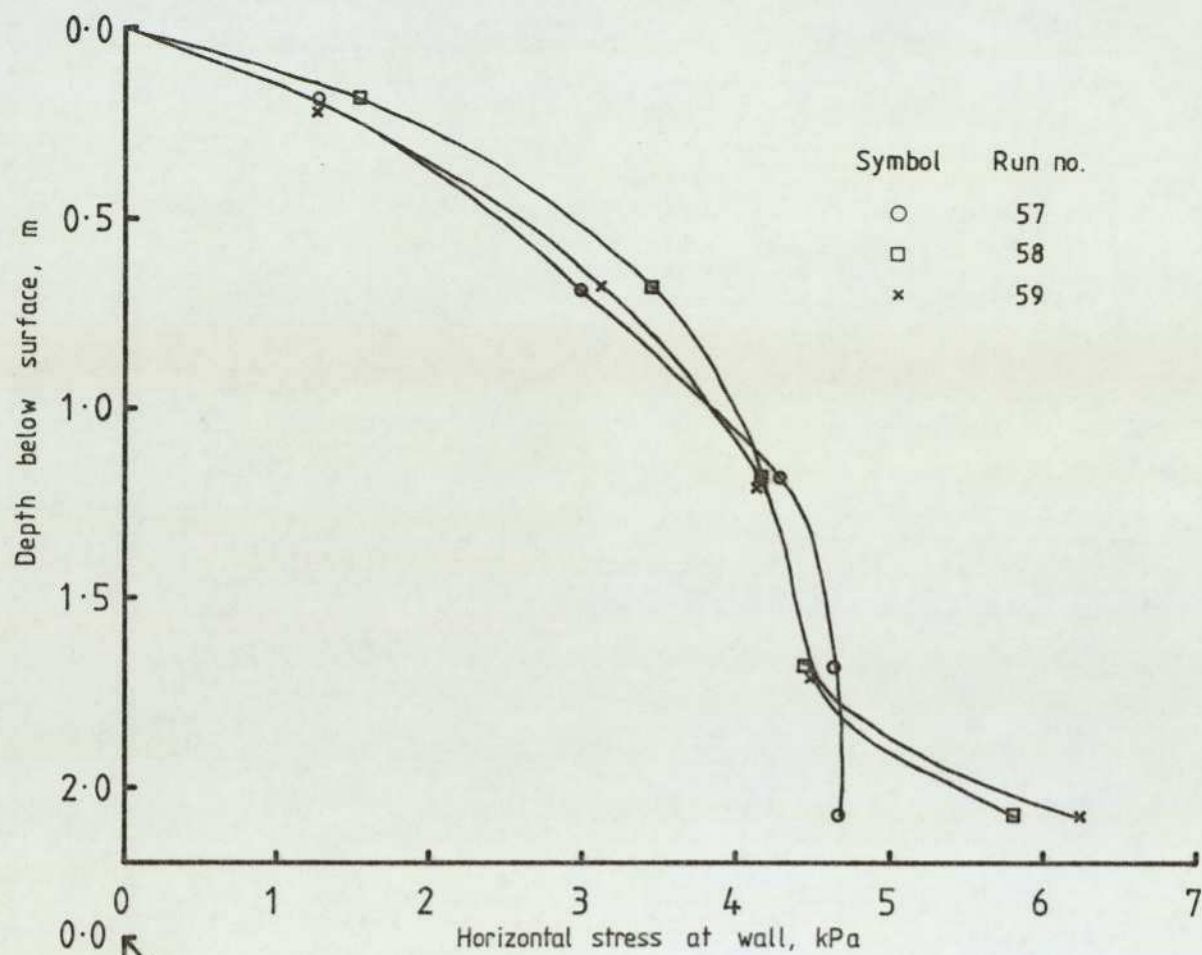


Fig. 6.12. Stress distributions on the main wall. Top, $3A^{1/2}$ configuration. Bottom $3F^{1/2}$ configuration.

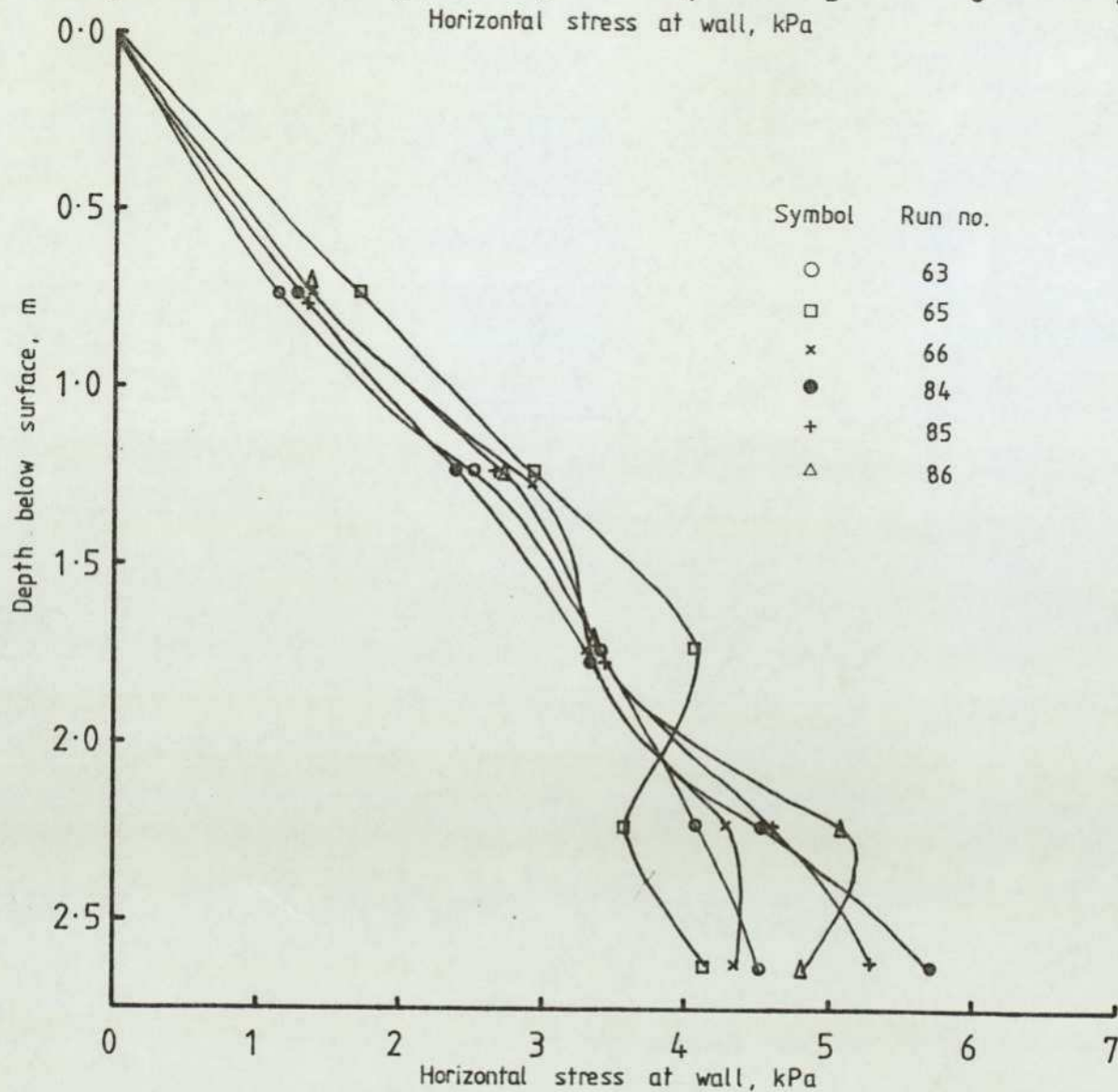
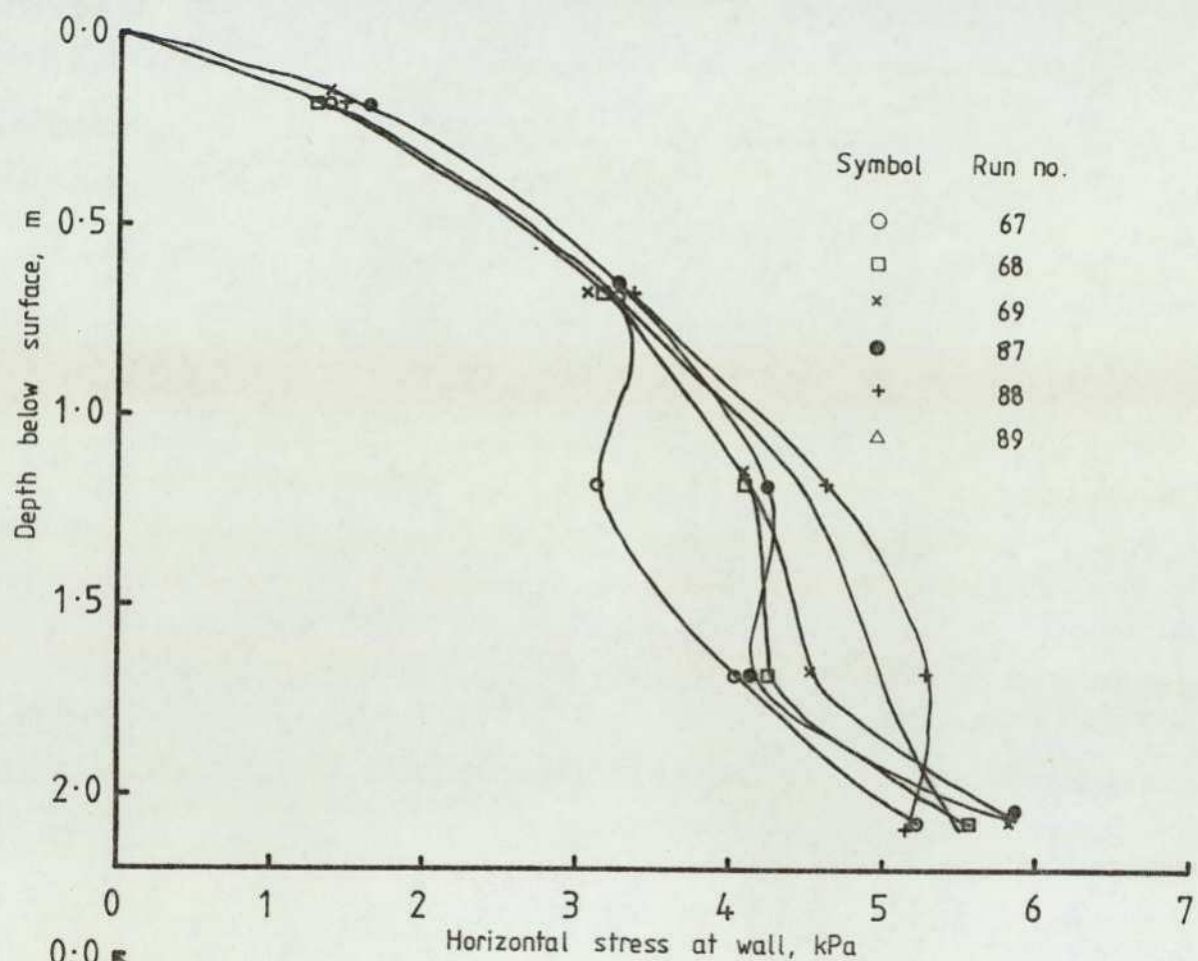


Fig. 6.13. Stress distributions on the main wall. Top, $2A^{1/2}$ configuration. Bottom, $2F_k$ configuration.

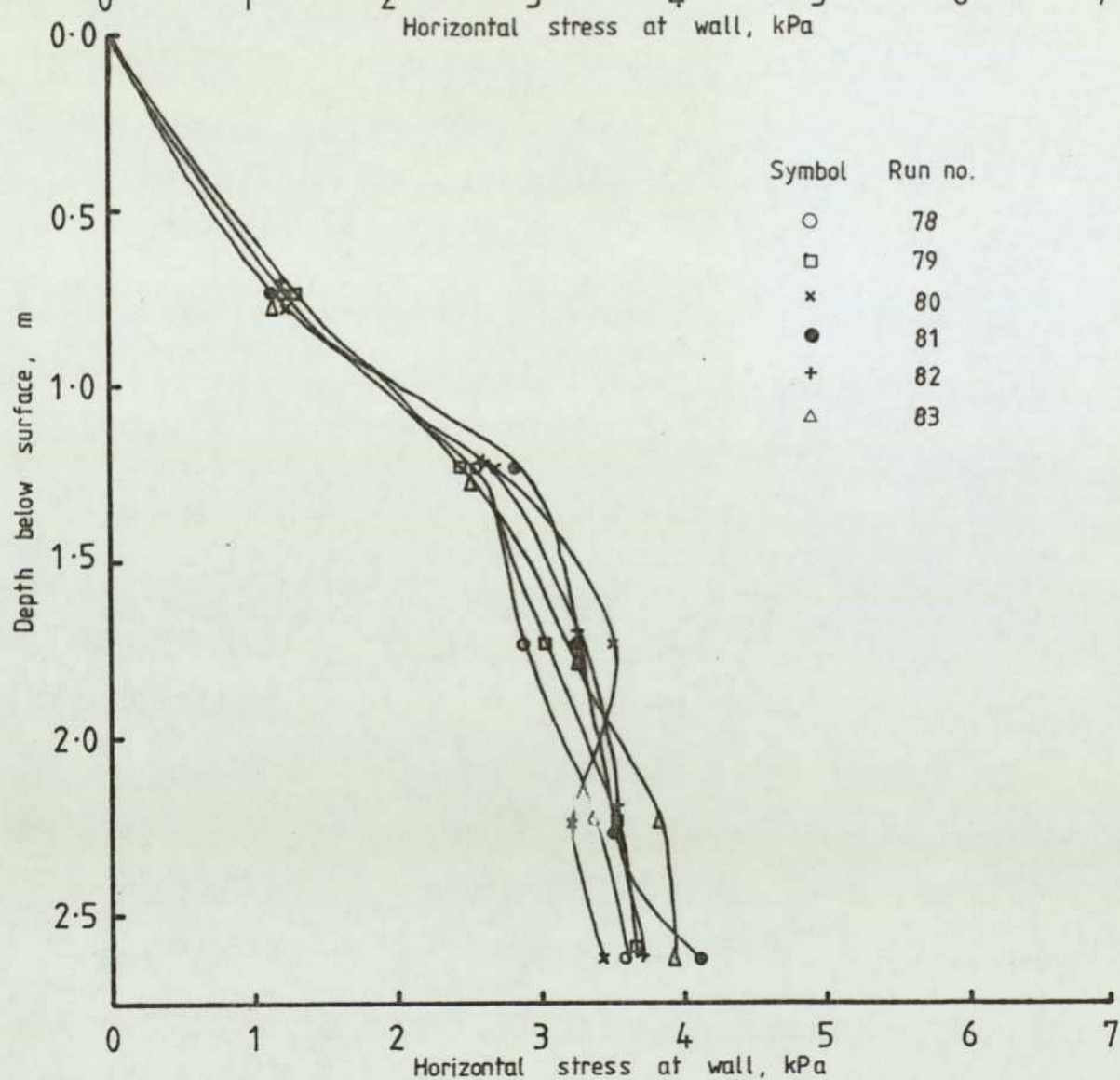
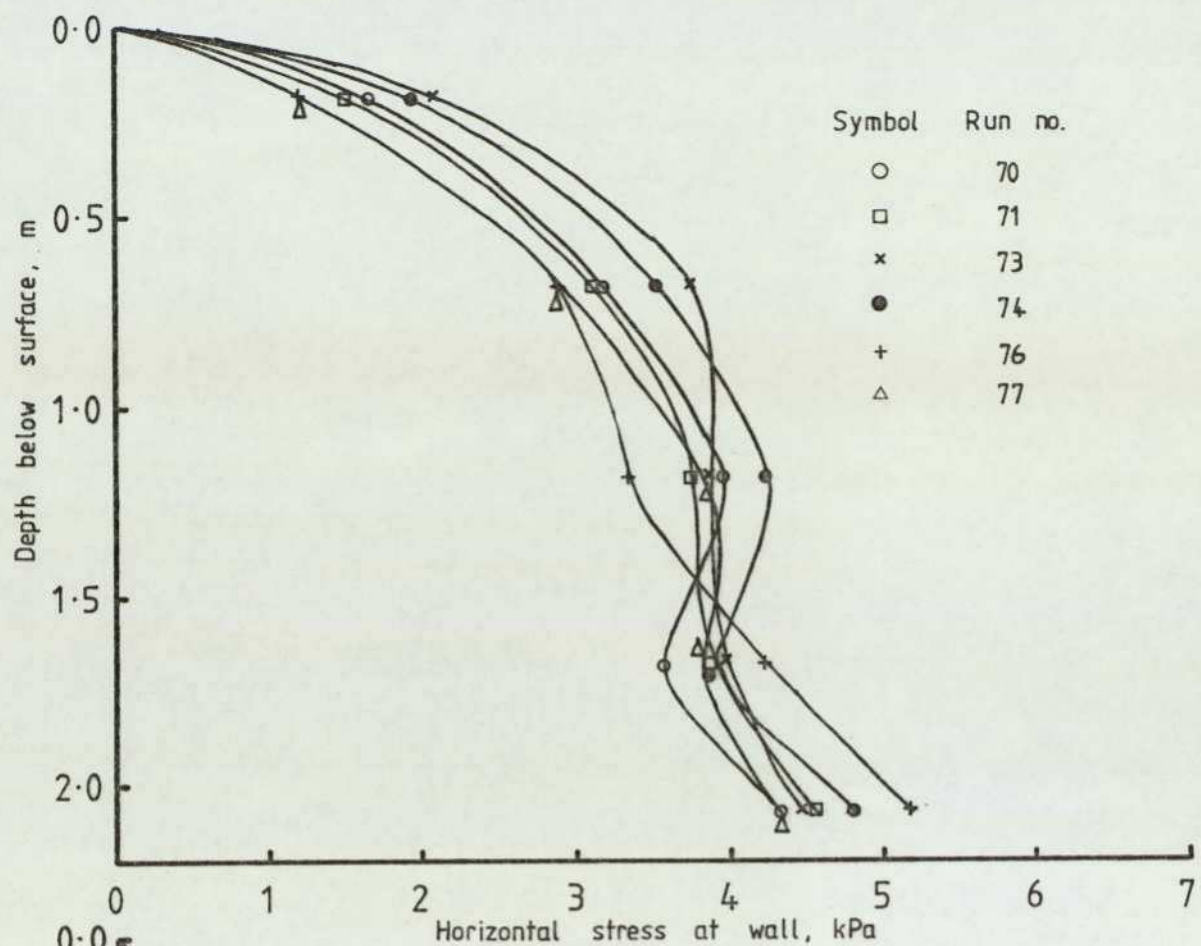


Fig. 6.14. Stress distributions on the main wall. Top, $1A^{1/2}$ configuration. Bottom, $1F^{1/2}$ configuration.

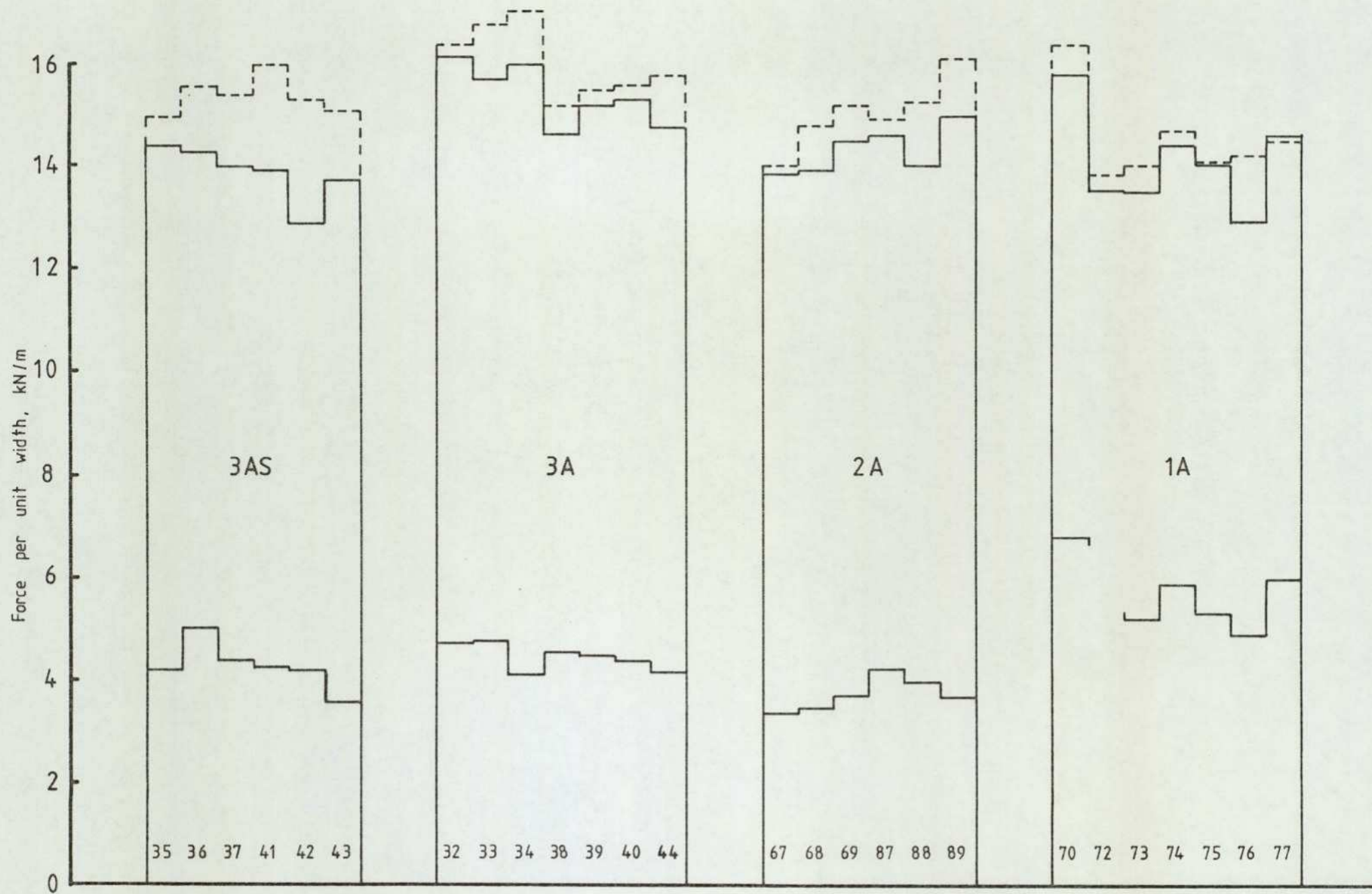


Fig. 6.15 . Horizontal (upper columns) and vertical (lower columns) forces per unit width on the articulated section of the main wall. Broken lines show forces calculated from the stress distributions. Figures show run numbers.

