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Department of Electrical and Electronic Engineering

ELECTROMECHANICAL STIFFNESSES AND DAMPINGS

IN

LINEAR INDUCTION MACHINES

by

Sai Wing Leung. B.Sc.

A thesis submitted for the degree of Doctor of Philosophy
of The City University, London.

February, 1981

	<u>CONTENTS</u>	Page
ABSTRACT		1
ACKNOWLEDGEMENTS		2
LIST OF SYMBOLS		3

CHAPTER 1

GENERAL INTRODUCTION

1.1.	Introduction	8
1.2.	Outline of the thesis	12

CHAPTER 2

THEORETICAL ANALYSIS

2.1.	Introduction	17
2.2.	Real model	18
2.3.	Mathematical model	19
2.4.	Representation of the stator windings by a thin current sheet	20
2.5.	Governing field equations	29
2.6.	Roll, pitch, yaw, lateral and normal displacements in the mathematical model	31
2.7.	Stator linear and angular velocity	35
2.8.	Field equations	36
2.9.	Solutions of the electromagnetic field equations	39
2.10.	Boundary conditions and boundary equations	45
2.11.	Determination of the field quantities	50
2.12.	Magnetic flux density and power flow	54

2.13.	Three axis forces	55
2.14.	Roll, pitch and yaw couples	60
2.15.	Induced voltage in the primary windings and constant voltage operation	61

CHAPTER 3

MECHANICAL EQUATIONS OF THE LINEAR INDUCTION MOTOR

3.1.	Introduction	68
3.2.	Mechanical equations having 1 degree of freedom	68
3.3.	Mechanical equations having 2 degrees of freedom	70
3.4.	Mechanical equations having 6 degrees of freedom	72

CHAPTER 4

THEORETICAL RESULTS

4.1.	Introduction	76
4.2.	Flux and force distributions	78
4.3.	Flux pattern across the air gap	88
4.4.	Stiffness coefficients	95
4.5.	Damping coefficients	98
4.6.	Relationship between input voltage and constant current excitation	100

CHAPTER 5

EXPERIMENTAL TESTS AND RESULTS - STATIC MACHINE

5.1.	Introduction	105
5.2.	Description of the static machine	105
5.3.	Force measurement techniques	107
5.4.	Experimental results	118
5.5.	Flux measurements and results	124

CHAPTER 6

EXPERIMENTAL TESTS AND RESULTS - DYNAMIC TEST

6.1.	Introduction	144
6.2.	Description of the dynamic machine and test rig	144
6.3.	Force measurement techniques and results	150
6.4.	Flux measurements and results	165

CHAPTER 7

DISCUSSION, FURTHER WORK AND CONCLUSION

7.1.	Discussion	188
7.2.	Further work	194
7.3.	Conclusion	196

REFERENCES AND BIBLIOGRAPHY	198
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APPENDICES

Appendix A1.	Boundary equations	204
Appendix A2.	Current sheet density distribution in the z direction	207
Appendix A3.	Computer program	214

PUBLICATION	223
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ABSTRACT

/ The thesis is concerned with the study of the performance of the linear induction motor, and its electromechanical stiffnesses and dampings. The three axis forces, magnetic flux density, roll, pitch and yaw couples are estimated under different stator symmetrical and asymmetrical positions, and the corresponding flux density and force distributions are plotted.

The three axis forces and the three rotational couples generated by the machine, in six different degrees of freedom, are related to the mechanical vibration system, and represented by the equivalent mechanical springs and damping devices, hence the electromechanical stiffness and damping coefficients are derived.

In order to verify the validity of the theoretical work, an experimental machine is designed and built. Both theoretical and experimental results are shown for comparison purposes.

In the theoretical analysis, a three-dimensional analytical approach is used. The field quantities are obtained by solving the field equations using the method of separation of variables. In the mathematical model, a constant-current source is assumed and represented by a thin current sheet. The terminal voltage of the stator is then determined in order to relate the constant-current drive to a constant-voltage drive machine. The forces generated by the secondary conductor are evaluated by using the Maxwell's second stress tensor.

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LIST OF SYMBOLS

$\vec{A}$	= Magnetic vector potential, Wb/m
A_x, A_y, A_z	= x, y and z components of magnetic vector potential A, Wb/m
a_{mnx}, a_{mnz}	= Fourier coefficients used to define the current sheet in a double Fourier series
$\vec{B}$	= Magnetic flux density, Tesla
B_x, B_y, B_z	= x, y and z components of the magnetic flux density $\vec{B}$
b_{mx}, b_{mz}	= Fourier coefficients
C_{mnx}, C_{mnz}	= Constants defined in section 2.9.
$C_{i,j}$	= Damping coefficient in i^{th} row and j^{th} column
[C]	= 6 X 6 damping coefficient matrix
d_{mnx}, d_{mnz}	= Constants defined in section 2.9.
$\vec{E}$	= Electric field intensity, V/m
E_x, E_y, E_z	= x, y and z components of electric field intensity $\vec{E}$, V/m
E_{mnx}, E_{mnz}	= Constants defined in section 2.9.
$\vec{F}$	= Force vector, N
F_x, F_y, F_z	= x, y and z components of force vector $\vec{F}$, N
$\vec{f}$	= Force-density stress vector, N/m ²
f_x, f_y, f_z	= x, y and z components of force density vector $\vec{f}$, N/m ²
f	= Frequency, Hz
$g(x,z)$	= Air gap length, variable of co-ordinate x and z, m
g_o	= The average air gap length, m
$\vec{H}$	= Magnetic field strength, A/m

H_x, H_y, H_z	= x, y and z components of magnetic field strength $\vec{H}$, A/m
I	= Integer defined in section 2.13.
I	= r.m.s. phase current, A
II	= Integer defined in section 2.13.
I_x, I_y, I_z	= Moment of inertia of the stator along the x, y and z axis
$\hat{i}, \hat{j}, \hat{k}$	= Unit vectors in x, y and z directions
i, j, k	= Integer constants used in numerical integration
I, J, K	= Integer constants used in numerical integration
$\vec{J}$	= Current density vector, A/m ²
j	= Square root of minus one
K	= Integer defined in section 2.13.
[K]	= Stiffness coefficient matrix
KK	= Integer defined in section 2.13.
$K_{i,j}$	= Stiffness coefficient in i th row and j th column
l_{x1}, l_{x2}, l_z	= Length defined in section 2.3., m
L_x, L_z	= Length defined in section 2.3., m
[M]	= Mass matrix defined in section 3.3. kg
$\vec{M}$	= Moment vector, N m
M_r, M_p, M_y	= Moment of roll, pitch and yaw, N m
m	= Mass of stator, kg
m	= Integer used in the Fourier Series
N	= Number of turns
N_{eff}	= Effective number of turns
n	= Integer represents the n th harmonic in the x axis
o	= The origin of the Cartesian co-ordinates
$\vec{P}$	= Poynting vector, W/m ²

P_x, P_y, P_z	= x, y and z components of the Poynting vector P , W/m^2
p	= Pole pitch, m
q	= Number of phase
Re	= Real part of
$\vec{R}$	= Space vector, $\vec{R} = (x, y, z)$
S, S_1, S_2	= Surfaces of integration
s	= Per unit slip
$\vec{T}$	= Maxwell's second stress tensor
t	= time, s
V_{in}	= Total induced voltage per phase, V
$V_{i/p}$	= Input voltage, V
V_{AB}	= Voltage induced in the coil AB, V
$\vec{V}$	= Velocity vector, m/s
V_x, V_y, V_z	= x, y and z components of velocity vector $\vec{V}$, m/s
V_{sy}	= Synchronous speed, m/s
W_o	= Width of overhang winding, m
x	= Longitudinal space co-ordinate
y	= Normal direction space co-ordinate
z	= Transverse space co-ordinate

α_n	= n^{th} harmonic propagation constant in x direction
β_{mn}	= Constant defined in chapter 2
γ_m	= m^{th} harmonic, constant defined in chapter 2
δ	= Thickness of the secondary conductor, m
λ	= Wavelength, m
ω	= Angular frequency, rad/s
$\vec{\omega}$	= Angular velocity vector, $\vec{\omega} = (\dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_p)$
ψ	= Flux, Wb
$\vec{\theta}$	= Angular displacement, $\vec{\theta} = (\theta_x, \theta_y, \theta_p)$
ϕ	= Electric scalar potential, V/m
μ_0	= Permeability of free space, $4\pi \times 10^{-7}$
μ_r	= Relative permeability
$\cdot$	= $\frac{d}{dt}$, derivative with respect to time
∇^2	= Laplacian, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
$*$	= Complex conjugate
$'$	= Derivative with respect to the corresponding function
$[]^T$	= Transpose of the matrix

CHAPTER 1

CHAPTER 1. GENERAL INTRODUCTION

1.1. Introduction

The induction machine is one of the most commonly used devices in electrical engineering. Its ability to produce force on the secondary member at a distance without either mechanical or electrical contact, its high efficiency and good power to weight ratio are of great value to users and designers. Investigation and study of induction motors have been going on since the first invention by Tesla in 1880.

In the last twenty years the increasing needs for improvements in existing transport systems and the unacceptability of aircraft operation within city centres have led to the consideration of High Speed Ground Transportation, up to 300 km/h. The present wheel driven systems become impracticable, because of the considerable mechanical engineering problems which would be involved in the suspension system, and the contact between the wheel and track. These problems may be overcome by making the vehicle airborne, it has been suggested, either by air cushion or by electromagnetic levitation as the means for hovering suspension and propulsion by a linear motor. Attention has been focused on the linear version of the induction motor; it has been considered as one of the best offers for future ground transport due to the following outstanding features:

1. It is not necessary for the rotor receiving the energy, for production of forces, to have mechanical contact with its source. The forces generated by the rotor are due to electromagnetic induction -with no electrical or mechanical contact.
2. Because of the lack of contact between the primary and secondary members, there is no mechanical friction between wheel and track -

therefore it is capable of running at high speed.

3. The system is chemical pollution free.

4. It has a low noise level when compared with an air cushion for hovering suspension.

5. Simplicity in construction of the track.

The track can be in the form of an aluminium sheet, backed with steel.

The maintenance of such a track would be negligible when compared with that of the existing railway track which requires frequent maintenance as a result of the high stress concentration due to the small area of contact between wheel and rail.

In order to describe the linear version of the induction motor (Linear Induction Motor), it may be looked upon in the simplest form as an unrolled induction motor^{1,2}. However, when we come to the performance analysis of the linear induction motor, it would not be so easy to describe when compared with the conventional induction motor.

The important feature of the linear machine is the discontinuity of the motor (in High Speed Ground Transport, a short stator machine is usually suggested) which leads to entry and exit end effects. These end effects arise from the discontinuous nature of the excitation of the stator windings and the stator iron. Especially for high speed or high frequency operation, the end effect is serious in that there is a tail of magnetic flux after the exit end. Besides, the linear induction motor does not develop forces in just one direction, but in all three directions. Force in the traction direction is for propulsion, force in the normal direction can be designed for levitation purposes, and force in the lateral direction can be designed for sideways stability.

Although the linear induction motor and the conventional induction machine have the same basic principle, their analyses are quite different from each other.

The history of the linear machine dates back to the end of the nineteenth century. The first machine was considered in the textile industry in 1895 for propelling the shuttle through the warp without any electrical or mechanical contact³. In the next few decades, the linear induction motor was suggested for high speed ground transport, but due to the presence of the existing transport system followed by the development of the aeroplane - which seemed to be a more convenient method - there was a lack of interest in linear induction motors. In 1946, a large linear induction motor^{4,5} was built in the U.S.A. by Westinghouse which was the first major application of the linear induction motor. It was used as an aircraft accelerator, known as an 'Electropult', which consisted of a short stator as a carrier for the aircraft and a long 'squirrel cage' type rotor.

It was not until the middle of the 1950's that most of the analysis of the linear induction motor started. By that time, it was analysed for the liquid metal pump and the self oscillating linear induction motor^{6,7} for shuttle propulsion. In the last twenty years, it became clear that the linear machine could advantageously be used in many applications, and investigation were made in many countries, including the possibility of using the linear induction motor for high speed ground transport.

The object of the present project is to study the performance of the single-sided linear induction motor and its electromechanical stiffness and damping. It includes the prediction of the following:

- (a) flux density;
- (b) three axis forces;
- (c) flux and force distributions;
- (d) terminal voltage;
- (e) power flow distribution across the air gap;
- (f) roll, pitch and yaw couples;
- (g) electromechanical stiffness and damping coefficients in six degrees of freedom.

with the variation of

- (a) slip - speed of the motor;
- (b) air gap length;
- (c) lateral displacement of the stator;
- (d) different angles of roll, pitch and yaw;
- (e) different velocity in six degrees of freedom so that the electromechanical stiffness and damping coefficients can be found.

The electromechanical stiffness and damping coefficients will provide data for a further study of the stability of the machine. The results can be used for designing the guidance of the linear induction motor.

1.2. Outline of the thesis

When an object is in motion, it has six degrees of freedom, the three translational motions along the three axes of the Cartesian co-ordinate system, and three rotational motions, that is, rotations about the three axes.

In the linear induction motor, which is a rigid body, the three translational motions along the three axes (x, y and z axes) are referred to as the traction, normal and lateral directions. The three rotational motions along the three axes are referred to as roll, pitch and yaw. See Fig.1.2.1.

In the analysis, the three axial forces and the three rotational couples are found with respect to different positions, linear velocity and rotational angular velocity of the stator. Hence the electromechanical stiffness and damping coefficients in each of the six degrees of freedom can be found. In order to calculate the stiffness and damping coefficients of the linear induction motor in each of the six degrees of freedom, a three dimensional analysis is essential. An analytical approach is used in the present investigation. By assuming a constant current source and a thin current sheet representation for the stator windings, the field equations in different regions (air gap, secondary conductor and back iron regions) are solved by the method of separation of variables, and the flux and force distributions are also plotted.

In the analysis, both the end effect and the lateral edge effect, which are due to the discontinuity of the stator are considered, and the skin effect of the secondary conductor is included.

Boon Teck Ooi⁸ has published a paper about the electromechanical stiffness and damping coefficients in repulsive magnetic levitation.

In the paper, the stiffness and damping for each of three degrees of freedom were found. Besides, there are several papers which have investigated one or two degrees of freedom of the linear induction motor.

K. Oberretl⁹ published a paper about the three-dimensional analysis of the linear induction motor, taking into account the edge effects and the distribution of the winding. The paper used a three-dimensional analytical approach and the Fourier series representation for the current sheet, so both the lateral edge effect and the longitudinal end effect are considered.

Besides the two papers mentioned above, there are several machine analysis techniques which have been used to estimate the performance of the linear induction motor. They included different approaches and these varied from one dimensional to three dimensional analyses: one and two dimensional approaches by Yamamura^{10,11}, using the Fourier transformation; the transfer matrix analysis by Alden & Nolan¹³; the pole-by pole analysis of Lipo¹⁴; generalized machine theory by Boon Teck Ooi¹⁵; the finite difference method by Nasar¹⁶; numerous papers study the linear induction motor in one or two dimensions and stability in one or two degree of freedom (refs.17-28).

The work in this thesis is closely related to the two papers by Boon Teck Ooi and K. Oberretl(refs.8&9). It attempts to complete the investigation into the stiffness and damping of the linear induction motor in all degrees of freedom.

An experimental linear induction motor was designed and built, and experimental results were obtained. The six by six stiffness coefficient matrix was verified by comparison between the theoretical

and experimental results. Some of the damping coefficients were obtained from the experimental results. The magnetic flux densities in all three directions were measured for comparison purposes.

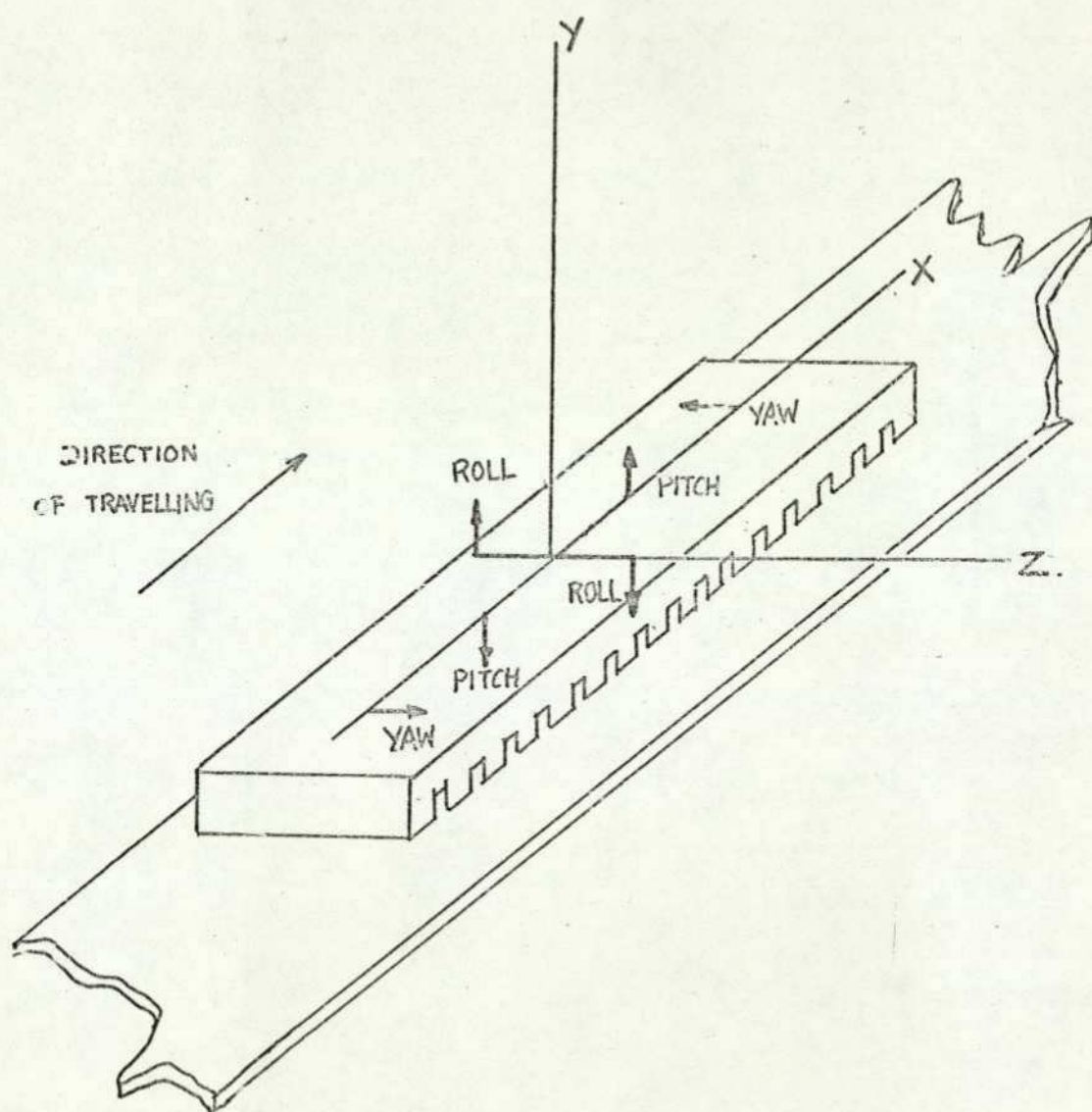


Fig.1.2.1. LIM showing roll , pitch and yaw couples

CHAPTER 2

CHAPTER 2 THEORETICAL ANALYSIS

2.1. Introduction

The model used in this project was a single-sided, longitudinal linear induction machine having an aluminium sheet as the secondary conductor. The chosen model can be with or without backing iron. In the mathematical model, the relative permeability in the back iron region is chosen as $\mu_r=1$ if the machine has no backing iron, otherwise the relative permeability of the material is used. The primary 3-phase winding is represented by a thin current sheet, where the m.m.f. distribution of the current sheet is the same as would be produced by the 3 - phase winding.

It is assumed that there are infinite arrays of machines in both the traction and the lateral directions(x and z directions) and the discontinuous current sheet in the x and z directions is represented by a double Fourier series. The whole mathematical model is divided into four regions in the normal direction(y direction): the stator iron region, air gap region, secondary conductor region and the back iron region. The stator iron region is assumed to have infinite permeability, and only the field equations in the air gap, secondary conductor and the back iron regions are considered. By using the method of separation of variables in the air gap, secondary conductor and the back iron regions(namely regions 1, 2 and 3), the field equations are solved, thus flux and forces can be found. The forces are calculated by using Maxwell's second stress tensor.

The three axis forces and the three rotational couples are found in varied positions and varied roll, pitch and yaw angles, the electromechanical stiffnesses are thus obtained.

Also, the three axis forces and the three rotational couples are found with varied linear velocity and with varied angular velocity of the stator, hence the electromechanical damping coefficients are obtained.

By using Maxwell's Second Stress Tensor, and taking the surface of the secondary conductor as the surface of integration, the force distribution on the secondary conductor can be plotted.

2.2. Real model

The real model used in the analysis is shown in Fig.2.2.1. and Fig.2.2.2.. Fig.2.2.1. represents the model seen in the z direction and Fig.2.2.2. represents the model seen in the x direction.

The model consists of the stator iron, stator winding, secondary conductor and back iron. The stator is made of laminated iron with slots on one side, and the winding is placed in the slots so that it can produce a travelling m.m.f.. The secondary conductor is made of aluminium. The back iron is made of magnetic material, which is in a laminated form.

The model used is a short stator machine, which is a familiar model usually suggested for high speed ground transport. The stator is installed in the vehicle and the aluminium sheet is used as the track.

The stator windings are half filled at both ends, series connected and supplied by a 3-phase balanced current. Usually, machines are

supplied by a constant voltage source, but in this analysis a 3-phase current source is assumed.

The co-ordinates used in the analysis are shown in Fig.2.2.1. and Fig.2.2.2..

2.3. Mathematical model

The mathematical model used in the analysis is shown in Fig.2.3.1. and Fig.2.3.2.. They represent the model seen in the z and x directions. This model is commonly used whenever an analytical approach is applied^{10,18,30}, since it gives no discontinuity in either the stator iron or the back iron regions, though certain assumptions are made.

In the mathematical model, it is assumed that there are infinite arrays of machines in both the x and z directions, so that the m.m.f. produced by the stator windings can be represented by a double Fourier series. The distance between adjacent machines in the x direction (L_x) must be taken sufficiently long so that there are no interactions between the machines. The distance taken between adjacent machines in the z direction (L_z) is the same as the width of the secondary conductor sheet. This will give neither flux nor current flow across the boundaries between machines in the z direction.

The stator iron is assumed to have infinite permeability and the stator windings are assumed to be in the form of a thin current sheet.

The current distribution in the x-z plane of the linear induction motor, at $y = g$, is different from that of a conventional machine, due to the discontinuity of the stator winding in the x direction,

$$I = r.m.s. \text{ value of the phase current}$$

$$N_{eff} = \text{effective number of turns per pole per phase}$$

$$F_p = \text{pitch factor}$$

$$F_b = \text{breadth factor}$$

$$N_{eff} = N F_p F_b$$

$$j = \frac{6 \sqrt{2} I N_{eff}}{p}$$

where

$$\vec{j} = j \exp j(\omega t - \frac{p}{\tau} x)$$

(2.1.)

The current flow in a continuous stator winding may be represented by a current $10, 11, 31$ sheet. This takes the form:

2.4. Representation of stator winding by a thin current sheet

The machine is supplied from a constant current source and a uniform current distribution is assumed within the active zone. The current distribution is reduced to half at the two ends due to the half-filled end slots.

$$y = g \quad \text{to} \quad y = +\infty$$

and the stator iron is extended from

$$y = 0 \quad \text{to} \quad y = -\infty$$

The stator iron and the back iron are assumed to be infinite in length and width. The back iron region is extended from

Several different waveforms can be used to represent the current distribution in the z direction. They are examined in Appendix A1. The current distribution in the z direction for the J_x and J_z components (current flow in the x and z directions) are assumed as shown in Fig.2.4.1. and Fig.2.4.2.. The dimensions of the machine are also shown in Fig.2.4.1..

The current flow in the x direction (J_x) is only due to the overhang winding. The current flow in the z direction (J_z) is mainly due to the winding in the slots within the active zone. There is also a J_z component in the overhang region. The current flow in the y direction (J_y) is assumed to be zero.

The current sheet distributions in the z direction in the overhang region have the form of a sinusoidal decay. This is one of the best representations of the stator windings when an infinite wide stator iron is assumed⁹.

In Fig.2.4.3., the current distributions of both J_x and J_z are shown in the z direction. They are both assumed to be of uniform magnitude, except at both ends, where the magnitude of the current is reduced to half because of the half-filled end slots.

The current distribution of J_x in the z direction (Fig.2.4.1.) can be represented by a Fourier series $F_1(z)$, with period L_z ,

$$F_1(z) = \sum_{m=1}^{\infty} b_{mx} \sin \gamma_m z \quad (2.2.)$$

The current distribution of J_z in the z direction (Fig.2.4.2.) can be represented by a Fourier series $F_2(z)$, with period L_z ,

$$F_2(z) = \sum_{m=1}^{\infty} b_{mz} \sin \gamma_m z \quad (2.3.)$$

where b_{mx} and b_{mz} are Fourier coefficients, and

$$b_{mx} = \frac{2}{Lz} \int_{-Lz/2}^{Lz/2} F_1(z) \sin \gamma_m z \, dz$$

$$= \frac{1}{Lz} \left[\frac{-2\gamma_m \sin(\gamma_m a) + \frac{\pi}{a}}{(\frac{\pi}{2a})^2 + \gamma_m^2} - \frac{2\gamma_m \sin(\gamma_m b) - \frac{\pi}{b} \cos(m\pi)}{(\frac{\pi}{2b})^2 - \gamma_m^2} \right]$$

$$b_{mz} = \frac{2}{Lz} \int_{-Lz/2}^{Lz/2} F_2(z) \cos \gamma_m z \, dz$$

$$= \frac{1}{Lz} \left[\frac{2}{\gamma_m} (\cos m\pi - 1) + \frac{(\frac{\pi}{a}) \sin(\gamma_m a) - 2\gamma_m}{(\frac{\pi}{2a})^2 - \gamma_m^2} + \frac{2\gamma_m \cos m\pi + (\frac{\pi}{b}) \sin(\gamma_m b)}{(\frac{\pi}{2b})^2 - \gamma_m^2} \right]$$

$$a = \frac{Lz}{2} - Z_0 - \frac{Lz}{2}$$

$$b = \frac{Lz}{2} - Z_0 + \frac{Lz}{2}$$

$$k = Lz - b$$

$$\gamma_m = \frac{m\pi}{Lz}$$

z_0 represents the lateral displacement of the stator to one side of the machine, as shown in Fig.2.4.4.. The corresponding J_x and J_z distributions in the z direction are also plotted in the diagram.

Both J_x and J_z are assumed to have the same distribution in the x direction, as shown in Fig.2.4.3.. By using the Fourier series, this distribution can be represented by a function $F(x)$, period L_x ;

$$F(x) = \sum_{n=1}^{\infty} a_n \cos \alpha'_n x \quad (2.4.)$$

where a_n is the Fourier coefficient, and

$$\begin{aligned} a_n &= \frac{2}{L_x} \int_{-L_x/2}^{L_x/2} F(x) \cos \alpha'_n x \, dx \\ &= \frac{2}{(2n-1)\pi} \left[\sin \left(\frac{\alpha'_n L_{x1}}{2} \right) + \sin \left(\frac{\alpha'_n L_{x2}}{2} \right) \right] \\ \alpha'_n &= \frac{(2n-1)\pi}{L_x} \end{aligned}$$

The current distribution of J_x in the x-z plane can be represented by a series,

$$\begin{aligned} J_x(x, z, t) &= j \bar{J} \bar{F}_1(z) F(x) \\ &= j J \exp j(\omega t - \frac{\pi}{p} x) \cdot \sum_{m=1}^{\infty} b_{mx} \sin \alpha_m z \cdot \sum_{n=1}^{\infty} a_n \cos \alpha'_n x \\ &= \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} a_{mnx} \exp j(\alpha_n x - \omega t) \cdot \sin \alpha_m z \quad (2.5.) \end{aligned}$$

where $a_{mnx} = j J a_n b_{mx}$

$$\alpha_n = \left(\frac{2n-1}{L_x} - \frac{1}{p} \right) \pi$$

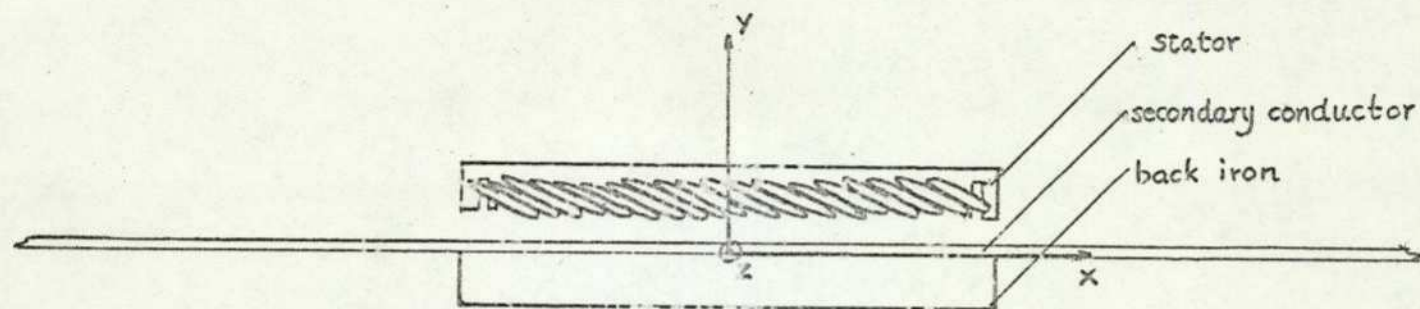


Fig. 2.2.1. Real model looking in the z direction

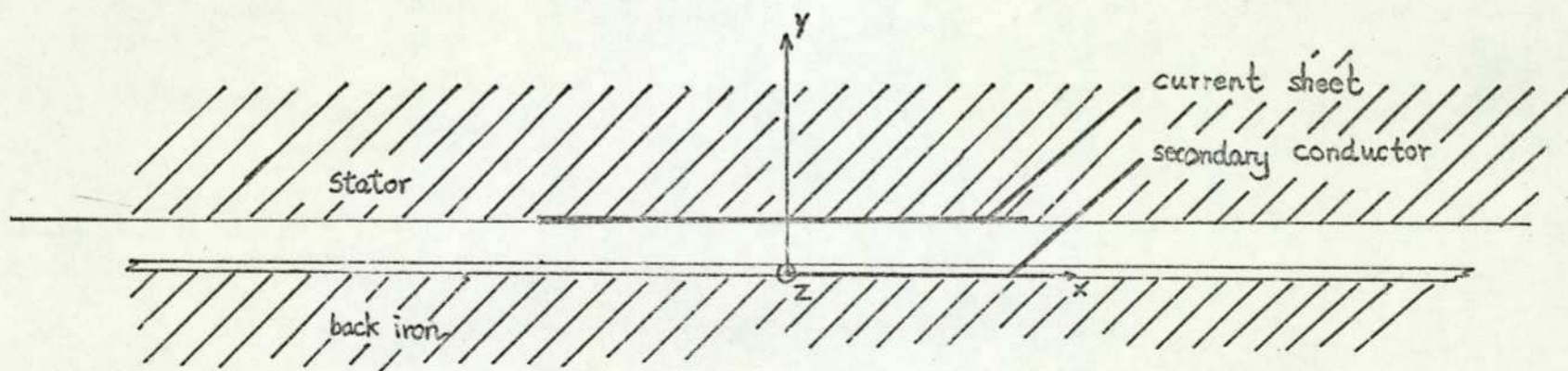


Fig. 2.3.1. Mathematical model looking in the z direction

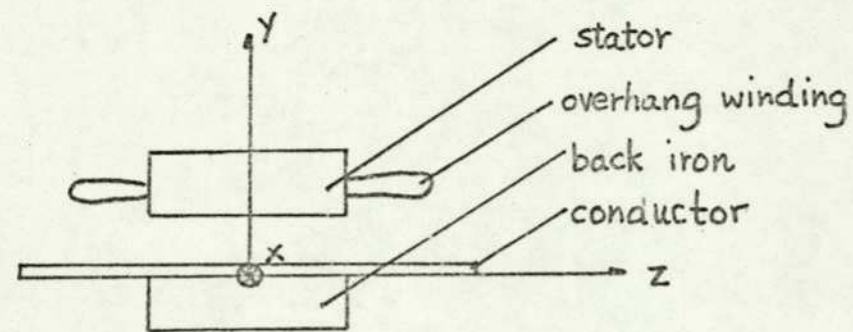


Fig. 2.2.2. Real model looking in the x direction

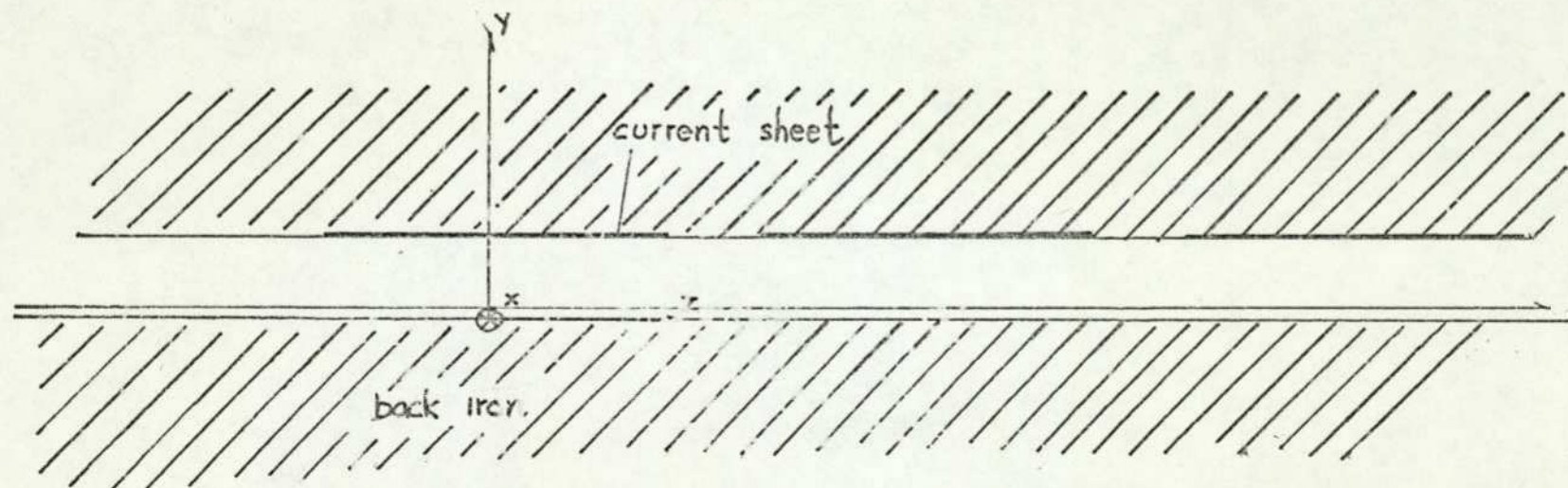


Fig. 2.3.2. Mathematical model looking in the x direction

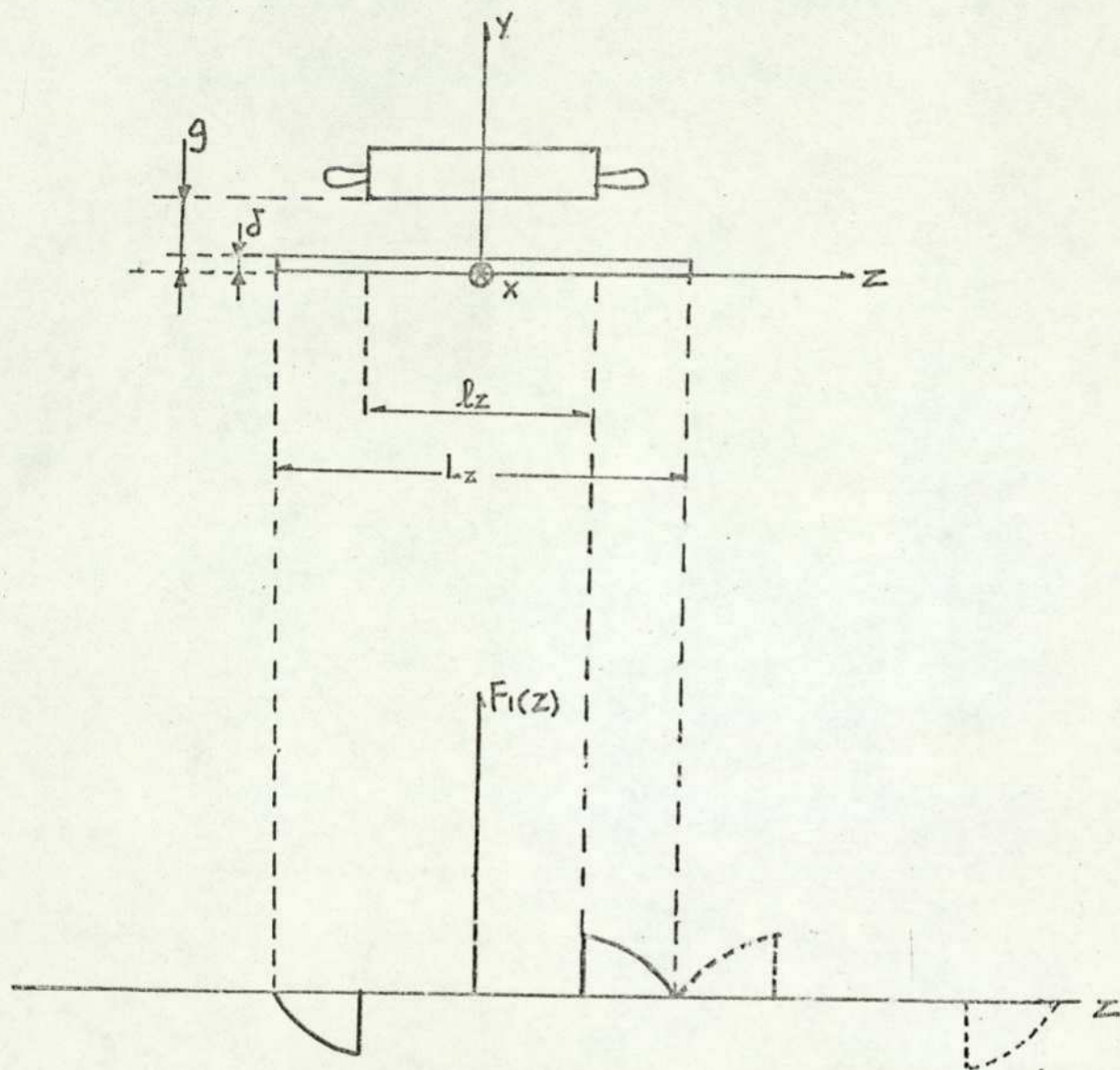


Fig. 2.4.1. J_x distribution in the z direction

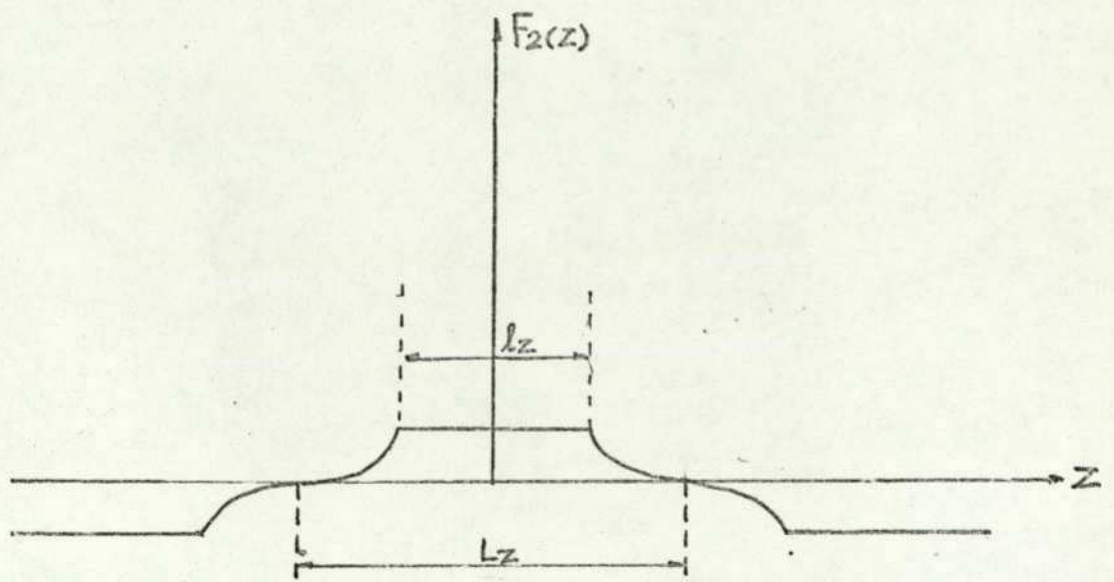


Fig. 2.4.2. J_z distribution in the Z direction

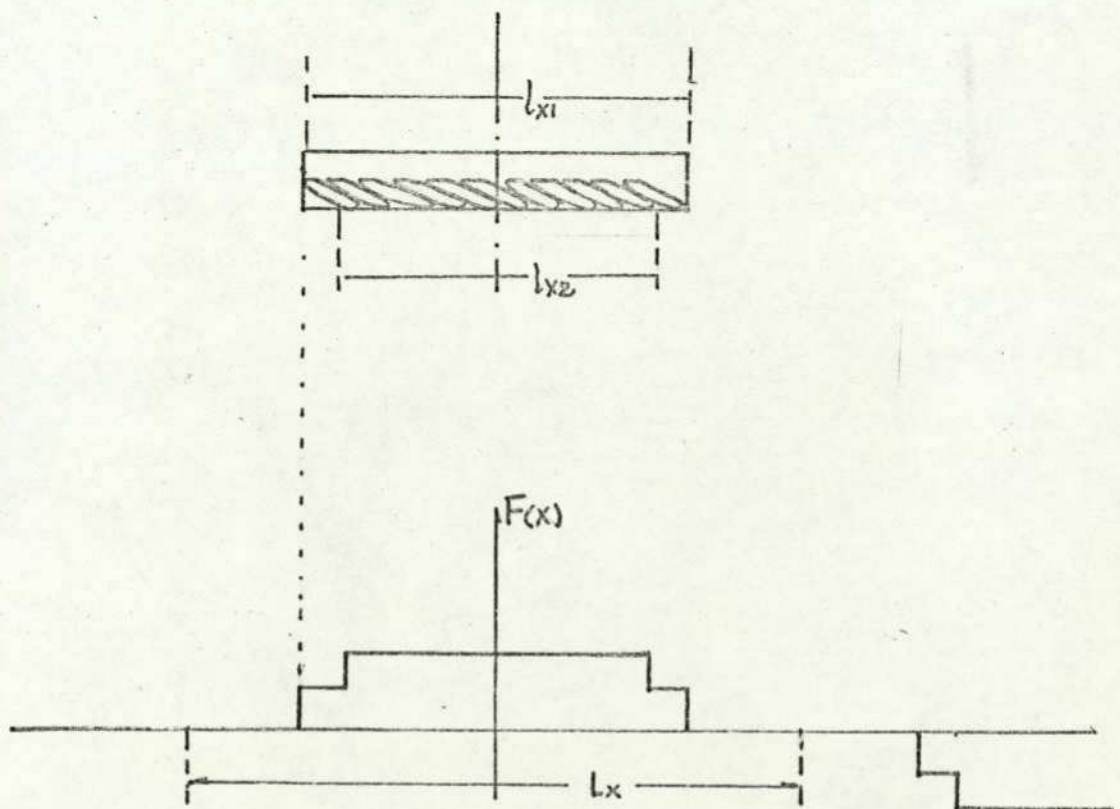
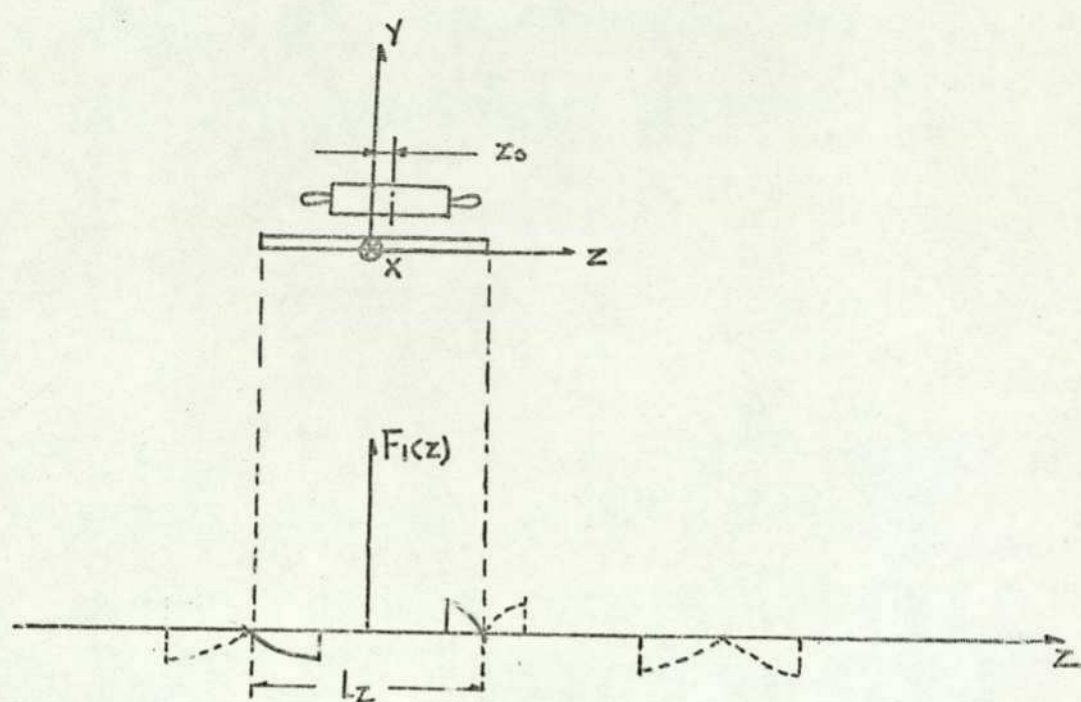
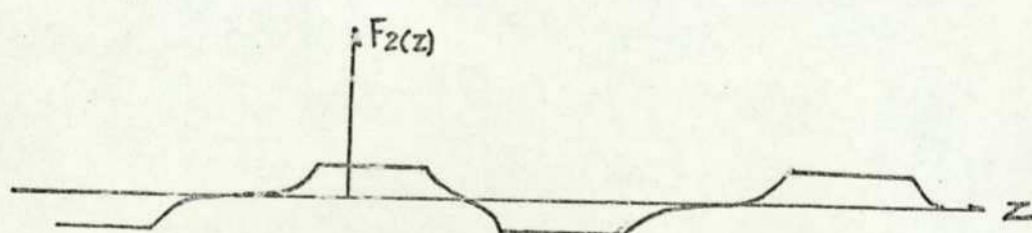


Fig. 2.4.3. J_x distribution in the X direction



J_x distribution



J_z distribution

Fig. 2.4.4. L.I.M. with lateral displacement z_0 and the corresponding J_x and J_z distribution

Similarly the J_z component can be represented in the x-z plane by a double Fourier series,

$$\begin{aligned} J_z &= j F_z(z) F(x) \\ &= \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} a_{mnz} \exp j(\alpha_n x - \omega t) \cos \gamma_m z \end{aligned} \quad (2.6.)$$

where $a_{mnz} = j a_n b_{mz}$

2.5. Governing equations

Electromagnetic fields are governed by the following Maxwell's equations^{32,34},

$$\nabla \times \bar{E} = -\dot{\bar{B}} \quad (2.7.)$$

$$\nabla \times \bar{H} = \bar{J} \quad (2.8.)$$

$$\nabla \cdot \bar{B} = 0 \quad (2.9.)$$

$$\nabla \cdot \bar{D} = \rho \quad (2.10.)$$

Also, Ohm's Law for an isotropic medium moving with velocity $\bar{V}$ is:

$$\bar{J} = \sigma (\bar{E} + \bar{V} \times \bar{B}) \quad (2.11.)$$

In order to set up the electromagnetic field equations in this analysis, a magnetic vector potential $\bar{A}$ is defined,

$$\bar{B} = \nabla \times \bar{A} \quad (2.12.)$$

and $\nabla \cdot \bar{A} = 0$ is assumed. (2.13.)

For a medium with constant permeability μ ,

$$\bar{B} = \mu \cdot \bar{H} \quad (2.14.)$$

eqns. 2.7. and 2.12. give,

$$\begin{aligned} \nabla \times \bar{E} &= -\dot{\bar{B}} \\ \text{i.e.} \quad \bar{E} &= -\dot{\bar{A}} - \nabla \phi \end{aligned} \quad (2.15.)$$

eqns. 2.8. and 2.11. give,

$$\nabla \times \bar{H} = \sigma (\bar{E} + \bar{V} \times \bar{B}) \quad (2.16.)$$

eqns. 2.12., 2.14., 2.15. and 2.16. give

$$\begin{aligned} \nabla \times \nabla \times \bar{A} &= \mu \sigma (-\dot{\bar{A}} + \bar{V} \times \nabla \times \bar{A} - \nabla \phi) \\ \text{i.e.} \quad \nabla^2 \bar{A} &= \mu \sigma (\dot{\bar{A}} - \bar{V} \times \nabla \times \bar{A} + \nabla \phi) \end{aligned} \quad (2.17.)$$

Equation 2.17. is the field equation used in the analysis.

As defined in the previous section, the current sheet has two components only, $\bar{J}(x, z, t) = (J_x, 0, J_z)$

but in the secondary conductor region, there may be a small amount of current flow in the y direction. For simplicity of the analysis, it is assumed that there is no J_y component anywhere.

From eqns. 2.8. and 2.12.,

$$\nabla^2 \bar{A} = -\mu \bar{J} \quad (2.18.)$$

since $\bar{J}$ has two components in x and z directions only, the magnetic vector potential $\bar{A}$ is also assumed to have two components in the same directions, i.e.

$$\bar{A} = (A_x, 0, A_z)$$

2.6. Roll, pitch, yaw, lateral and normal displacement in the mathematical model

In the mathematical model, the roll, pitch, yaw and the linear displacement of the stator are represented either by a change in the air gap length or a change in the current distribution of the current sheet in the z direction.

Roll: This is represented by a change in the air gap length g , such that it is a function of the z co-ordinate (see Fig.2.6.1.).

$$g(z) = g_0 - \theta_x z \quad (2.19.)$$

where g_0 is the average original gap length,

θ_x is the angle of roll.

With roll, the stator is tilted and the current sheet produces a component J_y flowing in the y direction. If the angle of roll is small, it may be assumed that the current flow in the y direction is also small, so that J_y can be neglected.

In this analysis, the effect of roll is to shorten the air gap on one side of the machine, and to lengthen it on the other side.

Pitch: This can be represented by a change in the air gap length as a function of the x co-ordinate.(see Fig.2.6.2.)

$$g(x) = g_0 + \theta_z x \quad (2.20.)$$

Yaw: This can be represented by a change of a lateral displacement of the current sheet, z_o , so that it is a function of the x co-ordinate (see Fig.2.6.3.)

$$z_o = \theta_y x \quad (2.21.)$$

Also, it is assumed that the angle of yaw is small, so that the change of the lateral displacement of the current sheet is the only result from yaw.

The positive directions of the rotation for roll, pitch and yaw are defined by the right hand screw rule, the axes of rotation being the x, y and z axes respectively.

Traction direction displacement:

There is no electromagnetic effect on the displacement of the machine in the x direction except in the change of the origin of the Cartesian co-ordinate.

Normal direction displacement:

In the mathematical model, this can be represented by a change in the air gap length g,

$$g = g_o + y_o \quad (2.22.)$$

Lateral displacement:

In the mathematical model, this can be represented by a change in the sideways displacement of the current sheet z_o (see Fig.2.4.4.).

$$z_o(x) = z_o \quad (2.23.)$$

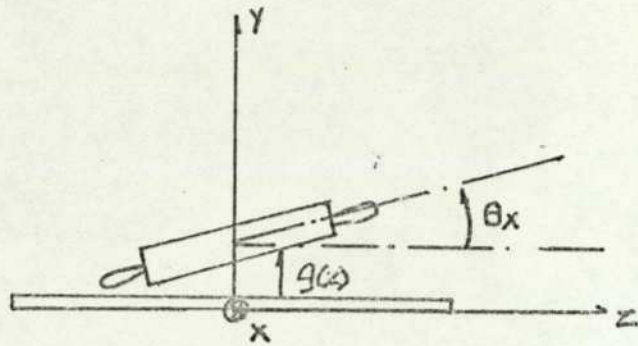


Fig. 2.6.1. Roll, represented by a change in air gap length $g(z) = g_0 - \theta_x \cdot z$

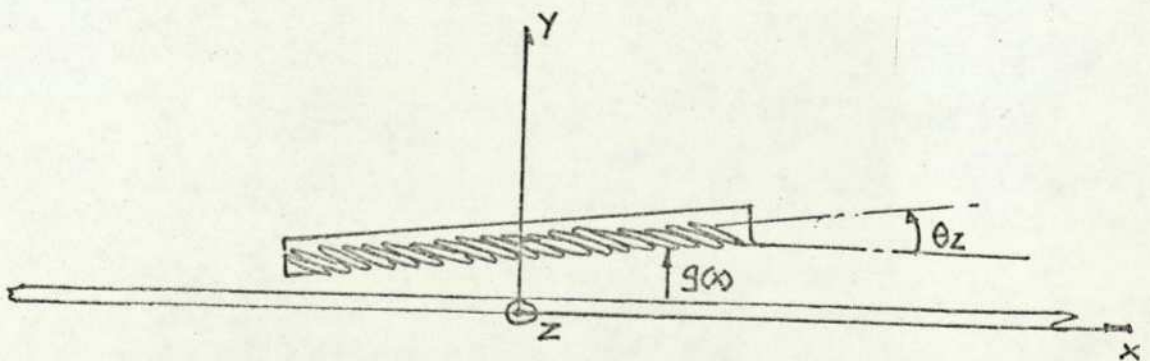


Fig. 2.6.2. Pitch represented by a change in air gap length $g(x) = g_0 + \theta_z \cdot x$

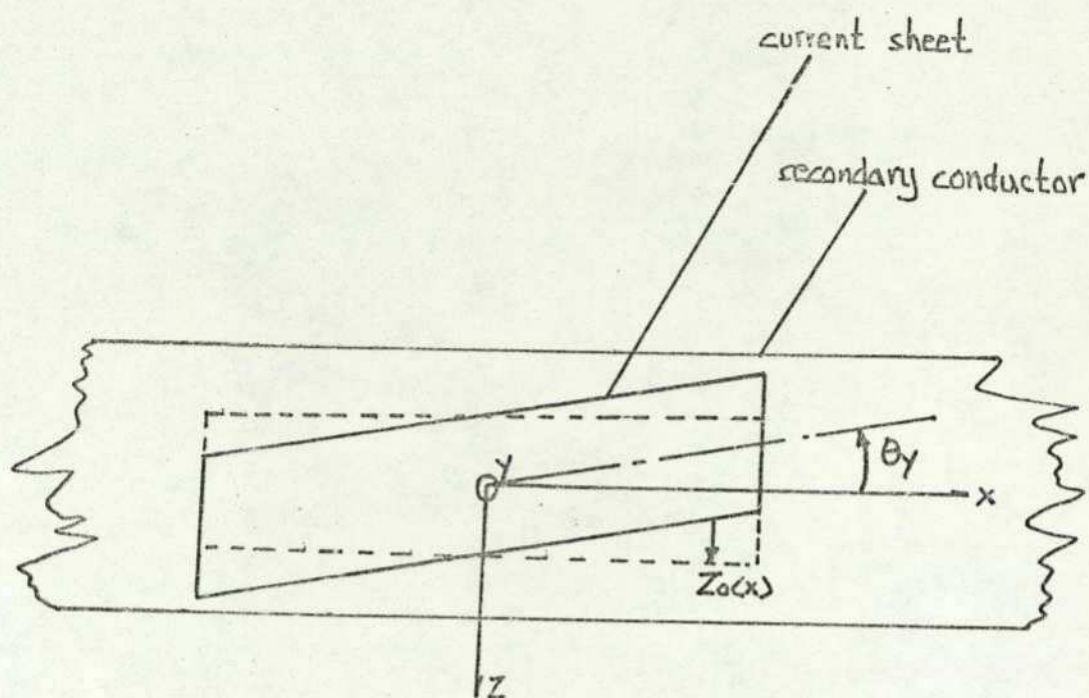


Fig. 2.6.3. Yaw, represented by a lateral displacement of the current sheet z_0 .

$$z_0 = \theta_y \cdot x$$

In general these rotational angular displacements(roll, pitch and yaw) and linear displacements can be represented by two equations $g(x,z)$ and $z_0(x)$, where

$$g(x,z) = \theta_z x - \theta_x z + y_0 \quad (2.24.)$$

and
$$z_0(x) = \theta_y z + z_0 \quad (2.25.)$$

$g(x,z)$ is substituted in the boundary conditions after the field equations are solved, and $z_0(x)$ is used as one of the parameters in the current sheet equations(eqn.2.4. and eqn.2.5.) in solving the field equations.

2.7. Stator linear velocity and angular velocity

In the analysis, the stator is assumed to be stationary and the rotor moving. The origin of the co-ordinate system is placed on the bottom surface of the secondary conductor(see Fig.2.2.1.). Whenever there is a motion in the stator, either a linear motion or rotational motion, it is reflected into the secondary conductor so that the conductor moves at the same velocity but travels in the opposite direction.

The linear velocity is defined by a velocity vector $\bar{V}$, where

$$\bar{V} = (V_x, V_y, V_z)$$

This is used so that it enables the electromechanical damping coefficients to be calculated.

The rotational angular velocity is defined by $\bar{\omega}$, where

$$\bar{\omega} = (\omega_x, \omega_y, \omega_z)$$

$$= (\dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_z)$$

The angular velocity of the stator can be expressed in terms of the linear velocity by the equation³³,

$$\bar{V} = \bar{\omega} \times \bar{R}$$

where $\bar{R} = (x, y, z)$

In general, the linear velocity and the rotational velocity can be included in one equation and expressed in terms of the linear velocity only.

$$\bar{V}_{\text{total}} = \bar{V} + \bar{\omega} \times \bar{R} \quad (2.26.)$$

2.8. Field equations

The electromagnetic field equations are defined in three regions; the air gap region, the secondary conductor region and the back iron region. These are shown in Fig.2.8.1. and labelled region 1, region 2 and region 3 respectively. The field equations in these regions are derived from the governing equation(eqn.2.17.),

$$\nabla^2 \bar{A} = \mu \sigma \{ \dot{\bar{A}} - \bar{V} \times \nabla \times \bar{A} + \nabla \phi \}$$

and they are discussed separately as follows:

Region 1 : The air gap region.

Since the conductivity in air is zero, $\sigma = 0$, thus the governing equation (eqn.2.17.) is reduced to a Laplacian equation,

$$\nabla^2 \bar{A}_1 = 0 \quad (2.27.)$$

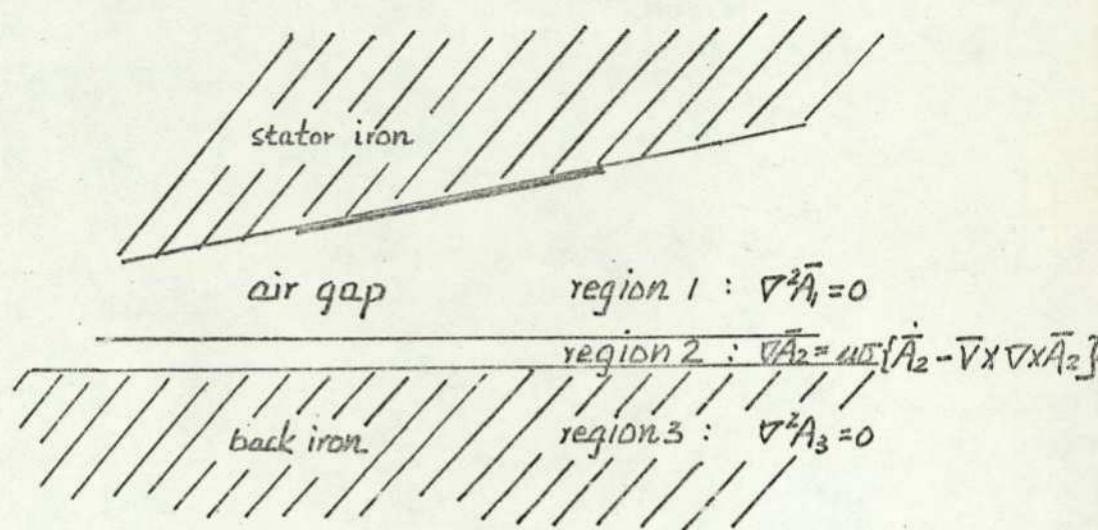


Fig.2.8.1. The three regions defined in the mathematical model.

where $\bar{A}_1 = (A_{1x}, 0, A_{1z})$

Region 2 : The secondary conductor region.

By putting $\bar{A}_2 = (A_{2x}, 0, A_{2z})$

and $\bar{V} = (V_x, V_y, V_z)$

into eqn.2.17., the field in region 2 is defined by the Poisson equation,

$$\nabla^2 \bar{A}_2 = \mu \sigma \left\{ \dot{\bar{A}} - \bar{V} \times \nabla \bar{A} + \nabla \phi \right\}$$

The term $\nabla \phi$ is customarily associated with the electrostatic effects of the charges on a conductor surface, and is usually ignored in the analysis of linear induction motor^{9,10}.

hence

$$\nabla^2 \bar{A}_2 = \mu \sigma \left\{ \dot{\bar{A}} - \bar{V} \times \nabla \bar{A} \right\}$$

i.e.

$$\nabla^2 A_{2x} = \mu \sigma \left\{ \dot{A}_{2x} + V_y \frac{\partial A_{2x}}{\partial y} + V_z \left(\frac{\partial A_{2x}}{\partial z} - \frac{\partial A_{2z}}{\partial x} \right) \right\}$$

$$\nabla^2 A_{2z} = \mu \sigma \left\{ \dot{A}_{2z} - V_x \left(\frac{\partial A_{2x}}{\partial z} - \frac{\partial A_{2z}}{\partial x} \right) + V_y \frac{\partial A_{2z}}{\partial y} \right\}$$

$$\text{and } A_{2y} = 0 \quad (2.28.)$$

Region 3 : The back iron region.

The back iron is laminated, zero conductivity being assumed in this region. The field equation is the same as region 1. a Laplacian

equation,

$$\nabla^2 \bar{A}_3 = 0 \quad (2.29.)$$

2.9. Solutions of the electromagnetic field equations

The electromagnetic field equations in the three regions are solved by the method of separation of variables³⁴ in terms of the magnetic vector potential $\bar{A}$. In order to match the given boundary conditions on the interface between the stator iron and the air gap regions, the particular solutions and the general solutions of the field equations in the three regions are in the form of a double Fourier series, since the defined current sheet at the boundary is in terms of a double Fourier series.

The solutions of the electromagnetic field equations in the three regions are determined as follow:

Region 1. The air gap region,

from eqn.2.27., the field equation in this region is,

$$\nabla^2 \bar{A}_1 = 0$$

$$\text{i.e.} \quad \frac{\partial^2 \bar{A}_{1x}}{\partial x^2} + \frac{\partial^2 \bar{A}_{1x}}{\partial y^2} + \frac{\partial^2 \bar{A}_{1x}}{\partial z^2} = 0$$

$$\text{and} \quad \frac{\partial^2 \bar{A}_{1z}}{\partial x^2} + \frac{\partial^2 \bar{A}_{1z}}{\partial y^2} + \frac{\partial^2 \bar{A}_{1z}}{\partial z^2} = 0$$

By assuming the solution,

$$\bar{A}_1 = X_1(x, t) Y_1(y) Z_1(z)$$

the general solutions for A_{1x} and A_{1z} are:

$$A_{1x} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ (C_{1mn} \exp \beta_{mn} y + C_{2mn} \exp -\beta_{mn} y) \exp j(\alpha_n x - \omega t) \sin \delta_m z \right. \\ \left. + (C'_{1mn} \exp \beta_{mn} y + C'_{2mn} \exp -\beta_{mn} y) \exp j(\alpha_n x - \omega t) \cos \delta_m z \right\}$$

$$A_{1z} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ (C_{3mn} \exp \beta_{mn} y + C_{4mn} \exp -\beta_{mn} y) \exp j(\alpha_n x - \omega t) \cos \delta_m z \right. \\ \left. + (C'_{3mn} \exp \beta_{mn} y + C'_{4mn} \exp -\beta_{mn} y) \exp j(\alpha_n x - \omega t) \sin \delta_m z \right\} \quad (2.29)$$

where

$$\beta_{mn}^2 = \alpha_n^2 + \delta_m^2$$

Region 3. The back iron region.

The electromagnetic field equation in this region (eqn. 2.28.)

is the same as in region 1 - a Laplacian equation. The solution will be the same as in region 1, except with different constants C.

The general solution in this region is,

$$A_{3x} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ (C_{mn} \exp \beta_{mn} y + C'_{mn} \exp -\beta_{mn} y) \exp j(\alpha_n x - \omega t) \sin \delta_m z \right. \\ \left. + (C'_{mn} \exp \beta_{mn} y + C_{mn} \exp -\beta_{mn} y) \exp j(\alpha_n x - \omega t) \cos \delta_m z \right\}$$

$$A_{3z} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ (C_{mn} \exp \beta_{mn} y + C'_{mn} \exp -\beta_{mn} y) \exp j(\alpha_n x - \omega t) \cos \delta_m z \right. \\ \left. + (C'_{mn} \exp \beta_{mn} y + C_{mn} \exp -\beta_{mn} y) \exp j(\alpha_n x - \omega t) \sin \delta_m z \right\}$$

(2.30)

Region 2. The secondary conduction region.