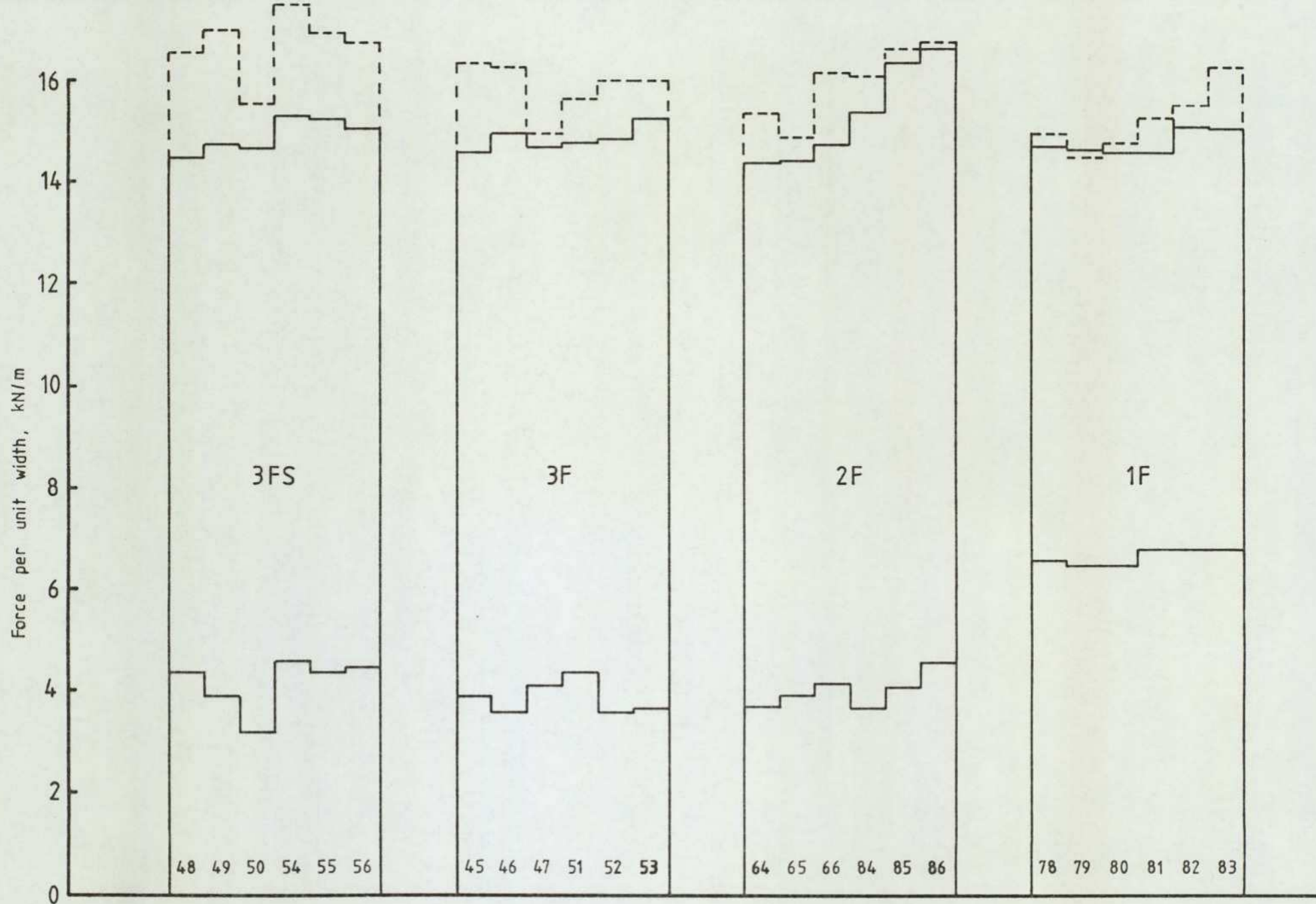


Fig. 6-16. Horizontal (upper columns) and vertical (lower columns) forces per unit width on the articulated section of the main wall. Broken lines show forces calculated from the stress distributions. Figures show run numbers.

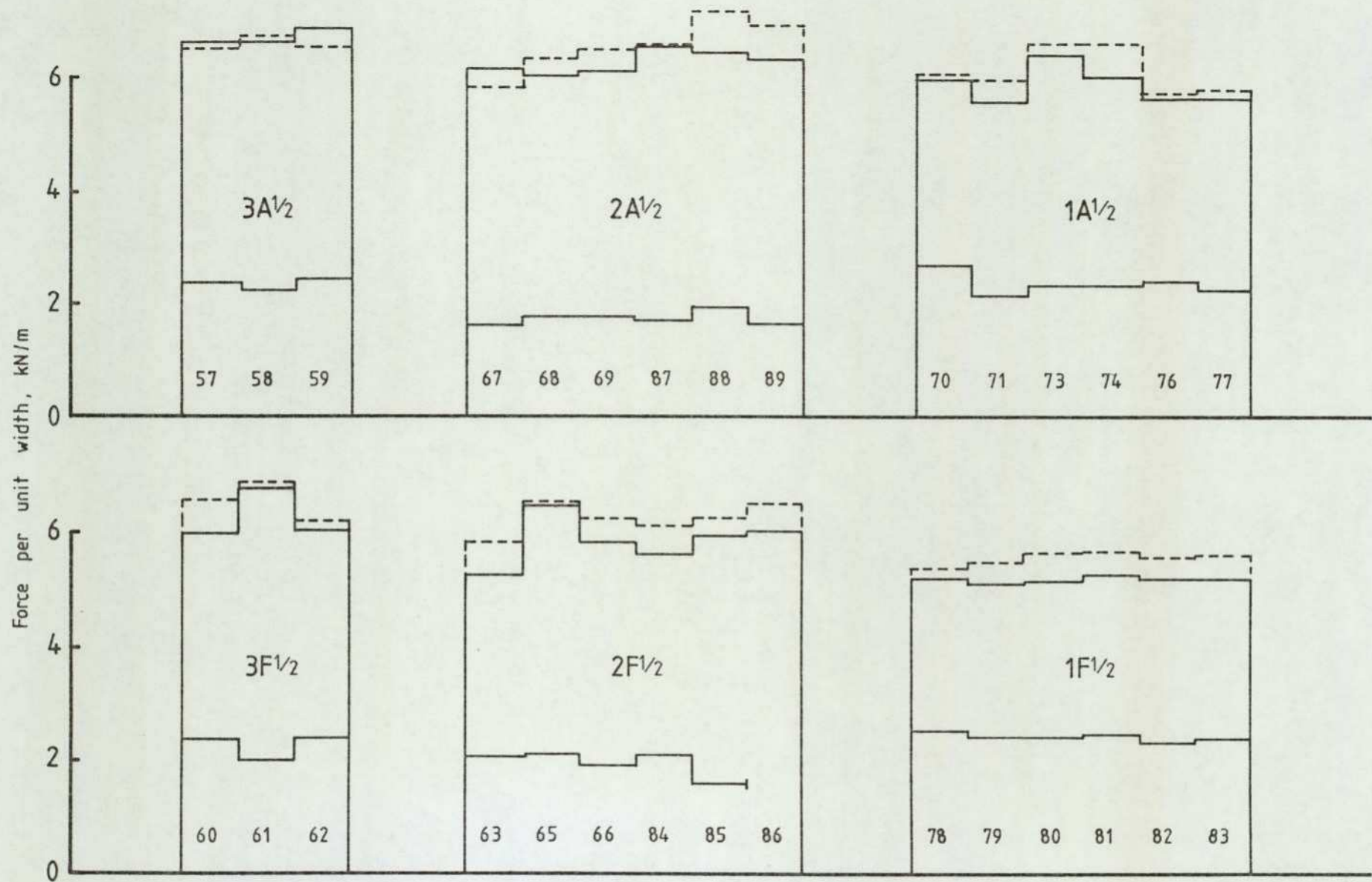


Fig. 6.17 . Horizontal (upper columns) and vertical (lower columns) forces per unit width on the articulated section of the main wall. Top, A^{1/2} configurations. Bottom, F^{1/2} configurations. Broken lines show forces calculated from the stress distributions. Figures show run numbers.

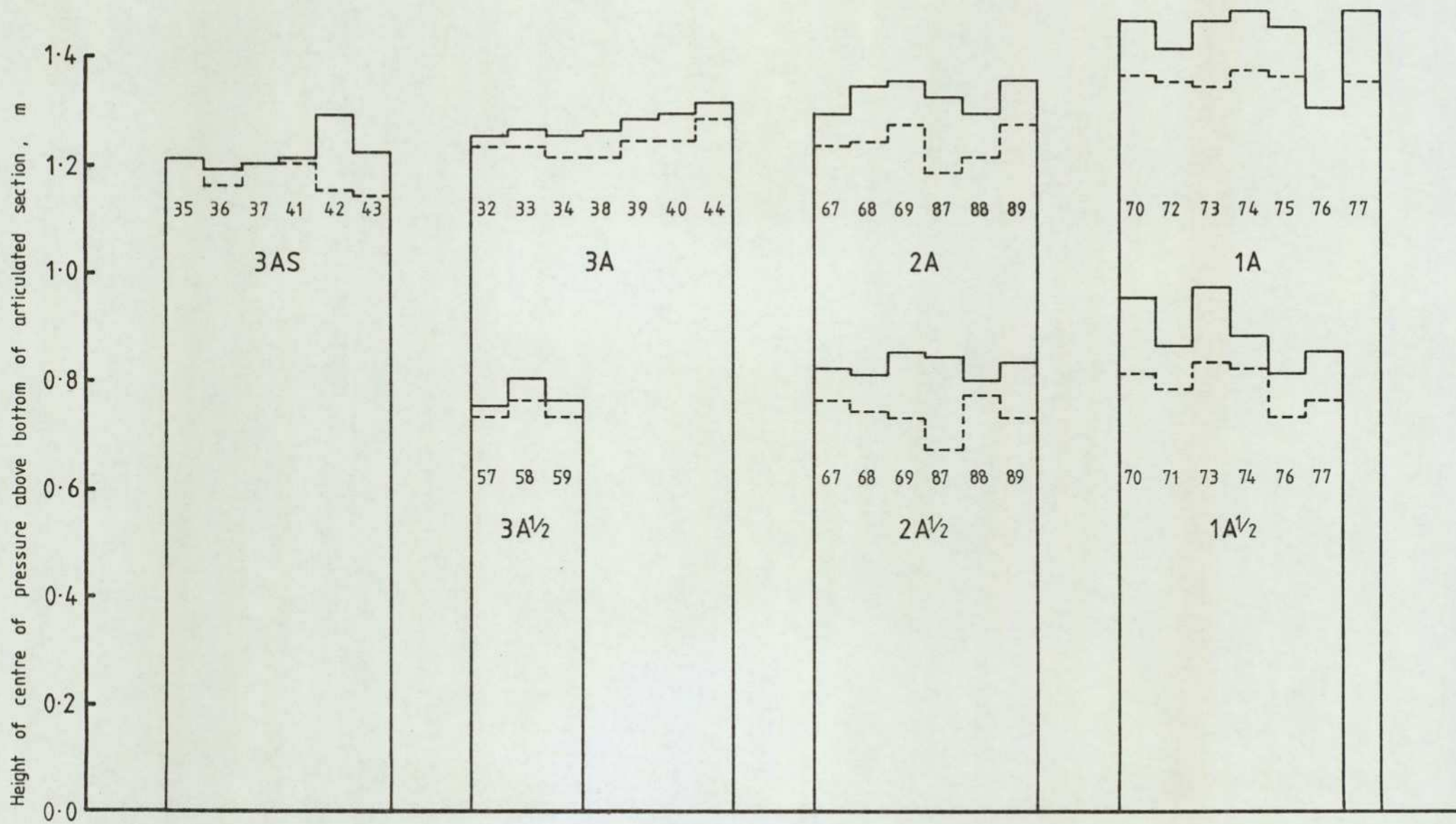


Fig. 6-18. Centres of pressure on the articulated section of the main wall. Broken lines show centres of pressure calculated from the stress distributions. Figures show run numbers.

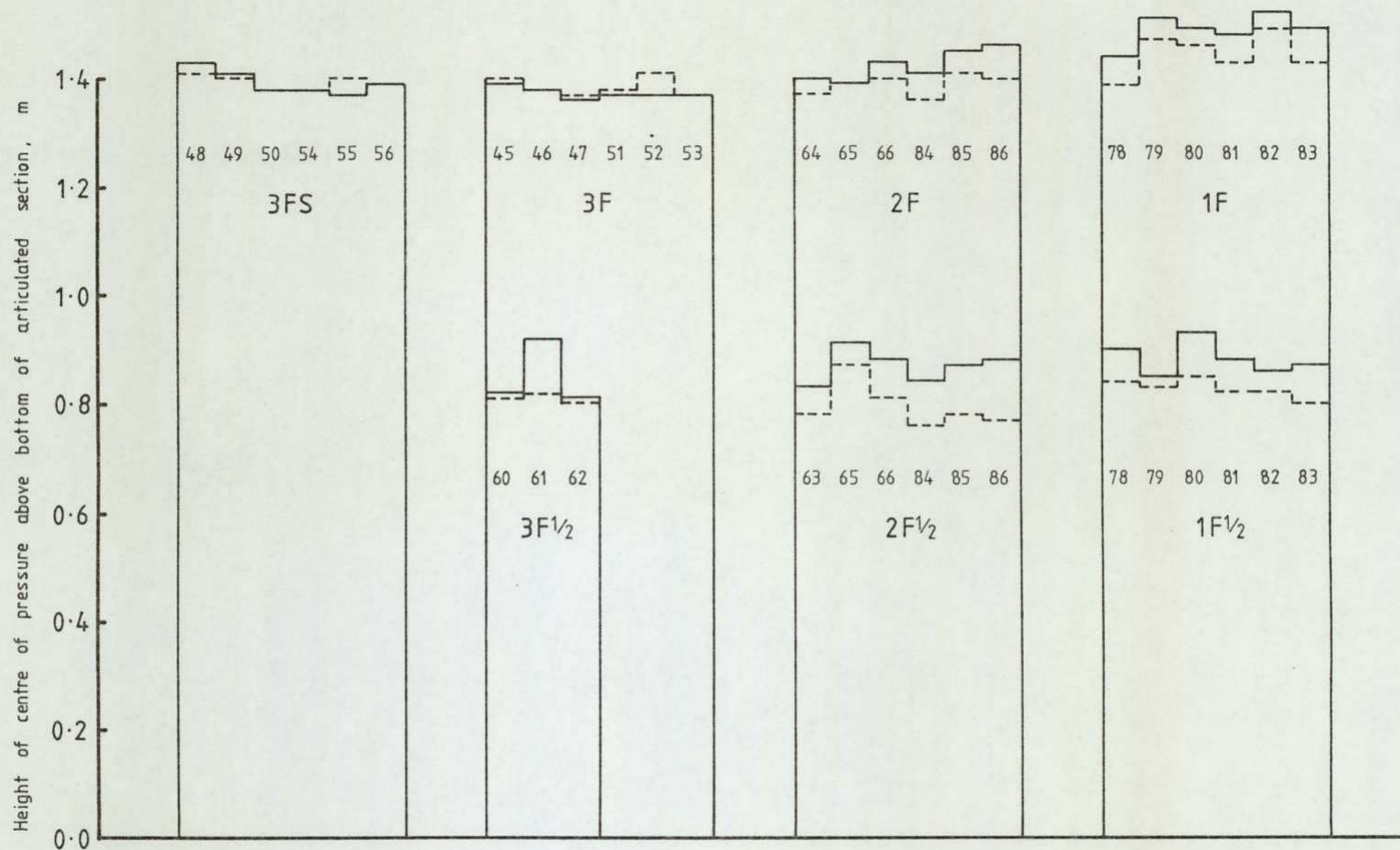


Fig. 6-19. Centres of pressure on the articulated section of the main wall. Broken lines show centres of pressure calculated from the stress distributions. Figures show run numbers.

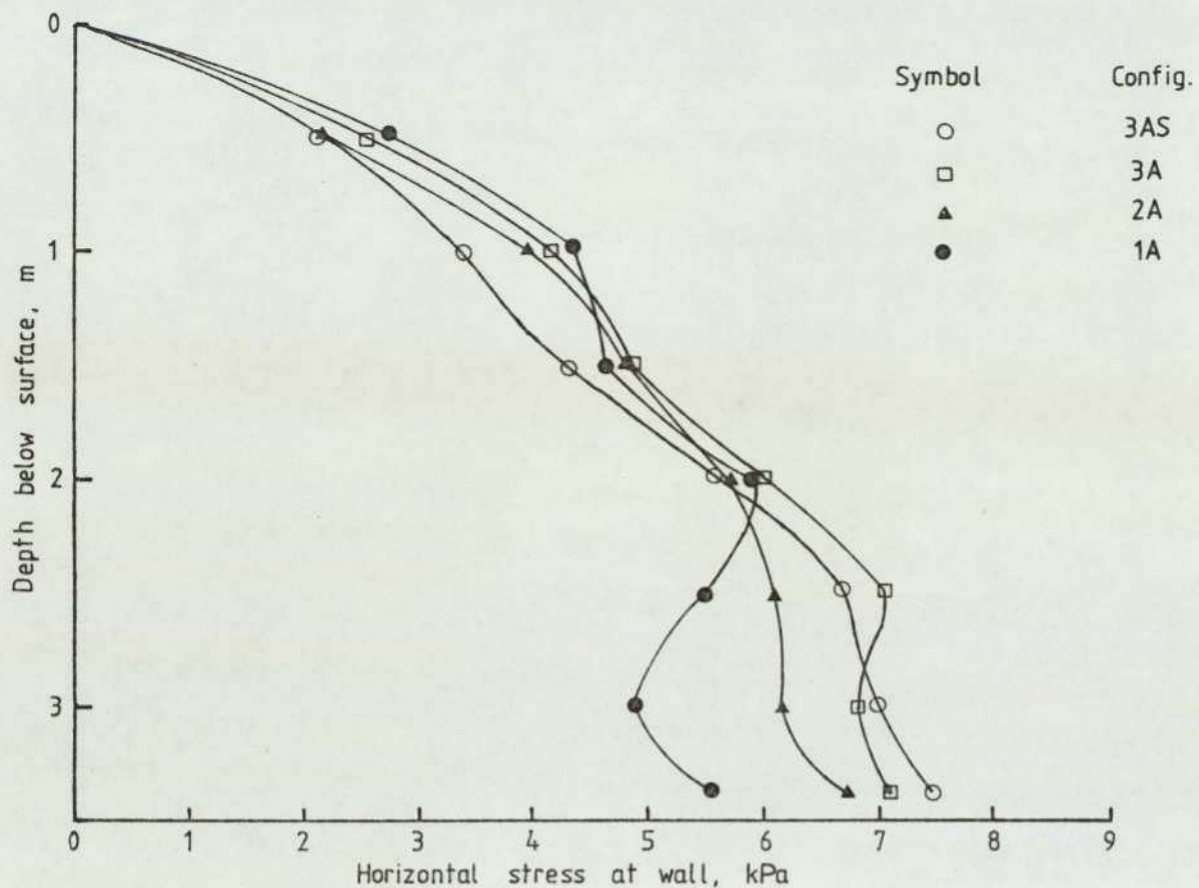


Fig. 6-20. Stress distributions averaged over all the runs in each A configuration.

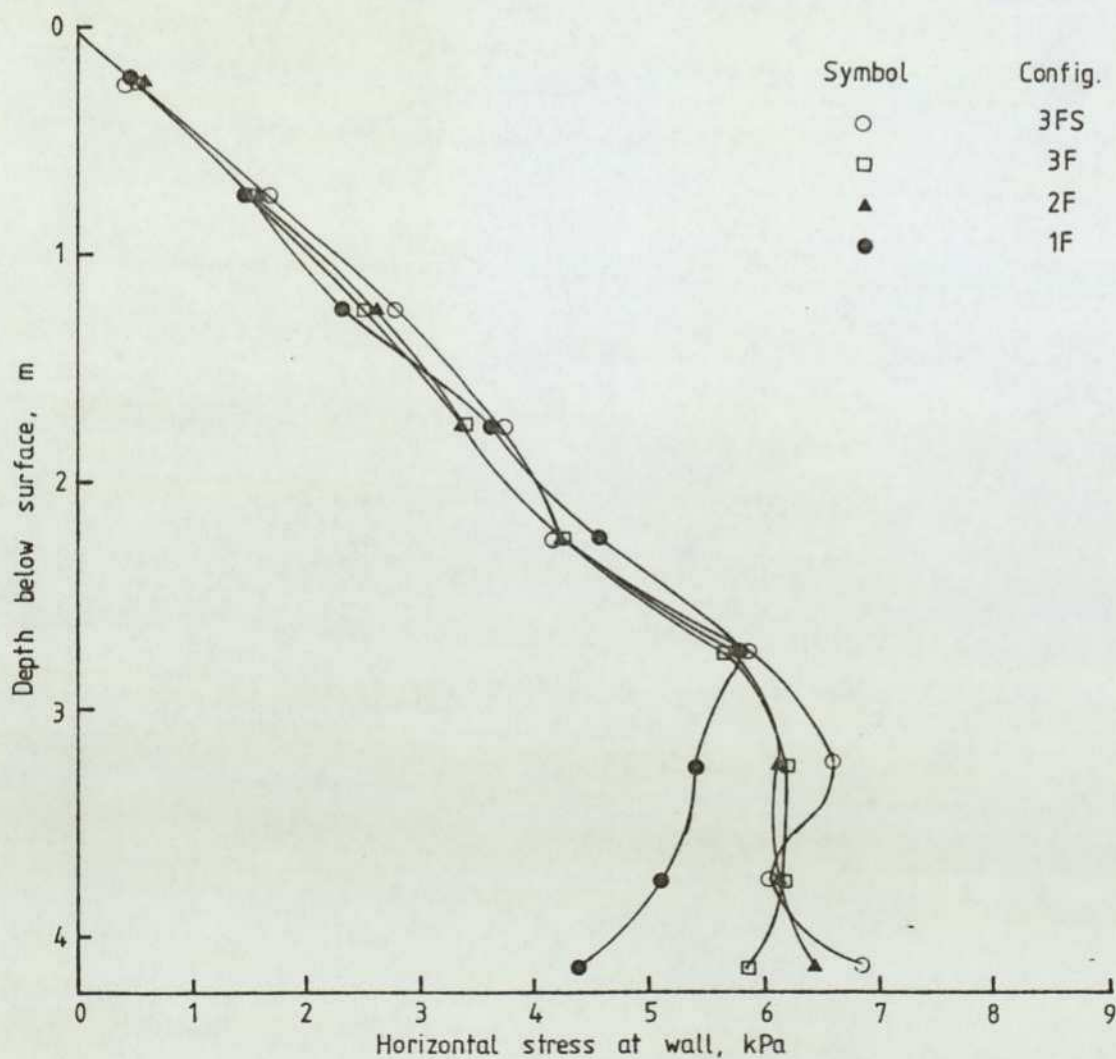


Fig. 6-21. Stress distributions averaged over all the runs in each F configuration.