The field equations in region 2 are:

$$\nabla^2 A_{2x} = \mu \sigma \left\{ \dot{A}_{2x} + V_y \frac{\partial A_{2x}}{\partial y} + V_z \left(\frac{\partial A_{2x}}{\partial z} - \frac{\partial A_{2z}}{\partial x} \right) \right\}$$

$$\nabla^2 A_{2z} = \mu \sigma \left\{ \dot{A}_{2z} - V_x \left(\frac{\partial A_{2x}}{\partial z} - \frac{\partial A_{2z}}{\partial x} \right) + V_y \frac{\partial A_{2z}}{\partial y} \right\}$$

In the above equations, A_{2x} and A_{2z} appear in both equations and they are said to be coupled, which leads to problems in solving these equations. This can be overcome in a two dimensional analysis by assuming that A_{2x} is small and can be neglected, only the A_{2z} component being considered. In a three dimensional analysis, the A_{2x} component is also considered and can not be neglected (as

otherwise the analysis would be in two dimensions only). In order to determine the solutions of the field equations in a three dimensional analysis, the derivative of A_{2x} with respect to z is assumed to be small when compared with the derivative of A_{2z} with respect to x , i.e.

$$\frac{\partial A_{2x}}{\partial z} \ll \frac{\partial A_{2z}}{\partial x}$$

The field equations in region 2 will then be reduced to,

$$\nabla^2 A_{2x} = \mu \sigma \left\{ \dot{A}_{2x} + V_y \frac{\partial A_{2x}}{\partial z} - V_z \frac{\partial A_{2z}}{\partial x} \right\} \quad (2.31.)$$

$$\nabla^2 A_{2z} = \mu \sigma \left\{ \dot{A}_{2z} + V_x \frac{\partial A_{2z}}{\partial x} - V_y \frac{\partial A_{2x}}{\partial y} \right\} \quad (2.32.)$$

Eqn.2.32. contains only A_{2z} which is ready to be solved by the method of separation of variables. But eqn.2.31. still contains both A_{2x} and A_{2z} . Its solution will be discussed later in this section.

Let

$$A_{2z} = A_{2z}(x, y, z, t)$$

$$= X_{2z}(x, t) Y_{2z}(y) Z_{2z}(z)$$

eqn.2.32. becomes,

$$X_{2z}'' Y_{2z} Z_{2z} + X_{2z} Y_{2z}'' Z_{2z} + X_{2z} Y_{2z} Z_{2z}'' = \mu \sigma \left\{ \frac{d}{dt} (X_{2z} Y_{2z} Z_{2z}) + V_x X_{2z}' Y_{2z} Z_{2z} + V_y X_{2z} Y_{2z}' Z_{2z} \right\} \quad (2.33.)$$

The supplied current is assumed sinusoidal hence all the field quantities are sinusoidal in time.

The solutions for X_{2z} and Z_{2z} are,

$$X_{2z}(x, t) = \sum_{n=-\infty}^{\infty} \exp j(\alpha_n x - \omega t)$$

$$Z_{2z}(z) = \sum_{m=1}^{\infty} \cos \delta_m z$$

Thus

$$X_{2z}' = j \alpha_n X_{2z}$$

$$X_{2z}'' = -\alpha_n^2 X_{2z}$$

$$\frac{d}{dt} X_{2z} = -j\omega X_{2z}$$

and

$$Z_{2z}'' = -\gamma_m^2 Z_{2z}$$

Substitute the above four equations into eqn.2.33.:

$$(-\alpha_n^2 Y_{2z} + Y_{2z}'' - \gamma_m^2 Y_{2z}) \cdot X_{2z} Z_{2z} = \mu\sigma (-j\omega Y_{2z} + j\alpha_n V_x Y_{2z} + V_y Y_{2z}') X_{2z} Z_{2z}$$

i.e.

$$Y_{2z}'' + Y_{2z}'(-\mu\sigma V_y) + (-\alpha_n^2 - \gamma_m^2 + j\omega\mu\sigma - j\mu\sigma V_x \alpha_n) Y_{2z} = 0$$

$$Y_{2z}'' + d_{1mn} Y_{2z}' + d_{2mn} Y_{2z} = 0$$

where

$$d_{1mn} = -\mu\sigma V_y$$

$$d_{2mn} = -\alpha_n^2 - \gamma_m^2 + j\mu\sigma(\omega - V_x \alpha_n)$$

The general solution for Y_{2z} is,

$$Y_{2z} = \{ C_{1mn} \exp E_{1mn} y \cos E_{2mn} z + C_{2mn} \exp E_{1mn} y \sin E_{2mn} z \}$$

and the general solution for A_{2z} is,

$$A_{2z} = X_{2z}(x, t) Y_{2z}(y) Z_{2z}(z)$$

$$= \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \{ C_{1mn} \cos E_{2mn} z + C_{2mn} \sin E_{2mn} z \} \exp E_{1mn} y \exp j(\alpha_n x - \omega t) \cos$$

(2.34.)

where

$$E_{1mn} = -\frac{d_{1mn}}{2}$$

$$E_{2mn} = \sqrt{4d_{2mn}^2 - d_{1mn}^2}/2$$

The field equation of A_{2x} in the secondary conductor region is (eqn.2.32.):

$$\nabla^2 A_{2x} = \mu\sigma \left\{ \dot{A}_{2x} + V_y \frac{\partial A_{2x}}{\partial y} - V_z \frac{\partial A_{2z}}{\partial x} \right\}$$

It is rewritten in the following form:

$$\nabla^2 A_{2x} - \mu\sigma \left\{ \dot{A}_{2x} + V_y \frac{\partial A_{2x}}{\partial y} \right\} = -\mu\sigma V_z \frac{\partial A_{2z}}{\partial x} \quad (2.35.)$$

The complementary solution on the R.H.S. of the equation is assumed to be as follows:

$$A_{2x} = X_{2x}(x, t) Y_{2x}(y) Z_{2x}(z)$$

and

$$X_{2x} = \sum_{n=-\infty}^{\infty} \exp j(\alpha_n x - \omega t)$$

$$Z_{2x} = \sum_{m=1}^{\infty} \sin \gamma_m z$$

hence,

$$X_{2x}'' = -\alpha_n^2 X_{2x}$$

$$Z_{2x}'' = -\gamma_m^2 Z_{2x}$$

The above equations into eqn. 2.35. give:

$$(-\alpha_n^2 Y_{2x} + Y_{2x}'' - \gamma_m^2 Y_{2x}) X_{2x} Z_{2x} - \mu \sigma (-j\omega X_{2x} Y_{2x} Z_{2x} + V_y X_{2x} Y_{2x}' Z_{2x}) = -\mu \sigma \frac{\partial A}{\partial z}$$

$$\text{i.e. } \{ Y_{2x}'' + Y_{2x}'(-\mu \sigma V_y) + (-\alpha_n^2 - \gamma_m^2 + j\mu \sigma \omega) Y_{2x} \} X_{2x} Z_{2x} = -\mu \sigma X_{2z}' Y_{2z} Z_{2z}$$

$$Y_{2x}'' + Y_{2x}' d_{1mn} + Y_{2x} d_{2mn} = \frac{-\mu \sigma V_z Z_{2z}}{Z_{2x}} \quad (2.36.)$$

where

$$d_{1mn} = -\mu \sigma V_y$$

$$d_{2mn} = -\alpha_n^2 - \gamma_m^2 + j\mu \sigma \omega$$

and

$$X_{2x} = X_{2z}$$

The complementary function for the solution of Y_{2x} on the R.H.S. of eqn. 2.36. is:

$$(C_{1mn} \cos E_{2mnx} y + C_{10mn} \sin E_{2mnx} y) \exp F_{1mn} y$$

The particular integral of eqn. 2.36. is:

$$\frac{-j\alpha_n V_z Z_{2z}}{(E_{2mnx}^2 - E_{2mny}^2) Z_{2x}} (C'_{1mn} \cos E_{2mnx} y + C'_{10mn} \sin E_{2mnx} y) \exp F_{1mn} y$$

where

$$E_{1mn} = \frac{-d_{1mn}}{2}$$

$$E_{2mnx} = \frac{\sqrt{4d_{2mnx}^2 - d_{1mn}^2}}{2}$$

and the complete general solution of A_{2x} is:

$$A_{2x} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ (C_{qmn} \cos E_{2mnx} y + C_{omn} \sin E_{2mnx} y) \sin \delta_m z \right.$$

$$\left. + \frac{j \alpha_n V_z}{E_{2mnz}^2 E_{2mnx}^2} (C_{qmn}' \cos E_{2mnx} y + C_{omn}' \sin E_{2mnx} y) \cos \delta_m z \right\}$$

$$\cdot \exp j(\alpha_n x - \omega t) \cdot \exp E_{1mn} y$$

(2.37.)

2.10. Boundary conditions and boundary equations

The electromagnetic field at the boundary between two media can be equated in terms of tangential and normal components³².

The equations for the tangential components, with a current sheet between the two media are:

$$H_{1x} - H_{2x} = -J_z$$

and $H_{1z} - H_{2z} = J_x$

see Fig.2.10.1. and Fig.2.10.2..

In terms of magnetic vector potential $\bar{A}$,

$$\frac{1}{\mu_0 \mu_{r1}} \frac{\partial A_{1z}}{\partial y} - \frac{1}{\mu_0 \mu_{r2}} \frac{\partial A_{2z}}{\partial y} = -J_z$$

and

$$-\frac{1}{\mu_0 \mu_{r1}} \frac{\partial A_{1x}}{\partial y} + \frac{1}{\mu_0 \mu_{r2}} \frac{\partial A_{2x}}{\partial y} = J_x$$

where suffixes 1 and 2 refer to medium 1 and medium 2 respectively, and J_x and J_z are components of current density between the two media.

In the normal direction, the component of the flux density B_{y1} in medium 1 should be equal to the normal component of the flux density in medium 2 at the boundary (see Fig.2.10.3.), i.e.

$$B_{y1} = B_{y2}$$

In terms of the magnetic vector potential $\bar{A}$,

$$\frac{\partial A_{1x}}{\partial z} - \frac{\partial A_{1z}}{\partial x} = \frac{\partial A_{2x}}{\partial z} - \frac{\partial A_{2z}}{\partial x}$$

Since A_x and A_z are independent variables, they can be written as two equations,

$$\frac{\partial A_{1x}}{\partial z} = \frac{\partial A_{2x}}{\partial z}$$

and

$$\frac{\partial A_{1z}}{\partial x} = \frac{\partial A_{2z}}{\partial x}$$

There are three defined regions, region 1, region 2 and region 3 (see Fig.2.10.4.) and one undefined region, the stator iron. Between these regions, there are three boundaries,

1. between stator iron and the air gap region($y=g$)
2. between the air gap and the secondary conductor($y=\delta$)
3. between the secondary conductor and the back iron region ($y = 0$).

Also, there is one more boundary condition at $y = -\infty$, where the magnetic vector potential $\overline{A}$ needs to be defined. The boundary conditions at various interfaces are as follows:

Between stator iron and air gap region ($y = g$):

The permeability in the stator iron is assumed to be infinite, so that there is no normal component on the stator surface to be equated.

The tangential components are,

$$-\frac{1}{\mu_0} \frac{\partial A_{1x}}{\partial y} = J_x(z_0) \quad \text{at } y=g(x,z) \quad (2.38.)$$

$$\frac{1}{\mu_0} \frac{\partial A_{1z}}{\partial y} = -J_z(z_0) \quad \text{at } y=g(x,z) \quad (2.39.)$$

Boundary between the air gap region and the secondary conductor region($y = \delta$):

Normal component are,

$$\frac{\partial A_{1x}}{\partial z} = \frac{\partial A_{2x}}{\partial z} \quad \text{at } y=\delta \quad (2.40.)$$

and $\frac{\partial A_{1z}}{\partial x} = \frac{\partial A_{2z}}{\partial x} \quad \text{at } y = \delta \quad (2.41.)$

Tangential components are:

$$\frac{\partial A_{1x}}{\partial y} = \frac{\partial A_{2x}}{\partial y} \quad \text{at } y=\delta \quad (2.42.)$$

$$\frac{\partial A_{1z}}{\partial y} = \frac{\partial A_{2z}}{\partial y} \quad \text{at } y=\delta \quad (2.43.)$$

Boundary between the secondary conductor region and the back iron region ($y = 0$):

Normal components:

$$\frac{\partial A_{2x}}{\partial z} = \frac{\partial A_{3x}}{\partial z} \quad \text{at } y = 0 \quad (2.44.)$$

$$\frac{\partial A_{2z}}{\partial x} = \frac{\partial A_{3z}}{\partial x} \quad \text{at } y = 0 \quad (2.45.)$$

Tangential components:

$$\frac{\partial A_{2x}}{\partial y} = \frac{1}{\mu_{r3}} \frac{\partial A_{3x}}{\partial y} \quad \text{at } y = 0 \quad (2.46.)$$

$$\frac{\partial A_{2z}}{\partial y} = \frac{1}{\mu_{r3}} \frac{\partial A_{3z}}{\partial y} \quad \text{at } y = 0 \quad (2.47.)$$

Boundary conditions at $y = -\infty$:

Since the flux density is assumed to be zero,

$$A_{3x} = 0 \quad \text{at } y = -\infty \quad (2.48.)$$

$$\text{and } A_{3z} = 0 \quad \text{at } y = -\infty \quad (2.49.)$$

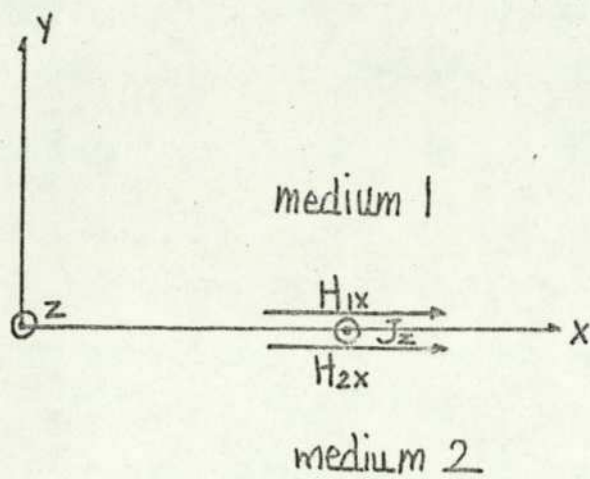


Fig. 2.10.1. Boundary with J_z component between two media

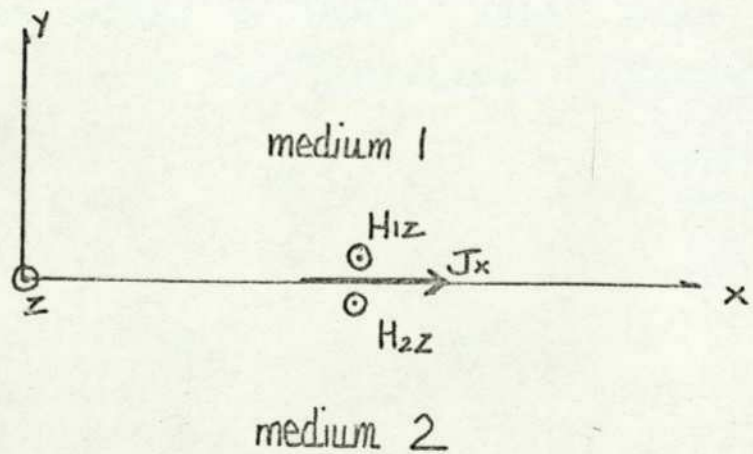


Fig. 2.10.2. Boundary with J_x component between two media

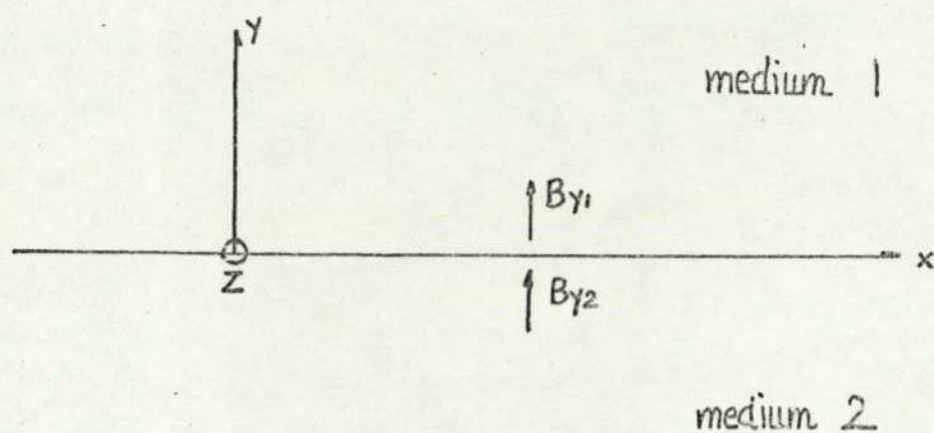


Fig. 2.10.3. Boundary condition in normal direction
 $B_{y1} = B_{y2}$

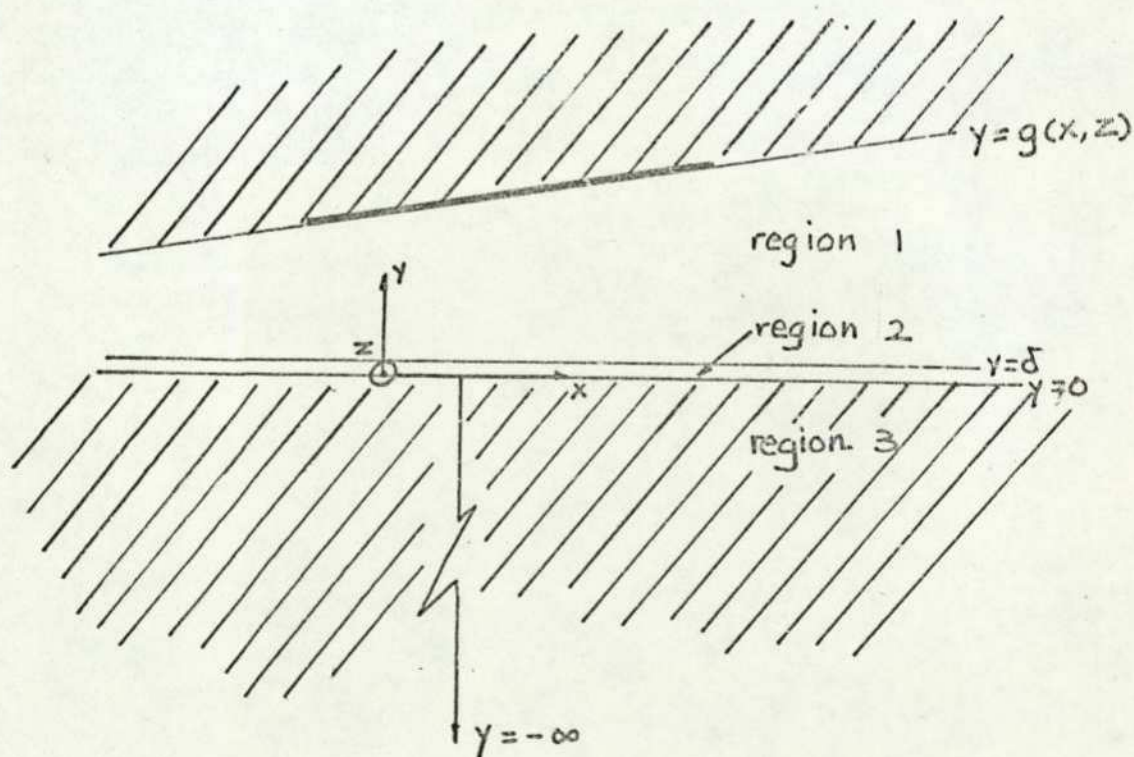


Fig. 2.10.4. The three regions and their boundaries

2.11. Determination of the field quantities

In section 2.9., the magnetic vector potential in the three regions ($\bar{A}_1$, $\bar{A}_2$ and $\bar{A}_3$) are obtained in terms of constants C_{mn} and C'_{mn} in six different equations (eqn.2.29., eqn.2.30. and eqn.2.31.). There are twenty two unknown constants in these equations ($C_{1mn}, C_{2mn}, C_{3mn} \dots \dots \dots C_{12mn}$ and $C'_{1mn}, C'_{2mn} \dots \dots \dots C'_{10mn}$). By substitution of these six equations into the twelve equations obtained from the boundary conditions in the last section (eqn.2.38. to eqn. 2.49.) and by equating the n^{th} and m^{th} harmonic of the $\sin \chi_{mz}$ and $\cos \chi_{mz}$ terms separately, twenty two equations can be obtained (see Appendix A2). The values of the twenty two unknown constants C can be evaluated from these twenty two equations. The long winded manipulation of evaluation of these constants is not shown in the thesis. However, after the constants C have been determined and substituted back into eqn.2.29., eqn.2.30. and eqn.2.31., the final solutions of the magnetic vector potential $\bar{A}$ in the three regions are as follows:

$$\bar{A}_1 = (A_{1x}, 0, A_{1z})$$

$$\bar{A}_2 = (A_{2x}, 0, A_{2z})$$

$$\bar{A}_3 = (A_{3x}, 0, A_{3z})$$

$$A_{1x} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \left[\frac{\beta_{mn} (E_{3mnx} - E_{2mnx} \cot E_{2mnx} \delta)}{E_{4mnx}} \cdot \cosh \beta_{mn} (y - \delta) + \sinh \beta_{mn} (y - \delta) \right] \cdot \frac{\mu_0 a_{mnx} \exp j(\alpha_n x - \omega t) \sin \delta_m z}{\Delta_{mnx} \beta_{mn}} \right. \\ \left. - \frac{V_z a_{mnz} (P_z(\delta) - P_z(\delta))}{V_x \Delta_{mnx} \Delta_{mnz}} \cdot \cosh \beta_{mn} (y - \delta) \cdot \exp j(\alpha_n x - \omega t) \cosh \delta_m z \right\}$$

$$A_{1z} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} - \left\{ \frac{\beta_{mn} (E_{3mnz} - E_{2mnz} \cot E_{2mnz} \delta)}{E_{4mnz}} \cdot \cosh \beta_{mn} (y - \delta) + \sinh \beta_{mn} (y - \delta) \right\} \frac{\mu_0 a_{mnz} \exp j(\alpha_n x - \omega t) \cos \delta_m z}{\Delta_{mnz} \beta_{mn}}$$

(2.50)

$$A_{2x} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \frac{\exp E_{1mn}(y-\delta) P_z(y) M_0 A_{mnx} \exp j(\alpha_n x - \omega t) \sin \delta_m Z}{\Delta_{mnx}} \right. \\ \left. - \frac{V_z A_{mnz} \exp E_{1mn}(y-\delta)}{V_x \Delta_{mnz} \sigma} \left[P_z(y) - P_x(y) \cdot \left(1 - \frac{P_z(\delta) - P_x(\delta)}{\Delta_{mnx}} \beta_{mn} \sinh \beta_{mn}(g-\delta) \right) \cos \delta_m Z \exp j(\alpha_n x - \omega t) \right] \right\}$$

$$A_{2z} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ - \frac{\exp E_{1mn}(y-\delta) P_z(y) M_0 A_{mnz} \exp j(\alpha_n x - \omega t) \cos \delta_m Z}{\Delta_{mnz}} \right\} \quad (2.51.)$$

$$A_{3x} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \frac{-E_{2mnx} M_0 A_{mnx} \exp \beta_{mn}(y - E_{1mn}\delta) \sin \delta_m Z}{\sin E_{2mnx} \cdot E_{4mnx} \Delta_{mnx}} \exp j(\alpha_n x - \omega t) - \frac{V_z A_{mnz}}{\sigma V_x \Delta_{mnz}} \left[\frac{E_{2mnz}}{\sin E_{2mnz} \cdot \delta E_{4mnz}} \right. \right. \\ \left. \left. - \frac{E_{2mnx}}{E_{4mnx} \sin E_{2mnx} \delta} \left(1 - \frac{P_z(\delta) - P_x(\delta)}{\Delta_{mnx}} \beta_{mn} \sinh \beta_{mn}(g-\delta) \right) \right] \exp \beta_{mn} y \cos \delta_m Z \exp j(\alpha_n x - \omega t) \right\}$$

$$A_{3z} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{E_{2mnz} M_0 A_{mnz} \exp(\beta_{mn} y - E_{1mn}\delta) \cos \delta_m Z \exp j(\alpha_n x - \omega t)}{\sin E_{2mnz} \delta E_{4mnz} \Delta_{mnz}} \quad (2.52.)$$

where $E_{3max} = E_{3min}$

$$= (E_{1min} - \frac{\beta_{mn}}{\mu_r})$$

$$E_{4max} = \left\{ (E_{1min} E_{3max} + E_{2max}^2) - \frac{\beta_{mn} E_{2max}}{\mu_r} \cot E_{2max} \delta \right\}$$

$$E_{4max} = \left\{ (E_{1min} E_{3min} + E_{2min}^2) - \frac{\beta_{mn} E_{2min}}{\mu_r} \cot E_{2min} \delta \right\}$$

$$P_x(y) = \frac{E_{3max} \sin E_{2max} y - E_{2max} \cos E_{2max} y}{\sin E_{2max} \delta \cdot E_{4max}}$$

$$P_z(y) = \frac{E_{3min} \sin E_{2min} y - E_{2min} \cos E_{2min} y}{\sin E_{2min} \delta \cdot E_{4min}}$$

$$\Delta_{mnx} = \left\{ \frac{\beta_{mn} (E_{3max} - E_{2max} \cot E_{2max} \delta) \cdot \sinh \beta_{mn} (g - \delta) + \cosh \beta_{mn} (g - \delta)}{E_{4max}} \right\}$$

$$\Delta_{mnz} = \left\{ \frac{\beta_{mn} (E_{3min} - E_{2min} \cot E_{2min} \delta) \cdot \sinh \beta_{mn} (g - \delta) + \cosh \beta_{mn} (g - \delta)}{E_{4min}} \right\}$$

2.12. Magnetic flux density and power flow

The magnetic flux density follows directly from the definition of the magnetic vector potential $\bar{A}$ (eqn.2.12.),

$$\bar{B} = \nabla \times \bar{A}$$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & 0 & A_z \end{vmatrix}$$

i.e.

$$B_x = \frac{\partial A_z}{\partial y}$$

$$B_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}$$

$$B_z = -\frac{\partial A_x}{\partial y}$$

Each component of $\bar{B}$ requires the differentiation of A_x and A_z with respect to the three co-ordinates x , y and z . Since A_x and A_z are in terms of the Fourier series, they can be differentiated term by term.

The net power flow across the air gap can be found by using the Poynting Vector, where

$$\bar{P} = \bar{E} \times \bar{H}$$

The active power flow in the y direction is,

$$P_y = \text{Re} (E_z \cdot H_x^* - H_z^* \cdot E_x)$$

$$= -\frac{1}{\mu_0} \text{Re} (j\omega A_z \frac{\partial A_z}{\partial y} + j\omega A_x \frac{\partial A_x}{\partial y}) \quad (2.53.)$$

The total power flow across the air gap from the primary to the secondary of the machine is,

$$P_{y_{\text{total}}} = \int_{-l_z/2}^{l_z/2} \int_{-l_x/2}^{l_x/2} -\frac{1}{\mu_0} j\omega \text{Re} (A_z \frac{\partial A_z}{\partial y} + A_x \frac{\partial A_x}{\partial y}) dx dz \quad (2.54.)$$

2.13. The three axis forces

The electromagnetic force of the machine is found by using the Maxwell's second stress tensor^{29,34}. The forces generated on the secondary conductor are due to the interaction between the current $\vec{J}$, and the magnetic flux $\vec{B}$ passing through the conductor, i.e.

$$\vec{F} = \vec{J} \times \vec{B}$$

The total force generated is found by a volume integration over the whole secondary conductor,

$$\vec{F} = \int_V (\vec{J} \times \vec{B}) \cdot d\vec{v} \quad (2.55.)$$

The vector $(\vec{J} \times \vec{B})$ can be written in the form of a divergence of another Tensor, $\vec{T}$, i.e.

$$\text{div } \vec{T} = \vec{J} \times \vec{B}$$

Substitute this into eqn.2.55. and by using the Divergence Theorem, gives,

$$\begin{aligned} \vec{F} &= \int_V (\text{div } \vec{T}) \cdot d\vec{v} \\ &= \int_S \vec{T} \cdot d\vec{s} \end{aligned} \quad (2.56.)$$

The volume integration becomes a surface integration, where

$$\vec{T} = \begin{vmatrix} \frac{\mu}{2}(H_x^2 - H_y^2 - H_z^2) & \mu H_x H_y & \mu H_x H_z \\ \mu H_y H_x & \frac{\mu}{2}(-H_x^2 + H_y^2 - H_z^2) & \mu H_y H_z \\ \mu H_z H_x & \mu H_z H_y & \frac{\mu}{2}(H_x^2 - H_y^2 + H_z^2) \end{vmatrix}$$

and S represents all the surfaces enclosing the secondary conductor, where the forces are produced.

In finding the three axis forces, the choice of surface of the integration is quite arbitrary, so long as it encloses the secondary conductor where the forces are produced. In this analysis, the surface of integration is chosen to be the same as the surface of the secondary conductor, thus the exact force density can be found. If some other surfaces are chosen as the surface of integration, though the total force after the surface integration will be the same, the individual force density on the chosen surface does not represent the actual force density distribution on the secondary conductor.

In this analysis, only two surfaces, S_1 and S_2 , are used as the surface of integration (see Fig.2.13.1.), where

$$\begin{aligned} S_1 : y &= \delta \\ S_2 : y &= 0 \end{aligned} \quad \begin{cases} -L_x/2 \leq x \leq L_x/2 \\ -L_z/2 \leq z \leq L_z/2 \end{cases}$$

since in the mathematical model used, there is no flux crossing the surface $dx \cdot dy$ (at $z = L_z/2$ and $z = -L_z/2$) and the surfaces $dy \cdot dz$ (at $x = L_x/2$ and $x = -L_x/2$).

From eqn.2.56.,

$$\vec{F} = \int_{S: S_1 + S_2} \vec{T} \cdot d\vec{s} = (F_x, F_y, F_z)$$

The total three axis forces are:

$$F_x = \int_{S_1 + S_2} \frac{1}{2} \mu H_x H_y \, dx \, dz$$

$$F_y = \int_{S_1 + S_2} \frac{1}{4} \mu (-H_x^2 + H_y^2 - H_z^2) \, dx \, dz$$

$$F_z = \int_{S_1+S_2} \frac{1}{2} \mu H_y H_z dx dz$$

where F_x , F_y and F_z are the time averaging forces in the x, y and z directions respectively.

Differentiating $\bar{F}$ with respect to x and z gives the force density distribution function on the secondary conductor surface.

$$\begin{aligned} \frac{\partial^2 \bar{F}}{\partial x \partial z} &= \bar{f}(x, z) \\ &= (f_x, f_y, f_z) \end{aligned}$$

and

$$\begin{aligned} f_x &= \frac{1}{2} \mu H_x H_y \Big|_{y=\delta} - \frac{1}{2} \mu H_x H_y \Big|_{y=0} \\ f_y &= \frac{1}{4} \mu (-H_x^2 + H_y^2 - H_z^2) \Big|_{y=\delta} - \frac{1}{4} \mu (-H_x^2 + H_y^2 - H_z^2) \Big|_{y=0} \\ f_z &= \frac{1}{2} \mu H_y H_z \Big|_{y=\delta} - \frac{1}{2} \mu H_y H_z \Big|_{y=0} \end{aligned}$$

In finding the three axis forces, the integration of $\bar{F}$ over S_1 and S_2 is evaluated by numerical integration in this analysis. A three-point Simpson's Rule is used to integrate along the x and the z directions.

$$\begin{aligned} \bar{F} &= \int_{S_1+S_2} f(x, z) dx dz \\ &= \int_{S_1} f(x, z) dx dz + \int_{S_2} f(x, z) dx dz \\ &= \frac{1}{3} \sum_{k=1}^K \sum_{i=1}^I \left\{ KK \cdot II \cdot f(x_i, z_k) \Delta x \Delta z + KK \cdot II \cdot f(x_i, z_k) \Delta x \Delta z \right\} \end{aligned}$$

(2.57.)

where I and K are the number of intervals taken along the x and z directions for the numerical integration.

and

$$\Delta x = Lx/I$$

$$\Delta z = Lz/K$$

$$Xi = -Lx/2 + \Delta x \cdot i$$

$$Zk = -Lz/2 + \Delta z \cdot k$$

$$KK = KK(k)$$

$$= 1$$

$$k = 1, K$$

$$= 4$$

$$k = 2, 4, 6 \dots$$

$$= 2$$

$$k = 3, 5, 7 \dots$$

$$II = II(i)$$

$$= 1$$

$$i = 1, I$$

$$= 4$$

$$i = 2, 4, 6 \dots$$

$$= 2$$

$$i = 3, 5, 7 \dots$$

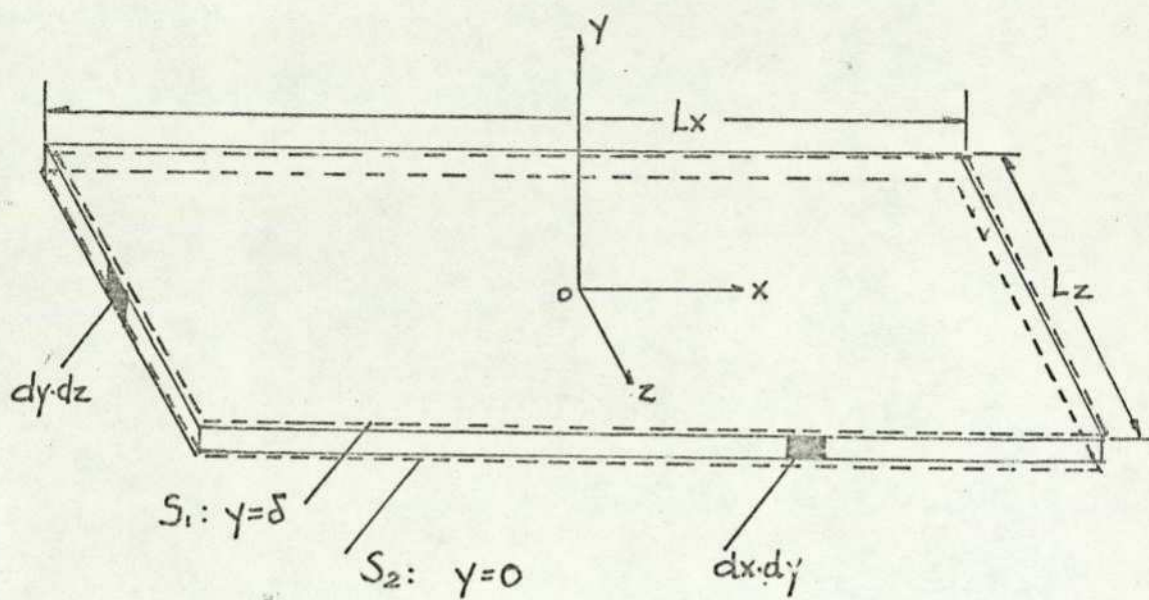


Fig. 2.13.1. Surface of integration S_1 and S_2

2.14. Roll, pitch and yaw couples

The equation for the moment is ²⁹,

$$\bar{M} = \bar{F} \times \bar{R}$$

where $\bar{R} = (x, y, z)$

The moment of a thin plate having a force density distribution $\bar{f}$ on the surface along the x and z directions is:

$$\begin{aligned}\bar{M} &= \int_S (\bar{f} \times \bar{R}) dx dz \\ &= (M_x, M_y, M_z)\end{aligned}$$

$$\begin{aligned}\text{and } M_x &= \int_S (f_y \cdot z - f_z \cdot y) dx dz \\ M_y &= \int_S (f_z \cdot x - f_x \cdot z) dx dz \\ M_z &= \int_S (f_x \cdot y - f_y \cdot x) dx dz\end{aligned}$$

In this analysis, the total moments of the three rotational couples are found by numerical integration, using the same method to evaluate the three axis forces as in the last section. A three point Simpson's Rule is used, thus

$$\begin{aligned}M_x &= \frac{1}{3} \sum_{k=1}^K \sum_{l=1}^I II \cdot KK \cdot (f_y(x_l, z_k) \cdot z_k - f_z(x_l, z_k) \cdot y_{lk}) \Delta x \Delta z \\ M_y &= \frac{1}{3} \sum_{k=1}^K \sum_{l=1}^I II \cdot KK \cdot (f_z(x_l, z_k) \cdot x_l - f_x(x_l, z_k) \cdot z_k) \Delta x \Delta z \\ M_z &= \frac{1}{3} \sum_{k=1}^K \sum_{l=1}^I II \cdot KK \cdot (f_x(x_l, z_k) \cdot y_{lk} - f_y(x_l, z_k) \cdot x_l) \Delta x \Delta z\end{aligned}$$

(2.58.)

2.15. Induced voltage in the primary windings and the constant voltage operation

In the previous analysis, a constant current source excitation is assumed, and the primary resistance and leakage reactance were ignored. This section relates constant current operation to constant voltage operation by means of the input impedance of the machine. A basic circuit diagram is shown in Fig.2.15.1.

The primary leakage reactance is mainly due to the primary slot leakage and the overhang leakage, thus, only these leakage reactances are considered. The primary resistance R_1 and the primary reactance X_1 can be obtained from the conventional equations^{31,35,36}.

$$R_1 = \frac{\rho L_t N}{a}$$

$$= \frac{2.1 \cdot N^2 L_t}{10^6 a} \text{ ohms at } 75^\circ\text{C} \quad (2.59.)$$

where N = number of turns in series,

L_t = mean length of turn in cm,

and a = total copper cross-section of winding in square cm.

$$X_1 = 2 \pi \cdot f \cdot L_c \cdot \text{number of coils per phase} \quad (2.60.)$$

where $L_c = \mu_s N_s^2 (\lambda L_i + 2 \sqrt{q} (L_m - L_i))$

N_s = number of turns per slot,

L_i = length of conductor in active zone,

L_m = mean length of a half turn,

q = number of slots per pole per phase,

and λ = slot permeance ratio.

The primary input voltage $V_{i/p}$ is,

$$V_{i/p} = I_1 R_1 + j I_1 X_1 + V_{in}/\sqrt{2} \quad (2.61.)$$

where I_1 = primary input current

V_{in} = induced voltage in the primary winding.

The primary induced voltage is now determined.

The voltage induced in the coil AB of the primary winding
(see Fig. 2.15.2.) is,

$$V_{AB} = -N \frac{d\psi}{dt}$$

where N is the number of turns of coil AB

and ψ is the flux passing through the coil AB.

$$\begin{aligned} V_{AB} &= -N \frac{d}{dt} \int_S \vec{B} \cdot d\vec{s} \\ &= -N \frac{d}{dt} \int_S (\nabla \times \vec{A}) \cdot d\vec{s} \\ &= -j\omega N \int_S (\nabla \times \vec{A}) \cdot d\vec{s} \end{aligned}$$

by Stokes' Theorem,

$$\begin{aligned} V_{AB} &= -j\omega N \oint_{AB} \vec{A} \cdot d\vec{l} \\ &= -j\omega N \oint_{AB} A_x \cdot dx + A_z \cdot dz \end{aligned} \quad (2.62.)$$

where the path of integration is around the coil AB shown in Fig. 2.15.2., S is the surface enclosed by the coil AB and $d\vec{l}$ is the unit length along the coil AB.

The magnetic vector potential A_x and A_z , where the coil AB is located, can be obtained by substituting,

$$y = g_0$$

$$V_y = 0$$

$$V_z = 0$$

$$\text{and } z_0 = 0$$

into eqn. 2.50., that is, the stator is in a steady state condition without any lateral displacement, and is travelling in the x direction only with a velocity V_x . The value of A_x and A_z would be in the form of:

$$A_x = \sum_{m=-1}^{\infty} \sum_{n=-\infty}^{\infty} K_{xmn} \cdot \exp j(\alpha_n x - \omega t) \cdot \sin \gamma_m z$$

and

$$A_z = \sum_{m=-1}^{\infty} \sum_{n=-\infty}^{\infty} K_{zmn} \cdot \exp j(\alpha_n x - \omega t) \cdot \cos \gamma_m z$$

where K_{xmn} and K_{zmn} are constants and depend on the machine parameters and the machine dimensions.

By integrating eqn. 2.62. in six different sections,

$$\begin{aligned} V_{AB} &= -j\omega N \int \bar{A} \cdot d\bar{l} \\ &= -j\omega N \int (\bar{A} \cdot d\bar{l}_1 + \bar{A} \cdot d\bar{l}_2 + \bar{A} \cdot d\bar{l}_3 + \bar{A} \cdot d\bar{l}_4 + \bar{A} \cdot d\bar{l}_5 + \bar{A} \cdot d\bar{l}_6) \end{aligned}$$

Since the coil AB is symmetrical about the x axis, thus

$$\int \bar{A} \cdot d\bar{l}_1 = \int \bar{A} \cdot d\bar{l}_3$$

$$\int \bar{A} \cdot d\bar{l}_4 = \int \bar{A} \cdot d\bar{l}_6$$

$$\begin{aligned} V_{AB} &= \int_{l_1} 2(\bar{A}_x \cdot d\bar{x} + \bar{A}_z \cdot d\bar{z}) + \int_{l_2} \bar{A}_z \cdot d\bar{z} + \int_{l_4} 2(\bar{A}_x \cdot d\bar{x} + \bar{A}_z \cdot d\bar{z}) \\ &\quad + \int_{l_5} \bar{A}_z \cdot d\bar{z} \end{aligned}$$

$$\begin{aligned}
 V_{AB} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \int_{l_1: Z = -m(X-X_0) - C_1} (2K_{xmn} \exp(j(\alpha_n X - \omega t)) \sin \gamma_m Z \cdot dX + 2K_{zmn} \exp(j(\alpha_n X - \omega t)) \cos \gamma_m Z \cdot dZ) \right. \\
 + \int_{l_2: X = X_0 - X_1} (K_{xmn} \exp(j(\alpha_n X - \omega t)) \cos \gamma_m Z \cdot dZ) \\
 + \int_{l_4: Z = -m(X-X_0) + C_1} (2K_{xmn} \exp(j(\alpha_n X - \omega t)) \sin \gamma_m Z \cdot dX + 2K_{zmn} \exp(j(\alpha_n X - \omega t)) \cos \gamma_m Z \cdot dZ) \\
 \left. + \int_{l_5: X = X_0 + X_1} (K_{zmn} \exp(j(\alpha_n X - \omega t)) \cos \gamma_m Z \cdot dZ) \right\}
 \end{aligned}$$

The procedure of all the mathematical evaluations of the value V_{AB} is not shown in this thesis, but the final equation after the integration is:

$$\begin{aligned}
 V_{AB} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \frac{4 \exp(j(\alpha_n X_0))}{(m \gamma_m)^2 - \alpha_n^2} \left[K_{xmn} (-m \gamma_m \cos \gamma_m C_1 + m \gamma_m \cos \alpha_n X_1 \cos \gamma_m l_z/2) \right. \right. \\
 \left. \left. + \alpha_n \sin \alpha_n X_1 \sin \gamma_m l_z/2 \right] \right. \\
 \left. - j m K_{zmn} (-\alpha_n \cos \gamma_m C_1 + \alpha_n \cos \alpha_n X_1 \cos \gamma_m l_z/2 + m \gamma_m \sin \alpha_n X_1 \sin \gamma_m l_z/2) \right. \\
 \left. + \frac{j 4 K_{zmn}}{\gamma_m} \left[\exp(j \alpha_n X_0) \sin \alpha_n X_1 \sin \gamma_m l_z/2 \right] \right\} \exp(-j \omega t) (j \omega) \\
 (2.63.)
 \end{aligned}$$

where $l_z/2 = m X_1 - C_1$

V_{AB} is the voltage induced in a coil in the primary winding depends on its location, x_0 , along the x axis. When the primary windings are series connected, the total induced voltage per phase is:

$$V_{in} = \sum_1^{\text{total no. of coils per phase}} V_{AB}(x_0)$$

The total input voltage $V_{i/p}$ is directly proportional to the primary input current I_1 by assuming a constant permeability. The input impedance Z_{in} can be found by the equation:

$$Z_{in} = R_1 + jX_1 + \frac{V_{in}(s)}{I_1 \sqrt{2}}$$

where V_{in} is a slip-dependent variable.

For constant voltage operation, the primary input current I_1 can be found by,

$$I_1 = \frac{V_{in}}{Z_{in}}$$

and hence the forces and the flux densities at this particular input voltage can be obtained since $I_1 (J_1)$ is known.

Both forces and flux densities are evaluated using an arbitrary current sheet density J_1 , and their values for a constant input voltage can be obtained accordingly by the relationships:

$$\begin{aligned} \bar{F} &\propto I_1^2 \\ \bar{B} &\propto I_1 \end{aligned}$$

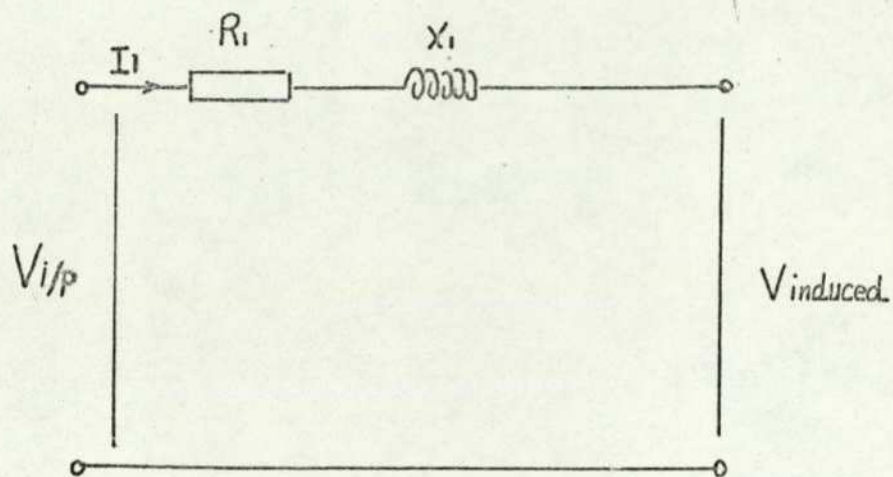


Fig. 2.15.1. Equivalent circuit

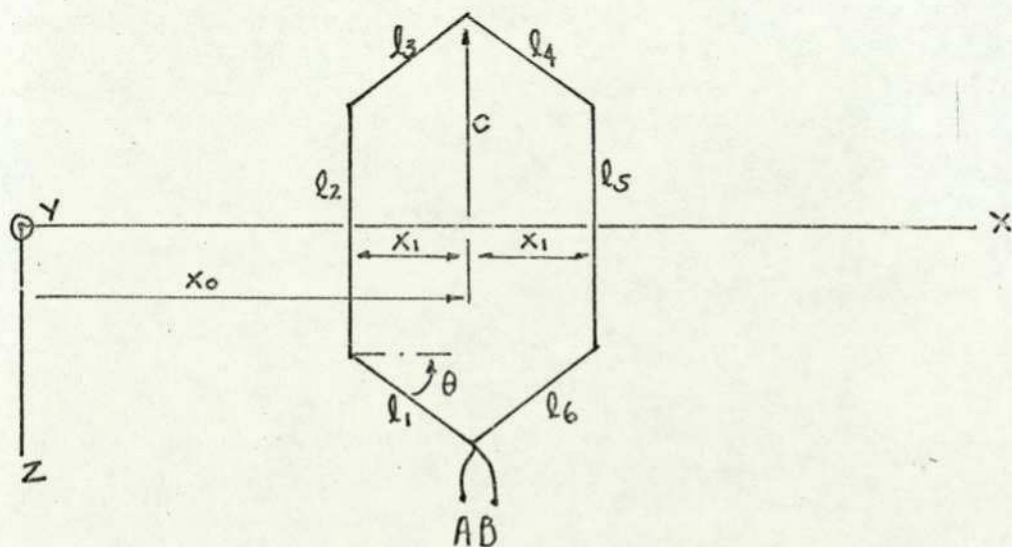


Fig. 2.15.2. Coil AB

CHAPTER 3

CHAPTER 3. MECHANICAL EQUATIONS OF THE LINEAR INDUCTION MOTOR

3.1. Introduction

In the first chapter, the motions of the linear induction motor in six degrees of freedom was briefly discussed. This chapter explained how the mechanical equations of these six degrees of freedom are derived. In the next section examples are given in one degree of freedom in the vertical direction and when the machine pitches, and they are discussed separately. Motion in two degrees of freedom is discussed in section 3.3., and an example is given(vertical motion and pitch occurring at the same time), in order to show how the mechanical equations of the machine in more than one degree of freedom are formulated. In section 3.4. the mechanical equations of six degrees of freedom are discussed and formulated.

3.2. Mechanical equations for one degree of freedom

Fig.3.2.1. depicts a linear induction motor, the machine being allowed to move freely in the vertical direction only. The electromagnetic forces generated in this direction(y direction) can be simulated in terms of mechanical components by a spring with stiffness K_{22} and dash pot with damping coefficient C_{22} . The mass of the stator is assumed to be concentrated at its centre of gravity. The mechanical equation for this type of vertical motion is usually represented in the form^{29,30},

$$m \ddot{y} + C_{22} \dot{y} + K_{22} y = F_{ye} \quad (3.1.)$$

where F_{ye} is the external force applied to the system, and in this case,

$$F_{ye} = mg$$

where g = gravitational acceleration

C_{22} corresponds to the rate of change of the force in the vertical direction with respect to the velocity in the same direction.

K_{22} corresponds to the rate of change of the force in the vertical direction with respect to the displacement in the same direction.

i.e.

$$C_{22} = \frac{\partial F_y}{\partial \dot{y}}$$

and

$$K_{22} = \frac{\partial F_y}{\partial y}$$

If the machine is not allowed to move in the y direction, but instead, allowed to rotate along the z axis, as shown in Fig.3.2.2., the electromagnetic couple generated by the motor in this rotational direction can be simulated by a torsional spring and a dash pot with stiffness coefficient K_{66} and damping coefficient C_{66} . This rotational motion is represented by the equation:

$$I_z \ddot{\theta}_z + C_{66} \dot{\theta}_z + K_{66} \theta_z = T_{ze} \quad (3.2.)$$

where I_z is the moment of inertia of the stator along the z axis,

θ_z is the angle of rotation(angle of pitch),

and T_{ze} is the external torque applied to the system.

In the configuration given, $T_{ze} = 0$

The stiffness and damping coefficients, K_{66} and C_{66} are defined as,

$$C_{66} = \frac{\partial T}{\partial \dot{\theta}_z}$$

and

$$K_{zz} = \frac{\partial T_z}{\partial \dot{\theta}_z}$$

3.3. Mechanical equations for two degrees of freedom

In the previous section, the motion for one degree of freedom was discussed and the mechanical equations were formulated. If the same machine is now allowed to move freely in the vertical direction, as well as to rotate about the z axis, the force generated by the motor would be coupled together in these two degrees of freedom. In order to describe the mechanical motion in these two degrees of freedom, two simultaneous differential equations are needed. They are:

$$m\ddot{y} + C_{22}\dot{y} + C_{26}\dot{\theta}_z + K_{22}y + K_{26}\theta_z = F_{ye} \quad (3.3.)$$

and
$$I_z\ddot{\theta}_z + C_{62}\dot{y} + C_{66}\dot{\theta}_z + K_{62}y + K_{66}\theta_z = T_{ze} \quad (3.4.)$$

The first equation represents the motion in the y direction. Compared with eqn.3.1., two extra terms are introduced, $C_{26}\dot{\theta}_z$ and $K_{26}\theta_z$. They are forces due to the cross coupling between the two degrees of freedom, and C_{26} and K_{26} are defined as:

$$C_{26} = \frac{\partial F_y}{\partial \dot{\theta}_z}$$

the rate of change of vertical force in y direction with respect to the rotational velocity in z direction.

and

$$K_{26} = \frac{\partial F_y}{\partial \theta_z}$$

the rate of change of vertical force in y direction with respect to the rotational displacement about the z axis.

In eqn.3.4., there are also two extra terms, $K_{62}y$ and $C_{62}\dot{y}$. They are the torque generated along the z axis due to the displacement of the machine, and due to the velocity of the machine moving in the y direction. They are defined as:

$$K_{62} = \frac{\partial T_z}{\partial y}$$

and

$$C_{62} = \frac{\partial T_z}{\partial \dot{y}}$$

The two simultaneous differential equations (eqns.3.3. and 3.4.) can be rewritten in matrix form as follow:

$$\begin{bmatrix} m & \Delta \\ 0 & I_z \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \ddot{\theta}_z \end{bmatrix} + \begin{bmatrix} C_{22} & C_{26} \\ C_{62} & C_{66} \end{bmatrix} \begin{bmatrix} \dot{y} \\ \dot{\theta}_z \end{bmatrix} + \begin{bmatrix} K_{22} & K_{26} \\ K_{62} & K_{66} \end{bmatrix} \begin{bmatrix} y \\ \theta_z \end{bmatrix} = \begin{bmatrix} F_{ye} \\ T_{ze} \end{bmatrix}$$

or

$$[M]\ddot{[R]} + [C]\dot{[R]} + [K][R] = [F_e] \quad (3.5.)$$

where $[M]$ is the mass matrix including the mass of the machine and the moment of inertia,

$[C]$ is the damping coefficient matrix,

$[K]$ is the stiffness coefficient matrix,

and $[R]$ is the positional matrix.

3.4. Mechanical equations for six degrees of freedom

In order to describe the mechanical motion of the stator in six degrees of freedom, six simultaneous differential equations are needed. It will be of a form, similar to eqn.3.5.:

$$[M]\ddot{[R]} + [C]\dot{[R]} + [K][R] = [F_e] \quad (3.6.)$$

where

$$[M] = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_x & 0 & 0 \\ 0 & 0 & 0 & 0 & I_y & 0 \\ 0 & 0 & 0 & 0 & 0 & I_z \end{bmatrix}$$

$$[R] = [x, y, z, \theta_x, \theta_y, \theta_z]^T$$

$$[C] = C_{ij} \quad , \quad \text{a } 6 \times 6 \text{ damping coefficient matrix}$$

$$[K] = K_{ij} \quad , \quad \text{a } 6 \times 6 \text{ stiffness coefficient matrix}$$

$$[F_e] = [F_{xe}, F_{ye}, F_{ze}, T_{xe}, T_{ye}, T_{ze}]$$

and

m is the mass of the stator,

I_x , I_y and I_z are the moments of inertia of the stator along the x , y and z axes,

θ_x , θ_y and θ_z are the angles of rotation along the x , y and z axes, i.e. angles of roll, pitch and yaw,

$$C_{ij} = \frac{\partial F_i}{\partial \dot{R}_j} \quad \text{the rate of change of force or torque in the}$$

i^{th} direction with respect to the change of velocity or angular velocity in the j^{th} direction,

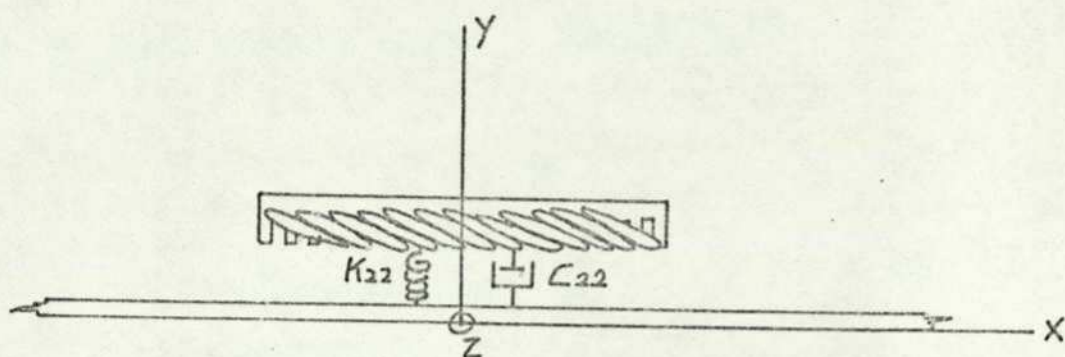


Fig.3.2.1. Simulation of electromagnetic force in the normal direction by a mechanical spring and a dash pot.

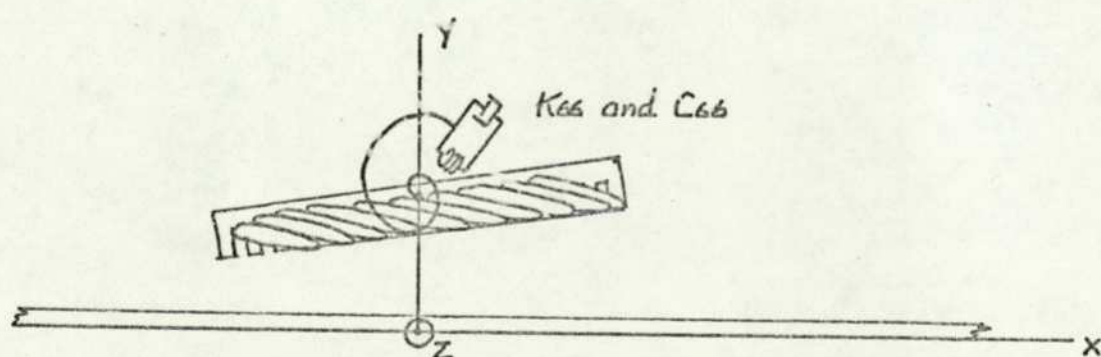


Fig.3.2.2. Simulation of pitch couple by a mechanical spring and a dash pot

$K_{ij} = \frac{\partial F_i}{\partial R_j}$ the rate of change of force or torque in the i^{th} direction with respect to the change of the displacement or angular displacement in the j^{th} direction.

$[K][R]$ are forces or torques which generated due to the changes in the position of the stator, and $[C][\dot{R}]$ are the forces or torques which generated due to the changes in velocity or angular velocity of the stator in six degrees of freedom.

In order to relate the three axis forces and the three rotational couples to the analogous mechanical vibration system in six degrees of freedom, all the forces and the rotational couples have to be evaluated for different stator positions and different velocities. From the definition of K_{ij} , differentiating the electromagnetic forces or couples F_i with respect to the displacement R_j gives the stiffness of the analogous mechanical spring. Similarly, differentiating the forces or couples F_i with respect to the velocity $\dot{R}_j$ gives the damping coefficient of the analogous dash pot.

CHAPTER 4

CHAPTER 4. THEORETICAL RESULTS

4.1. Introduction

In this chapter, theoretical results are obtained for the test machine, a specification of which is given below,

length of stator	$l_x = 0.53\text{m}$
width of stator	$l_z = 0.08\text{m}$
air gap length	$g_o = 0.01\text{m}$
thickness of the track	$\delta = 0.003175\text{m}$
conductivity of the track	$\sigma = 35000000 \text{ S/m}$
supply frequency	$f = 350 \text{ Hz}$
number of phases	$q = 3$
current sheet density	$J_1 = 45000 \text{ A/m}$
pole pitch	$p = 0.113\text{m}$
synchronous speed	$V_{sy} = 79\text{m/s}$
width of overhang winding	$W_o = 0.04\text{m}$
width of conductor	$L_z = 0.24\text{m}$

The length between adjacent machines in the x direction, must be sufficiently long to avoid interactions between machines, as explained in section 2.3.. A length of $L_x = 1.25\text{m}$, proved sufficient.

The distance between adjacent machines in the z direction should be the same as the track width. In order to avoid indeterminate values in the double Fourier Series of the form 0/0 in the computer programming, L_z was made slightly smaller than the actual width of the conductor, so that it would not be in an integral multiple of l_z , $L_z = .2395$ was chosen.

All the calculations were based on the values:

$$g_o = 0.01\text{m}$$

$$z_o = 0.0\text{m}$$

$$\theta_x = 0.0 \text{ rad.}$$

$$\theta_y = 0.0 \text{ rad.}$$

$$\theta_z = 0.0 \text{ rad.}$$

$$V_x = V_{sy} \cdot s \text{ m/s}$$

$$V_y = 0.0 \text{ m/s}$$

$$V_z = 0.0 \text{ m/s}$$

$$\text{and } \omega_x = \omega_y = \omega_z = 0.0 \text{ rad/s}$$

unless otherwise stated.

The machine was assumed to have half-filled end slots at both ends, l_{x1} and l_{x2} were taken as:

$$l_{x1} = 0.48\text{m}$$

$$l_{x2} = 0.313\text{m}$$

The number of terms used in the double Fourier Series in the x and z directions were,

$$N = -21 \text{ to } +21$$

$$M = 0 \text{ to } +31$$

$$\text{instead of } N = -\infty \text{ to } +\infty$$

$$\text{and } M = 0 \text{ to } +\infty$$

which allows space harmonics of 21 terms in the x direction and 31 terms in the z direction to be considered.

In the numerical integration of the total three axis forces and the three rotational couples(see sections 2.12. and 2.13.), the number of intervals taken in the x and z directions were:

$$I = 51$$

$$\text{and } K = 25$$

Carter's Coefficient^{35,36,37} was used for the determination of the effective air gap length of the machine.

$$g_{\text{effective}} = \frac{SP(5g + SW)}{SP(5g+SW) - SW^2} \cdot g$$

where g is the actual air gap length,

/ SW is the slot width

SP is the slot pitch.

The values of SW and SP were assumed to be:

$$SW = 0.011m$$

$$\text{and } SP = 0.01667m$$

All the results were computed by using the ULCC CDC 7600 computer. The graphs of the flux patterns were plotted by using the hardware package DIMFILM. The three dimensional plotting of the flux and force distributions were plotted by using the hardware package SYMUV(see University of London Computer Centre users guide and DIMFILM users guide). The language used is FORTRAN. The computer programs are included in Appendix A3.

4.2. Flux and force distributions

The flux density and force density distributions of the machine, with the given dimensions and parameters, are plotted in Fig.4.2.1. to Fig.4.2.6.. The first three diagrams show the three components of flux density $\vec{B}$, i.e. B_x , B_y and B_z in the air gap at $y=0.003175m$, with $s = 1.0$ and $s = 0.2$. Fig.4.2.4. to Fig.4.2.6. show the three axis force distributions on the secondary conductor sheet with $s = 1.0$ and $s = 0.2$. All the graphs were plotted on the x - z plane between,

$$-0.75m \leq x \leq 0.75m$$

$$-0.12m \leq z \leq 0.12m$$

The total three axis forces at $s = 0.2$ were,

$$F_x = 5.9\text{N}$$

$$F_y = 32.0\text{N}$$

and $F_z = 0.0\text{N}$

The lateral force on one side of the machine, F_{zh} , was found by integration of f_z over half of the secondary conductor plate, between

$$-0.75\text{m} \leq x \leq 0.75\text{m}$$

$$0.0\text{m} \leq z \leq 0.12\text{m}$$

and $F_{zh} = 1.5\text{N}$.

The three rotational couples were,

$$M_r = 0.0 \text{ N-m}$$

$$M_y = 0.0 \text{ N-m}$$

$$M_p = 0.5 \text{ N-m}.$$

Since the machine is placed in a symmetrical position, the moments of roll and yaw are zero.

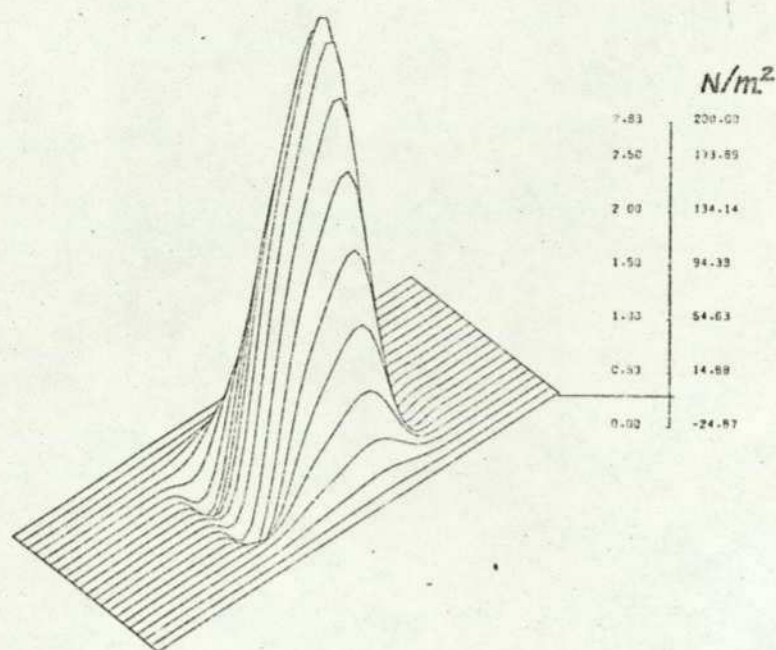
The power flow across the air gap at $y = .00527\text{m}$ was plotted in Fig.4.2.7.. It represents the real power flow across the air gap.

The total power flow across the air gap was,

$$P_y = 359.0\text{W} \text{ at } s = 1.0$$

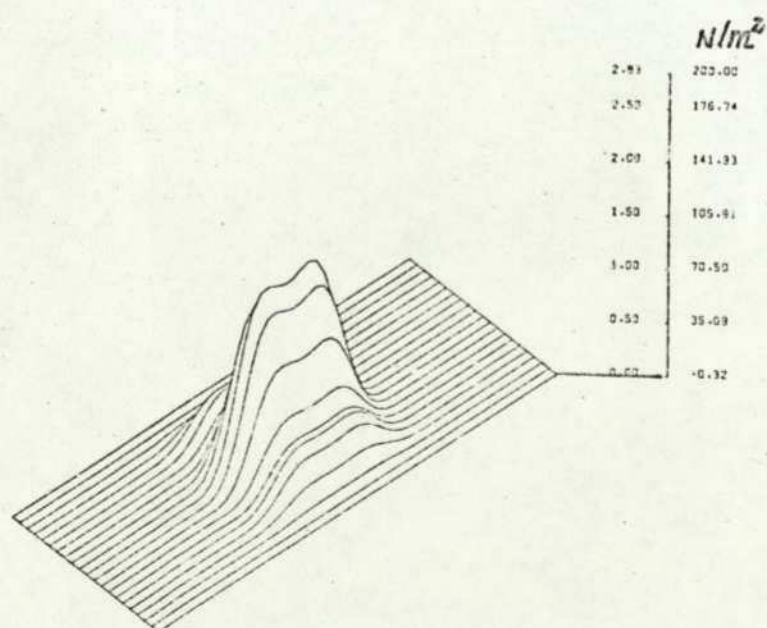
and $P_y = 737.0\text{W} \text{ at } s = 0.2$

Fig.4.2.8. shows the flux density distribution B_y along the x axis ($z = 0.0$) with varied slip s at $y = 0.009175\text{m}$



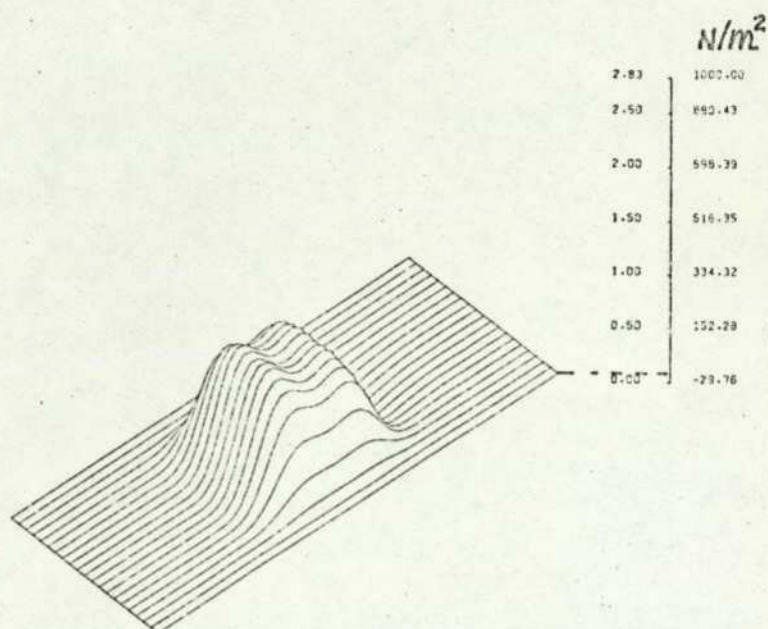
FORCE DISTRIBUTION OF LINEAR INDUCTION MOTOR - Fx
 AZIMUTH = 43 ALTITUDE = 45
 WIDTH = 5.10 HEIGHT = 4.00
 • BEFORE FORESHORTENING 19/05/80

Fig. 4.2.1.a Fx-Traction force distribution $S=2$



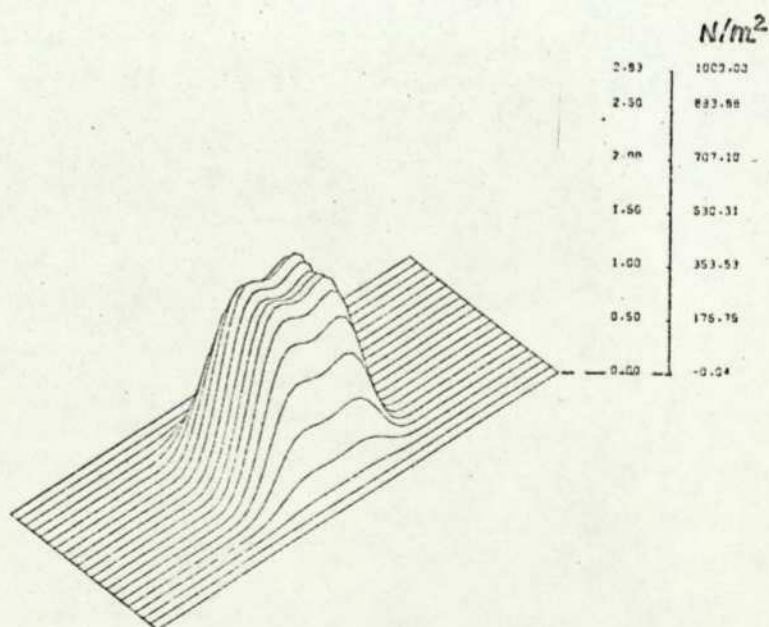
FORCE DISTRIBUTION OF LINEAR INDUCTION MOTOR - Fx
 AZIMUTH = 43 ALTITUDE = 45
 WIDTH = 5.10 HEIGHT = 4.00
 • BEFORE FORESHORTENING 20/05/80

Fig. 4.2.1.b Fx-Traction force distribution $S=1.0$



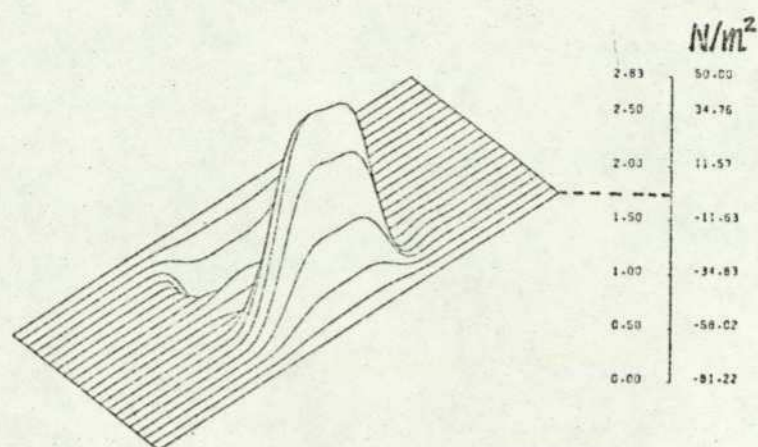
FORCE DISTRIBUTION OF LINEAR INDUCTION MOTOR - FY
 AZIMUTH = 43 ALTITUDE = 45
 WIDTH = 5.10 HEIGHT = 4.00
 BEFORE FORESHORTENING 19/05/80

Fig. 4.2.2.a F_y - levitation force distribution $S=0.2$



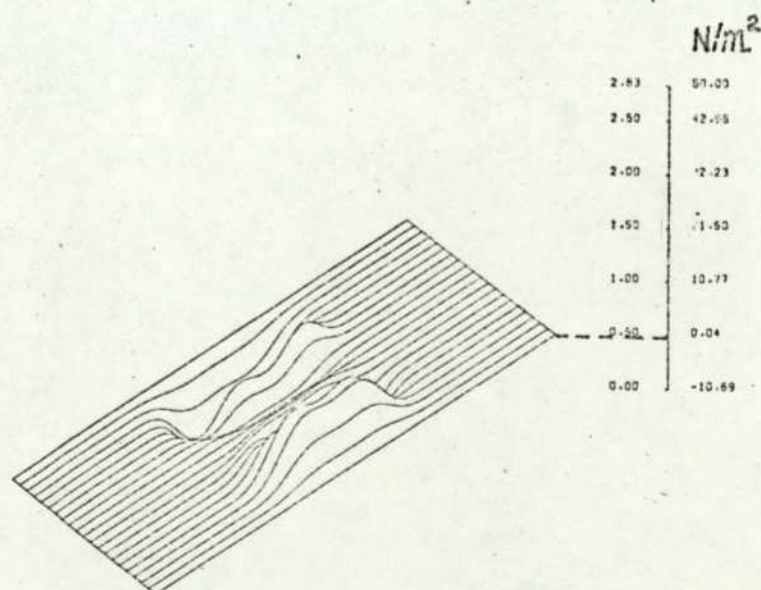
FORCE DISTRIBUTION OF LINEAR INDUCTION MOTOR - FY
 AZIMUTH = 43 ALTITUDE = 45
 WIDTH = 5.10 HEIGHT = 4.00
 BEFORE FORESHORTENING 20/05/80

Fig. 4.2.2.b F_y - levitation force distribution $S=1.0$



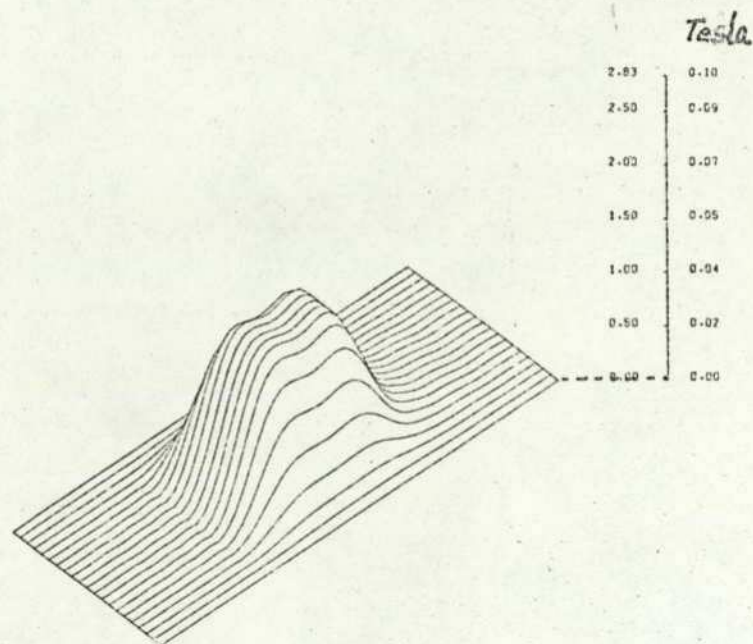
FORCE DISTRIBUTION OF LINEAR INDUCTION MOTOR - FZ
 AZIMUTH = 43 ALTITUDE = 45
 WIDTH = 5.10 HEIGHT = 4.00
 * BEFORE FORESHORTENING 19/06/80

Fig. 4.2.3. a. F_z - Lateral force distribution $S=0.2$



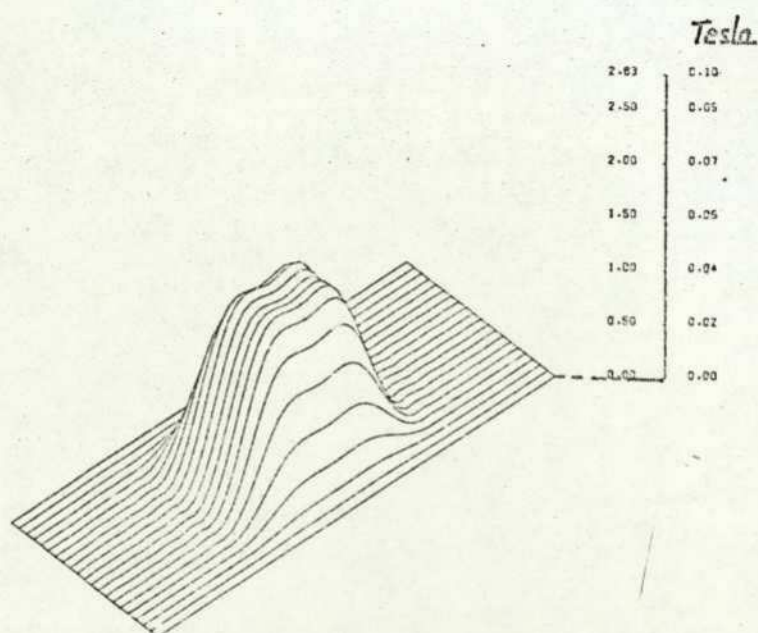
FORCE DISTRIBUTION OF LINEAR INDUCTION MOTOR - FZ
 AZIMUTH = 43 ALTITUDE = 45
 WIDTH = 5.10 HEIGHT = 4.00
 * BEFORE FORESHORTENING 20/06/80

Fig. 4.2.3. b. F_z - Lateral force distribution $S=1.0$



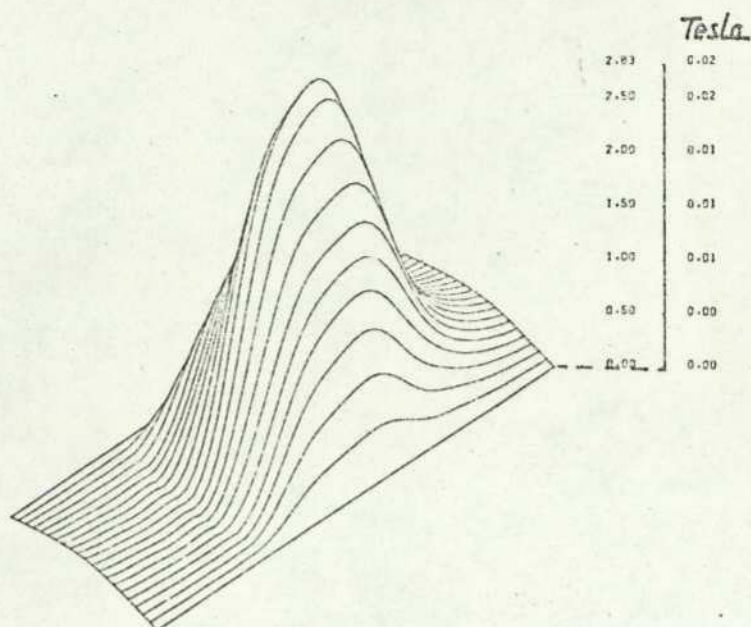
FLUX DISTRIBUTION OF LINEAR INDUCTION MOTOR - Bx
 AZIMUTH = 43 ALTITUDE = 45
 *WIDTH = 5.10 *HEIGHT = 4.00
 * BEFORE FORESHORTENING 19/06/80

Fig. 4.2.4. a Bx - Flux density distribution S=0.2



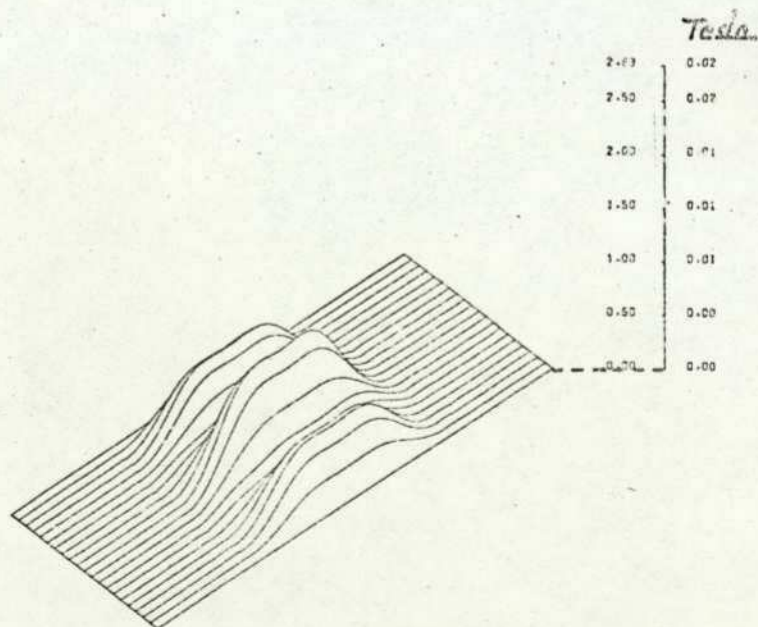
FLUX DISTRIBUTION OF LINEAR INDUCTION MOTOR - Bx
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 *WIDTH = 5.10 *HEIGHT = 4.00
 * BEFORE FORESHORTENING 20/06/80

Fig. 4.2.4. b Bx - Flux density distribution S=1.0



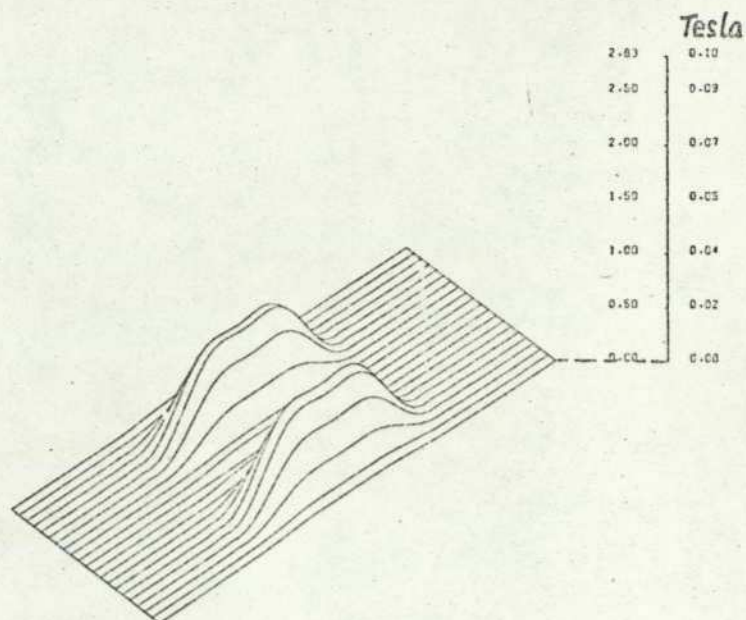
FLUX DISTRIBUTION OF LINEAR INDUCTION MOTOR - BY
 AZIMUTH = 43 ALTITUDE = 45
 *WIDTH = 5.10 *HEIGHT = 4.00
 * BEFORE FORESHORTENING 19/05/80

Fig. 4.2.5.a *By - Flux density distribution. $S = 0.2$*



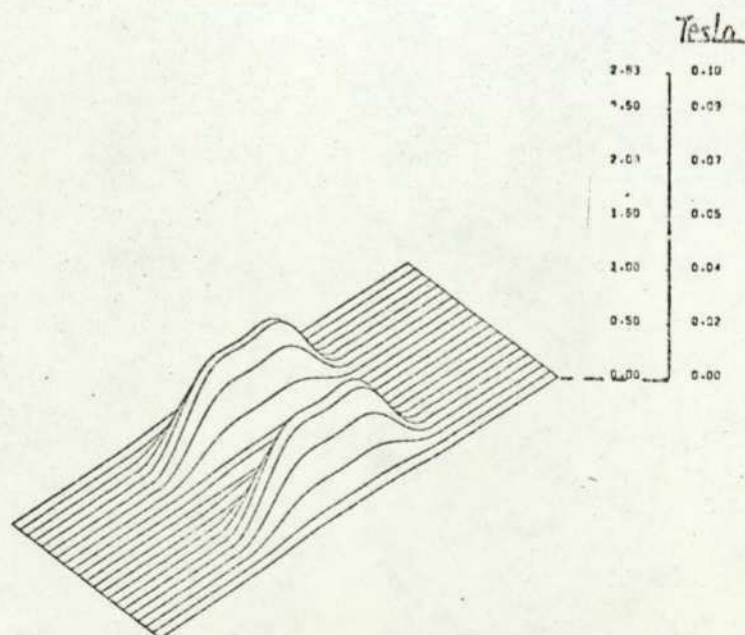
FLUX DISTRIBUTION OF LINEAR INDUCTION MOTOR - BY
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 *WIDTH = 5.10 *HEIGHT = 4.00
 * BEFORE FORESHORTENING 20/05/80

Fig. 4.2.5.b *By - Flux density distribution $S = 1.0$*



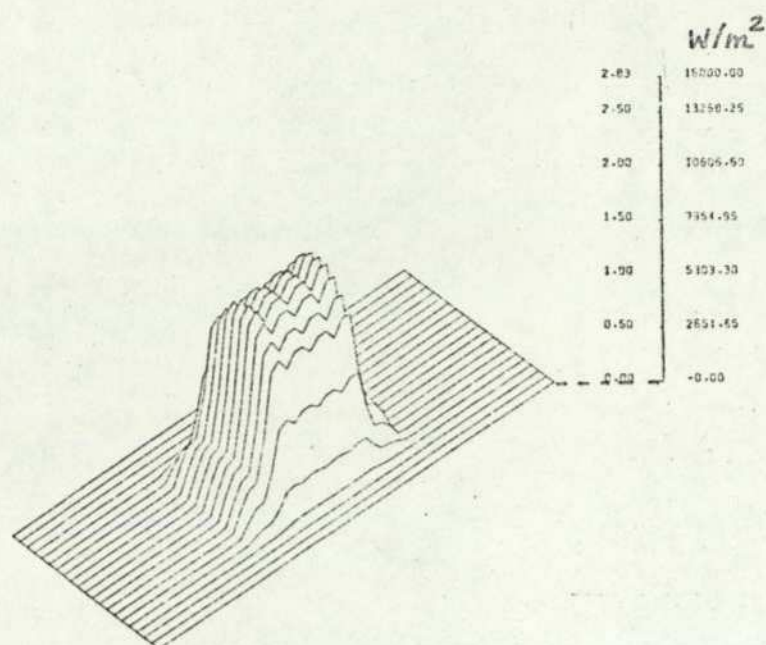
FLUX DISTRIBUTION OF LINEAR INDUCTION MOTOR - BZ
 AZIMUTH = 43 ALTITUDE = 45
 WIDTH = 5.10 HEIGHT = 4.00
 * BEFORE FORESHORTENING 19/06/80

Fig.4.2.6.a Bz - Flux density distribution $S=2.2$



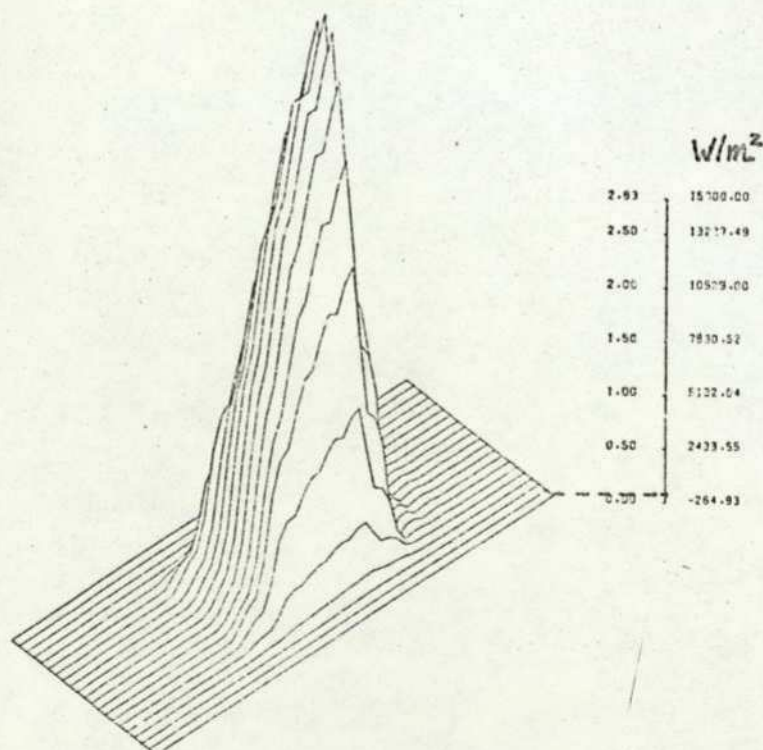
FLUX DISTRIBUTION OF LINEAR INDUCTION MOTOR - BZ
 AZIMUTH = 43 ALTITUDE = 45
 WIDTH = 5.10 HEIGHT = 4.00
 * BEFORE FORESHORTENING 20/05/80

Fig.4.2.6.b Bz - Flux density distribution $S=1.0$



POWER DISTRIBUTION -----IPY
 AZIMUTH = 43 ALTITUDE = 45
 *WIDTH = 5.10 *HEIGHT = 4.00
 * BEFORE FORESHORTENING 19/06/80

Fig. 4.2.7.a Power flow across the air gap $S=1.0$



POWER DISTRIBUTION -----IPY
 AZIMUTH = 43 ALTITUDE = 45
 *WIDTH = 5.10 *HEIGHT = 4.00
 * BEFORE FORESHORTENING 20/06/80

Fig 4.2.7.b Power flow across the air gap $S=0.2$

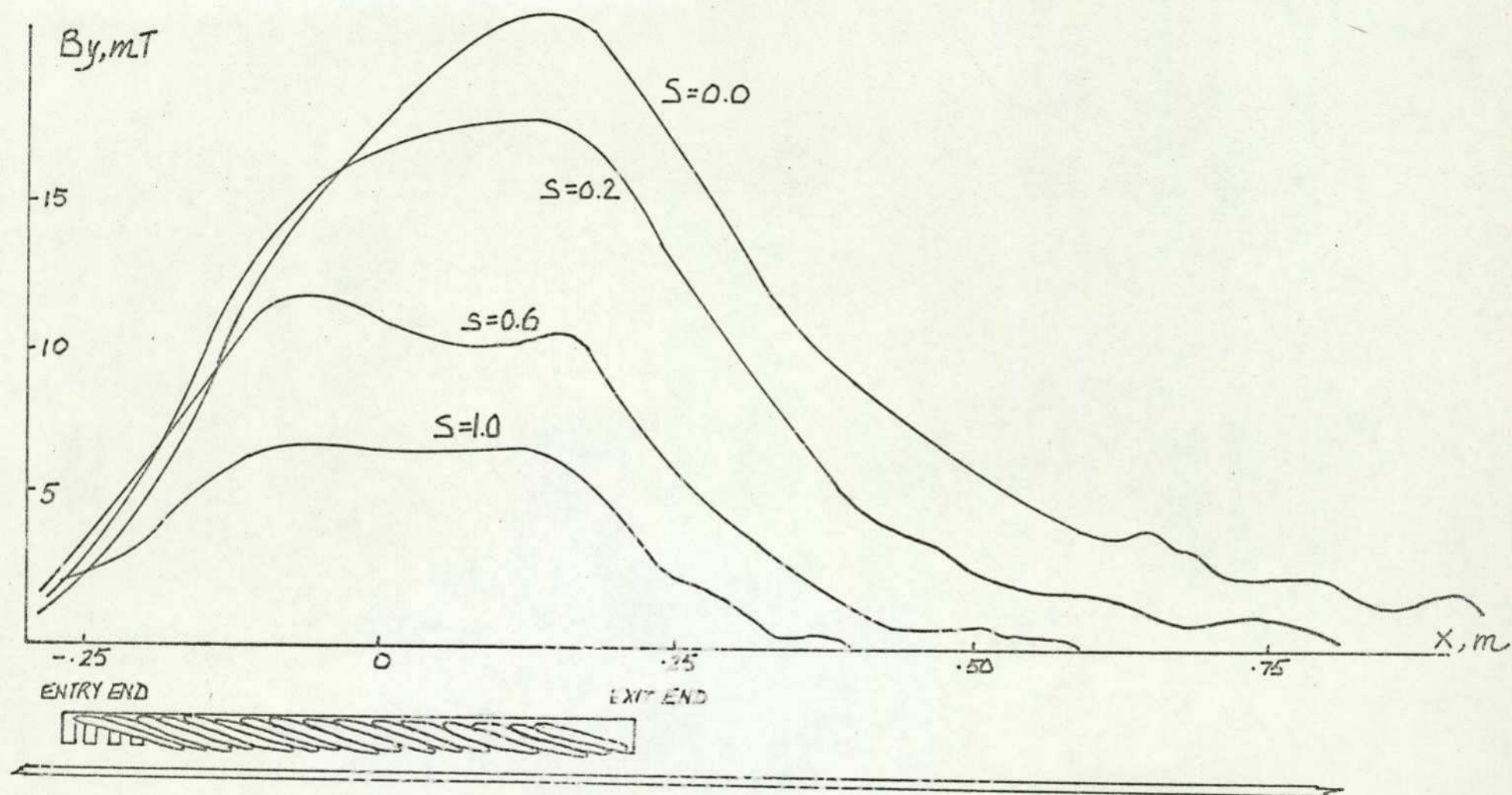


Fig. 4.2.8. B_y -Flux density distribution along X -axis

4.3. Flux pattern across the air gap

Fig.4.3.1. to Fig.4.3.11. show the sections of the three dimensional field distribution in the air gap, secondary conductor and the back iron regions. These flux distribution diagrams were plotted by contouring the A_z component between,

$$-0.24m \leq x \leq 0.24m$$

$$-0.0264m \leq y \leq 0.01375m$$

and at $z = 0.0m$.

This was only possible for $z = 0.0m$ since, in this case, one component of $\vec{B}$ is equal to zero ($B_z = 0.0$). The flux patterns were plotted at $t = 0.0$ and from $s = 0.0$ to $s = 1.0$ at 0.1 slip intervals.

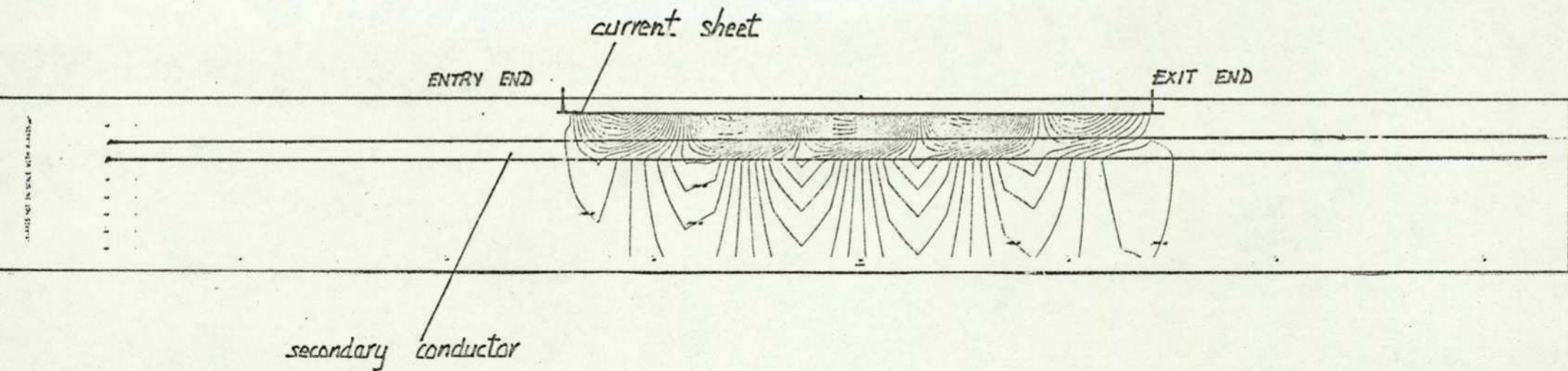


Fig 4.3.1.

Flux distribution

$S = 1.0$

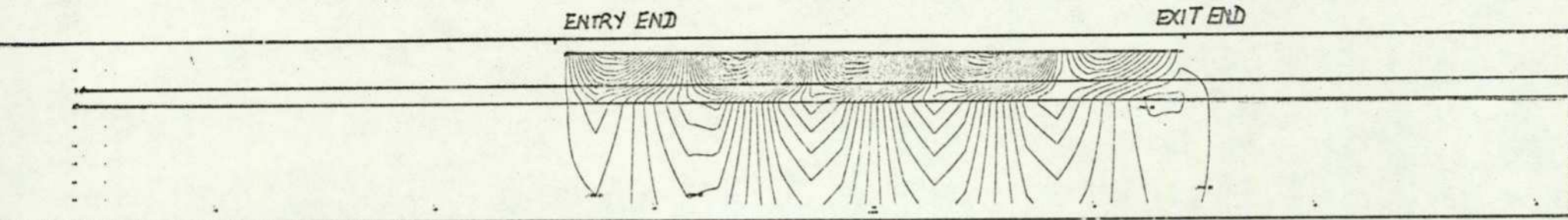


Fig 4.3.2. Flux distribution $S = .9$

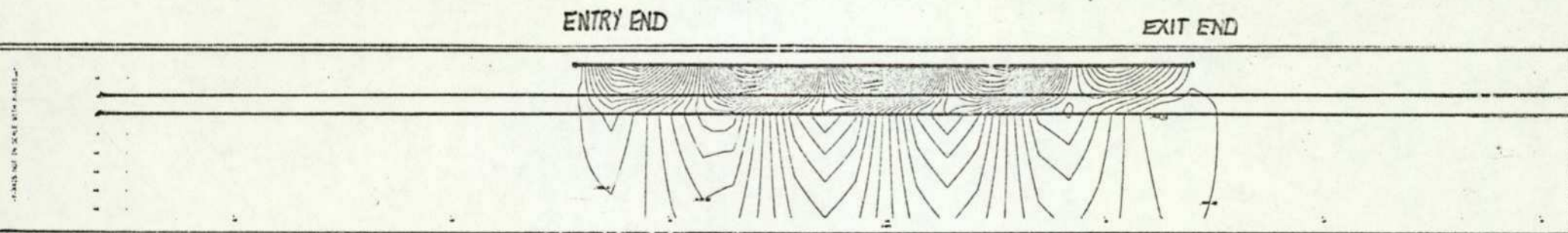


Fig 4.3.3. Flux distribution $S = .8$

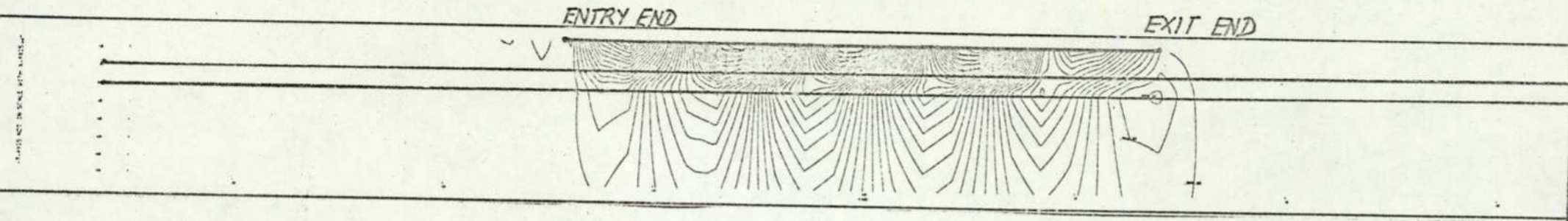


Fig. 4.3.4. Flux distribution $S = .7$

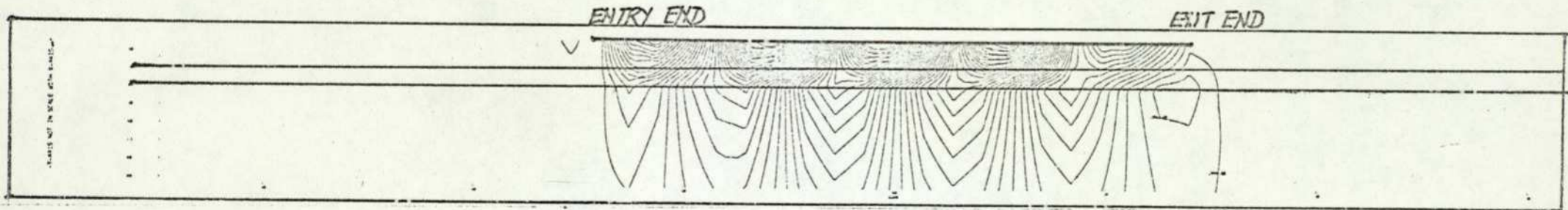


Fig. 4.3.5. Flux distribution $S = .6$

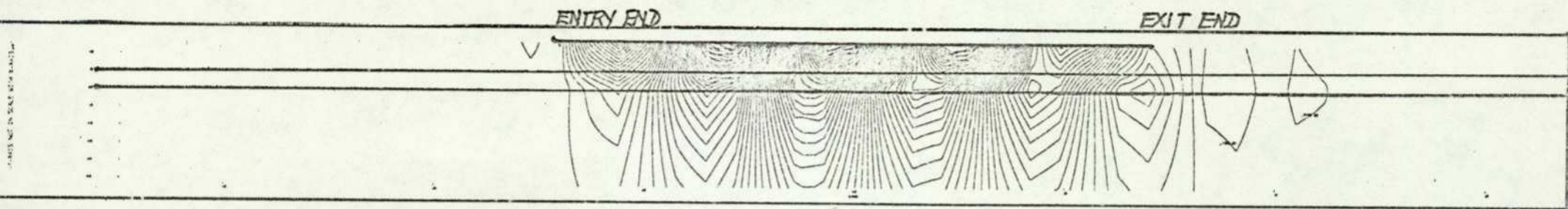


Fig. 4.3.6. Flux distribution $S = .5$

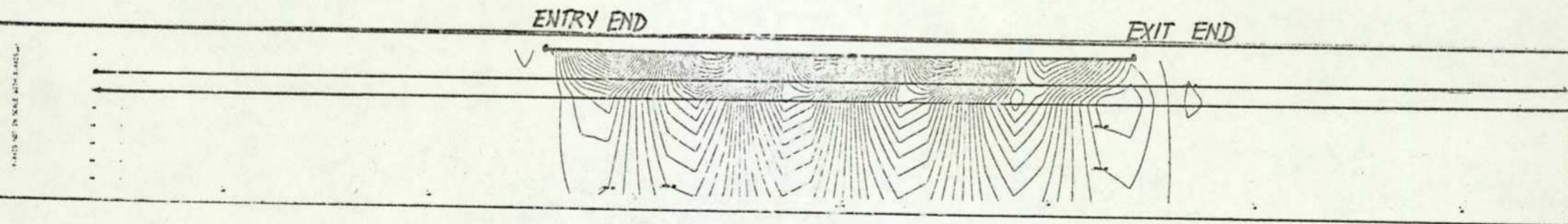


Fig 4.3.7. Flux distribution $S = .4$

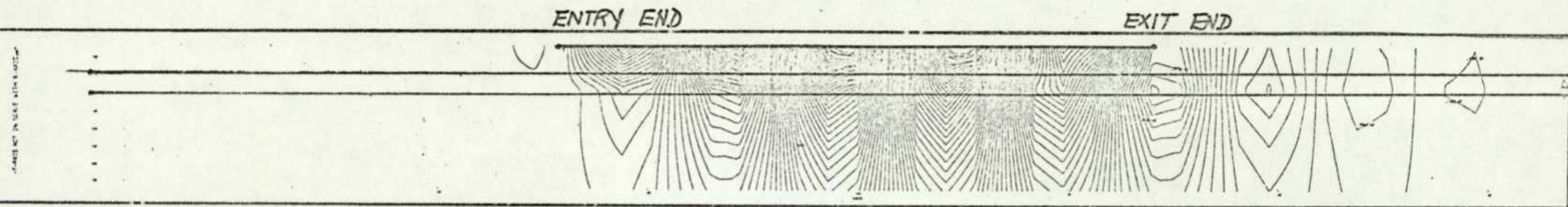


Fig. 4.3.8. Flux distribution $S=0.3$

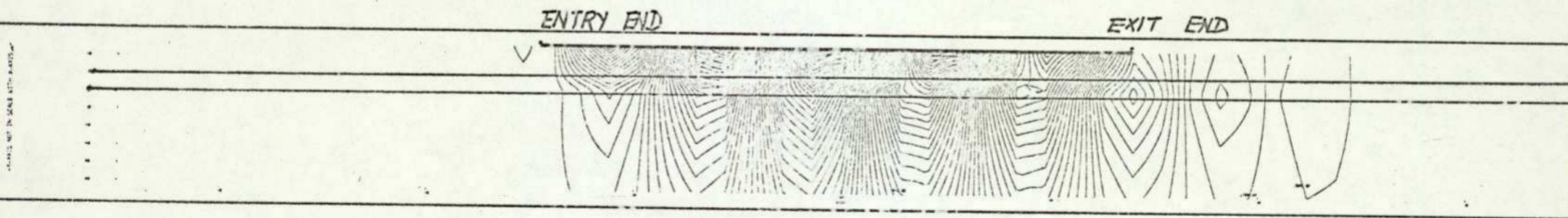


Fig. 4.3.9. Flux distribution $S=0.2$

PLATE NOT IN SCALE WITH EARTH

ENTRY END

EXIT END

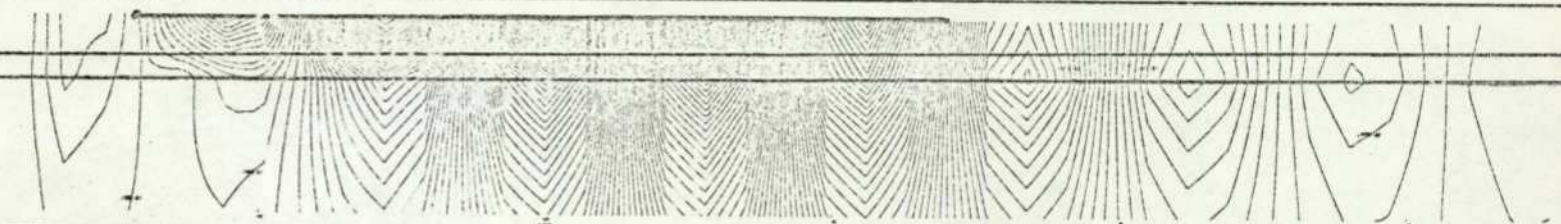


Fig. 4.3.IA. Flux distribution $S = .1$

- 76 -

ENTRY END

EXIT END

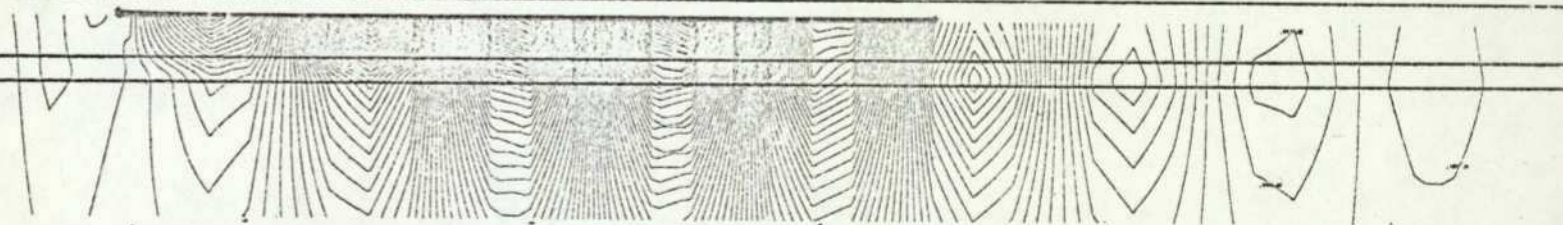


Fig. 4.3.II. Flux distribution $S = 0.0$

4.4. Electromechanical stiffness coefficients

The three axis forces and the three rotational couples are plotted in Fig.4.4.1., with the variation of different positions of the stator. There are six variables $\overline{F}_i$, for the vertical axes on the graph (F_x, F_y, F_z, M_x, M_y and M_z). They are plotted against the variables of another six variables $\overline{R}_j$ ($x_o, y_o, z_o, \theta_x, \theta_y$ and θ_z), on the horizontal axis. In order to complete the variations of $\overline{F}_i$ with respect to each of the variables of $\overline{R}_j$, 36 graphs are required. They are presented in a form of a six by six matrix.

The plotting of each graph in Fig.4.4.1. is $\overline{F}_i$ against $\overline{R}_j$, where i is the number of row and j is the number of column of the matrix. The dotted lines represent forces or couples on half of the secondary conductor;

$$z = 0.0 \quad \text{to} \quad L_z/2$$

$$\text{and} \quad x = -L_x/2 \quad \text{to} \quad L_x/2$$

Since they are of the same magnitude and in opposite direction on both sides of the conductor plate, the total net forces and couples on these graphs would be zero.

By differentiating the variables on the vertical axis $\overline{F}_i$, with respect to the variables on the horizontal axis $\overline{R}_j$, give the electromechanical stiffness coefficients, as defined in section 3.4..

When the machine is in a steady state and a symmetrical position ($V_y = V_z = \theta_x = \theta_y = \theta_z = 0.0$), the electromechanical stiffness coefficient matrix is:

$$\begin{bmatrix} 0 & -200 & 10 & 0 & 0 & 25 \\ 0 & 667 & 0 & 0 & 0 & -40 \\ 0 & 0 & -13 & -35 & 0 & 0 \\ 0 & 0 & 0 & .42 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 & 0 \\ 0 & 60 & 4 & 0 & 0 & 2 \end{bmatrix}$$

The values on the diagonal of the stiffness coefficient matrix represent the individual stiffness in six degrees of freedom when there is no coupling. The negative stiffness represents an unstable state. As explained in section 2.6., there is no electromagnetic effect when the stator is shifted in the x direction, thus

$$\frac{\partial F_i}{\partial R_i} = 0.0 \quad i = 1, \dots, 6$$

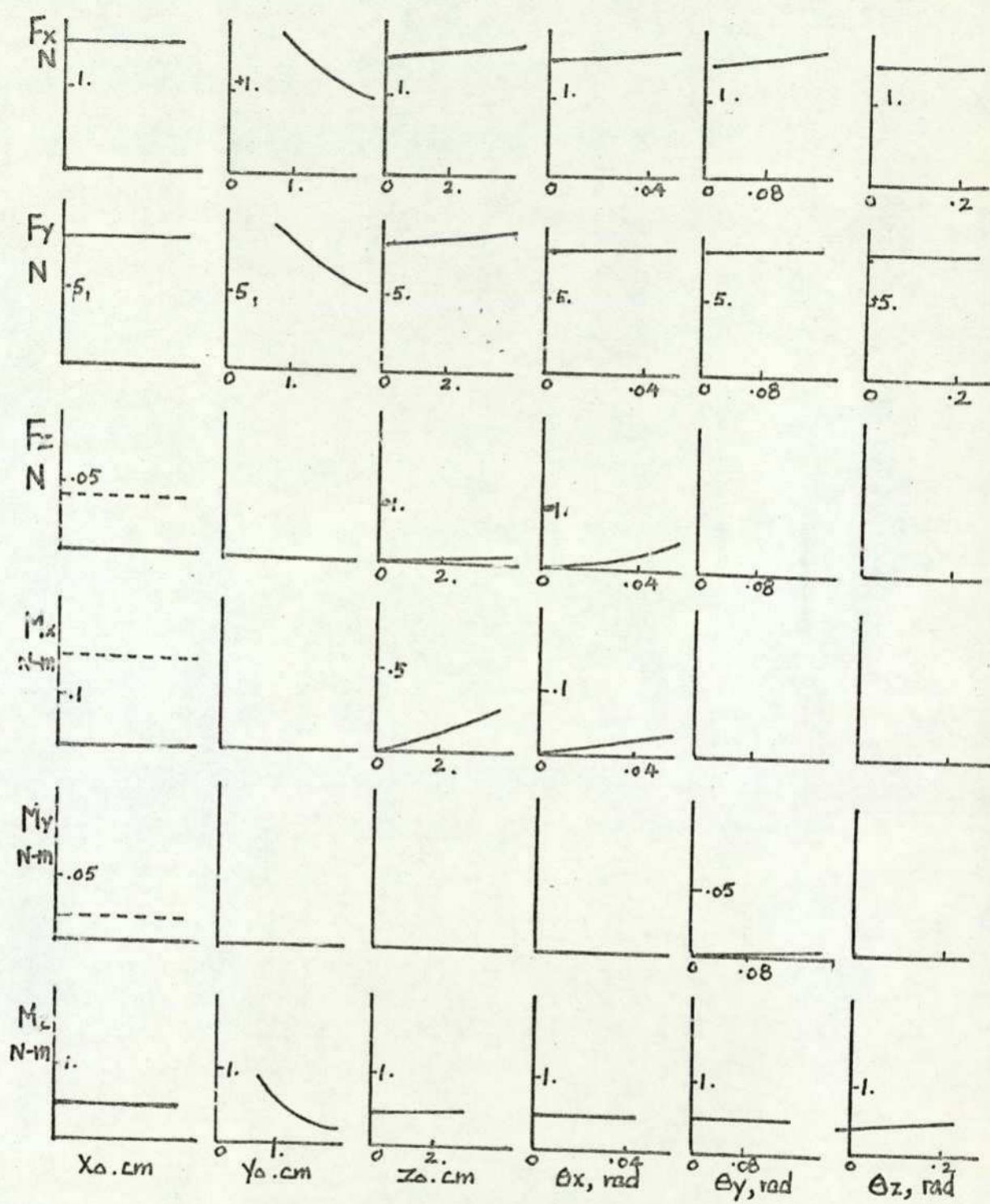


Fig. 4.4.1. F_i against R_j

4.5. Electromechanical damping coefficients

The three axis forces and the three rotational couples are plotted in Fig. 4.5.1. in the form of a six by six matrix. Each graph represents the variation of $\overline{F_i}$ against $\overline{\dot{R}_j}$, where i is the number of row and j is the number of column in the matrix, and

$$\overline{F_i} \quad i=1,6 = (F_x, F_y, F_z, M_x, M_y, M_z)$$

$$\overline{\dot{R}_j} \quad j=1,6 = (V_x, V_y, V_z, \omega_x, \omega_y, \omega_z)$$

By differentiating the variables on the vertical axis $\overline{F_i}$, with respect to the variables on the horizontal axis $\overline{\dot{R}_j}$, give the electromechanical damping coefficients.

i.e.

$$C_{ij} = \frac{\partial F_i}{\partial \dot{R}_j}$$

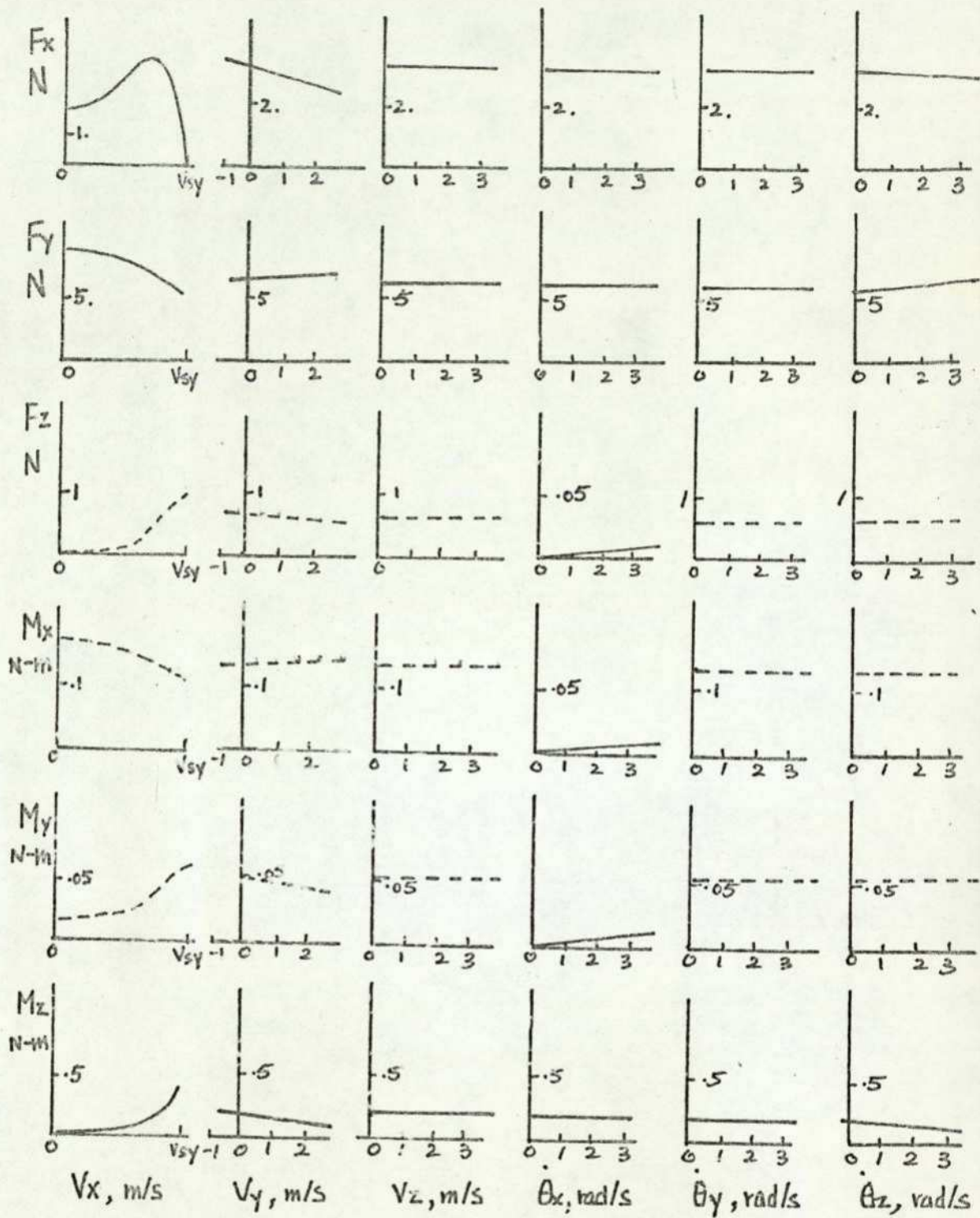


Fig. 4.5.1.

Fl against $\dot{R}_j$, $S=0.2$

4.6. Relationship between input voltage and constant current excitation

As described in section 2.15., the equation to relate the constant primary current excitation to the input terminal voltage was,

$$V_{i/p} = I_1(R_1 + jX_1) + V_{in}/\sqrt{2}$$

By substituting the following parameters,

$$\sigma, \text{ conductivity of copper} = 5.9 \times 10^7 \text{ S/m}$$

$$N_t, \text{ no. of turns per phase in series} = 400$$

$$l_t, \text{ mean length of turn} = 0.44 \text{ m}$$

$$\text{cross-section of the copper wire} = 0.00000177 \text{ m}^2$$

$$SP, \text{ slot depth} = 0.04 \text{ m}$$

and the parameters given in section 4.1. into eqn.2.59. and eqn.2.60.

R_1 and X_1 were found to be:

$$R_1 = 1.68 \, \Omega$$

$$X_1 = 31.2 \, \Omega$$

and the primary impedance was:

$$Z_1 = 31.24 \angle -86.9^\circ \, \Omega$$

The total induced voltage of the primary windings with variation of slip was obtained from eqn.2.61.. This was added vectorially to the primary resistance and leakage reactance voltage drops, to give the total terminal input voltage(see eqn.2.60.).

The input impedance is plotted as shown in Fig.4.6.1. in order to relate the magnitude of the terminal voltage to the magnitude of the primary current at varied slip.

As an example, the voltage induced in the stator windings is calculated at $s = 0.2$. Fig.4.6.2. shows the location of one phase of the three phase windings in the stator. The corresponding voltage induced in these coils were:

$$V_1 = 12.53 \angle 56.8^\circ$$

$$V_2 = 15.82 \angle 85.8^\circ$$

$$V_3 = 25.93 \angle 58.2^\circ$$

$$V_4 = 27.50 \angle 88.3^\circ$$

$$V_5 = 29.50 \angle 61.8^\circ$$

$$V_6 = 29.85 \angle 92.1^\circ$$

$$V_7 = 28.83 \angle 64.7^\circ$$

$$V_8 = 27.85 \angle 95.3^\circ$$

when $J_1 = 45000 \text{ A/m}$ or $I_1 = 5.68 \text{ A}$ (for experimental results, see Chapter 5).

The input primary negative yellow phase current was taken as the reference phasor, since the coil with the yellow phase current supply is situated at $x = 0.0$.

The total induced voltage is:

$$V_{in} = \sum_1^8 V_{AB}$$

$$V_{in}/\sqrt{2} = 190 \angle 76.3^\circ$$

for a series connection, the terminal voltage is:

$$V_{i/p} = I_1(R_1 + jX_1) + V_{in}/\sqrt{2}$$

$$= 208 \angle 79.0^\circ$$

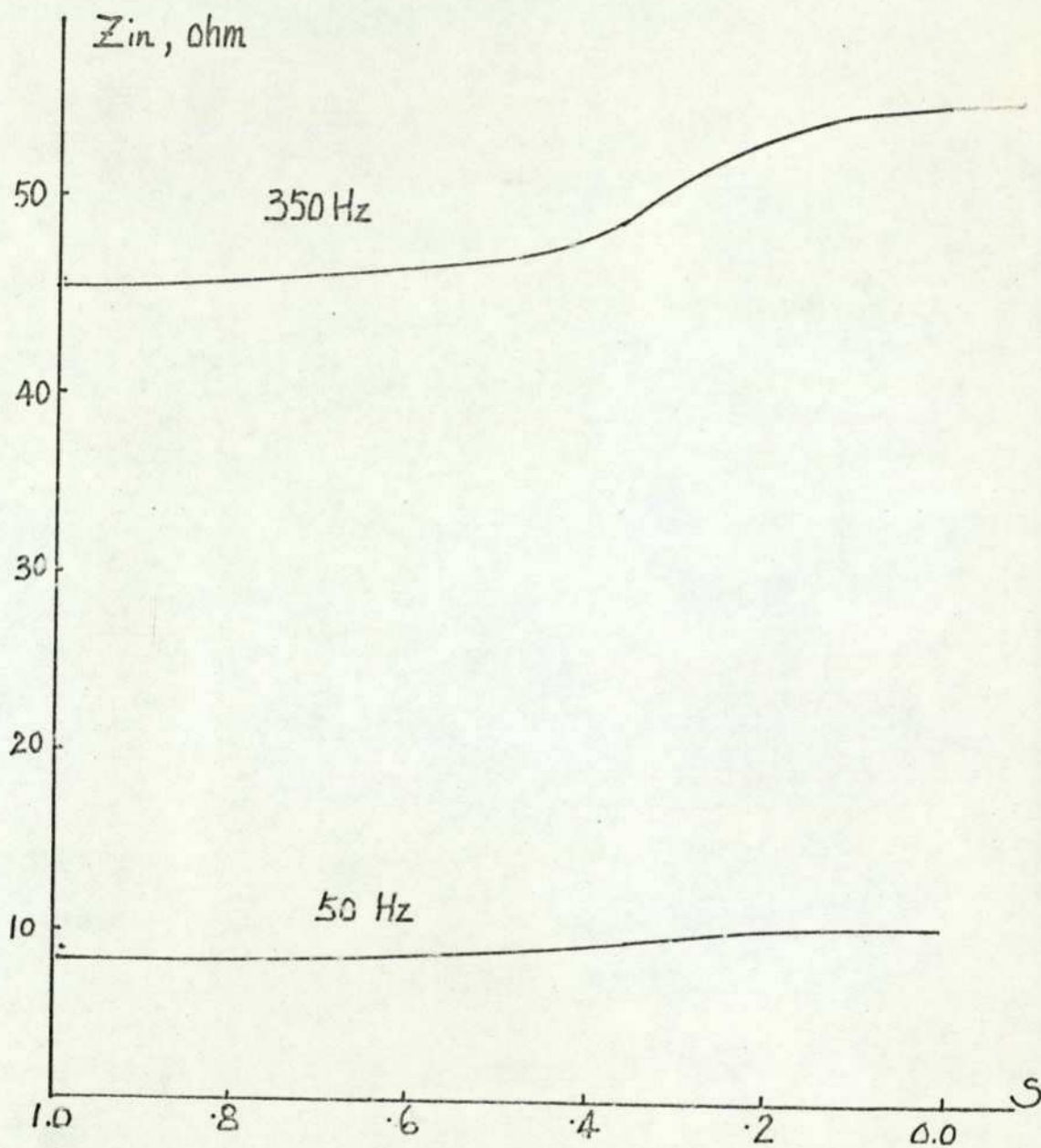


Fig. 4.6.1. Input impedance against slip

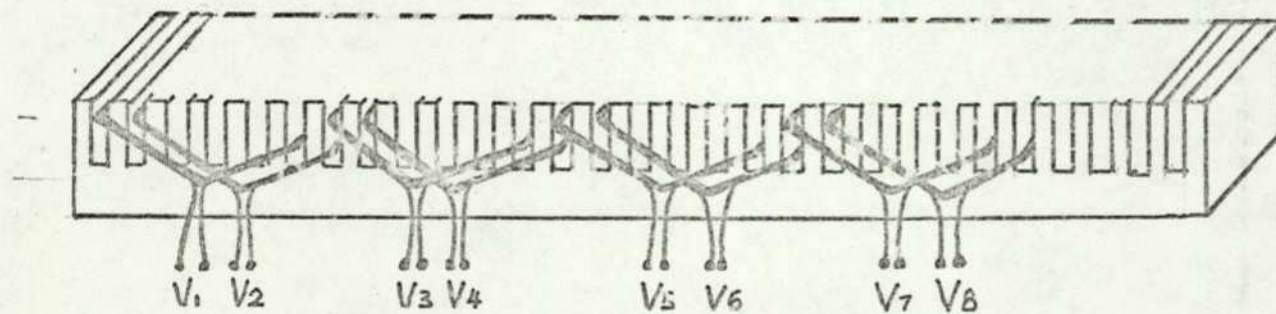


Fig. 4.6.2. Location of the red phase coils

CHAPTER 5

CHAPTER 5. EXPERIMENTAL RESULTS - STATIC TEST

5.1. Introduction

The purpose of the experimental test is to verify the theoretical results obtained in the last chapter. The tests include the measurement of the three axis forces and the three rotational couples, terminal voltages and the measurement of the flux densities at varied parts of the machine.

The experiments were carried out in two parts - static and dynamic. This chapter will be devoted to the static test only, the dynamic test arrangement and results being discussed in the next chapter. The static tests were performed in order to verify the predicted flux density distributions along the longitudinal and transverse axes, and the forces developed at standstill. The dynamic tests were carried out in order to give additional experimental results to correlate with the predicted three axis forces under dynamic conditions.

The construction of the apparatus and the method of force measurements will be discussed in a later section.

5.2. Description of the static test LEM

The machine under test was a single sided linear induction motor without back iron, and was designed to have a maximum synchronous speed of 285km/h(at 350 Hz). The machine was supplied by a generator of 5KVA, 7.5A, 450V, with variable frequency control.

Full details of stator dimensions are given in table 5.2.1. and

diagrams of the stator are shown in Fig.5.2.2.

TABLE 5.2.1. STATIC TEST MACHINE DIMENSIONS

stator width	0.08m
stator length	0.55m
core depth	0.038m
slot width	0.013m
slot depth	0.038m
slot pitch	0.019m
number of turns	50 turns per coil
rotor conductivity	35000000 S/m
width of overhang winding	0.05m
pole pitch	0.113m
number of phase	3
synchronous speed	285km/h
winding	2 slots p.p.p.p.
slot type	open

The stator was 80mm wide, and comprised of 126 sheets of laminated iron, each of 0.63mm thick. The laminations were pressed together and secured by six bolts. The rotor conductor sheet was made of aluminium of 3.175mm thick, 80mm long and 240mm wide.

The machine was supported by four bolts to allow different stator positions in roll, pitch and vertical displacement by adjusting the positions of the lock nuts to give different air gap lengths between the stator and the conductor plate. The yaw and lateral displacements were achieved simply by rotating the whole stator, or by moving the stator in the lateral direction respectively.

5.3. Force measurement techniques

In order to measure the three axis forces and the three rotational couples under static conditions, a mechanical system using strain gauges was designed. The basic principle³⁸ of strain gauge measurement is that a material is subjected to stress, which causes it to stretch by a small amount, and changes its resistivity. The amount of force applied to the material is thus related to the electrical output signal. The arrangement of the force measurement system is shown in Fig.5.3.1..

The measurements were taken from the secondary conductor plate instead of the stator. The conductor plate was firmly fixed to and supported by three square aluminium bars, B_{23} , B_{45} and B_{67} . an additional bar B_1 was attached to B_{67} through a double row self-aligning bearing.

Two pairs of strain gauges were attached to the four surfaces of each bar, which enable forces to be measured in the y and z directions. The positions of the strain gauges in the three horizontal bars are shown in Fig.5.3.2.. The strain gauges were fixed onto the bars in pairs, one pair(gauges A and B) on the top and bottom surfaces for measuring forces in the y direction, and another pair (gauges C and D) on the other surfaces for measuring forces in the z direction. This arrangement will give independent force measurement in both y and z directions as well as temperature compensation.

The total three axis forces will be the summation of the three axis forces measured from each of the aluminium bar,i.e.

$$F_x = F_1$$

$$F_y = F_2 + F_5 + F_7$$

$$F_z = F_3 + F_4 + F_6$$

(5.1.)

The three rotational couples will be the differences in forces measured from each bar, and then multiplied by the distance between the corresponding bars, i.e.

$$\begin{aligned} M_x &= (F_5 - F_2) \cdot L_1 \\ M_y &= (F_3 + F_4 - F_6) \cdot L_2 \\ M_z &= (-F_2 - F_5 + F_7) \cdot L_2 \end{aligned} \quad (5.2.)$$

The self aligning bearing was introduced between bars B_1 and B_{67} , in order to separate, or at least to reduce, the coupling between the three axis forces and the three rotational couples in the measurement system, so that they are independent of each other.

The rotor sheet was connected to a framework by means of bars B_1 , B_{23} and B_{45} . Bar B_1 was solidly fixed to the framework, while bars B_{23} and B_{45} were fixed to the framework by means of a thin metal plate (see Fig.5.3.3.). The metal plate was flexible in the x direction, so that the forces in the x direction would be picked up by the strain gauges in bar B_1 only.

All the aluminium bars carrying strain gauges were 25cm. long and 1/2 inch square cross sectional area. They were designed to carry a maximum loading in y and z directions of 50N (maximum yield 510 microstrains). The strain gauges were single filament foil type, having a resistance of 120Ω each and a gauge factor of 2.0. The location of the strain gauges was 3.0cm away from the secondary conductor, giving a change in resistance of 0.096Ω in the corresponding strain gauges when a force of 50N was exerted at the end of the bars in corresponding directions.

In order to convert the signals from the strain gauges to a measurable level (there was only a change of $160 \mu V$ in the strain gauges with a 10V d.c. supply and 1N of force exerted at the end of

of the bar), a d.c. amplifier was built. The circuitry was designed to include the multiplier, summing and inverting operational amplifiers (see eqns.5.1. and 5.2.), so that a direct reading of the three axis forces and the three rotational couples were possible.

The basic circuit is shown in Fig.5.3.5., and it can be divided into four stages. The first stage of the amplifier is in the form of a typical Wheatstone bridge but operates in an unbalanced state. There are seven Wheatstone bridges for the seven input signals(F_1 to F_7) and each bridge carries a pair of strain gauges(R_1). Any changes in resistance in the strain gauges are picked up by the voltage followers. The Wheatstone bridges are supplied by a stabilized d.c. regulator of $\pm 2.8V$.

The second stage of the amplifier is a voltage follower. The voltage followers are included in the circuit for impedance matching purposes. They have an overall gain of -1 with a high input impedance and low output impedance.

The third stage of the amplifier includes inverting operational amplifiers. They have an overall gain of -1.

The final stage includes the summing points and the output amplifiers. The summing points are connected according to eqns.5.1. and 5.2., through the inverter stage if necessary, for addition and subtraction purposes. The overall gain of the final stage amplifiers can be up to -1000, depending on the feedback resistors R_6 . The resistors R_6 are $1M\Omega$ variable preset type, so that the final output voltages can be adjusted and calibrated as a direct readout in N or N-m of the three axis forces and the three rotational couples.

The supply for the d.c. amplifier is a stabilized $\pm 12V$ dual output voltage regulator. The values of the components used are:

R_1 : strain gauges, 120Ω nominal.

R_2 : multiturn preset resistors, $2.2k\Omega$

- R_3 : input resistors for voltage followers, $2.2k\Omega$
- R_4 : input resistors for invertors and final amplifiers, $1k\Omega$
- R_5 : feedback resistors of invertors, preset type, $1k\Omega$
- R_6 : feedback resistors for final amplifiers, $1M\Omega$
- R_7 : bias resistors, $1k\Omega$
- R_8 : input resistors, preset type, $1k\Omega$

The output voltages from the final stage amplifiers are connected to six digital voltmeters, which enable the three axis forces and the three rotational couples to be measured simultaneously.

After the amplifier was constructed and tested, the output voltages were found to have a substantial noise level. In order to eliminate the unwanted noise signals, a low pass filter consisting of a simple R-C circuit, was added to each of the final stage output amplifier.

The output voltages were adjusted by means of the feedback resistors R_6 , and using dead weight calibration.

The overall static test setup is shown in Fig.5.3.6. and Fig.5.3.7..