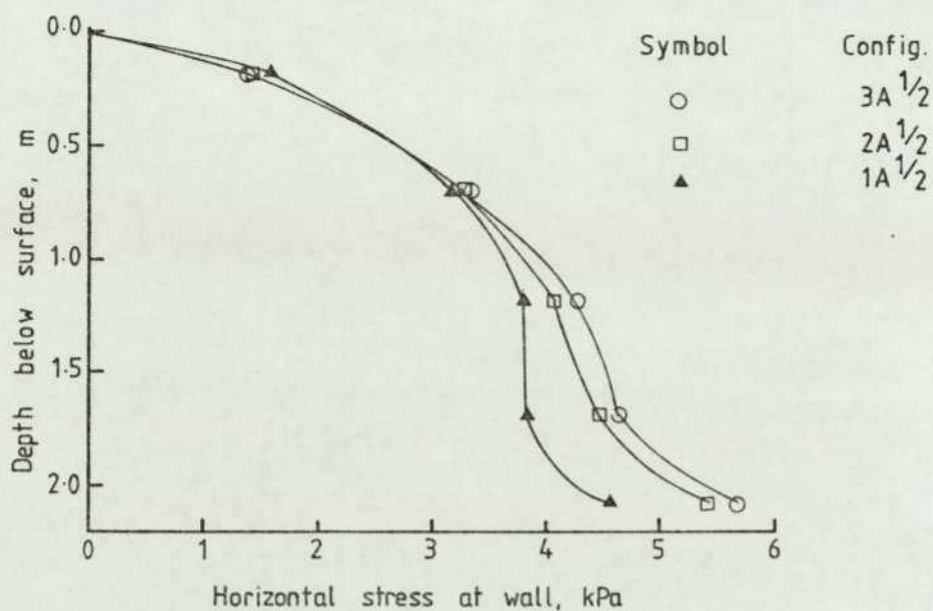


Fig. 6.22. Stress distributions averaged over all the runs in each A^{1/2} configuration.

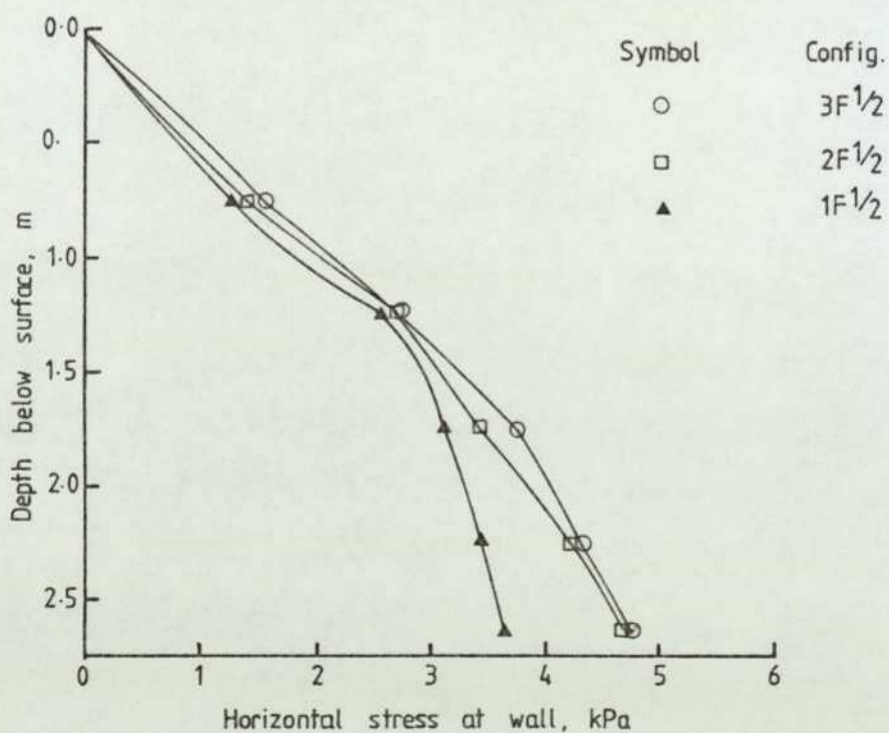


Fig. 6.23. Stress distributions averaged over all the runs in each F^{1/2} configuration.

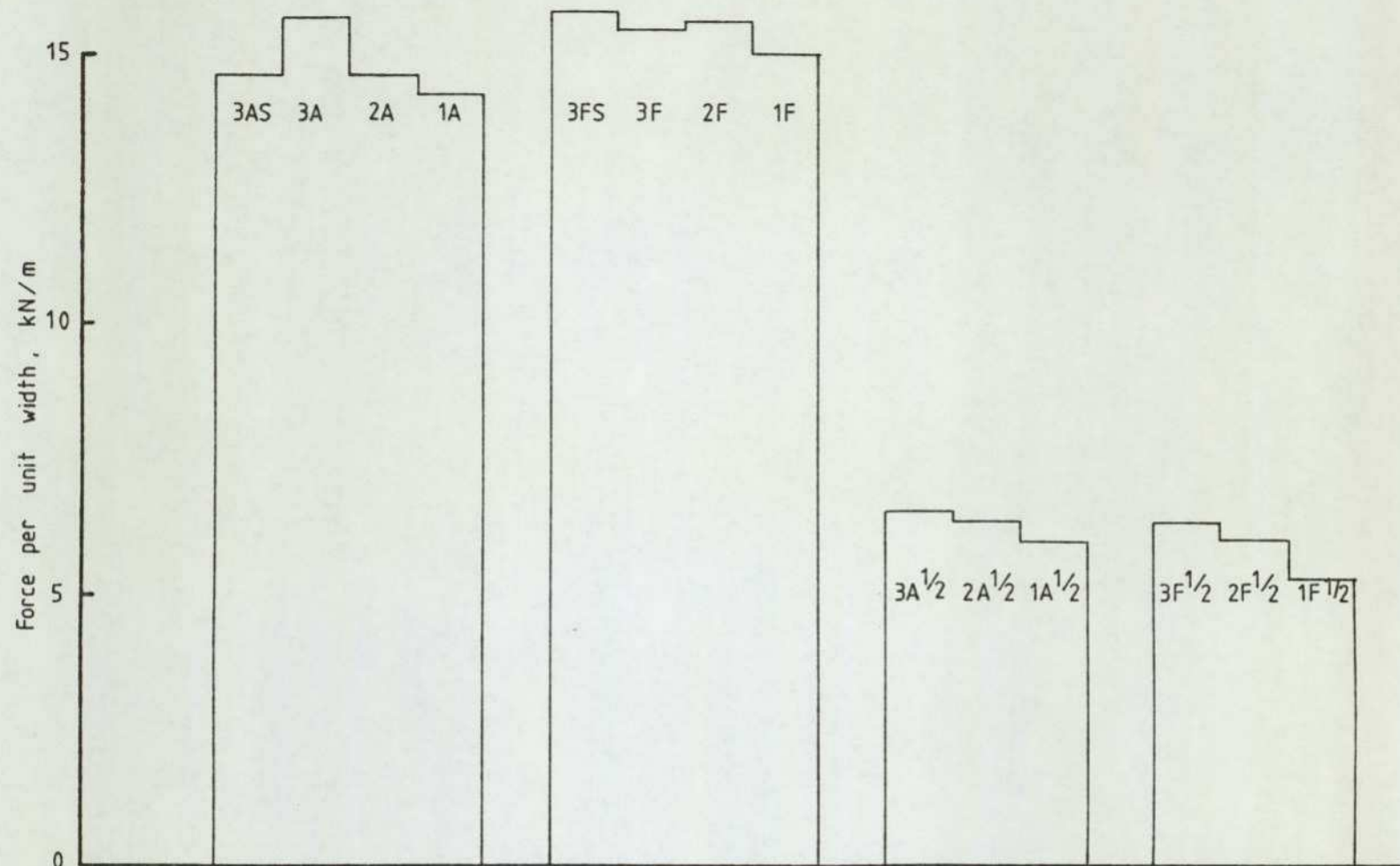


Fig. 6-24. Horizontal forces per unit width on the articulated section of the main wall averaged over all the runs in each configuration.

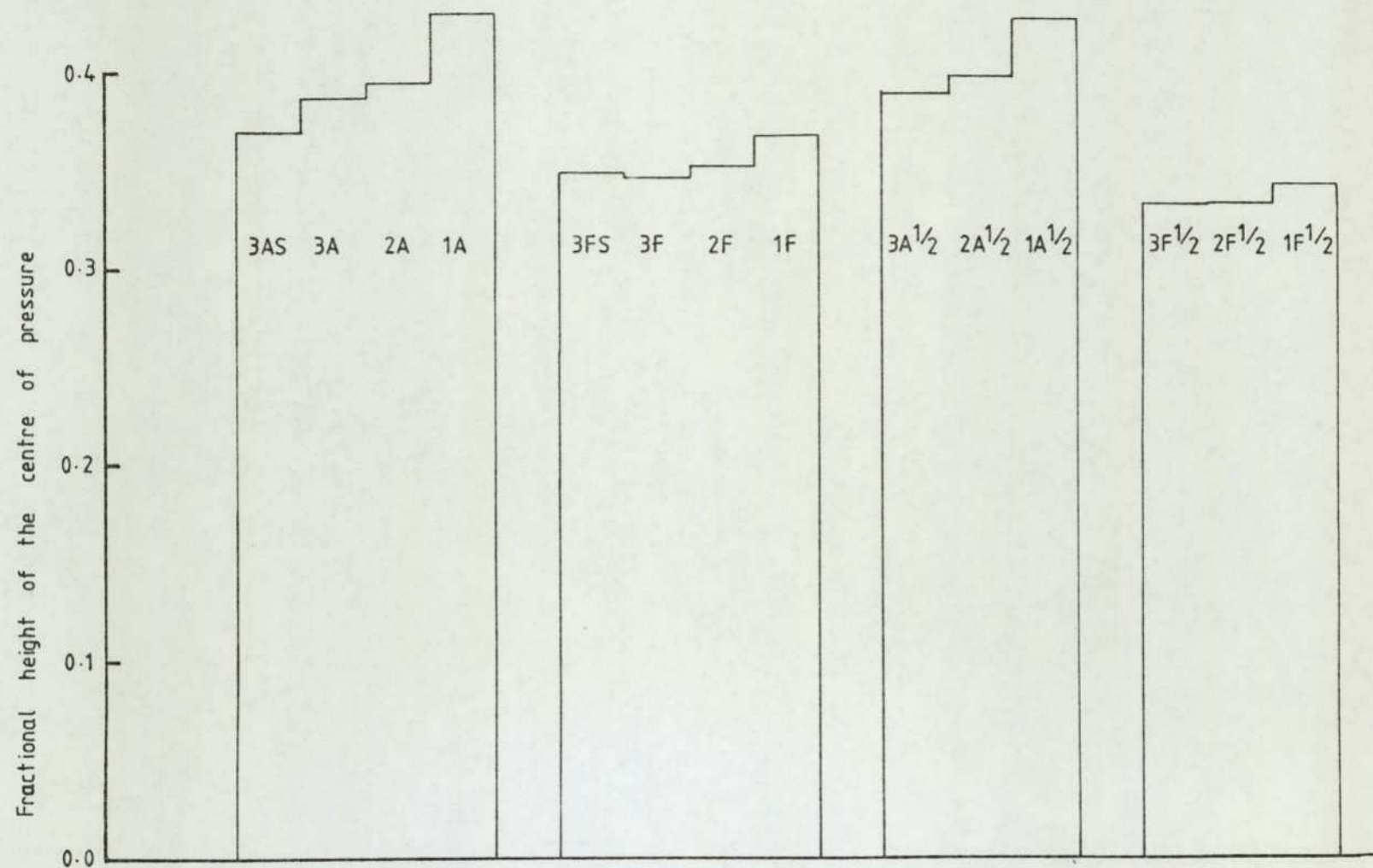


Fig. 6.25. Fractional heights of the centres of pressure on the articulated section of the main wall averaged over all the runs in each configuration.

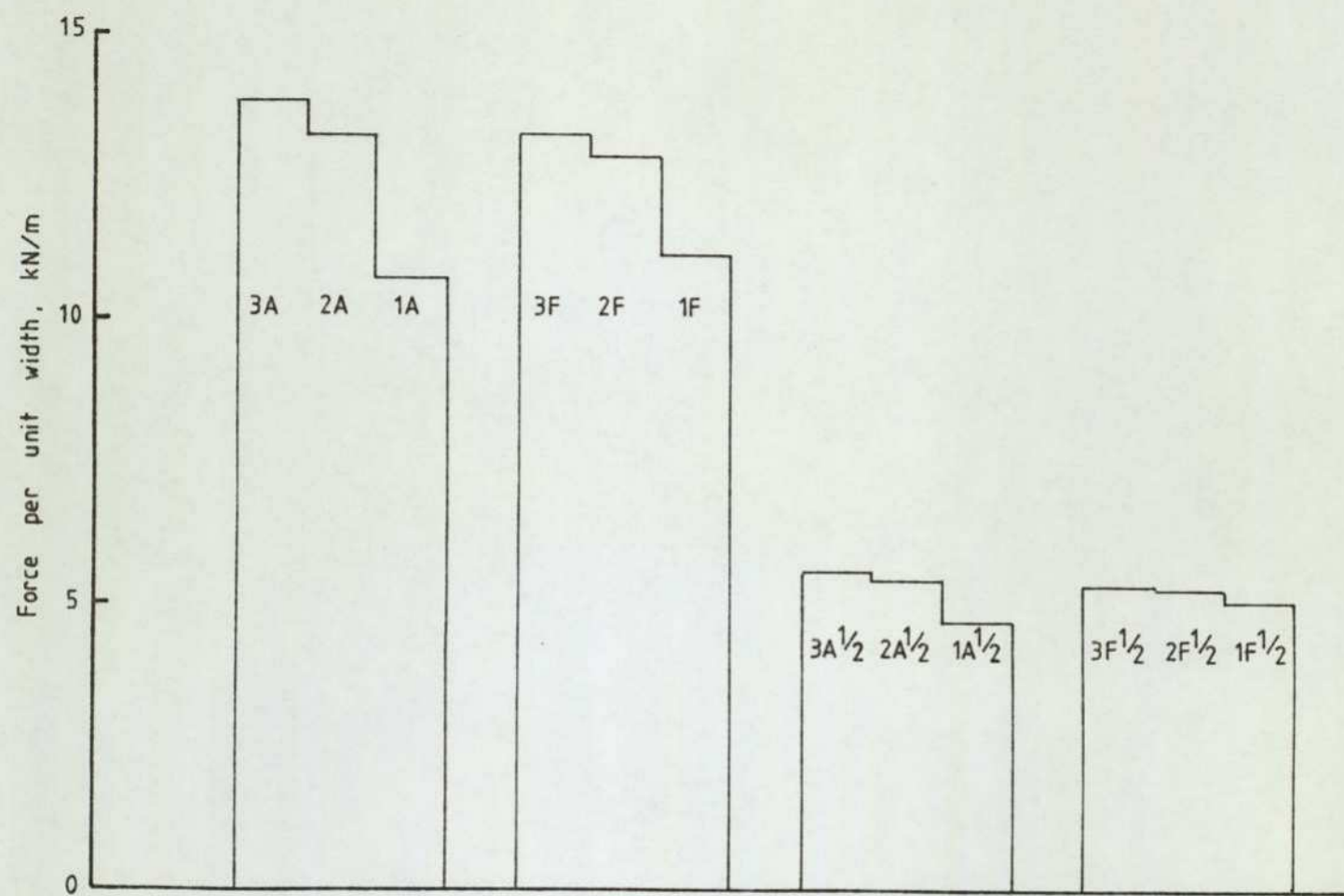


Fig. 6-26. Horizontal forces per unit width calculated using a combination of Coulomb's method and Janssen's formula .

CHAPTER 7

COMPARISON OF MEASURED LOADS WITH THOSE CALCULATED
USING THE FINITE ELEMENT MODEL

7.1 Finite element discretisation

To test the finite element model proposed in Chapter 4, it was used to calculate the wall loads for the same silo configurations as those used in the practical tests of Chapter 6. The way in which the cross section of the silo was divided into elements and the sizes of the load increments are shown in Figs 7.1 to 7.4. Most of the elements were of a similar type to those used in the tests of Sect. 4.4 i.e. quadrilaterals divided into four triangles. The loading was divided into ten increments for the A and A $\frac{1}{2}$ configurations and twelve increments for the F and F $\frac{1}{2}$ configurations. The triangular prism of grain that was already present on the floor at the start of loading in the practical tests was represented by the first load increment.

7.2 Stress/strain properties of the grain and boundary friction

The stress/strain properties of the grain used were those defined in Table 3.1. The coefficient of friction between the grain and the main wall was taken as the average of the values mobilised in the experimental runs with two and three main wall modules in use i.e. 0.27 (Sect.6.3.4). This value was used for the coefficients of friction at the back wall and at the side walls. As the initial runs of the program indicated that the shear stress mobilised at the floor was low compared with the normal stress (Sect. 4.4.2) it was assumed that no slip would occur between the grain and the floor and so the element nodes along the floor were assumed to be fixed in position. This assumption was later confirmed because the calculated shear stresses at the floor were very low compared with the normal stresses.

7.3 Bulk density of the grain

In the standard bushel weight apparatus used in this work (W. & T. Avery Ltd.) the grain falls from a height of only about 200mm into the measuring vessel whereas in the test silo the grain may fall from up to about 4.5m and so could result in a different arrangement of the grain particles in the bulk. However, experiments to measure the in-situ porosity of the grain in the surface layers at different stages of filling (See Appendix) showed that the in-situ porosity was less than 3% different from the porosity of a sample which had been poured lightly into a relatively small container. This was probably because the grain first hit the heap already deposited and then rolled or slid to its final position. Thus for locations in the silo other than at the peak of the heap, the fall of the grain was broken and the porosity was independent of the height from which it had fallen.

As any particular volume of grain is buried during the filling process its bulk density will change as it is compressed. However, from the stress/strain properties measured in Chapter 3, it can be shown that, even at the maximum stresses to be encountered (say a hydrostatic stress of about 16kPa) the compressibility of the grain would result in a volume change and hence a change in the bulk density of only about 1%. It seems reasonable therefore to use the bulk density measured in the standard apparatus in the following calculations.

7.4 Deflections of the main wall

Fig. 6.2 shows linear relationships between the deflection of the wall framework at five heights above the

ground and the height which the grain had reached. Each individual line on this figure is in fact a time history of the deflection of each measuring station on the wall. The information contained in this figure can be presented in a different way (e.g. Fig. 7.5 for the A configurations) where each line on the figure is a piecewise linear representation of the deflected shape of the wall at any instant of time. The instants of time chosen correspond to those at the end of each load increment in the computer simulation.

Fig. 7.5 was used to determine the boundary displacements fed into the computer program for runs in the A configurations. For instance, node 41 was situated 1.1m above the ground (Fig. 7.1), so a horizontal line with ordinate 1.1m was drawn on Fig. 7.5. Until the fifth load increment (step 5) there was no movement at this height so, for steps 1 to 4, the horizontal displacement of node 41 was set to zero. At the end of step 5 the deflection of the steel frame was 0.20mm and so the change in horizontal displacement was also 0.20mm because the displacement at the start of the step was zero. At the end of step 6 the deflection was 0.41mm, giving a change in horizontal displacement for this step of 0.22mm. This change in horizontal displacement then remained the same for subsequent steps up to the final step, number 10. For the F configurations a similar procedure was adopted.

A problem arises with the deflection that should be assigned to the uppermost boundary in the current top layer. The displacements of the boundary nodes are those which occur between the start of a step, just before a new layer is added, and the end of a step, after the addition of a

new layer. Obviously, deflections of the wall at any particular level that occur before the grain has reached that level have no effect (except on the dimensions of the silo where the effect is negligible). In a real continuous system, grain would reach the position of the uppermost boundary node at a time corresponding to the end of the step and there would be no displacement during the step occurring after the grain had reached that level. Because of this the displacement of the uppermost boundary node in the current top layer was always set to zero.

The small extra deflection of the plywood panels with respect to the steel frame was included as follows. The computer program was run without this deflection and the average stress over the area of each panel was determined as the mean of the horizontal stress at each node situated on that panel. The deflection of the centre of each panel was calculated using a rate of $52\mu\text{m/kPa}$ (Sect. 6.2.1) and the extra displacement of each node on the panel was taken as half this centre deflection. The computer program was then run again using the sum of the displacements from the steelwork and the plywood panels.

The boundary displacements for the configurations using springs in the main wall struts (3AS and 3FS) were calculated by assuming initially that the articulated section of the main wall deflected by rotating as a rigid slab about the lower pin joint. From the geometry of the main wall and struts (Fig. 5.2) it can be shown that the horizontal movement Δx of a point distance y above the lower pin joint is given by

$$\Delta x = \frac{y\Delta l}{2.0} \quad \dots (7.1)$$

where Δl is the change in strut length. Fig. 6.3 was used to calculate Δl for each load increment, for instance, in the 3AS configuration the strut length had changed by 42mm up to the end of step 7 (grain height 2.7m). The change in strut length during step 8 was therefore 5.6mm. The horizontal displacements of the boundary nodes due to this rigid slab displacement were calculated by using this value of Δl along with the height of each boundary node above the lower pin joint for y in Eqn 7.1. Again the displacement of the uppermost boundary node in the current top layer was set to zero. It is to be expected that the large rotation of the wall will swamp any extra effect caused by relatively slight bending of the steelwork, but for completeness the total displacements of the boundary nodes were obtained by adding the deflections of the steelwork and the plywood panels measured when the springs were absent.

7.5 Results and discussion

7.5.1 No rotation of the main wall

The stress distributions calculated with the finite element program for the runs with no springs in the struts are shown in Figs 7.6 to 7.9. The calculated stress distributions exhibit the following similar characteristics to the measured ones (Figs 6.20 to 6.22):-

- a) The horizontal stresses increase reasonably linearly from the grain surface but the rate of increase falls near the floor.
- b) The rate of stress increase near the surface is greater in the A and A $\frac{1}{2}$ configurations than in the F and F $\frac{1}{2}$ configurations.
- c) Reducing the side wall spacing reduces the stresses

on the main wall and the effect is greater near the floor.

d) The reduction in stress due to changing the side wall spacing is less in the A $\frac{1}{2}$ and F $\frac{1}{2}$ configurations than in the A and F configurations.