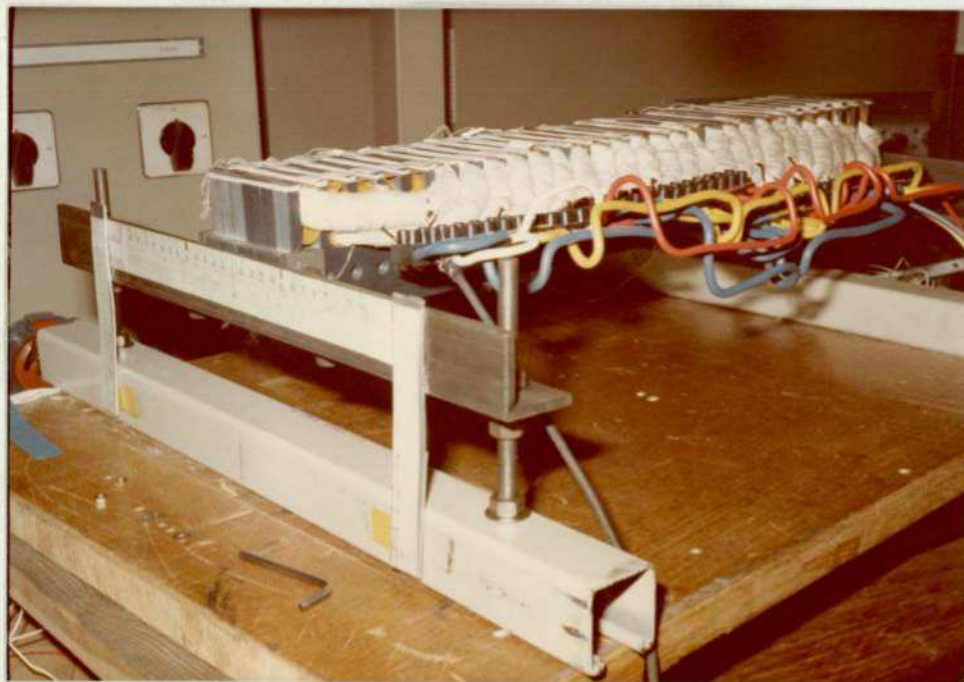
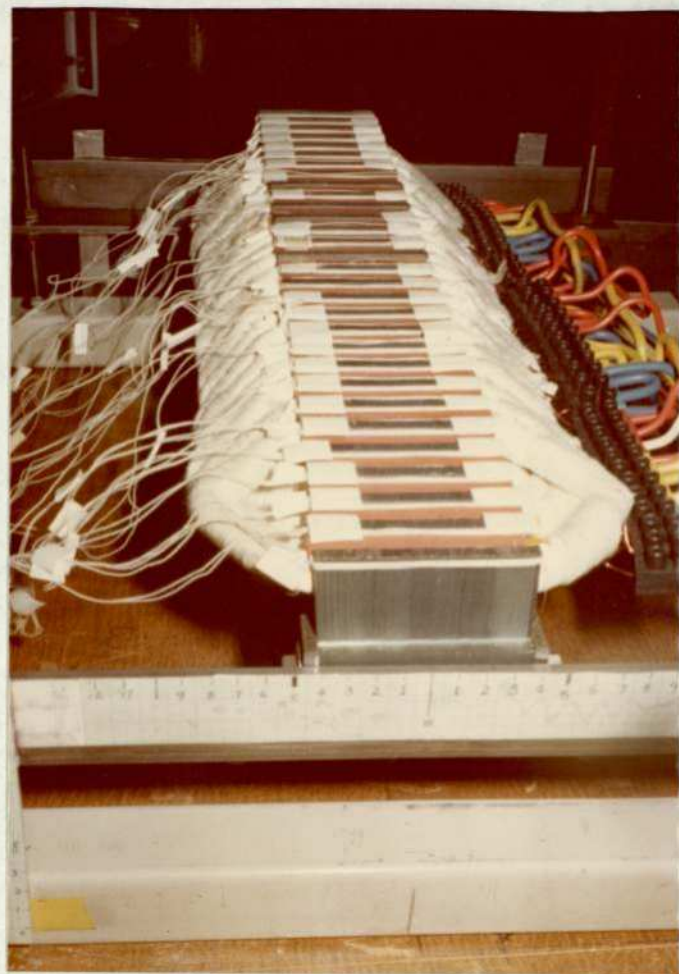


Fig. 5.2.2. *Static test stator*

$$F_x = F_1 \quad ; \quad F_y = F_2 + F_5 + F_7 \quad ; \quad F_z = F_3 + F_4 + F_6$$

$$M_x = (F_5 - F_2) \cdot L_1 \quad ; \quad M_y = (F_3 + F_4 - F_6) \cdot L_2 \quad ; \quad M_z = (F_7 - F_2 - F_5) \cdot L_2$$

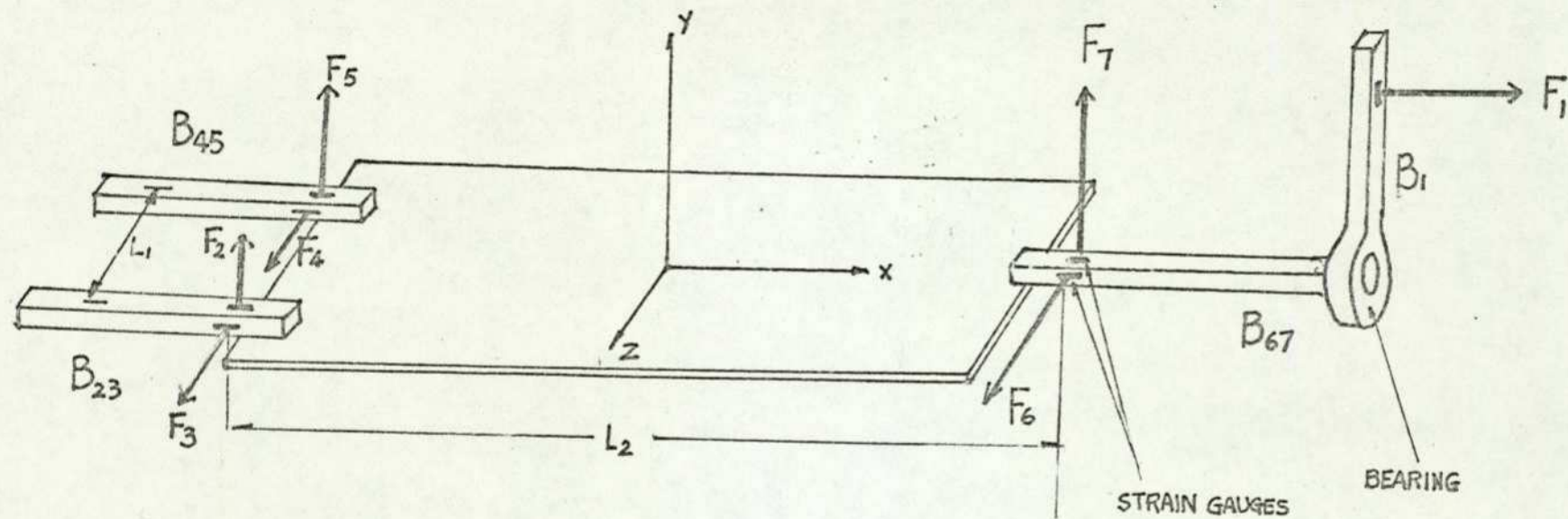


Fig.5.3.1. The arrangement of force measurement
— static test

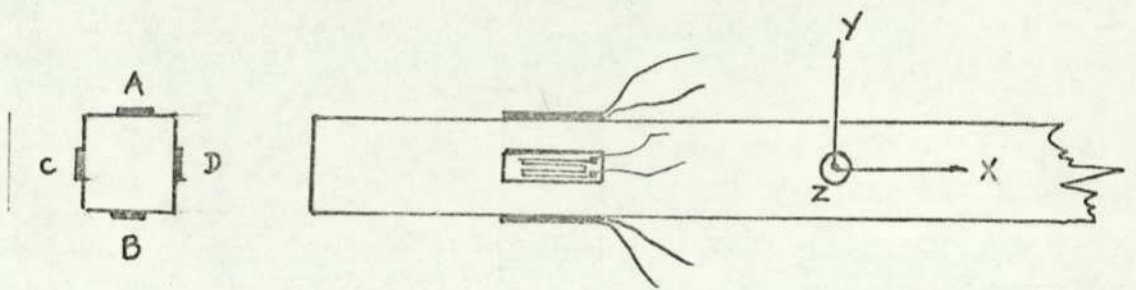


Fig. 5.3.2. Position of strain gauges on bars B₂₃, B₄₅ and B₆₇.

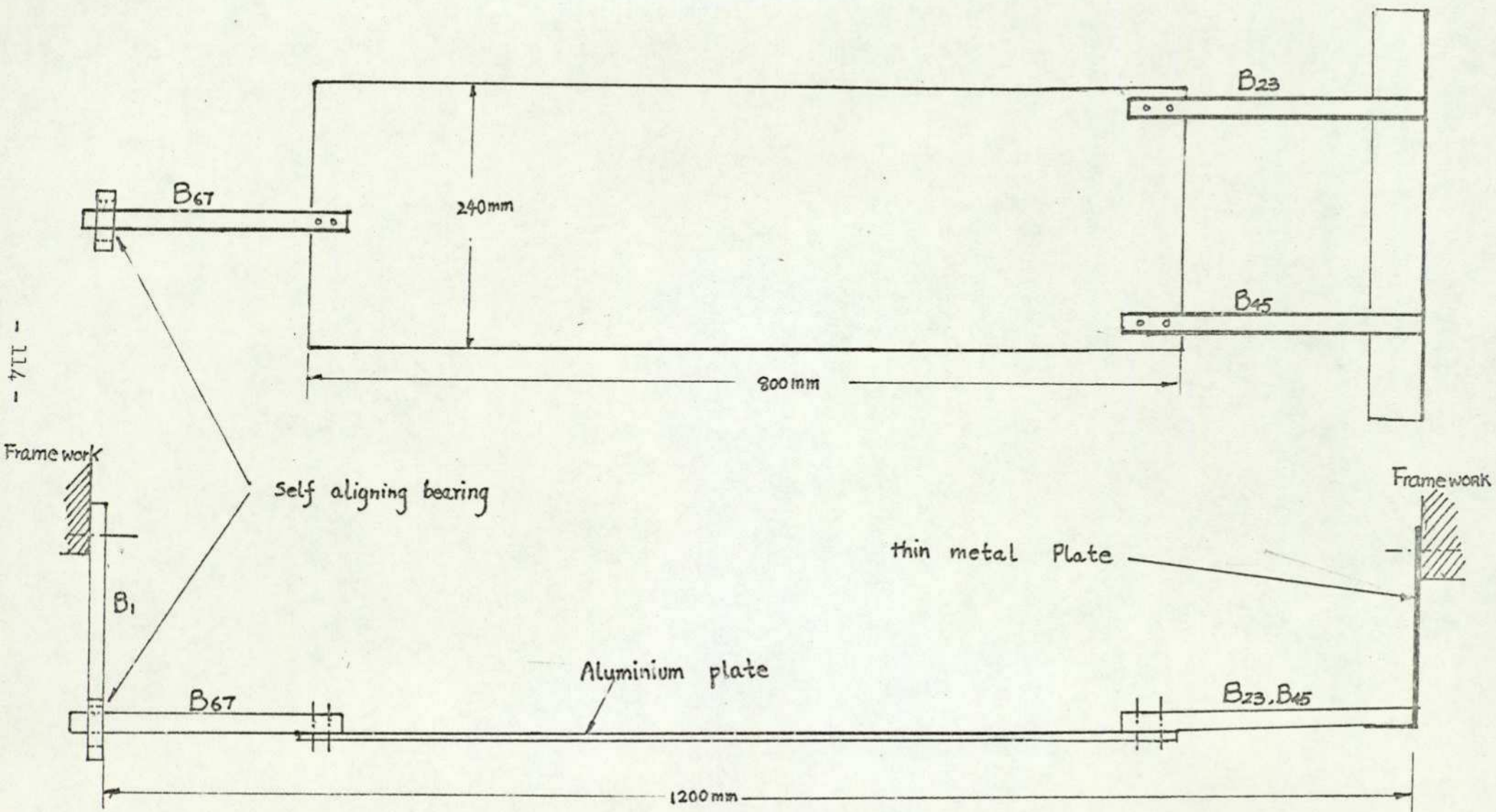


Fig. 5.3.3. The arrangement of the rotor plate.

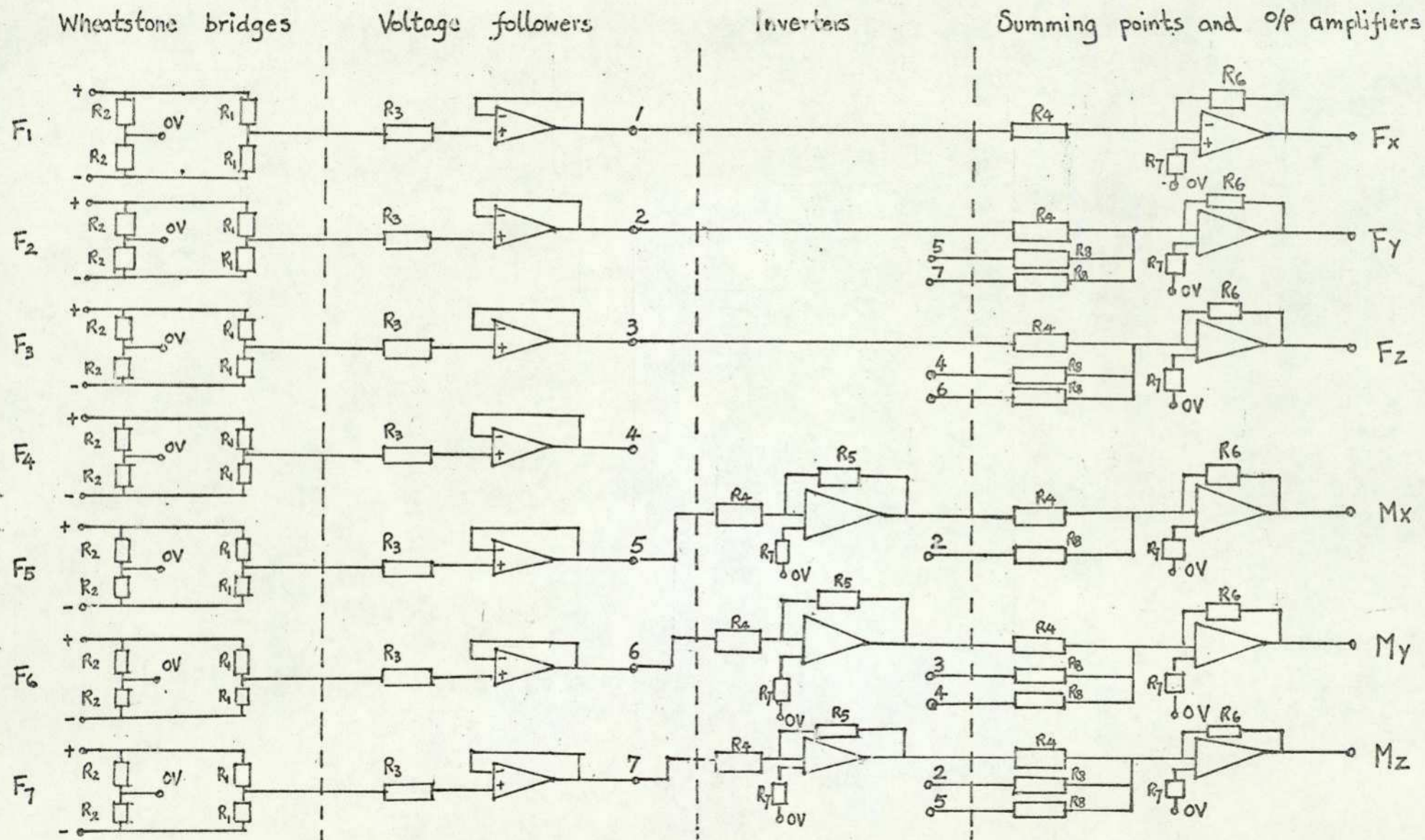


Fig. 5.3.5. The basic D.C. Amplifier circuit

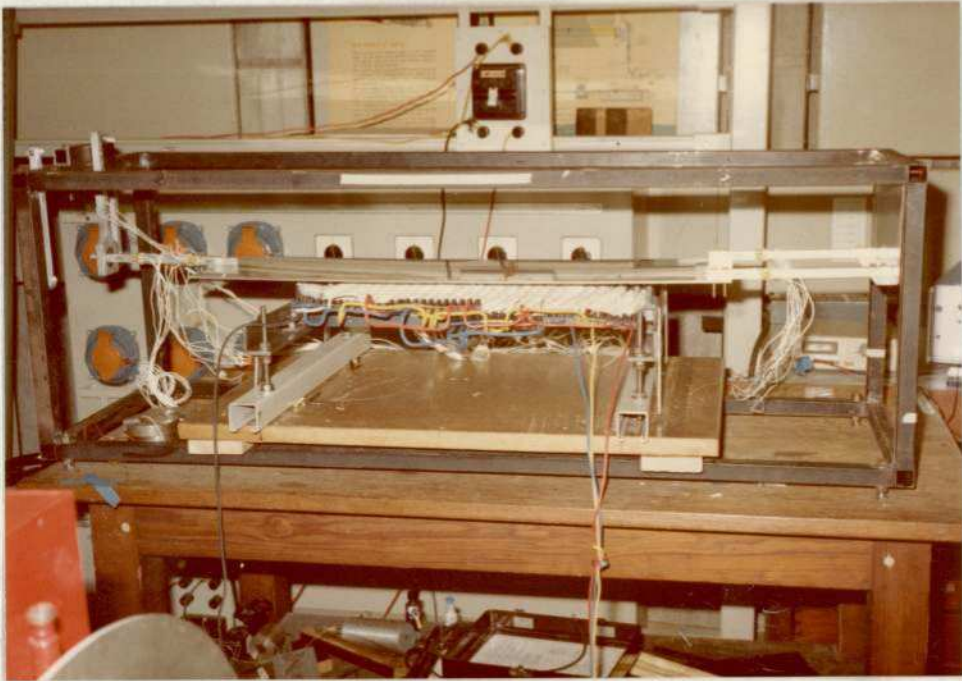


Fig. 5.3.6. Static test arrangement

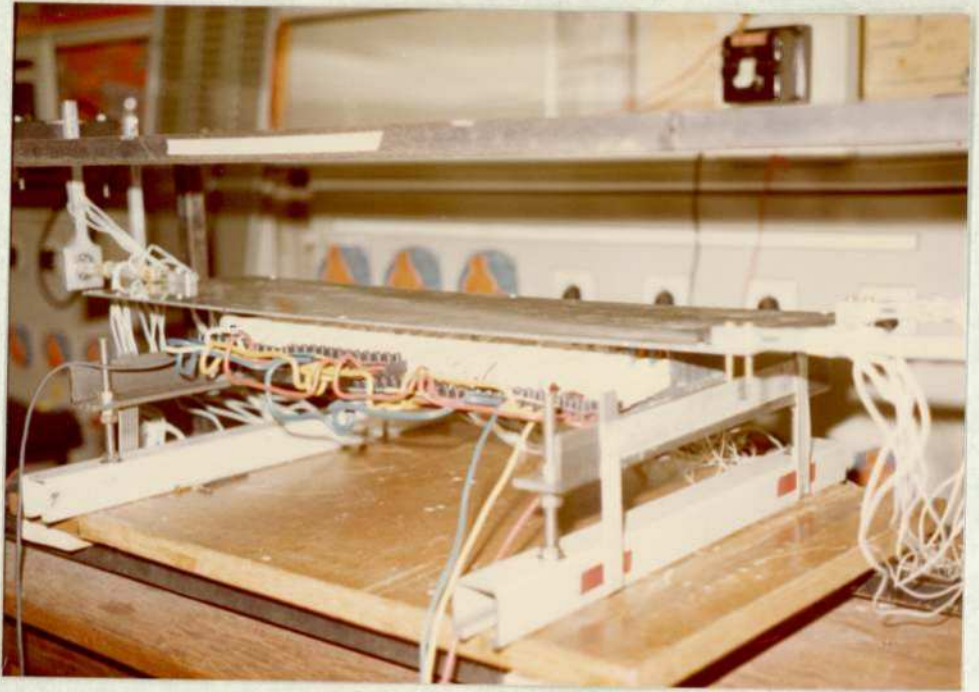


Fig. 5.3.7. Static test arrangement

5.4. Experimental results - static test

A description of the machine and the method of measurement has been given in the last section. The machine was tested statically with a constant current supply of 3.45A at 350Hz (equivalent current sheet density $J_1 = 24500 \text{ A/m}$). The same current was maintained throughout the test in order to simulate the constant current excitation in the theoretical model. The three axis forces and the three rotational couples were measured with different linear displacements and different angular displacements. The results were plotted in Fig.5.4.1. in the form of a six by six matrix.

When the machine was in a symmetrical position, the forces generated in the secondary conductor were also symmetrical about the x axis and identical on both sides of the conductor. Though there were changes in F_z , M_x and M_y on each side of the conductor when a variation in certain linear or angular displacements occurred, these could not be measured experimentally. In the theoretical computation, the forces and couples can be evaluated on one half of the rotor only. Some forces and couples have equal magnitude in both halves of the machine but act in opposite directions, thus cancelling each other. There were no net changes in F_z , M_x and M_y with the variations of x_0 , y_0 and θ_z .

The values of M_y and M_z are too small for measurement with the variations of the three linear displacements and the three angular displacements. No accurate reading could be obtained and they are not included in Fig.5.4.1.. However, it can be seen from the theoretical results (Fig.4.4.1.) that the values of M_y and M_z are also small.

By differentiating the curves in Fig.5.4.1. with respect to the variables in the horizontal axes(see section 3.3.), the experimental stiffness coefficient matrix was found:

$$[K] = \begin{bmatrix} 0 & 88.7 & 9.32 & 2.3 & 2.5 & 0 \\ 0 & 863 & 10.0 & 1.6 & 0 & 12.3 \\ 0 & 0 & -14.6 & 2.2 & 0 & 0 \\ 0 & 0 & 6.28 & .55 & 0 & 0 \\ 0 & 0 & 0 & 0 & -.75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

at $s = 1.0$

$$V_x = 0.0m/s$$

$$y_o = 1.0cm$$

$$z_o = 0.0cm$$

and $\theta_x = \theta_y = \theta_z = 0.0 \text{ rad.}$

The normal force F_y and traction F_x were also measured at supply frequencies of 50, 150, 250 and 350 Hz. The variations of these forces against frequency are plotted in Fig.5.4.2. and Fig 5.4.3.. Results in Fig.5.4.2. were obtained with a constant current supply of 3.5A and an air gap length of 1.32cm, the results shown in Fig.5.4.3. were obtained with the same current supply and an air gap length of 1.0cm. Theoretical results are also plotted on the same graphs for comparison purposes.

Terminal voltages were also measured with different supply frequency. Both theoretical and experimental results are plotted in Fig.5.4.4. together with the corresponding phase angles. The theoretical results were obtained from eqn.2.6.1. in section 2.15.

Individual terminal voltage of each of the red phase windings were also measured at 350Hz. The location of each coil can be referred to Fig.4.6.2.. Both theoretical and measured values of the terminal voltage are listed below,

Terminal voltage	Theoretical	experimental
V_1	18.3	21.1
V_2	20.2	23.2
V_3	23.4	25.5
V_4	24.6	25.2
V_5	23.8	24.2
V_6	24.6	22.6
V_7	23.5	23.0
V_8	23.7	18.9
total voltage	160.0	152.0

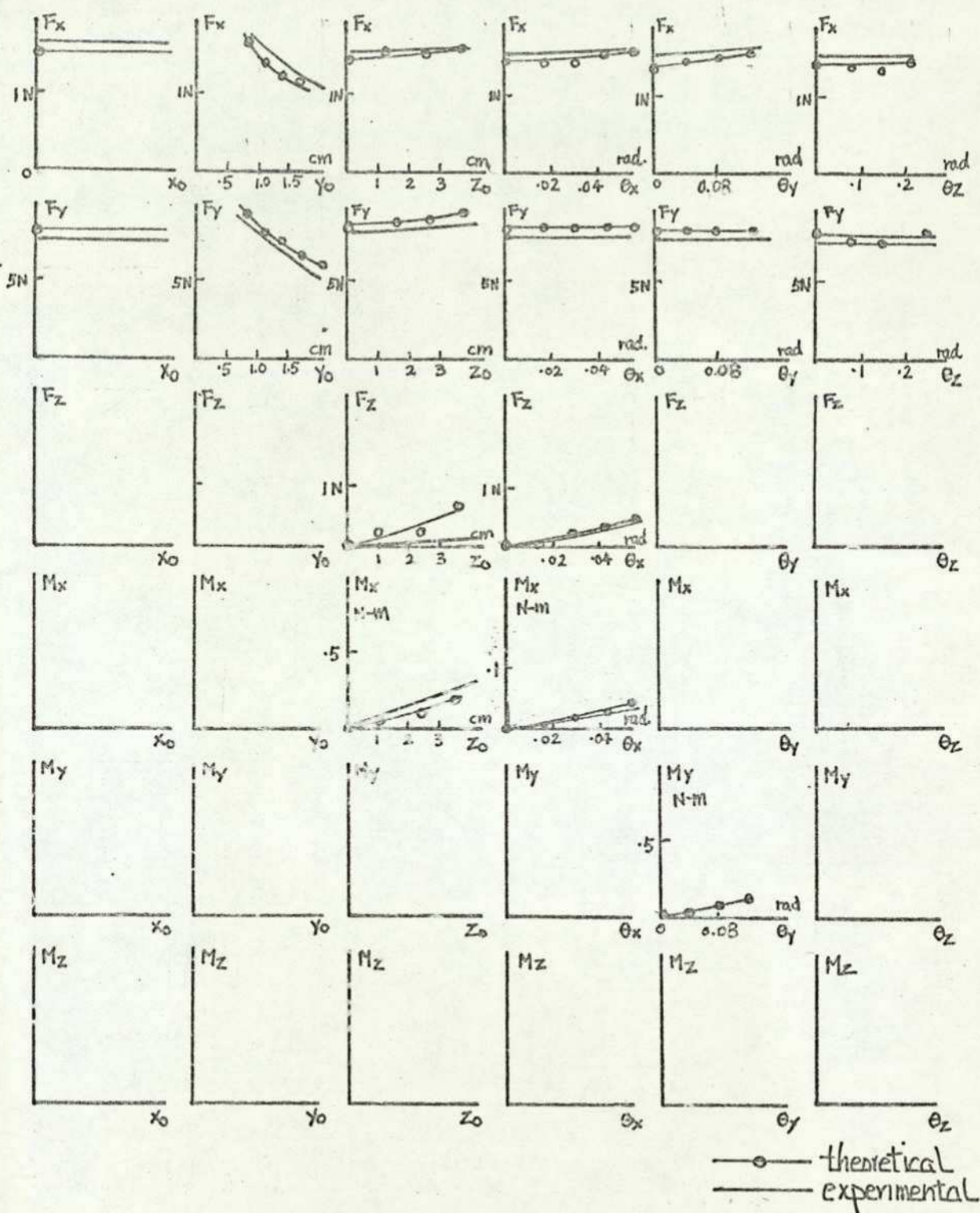


Fig. 5.4.1. F_i against R_j

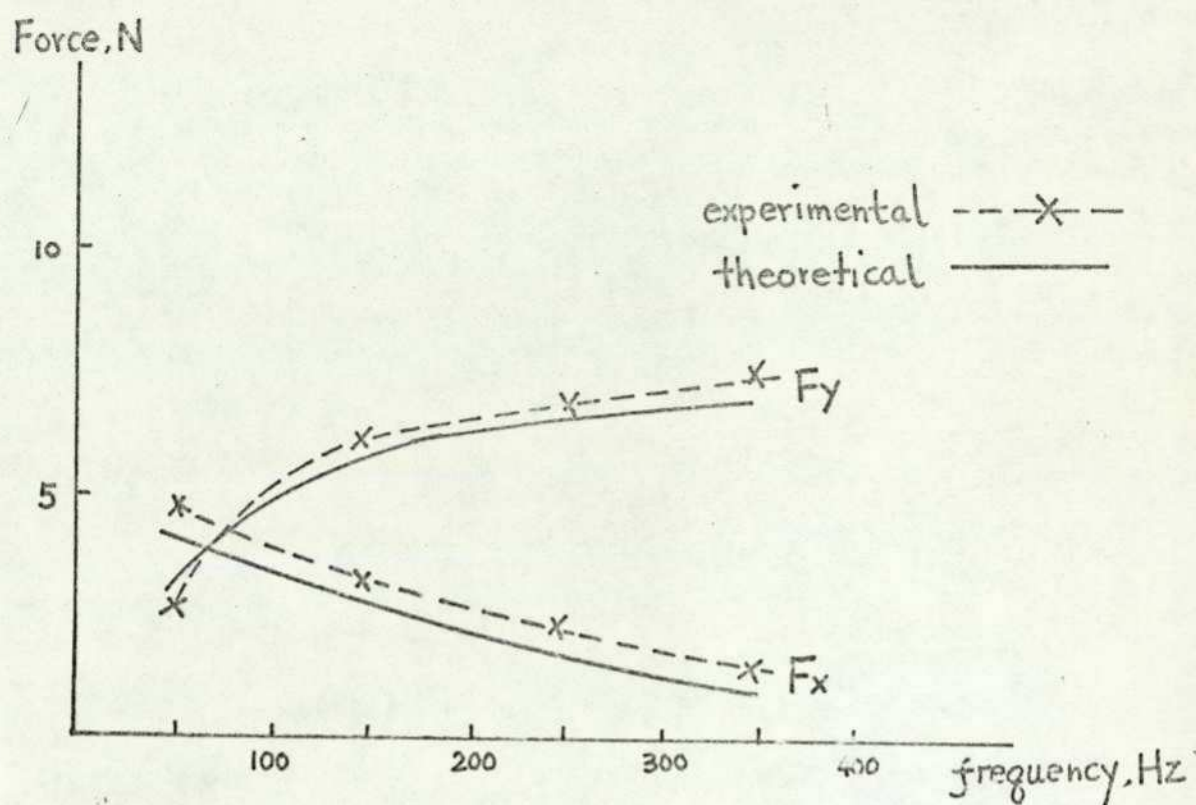


Fig. 5.4.2. F_x and F_y against frequency, $g = 1.32 \text{ cm}$

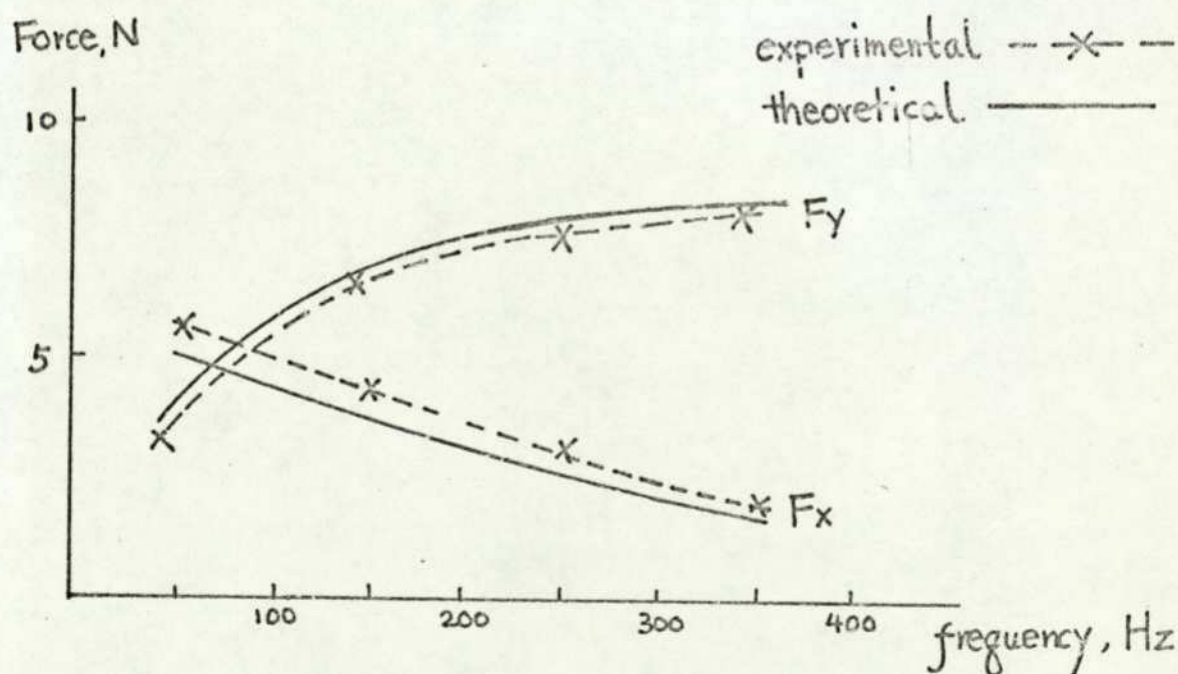


Fig. 5.4.3. F_x and F_y against frequency, $g = 1.0 \text{ cm}$

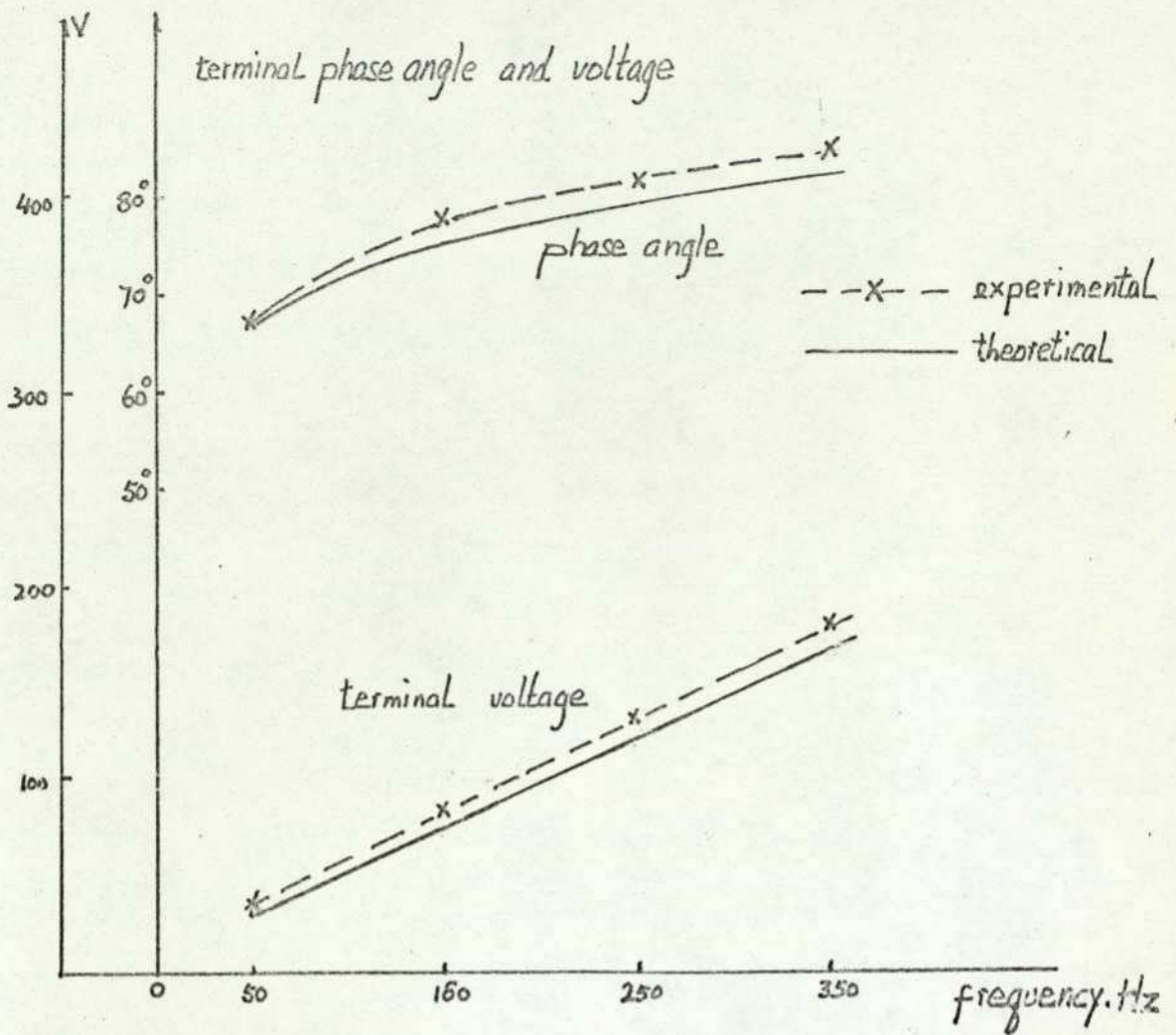


Fig. 5.4.4. Terminal voltage against frequency

5.5. Flux measurements and results

The air gap flux densities were measured by means of search coils. Three search coils were used and wound, such that their axes were mutually perpendicular to each other. Fig.5.5.1. shows the arrangements of the three search coils. Ideally, the area of the search coils should be as small as possible, and large enough to produce a measurable signal for the flux density expected. Search coils of area 0.36cm^2 and with 13 turns each were used, such that they would produce an induced voltage of 0.72mV for 1mT at 350Hz .

Due to the construction of the search coils, in practice, it was difficult to ensure that all the search coils were of equal area. To overcome this, a -core was made having a 2cm air gap length and a $3.8 \times 2.0\text{ cm}$ cross section. This produced a known flux density across the 2cm gap, and the induced voltages in the search coils were calibrated against the known flux density produced by the -core. Fig.5.5.2. shows the search coils and -core used for the flux density calibration.

All the flux density measurements were carried out with constant current supply of 3.5A (24500.0A/m) and 350Hz , unless otherwise stated.

Fig.5.5.3. shows the flux density distribution along the z axis with an air gap length g_0 , of 1.0cm . The flux measurements were taken at the middle of the machine ($x = 0.0\text{cm}$). Theoretical results are also plotted for comparison purposes.

Fig.5.5.4. shows the flux density distribution along the z axis and air gap length $g_0 = 2.0\text{cm}$.

Fig.5.5.5. shows the flux density distribution along the z axis with air gap length $g_0 = 1.0\text{cm}$ and lateral displacement $z_0 = 3.0\text{cm}$.

Fig.5.5.6. shows the flux density distribution along the z axis

with an average air gap length $g_0 = 1.0\text{cm}$ and angle of roll, $\theta_x = .0333\text{ rad.}$

Fig.5.5.7. shows the flux density distribution along the x axis with angle of pitch $\theta_z = 0.0182\text{ rad.}$ and $z = 0.0\text{cm.}$ (with $z = 0.0$, there is no B_z component.)

Fig.5.5.8. shows the flux density distribution along the x axis with $z = 0.0$, and air gap length $g_0 = 1.0\text{cm.}$

Fig.5.5.9. shows the flux density distribution along the z axis with air gap length $g_0 = 1.0\text{cm}$ and supply frequency 150Hz.

The phase angles of B_x , B_y and B_z were measured with supply frequency $f = 150\text{Hz.}$ They are plotted in Fig.5.5.10. Theoretical results are also included for comparison purposes.

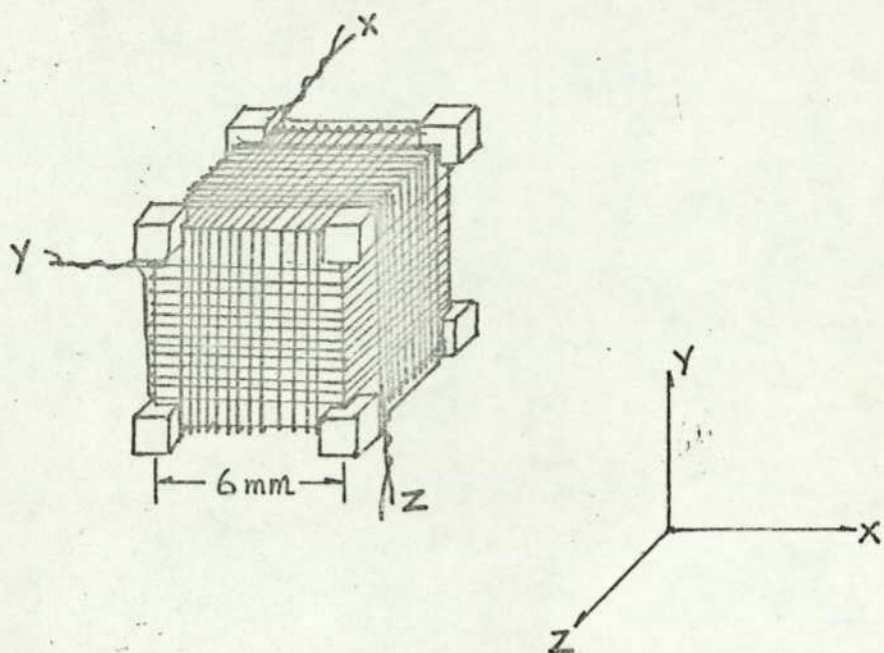


Fig. 5.5.1. Three axis search coil

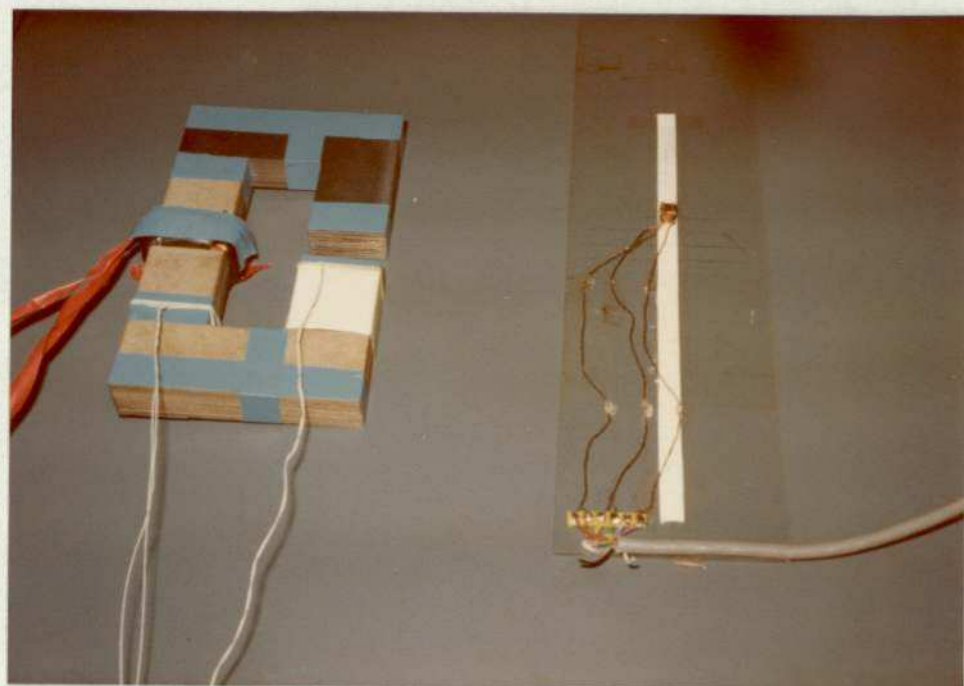


Fig. 5.5.2. C-core and search coils

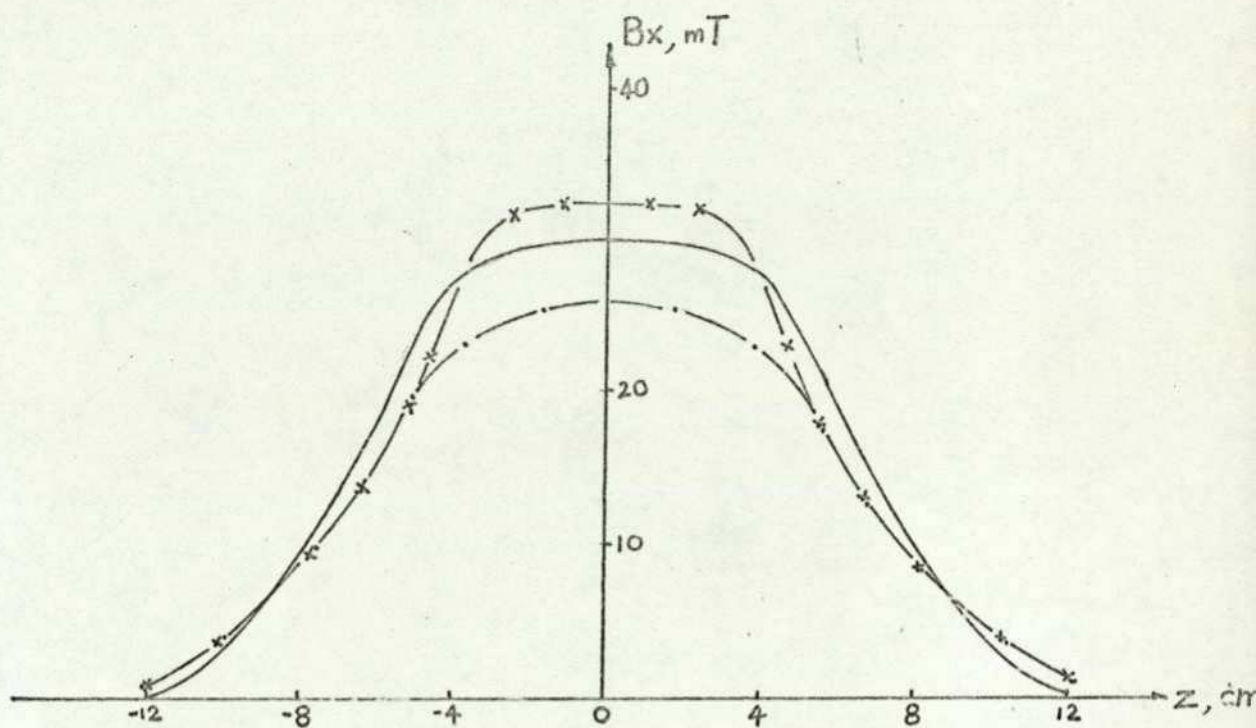


Fig. 5.5.3.(a) X component of flux density. $g = 1 \text{ cm}$

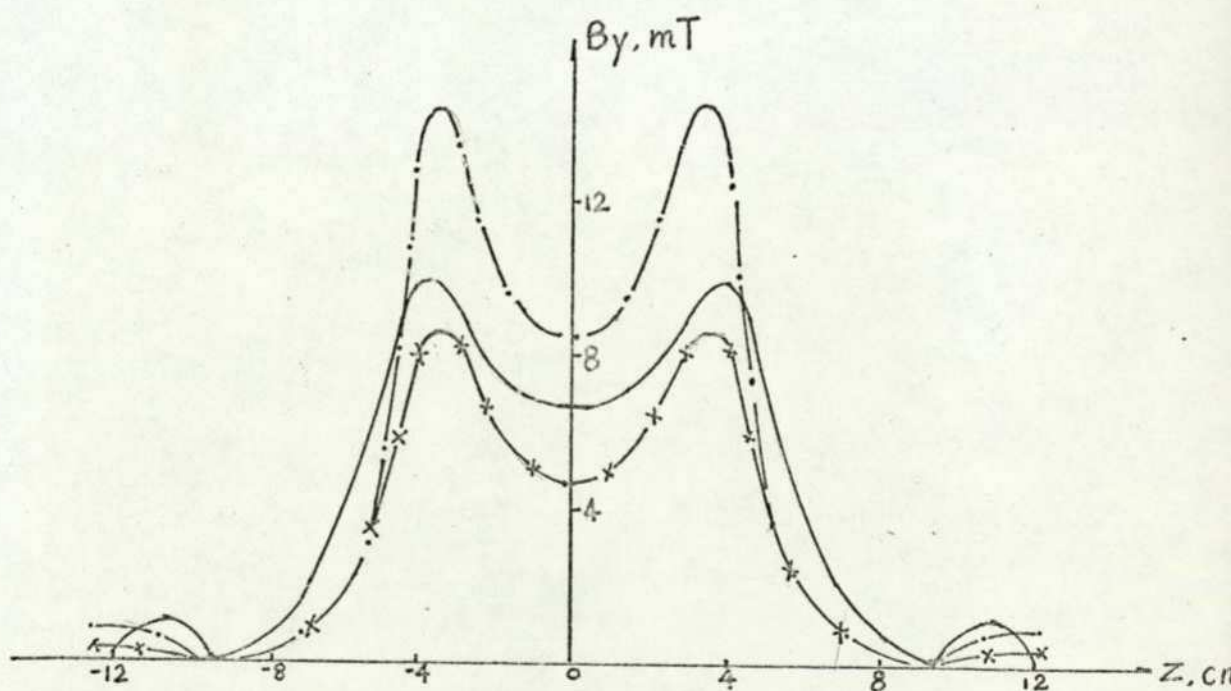
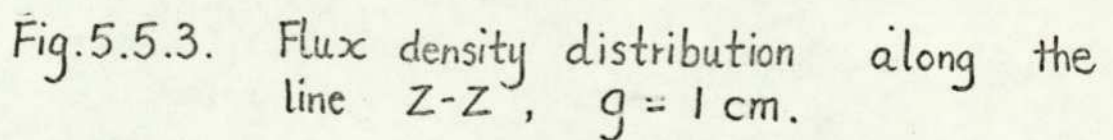
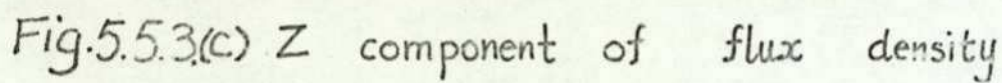


Fig 5.5.3.(b) Y component of flux density



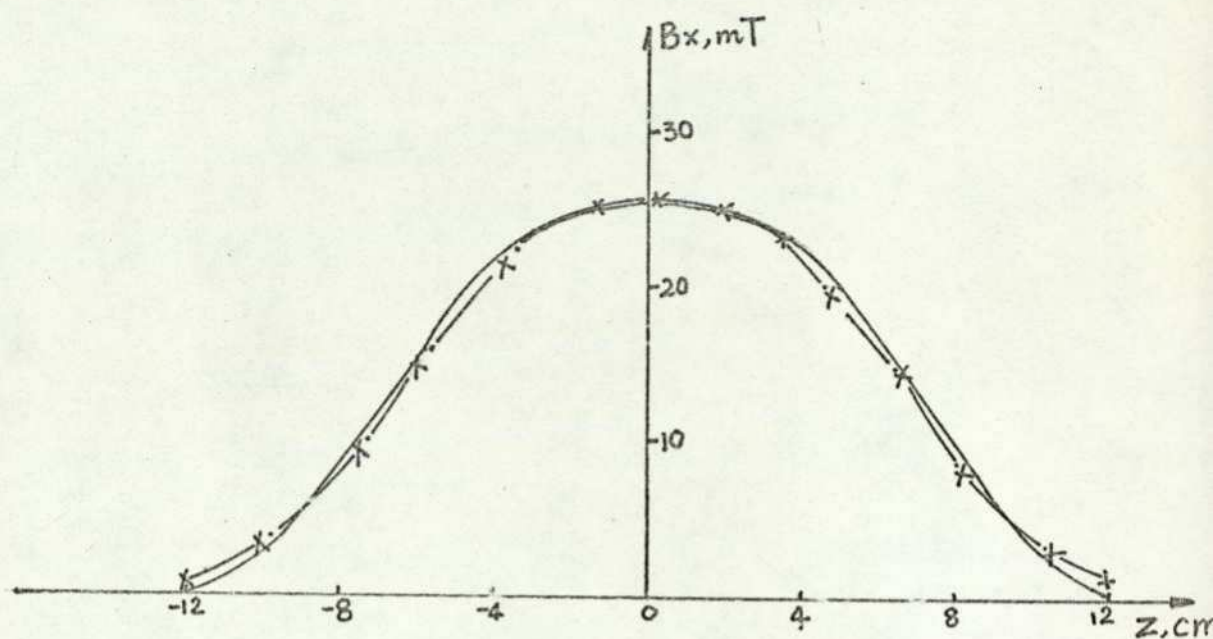


Fig.5.54(a) X component of flux density

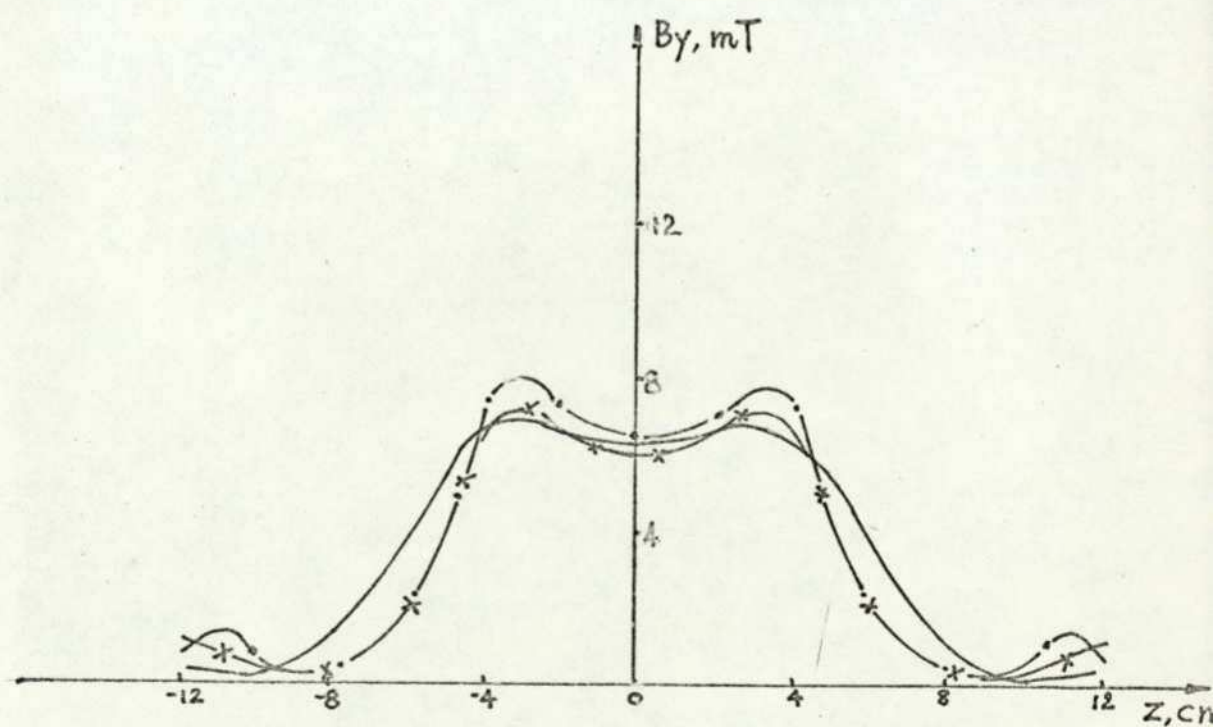


Fig.5.54(b) Y - component of flux density

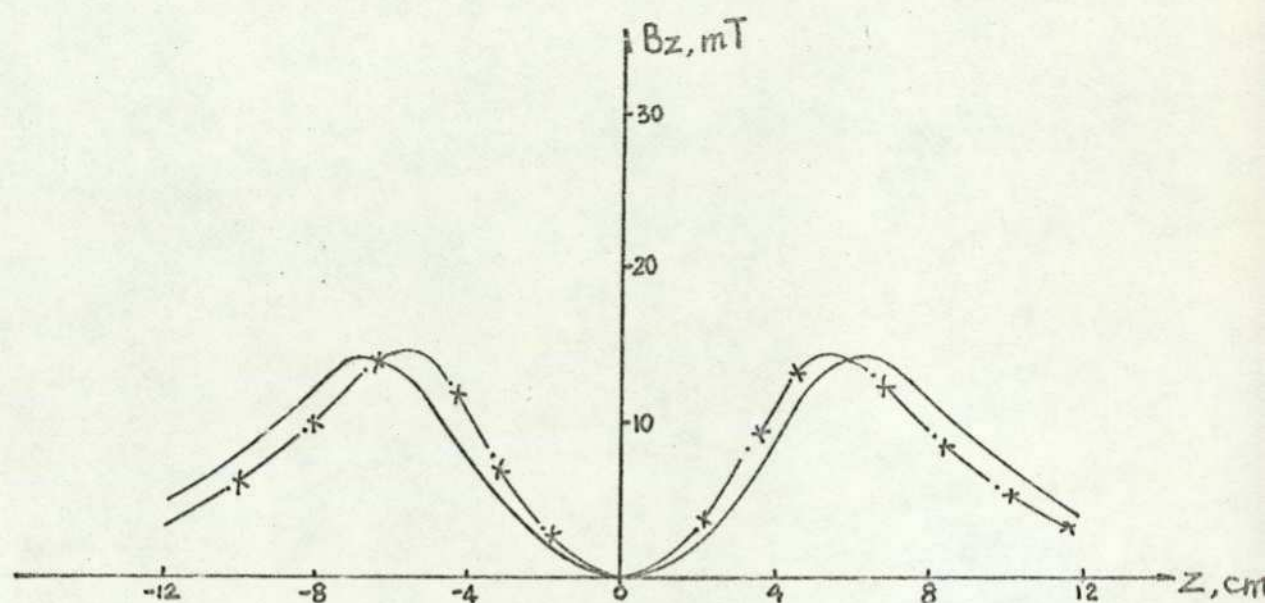


Fig.5.5.4.(c) Z component of flux density

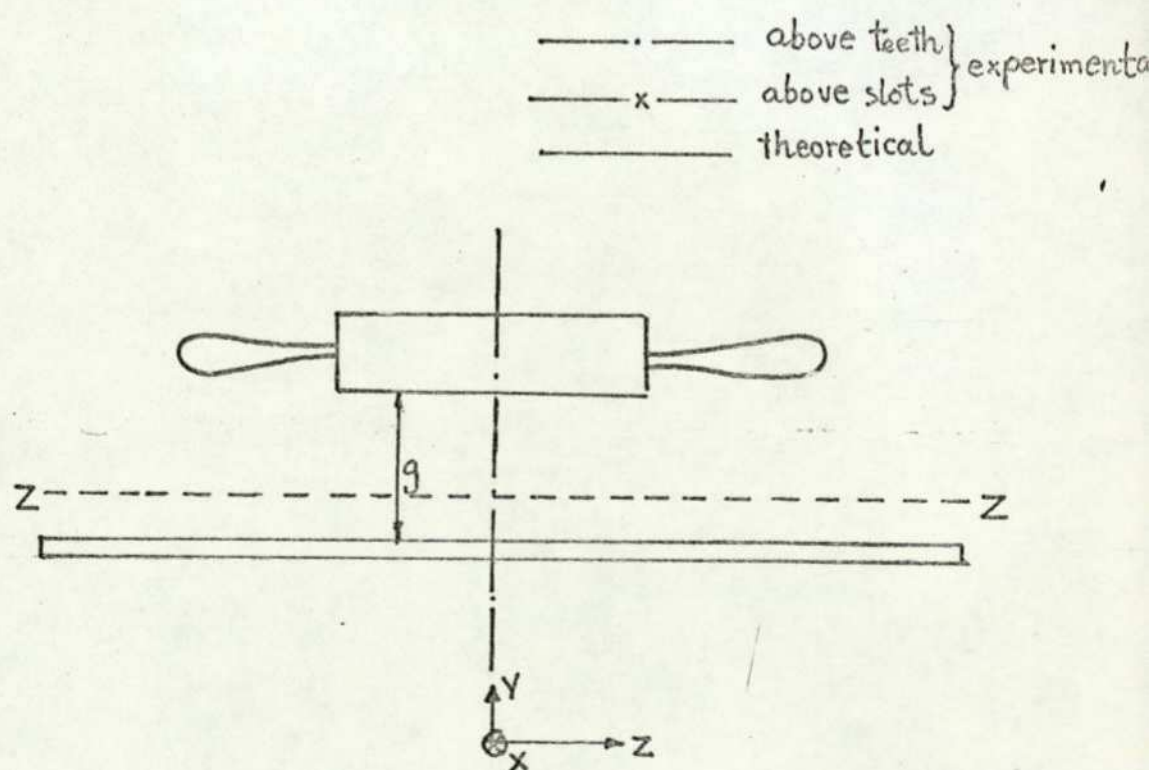


Fig. 5.5.4. Flux density distribution along the line Z-Z, $g = 2\text{cm}$.

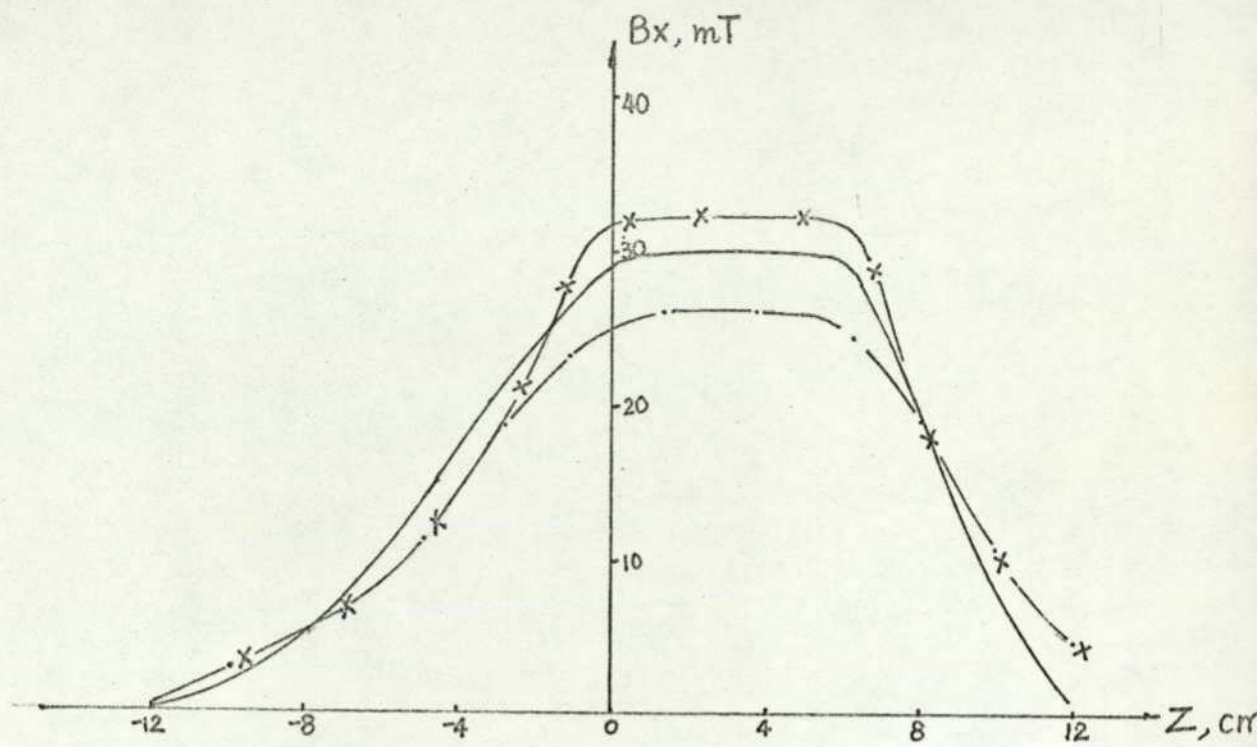


Fig 5.5.5(a) X component of flux density

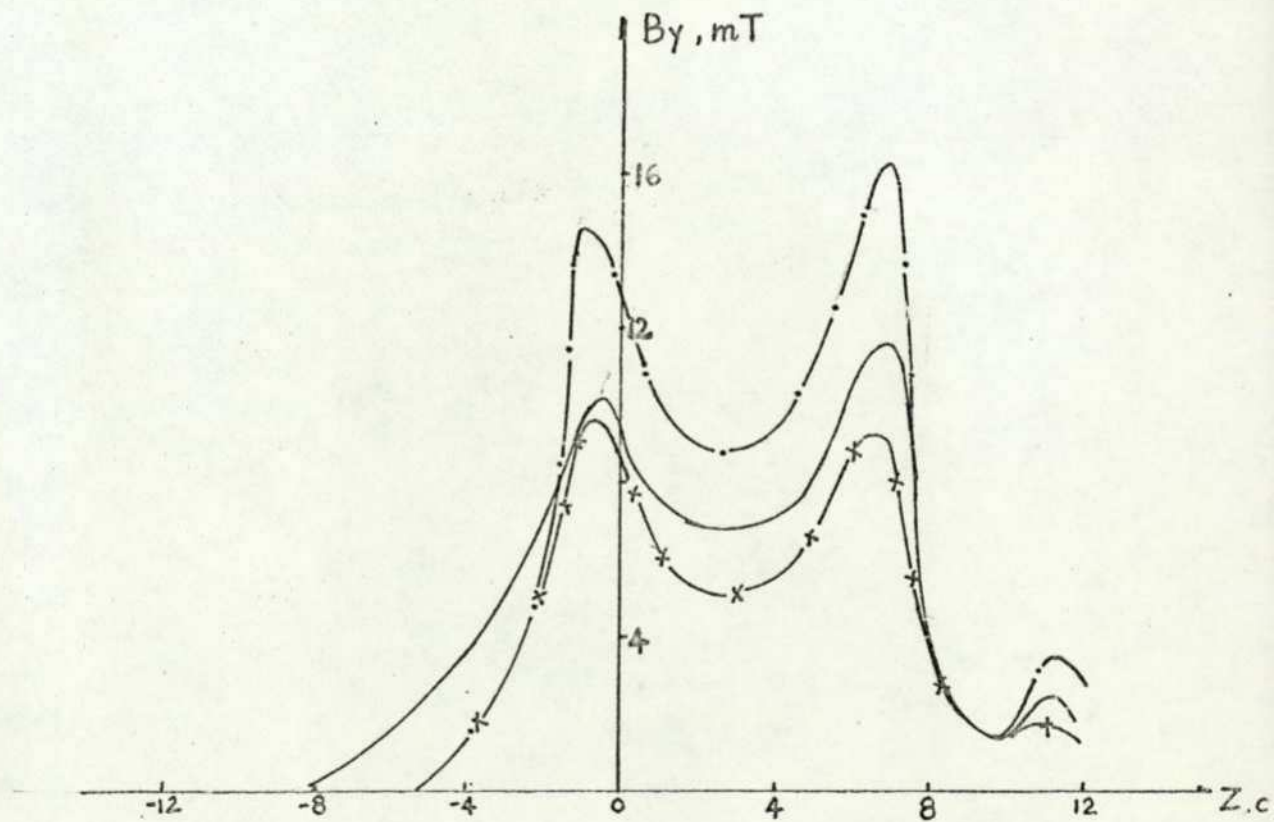


Fig 5.5.5(b) Y component of flux density

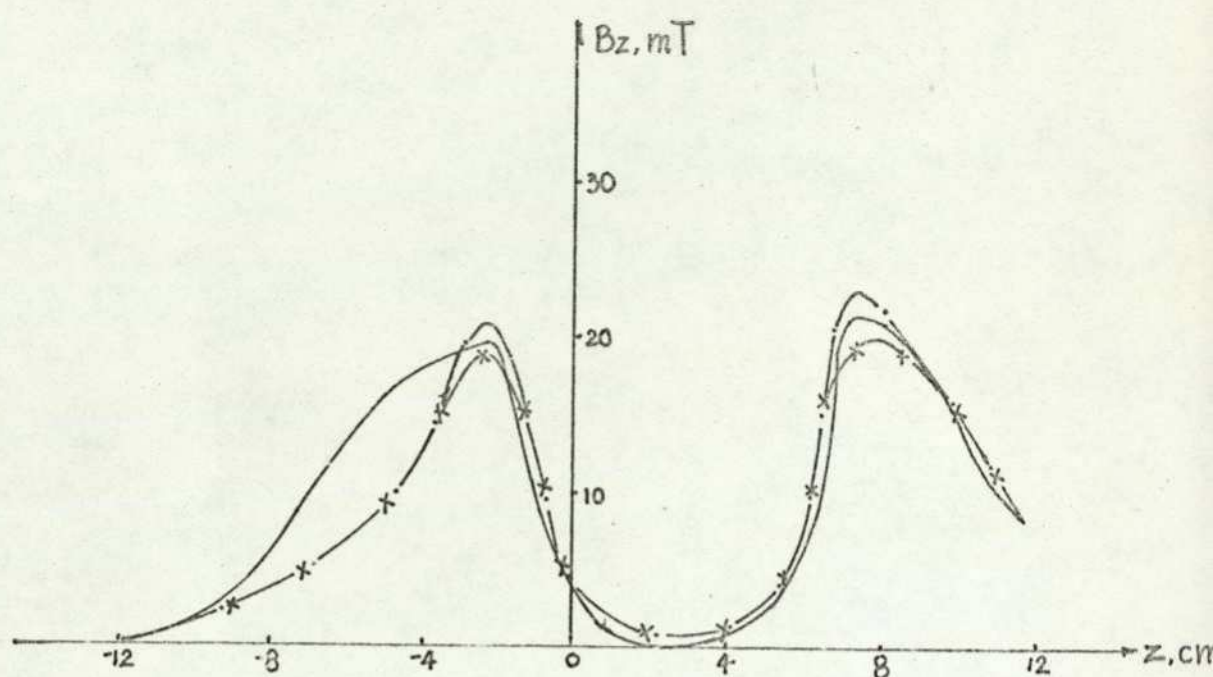


Fig 5.5.5(c) Z component of flux density

..... above teeth } experimental
x..... above slots }
 theoretical

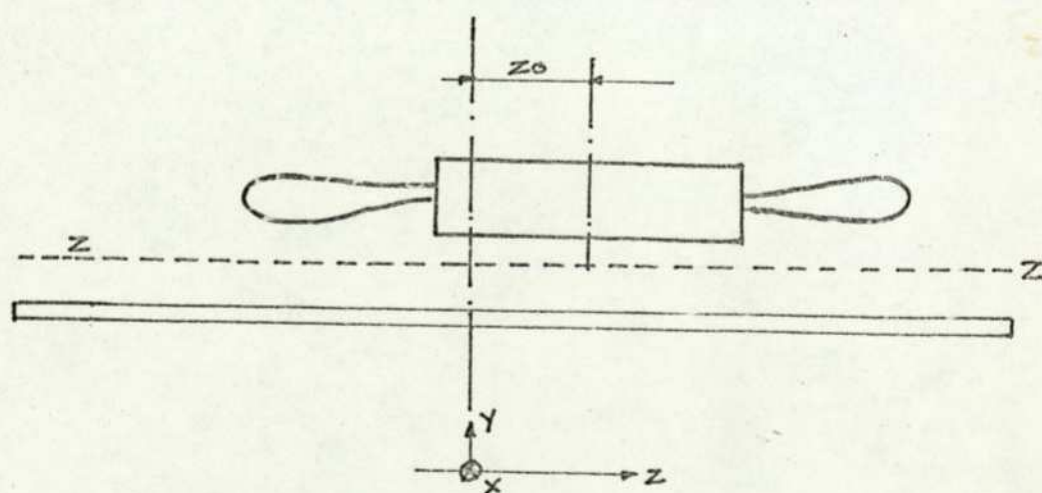


Fig. 5.5.5. Flux density distribution along the line Z-Z, $Z_0 = 3$ cm.