In general the calculated and measured stresses are very similar in magnitude. A closer examination shows that the agreement is excellent down to about half the grain depth and from there to the floor the calculated stresses become larger than the measured ones by an average of about 6%.

The larger magnitude of the calculated stresses is probably due to the silo being left for about an hour after filling before the measurements were taken (Sect. 6.1.2). A future part of this work will be an investigation of the dynamic loads on the test silo and, in preparation for this, a summing amplifier had been constructed which added the outputs of the overall force transducers in the correct proportions to give a signal which indicated the total horizontal force on the main wall. This device was being tested during the last few runs reported in Chapter 6 and recorded reductions in the total horizontal force of 9.7%, 4.3%, and 3.3% in the 2A configuration and 4.4% in the 2F configuration between the time that loading was completed and when the stress measurements were started. This reduction was probably caused by creep occurring in the grain bulk acting to relieve the stresses and it explains the greater part of the differences between the calculated and measured results. Other possible reasons for the measured loads being lower than the calculated ones are:-

a) Deflections in the side and back walls of the test silo. These were not measured but Figs 6.20 and 6.21 show

that even when the main wall was rotated by up to 2.1% of the grain height there was little effect on the horizontal stresses. Reductions in stress due to side and back wall movements are therefore expected to be slight.

b) Instrumentation errors. Sect. 6.3.1 showed that forces calculated by integrating the stress distributions were about 6% higher than those measured by the overall force transducers. Because it was impossible to apportion this difference between the two instrumentation systems, the true forces were taken as the average of the two figures and the stresses presented in Figs 6.20 to 6.22 were factored down by about 3% to suit. It is possible that integrating the stresses was the more accurate of the two systems in which case the measured loads should have been slightly higher.

c) Changes in stress/strain properties of the grain. Sect. 6.1.1 stated that the number of broken grains in the sample at the end of the experimentation was considerable while that at the start was negligible. This change slightly affected the shear strength and bulk density and may have affected the stress/strain properties of the material. Had the properties of the damaged sample been measured, they may have lead to lower calculated stresses.

The calculated values of the horizontal forces per unit width (Fig. 7.12) are nearly all slightly higher than the measured ones (Fig. 6.24) reflecting the higher calculated stresses. The relative magnitudes of the forces in each configuration at the same width are predicted well, for example the measured forces for the 3A, 3F, 3A $\frac{1}{2}$, and 3F $\frac{1}{2}$ configurations are in the ratio 1.00:0.99:0.42:0.41

and the calculated forces are in the ratio 1.00:1.02:0.43:0.43 for the same configurations. However, the effect of decreasing the side wall spacing is greater with the calculated forces. This could be because the shear stress produced by the side wall friction was assumed to vary linearly across the width of the silo (Sect. 4.3.6) whereas qualitative visual evidence suggests that the effect of the shear stress was only felt for a relatively short distance away from the side walls (Sect. 6.3.3). However, this fairly short penetration distance of the shear stress should not render the side wall friction part of the program completely invalid because the extra body force term induced by the shear stress depends on the rate of change of the shear stress across the width and not on its actual value. If the shear stress falls off rapidly near the side walls then the body force is high which compensates to some extent for the short distance over which the body force acts. As stated by Bransby and Smith (1975) the method of correction for the side wall friction should underestimate its effect near the side walls and overestimate it near the middle of the silo, however, it seems that the average of the effect across the width is overestimated.

The calculated positions of the centres of pressure (Fig. 7.13) are, like the measured ones (Fig. 6.25), slightly above one third of the height up the wall. The agreement between the calculated and measured heights is good but the increase in the heights calculated for the A and A $\frac{1}{2}$ configurations over those in the F and F $\frac{1}{2}$ configurations is greater in the measured figures than in the calculated ones. The increase in the heights of the centres of pressure

observed when the side wall spacing was reduced is also predicted well.

7.5.2 Rotation of the main wall

The calculated stress distributions (Figs 7.10 and 7.11) for the configurations with springs in the main wall struts were disappointing. The stress levels were much lower than those measured and the stress distributions were very uneven. A closer examination of the calculated stresses in each element showed that in many of them the octahedral shear stress was such that the material had gone past a state of failure (i.e. F_m was greater than 1.0) and that the large boundary displacements had given rise to tensile stresses in some elements. Obviously, in a continuous system F_m cannot exceed 1.0 as this would require infinite shear strains, nor can tensile stresses develop in the material. The computer program has features that seek to prevent both these occurrences (Sect. 4.3.4) but they do not become effective until the step after either F_m becomes greater than 1.0 or a tensile stress develops. For instance, if a tensile stress is calculated in an element the nodal forces causing that stress are re-distributed among the neighbouring elements on the next step. The stresses in the element are then set to zero and stress increments are added on to these zero stresses for subsequent steps. This results in stresses that are lower than those which occur in practice. If the load increments are small enough this approach gives only small errors. However, smaller increments require smaller elements and so more nodes and more equations. This leads to solution times which are unacceptably long and so there is clearly a need for further work to establish a more

realistic method for dealing with elements beyond failure and those where tensile stresses are about to develop.

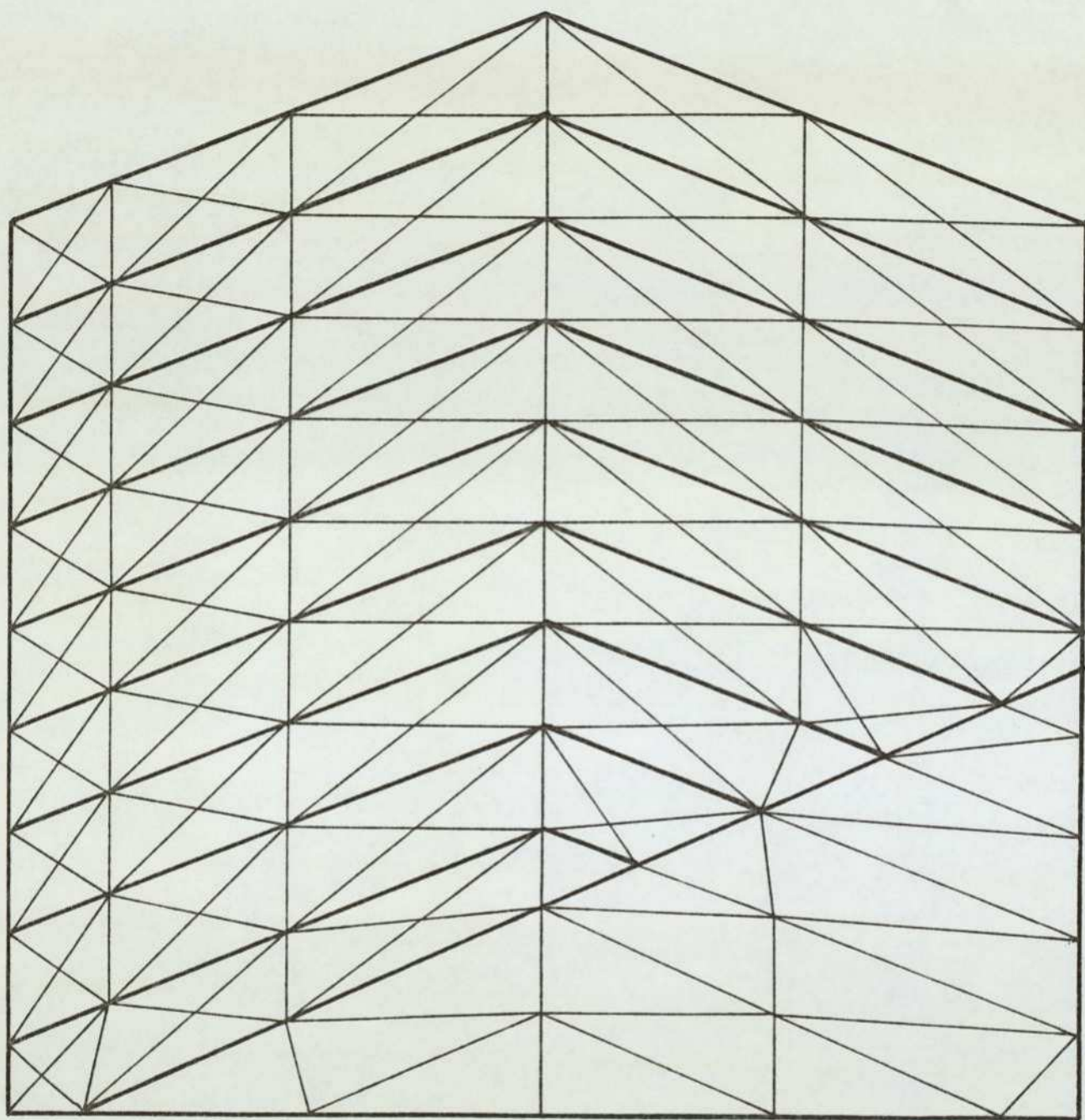


Fig. 7.1. Division into finite elements for the A configurations.
—— element boundaries ; —— load increments.

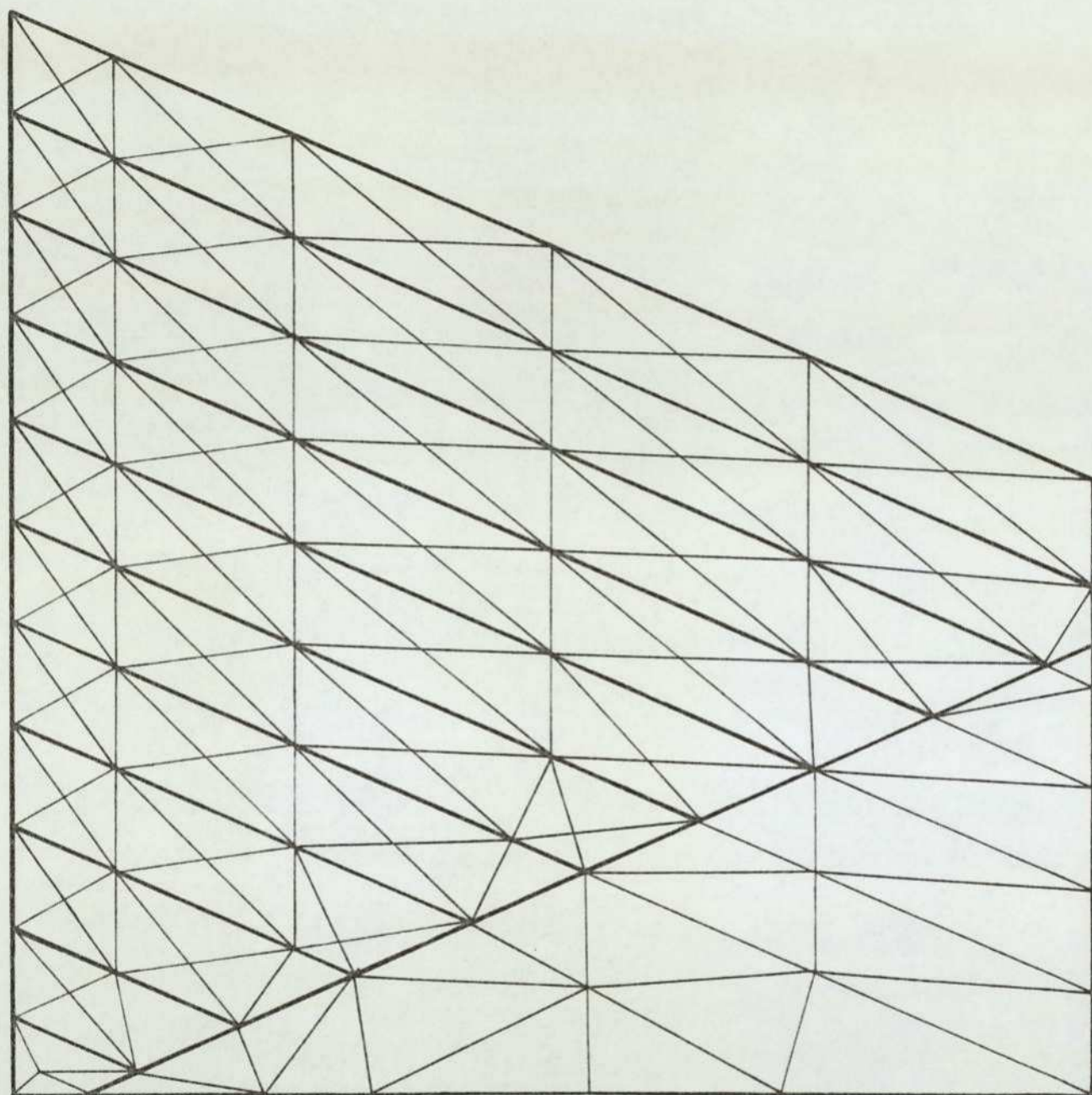


Fig. 7.2. Division into finite elements for the F configurations.
—— element boundaries ; ——— load increments.

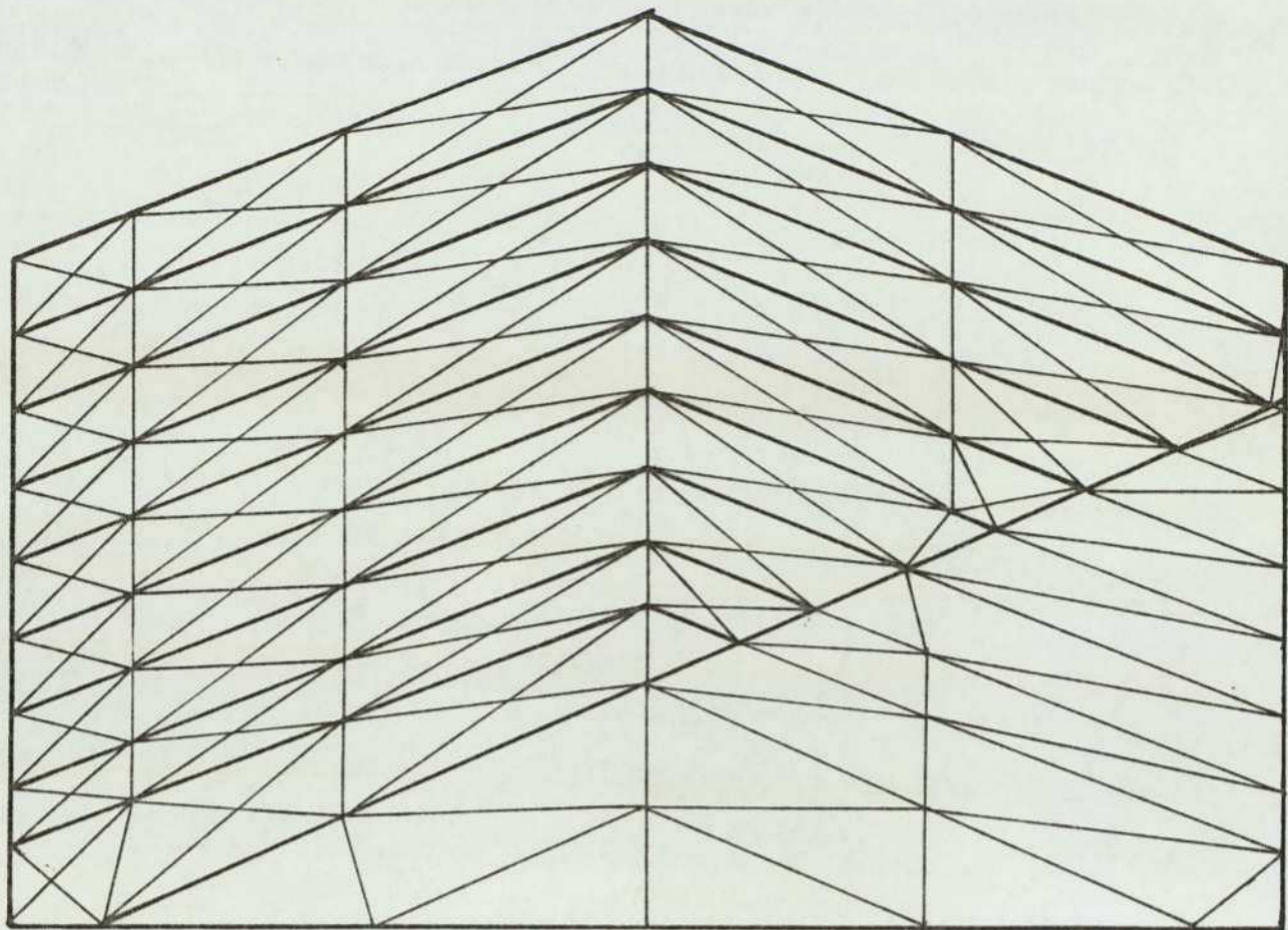


Fig. 7.3. Division into finite elements for the $A^{1/2}$ configurations. — element boundaries; — load increments.

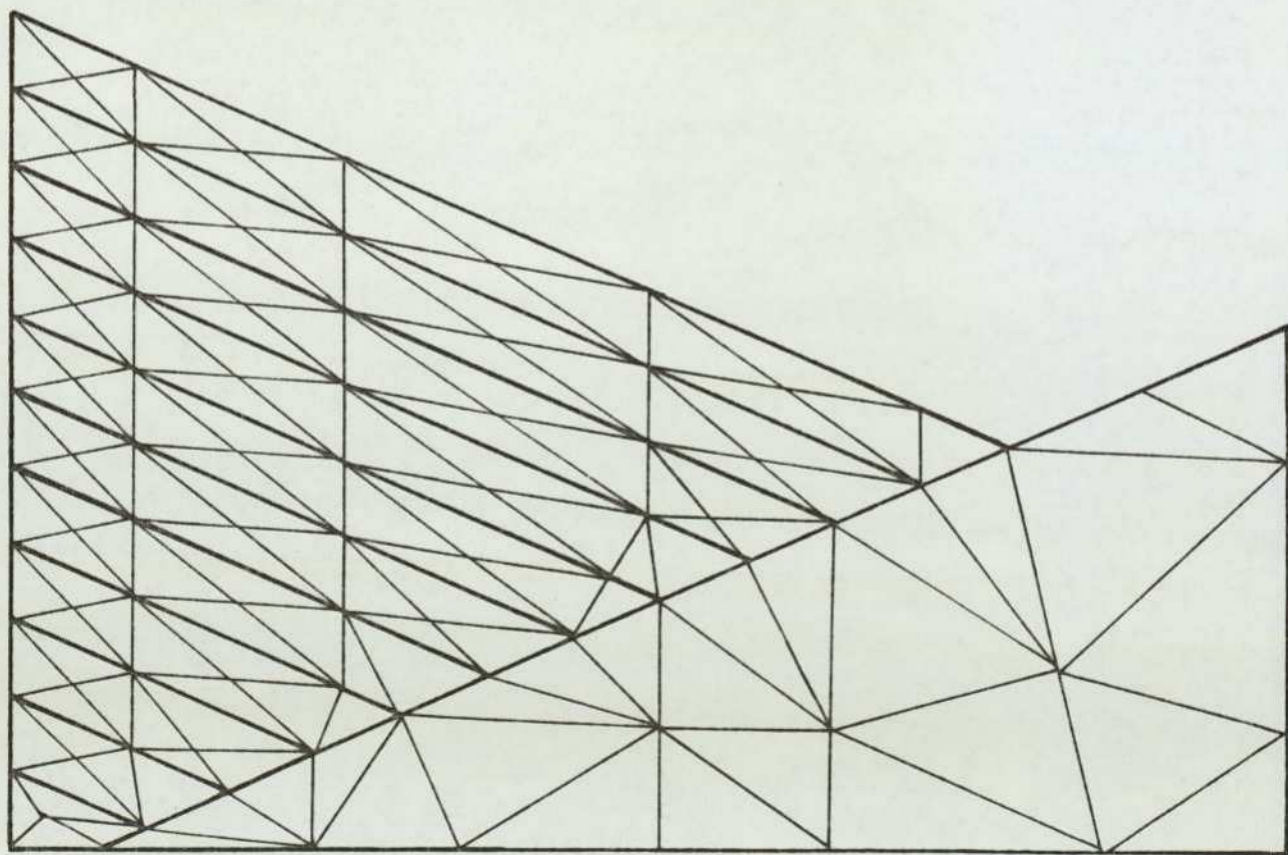


Fig. 7.4. Division into finite elements for the $F^{1/2}$ configurations. — element boundaries; — load increments.

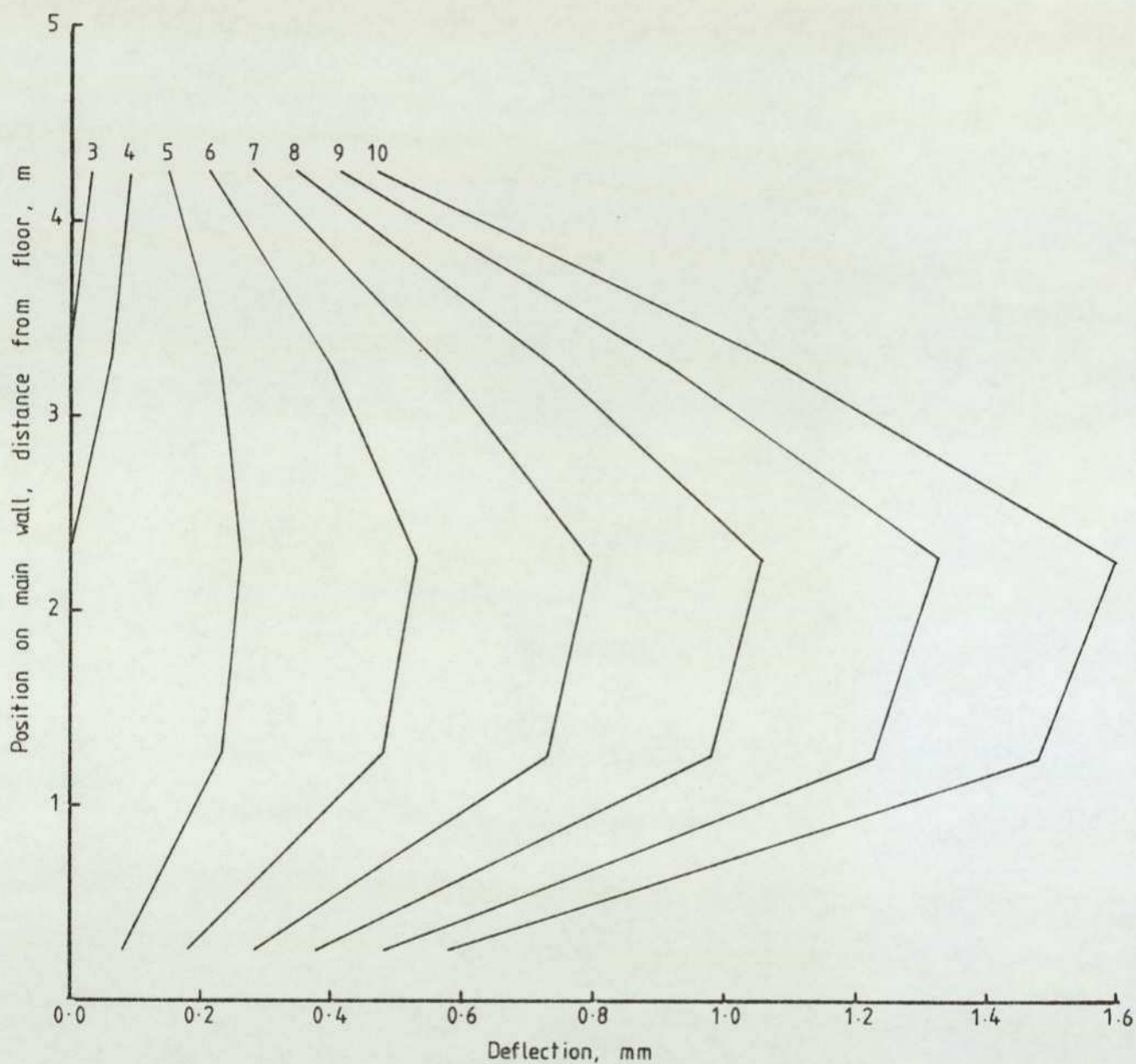


Fig. 7.5. Piecewise linear representation of the deflected shape of the steel frame of the main wall at each load increment in the 3A configuration. Numbers on lines are the load increments.

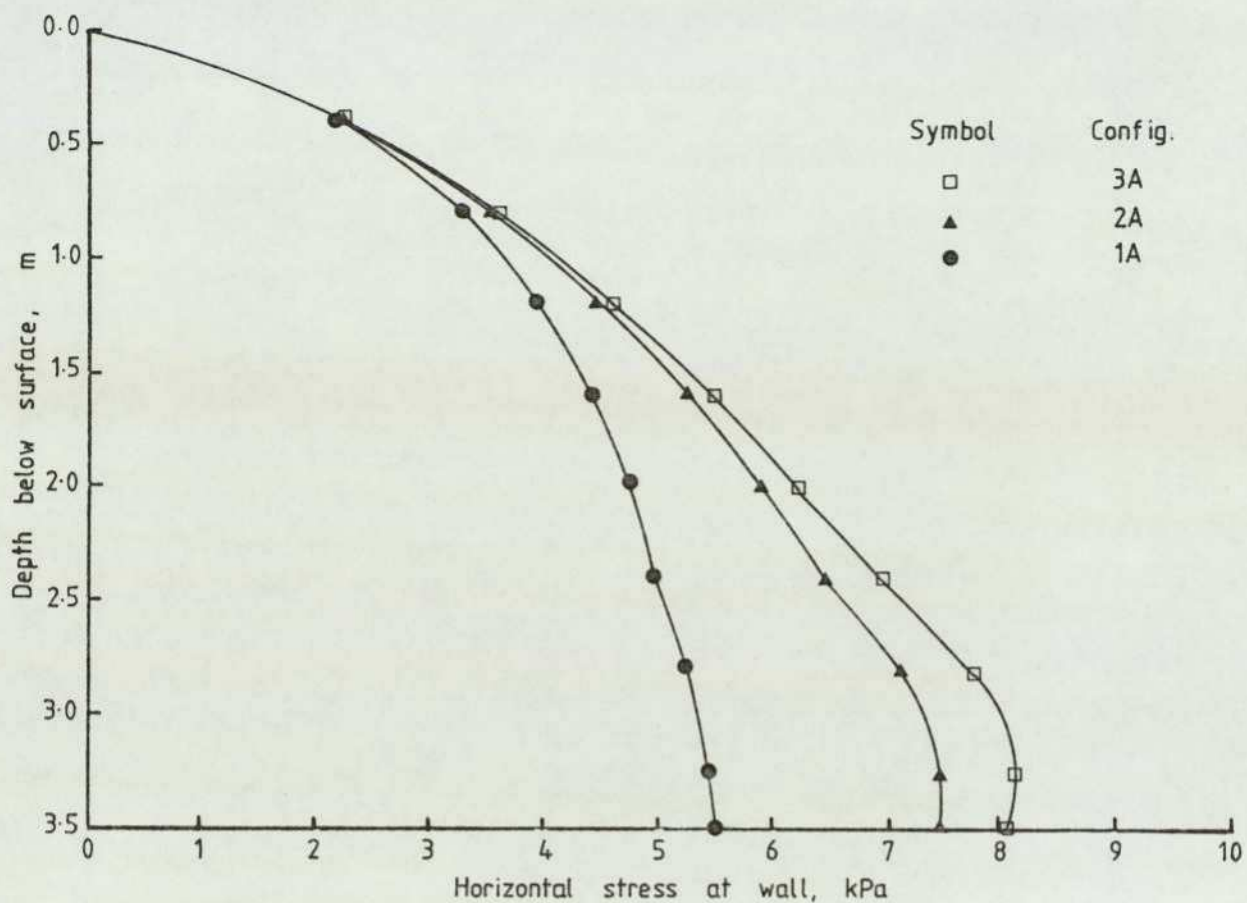


Fig. 7.6. Stress distributions calculated using the finite element program for the configurations of the practical tests.

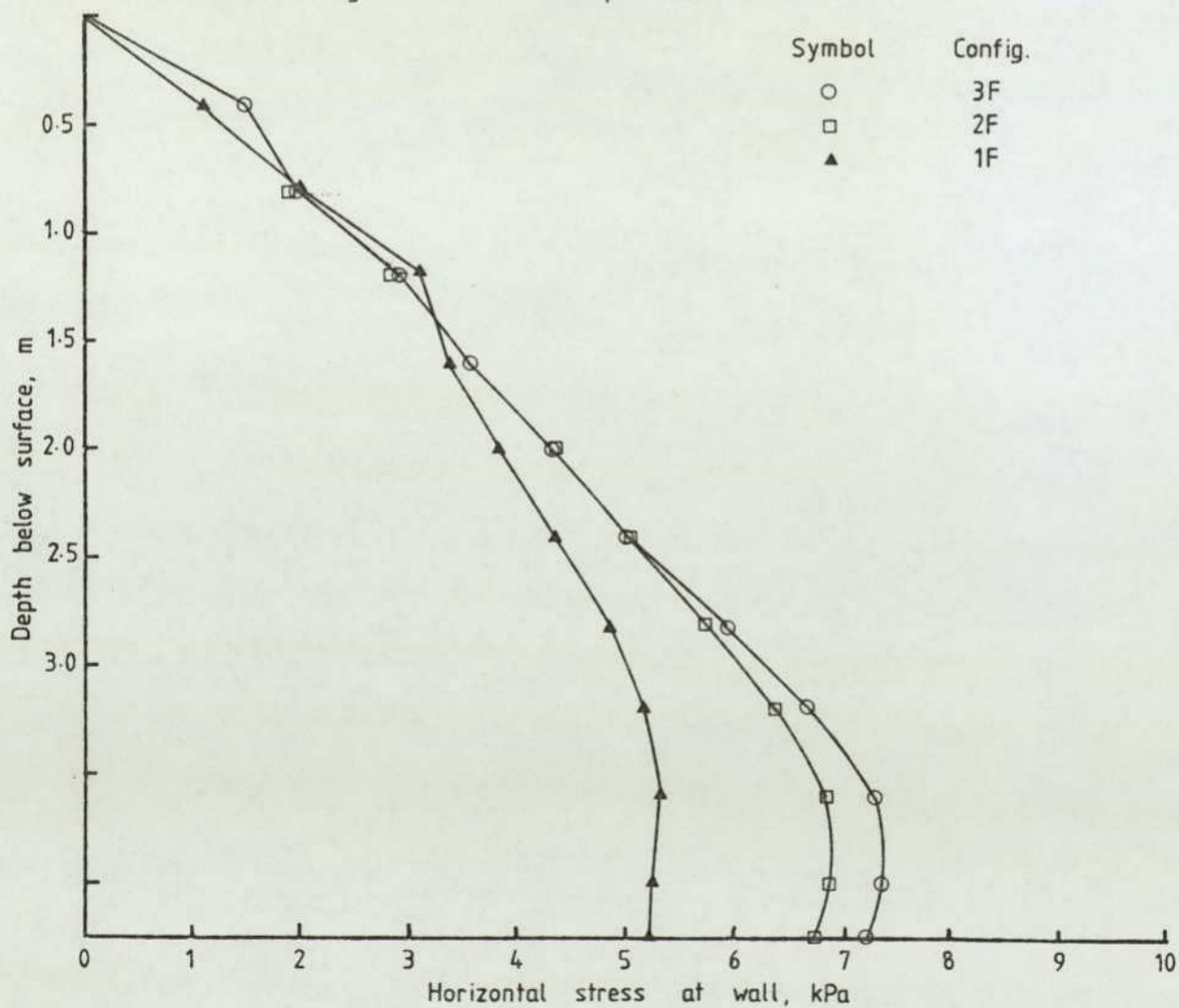


Fig. 7.7. Stress distributions calculated using the finite element program for the configurations of the practical tests.

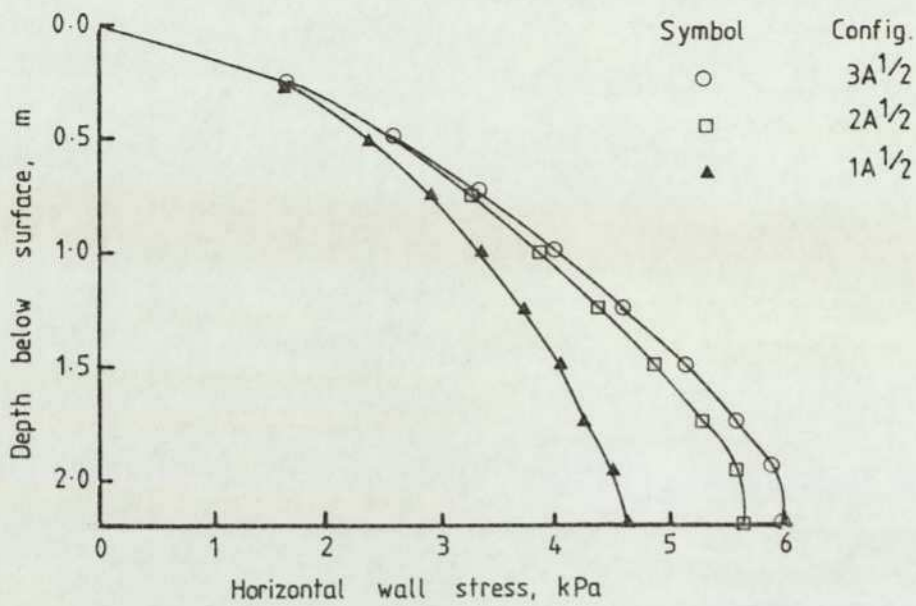


Fig. 7.8. Stress distributions calculated using the finite element program for the configurations used in the practical tests.

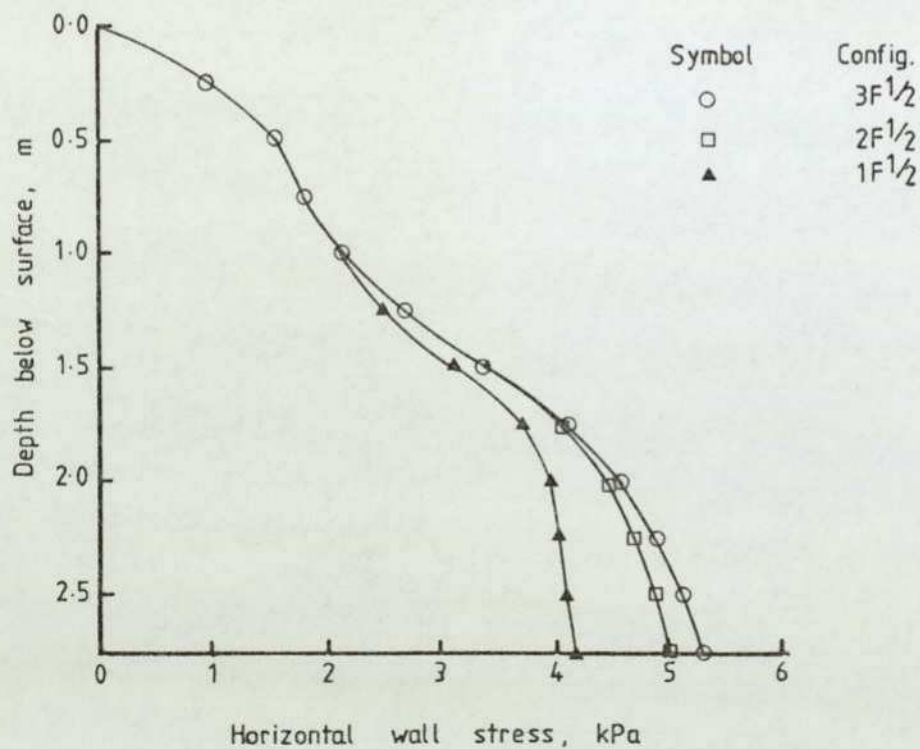


Fig. 7.9. Stress distributions calculated using the finite element program for the configurations used in the practical tests.

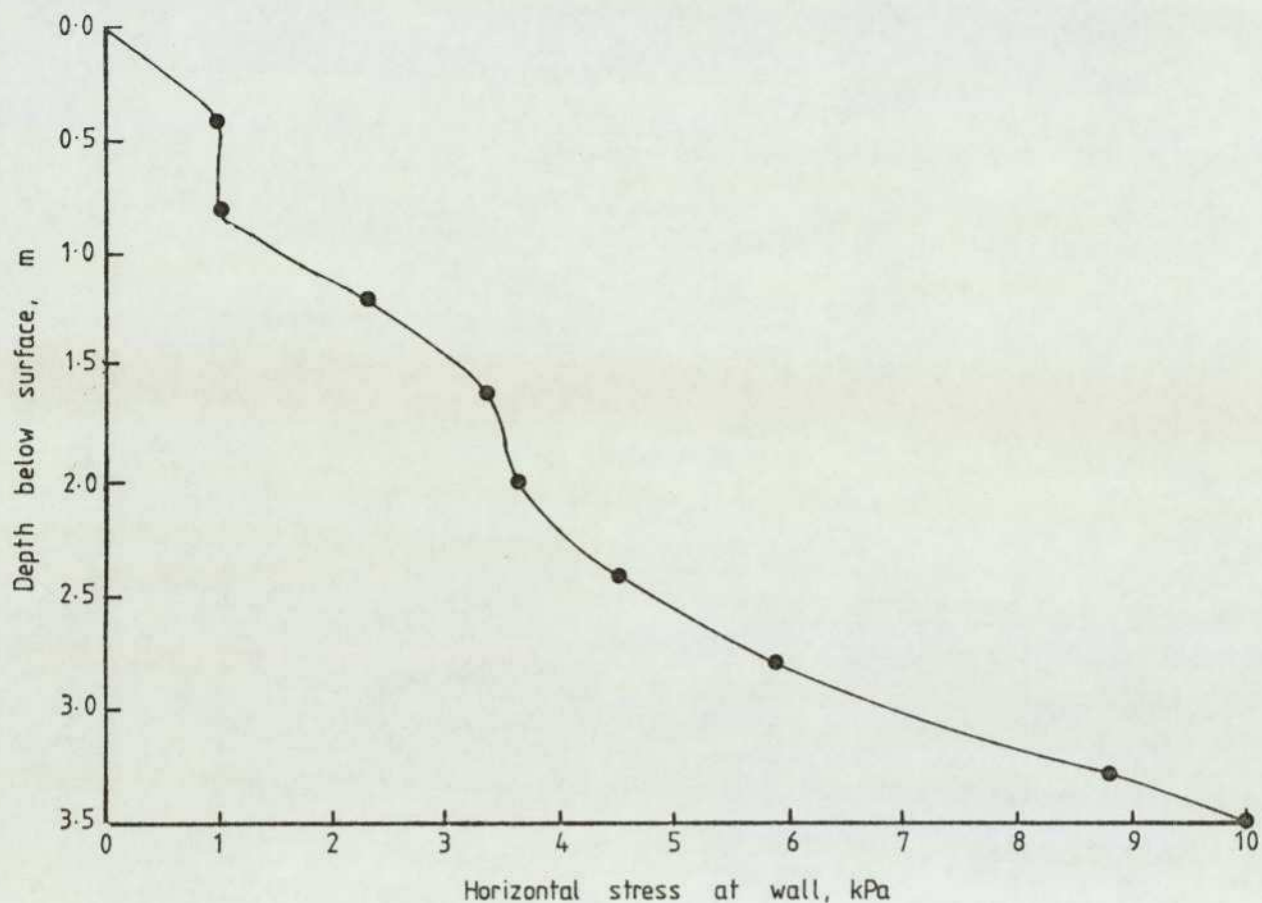


Fig. 7.10. Stress distribution calculated using the finite element program for configuration 3AS of the practical tests.

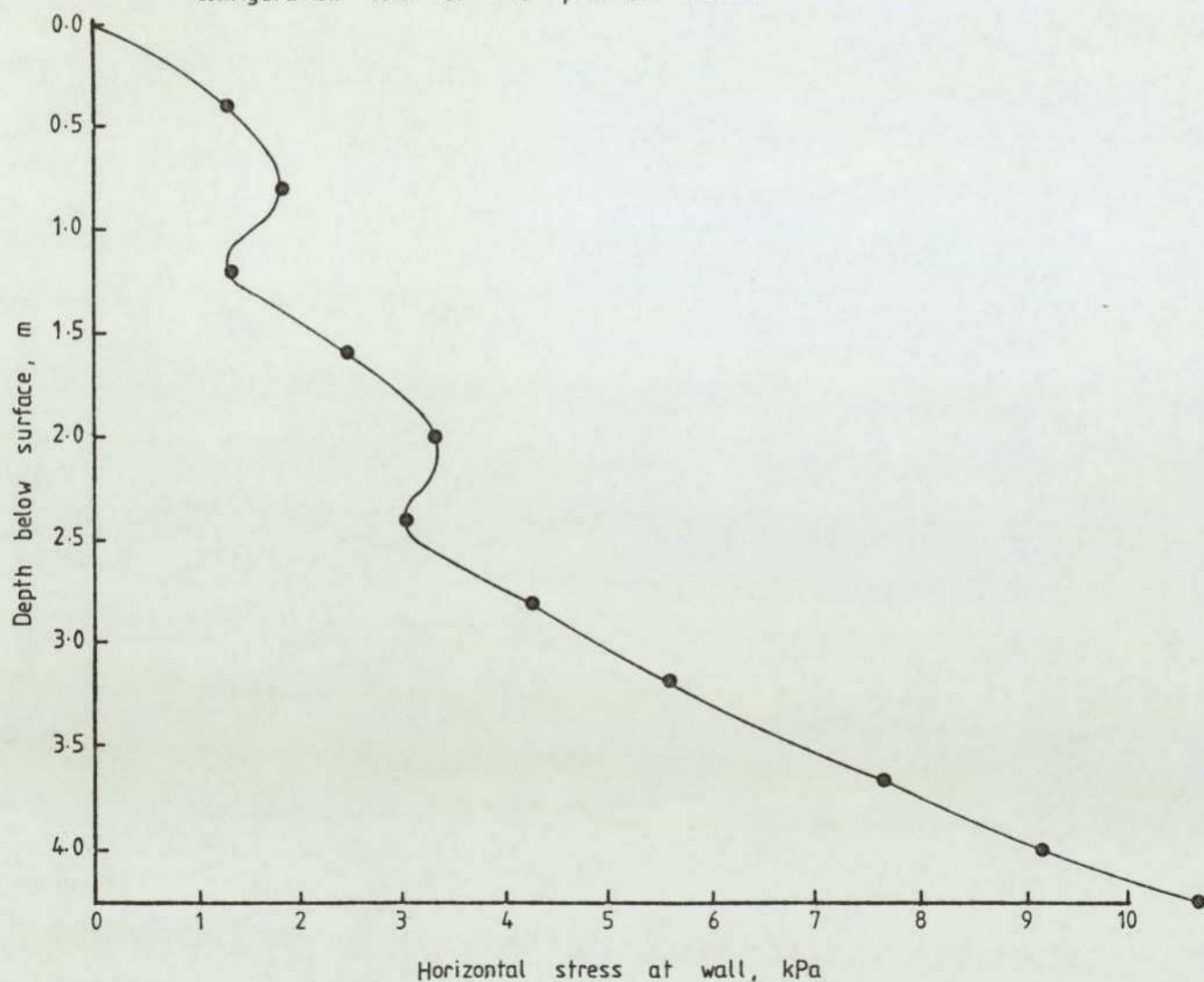


Fig. 7.11. Stress distribution calculated using the finite element program for configuration 3FS of the practical tests.