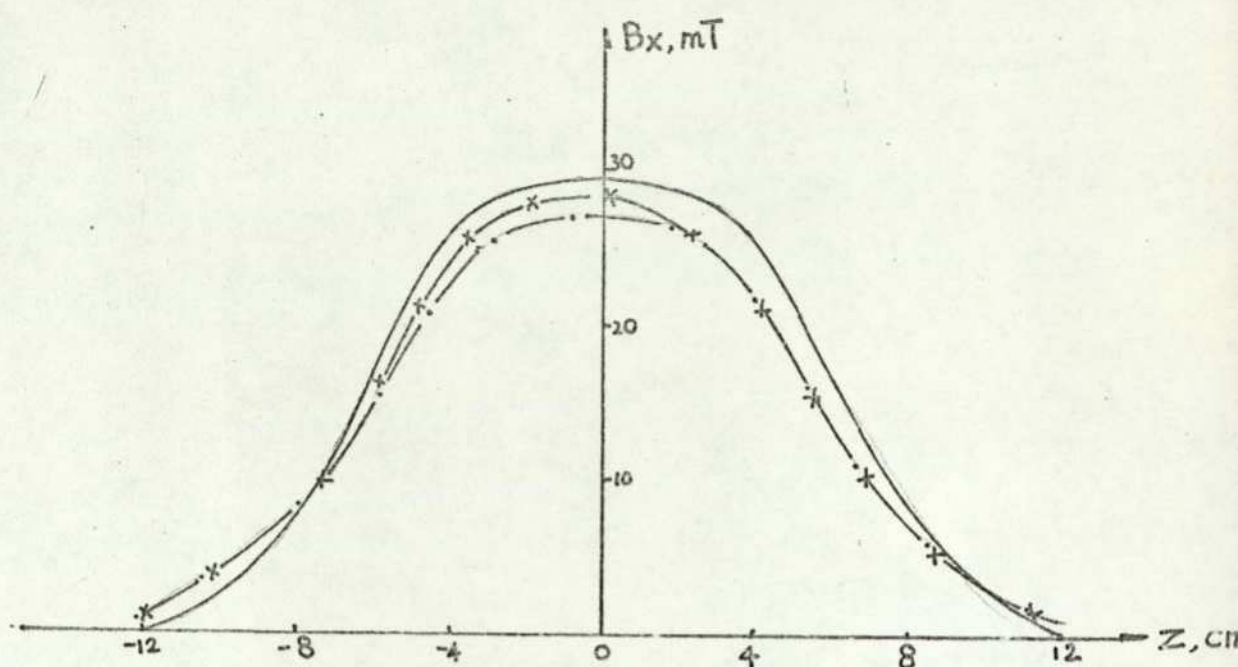


Fig.5.5.6.(a) x component of flux density

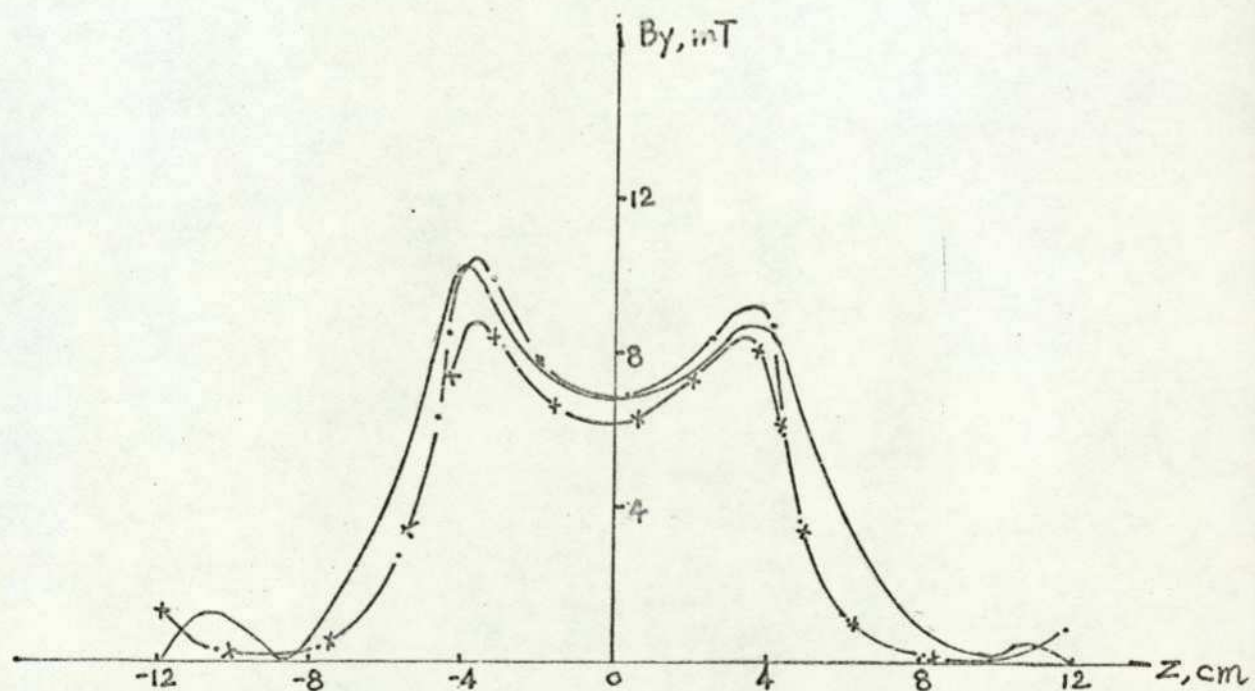


Fig.5.5.6.(b) y component of flux density

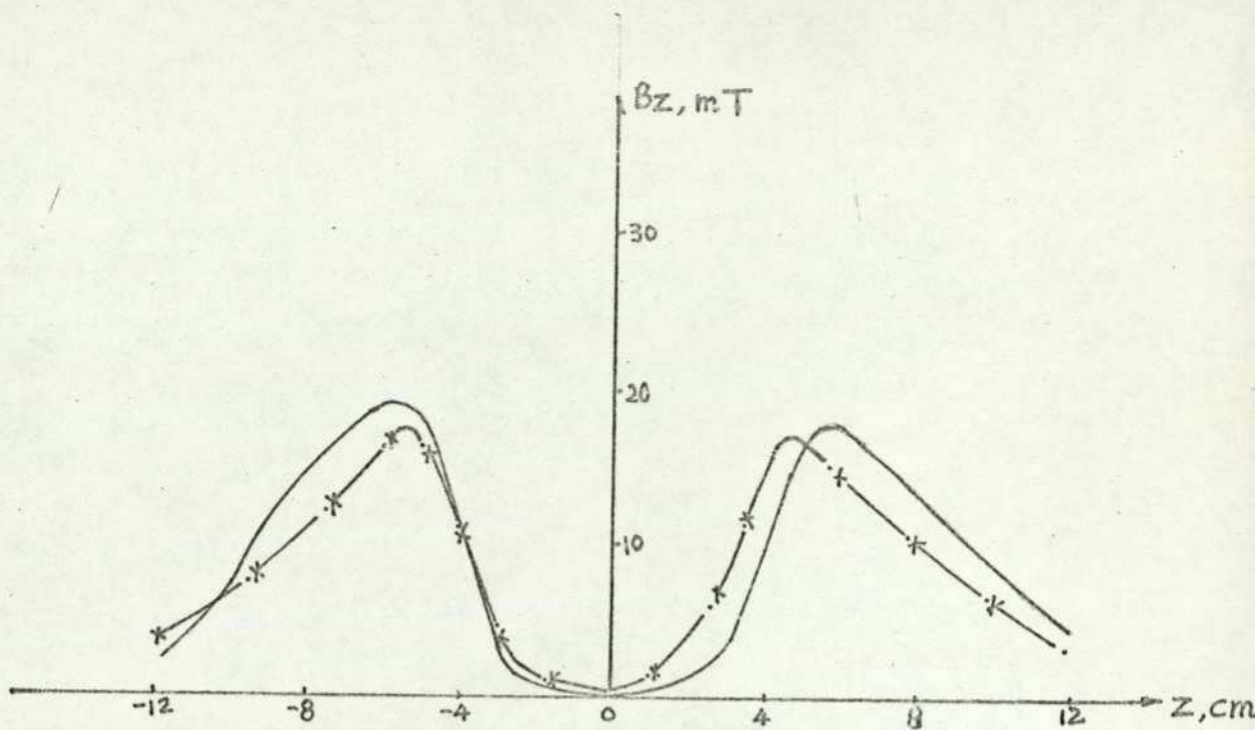


Fig. 5.5.6.(c) Z component of flux density

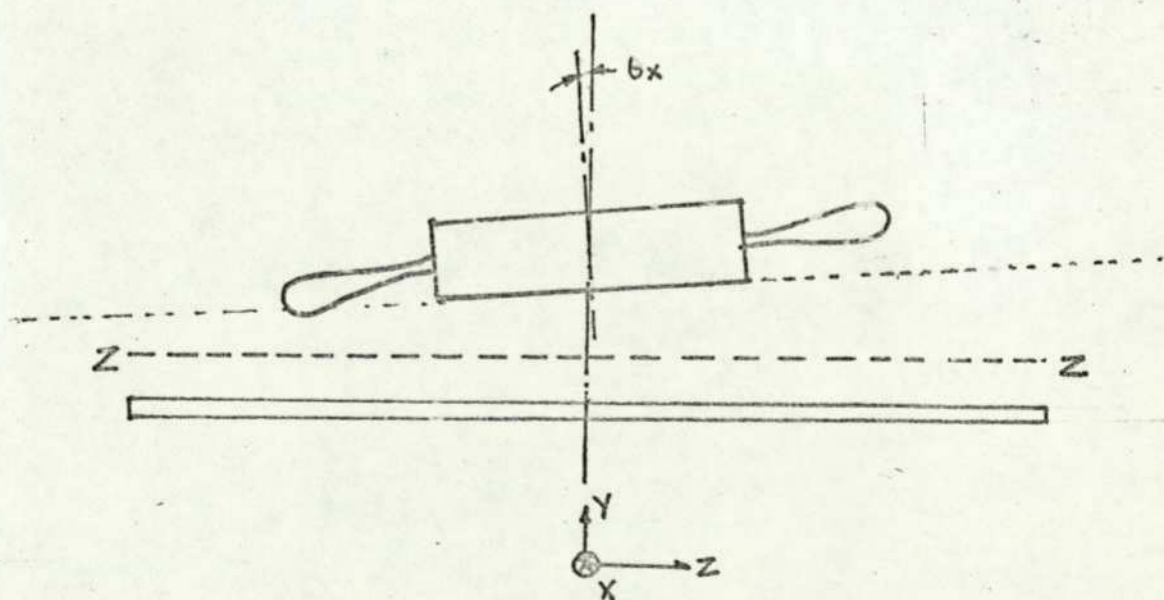


Fig. 5.5.6. Flux density distribution along the line $Z-Z$, $\theta_x = 0.033$ rad.

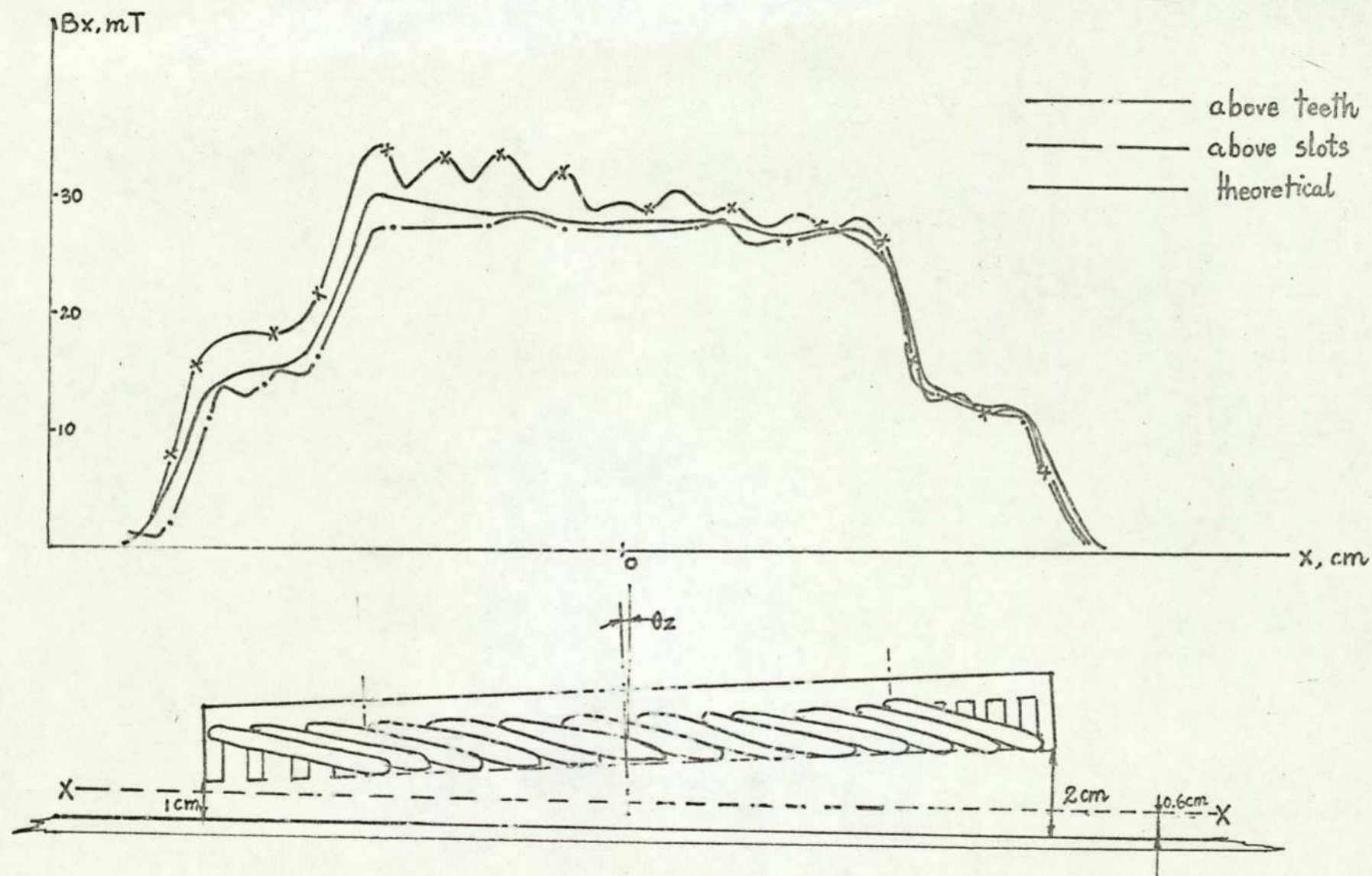


Fig. 5.5.7. (a) X-component of flux density distribution

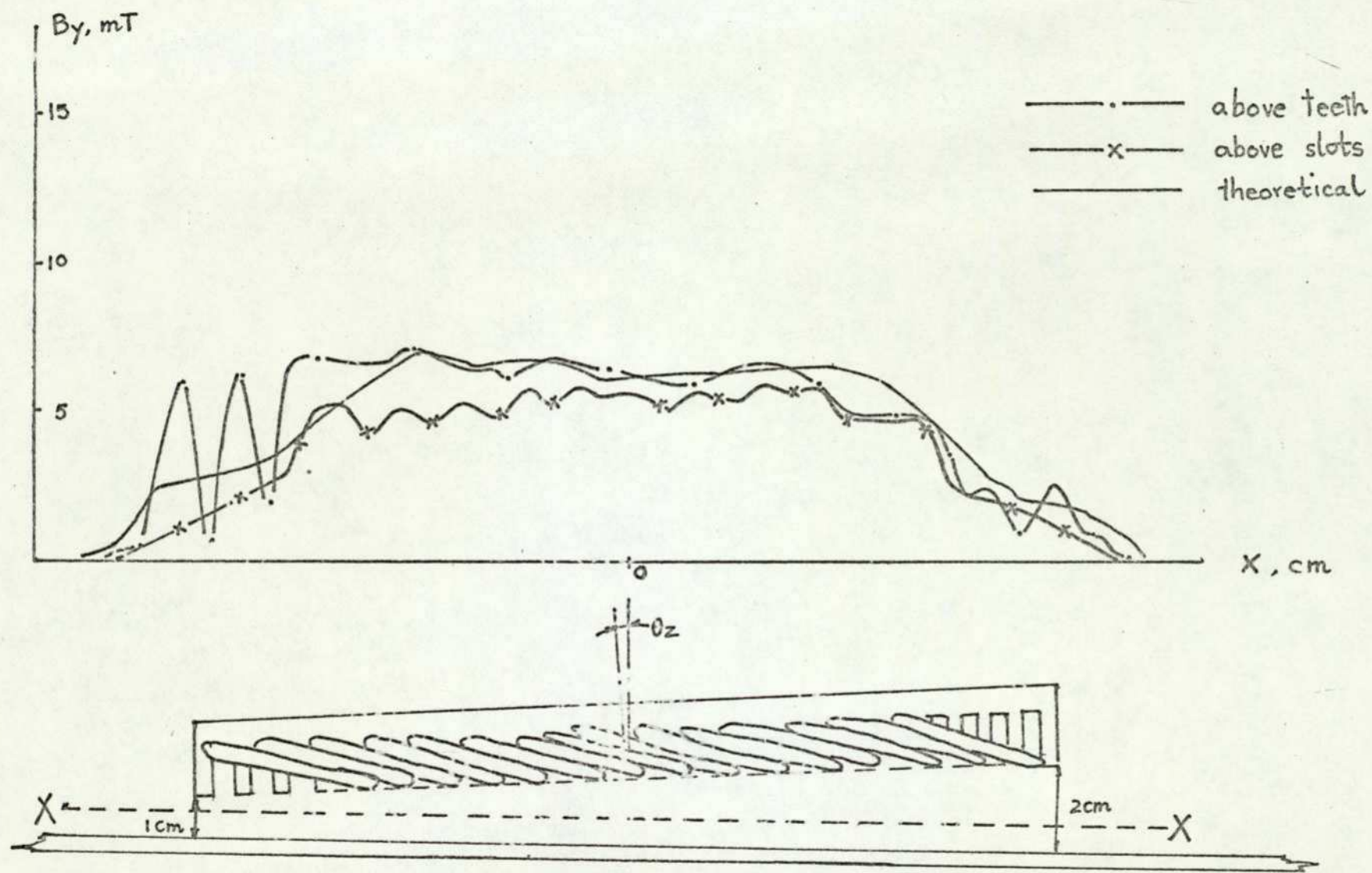


Fig. 5.5.7. (b) Y component of flux density distribution

along line $Y-Y$ $\theta_z = 0.0182$ rad.

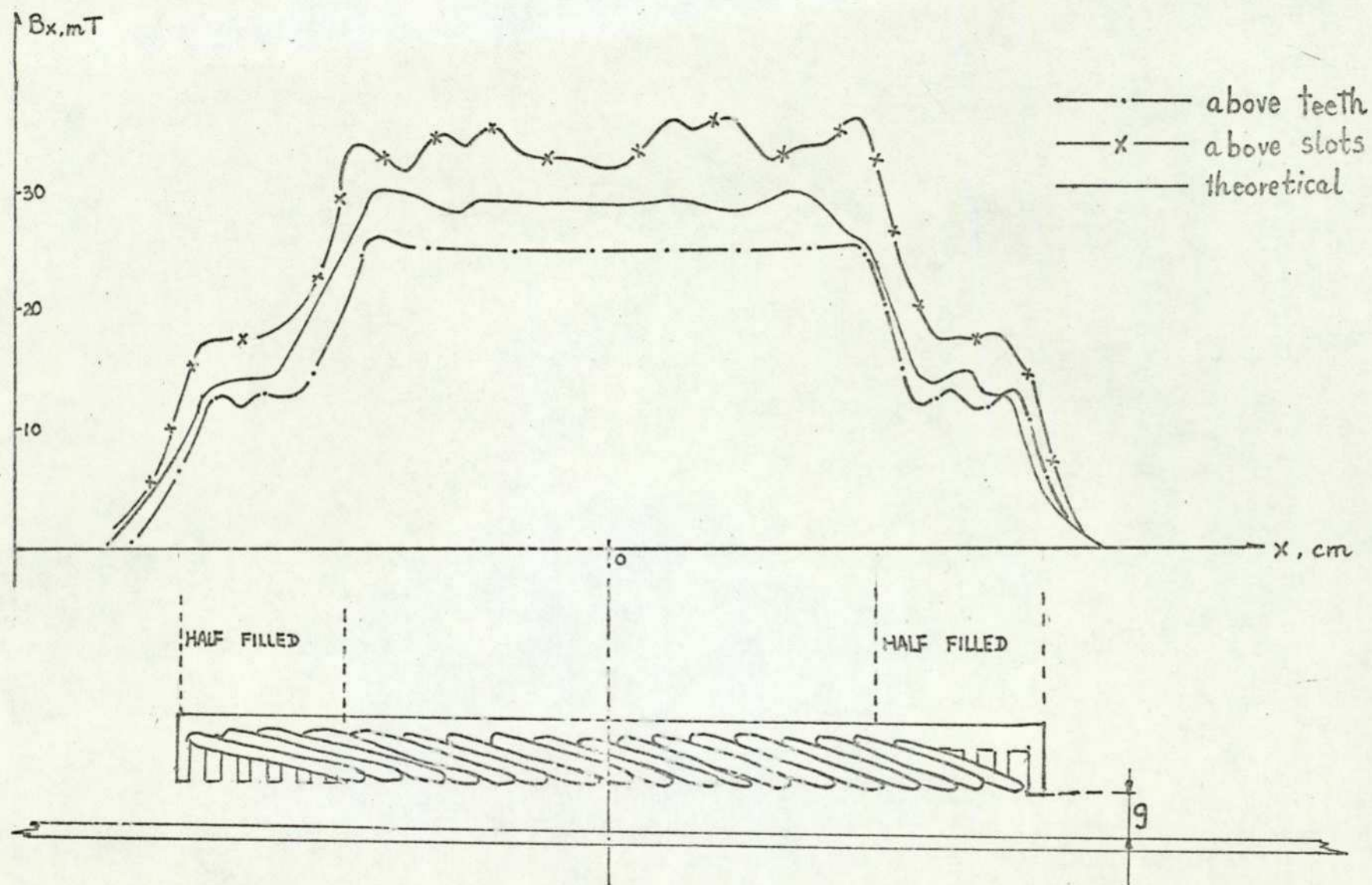


Fig.5.5.8. (a) x component of flux density distribution

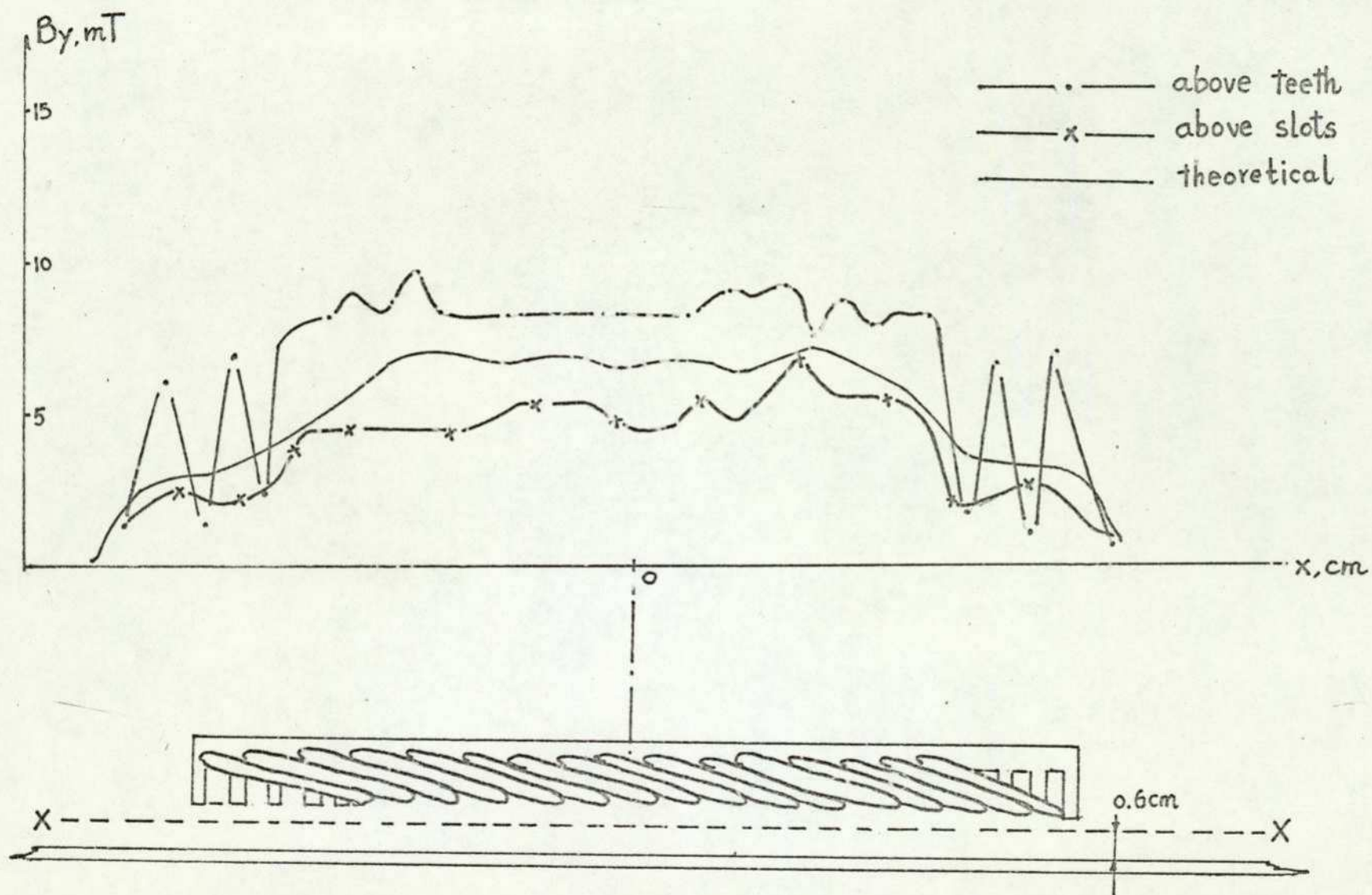


Fig. 5.5.8. (b) Y component of flux density distribution

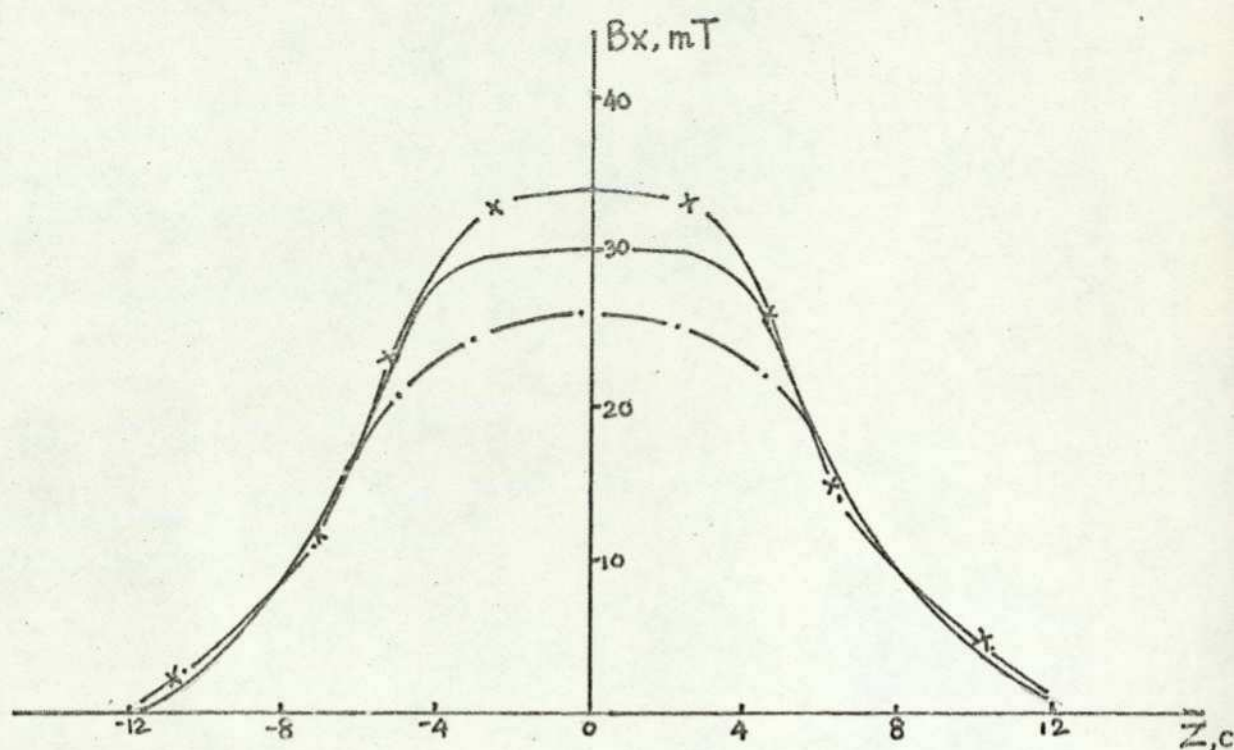


Fig.5.59.(a) X component of flux density

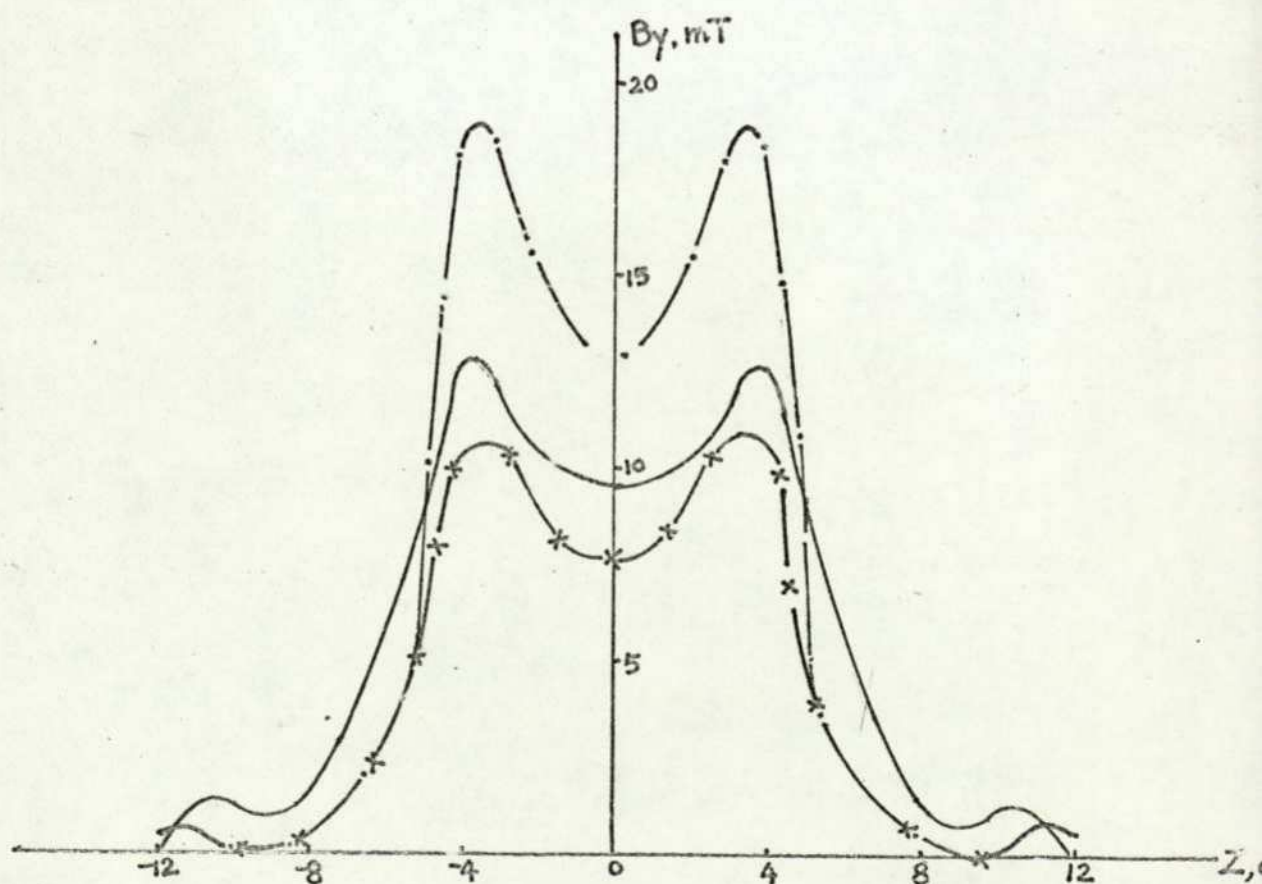


Fig.5.59.(b) Y component of flux density

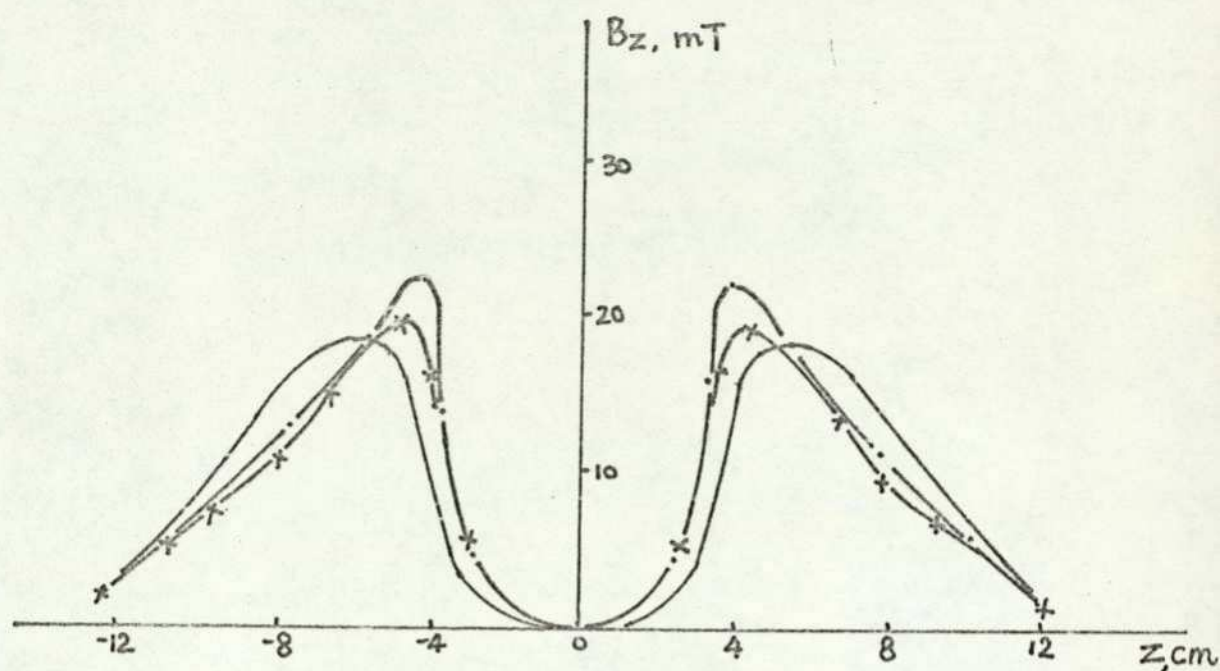


Fig.5.5.9(C). Z component of flux density

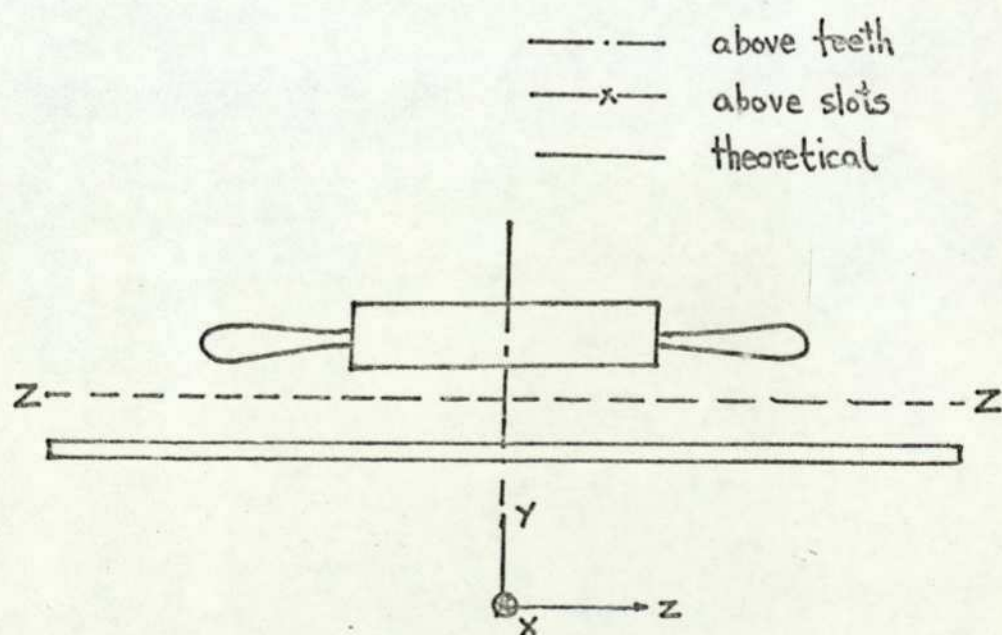


Fig.5.5.9. Flux density distributions along the line Z-Z, $g = 1\text{cm}$, $f = 150\text{Hz}$.

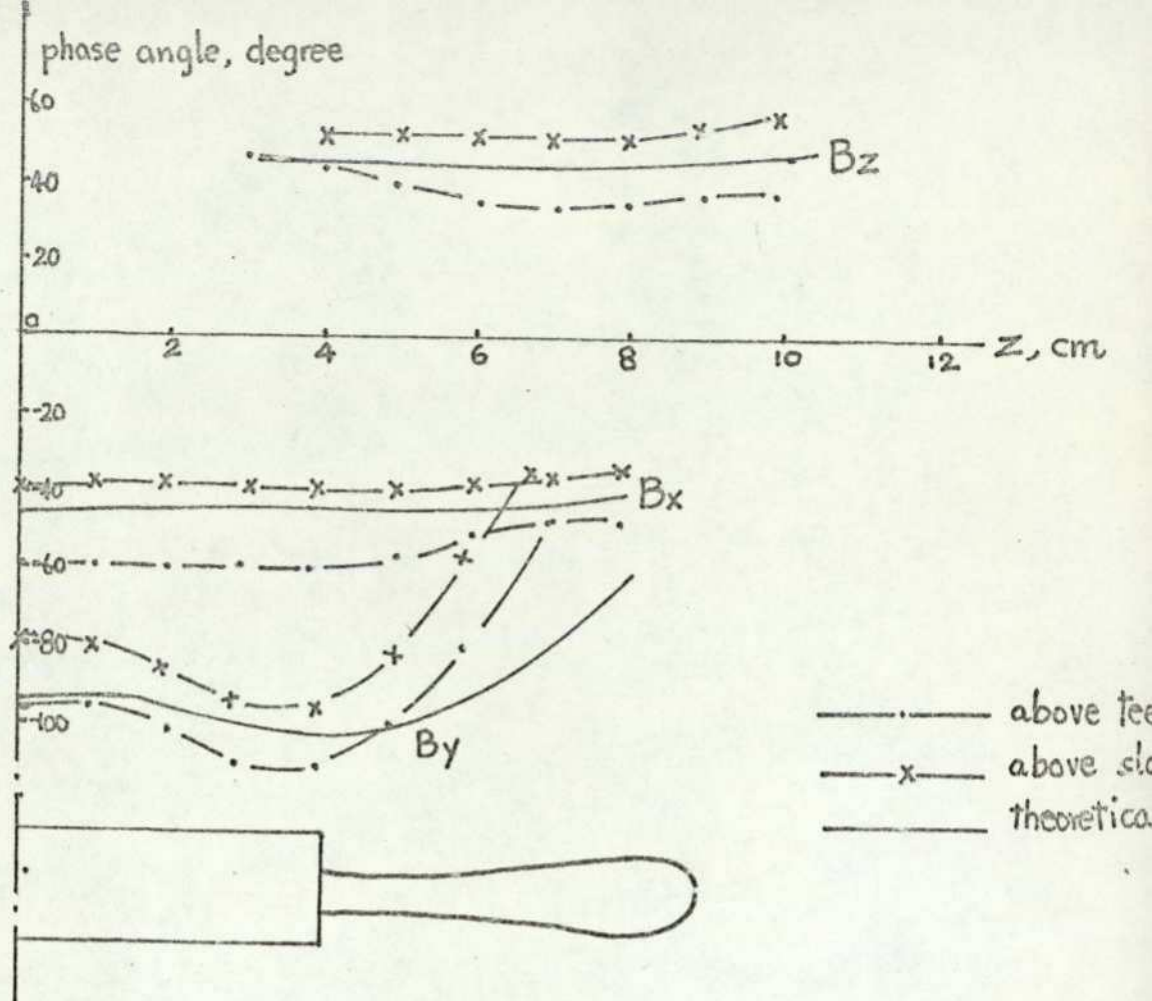


Fig. 5.5.10. Phase angle of flux densities B_x , B_y and

CHAPTER 6

CHAPTER 6. EXPERIMENTAL TESTS AND RESULTS - DYNAMIC TEST

6.1. Introduction

The three axis forces and the flux densities of the machine under static conditions are verified in the last chapter. This chapter is included in order to give additional data to correlate with the predicted flux and forces generated by the machine under dynamic conditions.

To facilitate measurements in case of a LDM under dynamic conditions, a continuous motion is required. This can be achieved by the construction of a rotating drum to produce a continuous moving track.

The arrangement of the dynamic test rig and machine will be discussed in the later section. Details of technical data and a drawing of the test rig are also included for further references.

6.2. Description of the dynamic machine and test rig

The LDM used in the dynamic test is basically similar to the static test machine described in Chapter 5. The main difference is that in the later case, stator has been made in the form of an arc shape in order to match the curvature of the secondary rotating track (see Fig.6.2.1.). Due to the limitations of the labour and manufacturing cost, it was impractical to make the stator core to the exact curvature required. The problem was overcome by making the stator core from several sections bolted together with two steel plates in order to form an arc machine(see Fig.6.2.2.). By using six section, it

was found that the overall error did not exceed 0.5mm with respect to the curvature required. The dynamic test machine is shown in Fig.6.2.3.. The stampings were obtained from Linear Motors Ltd. 'Standard A LAM 65mm NEWCOR' type laminations were used, 126 laminations being used in each section, stacked to form an overall width of 8cm and bolted together with two steel plates. Each bolt was insulated from the steel plates by non-conducting bushes, in order to avoid any circulating eddy currents which would distort the magnetic flux inside the stator core. The stator windings were identical to those of the static test machine, 50 turns per coil with 5/6 slot pitch and 2 slots per pole per phase.

Fig.6.2.4. shows the full assembled test rig. The diameter of the drum was initially designed to be 1.5m in order to achieve a peripheral speed of 289km/h(1000rev/min.). A smaller diameter would require a higher angular velocity for a given track velocity and vice versa. The rotating drum is constructed with non-conducting and non-magnetic materials(mainly fibre glass and wood) in order to minimize any unnecessary electromagnetic interference with the aluminium track.

The drum consists of three pieces of plywood, which are bolted together with six steel plates and two flanges. A layer of thin birch plywood is attached to the edges of the plywood discs and covered with fibre glass to form the drum surface. An aluminium track $1/8"$ thick and 24cm. wide is shruck on the surface of the drum and secured with epoxy resin adhesive. The shaft of the drum is supported by three bearings and coupled to a 10 H.P. D.C. driving motor through a flexible coupling. A concrete block was constructed specially by the Civil Engineering Department to give a suitable firm base support for the whole structure.

The sectional drawing of the test rig is shown in Fig.6.2.5..

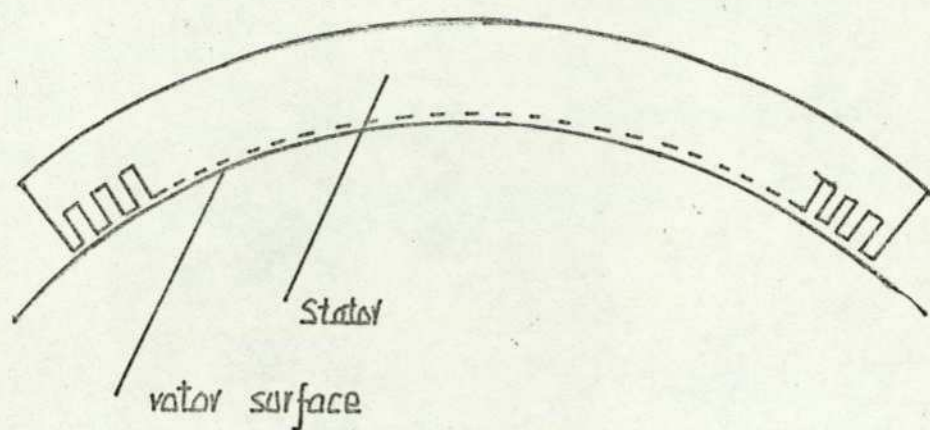


Fig. 6.2.1. Stator with exact curvature

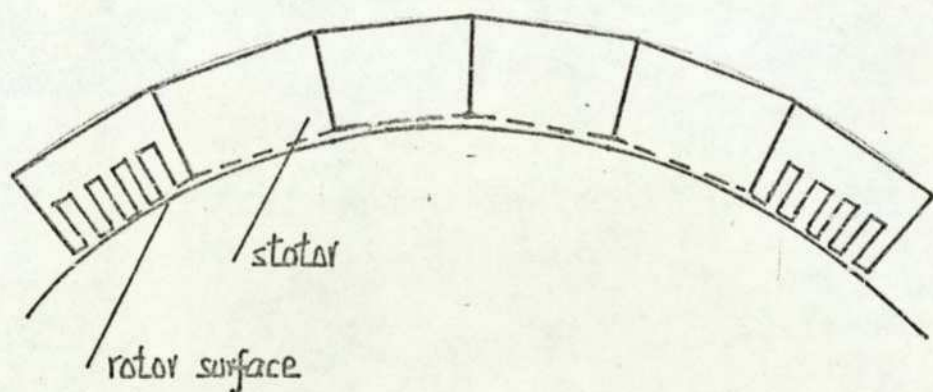


Fig 6.2.2. Stator in sections



Fig. 6.2.3. Stator - dynamic test

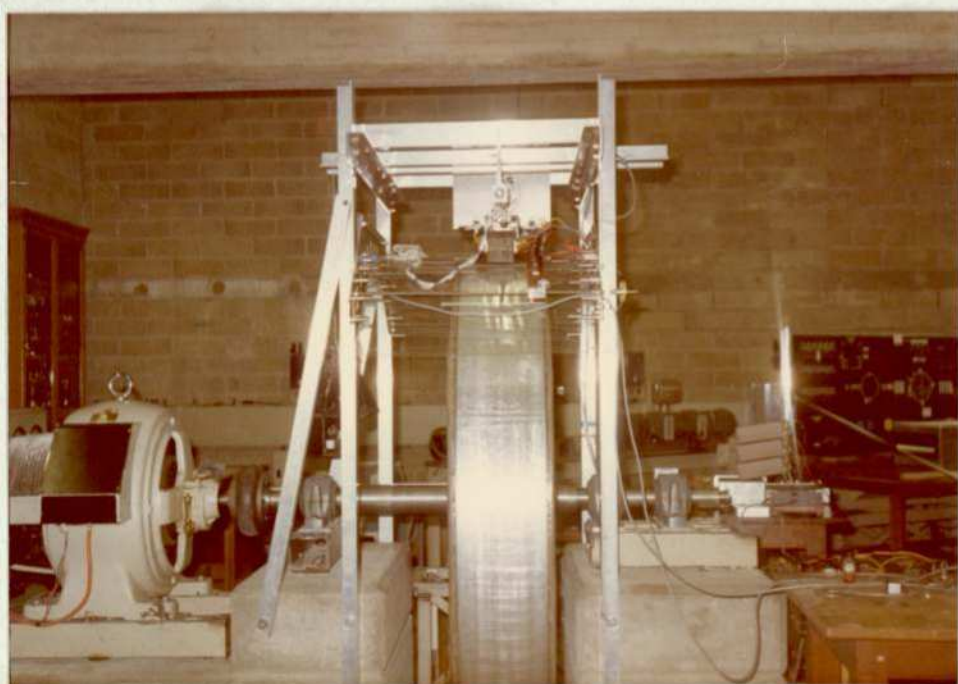
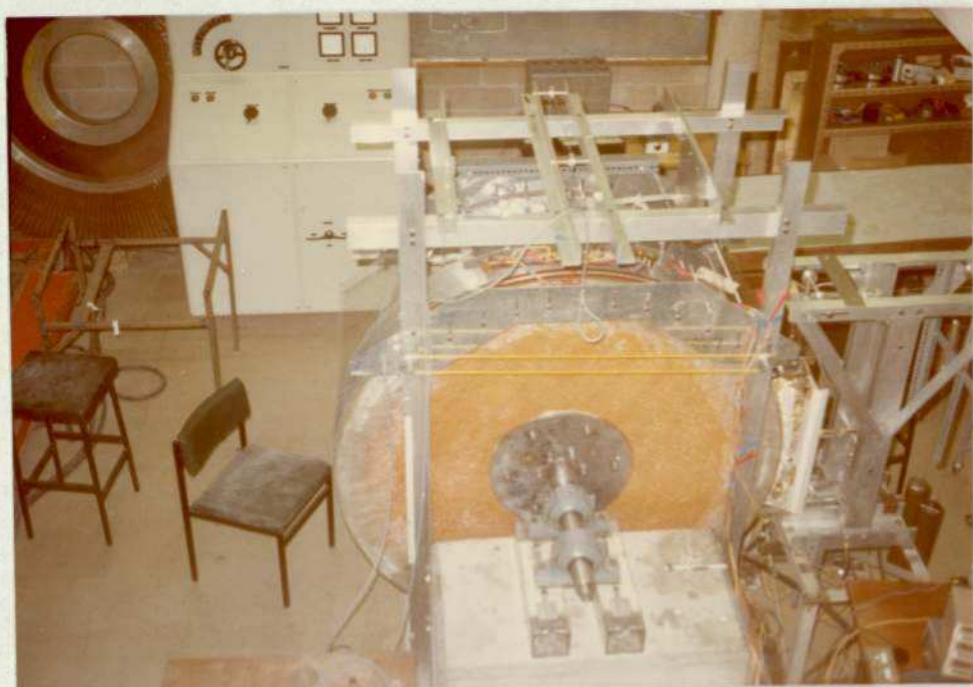


Fig. 6.2.4. Dynamic test rig

- 671 -

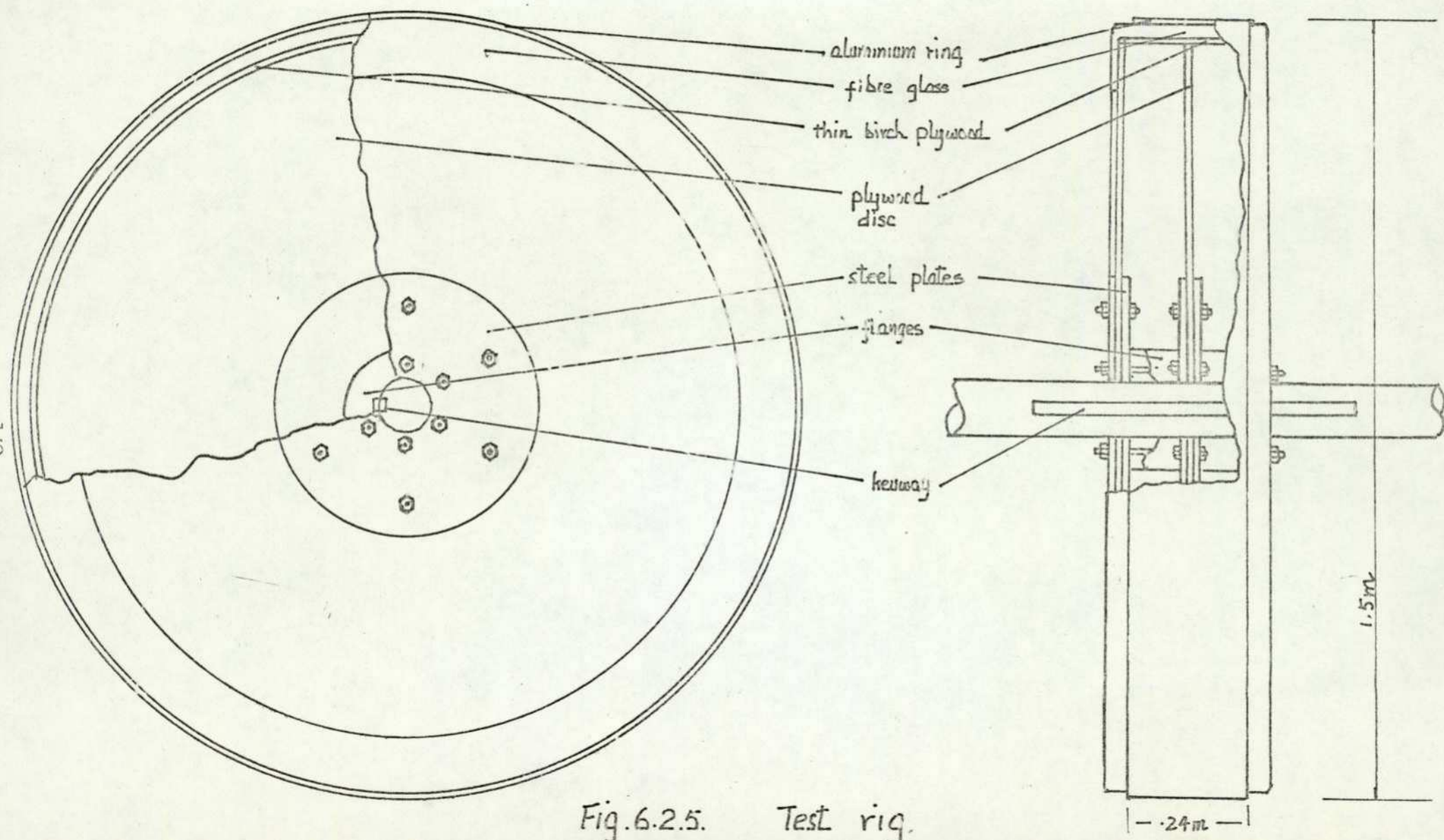


Fig.6.2.5. Test rig.

6.3. Force measurement techniques and results

The three axis forces were measured by using the same system described in section 5.3.(static test measurement). The basic setup of the dynamic force measurement system is shown in Fig.6.3.1.

The forces were measured from the stator. The stator is clamped to the end of the three aluminium bars by means of clamping plates so that the air gap length and the position of the stator can be adjusted. Bars B_{23} , B_{45} and B_{67} carry strain gauges which pick up forces in the y and z directions . Bar B_1 is joined with bar B_{67} through a self aligning bearing, which picks up forces in the x direction.(See section 5.3. for the description of the force measurement system in detail.)

Due to the heavy mass of the stator(28 kg), a counter weight system is necessary to balance the gravitational force of the stator. The stator is hung on the top of the drum by means of pulleys and counter weight, so as to reduce the initial stress in the three aluminium bars B_{23} , B_{45} and B_{67} . A thin perspex plate 3mm thick is fixed between the stator and the rotating aluminium surface in order to eliminate any aerodynamic drag force due to the moving conductor surface. The amplifier used in the static test was recalibrated so as to get a direct readout of the three axis forces and the three rotational couples in the dynamic test.

Fig.6.3.3. and Fig.6.3.4. show the traction force and levitation force(F_x and F_y) against slip s for a 50 Hz constant current supply of 5.0A, and $g_0 = 1.0\text{cm}$.

Fig.6.3.5. and Fig.6.3.6. show the traction and levitation forces against slip for a 150 Hz constant current supply of 5.0A, and $g_0 = 1.\text{cm}$.

Fig.6.3.7. and Fig.6.3.8. show the traction force and the levitation

force against slip for a 50 Hz constant current supply of 5.0A and $g_0 = 1.5\text{cm}$.

Fig.6.3.9. and Fig.6.3.10. show the traction and levitation forces against slip for a 50 Hz constant voltage supply of 54.4 volts, and $g_0 = 1.0\text{cm}$. The phase current was 5.0A at synchronous speed.

Fig.6.3.11. shows the three axis forces (F_x , F_y and F_z) against the lateral displacement z_0 for a 50 Hz constant current supply of 5.0A, $g_0 = 1.0\text{cm}$ and $s = 0.4$.

Fig.6.3.12. shows the traction force F_x and the moment of roll against the angle of roll θ_x for a 50 Hz constant current supply of 5.0A. $g_0 = 1.2\text{cm}$ and $s = 0.4$.

Fig.6.3.13. shows the moment of yaw M_y against the angle of yaw θ_y ; for a 50 Hz constant current supply of 5.0A, $g_0 = 1.2\text{cm}$ and $s = .4$.

Fig.6.3.14. shows the moment of pitch M_z against slip for a 50 Hz constant current supply of 5.0A and $g_0 = 1.0\text{cm}$.

Fig.6.3.15. shows the moment of pitch M_z against the angle of pitch θ_z for a 50 Hz constant current supply of 5.0A, $g_0 = 1.2\text{cm}$. and $s = .4$.

Fig.6.3.16. shows the traction force and the levitation force against the angle of pitch for a 50 Hz constant current supply of 5.0A, $g_0 = 1.2\text{cm}$ and $s = 0.4$.

Fig.6.3.17. shows the traction and levitation forces F_x and F_y against the air gap length g_0 for a 50 Hz constant current supply of 5.0A and $s = 0.4$.

Fig.6.3.18. shows the traction and levitation forces against the air gap length g_0 for a 150 Hz constant current supply of 5.0A and $s = 0.7$.

Fig.6.3.19. shows the measured Y-phase angle and current against slip for a 50 Hz constant voltage supply of 54.4 volts and $g_0 = 1.0\text{cm}$., and Fig.6.3.20. shows the corresponding power factor.