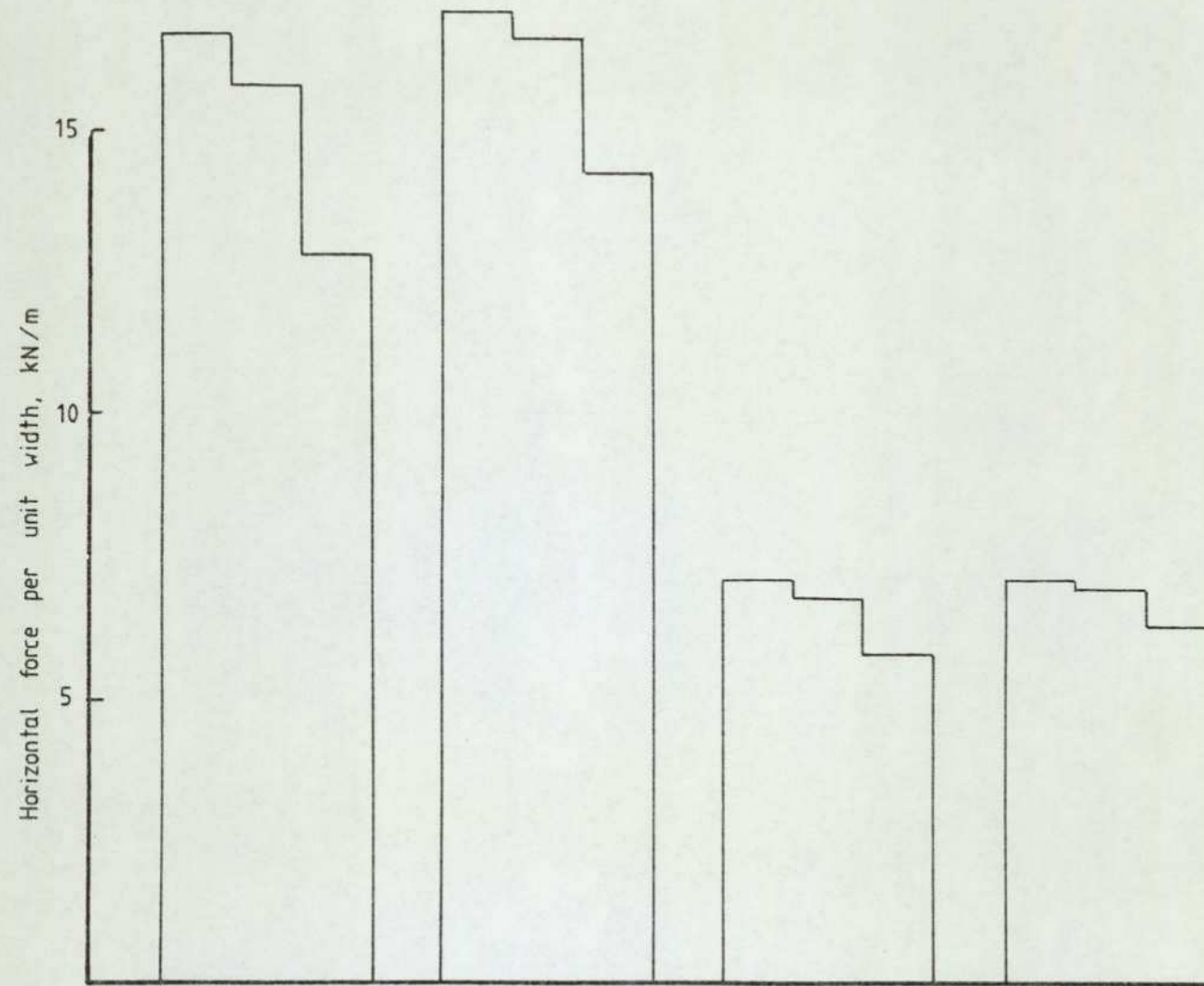


Fig. 7.12. Horizontal forces on the articulated section of the main wall per unit width for each configuration calculated with the finite element program.

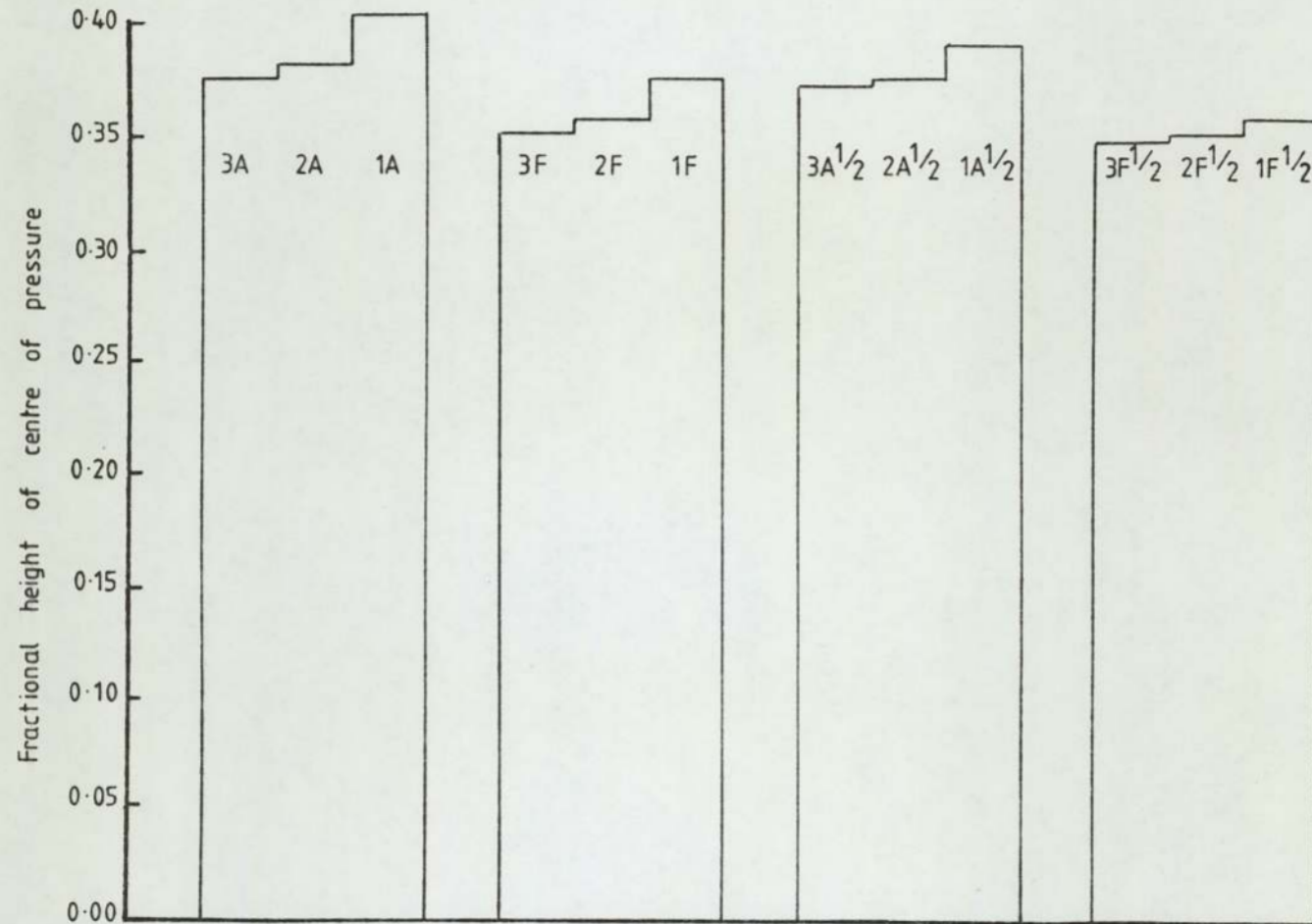


Fig. 7.13. Fractional height of the centre of pressure on the articulated section of the main wall for each configuration calculated with the finite element program.

CHAPTER 8

GENERAL CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

In the main body of this thesis, conclusions have been drawn where appropriate. However, since these conclusions are scattered throughout the text, the main points have been drawn together in this chapter which also contains some suggestions for further work.

8.1 Previous work

The traditional methods of calculating loads on the walls of grain storage structures can be divided into three main categories. These are:-

- a) Trial wedge methods.
- b) Methods assuming failure throughout the material.
- c) Methods that consider the equilibrium of thin horizontal layers.

Methods in the first two categories assume that the deflections of the walls of the structure will be large enough for the stored material to reach its strength limit. The stored material is assumed to be just at the point of failure, either along a surface within the material (the rupture surface) in the first category or throughout a certain volume of material (the zone of failure) in the second category. Methods in the third category rely on a knowledge of the ratio between the horizontal and vertical stress at all points in the material and, in principle, it is not necessary to assume failure. However, because there are no methods available to calculate this ratio for the general case, the value applying for failure conditions is nearly always used in practice.

The loads on a perfectly rigid retaining wall would be higher than those on a wall that deflects and it is to be expected that the loads would reduce with deflection

until the strength limit of the stored material was reached. Unfortunately, traditional methods of calculation can give no indication of the amount of wall deflection necessary to reach the strength limit. Experimental work with soils has shown that the necessary deflection is very small and would easily be accommodated by the design of most soil retaining walls. However, this deflection will depend on the stress/strain properties of the stored material and will be larger for more compressible materials such as agricultural grain. The larger deflection necessary to achieve active conditions coupled with the fact that the walls of agricultural storage structures would collapse with much smaller deflections than those allowed in say a gravity retaining wall for soils means that the design of structures for storing agricultural materials using methods assuming active failure conditions may be inappropriate.

If failure of the material cannot be assumed then the method of calculation must take into account all possible stress conditions within the material. This means that the problem ceases to be a relatively simple one where only strength properties of the stored material are required. Modern computer based methods can, in principle, handle the complex non-linear stress analysis problem that results but they need a more complete description of the way that the stored material responds to imposed stresses than the much simpler traditional methods.

As well as calculating the loads on the walls of storage structures, previous workers have also made measurements. The best way of measuring the stress distribution on a wall

is to mount separate stress measuring gauges on it but attention must be paid to the possible modification of the imposed stress by the presence of the gauge. Experimental work indicates that the measured stress will be highly sensitive to the deflection of the gauge and theoretical considerations indicate that the sensitivity will depend on the stress/strain characteristics of the stored material as well as the conditions at its boundaries.

Early measurements of stress distributions, when plotted on common axes and compared with standard design formulae, are very scattered. This could be due to a variety of causes such as:-

a) The simplifying assumptions made in the design formulae. For example the loads being independent of wall deflections and the proximity of side walls. Effects due to factors not taken account of in design formulae were probably very different for each range of experiments.

b) Unsuitable instrumentation. It seems that experimenters were often unaware of the effect of gauge deflection on the measured stresses. Also many experiments were carried out over seventy years ago when instrumentation may have been unreliable.

More recent experiments, especially those in which great care was taken to minimise gauge deflection and where the "fixed depth" technique was used to ensure the absence of a general lowering of stresses due to a bottom effect, show stresses to be above those calculated using traditional design methods. This is especially true of methods such as Coulomb's method which include the effect of main wall friction.

8.2 The new stress/strain model

A suitable technique with which to investigate the loads exerted by granular materials on retaining walls is the finite element method. As with all methods of stress analysis this requires a knowledge of the stress/strain characteristics of the material being analysed. For a realistic simulation of the behaviour of bulk granular materials it is necessary that the stress/strain law be a non-linear one. Previous workers, using soils, have derived incremental non-linear laws suitable for use with the finite element method. These were based on Hooke's law where the Young's modulus and Poisson's ratio were functions of the stresses in the material. Unfortunately, Hooke's law is incapable of representing one of the observed phenomena associated with granular materials, that is shear induced dilation.

A new incremental model has been developed in this thesis that not only relates the normal stresses to normal strains and the shear stresses to shear strains but also includes cross-coupling terms between normal and shear quantities. In this way shear induced dilation is included. The four stress dependent parameters needed to describe the behaviour of the material can be measured with suitable tests using a triaxial cell.

8.3 The finite element model

The new stress/strain model is formulated in terms of the octahedral normal stress (a measure of the average normal stress) and the octahedral shear stress (a measure of the average shear stress) and the corresponding octahedral strains. However, for inclusion in a finite element

stress analysis program the behaviour of the material must be specified in terms of relationships between the normal and shear stresses in each of the three coordinate directions. In this work the way in which these average stresses and strains would split into their Cartesian components was derived by assuming that a bulk of material would react in a symmetrical manner in response to certain applied strains. While the author feels that these assumptions are reasonable, there is no means of checking them with the basic triaxial equipment available. This may be an area in which useful work could be done with a true triaxial apparatus.

The computer program was used to calculate the stresses on the vertical walls of a rectangular section silo with a flat grain surface. The calculated stresses, assuming perfectly rigid walls were higher than those obtained using Coulomb's method by about 30%. The rate of change of the stresses with wall rotation was only about 11% for each per cent of rotation. This means that even with a rotation of 1%, which is a large deformation for most types of structure, the calculated stresses would still be about 19% greater than those given by Coulomb's method. An inspection of the stresses calculated within the stored material showed that this increase was due to the shear stress in most of the material being considerably lower than that required to cause failure.

Although the magnitudes of the stresses calculated without the assumption of failure (i.e. using the finite element program) are higher than those calculated using classical techniques, the sensitivity of the stresses to

wall friction is very similar whichever method is used. This is true both in the case of changing the front and back wall friction and in the case of changing the side wall friction. The position of the centre of pressure calculated by either method is very similar.

A limited investigation of the performance of the finite element program with circular section silos gave similar conclusions i.e. the stresses for a rigid walled silo were greater than those calculated by a classical method (Janssen's formula) by about 25% but the sensitivity to wall friction was about the same; the position of the centre of pressure was very similar whichever method was used.

Some initial work was done to determine the possibility of a simple formula being used to calculate the stresses, the parameters for this formula being derived from the results of the finite element program. It was found that, for the limited range of conditions considered (i.e. rectangular section silos with vertical walls, horizontal floors, and horizontal grain surfaces and where the wall deflection was a simple rotation), the stresses could be calculated by combining the effects of wall rotation, front and back wall friction, and side wall friction in a linear manner. Simple formulae, where the parameters could be obtained from a table or chart, would be very useful to a designer. It is suggested therefore that future work should be done with the finite element model to investigate the possibility of deriving such formulae to cover a wider range of silo configurations.

8.4 Measurements of wall loads

To assess the validity of the finite element model, a test silo was built so that the practical measurements of wall stresses could be made for comparison with the calculations. The silo was 4.5m high and 4.5m from front to back walls. Its width could be varied in steps of 1.0m up to a maximum of 6m. In this work however, the width was restricted to a maximum of 3m so that the main wall could be fully instrumented with the available equipment. As the main wall was pivoted about a point near the floor, it was possible to carry out measurements with either a virtually rigid wall or one with a controlled amount of compliance.

Considerable attention was paid to the design of the stress measuring gauges. A computer simulation showed that if the area over which the stress was measured (the face plate) was too small compared with the grain size, a random error would be introduced into the measurement. Calculations with the computer simulation showed that, with the particular sensing area chosen (106mm dia.), 99% of all readings would be within $\pm 3\%$ of the true value for equivalent grain diameters of up to about 8mm. Practical tests showed that a slightly lower figure, 95%, of the readings should lie within $\pm 3\%$ of the true value for wheat grains. This figure was felt to be acceptable.

Experiments were carried out to measure the effect of face plate deflection on the measured stress. It was found that if the face plate was allowed to move as a plane after the silo was filled, the stress reading changed by about 1.5% of the reading for zero deflection for each

micron of movement. This emphasises the importance of considering very carefully the effect of face plate deflection, for instance if the face plate were allowed to move by only one quarter of the amount usually recommended for soils (i.e. by $26\mu\text{m}$ for the 106mm diameter face plate used in this work) the error would be nearly 40%.

The stress measuring gauge designed by the author had an extremely small deflection (a maximum of about $25\mu\text{m}$) and, over this range, careful experiments showed that the change in measured stress with deflection was linear with hardly any hysteresis. Because of this linearity it was possible to develop a technique that measured the stress, the face plate deflection, and the sensitivity of measured stress to deflection. With this information it was possible to derive the stress that would have been measured had no deflection occurred.

The experimental measurements of wall loads consisted of filling the test silo in two basic configurations, one in which the grain sloped up from the main wall to a peak and then down to the back wall, and one in which the grain sloped down from the main wall to the back wall. Runs were carried out with two depths of grain, with three silo widths, and, at the maximum depth and width only, with a rotation of the main wall. As is to be expected, the loads increased with increasing depth and reduced with reducing width. The reduction of the total horizontal force on the wall as the width was reduced was fairly small, a maximum of 13% on reducing the internal width from 2.7m to 0.7m with a grain depth of 2.7m sloping downwards from the main wall. In the author's opinion the low sensitivity of the loads

to the silo width is due to the effect of the shear stress at the side walls falling rapidly with distance into the grain mass. The effect of wall rotation was also fairly small. In the case where the grain sloped up from the main wall the reduction of total horizontal force was only 7% for a 1.5% rotation and in the case where the grain sloped down from the main wall a slight increase, 2.4%, was observed for a rotation of 2.1% (however, a statistical test showed that it was not unlikely that this increase could have been due to random chance).

The position of the centre of pressure was much the same for all the experiments lying between 0.33 and 0.43 of the distance up the main wall.

By using Janssen's formula to correct Coulomb's method for the proximity of the side walls, it is possible to calculate the wall loads using the classical idea of active failure in the stored material and including the effect of side wall friction. Calculations using this technique gave horizontal forces on the main wall which were on average about 13% less than the measured values for the largest silo widths. The technique overestimated the effect of side wall proximity and, at the smallest width, the calculated values of horizontal force were an average of 21% lower than the measured ones.

8.5 Comparison between calculated and measured loads

There was good agreement between the shapes of the stress distributions calculated using the finite element model and those measured. There was also a good agreement in the relative values of the loads for the different configurations investigated. However, for the largest

silo widths the magnitudes of the stresses calculated using the finite element model were generally higher than those measured. On average the calculated value of the total horizontal force on the main wall with a silo width of 2.7m was about 13% higher than that measured. It was discovered in the later stages of experimentation that a significant proportion of this difference could be due to a reduction of stress occurring between the end of filling and taking the stress readings. This reduction is thought to be caused by creep in the stored material acting to relieve the stresses on it. It should be possible to measure parameters to describe creep, possibly on a small sample in a triaxial cell. It should also be possible to simulate the phenomenon within the finite element model and, if creep is assumed to commence from the moment any given volume of grain is stressed, the correspondence between the stresses calculated using the finite element model and those measured should be much closer. This possibility is left as a suggestion for further work.

Like the calculations using classical techniques, those with the finite element model overestimated the sensitivity of main wall stress to side wall proximity. This could be due to the non-linear nature of the variation of shear stress across the width of the silo rather than the linear variation assumed. The investigation of the shear stress distribution across a silo is also left as a suggestion for further work and might be approached experimentally or possibly, with the help of a three dimensional finite element model, in a theoretical manner.

The performance of the finite element model in simulating

the practical tests with wall rotations was disappointing. This is in contrast to its good performance when used to simulate a hypothetical silo with a flat grain surface and a rotating wall. Inspection of the stresses calculated when simulating the practical tests with wall rotations showed that in many of the elements the material had gone past a state of failure. Again this contrasts with the calculations in which the grain had a flat surface where the stresses in all the elements were considerably below a state of failure. This is not surprising because a volume of material with a sloping surface must be closer to a state of failure than one with a flat surface, even in the absence of any wall deflection. The finite element model has a fairly crude mechanism for preventing elements passing a state of failure. If it is to be useful in situations where a state of failure is approached closely then the computer program needs to be able to simulate this condition realistically. The author feels that this is an important area in which further work should be done.

APPENDIX

COMPARISON OF THE INITIAL VOID RATIO AND
ORIENTATION OF THE GRAIN IN THE TEST
SILO WITH THAT IN A TRIAXIAL SAMPLE

Introduction

If two samples of a granular material are tested then different values of a given mechanical property can be measured even if the individual grains comprising the samples are exactly similar. For instance, identical spherical bearings can be placed in a similar container so that they assume the minimum volume for a given number of spheres. This close packed array has a void ratio (void volume divided by solid volume) of 0.35 (Lambe and Whitman, 1969). Alternatively they could be packed in such a way that the maximum volume for a given number of spheres was obtained in which case the void ratio would be 0.92.

Rowe (1962) showed that the shear strength and volumetric strains of assemblies of identical spheres should depend on the initial packing arrangement and this theoretical work was confirmed by experiment. Rowe also extended his work to naturally occurring assemblies and measured the shear strengths and volumetric strains for identical sands at different initial void ratios. He found for example that a medium fine sand at 0.56 void ratio had a peak shear strength about 1.3 times that of the same sand at 0.72 void ratio while the rate of volume increase near failure was about 5.8 times greater. These high values of shear strength and volume change can be explained by the greater degree of interlocking between particles which occurs with the more dense packing. As well as causing differences in strains brought about by deviatoric stress changes, different packing arrangements might also give different behaviour under the action of hydrostatic stress changes. For instance, the bulk compressibility ($3a_{22}$ in the model

of Chapter 3) should be less for a more dense packing because each grain is confined by the closer proximity of its neighbours.

The main body of this thesis is concerned with calculating the stresses at the walls of a test silo using stress/strain properties measured on a relatively small sample of grain. Although there is some justification for assuming that the initial void ratio of the grain in the small sample is the same as that in the test silo (Sect. 7.3), they are filled in completely different ways and so a limited study was undertaken to compare the packing of the grain in the test silo with that in a triaxial sample. This involved removing samples of grain from the surface layers of the test silo without disturbing the packing of the individual grains. These samples were then analysed to determine their void ratios and hence to establish the initial void ratio (i.e. the void ratio before the samples were stressed by the weight of the material above them) throughout the silo. Similar tests were carried out on a sample from the triaxial cell and the results compared. The technique was also suitable for investigating the degree and direction of orientation of the samples and these properties were also compared. It must be noted that no attempt has been made to relate the initial void ratio or degree of orientation to the mechanical properties of the bulk. In the author's opinion the measurement of void ratio and orientation with various methods of filling, the development of techniques for reproducing these in test samples, and the relating of these quantities to the parameters of a stress/strain model forms a volume of work far greater than that which

could be contemplated here. However, the method of extracting samples from the test silo and the methods of analysing and presenting the data could be of use to anybody engaged in such a course of work.

Preparation of samples

The sample preparation technique was a two stage process. In the first stage about 200g of polyester resin (Type CL 201 PA, Tylon Ltd) with a black dye added was poured over an area of the grain surface approximately 100mm diameter. A ring, carefully placed on the surface, was used as a guide through which the resin was poured. During this time care was taken not to walk on the grain surface and access to the sampling area was by removable boards supported just above the surface on clips screwed to the inside of the two side walls. Access to these boards was gained by using a ladder.

After 24 hours the aggregated sample was removed. It was about 200mm long, decreasing in diameter away from the surface. It was strong enough to handle as all the grains were stuck together by a surface coat of resin but there were still air spaces between the grains. The sample was then placed in a plastic bag with a quantity of black resin sufficient to fill the air spaces and the bag was sealed with a rubber bung. The bag was then evacuated via a tube in the bung and the resin was allowed to set under vacuum for about an hour. It was found that this procedure gave a sample in which the original air spaces were completely filled with black resin. Although this technique was developed independently, the author subsequently found that a similar technique had been used by Arthur and Shamash

(1970) for extracting sand and gravel samples but in that work a different setting material was used and the vacuum impregnation stage was omitted. The orientation of the sample with respect to the silo was recorded by glueing a label on to it before it was taken from the silo. An arrow on the label showed the direction along the axis of the silo (i.e. perpendicular to the main wall) in the downward direction of the slope, and the angle of the slope was recorded.

Two or three samples were taken from the surface after which more grain was loaded into the silo and the process repeated with the grain at a different level. The locations from which the samples were taken are shown in Fig. A.1.

After all the samples had been prepared they were cut into slices using a bandsaw. The first cut was made vertical with respect to the original position of the sample and along the axis of the silo. This cut was about one third of the distance across the sample. The remaining two thirds were cut into slices using the arrow and angle of slope recorded on the label to ensure that the slices were horizontal with respect to the original position of the sample. During this process a few grains fell out of the resin. The resulting depressions were filled with white resin after which all the slices were polished on a belt sander. A typical slice is shown in Fig. A.2.