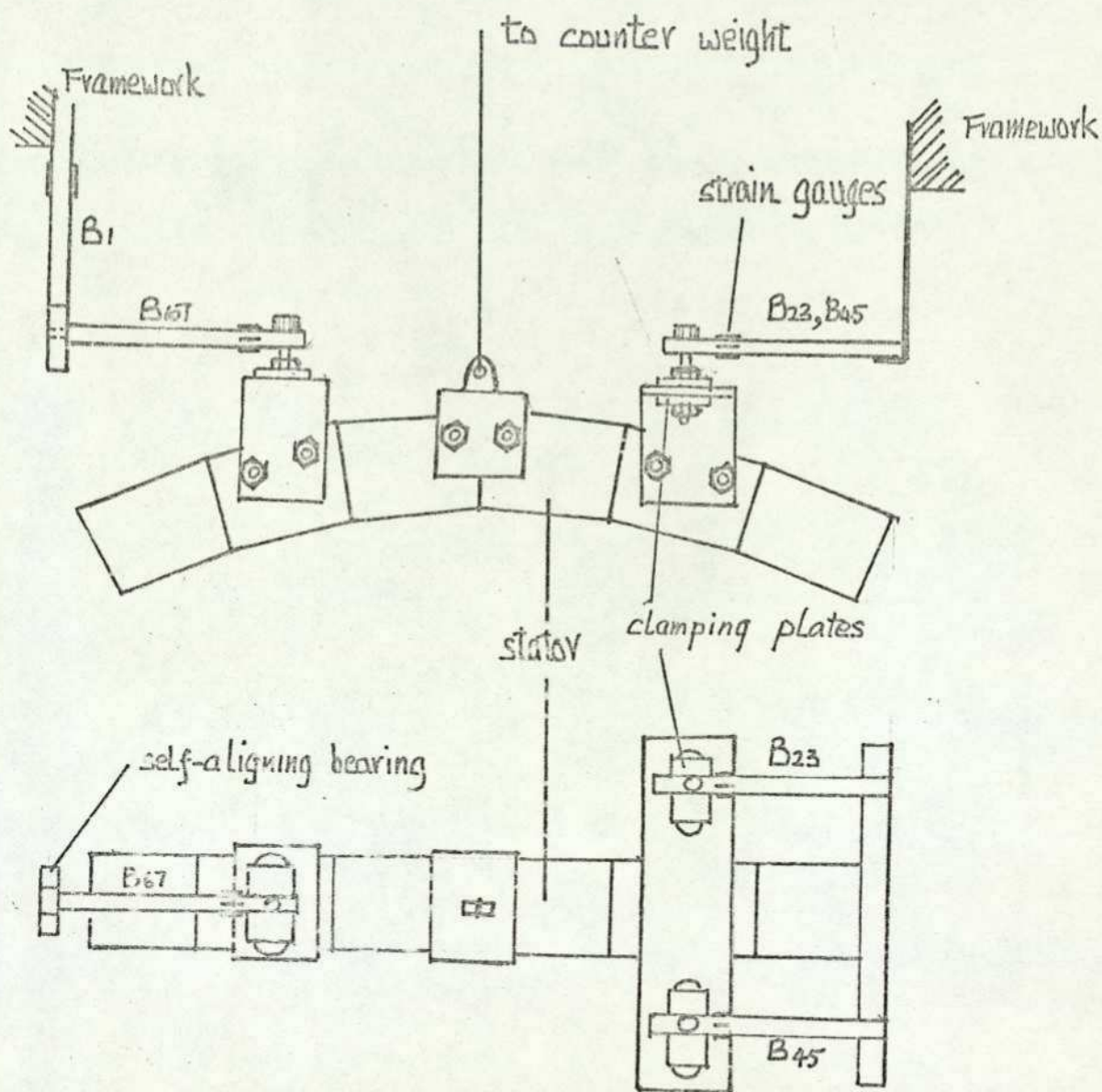


Fig. 6.3.1. Force measurement system

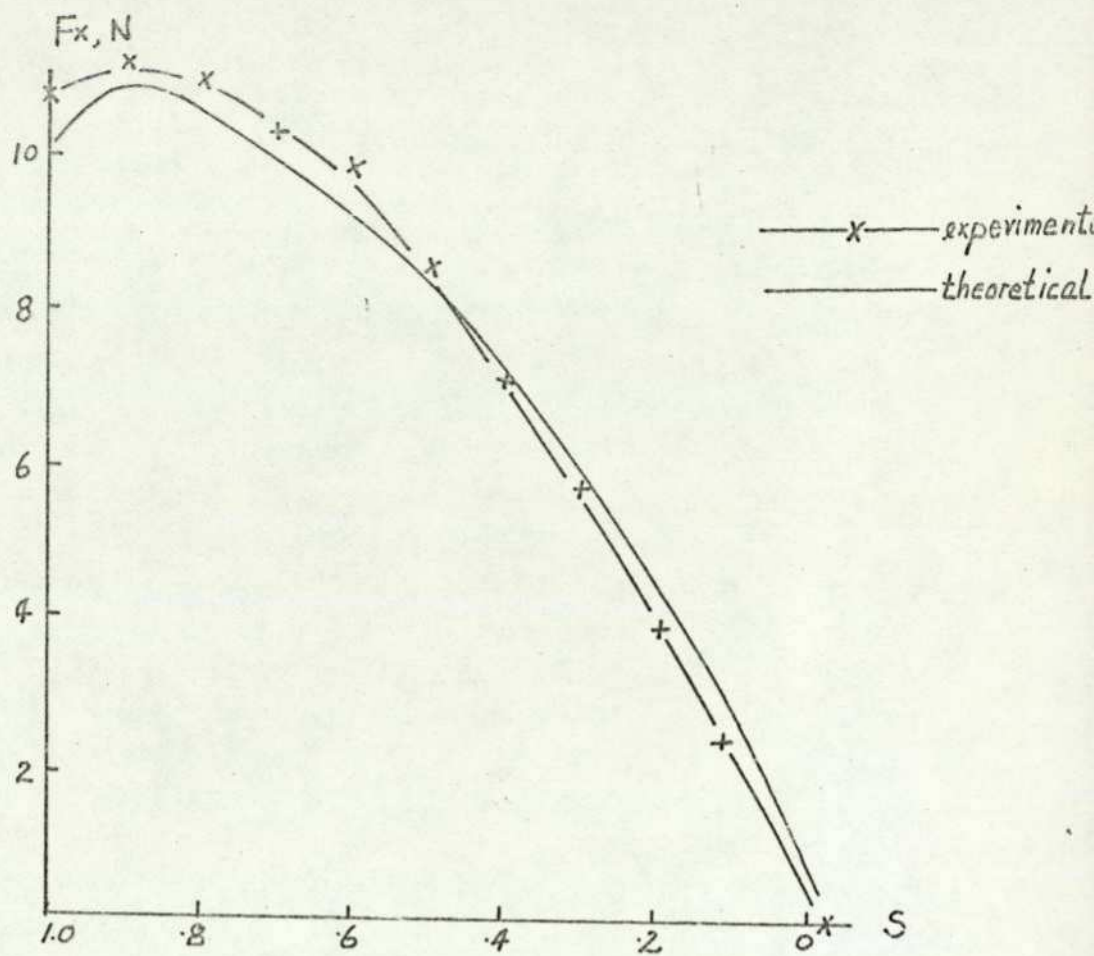


Fig. 6.3.3. F_x against slip, with a 50Hz constant current supply and $q_0 = 1.0\text{cm}$

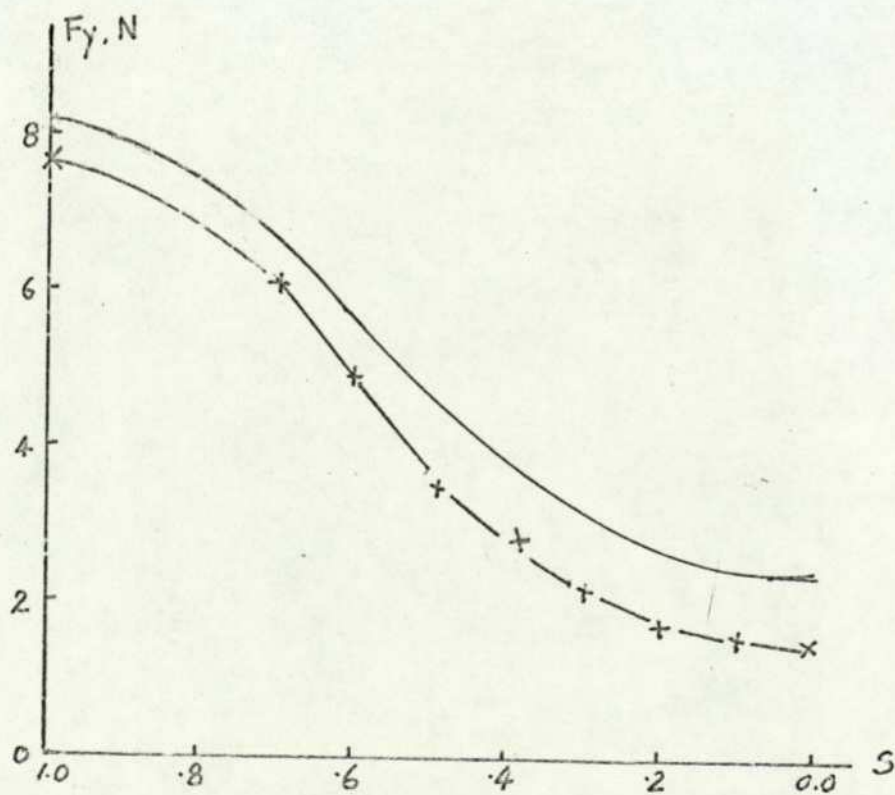


Fig. 6.3.4. F_y against slip, with a 50Hz constant current supply and $q_0 = 1.0\text{cm}$

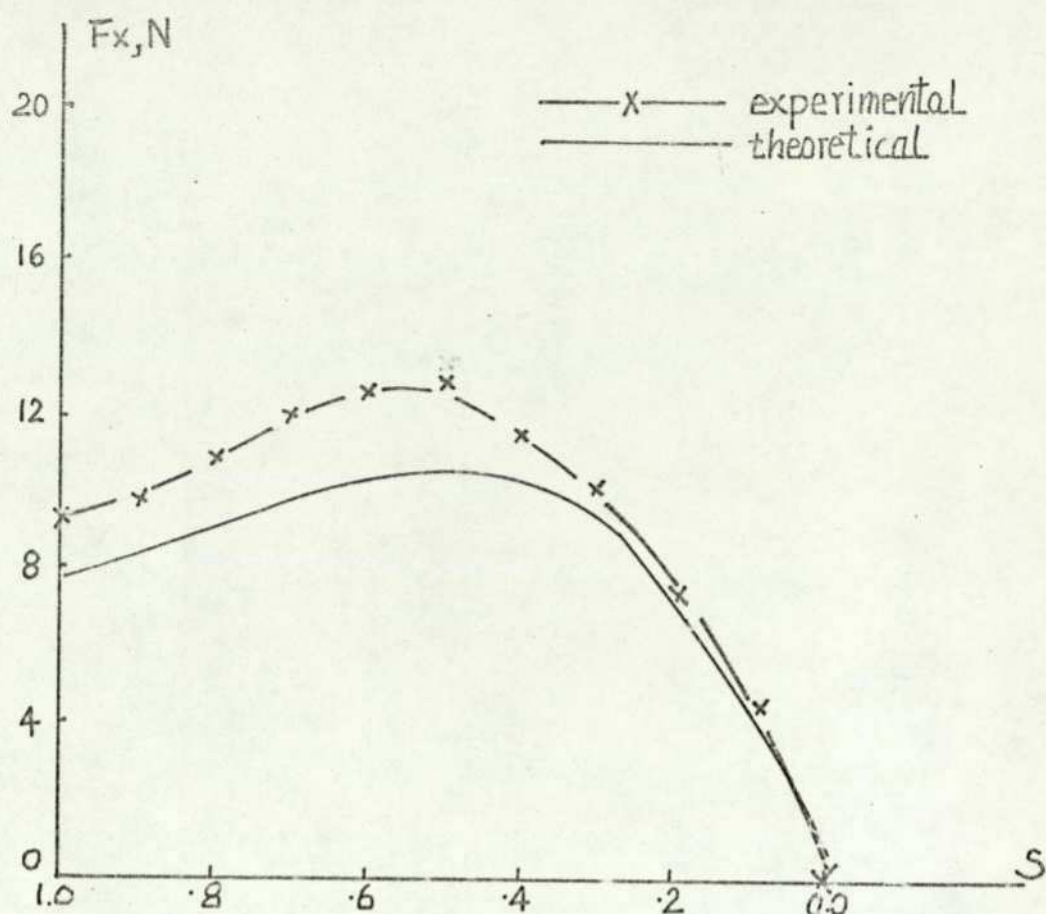


Fig. 6.3.5. F_x against slip. with a 150 Hz constant current supply and $g_0 = 1.0 \text{ cm}$.

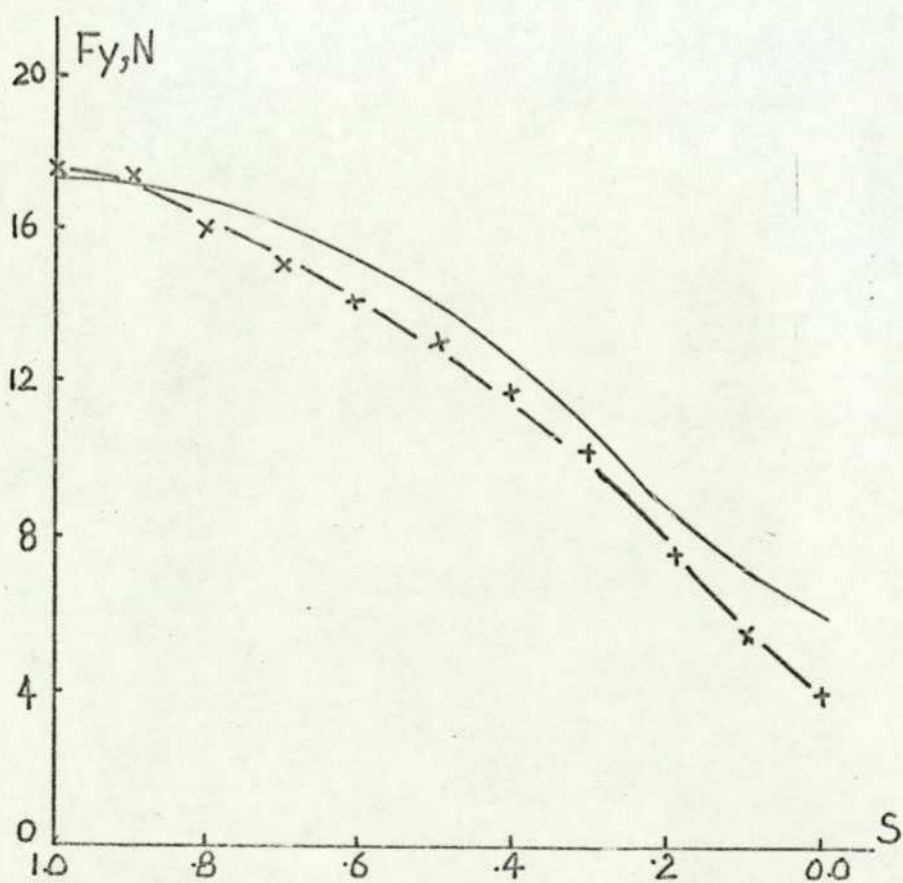


Fig. 6.3.6. F_y against slip. with a 150 Hz constant current supply and $g_0 = 1.0 \text{ cm}$.

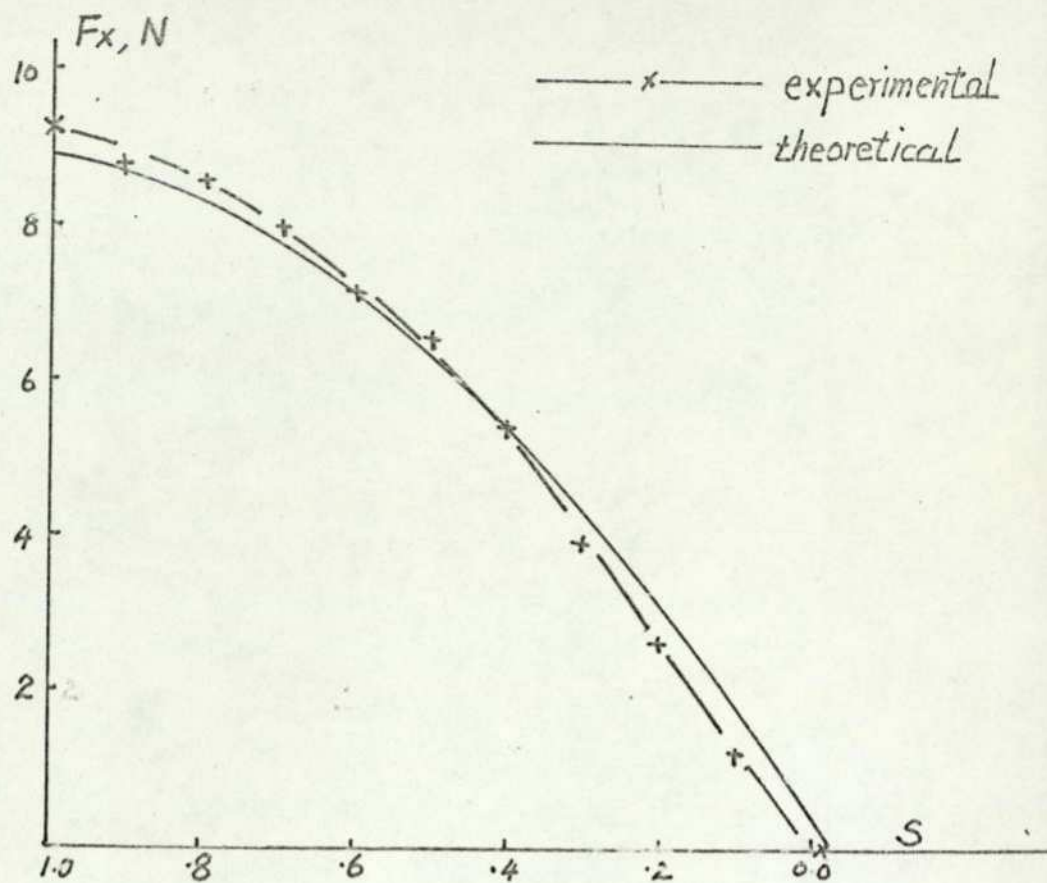


Fig. 6.3.7. F_x against slip. with a 50 Hz constant current supply and $g_0 = 1.5 \text{ cm}$

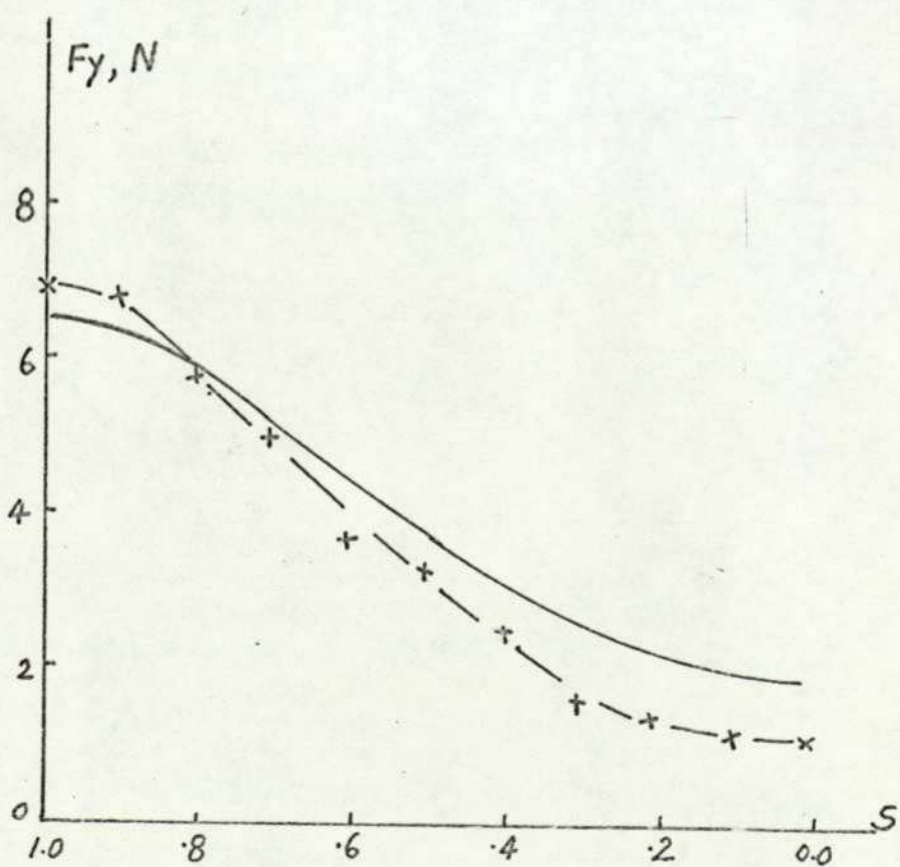


Fig. 6.3.8. F_y against slip with a 50 Hz constant current supply and $g_0 = 1.5 \text{ cm}$

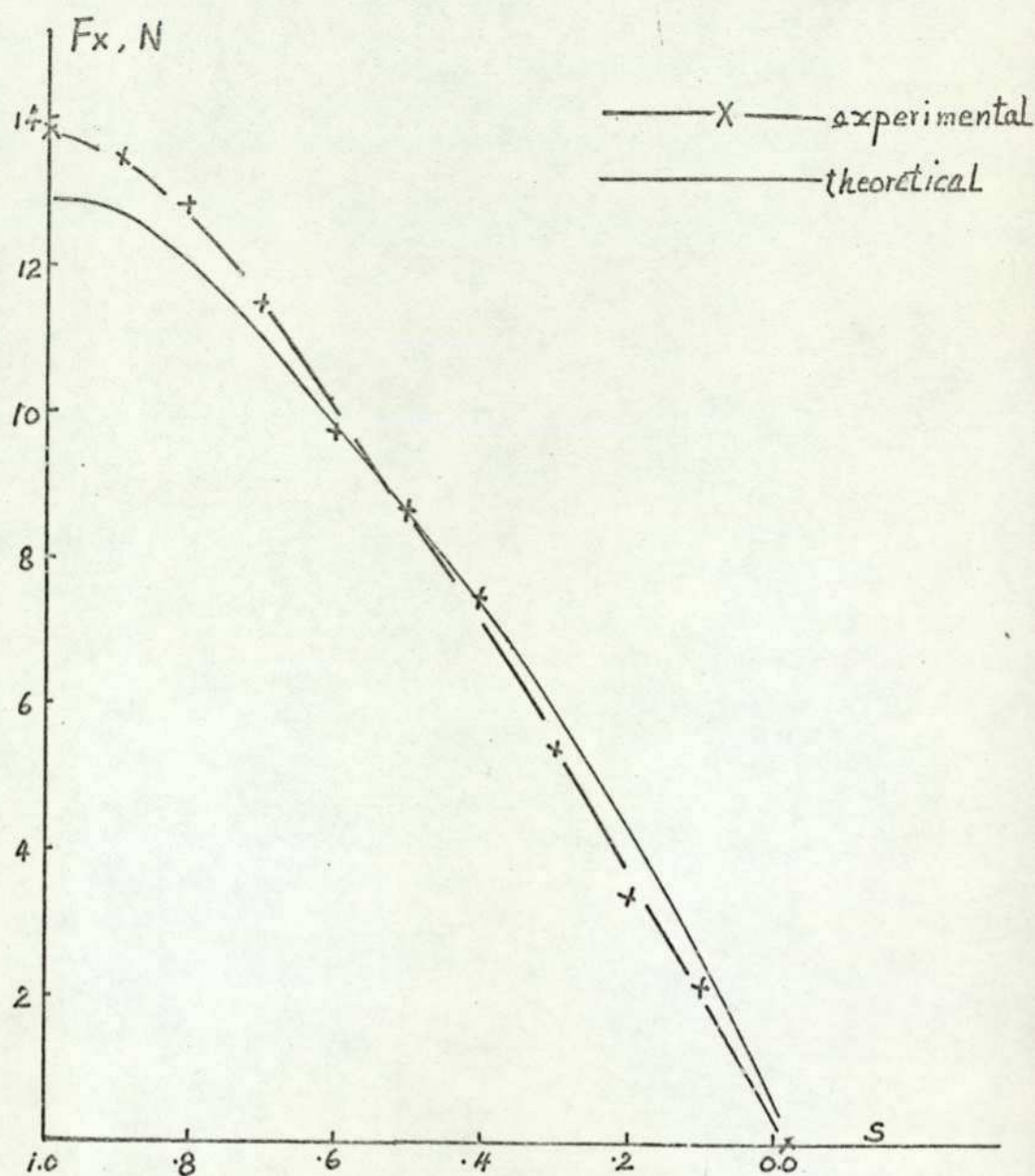


Fig. 6.3.9. F_x against slip with a 50Hz constant voltage supply and $g_0 = 1.0\text{cm}$

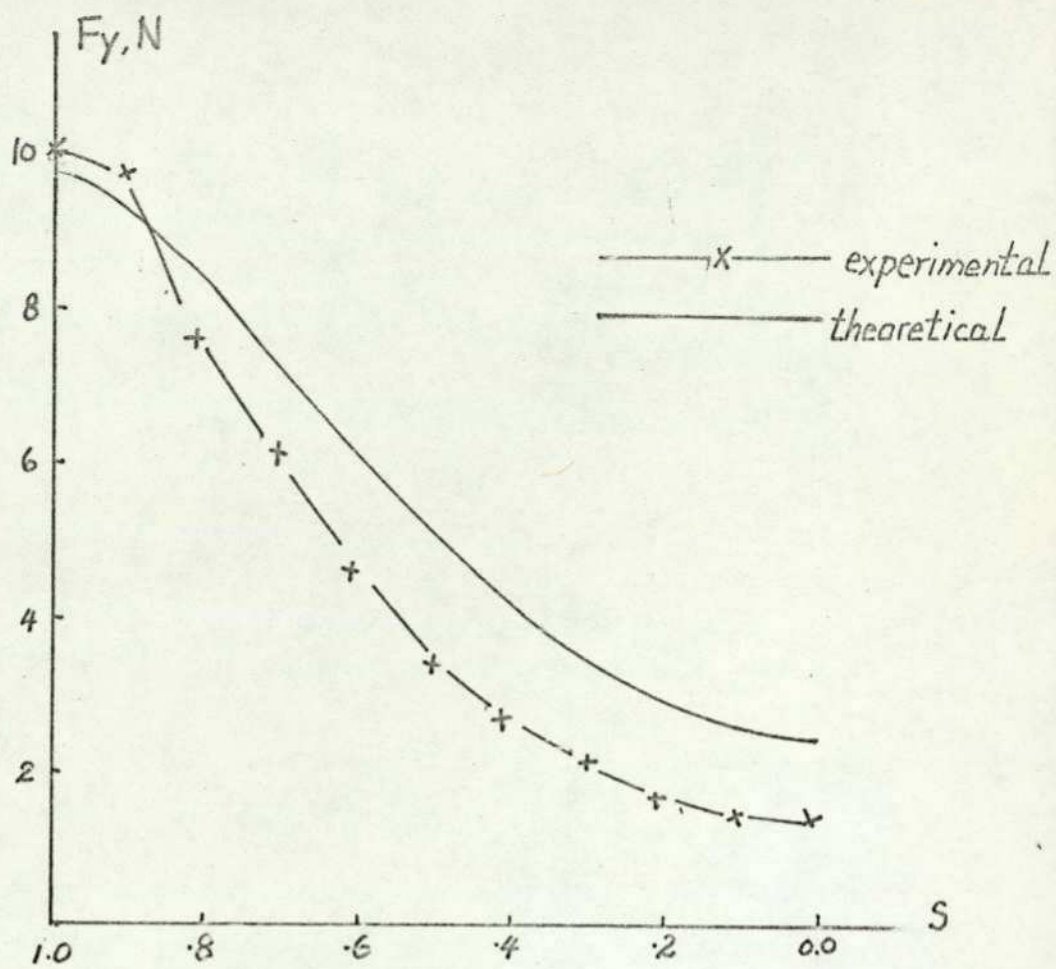


Fig 6.3.10. F_y against slip with a 50Hz constant voltage supply and $g_0 = 1.0\text{cm}$

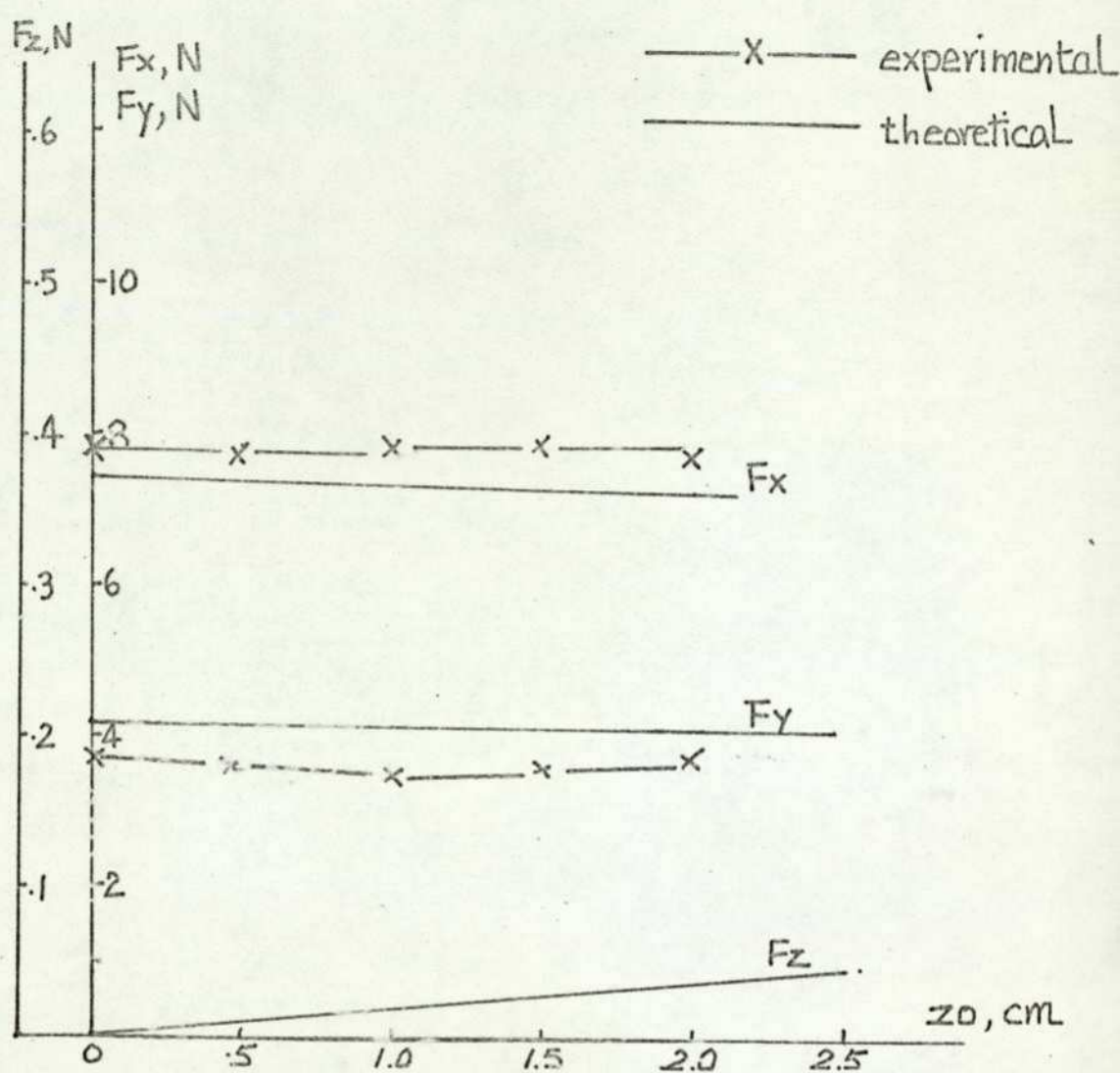


Fig 6.3.11. F_x , F_y and F_z against z_0 with a 50 Hz constant current supply and $G_0 = 1.0 \text{ cm}$, $S = 0.4$

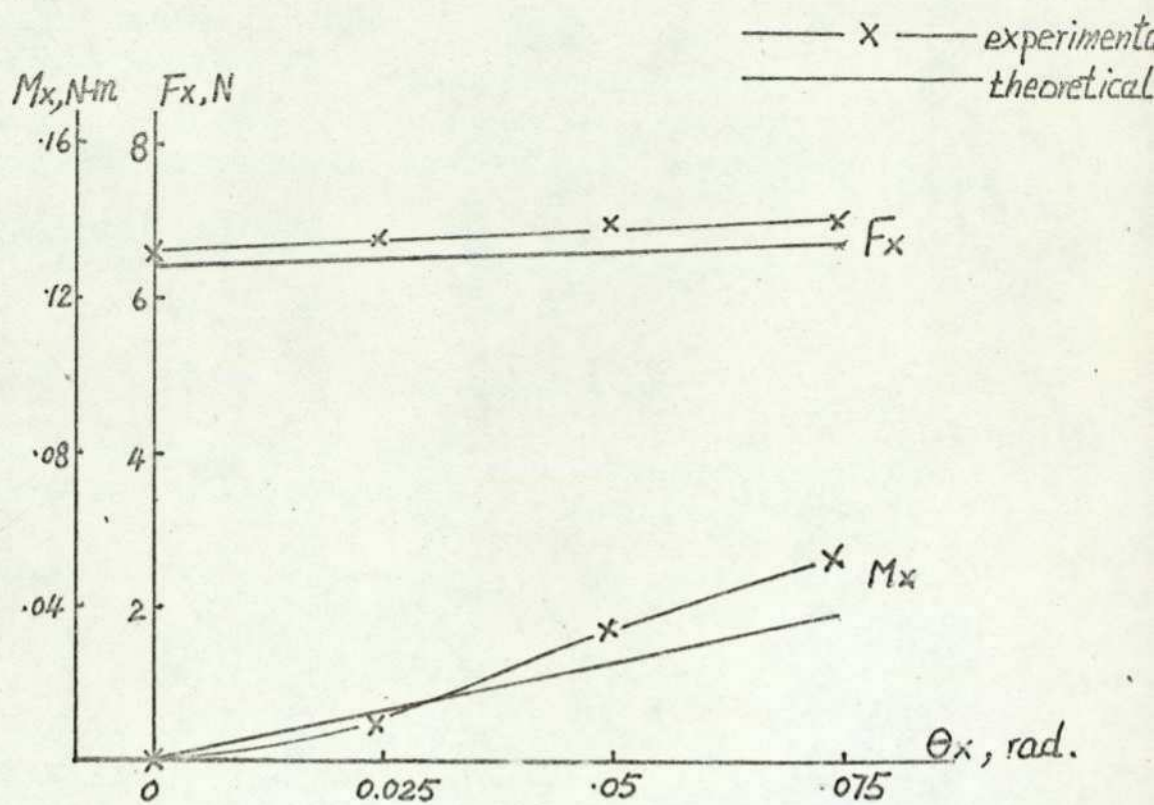


Fig. 6.3.12. F_x and M_x against θ_x with a constant current supply $G_0 = 1.2 \text{ cm}$ and $S = 0.4$

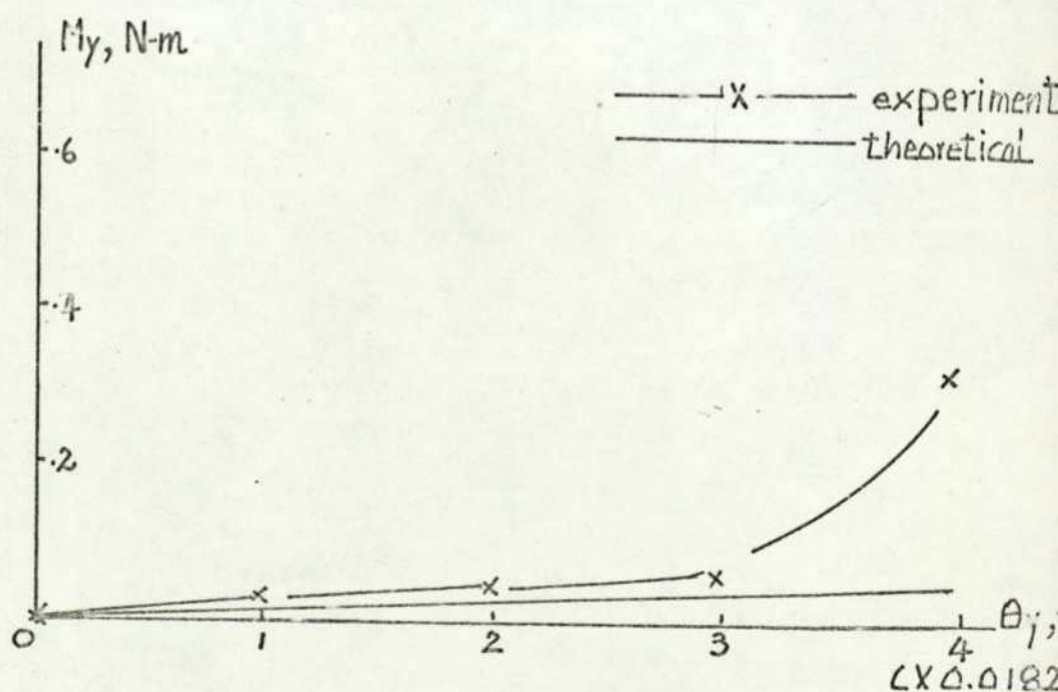


Fig. 6.3.13. M_y against θ_y with a 50Hz constant current supply. $G_0 = 1.2 \text{ cm}$ and $S = 0.4$

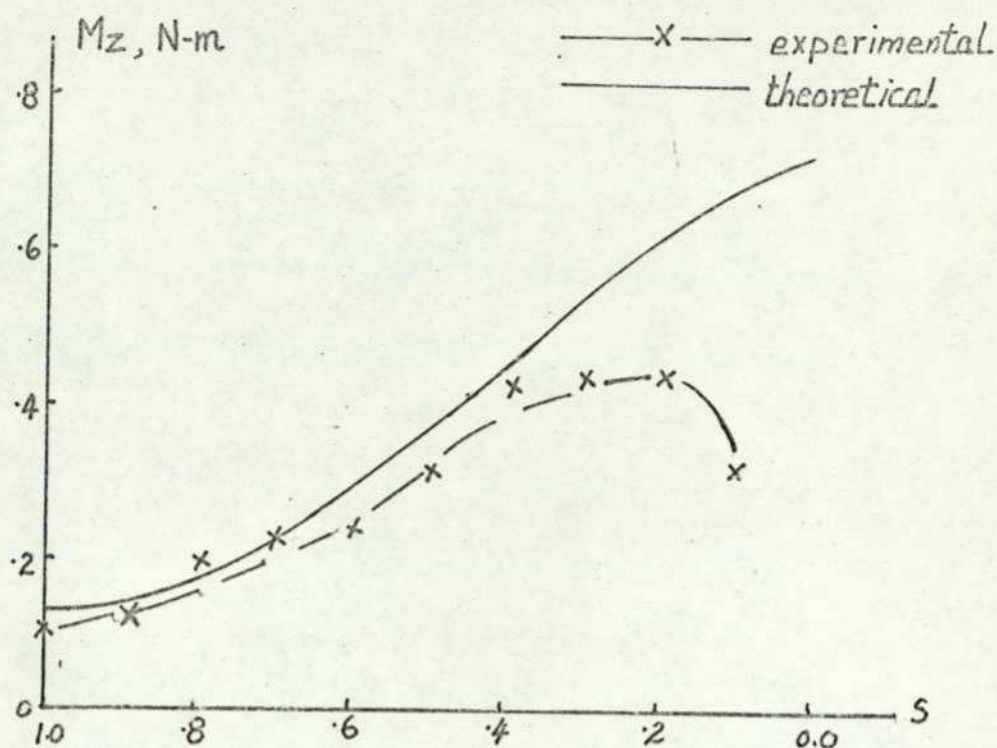


Fig. 6.3.14. M_z against slip with a 50 Hz constant current supply and $q_d = 1.0 \text{ cm}$

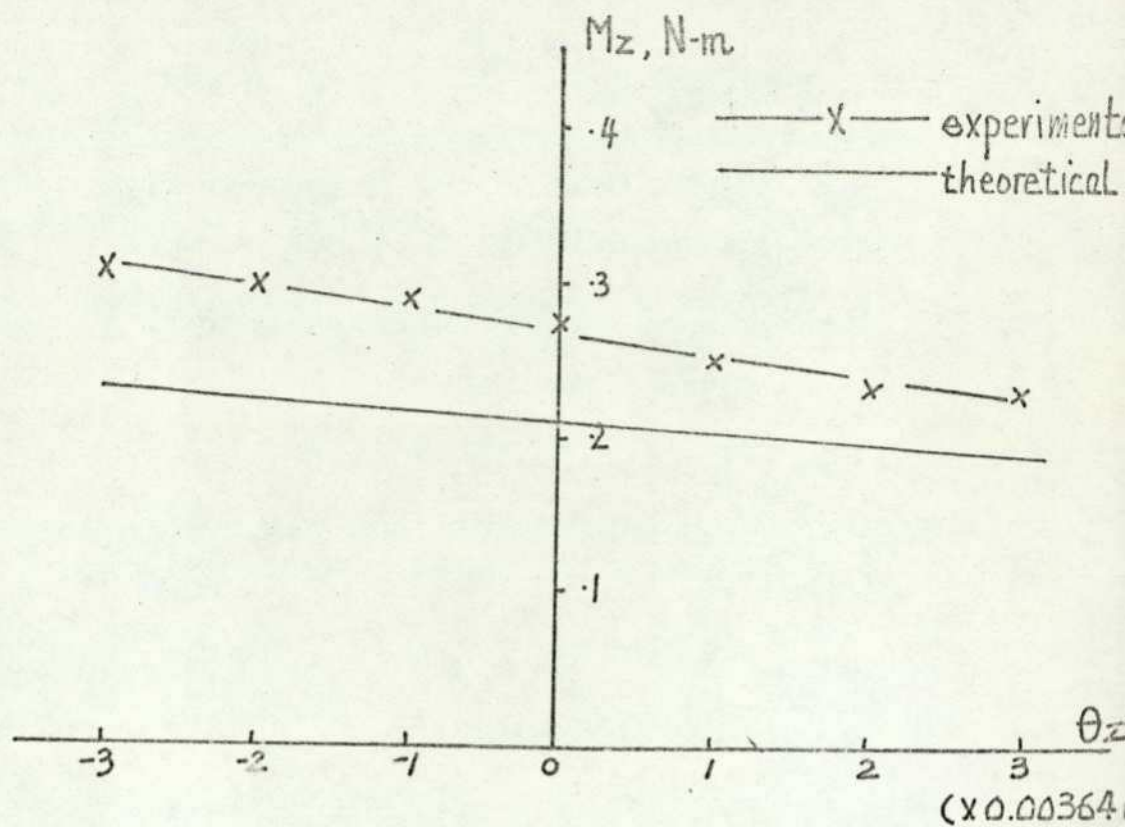


Fig. 6.3.15. M_z against θ_z with a constant current supply at 50Hz. $G_p = 1.2 \text{ cm}$ and $S = 0.4$.

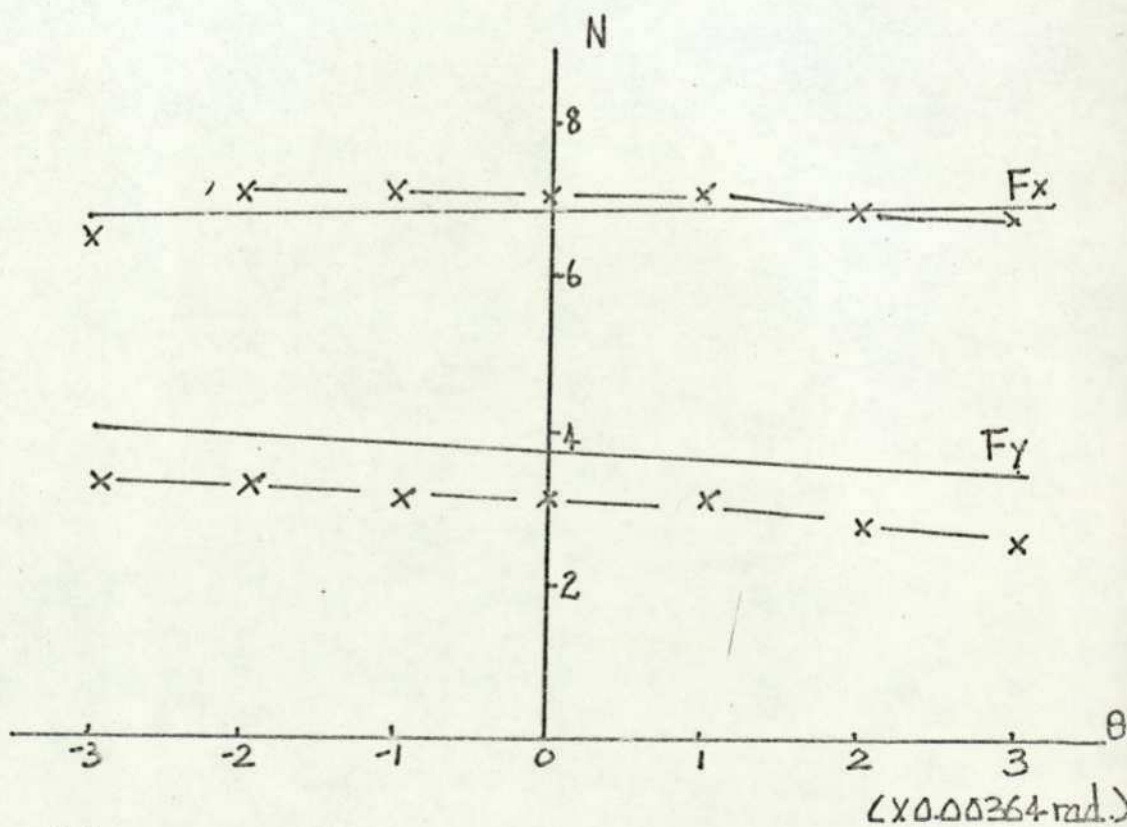


Fig. 6.3.16. F_x and F_y against θ_z with a 50 supply, $G_p = 1.2$ and $S = 0.4$.

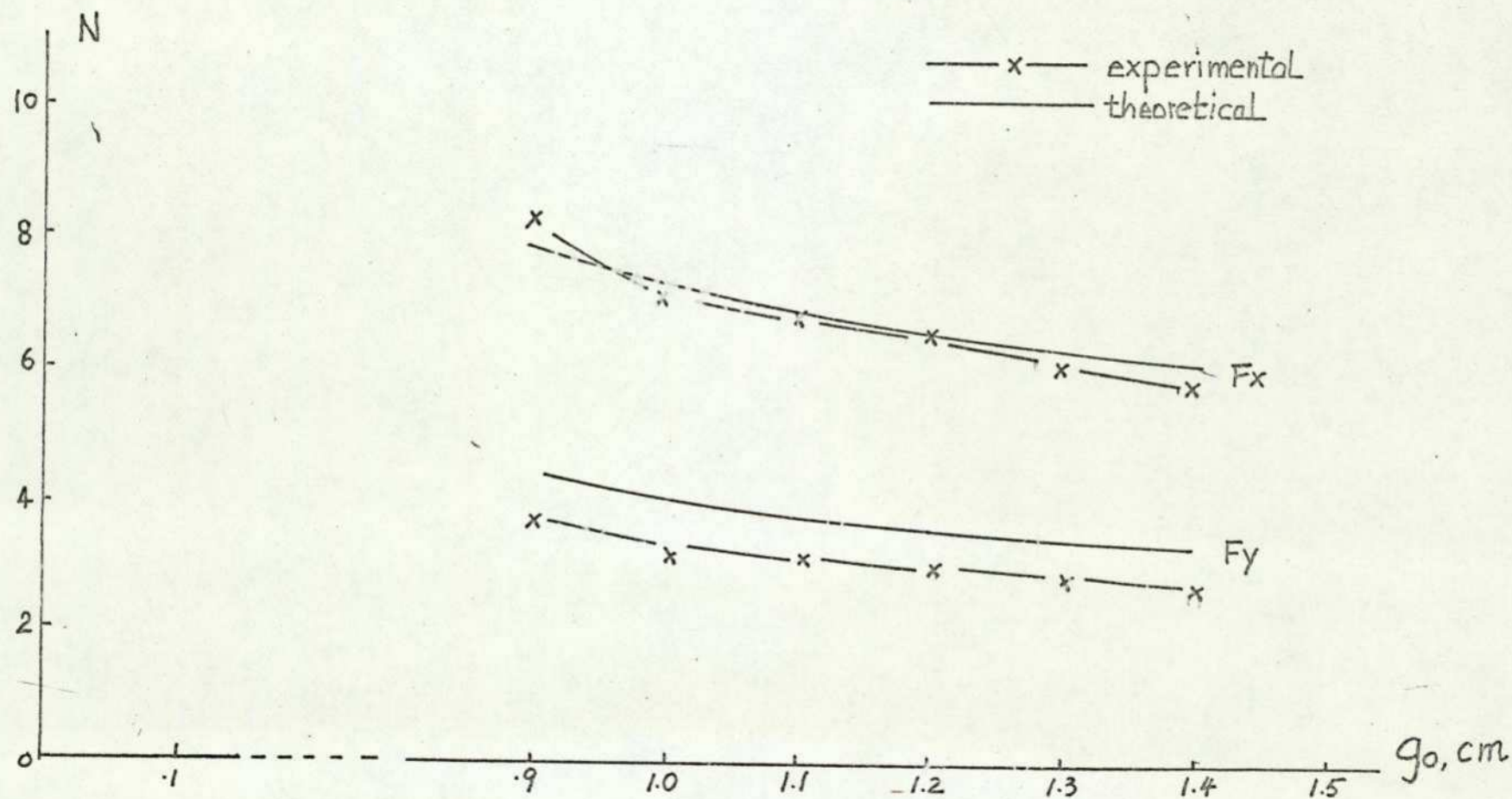


Fig 6.3.17. F_x and F_y against g_0 .. 50Hz and $S=0.4$

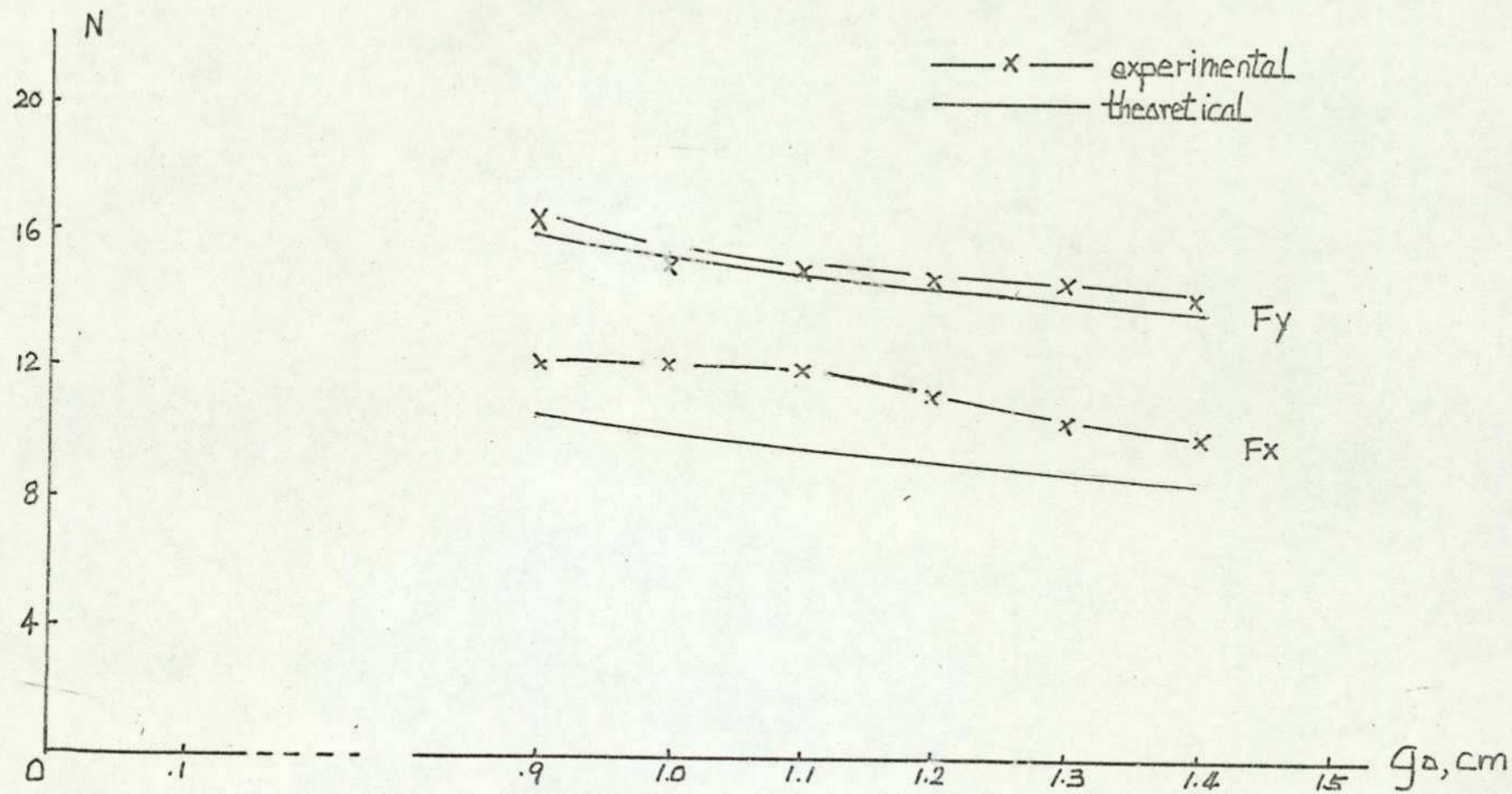


Fig. 6.3.18 F_x and F_y against Q_0 150 Hz and $S=0.7$

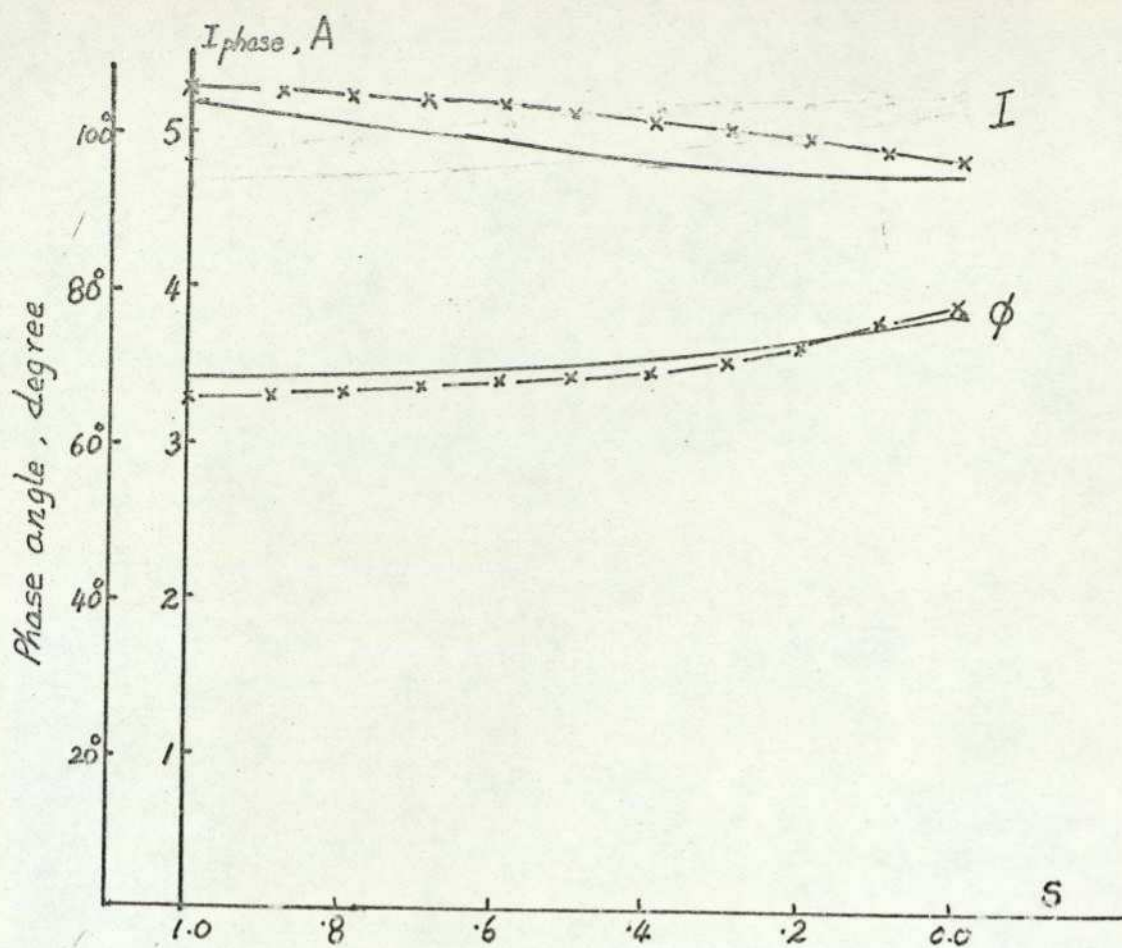


FIG. 6.3.19. Y-phase phase angle and current against slip

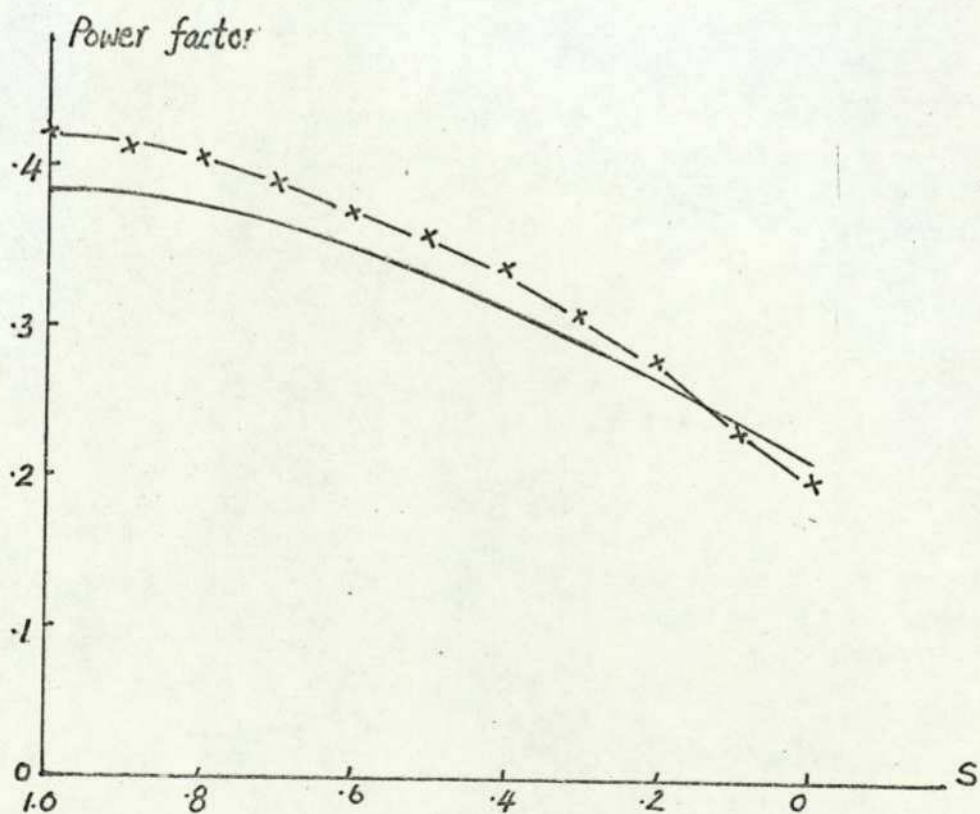


Fig 6.3.20. Power factor against slip

6.4. Flux measurements and results

The air gap flux density were measured by means of search coils. Three search coils were used and they were wound such that their axes were mutually perpendicular to each other. Fig.6.4.1. shows the arrangement of the search coils used in the dynamic test. Due to the limitation of space available between the moving conductor and the stator surface, the search coils were made thinner than those used in the static test. The perspex used to eliminate the aerodynamic drag effect was adjusted to have the same curvature as that of the drum surface. The search coils were mounted on top of the perspex, and the position in the air gap was adjusted by the pulleys, so that they could move in the x and z directions. The search coils were calibrated against a uniform known flux in the three axis directions. Figs.6.4.2. and 6.4.3. show the search coils mounting.

Fig.6.4.5. shows the flux density distribution along the z axis for a 50 Hz supply of 5.0A, $g_0 = 1.2\text{cm}$ and $s = 0.0$.

Fig.6.4.6. shows the flux density distribution along the x axis for a 50 Hz supply of 5.0A, $g_0 = 1.2\text{cm}$ and $s = 0.0$.

Fig.6.4.7. and Fig.6.4.8. show the flux density distribution along the z and the x axes for a 50 Hz supply of 5.0A, $g_0 = 1.2\text{cm}$ and $s = .7$.

Fig.6.4.9. and Fig. 6.4.10. show the flux density distribution along the z and x axes for a 150 Hz supply of 5.0A, $g_0 = 1.2\text{cm}$ and $s = 0.5$.

Fig.6.4.11. shows the flux density distribution along the z axis for a 150 Hz supply of 5.0A, $g_0 = 1.2\text{cm}$, $s = 0.5$ and $z_0 = 2\text{cm}$.

Fig.6.4.12. shows the flux density distribution along the z axis for a 150 Hz supply of 5.0A, $g_0 = 1.2\text{cm}$, $s = 0.5$ and $\theta_x = .075$ rad.

Fig. 6.4.13. shows the flux density distribution along the x axis for a 150 Hz supply of 5.0A, $g_0 = 1.2\text{cm}$, $s = 0.5$ and $\theta_z = .0112$ rad.

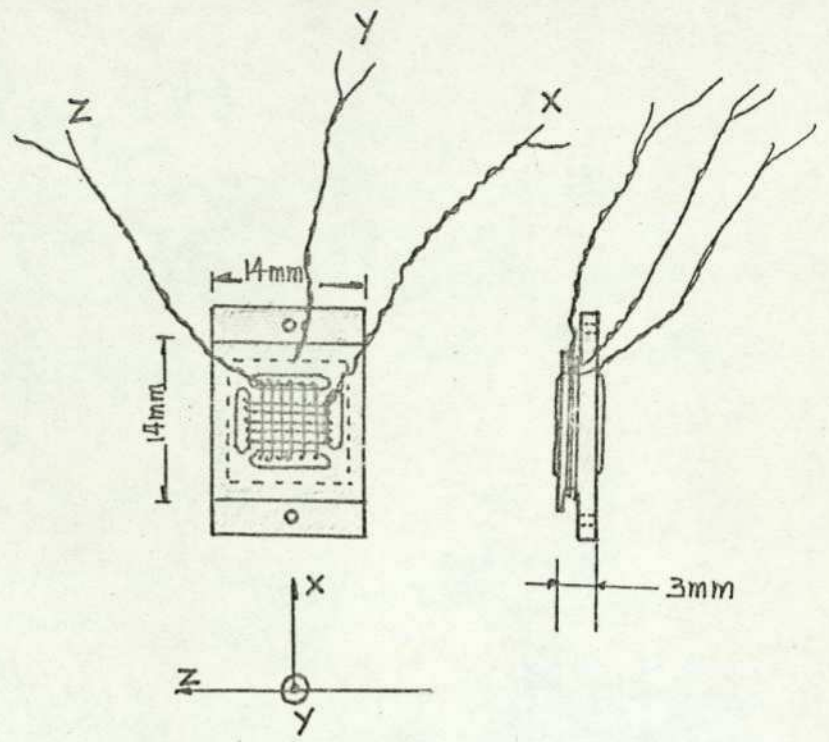


Fig. 6.4.1. Search coils arrangement

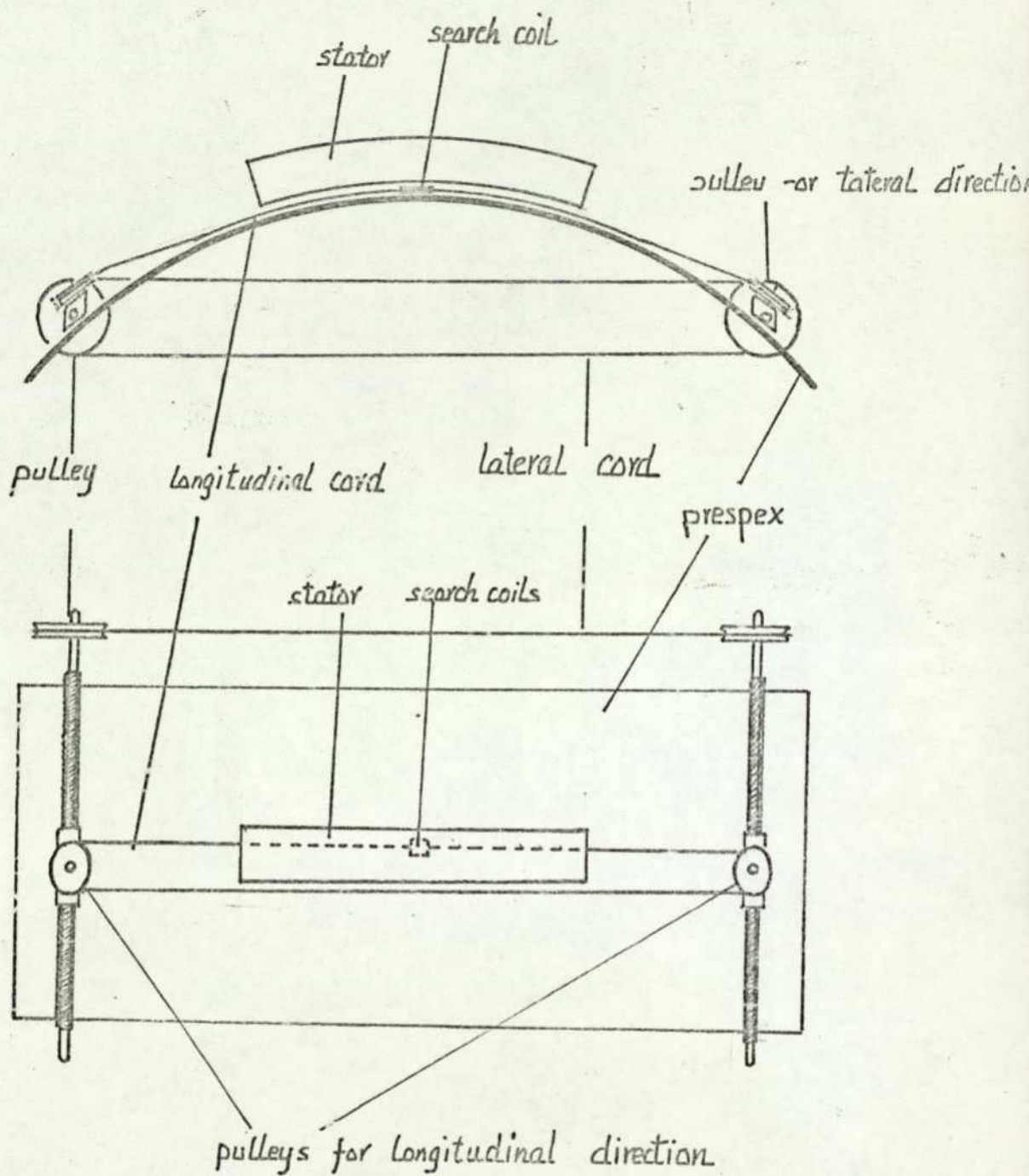


Fig. 6.4.2. Arrangement for flux measurement



Fig. 6.4.3. Search coil mounting

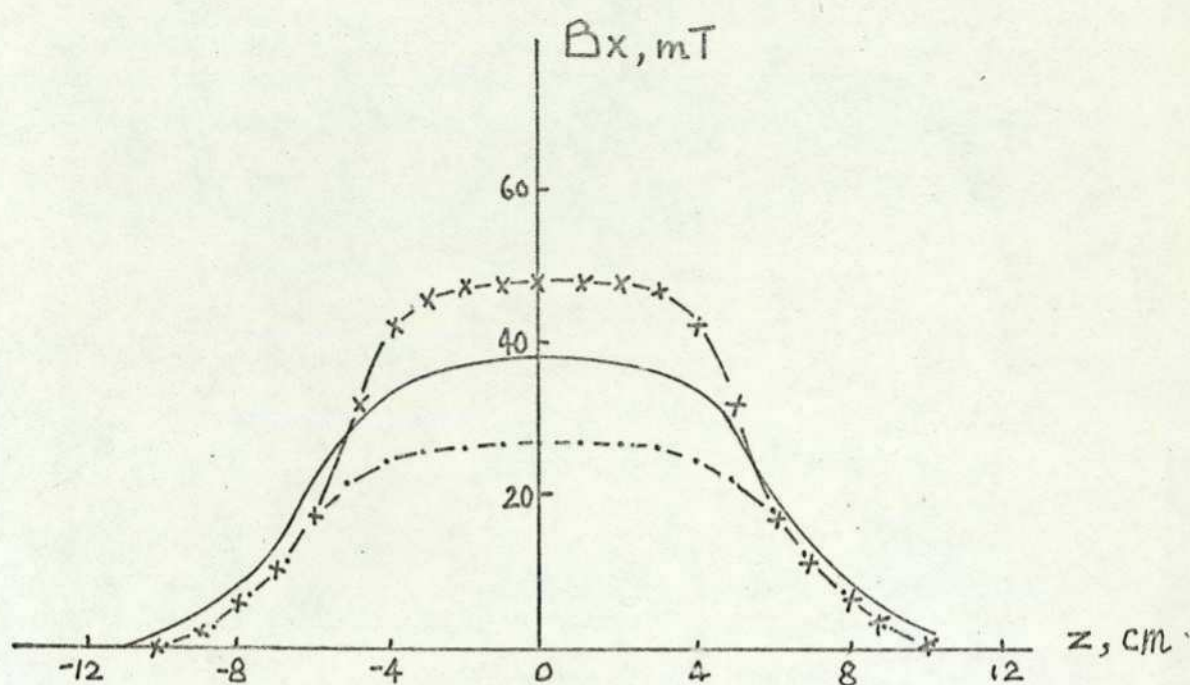


Fig.6.4.5.(a) X component of flux density

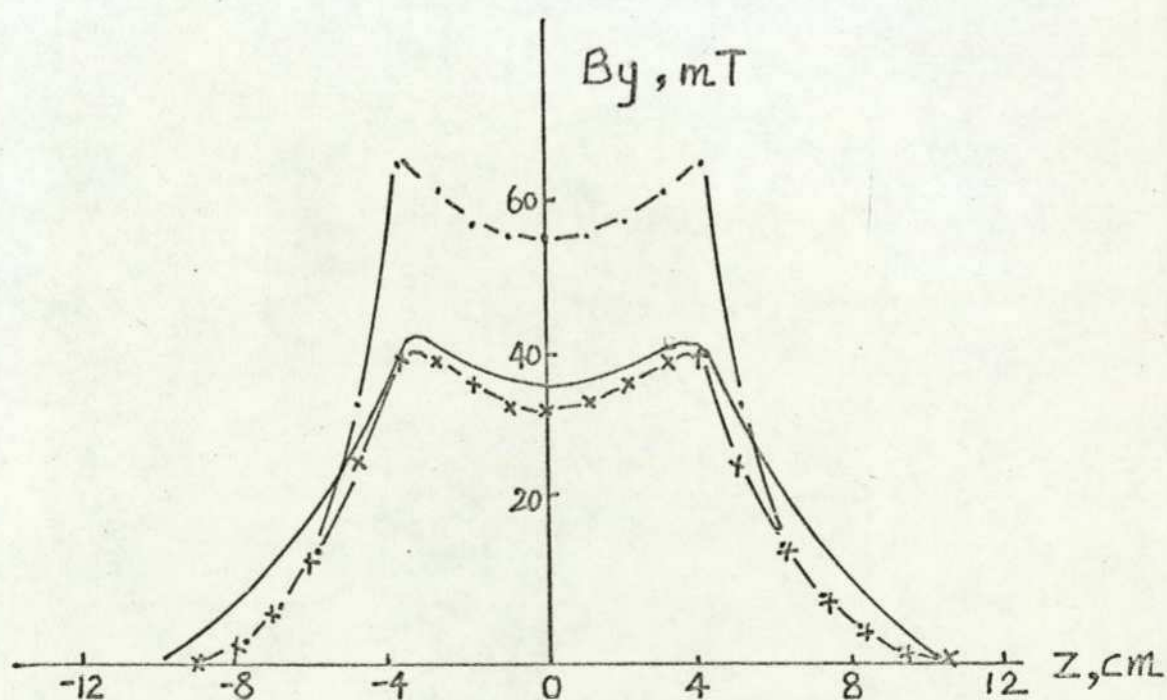


Fig.6.4.5.(b) Y component of flux density

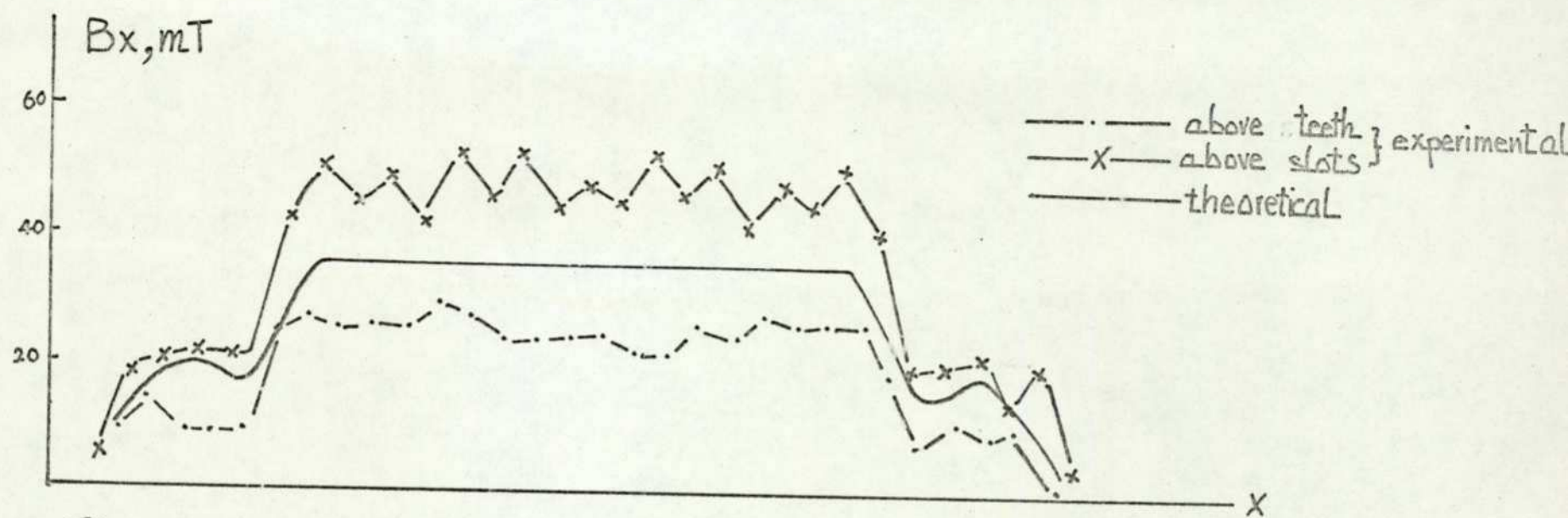
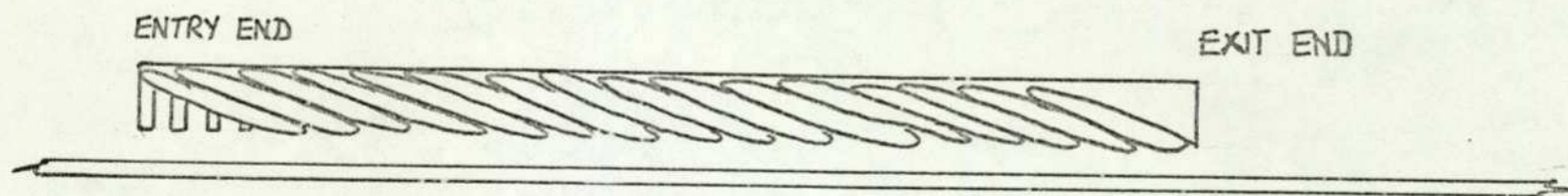
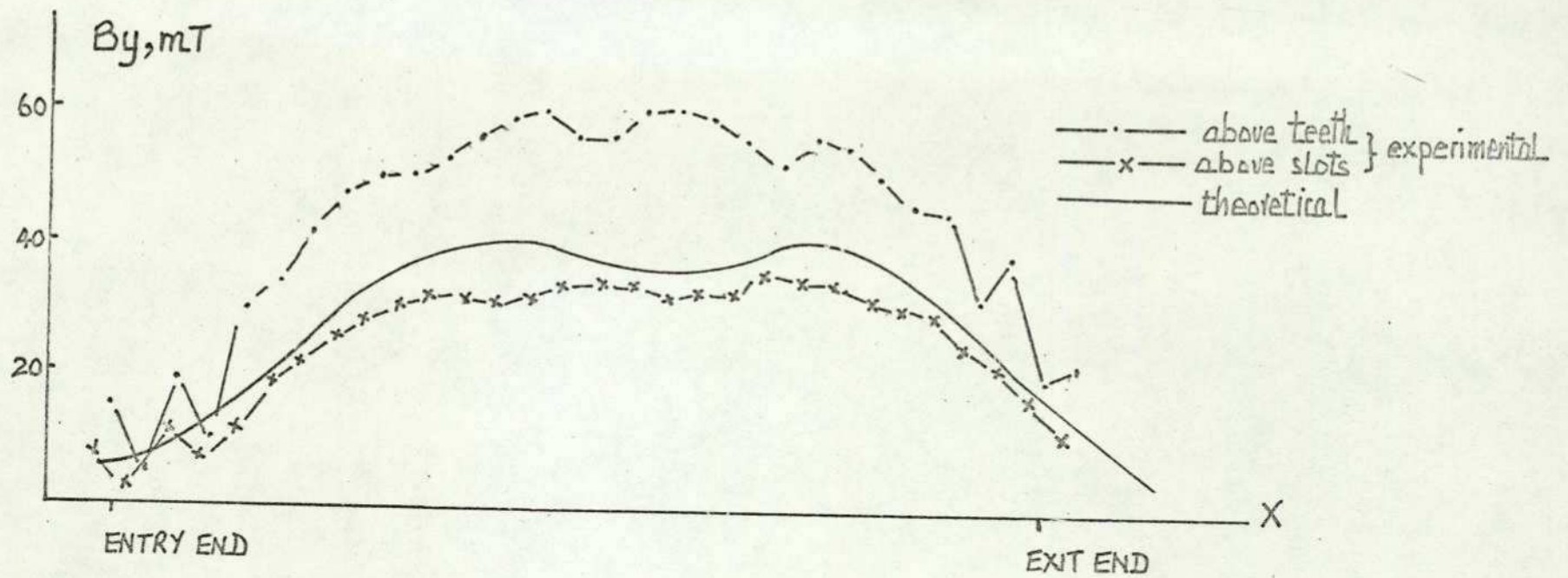


Fig. 6.4.6.(a) X component of flux density





(b) Y component of flux density

Fig. 6.4.6. Flux density distribution along the X axis
50 Hz and $S = 0.0$.