The sample from the triaxial cell was produced in a similar way, the resin being poured into the grain after it had been loaded into a rubber membrane inside a split mould in the normal way (Sect.3.4.3).

Void ratio

The volume fraction of a material consisting of two phases, α and β , is defined as

$$V_V = \frac{V_\alpha}{V_\alpha + V_\beta}$$

where V_V is the volume fraction of the α phase and V_α and V_β are the total volumes of the α and β phases in the sample. V_V is thus related to the void ratio, e , by the equation

$$e = \frac{V_V}{1 - V_V}$$

In this work the α phase is the air (or void) and the β phase is the grain. Underwood (1970) shows that the volume fraction, and hence the void ratio, can be estimated from measurements taken on planes cut through the sample. In this work the polished faces of each slice of the samples were divided into light coloured areas (the grains) and dark areas (the resin). If a rectangular grid is superimposed on one of these faces with a total number of grid points P_T and the number of points falling on the dark areas, P_α , are counted, the ratio P_α / P_T will be an estimate of V_V . Underwood shows that the expected value of this ratio equals the volume fraction.

The point counting procedure was carried out using the Quantimet Image Analyser (Imanco Ltd) in the soil microstructure laboratory at the City University. This device uses a video camera to scan a rectangular area of the sample on which it superimposes a grid of up to 500,000 points. The processor then counts the number of points that are darker or lighter than a particular level which is set beforehand by the operator. Setting this threshold can cause some difficulty and this has been suggested as the largest single cause of error in the device (Fisher, 1969).

However, with care it can be set to give reproducible readings by a technique known as the flicker method (Fisher, 1969). The device normally displays a view of the sample on a television screen. By pressing a button, this display can be changed so that the detected features (for instance those areas lighter than the threshold level) are displayed at one grey level and the rest of the sample is displayed at another. These two display levels are adjusted so that they correspond to the average levels of the grain and the resin seen through the optical system after which the button is pressed repeatedly. If the threshold is adjusted correctly the two displays, that of the face of the sample and that of the detected features on it, should look very similar. Any error in the threshold level shows up as a flicker and so the correct threshold setting is easily obtained.

The samples were cut into between three and five horizontal slices, most of which had two usable faces. Sometimes the faces were large enough to provide more than one separate area for analysis and so each sample from the test silo had between six and fourteen areas analysed in horizontal planes and between one and three areas analysed in vertical planes. The triaxial sample had nineteen usable horizontal faces providing one area each for analysis and the vertical face provided four areas.

A further feature of this particular Quantimet is the rotating stage on which the samples are placed. By measuring the angular variation of certain quantities, the orientation of the grains can be investigated (see next section). However, because the analysis area is rectangular, rotation

of the sample brings new parts of it into the field of view while other parts are removed. This leads to a slight variation in the measured quantity P_a with rotation and so, as the areas were analysed at each of nine 20° intervals of rotation, the mean value of P_a over these nine readings was taken.

An initial examination of the estimated void ratio, P_a / P_b , of the samples from the silo showed that, although the average values for samples 2 to 9 were about the same, that for sample 1 was considerably different. This is illustrated in Fig. A.3 (top and middle) which shows the relative frequency of observation of each class of void ratio. The mean of sample 1 was 0.603 whereas that of samples 2 to 9 was 0.447. A test of significance was carried out by examining the t statistic (Davies and Goldsmith, 1977) on the hypothesis that the two mean values were equal. This showed that the difference between the means was significant at the 0.1% level. There is no particular reason why the void ratio of sample 1 should have been so much higher than that of the other samples and it is likely that this reading was caused by incorrect setting of the threshold level. This view is reinforced by the fact that sample 1 was the first to be analysed and the author may not have gained sufficient experience in setting the level. Samples 2 to 9 only were therefore taken to be representative of the void ratio of the grain in the test silo.

Fig. A.3 (bottom) shows the relative frequency of each void ratio class measured in the triaxial sample. The mean value, 0.408, was lower than that of samples 2 to 9 and again a test was carried out to see if the difference was

significant. Examination of the t statistic showed that the difference in the mean values of 0.039 was significant at a level between 5% and 1% and so it was concluded that the void ratio of the material in the silo was slightly higher than that in the triaxial cell. This could be due to the fact that the grain in the silo arrived in position largely by sliding and the resulting dilation which occurred as the particles slid over one another increased the void volume whereas this sliding did not occur as the grain was loaded into the triaxial cell. The higher void ratio of the grain in the silo would be equivalent to a solid volume fraction and hence bulk density reduction of about 2.7% compared with the grain in the triaxial cell.

Preferred orientation

Fig. A.4 shows a set of ten straight lines of various lengths, ℓ_i , and directions, θ_i , drawn in a circle. One way of investigating the degree of orientation of the set is to measure the way in which the sum of the vertical heights of each line varies with the rotation, ϕ , of the figure (Foster, 1973). For Fig. A.4 as drawn (i.e. with $\phi = 0$) this sum is given by

$$V(0) = \sum_{i=1}^{10} |\ell_i \cos \theta_i|$$

and the variation with rotation is given by

$$V(\phi) = \sum_{i=1}^{10} |\ell_i \cos(\theta_i - \phi)|$$

$V(\phi)$ is obviously cyclic in nature and will repeat itself at 180° intervals of ϕ . A plot of $V(\phi)$ against ϕ (Fig. A.5) shows that the curve is approximately sinusoidal but the peak is flatter than the trough. The relatively sharp

trough is caused by the dominant influence of the two lines of length 30mm and 50mm which are in nearly the same direction (45° and 50°). A single straight line inclined at 45° would give a rectified sine curve with a peak at $\phi = 45^\circ$ and a sharp trough at $\phi = 135^\circ$. A measure of the orientation can be derived by expressing $V(\phi)$ as a Fourier series but first V must be made a function of $\psi = 2\phi$ so that the period of the function is 360° . V is now expressed as

$$V(\psi) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos k\psi + b_k \sin k\psi)$$

where

$$\frac{a_0}{2} = \frac{1}{2\pi} \int_0^{2\pi} V(\psi) d\psi$$

which is the average of V over one cycle and

$$a_k = \frac{1}{\pi} \int_0^{2\pi} V(\psi) \cos(k\psi) d\psi$$

and

$$b_k = \frac{1}{\pi} \int_0^{2\pi} V(\psi) \sin(k\psi) d\psi$$

If the orientation is large then the variation of V over the period will be large compared with its mean value. Conversely, if there is little orientation the variation will be small. The orientation can be considered to be proportional to the ratio of the amplitude of the fundamental component of the curve ($k = 1$) to the mean value or

$$\Omega = \frac{2C}{a_0} \sqrt{a_1^2 + b_1^2}$$

where C is a constant. The direction of the orientation is given by the phase angle of the fundamental component

$$\Phi = \tan^{-1} \left(\frac{b_1}{a_1} \right)$$

It is desirable that the orientation should be 1.0 for a set of parallel straight lines and in this case the values of θ_i are all equal. V is therefore given by

$$V(\phi) = |\ell_T \cos(\theta - \phi)|$$

where ℓ_T is the total length of the lines and θ is their inclination. The ratio between the fundamental amplitude and the mean value for a rectified sinusoid is 2/3 (Westman, 1964) and so for Ω to be 1.0, C must be 1.5 or, measuring Ω in per cent

$$\Omega = \frac{300}{a_0} \sqrt{a_1^2 + b_1^2}$$

Fig. A.5 shows the calculated values of the fundamental component and the mean value of $V(\phi)$ for the lines of Fig. A.4 which give $\Omega = 49\%$ and $\Phi = 38^\circ$.

As mentioned previously, the image analyser was equipped with a rotating stage coupled to the processor enabling an analysis to be carried out automatically for each 20° rotation of the sample. The device was programmed to measure the sum of the vertical heights of the detected features and a typical set of results, that obtained from the slices of sample 1, is shown in Figs A.6 and A.7. Because the values of V were only available at discrete intervals, the discrete Fourier series (Bendat and Piersol, 1971) was used in which the integrals defining the coefficients were replaced by summations i.e.

$$\frac{a_0}{2} = \frac{1}{9} \sum_{i=1}^9 V(\psi_i)$$

$$a_1 = \frac{2}{9} \sum_{i=1}^9 V(\psi_i) \cos \psi_i$$

$$b_1 = \frac{2}{9} \sum_{i=1}^9 V(\psi_i) \sin \psi_i$$

A coding convention was adopted so that slice 1 was nearest the surface and slice 5 was at the bottom of the sample. Face "a" of each slice was the upper and face "b" the lower face. In drawing the figures care was taken to ensure that they were the correct way round. For instance, the slices were placed on the stage so that the original direction of the silo axis was vertical on the television screen. The first set of measurements was therefore always in this direction. However, the rotation direction (clockwise or anticlockwise looking down on the original position of the sample in the silo) depended on whether face a or face b was being analysed. In Figs A.6 and A.7 positive values of ϕ are for directions of analysis in an anticlockwise direction looking down on the original position of the sample in the silo. The direction $\phi = 0$ corresponds to the axis of the silo in the horizontal slices and the vertical direction in the vertical slices.

It can be seen that the peak of the fundamental component always occurred near $\phi = 0$ for the horizontal slices. This indicates that the direction of preferred orientation for the horizontal planes of sample 1 was along the axis of the silo. For the vertical slices the peak occurred around $\phi = 90^\circ$ which shows that the orientation direction for a vertical plane through sample 1 was roughly horizontal. Polar diagrams (Figs A.8 and A.9) have been used to present the orientation data for the samples as a whole. For each

sample the orientations for the individual slices have been grouped into 10° class intervals according to their directions, Φ . The magnitudes, Ω , of the orientations in each class interval have been added together to produce a total magnitude for that direction. To compare the diagrams for different samples the total magnitudes have been divided by the number of areas analysed in that sample. This avoids the problem of producing a larger total magnitude simply because more areas have been analysed. The sum of the magnitudes for all the class intervals in each diagram is now the mean orientation magnitude for that sample. This figure has been written alongside each diagram.

The orientation diagrams for the horizontal slices show that nearly all of the samples had their grains orientated in a direction along the axis of the silo. This coincides with the direction they would have been travelling in just before they came to rest i.e. down the slope. In sample 9 however the orientation is less clearly defined but is at a considerable angle to the silo axis. This is probably because the samples were extracted in the period before the overhead conveyor (Sect. 5.1.5) was installed and the silo was loaded by using a telescopic shute on the end of the loading auger (Marchant, 1979). The end of this shute was constrained to move in a line across the silo but was moved by hand. When the silo became nearly full the end of the shute was nearly touching the peak of the heap in the silo and the grain had to be moved away from it by hand. This caused considerable disturbance to the surface of the heap (which was one reason why the telescopic shute was replaced) especially behind the peak and this was

probably the cause of the peculiar orientation of sample 9. The orientations for the vertical slices of all the samples have been grouped on one diagram (Fig. A.9, bottom). The directions of orientation for the samples on the right of the peak (Fig. A.1) have been reversed so that the slope direction is upwards from left to right. It can be seen that the orientation in a vertical plane is between horizontal and the slope direction (approximately 27° to the horizontal) which, like the orientation in horizontal planes, is in the direction of motion of the grains before they came to rest. It seems therefore that the grains tended to slide down the slope with their long axes parallel to the direction of slide. The magnitudes of the orientations were however generally low being in the range 6.9% to 17.1% but it is interesting to note that these small magnitudes which would go undetected with the naked eye are easily resolved with the image analyser.

The method of filling the triaxial sample (Sect. 3.4.3) should not give rise to any particular orientation direction in horizontal planes. Fig. A.10 (top), which shows the orientation for a typical horizontal slice, confirms this, there being no noticeable cyclic variation of V with ϕ . The horizontal orientation diagram for the whole sample (Fig. A.10, bottom left) shows that the orientation is very low (note the scale which is ten times that of the other diagrams) and the directions are variable. There is however some capability for orientation in a vertical plane as the filling of the triaxial sample proceeds by keeping the surface level. This is confirmed by Fig. 10 (middle and bottom right) which shows a small horizontal orientation in the vertical plane.

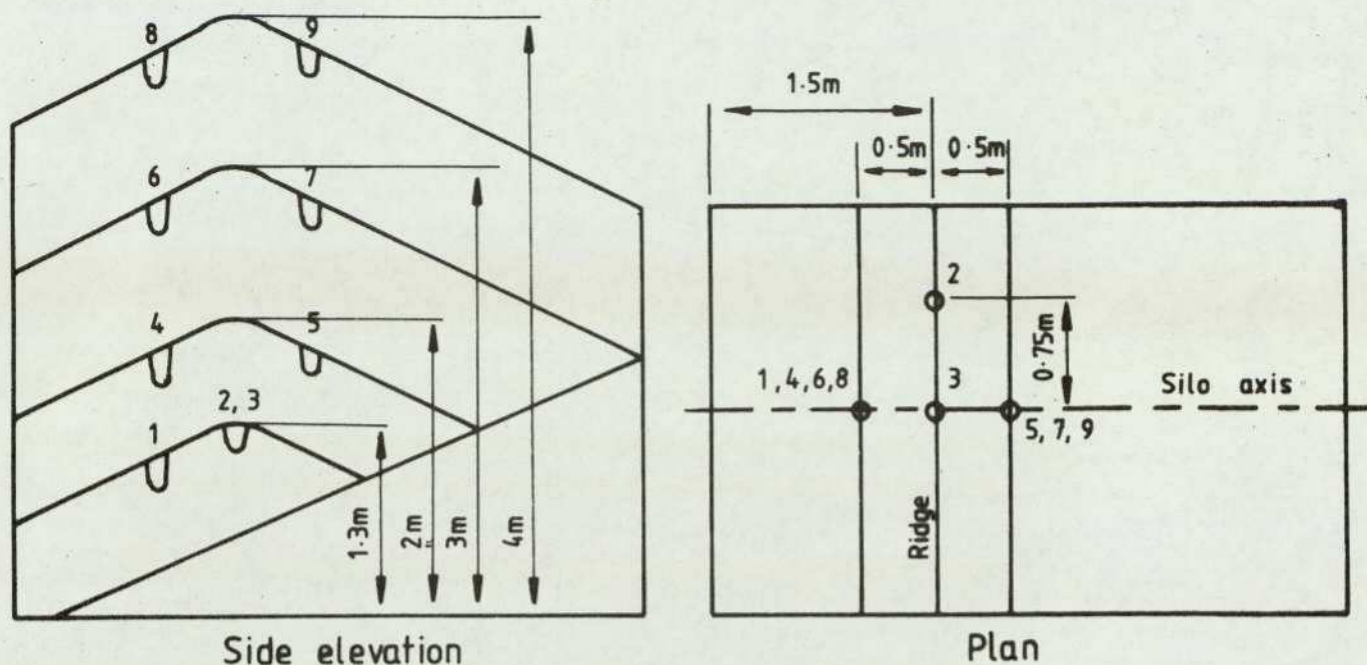


Fig. A.1. . Location of samples in test silo .

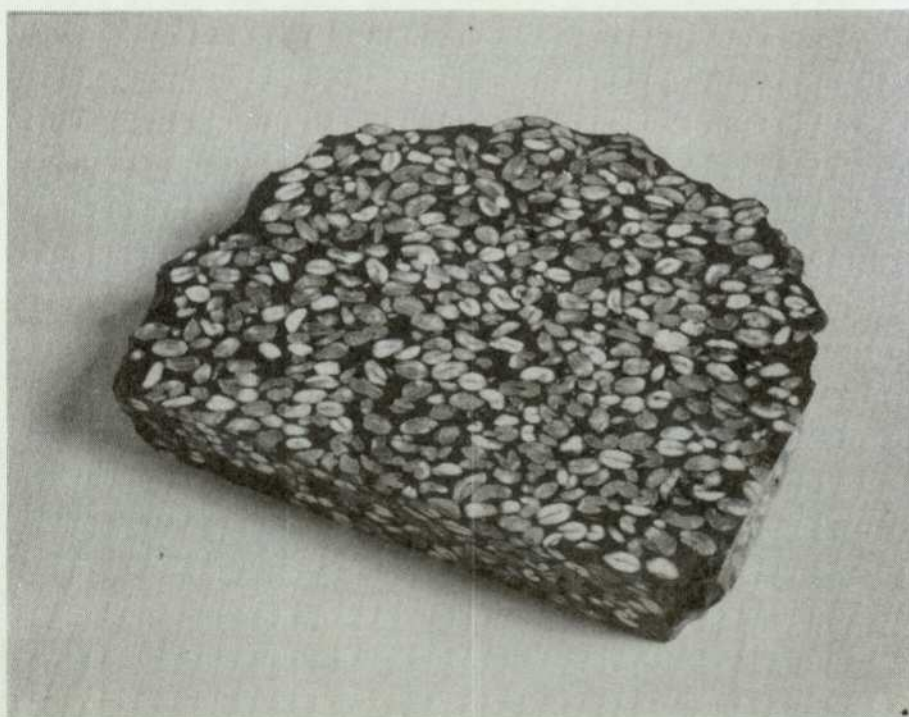


Fig. A.2. Typical slice of a sample after impregnation, cutting, and polishing.

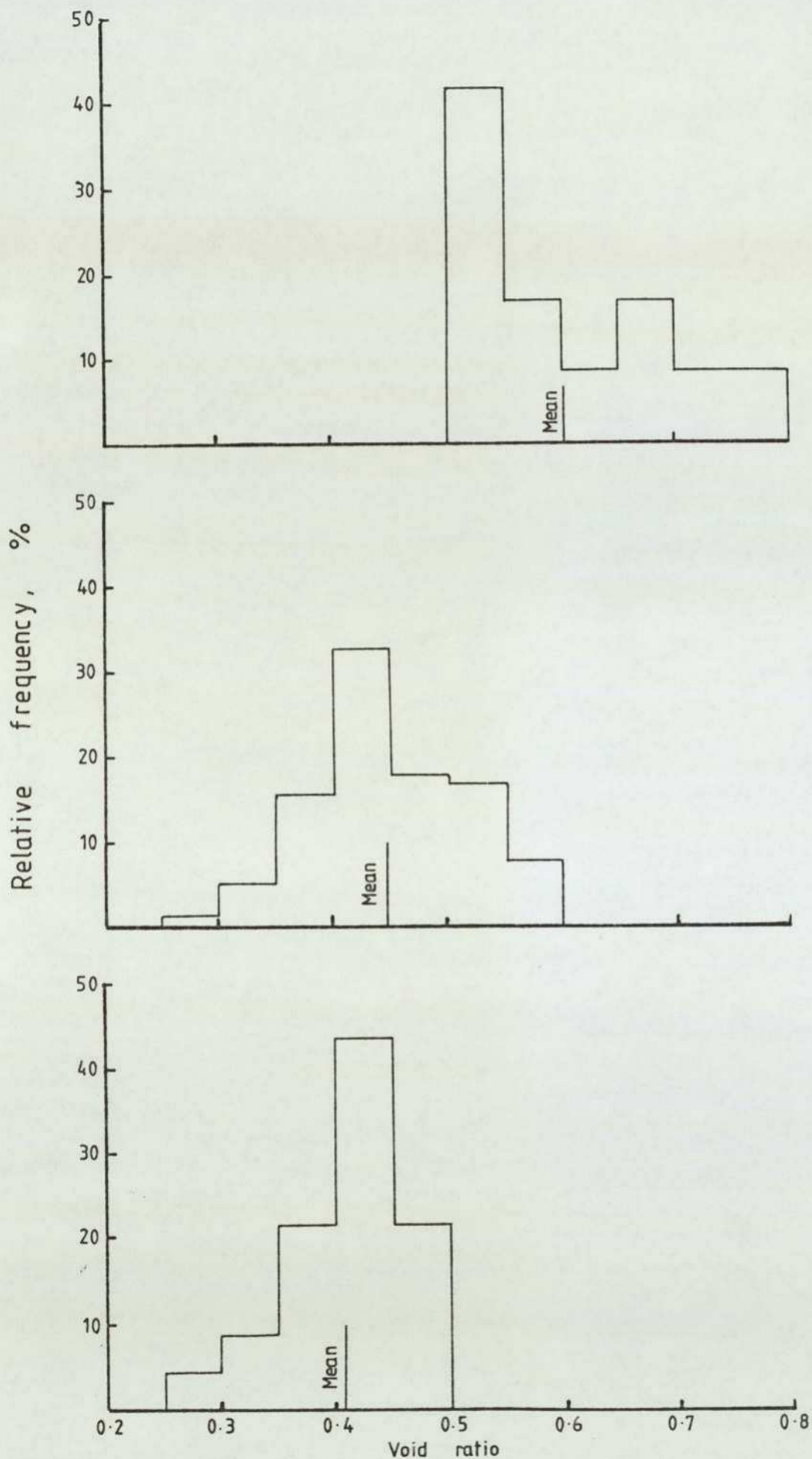


Fig. A-3. Histograms of estimated void ratio. Top, sample 1; Middle, samples 2 to 9; Bottom, triaxial sample.

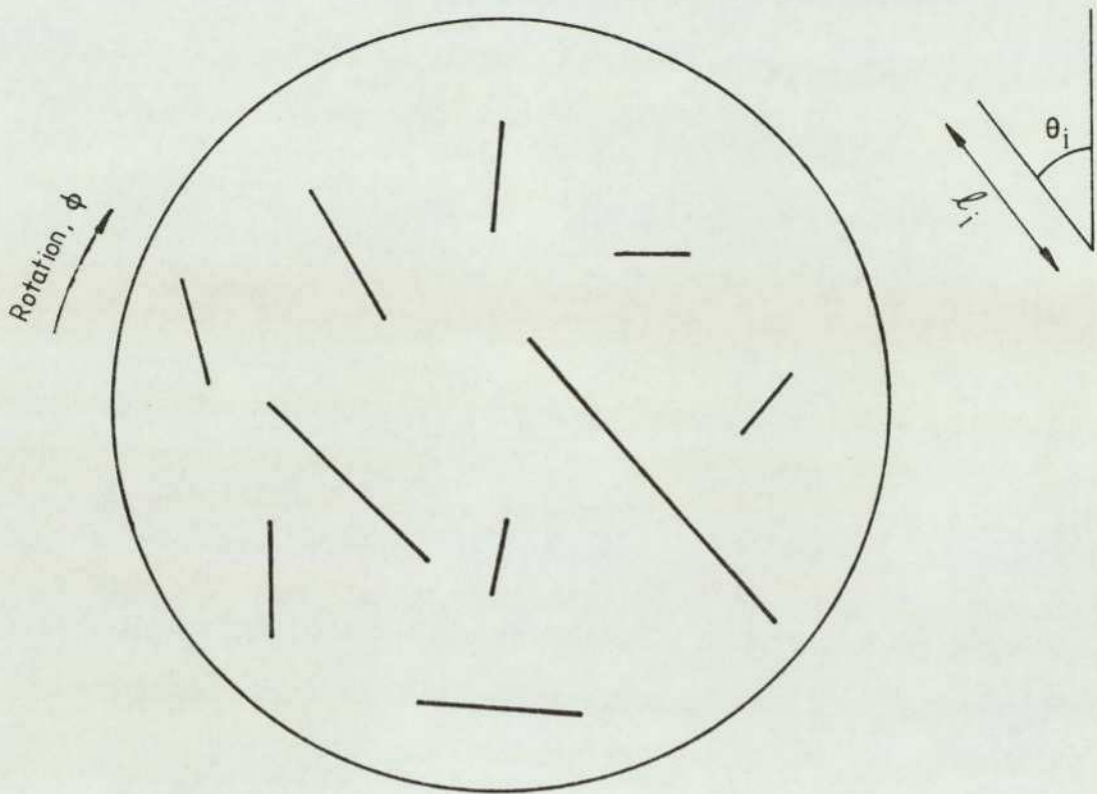


Fig. A-4. Set of lines of various lengths and orientations.