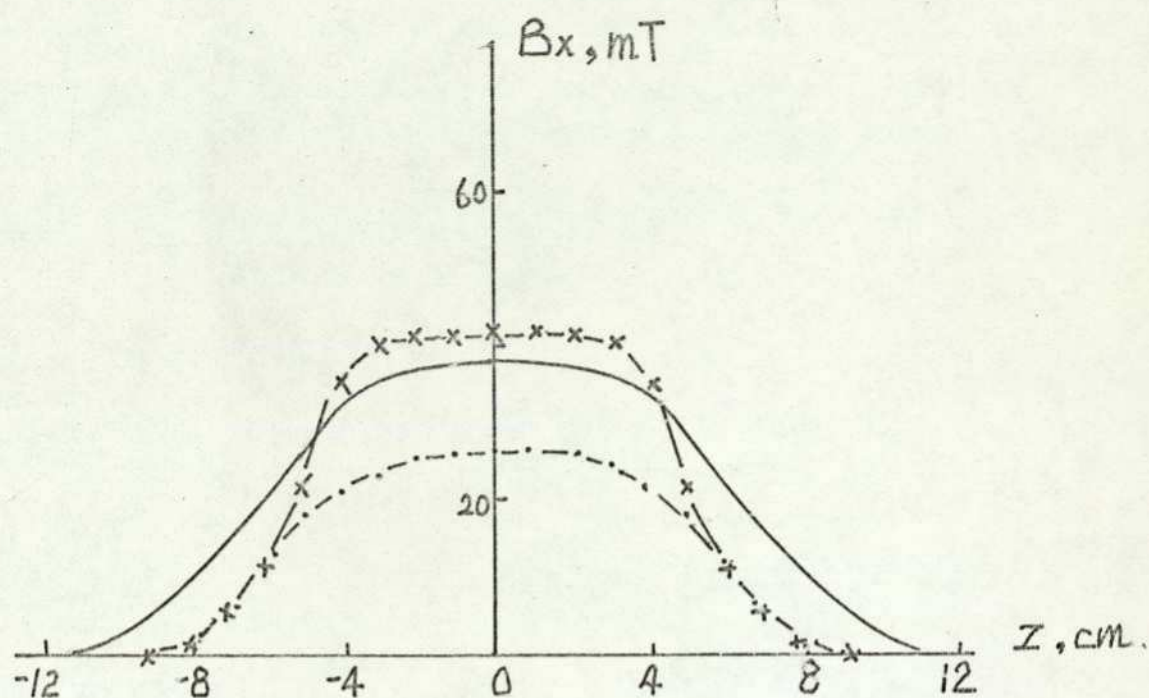


Fig. 6.4.7.(a) x component of flux density

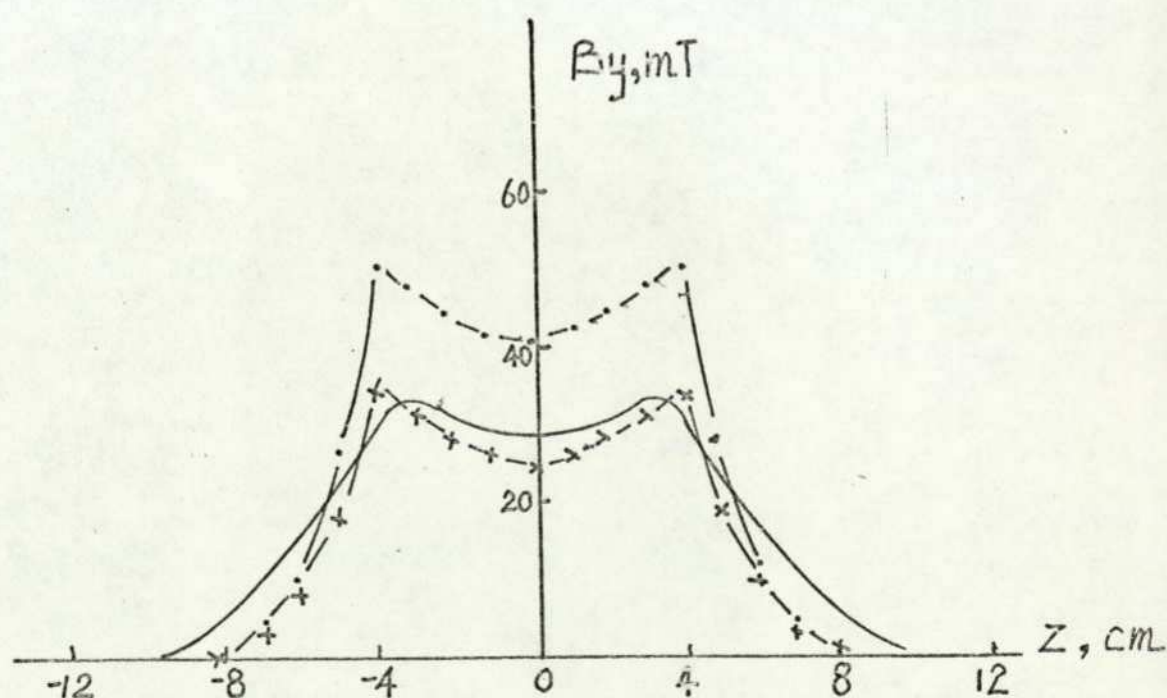


Fig. 6.4.7.(b) y component of flux density

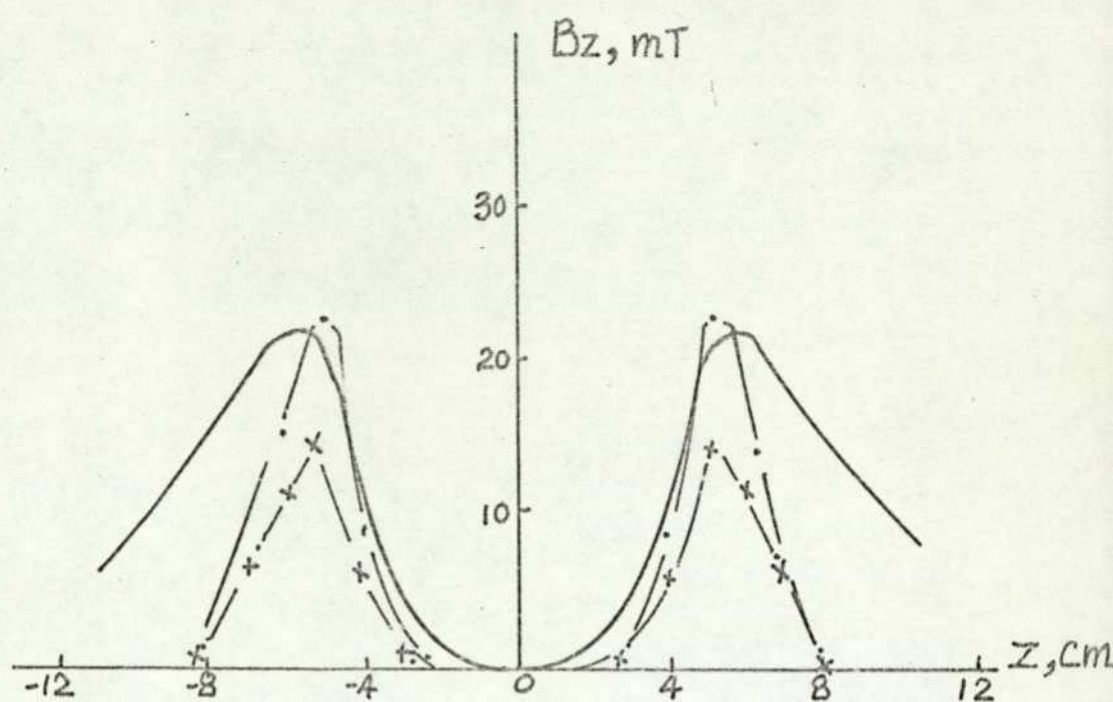


Fig. 6.4.7.(c) Z component of flux density

— · — — above teeth } experimental
 — x — — above slots }
 - - - - - theoretical

Fig. 6.4.7. Flux density distribution along the Z-axis 50Hz $S=0.7$

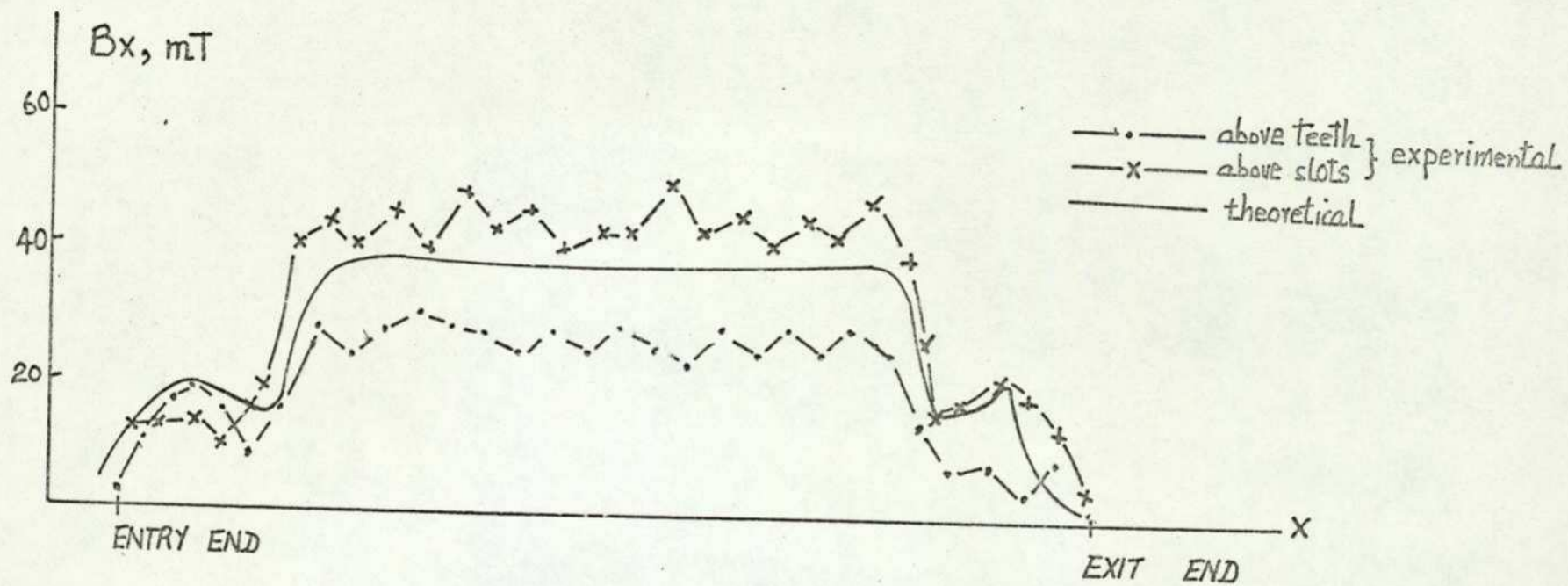
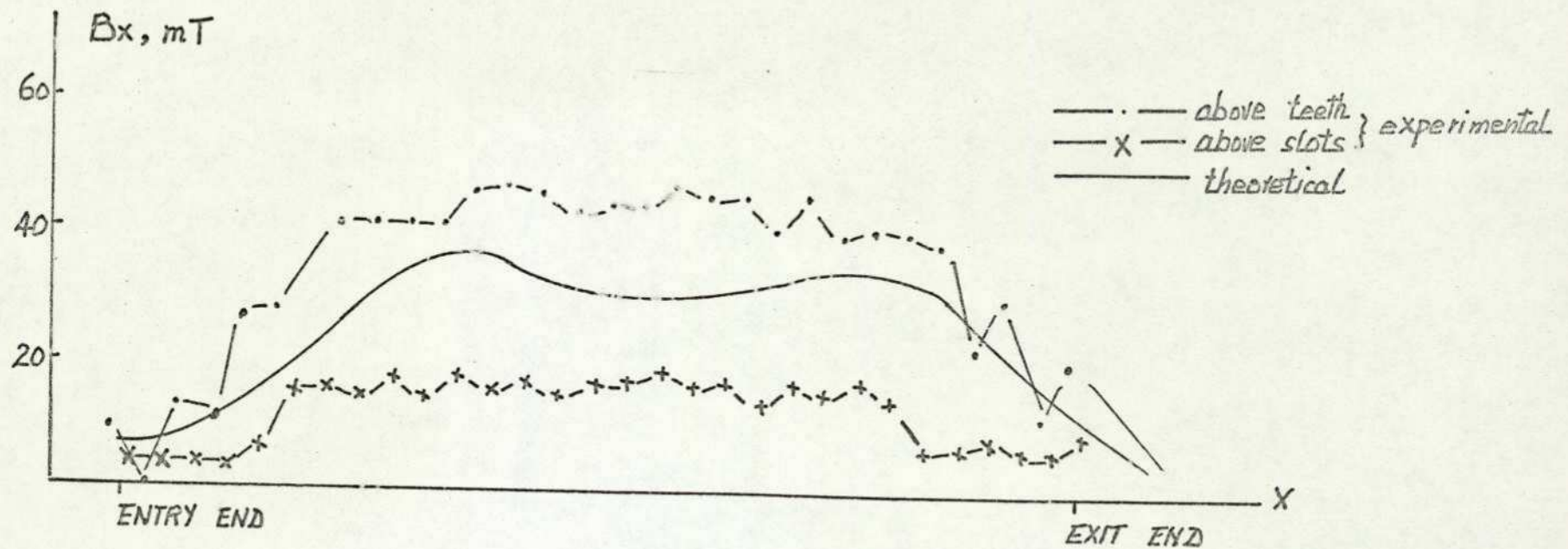


Fig. 6.4.8.(a) x component of flux density



(b) γ component of flux density

Fig. 6.4.8. Flux density distribution along the X axis
50 Hz. $S=0.7$.

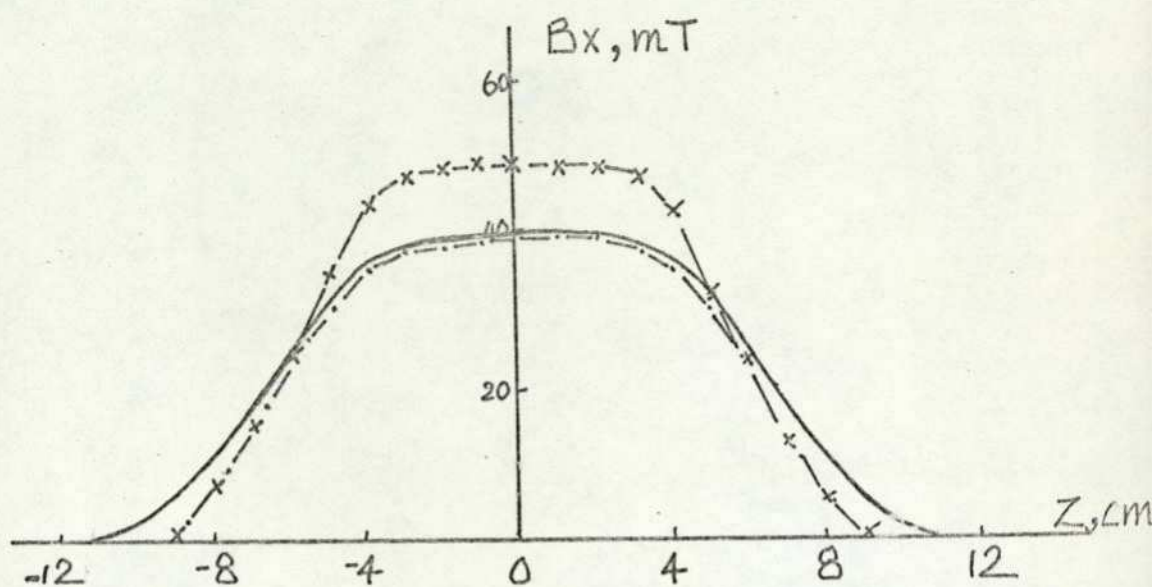


Fig.6.4.9.(a) x - component of flux density

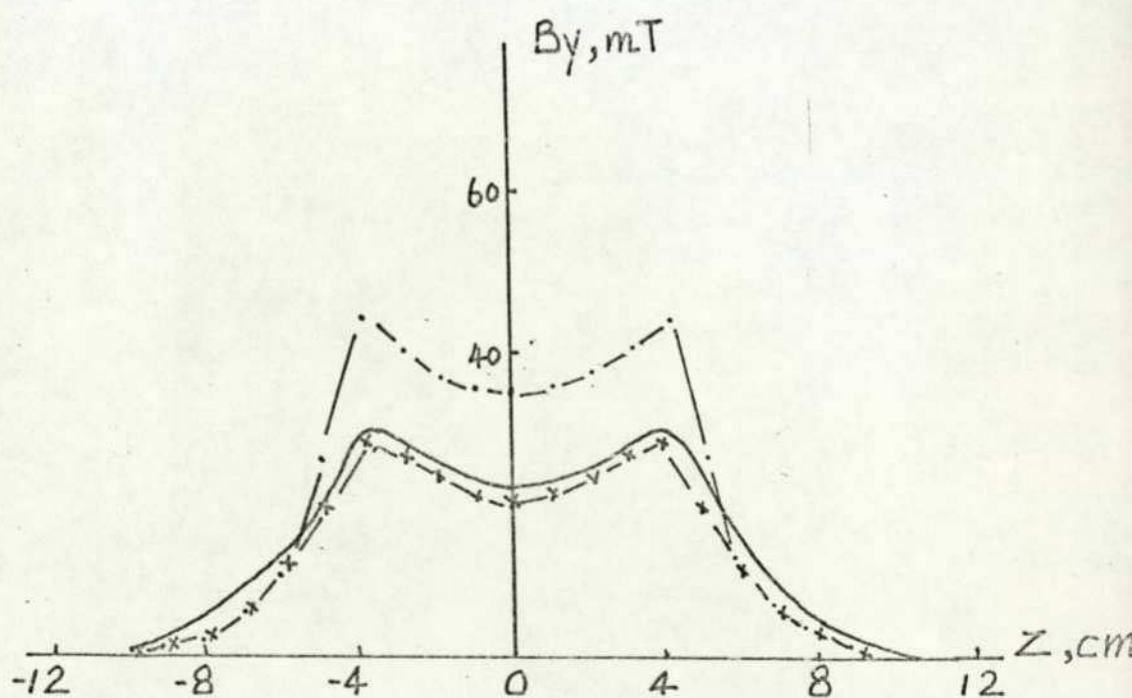


Fig.6.4.9.(b) y - component of flux density

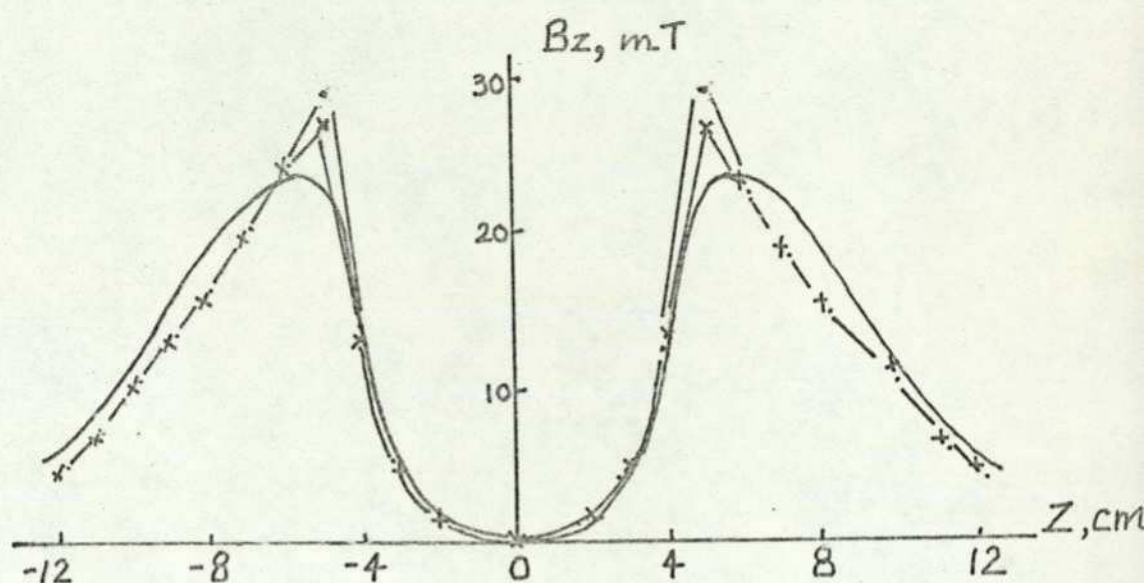


Fig. 6.4.9.(c) Z - component of flux density

—•— above teeth } experimental
 —x— above slots }
 — theoretical

Fig. 6.4.9. Flux density distribution along the Z axis 150 Hz $S = 0.5$

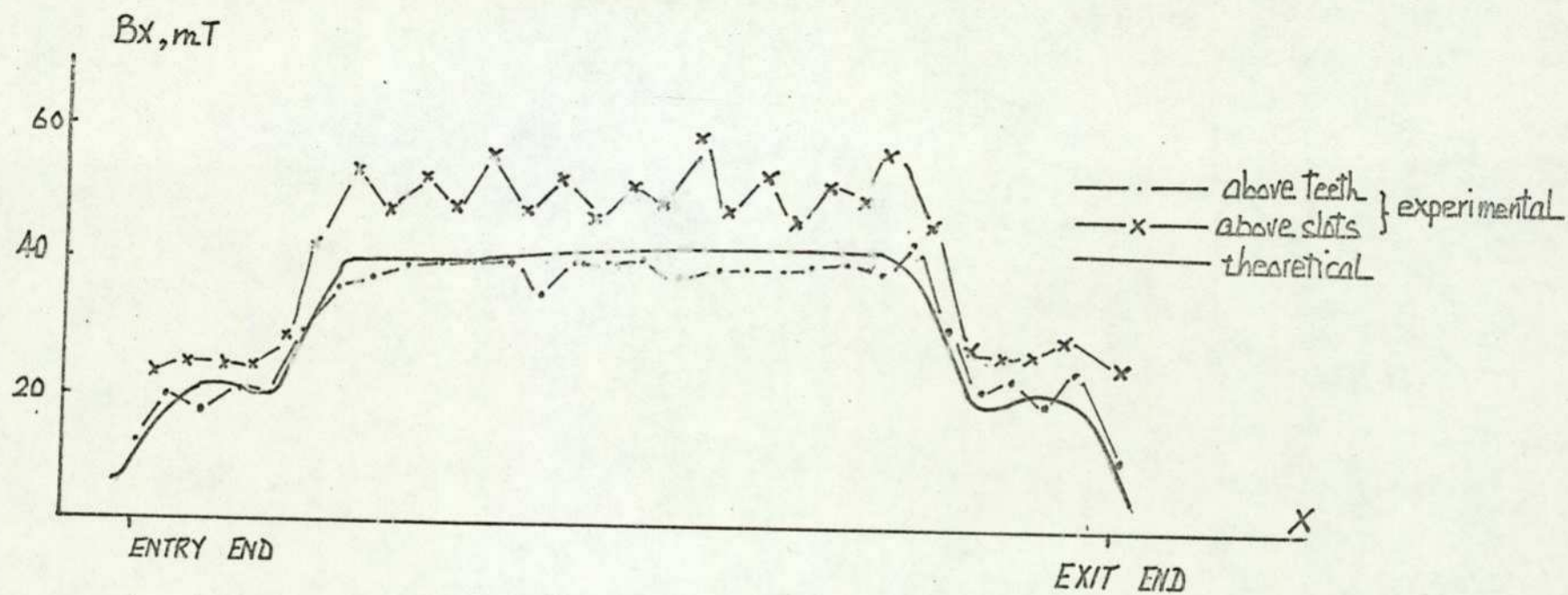


Fig. 6.4.10.(a) X component of flux density

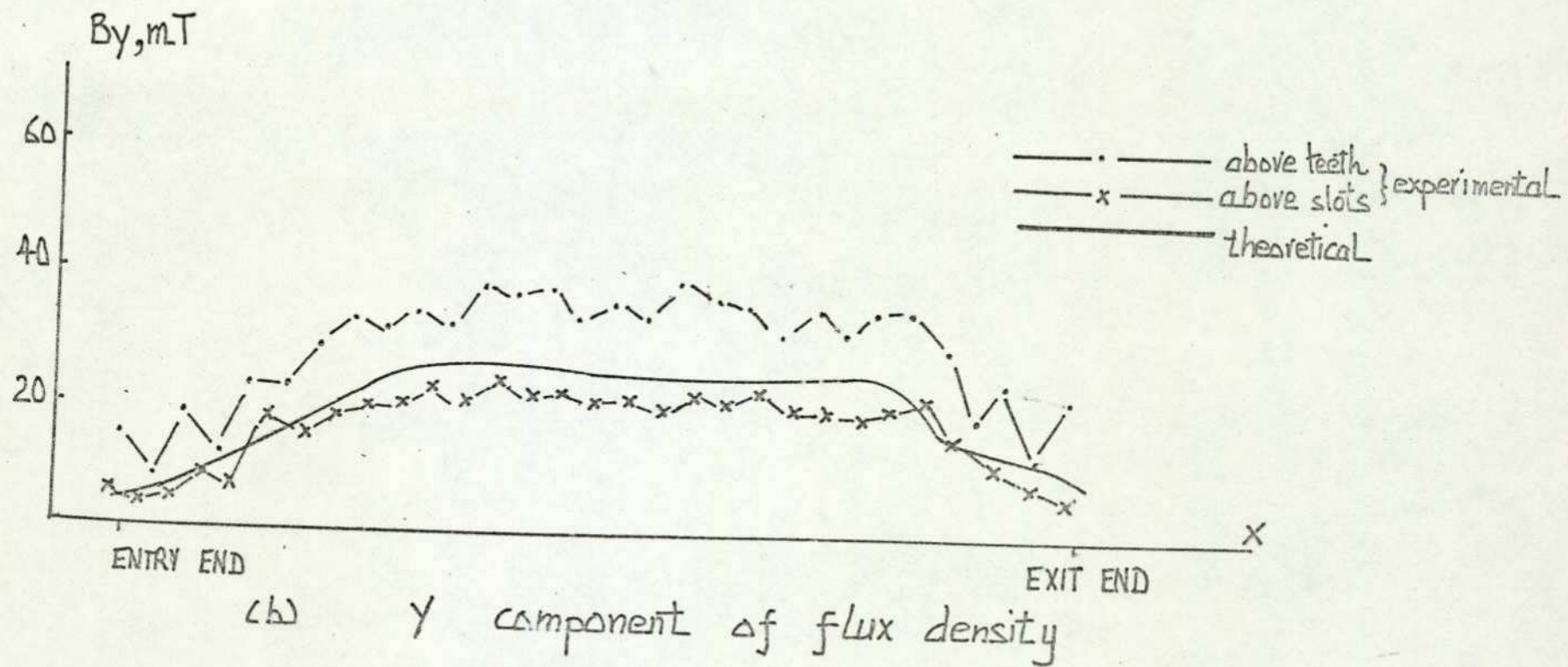


Fig. 6.4.10. Flux density distribution along the x axis
150Hz $S=0.5$

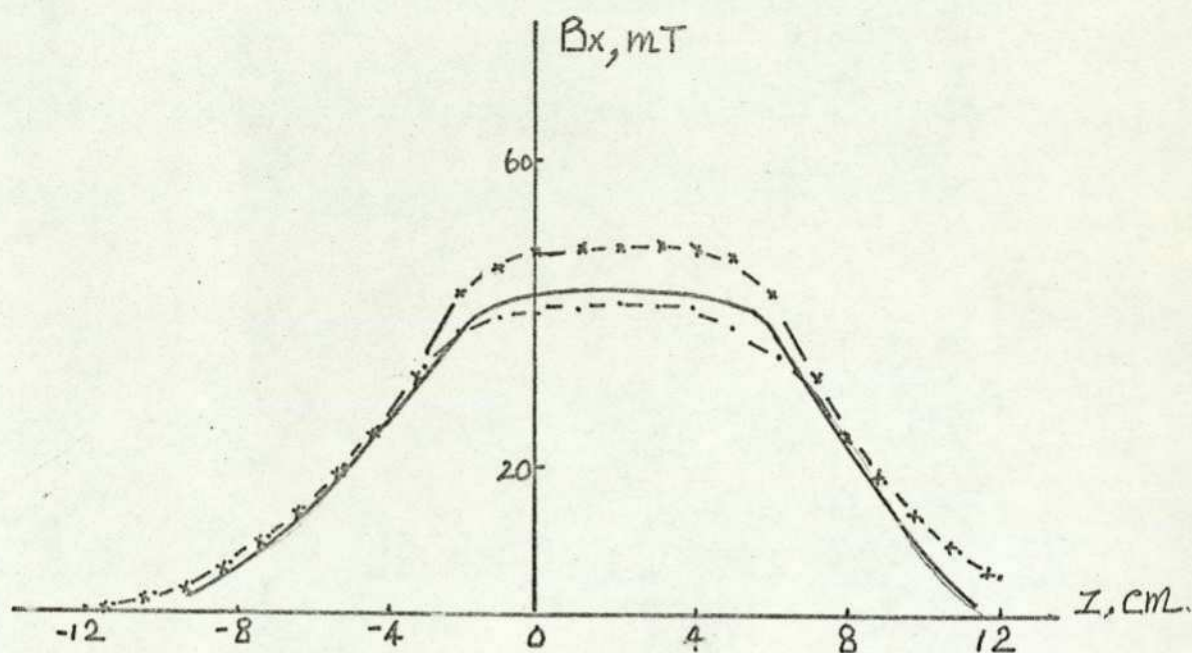


Fig.6.4.11.(a) x component of flux density

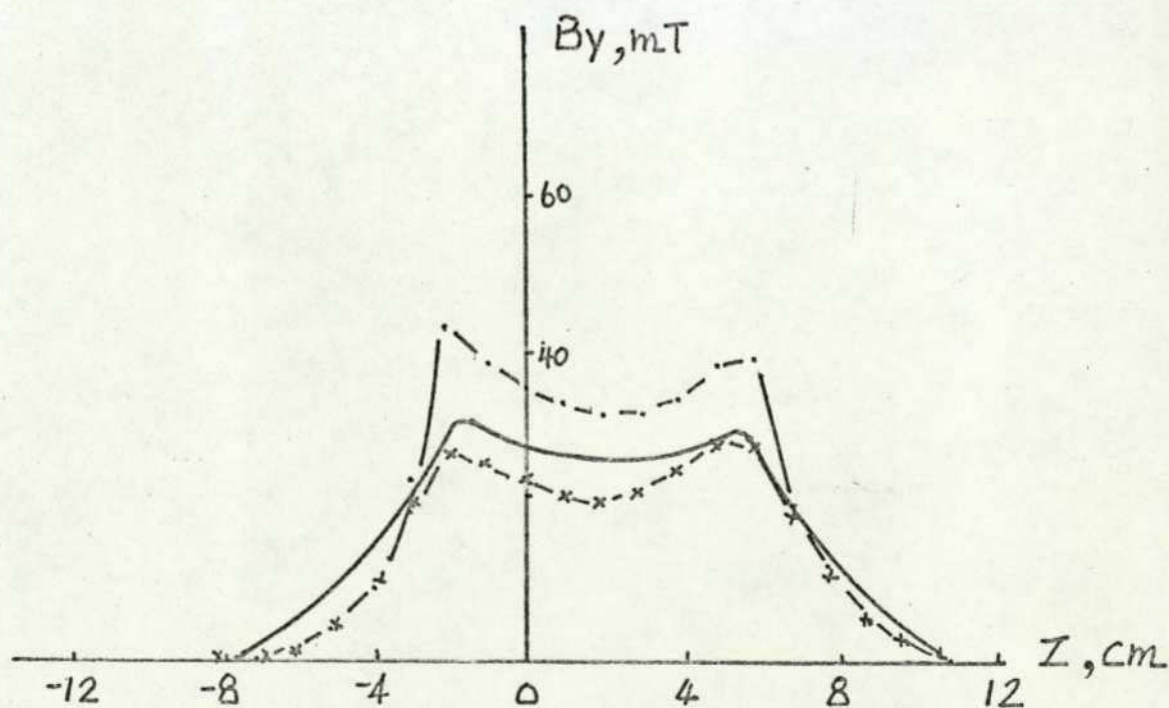


Fig.6.4.11.(b) y component of flux density

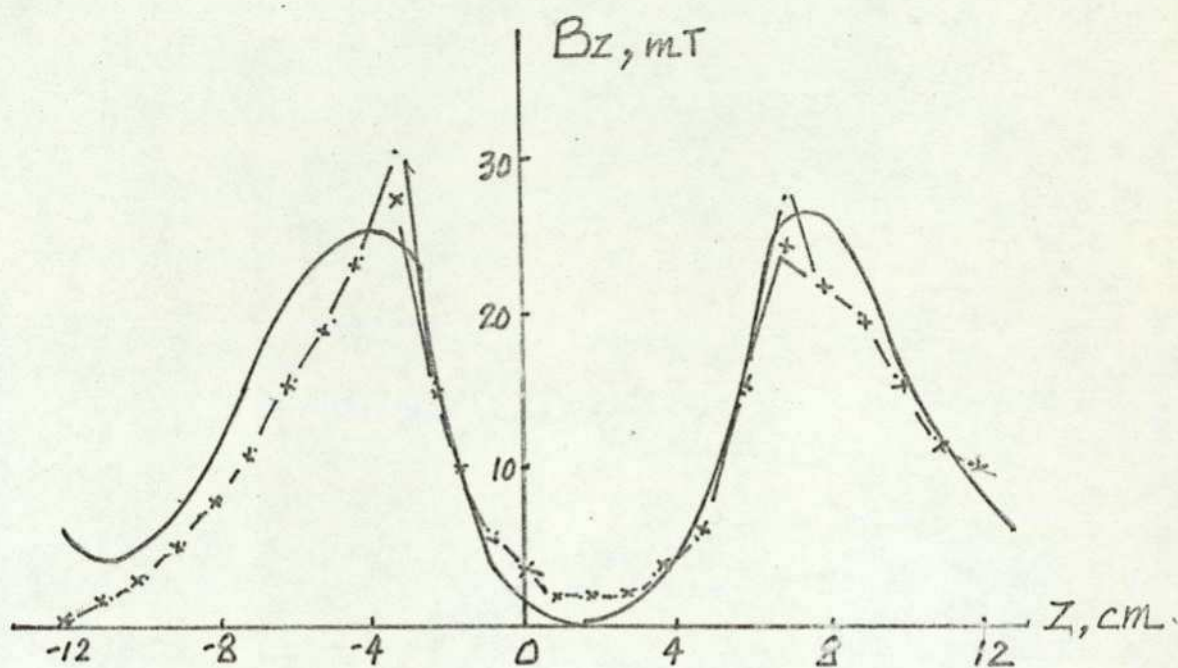


Fig. 6.4.11.(c) Z component of flux density

— x — above slots } experimental
 — x — above teeth }
 — theoretical

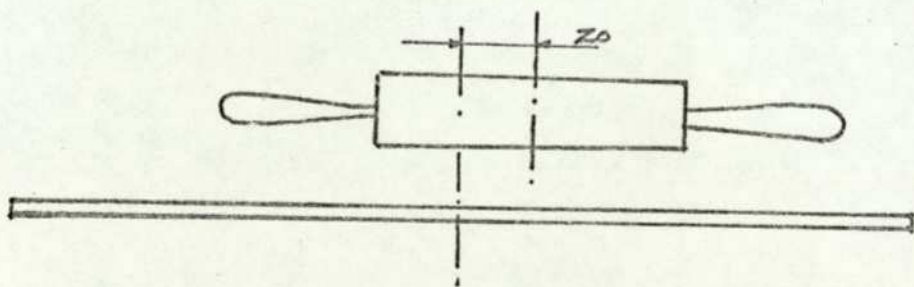


Fig. 6.4.11. Flux density distribution along the Z axis. 150Hz $Z_0 = 2\text{cm}$
 $S = 0.5$

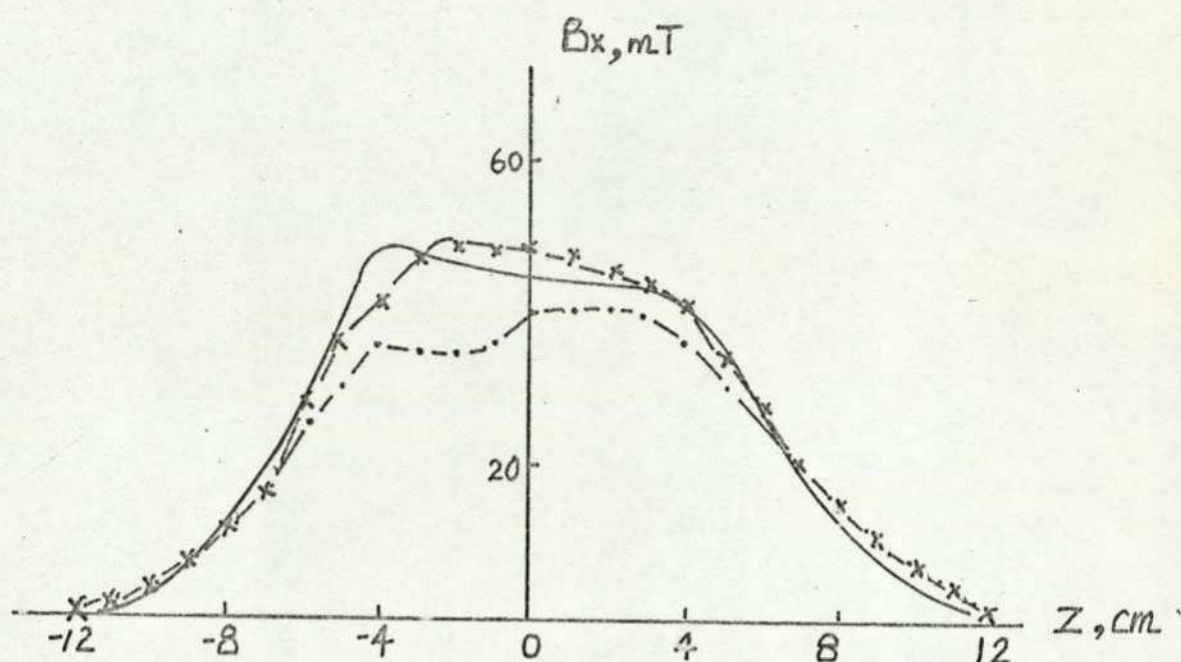


Fig. 6.4.12(a) x component of flux density

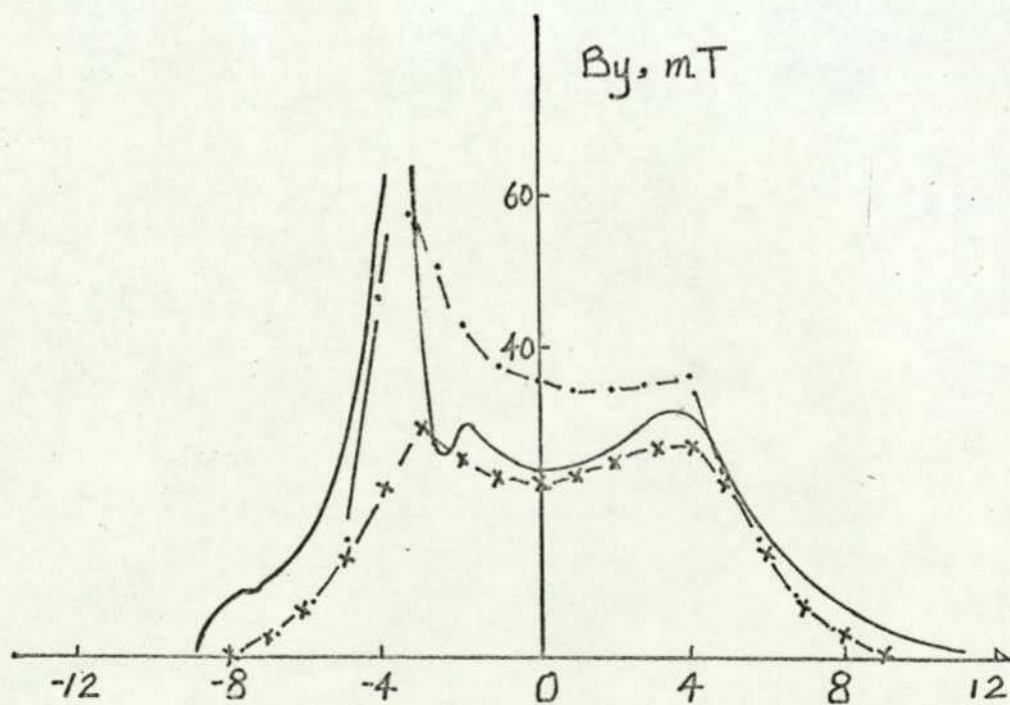


Fig. 6.4.12(b) y component of flux density

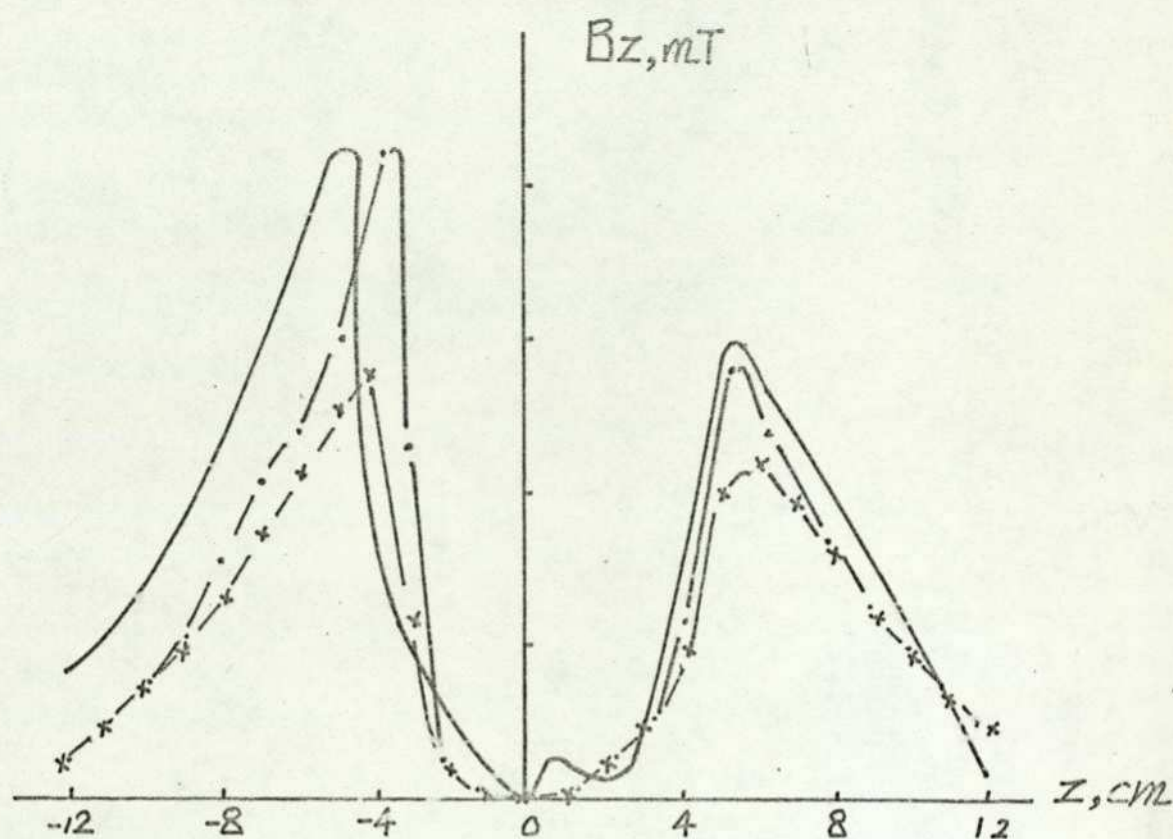


Fig. 6.4.12.(c) Z component of flux density

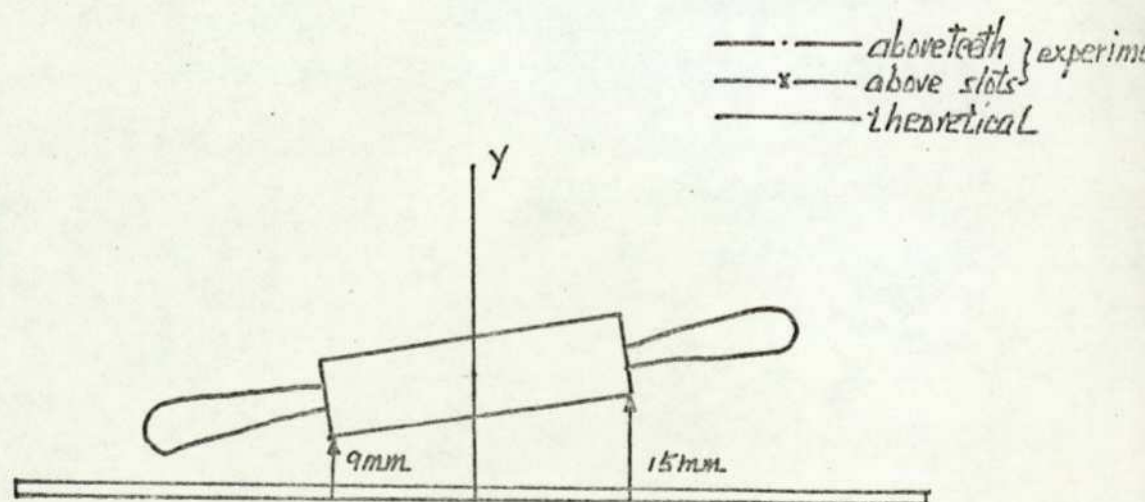


Fig. 6.4.12. Flux density distribution along the Z axis 150Hz. $S=0.5$ $A_x=0.075m$

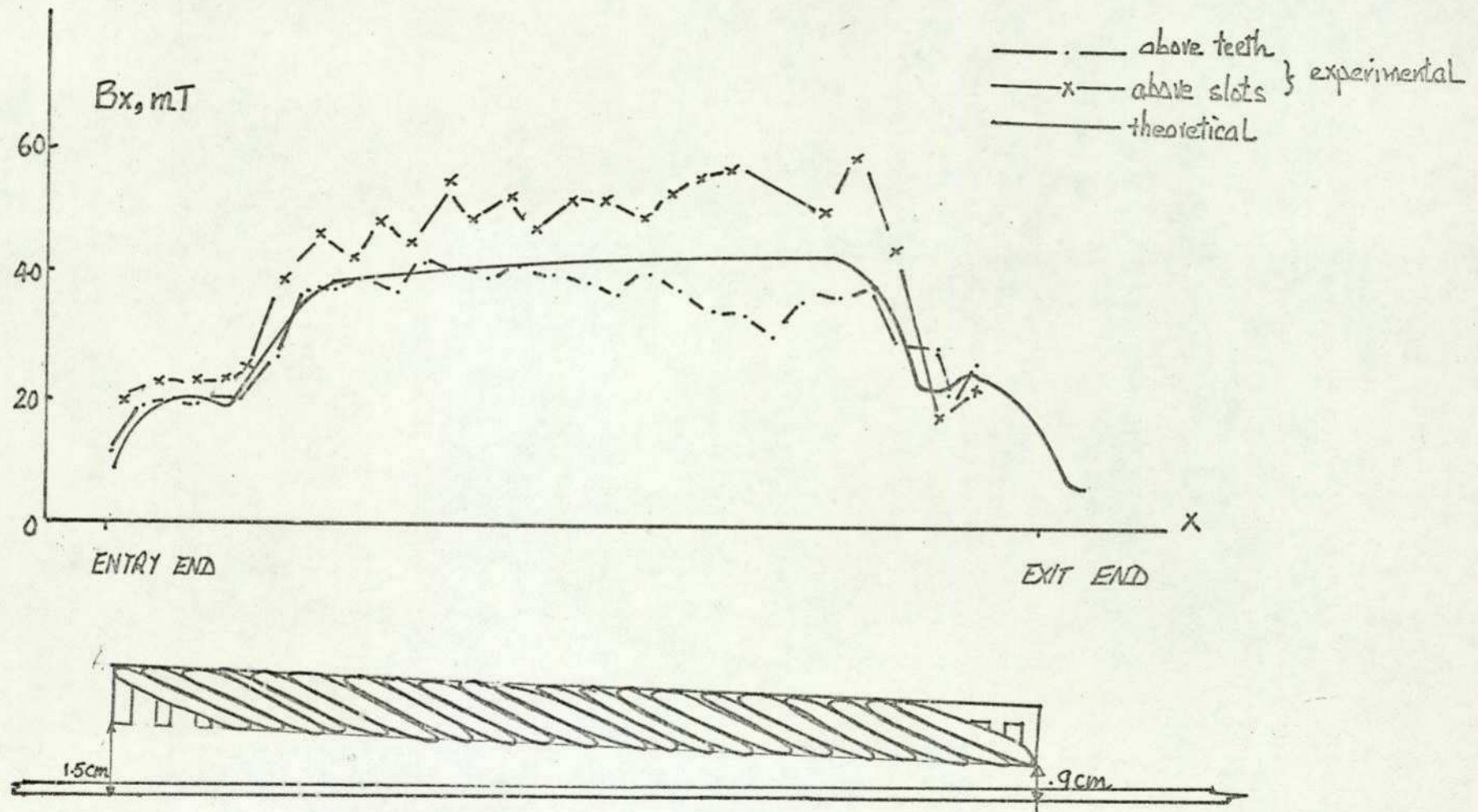


Fig. 4.13.(a) X component of flux density

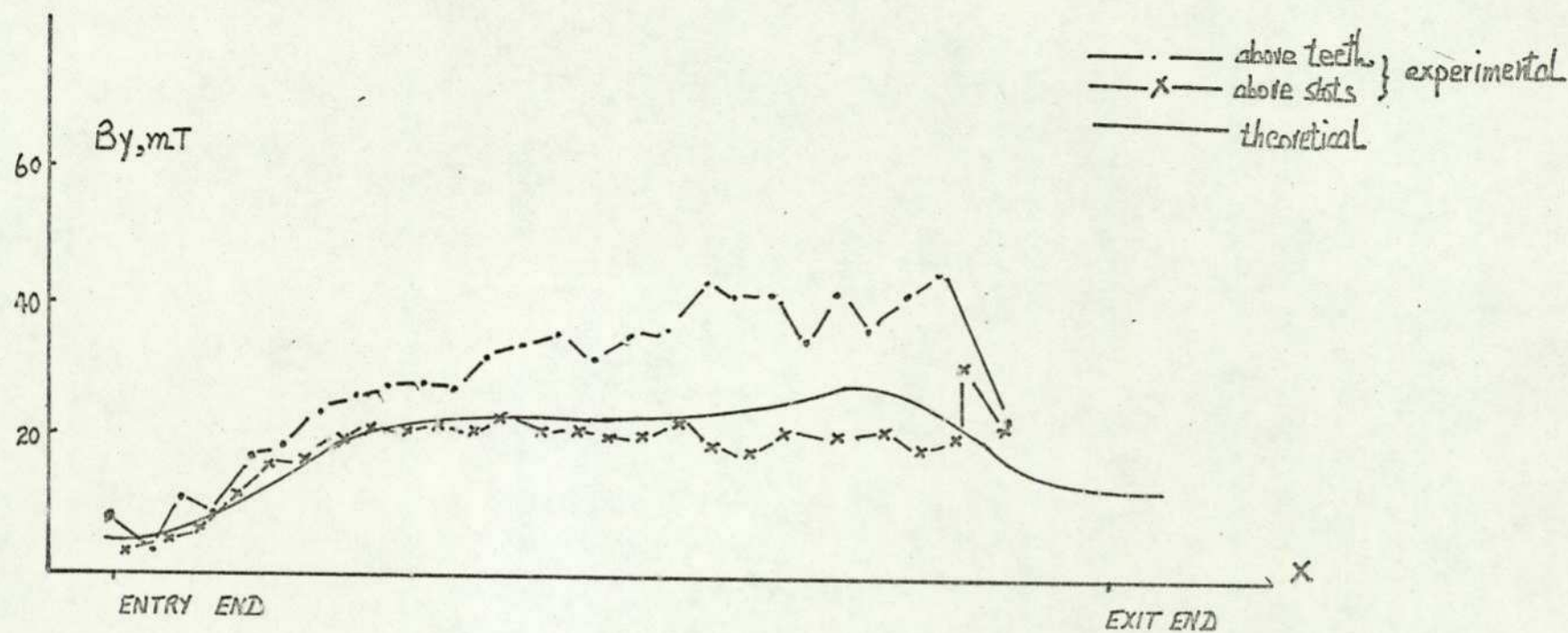


Fig. 6.4.13. Flux density distribution along the x axis
 150Hz , $S=0.5$ and $\theta_z=0.0112\text{rad}$.

CHAPTER 7

CHAPTER 7. DISCUSSION, FURTHER WORK AND CONCLUSION

7.1. Discussion

The work presented in this thesis has examined the electromechanical stiffness and damping coefficients of the linear induction motor in all six degrees of freedom. The three axis forces and the three rotational couples generated by the machine under different stator symmetrical and asymmetrical positions were studied. Their corresponding flux density distributions were also investigated.

The input impedance of the linear induction motor was used to relate the constant current drive machine to the constant voltage drive machine. A computer program was developed for the purposes of predicting the flux densities and forces in the machine. A listing of the computer program has been given in Appendix A3. In addition, the computed results were plotted in two and three dimensional diagrams to show the distributions of various components of flux density, force density and power flow in the machine. Results were verified with an experimental machine under various operating conditions.

In the mathematical model of Chapter 2, current sheet density distributions for both J_x and J_z were assumed to be extended to $\pm L_z/2$, as discussed in Appendix A1. This was the best waveform chosen from those described in the appendix for the 350 Hz supply frequency. However, the flux density distributions along the z axis were found to be frequency dependent as indicated in Fig.6.4.5. for 50 Hz. These experimental results suggest that a short $(1 - \sin \theta)$ exponential decay representation of the current sheet along the z axis would be more suitable for this particular frequency. At present, since there is no empirical formula to generalise the current sheet representation along the

z axis, the best representation would be derived from the experimental results to match the particular configuration and the parameters of the machine.

In the current sheet representation in the x axis, a sinusoidal travelling wave current density is assumed and the slot harmonic and slot effect are ignored. The flux measurement results show that there are large differences in values between the flux densities measured above teeth and above slots. Though the theoretical results fit in between these values, in order to obtain an accurate prediction of the flux densities in various parts of the machine, it is evident that the slot effect cannot be neglected. Slot effect can be included in this analysis by defining a current sheet density distributions J_x and J_z , the same as in Oberretl's paper⁹. It is achieved at the expense of a very high computer time, since a large number of terms of N in the double Fourier Series is required.

The stator iron is assumed to be extended to $\pm \infty$ in the x direction, hence flux beyond the exit end carried by the rotor would still interact with the artificial extended stator iron. It was expected that this assumption would cause discrepancy between the theoretical and the experimental results, especially when the machine is operating at high speed or near synchronous speed where a large amount of flux is present beyond the exit end. Fig.6.3.14. shows the pitch couple M_z against slip and Fig.6.3.10. shows the levitation force F_y against slip. The theoretical results in these diagrams diverge from the experimental results, these suggest an improved modelling of the finite stator iron length may prove more adequate.

As stated in Chapter 2, the angles of roll, pitch and yaw are assumed small, a variation of air gap g of 6mm in the x direction

means a angle of pitch of 0.64° , and a variation of 6mm in the active zone in the z direction means a angle of roll of 4.3° . As the stator iron is assumed to be extended to $\pm\infty$ in the x and z directions, this implies a limitation on the angles of roll and pitch. Negative air gap length must be avoid in the mathematical model when the machine is rolled and pitched.

The stiffness coefficients in the six degrees of freedom are calculated and verified by the experimental results in both static and dynamic conditions(Chapter 5 and Chapter 6). A high stiffness in the y direction K_{22} is obtained, 8.62N/cm with 350 Hz, 3.5A supply and with average air gap length g_0 of 1.0cm. Negative stiffness coefficients K_{33} and K_{55} were obtained in both experimental and theoretical results, which means an unstable system in the z direction and the rotational direction of yaw. As explained in Chapter 2, ther is no electromagnetic effect on the stiffness when there is a change in the position of the stator in the x direction, hence K_{11} is zero. Positive stiffness coefficients were obtained in roll and pitch; whereas in the later case, a pitch couples is presented at zero pitch angle due to the non--uniform distribution of F_y in the x direction because of the end effect(see Fig.6.3.15.).

Part of the cross coupling between the six degrees of freedom were verified. The effects on the levitation and propulsion forces when the machine is laterally shifted and the machine is rolled, pitched and yaw are illustrated in Chapter 5, Fig.5.4.1. There is also a substantial cross coupling between the lateral direction and the roll of the machine. As shown in Fig.5.4.1., a lateral displacement in the z direction causes the machine to generate a lateral force F_z and a moment of roll(-0.146N/cm and 0.0628N-m/cm experimental).

When the machine rolled, there is a change in the lateral force and the moment of roll(0.022N/rad. and 0.0055N-m/rad. experimental).

In the experimental measurement, when the machine is in a symmetrical position($\theta_x = \theta_y = \theta_z = z_o = 0.0$), there is a small amount of moment of roll and yaw present. They are caused by the asymmetrical configuration of the stator windings and the stator iron. In the experimental results, these values are subtracted from the other measured values in order to obtain a zero moment of roll and yaw when the machine is in the symmetrical condition.

The theoretical results of the lateral force variation with respect to the lateral displacement is shown in Fig.6.3.11. Experimental results were unobtainable due to the small magnitude of the lateral force(0.10N maximum).

The theoretical dampings of the linear induction motor are shown in Fig.4.5.1. The first column of the damping coefficient matrix is verified in Chapter 6, and corresponds to the changes of the three axis forces and the three rotational couples with respect to the changes in slip. The dampings in the other five degrees of freedom could not be verified due to the difficulty in the experimental work. In order to obtain the experimental damping in these five degrees of freedom, velocity has to be included or the machine has to be set into vibration in the corresponding degree of freedom. It is hoped that the theoretical results of the damping coefficient matrix will provide a general indication of the values of the dampings in the machine.