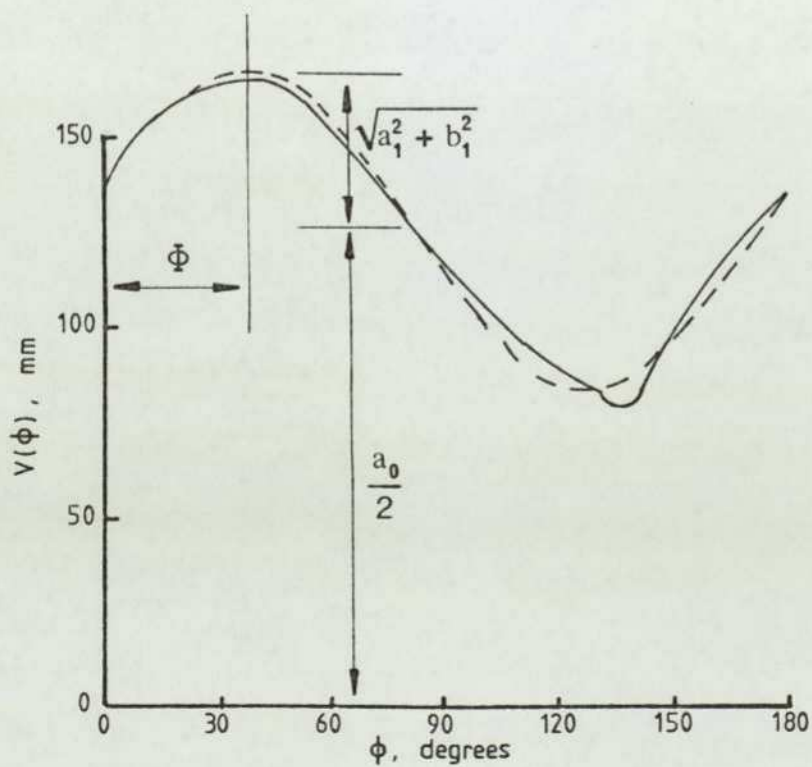


Fig. A-5. Sum of vertical heights of lines vs rotation. Fundamental component shown as a broken line.

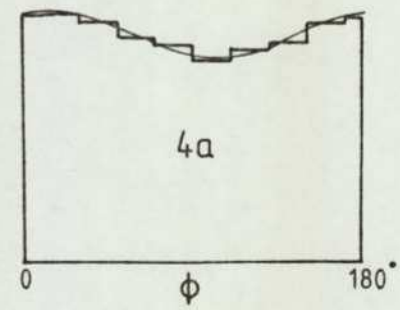
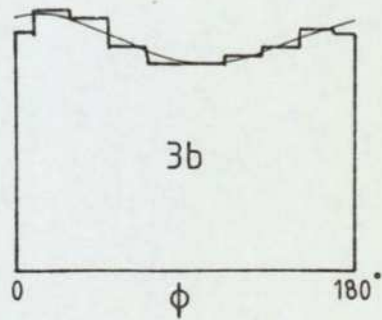
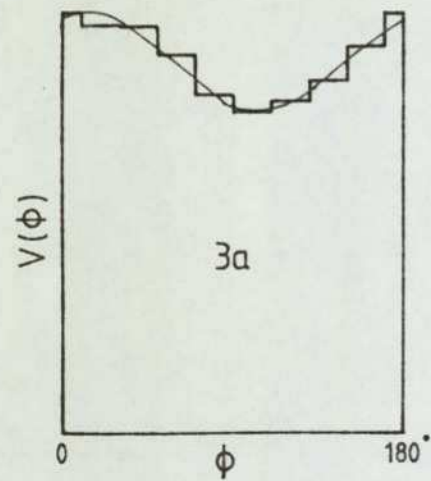
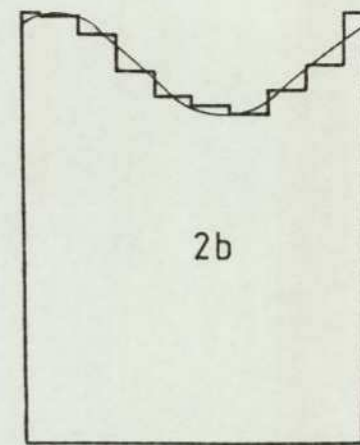
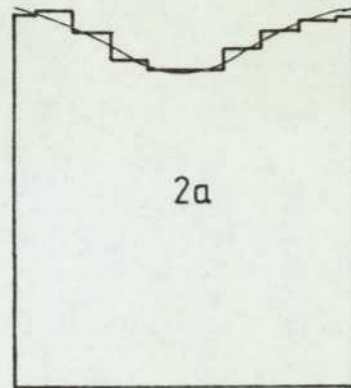
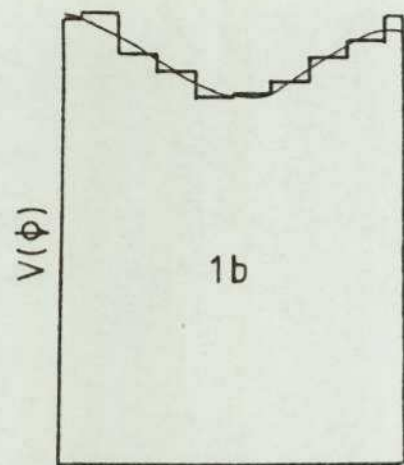


Fig. A-6. Measured values of $V(\phi)$ (arbitrary units) vs ϕ at 20° intervals of rotation for horizontal slices of sample 1.

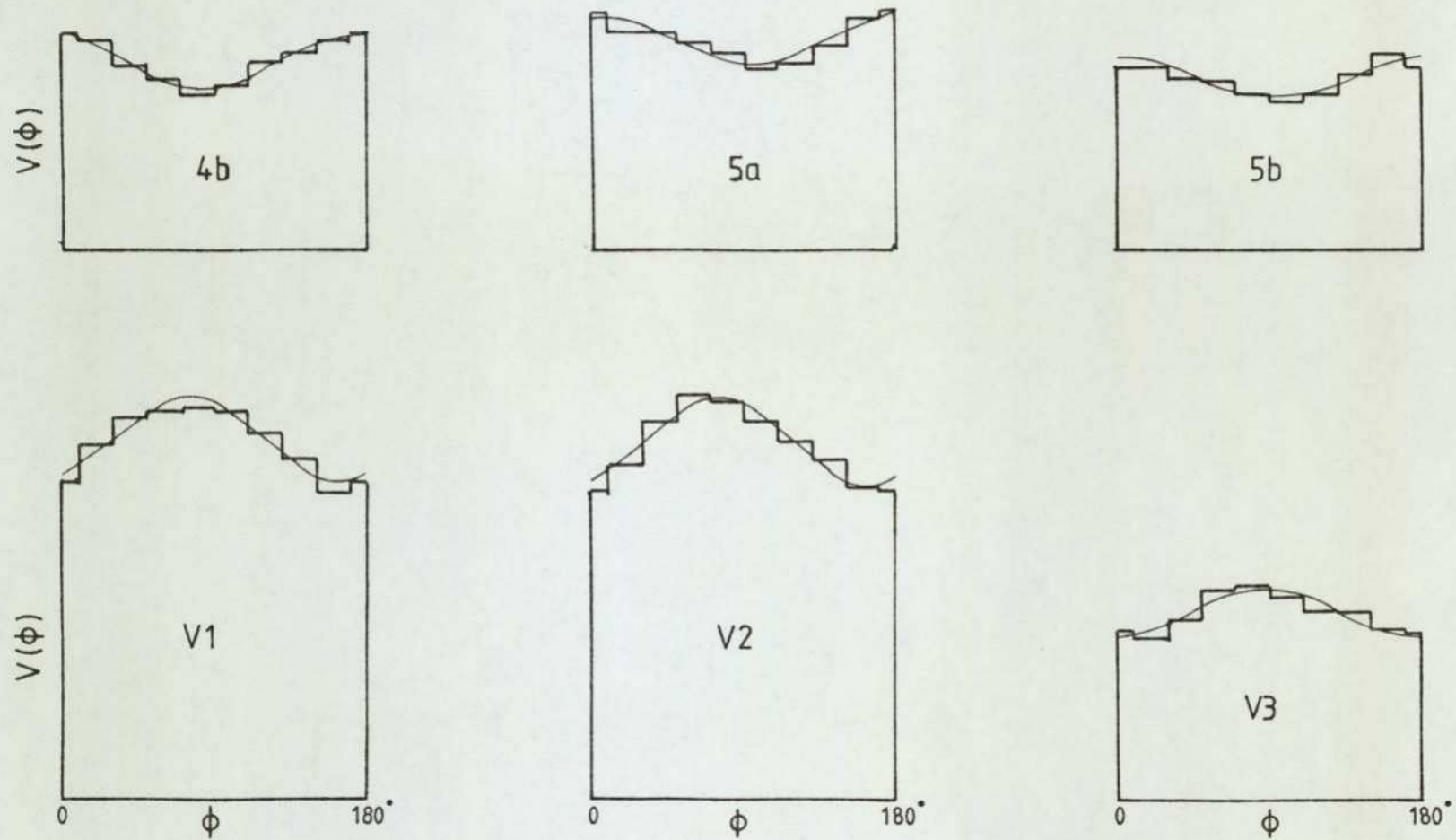


Fig. A-7. Measured values of $V(\phi)$ (arbitrary units) vs ϕ at 20° intervals of rotation for horizontal and vertical slices of sample 1.

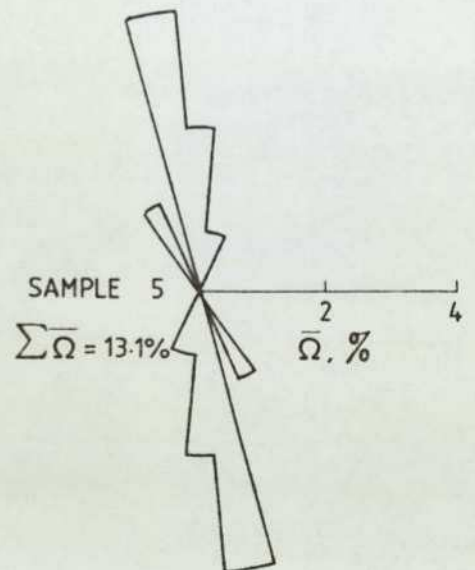
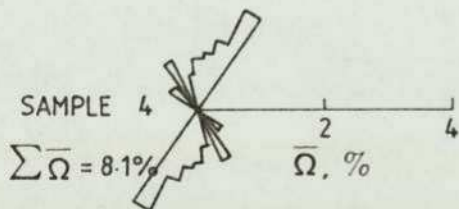
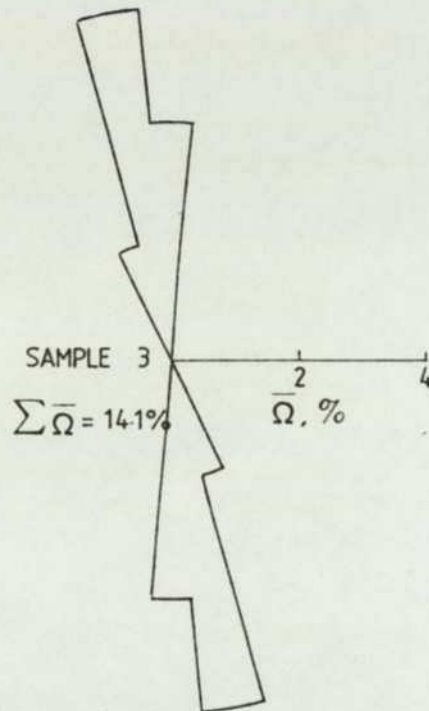
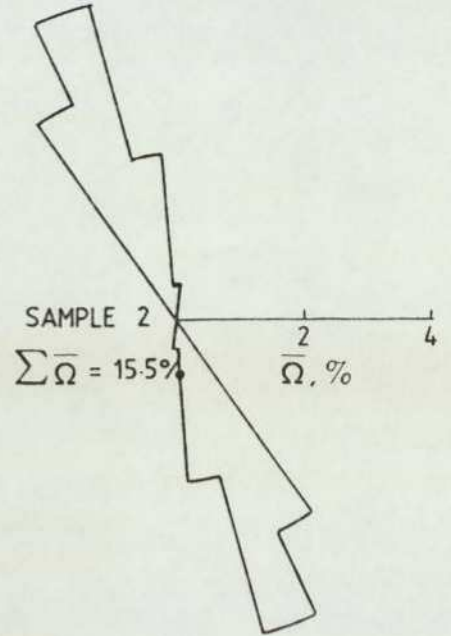
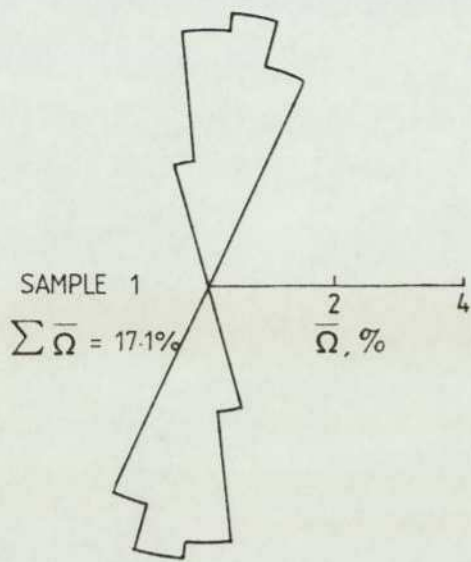


Fig. A-8. Horizontal orientation diagrams for samples 1 to 5. The radial distance corresponds to the mean orientation measured in all the slices of the sample in each 10° class interval. Vertical on page corresponds to direction in silo axis.

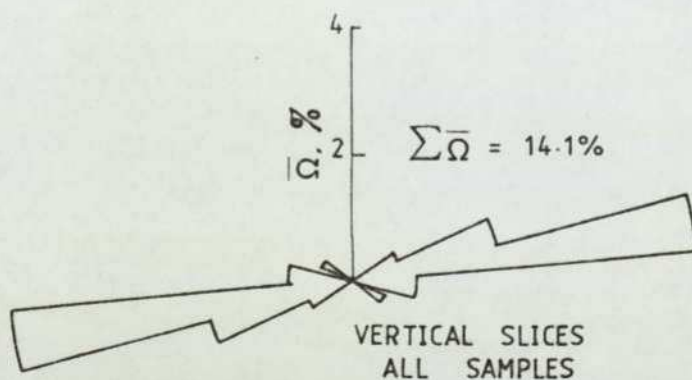
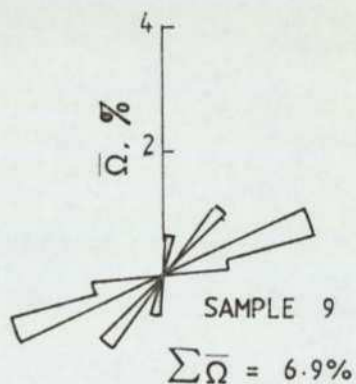
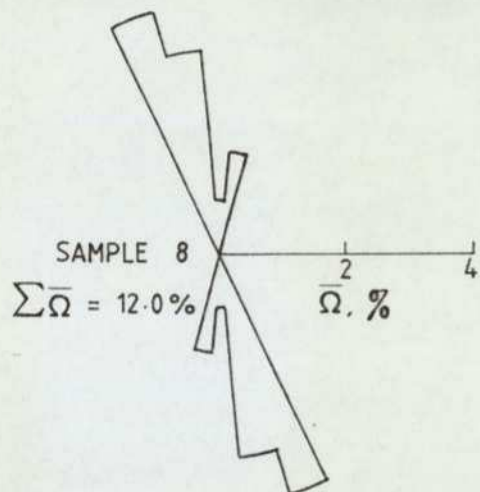
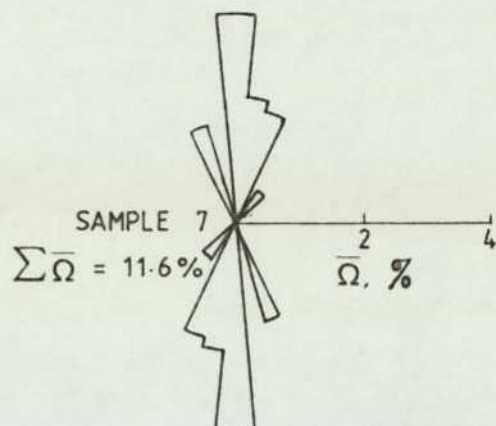
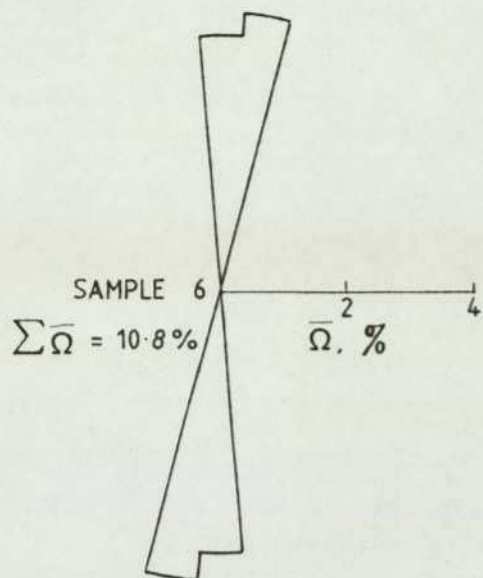


Fig. A-9. Horizontal orientation diagrams for samples 6 to 9 and a vertical orientation diagram for all samples from the silo.

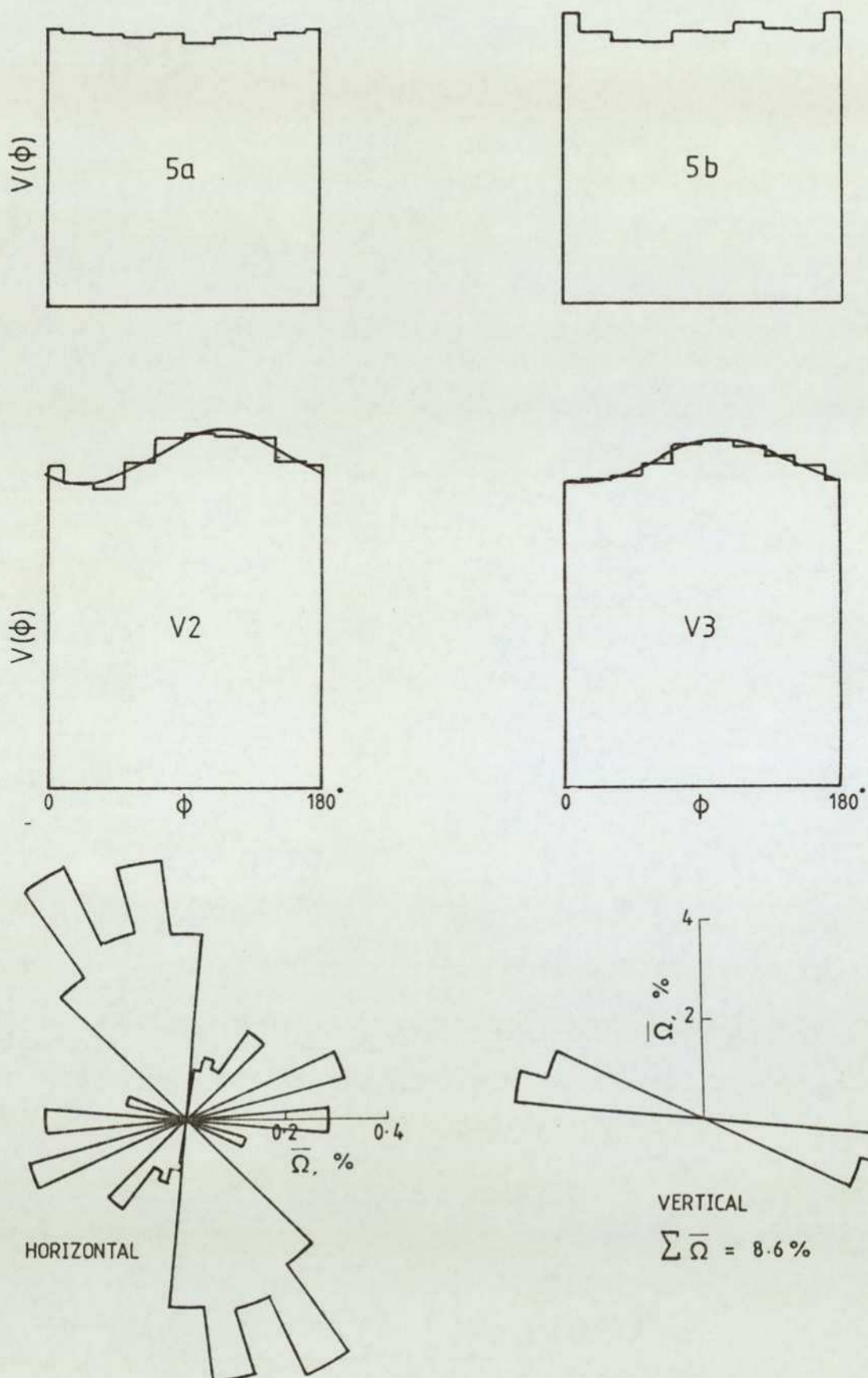


Fig. A-10. Orientation of the triaxial sample. Top, $V(\phi)$ vs ϕ for horizontal slice 5; Middle, vertical slice areas 2 and 3; Bottom, mean orientation in each 10° class interval.

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