The input impedance of the linear induction motor was calculated and is shown in Fig.4.6.1. The theoretical results and the experimental results of the Y-phase current for a constant voltage supply of 50 Hz under different slips is shown in Fig.6.3.19., and Fig.6.3.20. shows the corresponding power factor. Fig.6.3.9. and Fig.6.3.10. show the

forces when the machine is operated under a constant input voltage of 54.4volts, and the current is 5.0A at synchronous speed. Fig.6.3.5. and Fig.6.3.6. show the propulsion and levitation forces when the machine is operated under a constant current supply of 5.0A and with a terminal voltage of 54.4 volts at synchronous speed. These diagrams illustrate the differences between the performance of a constant voltage drive machine and a constant current drive machine, both the experimental and theoretical results are shown for comparison purposes.

In Chapter 4, the computed results were plotted in three dimensional diagrams to show the distribution of various components of flux density and force density distribution in the machine. Experimental results of flux density distributions were plotted in two dimensional diagrams in Chapter 5 along both the x and z directions; the corresponding computed results are also plotted. The power flow across the air gap is also included in Chapter 4 with a 350 Hz supply, at $s = 1.0$ and $s = 0.2$.

Flux distribution in the air gap, secondary conductor and two back iron regions are plotted with $s = 0.0$ to $s = 1.0$ at 0.1 slip intervals. They are shown in Fig.4.3.1. to Fig.4.3.11. These diagrams provide an easy insight of the flux pattern with respect to the variation of slip. It can be observed that the flux penetrates gradually through the secondary conductor as the machine approaches synchronous speed.

The flux density distributions along the x and z directions are included in Chapter 5 and Chapter 6 for both static and dynamic tests results. These results include different frequency, slip, air gap length, angles of roll, pitch and yaw, and the lateral displacement. Theoretical results are also shown in each diagram for comparison purposes. When the machine is rolled, pitch or laterally

displaced, the flux density distributions of the machine are shown in Fig.6.4.12., Fig.6.4.13. and Fig.6.4.11. respectively. These diagrams can be compared with Fig.6.4.9. and Fig.6.4.10. for a machine with zero angle of roll and pitch, and without any lateral displacement in the stator.

Chapter 5 and Chapter 6 give both the experimental and theoretical results of the static and the dynamic tests. There are discrepancies between the computed and the experimental values, the causes of these discrepancies can be summarized by the following factors:

1. the assumption of the infinite permeability in the stator iron,
2. sinusoidal balanced three phase supply was assumed; in practice, currents are different in the phases and they are also different for a three wire and a four wire system. For example, at 150 Hz supply and $s = 0.5$, the three phase currents are 4.25A, 5.0A and 5.5A.
3. stator iron was assumed to be extended to $\pm \infty$, in both the x and z directions in the mathematical model,
4. slot effect and slot harmonics were not taken into account in the mathematical model,
5. the representation of the stator windings by a thin current sheet,
6. the uncertainty of the J_x and J_z distributions in the z direction,
7. the uses of Carter's coefficient for the equivalent air gap length which may not be justified when applied to the linear induction motor with an aluminium as the secondary; especially a machine without back iron,
8. experimental errors.

7.2. Further work

In this thesis, a three dimensional analytical approach by using double Fourier Series is used to calculate the stiffnesses and dampings of the linear induction motor. Some assumptions are made and they were discussed in the previous section. In this section, some proposals for further work are made. These are for the purposes of extending the theory further in order to obtain a more accurate and practical model from the analysis.

In the mathematical model, a linear induction motor without back iron was considered, and all the experimental work was obtained with a linear induction machine without back iron. As discussed in Chapter 2, if the machine has back iron, the analysis can include this by substituting the relative permeability of the back iron (μ_{r3}) into the boundary conditions. However, this model is based on the assumption that the back iron is extended to $\pm \infty$ in the x and the z directions. When the machine is laterally shifted or the machine is subject to yaw, the change in position of the back iron produces a change in the reluctance in the magnetic circuit and this effect is not included in the analysis. This phenomenon can be taken into account by modelling the back iron in finite width; or to a first approximation, by consideration of the electromagnetic energy stored in the back iron as in Lowther's thesis⁷. Considering the finite width of the back iron would be a more accurate approach, but it would be a difficult task in an analytical analysis.

As discussed in the last section, when the machine is operated at high speed or near synchronous speed, then a large flux is present beyond the exit end and a finite stator length in the mathematical model would be more adequate. In dealing with the finite length and finite width in the stator iron and the back iron, a numerical solution would

be more appropriate in view of the boundary conditions, hence the finite-element or the finite-difference methods are recommended.

From the experimental results of the flux density measurement, there are large differences in values between the flux densities measured above teeth and above slots. As explained in the last section, though the theoretical results fit in between these values, in order to obtain an accurate prediction of the flux densities, the slot effect cannot be ignored. By defining the current sheet density distributions J_x and J_z in the x direction as in Oberretl's paper⁹, the slot effect can be included.

Besides the few points mentioned above, there are other fields of interest for further investigation of the linear induction motor. In the mathematical model, the relative permeability of the machine was assumed constant. In reality, parts of the iron in the machine are saturated and its permeability varies. This leads into the examination of the non-linear characteristic of the B-H curve and the saturation in the magnetic parts of the machine. Also, the electromechanical stiffnesses and dampings of the linear induction motor can be investigated in the other configurations, for examples, solid iron secondary and double-sided motor.

Futhermore, not all the damping coefficients are verified in this thesis. It would be interesting to have the proof of the damping coefficients in all six degrees of freedom including the effect of the cross coupling.

7.3. Conclusion

In general, the work described in this thesis has been concerned with the calculation of the electromechanical stiffnesses and dampings of the linear induction motor. Experimental results show close correlation with the predicted values. It is hoped that the theories and techniques developed in this thesis will increase the understanding of the stiffnesses and dampings of the linear induction machine, and provide data for further study of the stability and design of the machine.

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APPENDICES

Appendix A1. Boundary equations

Boundary conditions at $y = g$, eqn.2.29. into eqn. 2.38. and eqn.2.39.;;
equating $\sin \delta_m(z-z_0)$ and $\cos \delta_m(z-z_0)$ terms separately,

$$\beta_{mn} (C_{1mn} \exp \beta_{mn} g - C_{2mn} \exp -\beta_{mn} g) = \mu_0 a_{mnx} \quad (A.1.)$$

$$\beta_{mn} (C'_{1mn} \exp \beta_{mn} g - C'_{2mn} \exp -\beta_{mn} g) = 0 \quad (A.2.)$$

$$\beta_{mn} (C_{3mn} \exp \beta_{mn} g - C_{4mn} \exp -\beta_{mn} g) = -\mu_0 a_{mny} \quad (A.3.)$$

$$\beta_{mn} (C'_{3mn} \exp \beta_{mn} g - C'_{4mn} \exp -\beta_{mn} g) = 0 \quad (A.4.)$$

Boundary conditions at $y = \delta$, normal components, eqn.2.29.
and eqn.2.30. into eqn.2.40. and eqn.2.41., give

$$C_{1mn} \exp \beta_{mn} \delta + C_{2mn} \exp \beta_{mn} -\delta = (\exp E_{1mn} \delta) \cdot (C_{9mn} \cos E_{2mn} \delta + C_{10mn} \sin E_{2mn} \delta) \quad (A.5.)$$

$$C'_{1mn} \exp \beta_{mn} \delta + C'_{2mn} \exp \beta_{mn} -\delta = \frac{j a_n V_z \exp E_{1mn} \delta}{(E_{2mnz}^2 - E_{2mnx}^2)} (C'_{9mn} \cos E_{2mn} \delta + C'_{10mn} \sin E_{2mn} \delta) \quad (A.6.)$$

$$C_{3mn} \exp \beta_{mn} \delta + C_{4mn} \exp -\beta_{mn} \delta = \exp E_{1mn} \delta (C_{11mn} \cos E_{2mn} \delta + C_{12mn} \sin E_{2mn} \delta) \quad (A.7.)$$

$$C'_{3mn} \exp \beta_{mn} \delta + C'_{4mn} \exp -\beta_{mn} \delta = 0 \quad (A.8.)$$

Boundary conditions at $y = \delta$, tangential component, eqn.2.29. and
eqn.2.30. into eqn.2.42. and eqn.2.43., give

$$\beta_{mn} (C_{1mn} \exp \beta_{mn} \delta - C_{2mn} \exp -\beta_{mn} \delta) = (E_{2mnx}) (-C_{9mn} \sin E_{2mn} \delta + C_{10mn} \cos E_{2mn} \delta) \\ + \exp E_{1mn} \delta (C_{9mn} \cos E_{2mn} \delta + C_{10mn} \sin E_{2mn} \delta) \quad (A.9.)$$

$$\beta_{mn} (C'_{1mn} \exp \beta_{mn} \delta - C'_{2mn} \exp -\beta_{mn} \delta) = \frac{j a_n V_z}{(E_{2mnz}^2 - E_{2mnx}^2)} (E_{2mnz} (C'_{9mn} \sin E_{2mn} \delta \\ + C'_{10mn} \cos E_{2mn} \delta) + \exp E_{1mn} \delta (C'_{9mn} \cos E_{2mn} \delta + C'_{10mn} \sin E_{2mn} \delta)) \quad (A.10.)$$

$$\beta_{mn} (C_{3mn} \exp \beta_{mn} \delta - C_{4mn} \exp -\beta_{mn} \delta) = \exp E_{1mn} (C_{11mn} \cos E_{2mnz} \delta + C_{12mn} \sin E_{2mnz} \delta) \\ + E_{2mnz} (-C_{11mn} \sin E_{2mnz} \delta + C_{12mn} \cos E_{2mnz} \delta) \quad (A.11.)$$

$$\beta_{mn} (C_{3mn}' \exp \beta_{mn} \delta - C_{4mn}' \exp -\beta_{mn} \delta) = 0 \quad (A.12.)$$

Boundary conditions at $y=0$, normal components, eqn.2.30. and eqn.2.31. into eqn.2.44. and eqn.2.45., give

$$C_{9mn} = C_{5mn} + C_{6mn} \quad (A.13.)$$

$$\frac{j a V_z}{(E_{2mnz}^2 - E_{2mnx}^2)} C_{9mn}' = C_{5mn}' + C_{6mn}' \quad (A.14.)$$

$$C_{11mn} = C_{7mn} + C_{8mn} \quad (A.15.)$$

$$0 = C_{7mn}' + C_{8mn}' \quad (A.16.)$$

Boundary conditions at $y=0$, tangential components, eqn.2.30. and eqn.2.31. into eqn.2.46. and eqn.2.47., give,

$$E_{1mn} C_{9mn} + E_{2mnx} C_{10mn} = \frac{1}{\mu_r} \beta_{mn} (C_{5mn} - C_{6mn}) \quad (A.17.)$$

$$\frac{j a V_z}{(E_{2mnz}^2 - E_{2mnx}^2)} (E_{2mnz} C_{10mn}' + E_{1mn} C_{9mn}') = \frac{1}{\mu_r} \beta_{mn} (C_{5mn}' - C_{6mn}') \quad (A.18.)$$

$$E_{2mnz} C_{12mn} + E_{1mn} C_{11mn} = \frac{\beta_{mn}}{\mu_r} (C_{7mn} + C_{8mn}) \quad (A.19.)$$

$$0 = \frac{\beta_{mn}}{\mu_r} (C_{7mn}' + C_{8mn}') \quad (A.20.)$$

Boundary conditions at $y = -\infty$, eqn.2.30. into eqn.2.48. and eqn.2.49., give,

$$C_{6mn} = 0 \quad (A.21.)$$

$$C_{6mn}' = 0 \quad (A.22.)$$

$$C_{8mn} = 0 \quad (A.23.)$$

$$C_{8mn}' = 0 \quad (A.24.)$$

There are 24 simultaneous equations for solving the 22 unknown constants, C_{mn} 's (eqn.A.1. to eqn.A.24.). Two of these equations are dependent equations, eqn.A.16. and eqn.A.20. represent the same equation, i.e.

$$C_{7mn} = C_{8mn}$$

and eqn.A.4., eqn.A.8. and eqn.A.12. give two equations, i.e.

$$C'_{3mn} = 0$$

$$C'_{4mn} = 0$$

Thus, all these equations (eqn.A.1. to eqn.A.24.) reduce to a set of equations, which has 22 simultaneous equations, with 22 unknown constants, C_{mn} 's. They can be solved by conventional method, using method of elimination or by inversion of the matrix.

Appendix A2. Current density distribution in the z direction

Fig.A.2.1. shows the layout of the windings over the active zone and the overhang regions. The solid lines represent the upper layers of the stator coils, and the dotted lines represent the lower layers of the stator coils.

In the overhang region, the current flow in the x direction and the z direction are:

$$\begin{aligned} J_x &= J_{x \text{ upper layer}} + J_{x \text{ lower layer}} \\ &= \frac{J_1}{2} \cos \theta \exp j \left(\frac{\pi}{p} - \omega t + \frac{z - l_z}{w_0} \frac{\pi}{2} \right) \\ &\quad + \frac{J_1}{2} \cos \theta \exp j \left(\frac{\pi}{p} - \omega t - \frac{z - l_z}{w_0} \frac{\pi}{2} \right) \\ &= J_1 \cos \theta \cos \left(\frac{z - l_z}{w_0} \cdot \frac{\pi}{2} \right) \end{aligned}$$

where J_1 is the current sheet density in the active zone.

Similarly,

$$\begin{aligned} J_z &= \frac{J_1}{2} \sin \theta \exp j \left(\frac{\pi}{p} - \omega t + \frac{z - l_z}{w_0} \frac{\pi}{2} \right) \\ &\quad - \frac{J_1}{2} \sin \theta \exp j \left(\frac{\pi}{p} - \omega t - \frac{z - l_z}{w_0} \frac{\pi}{2} \right) \\ &= j J_1 \sin \theta \sin \left(\frac{z - l_z}{w_0} \cdot \frac{\pi}{2} \right) \end{aligned}$$

The current distributions are plotted in Fig.A.2.2., and they represent the real current waveforms. However, due to the assumption of the artificial extension of the stator iron over the overhang regions, both Oberretl⁹ and Preston & Reece's papers²⁵ suggested a $(1 - \sin \theta)$ sinusoidally decaying current waveform.

Fig.A.2.3. shows the current waveforms used by Oberretl, where the secondary width is greater than the width of the stator iron + overhang region.

These waveforms are used for the evaluation of the three axis forces since they give the best correlation with the experimental force measurements.

The Fourier coefficients b_{mx} and b_{mz} for these given waveforms are (see section 2.4.):

$$\begin{aligned}
 b_{mx} &= \frac{2}{Lz} \int_{-Lz/2}^{Lz/2} F_1(z) \sin \gamma_m z dz \\
 &= \frac{\left(\frac{Lz}{W_0}\right)^2}{\left(\frac{Lz}{W_0}\right)^2 - (2m-1)^2} \left[\frac{1}{(2m-1)} \sin \frac{\pi(2m-1)(Lz+W_0)}{Lz} - \frac{2W_0}{Lz} \cos \frac{\pi(2m-1)W_0}{Lz} \right] \\
 b_{mz} &= \frac{2}{Lz} \int_{-Lz/2}^{Lz/2} F_2(z) \sin \gamma_m z dz \\
 &= \frac{-2 \gamma_m b_{mx} W_0}{\pi}
 \end{aligned}$$

Fig.A.2.4. shows the current waveforms used by Preston & Reece, where the width of the secondary is smaller than the width of the stator iron + overhang region.

In the thesis of Lowther¹⁹, another waveform is suggested, where the current sheet density is sinusoidally decaying until it reaches the edges of the secondary conductor (see Fig.A.2.5.). This current waveform is adapted in the evaluation of the flux density since it gives the best results when compared with the experimental field measurements.

Fig.A.2.6. shows the current waveforms used when the stator is laterally displaced.

Fig.A.2.7. compares the theoretical flux density distribution B_x by using the waveforms shown in Fig.A.2.4. and Fig.A.2.6. with $z_0 = 3\text{cm}$. The experimental results are also shown in the same diagram. It is

clear that the waveforms suggested by Lowther would have a better correlation with the experimental flux measurements, since it gives a more accurate field representation in the overhang regions.

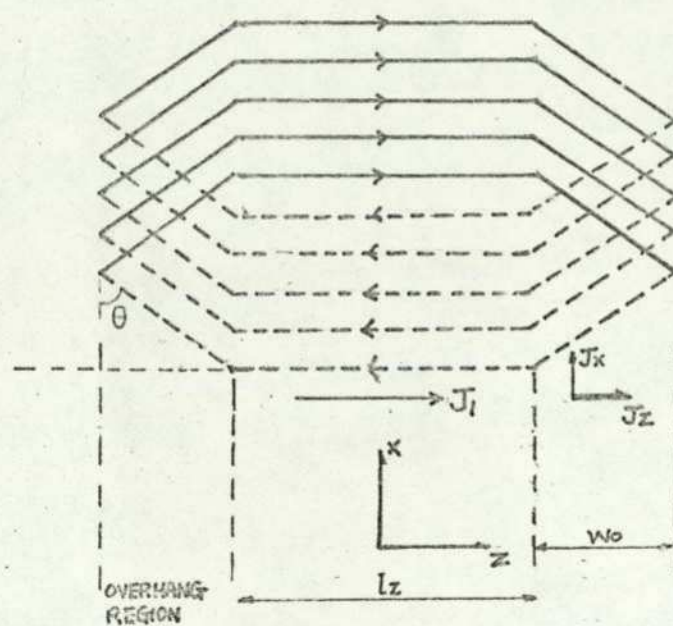


Fig. A.2.1. Layout of stator winding

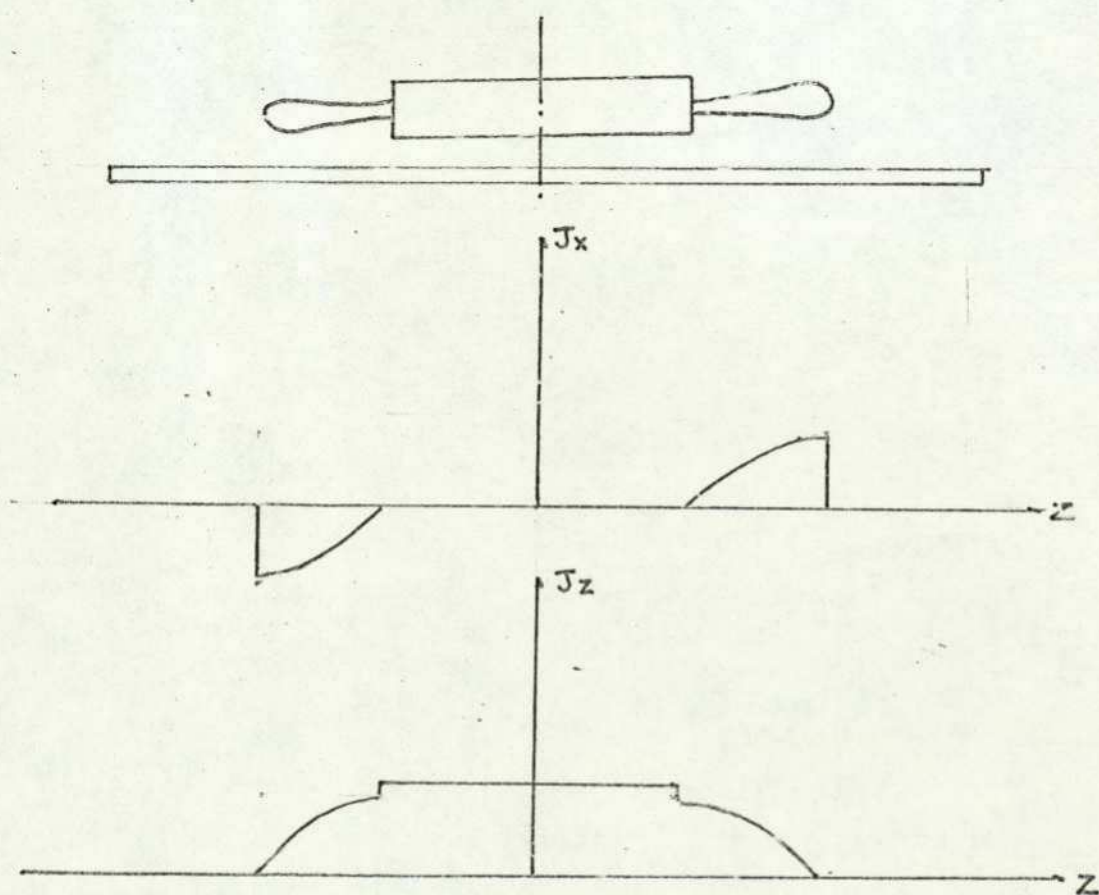


Fig. A.2.2. Current distribution

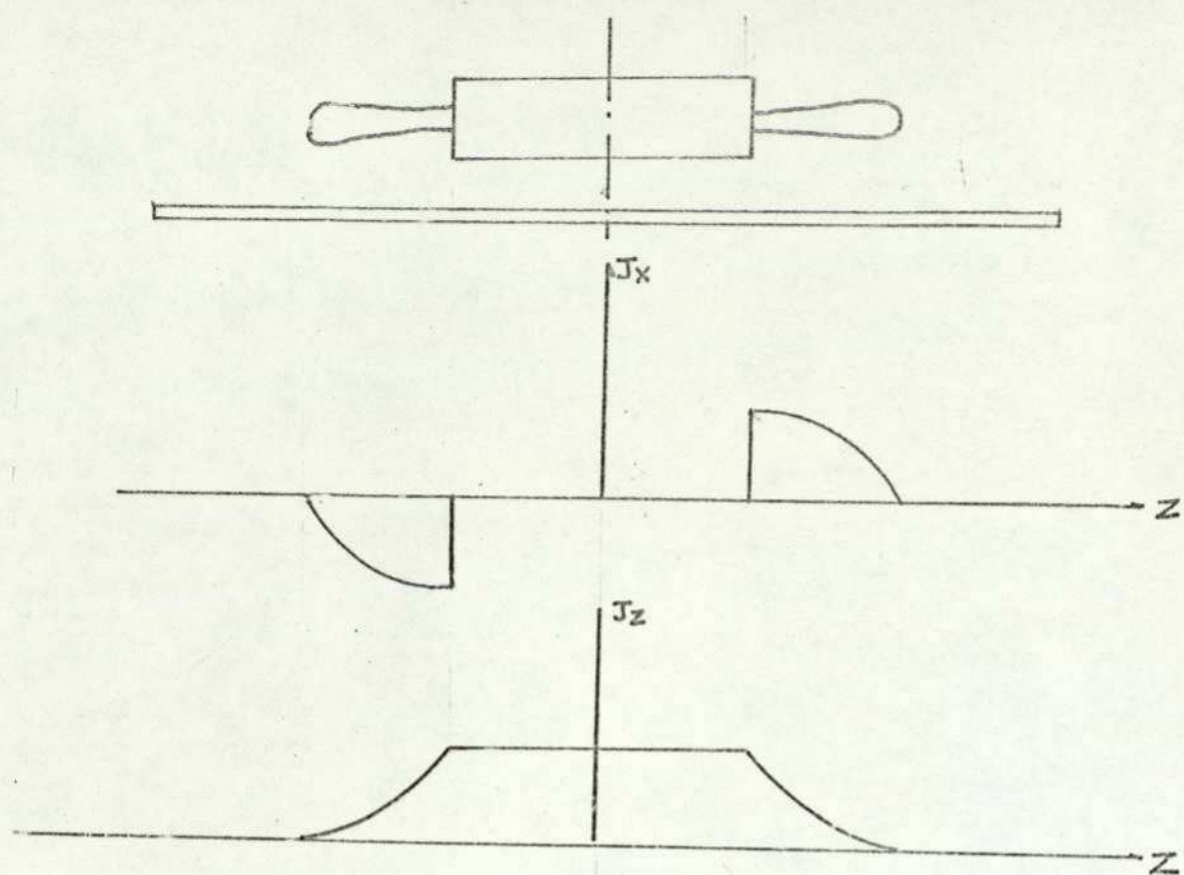


Fig. A.2.3. Current distribution

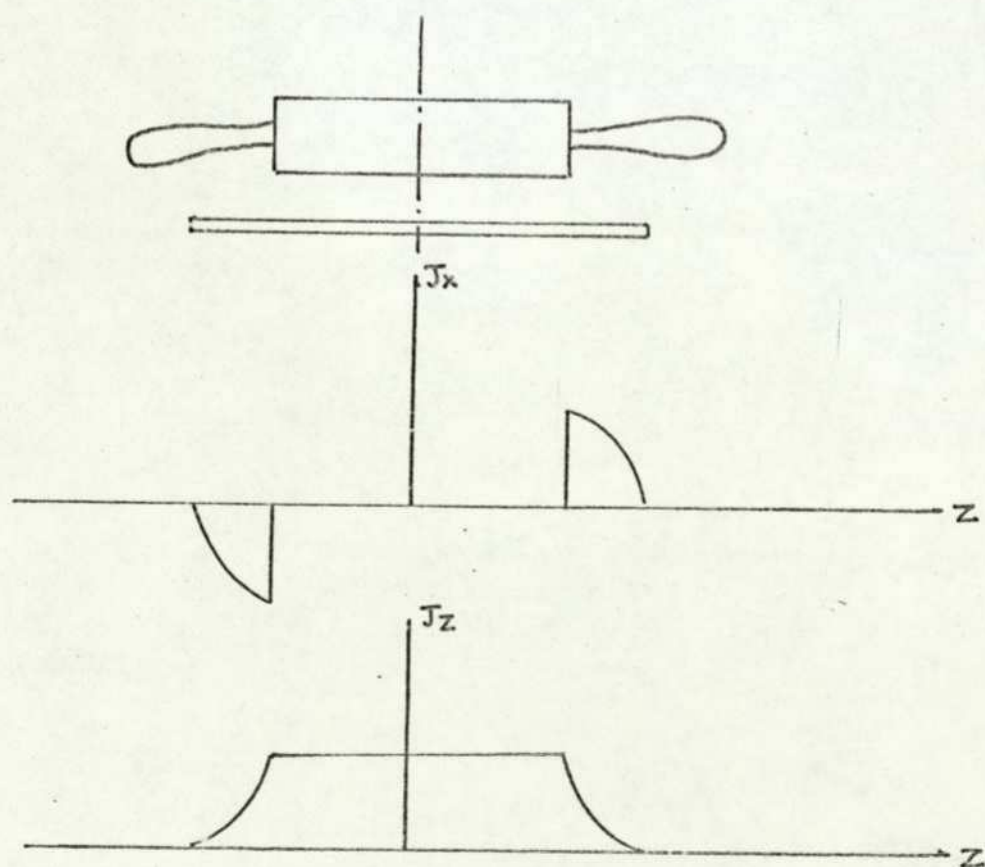


Fig. A.2.4. Current distribution

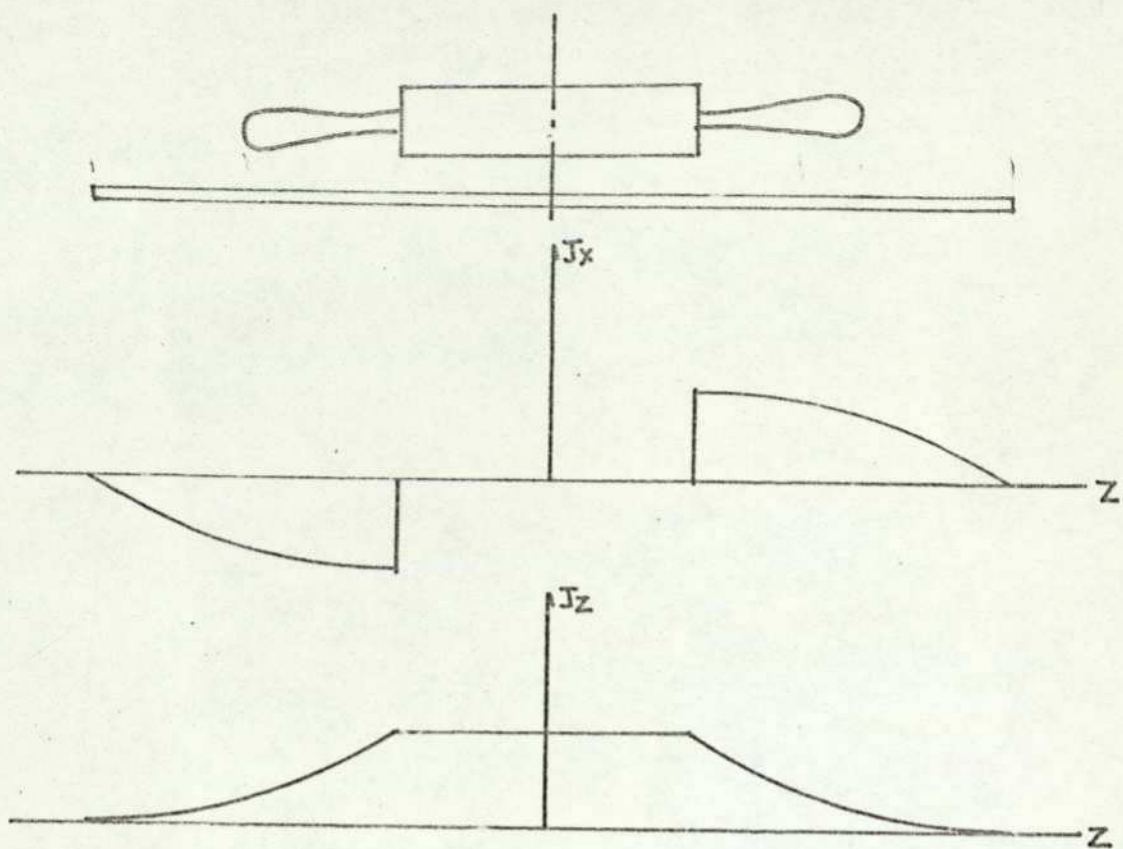


Fig. A.2.5. Current distribution

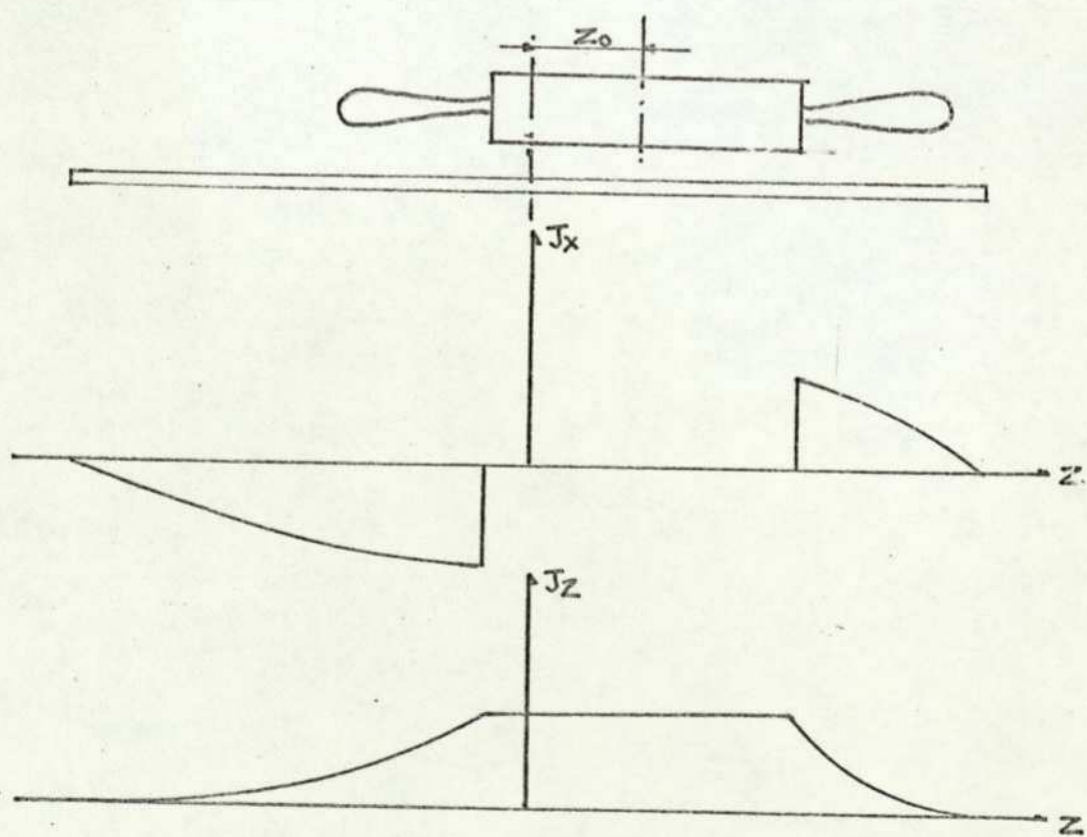


Fig. A.2.6. Current distribution with lateral displacement, z_0

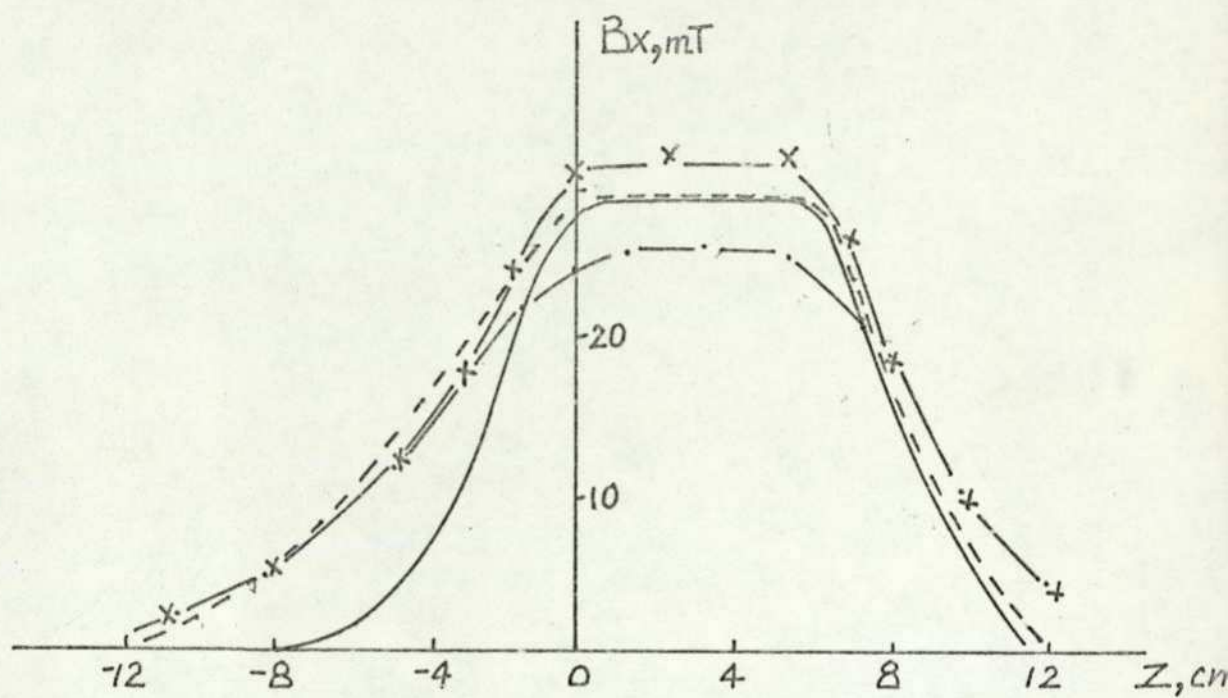


Fig. A.2.7. X component of flux density. $Z_0 = 3 \text{ cm}$

— · — above teeth } experimental
 — x — above slots }
 — theoretical. using
 waveform shown in Fig. A.2.4.
 - - - - - theoretical using
 waveform shown in Fig. A.2.6.

Appendix A3. Computer programs

In Chapter 2 and Chapter 4, an approach to calculate the flux densities and the forces has been outlined and discussed. Based on these theories, some computer programs have been developed. A listing of the computer program for the evaluation of the three axis forces and the three rotational couples is given below. The other subsidiary computer programs for the three dimensional plotting of the flux densities, force densities and the power flow across the air gap are similar to the program enclosed in this Appendix, hence they are not included.

JOB(GPEE514,J15,T9000,M7600) S W LEUNG JOB CC *SLOT 514*
 ATTACH(MAN,MANTRAP,ID=PUBLIC)
 LIBRARY(MAN)
 REQUEST(TAPE10,*PF)
 FTN(OPT=1,SL)
 LDSET(MAP=B/ZZZZMP,PRESFT=NGINF)
 LGO(PL=10000)
 CATALOG(TAPE10,DATA5,ID=GPEE126)
 J

PROGRAM LIM3(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE10)
 C THIS IS A 3 DIMENSIONAL ANALYSIS OF THE LINEAR INDUCTION MOTOR
 C THE 'FRCE' PLOTTING

COMPLEX BBX1,BBY1,BBZ1
 COMPLEX AX1,AZ1,EX2SQ,EX2,VX1,VX2,VX4,VX7,VX9
 COMPLEX QX1,QX2,QX3,QX4,QX5
 COMPLEX AAX1,AAY1,AAZ1,SUMX1,SUMY1,SUMZ1
 COMPLEX SUMX,SUMY,SUMZ
 COMPLEX JJ,Q1,Q2,Q3,Q4,Q5,E2,AA1
 COMPLEX CUJ,V1,V2,V4,V7,V9
 COMPLEX E2SQ
 COMPLEX AAX0,AAZ0,BRX0,BBY0,BBZ0
 COMPLEX SUMX0,SUMY0,SUMZ0,SUMX10,SUMY10,SUMZ10
 COMPLEX P2,P3,P4,P5,PZ,PX
 COMPLEX CK1,CK0,EX2SQ0,EX20,VX40,VX70,VX90
 COMPLEX QX20,QX30,QX40,QX50
 REAL LX,LLX,LZ,LLZ,K,J1,I17,I18,LX1

DIMENSION AA(50),CC(50), AN(50),AM(50),AMX(50)
 DIMENSION FFA(51,22)
 DIMENSION FFL(51,22)
 DIMENSION FFI(51,22),FL(51,22),FT(51,22)
 DIMENSION BX(51,22),BY(51,22),BZ(51,22)
 DIMENSION ZZZ(30),ZZT(30)
 DIMENSION ZFL(30),ZFT(30),ZFX(30)
 DIMENSION FFN(51,22)
 DIMENSION FFX(51,22)
 DIMENSION FFZ(51,22)
 DIMENSION FKZ(51,22)
 DIMENSION XX(51,22)
 DIMENSION AB(120)
 DIMENSION FFM(51,22),SMM(22)

C MACHINE PARAMETERS ARE AS FOLLOWS

C *****

W0=0.04
 LLX=1.25
 LX=0.53
 LX1=0.34
 LZ=0.10
 LZ=0.09
 LZ=0.08
 LLZ=0.235
 DD=0.003175
 C=0.013175
 C=0.01
 P=0.112
 SW=0.013


```

F=350.0
COND=35000000.0
UO=0.000001256
UR=2000.0
UR=1.0
J1=45000.0
J1=24500.0
P1=3.1416

```

```

11 READ(5,11) S,VY
   FORMAT(2F5.2)

```

```

S=1.0
S=0.9999
S=0.2

```

```

19 READ(5,19) MM,NN
   FORMAT(2I2)

```

```

12 READ(5,12) IX1,IX2,IY1,IY2,IZ1,IZ2
   FORMAT(6I2)

```

```

15 READ(5,15) K
   FORMAT(F6.2)

```

```

16 READ(5,16) B
   FORMAT(F10.5)

```

```

DO 35 I35=1,5

```

```

VZ=0.0
ZZO=0.0
VYO=0.0
VZO=0.0
RZ=0.0
B=0.0
THX=0.0
THY=0.0
THZ=0.0
VIX=0.0
VTY=0.0
VTZ=0.0

```

```

17 WRITE(6,17) K,MM,NN,B,VY
   FORMAT(////* K=*,F5.1,////* MM=*,I2,///
1* NN=*,I2,///* B=*,F10.5,10X,* VY=*,F5.2)

```

```

T=0.0

```

```

C *****

```

```

DX=LLX/50.0
DY=((LLZ/2.0)*B+C)/10.0
DZ=LLZ/20.0

```

```

14 WRITE(6,14) S,T,IX1,IX2,IY1,IY2,IZ1,IZ2
   FORMAT(////* S=*,F5.2,///* T=*,F6.3,///* X FROM *,I2,* TO *,
1,///* Y FROM *,I2,* TO *,I2,///* Z FROM *,I2,* TO *,I2)

```

```

NN1=(NN-1)/2+1

```

```

700 DO 700 I7=1,NN
     II7=FLOAT(I7-NN1)
     AA(I7)=((2.0*II7-1.0)/LLX+1.0/P)*P1
     AN(I7)=J1*(SIN((2.0*II7-1.0)*P1*LX/(LLX*2.0))
1+SIN((2.0*II7-1.0)*P1*LX1/(LLX*2.0)))
     1/((2.0*II7-1.0)*P1)
     CONTINUE
     KK2=2
     KK2=3
     KK1=3

```

```

KK2=4
KK1=2
KK1=1
KK2=5
D031 I31=KK1, KK2
GOTO 311
VTZ=FLOAT(I31)*1.0
VTY=FLOAT(I31)*1.0
VTX=FLOAT(I31)*1.0
VZO=FLOAT(I31)*1.0
VYO=FLOAT(I31-2)*1.0
S=FLOAT(I31-1)*0.1
THY=FLOAT(I31)*0.04123
THZ=FLOAT(I31-2)*0.005
THX=FLOAT(I31)*0.01
ZZO=FLOAT(I31-1)*0.01
ZZO=FLOAT(I31)*0.005
C=FLOAT(I31)*0.003+0.005
F=100.0*FLOAT(I31-1)+50.0
311 CONTINUE
IF(I35.EQ.1) VYO=FLOAT(I31-2)*1.0
IF(I35.EQ.2) VZO=FLOAT(I31)*1.0
IF(I35.EQ.3) VTX=FLOAT(I31)*1.0
IF(I35.EQ.4) VTY=FLOAT(I31)*1.0
IF(I35.EQ.5) VTZ=FLOAT(I31)*1.0

W=2.*P1*F
B=THZ
RZ=THX
VX=2.0*F*P*(1.0-S)
DO 600 I5=I21, I22
Z=-(I5-1.0)*DZ+LLZ/2.0
Z2=Z

DO 500 I3=IY1, IY2
Y=DD
Y0=0.0

DO 400 I1=IX1, IX2
X=-LLX/2.0+(I1-1.0)*DX
Z0=ZZO-THY*X
VZ=VZO-VTY*X
VY=VYO+VTZ*X-VTX*Z2
Z=Z2-LLZ/2.0-Z0
Z=Z2-LLZ/2.0

DO 800 I8=1, MM
I18=FLOAT(I8)
CC(I8)=I18*P1/LLZ
B1=(LLZ/(2.0*W0))**2
B2=P1*I18*((LLZ/2.0)+W0)/LLZ
B3=P1*I18*(LLZ/2.0)/LLZ
AM(I8)=(4.0/P1)*(B1/(B1-(I18)**2))*
1*((1.0/I18)*SIN(B2)-(2.0*W0/LLZ)*COS(B3))
2*SIN(CC(I8)*(LLZ/2.0-Z0))
AMX(I8)=-AM(I8)*CC(I8)*W0*2.0/P1
AMX(I8)=-AMX(I8)
XA=LLZ/2.0-Z0-LZ/2.0
XB=LLZ/2.0-Z0+LZ/2.0
XK=LLZ-XB-Z0/2.0
XK=LLZ-XB-Z0
XK=LLZ-XB
XD1=(P1/(2.0*XA))**2-CC(I8)**2
XD2=(P1/(2.0*XK))**2-CC(I8)**2
XD3=2.0*(COS(CC(I8)*(LLZ-Z0/2.0))-1.0)/CC(I8)

```


800

```

XD3=2.0*(COS(II8*P1)-1.0)/CC(18)
XD4=(P1/XA)*SIN(CC(18)*XA)
XD5=(P1/XK)*COS(II8*P1)
XD6=2.0*CC(18)*SIN(CC(18)*XB)
XDX4=2.0*CC(18)*SIN(CC(18)*XA)
XDX5=2.0*CC(18)*COS(II8*P1)
XDX6=(P1/XK)*SIN(CC(18)*XB)
AM(18)=(1.0/LLZ)*(XD3+(XD4-2.0*CC(18))/XD1+(XDX5+XDX6)/XD2)
AMX(18)=(1.0/LLZ)*((XDX4-(P1/XA))/XD1+(XD5+XD6)/XD2)
AMX(18)=0.73*AMX(18)
CONTINUE

```

```

SUMX1=(0.0,0.0)
SUMY1=(0.0,0.0)
SUMZ1=(0.0,0.0)
SUMX10=(0.0,0.0)
SUMY10=(0.0,0.0)
SUMZ10=(0.0,0.0)

```

```

E1=UO*COND*VY/2.0
G=B*X+RZ*Z2+C
B5=SP*(5*G+SW)
G=(B5/(B5-SW**2.0))*G+DD
CUJ=COND*UO*CMPLX(0.0,1.0)
E1SQ=E1**2
GD=G-DD
E1YD=E1*(Y-DD)
E1YD0=E1*(Y0-DD)

```

```

DO 200 N=1,NN
AAN=AA(N)
ANN=AN(N)
V1=CUJ*(W-VX*AAN)-E1SQ
VX1=CUJ*W-E1SQ
V2=CMPLX(COS(AAN*X-W*T),SIN(AAN*X-W*T))*UO
VX2=(0.0,1.0)*V2
AANSQ=AAN**2
SUMX=(0.0,0.0)
SUMY=(0.0,0.0)
SUMZ=(0.0,0.0)
SUMX0=(0.0,0.0)
SUMY0=(0.0,0.0)
SUMZ0=(0.0,0.0)

```

```

DO 100 M=1,MM
CCM=CC(M)
BBSQ=AANSQ+CCM**2
BR=SQRT(BBSQ)
E1B=(E1-RB/UR)
E1BS=E1*E1B
A=ANN*AM(M)
E2SQ=-BBSQ+V1
E2=CSQRT(E2SQ)
V4=E2SQ+E1BS
V7=E2*Y
V8=CCM*Z+P1/2.0
V9=CSIN(V7)
V11=COS(V8)
Q1=A*V2*V11
Q2=CCOS(E2*DD)
Q3=CSIN(E2*DD)
V4=V4-BB*Q2*E2/(UR*Q3)
Q4=((E1B-E2*Q2/Q3)*SINH(BB*GD)/V4)

```



```

AX=ANN*AMX(M)
EX2SQ=-BBSQ+VX1
EX2=CSQRT(EX2SQ)
VX4=EX2SQ+E1BS
VX7=EX2*Y
VX9=CSIN(VX7)
VX11=SIN(V8)
QX1=AX*VX2*VX11
QX2=CCOS(EX2*DD)
QX3=CSIN(EX2*DD)
VX4=VX4-BB*QX2*EX2/(UR*QX3)
QX4=((E1B-EX2*QX2/QX3)*SINH(BB*GD)/VX4)
QX5=BB*(QX4)+COSH(BB*GD)
PX=(E1B-EX2*QX2/QX3)/VX4
PZ=(E1B-E2*Q2/Q3)/V4
P2=-EX2/(VX9*VX4*QX5)
P3=-VZ/VX*(PZ-PX)*COSH(BB*GD)/(O5*QX5*COND)
P4=-VZ/(VX*O5*COND)*(-E2/(V9*V4)+EX2*(1.0-(PZ-PX)
1*BB*SINH(BB*GD)/QX5)/(VX9*VX4))
P5=PX/QX5
AAZ1=-PZ*Q1/O5
AZ1=-1/Q5
AAZ0=-EXP(E1YD0)*(-E2)*Q1/(Q3*V4*Q5)
AX1=AX*VX2*VX11/QX5+VZ/VX*(PZ-PX)*SINH(BB*GD)*A*V2
1*V11/(UO*O5*QX5*COND)
AAX0=EXP(E1YD0)*(-EX2)/(QX3*VX4*QX5)
60 BRX1=AZ1*Q1
BBX0=Q1*EXP(E1YD0)*(E2*BB/UR)/(V4*Q3*Q5)
BBY1=(0.0,-1.0)*AAN*AAZ1+CCM*(P5*AX*VX2*V11-P3*A*V2*VX11/UO)
BBY0=(0.0,-1.0)*AAN*AAZ0+CCM*(AAX0*AX*VX2*V11-P4*A*V2*VX11/
BRZ1=-AX1
BBZ0=QX1*EXP(E1YD0)*(EX2*BB/UR)/(VX4*QX3*QX5)-BB*P4*Q1/(UO*
SUMX=SUMX+BBX1
SUMY=SUMY+BBY1
SUMZ=SUMZ+BBZ1
SUMX0=SUMX0+BBX0
SUMY0=SUMY0+BBY0
SUMZ0=SUMZ0+BBZ0
100 CONTINUE

SUMX1=SUMX1+SUMX
SUMY1=SUMY1+SUMY
SUMZ1=SUMZ1+SUMZ
SUMX10=SUMX10+SUMX0
SUMY10=SUMY10+SUMY0
SUMZ10=SUMZ10+SUMZ0
200 CONTINUE

BBX=CABS(SUMX1)
BBY=CABS(SUMY1)
BBZ=CABS(SUMZ1)
BX0=CABS(SUMX10)
BY0=CABS(SUMY10)
BZ0=CABS(SUMZ10)

F1=(-(BBX**2)+(BBY**2)-(BBZ**2))/(UO*4.0)
F2=(BX0**2-BY0**2+BZ0**2)/(UO*4.0)
FLE=(F1+F2)
FA=F2
FN=F1/1.0
FX=REAL(SUMX1*CONJG(SUMY1)-SUMX10*CONJG(SUMY10))/(UO*2.0)
FZ=REAL(SUMY1*CONJG(SUMZ1)-SUMY10*CONJG(SUMZ10))/(UO*2.0)
FFA(I1,I5)=FA
FFL(I1,I5)=FLE
FFN(I1,I5)=FN

```

```

FFZ(11,15)=FZ
FFT(11,15)=X*FLE
FKZ(11,15)=FZ
FFM(11,15)=X*FZ
FFM(11,15)=X*FZ-Z2*FX
1+FX*THY*X
BX(11,15)=-BBX
BY(11,15)=-BBY
BZ(11,15)=-BBZ
IF(11.NE.26) GOTO 223
WRITE(6,222) 11,15,SUMX1,SUMY1,SUMZ1,BBX,BBY,BBZ,FX,FLE,FZ
222 FORMAT(/2112,/6F18.8,/3F18.8,/3F18.8)
223 CONTINUE
400 CONTINUE

500 CONTINUE

Z=Z2
DO 51 I51=1,24
FFM(2*I51+1,15)=2.0*FFM(2*I51+1,15)
FL(2*I51+1,15)=2.0*FFL(2*I51+1,15)
FT(2*I51+1,15)=2.0*FFT(2*I51+1,15)
XX(2*I51+1,15)=2.0*FFX(2*I51+1,15)
FFZ(2*I51+1,15)=2.0*FFZ(2*I51+1,15)
51 CONTINUE

DO 52 I52=1,25
FFM(2*I52,15)=4.0*FFM(2*I52,15)
FL(2*I52,15)=4.0*FFL(2*I52,15)
FT(2*I52,15)=4.0*FFT(2*I52,15)
XX(2*I52,15)=4.0*FFX(2*I52,15)
FFZ(2*I52,15)=4.0*FFZ(2*I52,15)
52 CONTINUE

FL( 1,15)=FFL( 1,15)
FT( 1,15)=FFT( 1,15)
XX( 1,15)=FFX( 1,15)
FL(51,15)=FFL(51,15)
FT(51,15)=FFT(51,15)
XX(51,15)=FFX(51,15)

SM=0.0
SF=0.0
ST=0.0
SX=0.0
SZ=0.0

DO 53 I53=1,51
SM=SM+FFM(I53,15)
SF=SF+FL(I53,15)
ST=ST+FT(I53,15)
SX=SX+XX(I53,15)
SZ=SZ+FFZ(I53,15)
53 CONTINUE

SMM(15)=SM*DX/3.0
ZZT(15)=SF*Z*DX/3.0
SF=SF*DX/3.0
ST=ST*DX/3.0
SX=SX*DX/3.0
ZFL(15)=SF
ZFT(15)=ST
ZFX(15)=SX
ZZZ(15)=SZ*DX/3.0

```



```

DO 55 I55=1,10
SMM(2*I55)=4.0*SMM(I55*2)
ZFL(2*I55)=4.0*ZFL(2*I55)
ZFT(2*I55)=4.0*ZFT(2*I55)
ZFX(2*I55)=4.0*ZFX(2*I55)
ZZT(2*I55)=4.0*ZZT(2*I55)
ZZZ(2*I55)=4.0*ZZZ(2*I55)
55 CONTINUE

DO 56 I56=1,9
SMM(2*I56+1)=2.0*SMM(2*I56+1)
ZFL(2*I56+1)=2.0*ZFL(2*I56+1)
ZFT(2*I56+1)=2.0*ZFT(2*I56+1)
ZFX(2*I56+1)=2.0*ZFX(2*I56+1)
ZZT(2*I56+1)=2.0*ZZT(2*I56+1)
ZZZ(2*I56+1)=2.0*ZZZ(2*I56+1)
56 CONTINUE

ZM=0.0
ZZFL=0.0
ZZFT=0.0
ZZFX=0.0
ZZZZ=0.0
ZZZT=0.0

DO 57 I57=1,21
ZZZT=ZZZT+ZZT(I57)*DZ/3.0
ZZZZ=ZZZZ+ZZZ(I57)*DZ/3.0
ZZFX=ZZFX+ZFX(I57)*DZ/3.0
ZZFT=ZZFT+ZFT(I57)*DZ/3.0
ZZFL=ZZFL+ZFL(I57)*DZ/3.0
ZM=ZM+SMM(I57)*DZ/3.0
57 CONTINUE

WRITE(6,54) VY,C,ZZFX,ZZFL,ZZZZ,ZZZT,ZM,ZZFT
54 FORMAT(///2F18.8,/6F18.8)
ZM=0.0
ZZFL=0.0
ZZFT=0.0
ZZFX=0.0
ZZZZ=0.0
ZZZT=0.0
SMM(11)=SMM(11)/2.0
ZFL(11)=ZFL(11)/2.0
ZFT(11)=ZFT(11)/2.0
ZFX(11)=ZFX(11)/2.0
ZZZ(11)=ZZZ(11)/2.0
ZZT(11)=ZZT(11)/2.0
DO 571 I571=1,11
ZM=ZM+SMM(I571)*DZ/3.0
ZZFL=ZZFL+ZFL(I571)*DZ/3.0
ZZFT=ZZFT+ZFT(I571)*DZ/3.0
ZZFX=ZZFX+ZFX(I571)*DZ/3.0
ZZZZ=ZZZZ+ZZZ(I571)*DZ/3.0
ZZZT=ZZZT+ZZT(I571)*DZ/3.0
571 CONTINUE
WRITE(6,541) ZZFX,ZZFL,ZZZZ,ZZZT,ZM,ZZFT
541 FORMAT(/* SUMMATION OVER HALF PLANE*,/6F18.8)
31 CONTINUE
35 CONTINUE
STOP
END
1
1.0

```

PUBLICATION

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ON THE ELECTROMECHANICAL STIFFNESS AND DAMPING OF LINEAR INDUCTION MOTORS

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ABSTRACT

The three axis forces and the three rotational couples of the linear induction motor are analysed as functions of position, linear velocity and rotational velocity of the stator. The electromechanical stiffness and damping coefficients are determined in each of the six degrees of freedom. A current-fed machine is assumed and a three-dimensional analytical approach is used. Both the end effect and the lateral edge effect are considered and the skin effect of the secondary conductor is included. The predicted results are verified on a test rig.

INTRODUCTION

An object in motion has six degrees of freedom: three translational motions and three rotational motions. The coordinate system used for the analysis in this paper is shown in Fig.1. The three translational motions are referred to the traction, normal and lateral (x , y and z) directions. The three rotational motions are referred to the roll, yaw and pitch (θ_x , θ_y and θ_z).

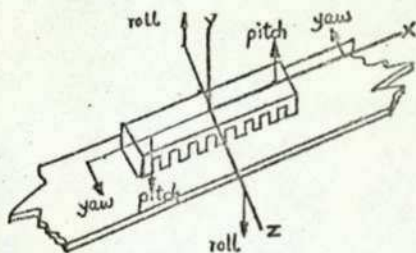


Fig. 1.

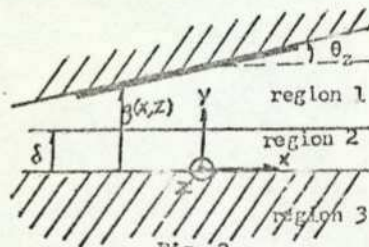


Fig. 2.

MATHEMATICAL MODEL

The mathematical model is divided into three regions: the air gap region, the secondary conductor region and the back iron region (regions 1,2,3 in Fig.2). The stator winding is represented by a thin current sheet and presented in the form of a double Fourier series by having infinite arrays of m.m.f. in the x and z directions. The stator and back iron are assumed to be infinite in length and in width, and the stator iron infinitely permeable. The field equations are formulated in terms of magnetic vector potential and solved analytically by separation of variables. The three components of electromagnetic forces are calculated by numerical surface integration of Maxwell's Second Stress Tensor over the surface of the secondary conductor¹. The total moments of the three rotational couples are found in a similar manner. The theoretical groundwork of this paper

follows that of Oberretl².

The current distribution for the J_x and J_z components are shown in Figs 3 and 4 and are represented by the following double Fourier series.

$$J_x = \sum_{n=-\infty}^{\infty} \sum_{m=1}^{\infty} j A_{mnx} \cdot \sin \gamma_m z \cdot \exp j(\alpha_n x - \omega t) \quad (1)$$

$$J_z = \sum_{n=-\infty}^{\infty} \sum_{m=1}^{\infty} A_{mnz} \cdot \cos \gamma_m z \cdot \exp j(\alpha_n x - \omega t) \quad (2)$$

$$\text{where } A_{mnx} = \frac{1}{L_z} \left[\frac{z}{\gamma_m} (\cos m\pi - 1) + \frac{\pi/2 \sin \gamma_m a - 2\gamma_m}{(\pi/2a)^2 - \gamma_m^2} + \frac{2\gamma_m \cos m\pi + \frac{\pi}{2} \sin \gamma_m b}{(\pi/2b)^2 - \gamma_m^2} \right] \\ \times \frac{z}{2\pi-1} \left[\sin \left\{ \frac{2\pi-1}{L_x} \cdot \frac{L_{x1}}{2} \right\} + \sin \left\{ \frac{2\pi-1}{L_x} \cdot \frac{L_{x2}}{2} \right\} \right]$$

$$A_{mnz} = \frac{1}{L_z} \left[\frac{-2\gamma_m \sin(\gamma_m a) + \pi/a}{(\pi/2a)^2 + \gamma_m^2} - \frac{2\gamma_m \sin(\gamma_m b) - \pi/b \cos m\pi}{(\pi/2b)^2 + \gamma_m^2} \right] \\ \times \frac{z}{2\pi-1} \left[\sin \frac{(2\pi-1)L_{x1}}{L_x} + \sin \frac{(2\pi-1)L_{x2}}{L_x} \right]$$

$$a = \frac{L_z}{2} - z_0 - \frac{L_z}{2} ; \quad b = \frac{L_z}{2} - z_0 - \frac{L_z}{2} ; \quad k = L_z - b ;$$

$$\alpha_n = \left(\frac{2\pi-1}{L_x} - \frac{1}{p} \right) \pi ; \quad \gamma_m = \frac{m\pi}{L_z}$$

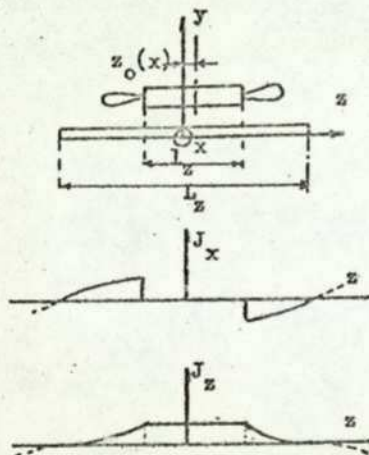


Fig. 3.

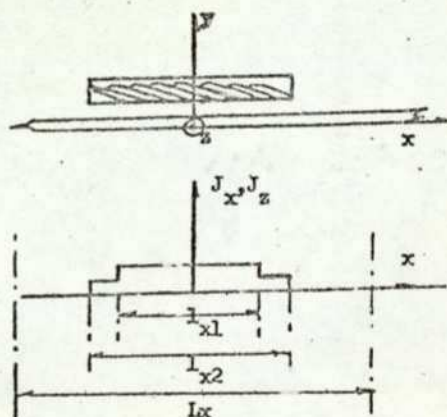


Fig. 4.

The three translational displacements and the three rotational displacements of the stator are represented either by a change in the air gap length or a change in the current distribution of the current sheet in the z direction; they are:

$$g(x, z) = \theta_x x - \theta_z z + y_0 \quad (3)$$

$$\text{and } z_0(x) = \theta_y x - z_0 \quad (4)$$

where $g(x, z)$ is substituted in the boundary conditions after the field equations are solved. θ_x , θ_y and θ_z are the angles of roll, yaw and pitch

and they are assumed small. $z_0(x)$ is used as one of the parameters in the current sheet equations, (1) and (2).

The linear velocity is defined by a velocity vector $\bar{V}$,

$$\bar{V} = (v_x, v_y, v_z)$$

and the rotational angular velocity is defined by $\bar{\omega}$,

$$\bar{\omega} = (\omega_x, \omega_y, \omega_z) = (\dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_z)$$

The angular velocity of the stator can be included and expressed in terms of the linear velocity by the equation:

$$\bar{V}_{\text{total}} = \bar{V} + \bar{\omega} \times \bar{R}$$

$$\text{where } \bar{R} = (x, y, z)$$

FIELD EQUATIONS

The field equations defined in the three regions (see Fig.6) are:

(a) The air gap region (region 1),

$$\nabla^2 A_1 = 0 \quad (5)$$

and the solutions are:

$$A_{1x} = \sum_{n=-\infty}^{\infty} \sum_{m=1}^{\infty} \left\{ [C_{1mn} \exp(\beta_{mn} y) + C_{2mn} \exp(-\beta_{mn} y)] \sin \gamma_m x \cdot \exp j(\alpha_n x - \omega t) \right. \\ \left. + [C'_{1mn} \exp(\beta_{mn} y) + C'_{2mn} \exp(-\beta_{mn} y)] \cos \gamma_m x \cdot \exp j(\alpha_n x - \omega t) \right\} \quad (6)$$

$$A_{1z} = \sum_{n=-\infty}^{\infty} \sum_{m=1}^{\infty} \left\{ [C_{3mn} \exp(\beta_{mn} y) + C_{4mn} \exp(-\beta_{mn} y)] \cos \gamma_m x \cdot \exp j(\alpha_n x - \omega t) \right. \\ \left. + [C'_{3mn} \exp(\beta_{mn} y) + C'_{4mn} \exp(-\beta_{mn} y)] \sin \gamma_m x \cdot \exp j(\alpha_n x - \omega t) \right\} \quad (7)$$

$$\beta_{mn}^2 = \alpha_n^2 + \gamma_m^2$$

(b) The conductor region (region 2),

$$\nabla^2 A_2 = \mu \sigma \left[\frac{\partial \bar{A}}{\partial t} - \bar{V} \times \nabla \times \bar{A} \right]$$

which gives

$$A_{2x} = \sum_{n=-\infty}^{\infty} \sum_{m=1}^{\infty} \left\{ [C_{11mn} \cos(E_{2mn} y) + C_{12mn} \sin(E_{2mn} y)] \cos \gamma_m x \right. \\ \left. \cdot \exp j(\alpha_n x - \omega t) \cdot \exp(E_{2mn} y) \right\} \quad (8)$$

$$E_{2mn} = -\mu \sigma V_y / 2$$

$$E_{2mn} = \frac{1}{2} \sqrt{4(-\alpha_n^2 - \gamma_m^2 + j\mu\sigma(\omega - V_x \alpha_n) + \mu\sigma V_y)}$$

$$A_{2x} = \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} \left\{ [C_{9mn} \cos(E_{2mn} y) + C_{10mn} \sin(E_{2mn} y)] \sin \gamma_m x \right. \\ \left. + \frac{j\alpha_n V_x}{E_{2mn}^2 - E_{2mn}^2} \{ C'_{9mn} \cos(E_{2mn} y) + C'_{10mn} \sin(E_{2mn} y) \} \right. \\ \left. \cdot \cos \gamma_m x \cdot \exp j(\alpha_n x - \omega t) \cdot \exp E_{1mn} y \right\} \quad (9)$$

(c) The back iron region (region 3),

$$\nabla^2 \bar{A}_3 = 0$$

The general solutions will be

$$A_{3x} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left[\{C_{5mn} \exp(\beta_{mn} y) + C_{6mn} \exp(-\beta_{mn} y)\} \sin \gamma_m x \cdot \exp j(\alpha_n x - \omega t) + \{C'_{5mn} \exp(\beta_{mn} y) + C'_{6mn} \exp(-\beta_{mn} y)\} \cos \gamma_m x \cdot \exp j(\alpha_n x - \omega t) \right] \quad (10)$$

$$A_{3z} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left[\{C_{7mn} \exp(\beta_{mn} y) + C_{8mn} \exp(-\beta_{mn} y)\} \cos \gamma_m x \cdot \exp j(\alpha_n x - \omega t) + \{C'_{7mn} \exp(\beta_{mn} y) + C'_{8mn} \exp(-\beta_{mn} y)\} \sin \gamma_m x \cdot \exp j(\alpha_n x - \omega t) \right] \quad (11)$$

BOUNDARY CONDITIONS

The following boundary conditions are used:

Boundary at the stator surface,

$$\frac{1}{\mu_0} \frac{\partial A_{1x}}{\partial y} = J_x(x_0) \quad ; \quad -\frac{1}{\mu_0} \frac{\partial A_{1x}}{\partial y} = J_x(x_0) \quad (12)$$

Boundary between region 1 and region 2,

$$\frac{\partial A_{1x}}{\partial x} = \frac{\partial A_{2x}}{\partial x} ; \quad \frac{\partial A_{1z}}{\partial x} = \frac{\partial A_{2z}}{\partial x} ; \quad \frac{\partial A_{1x}}{\partial y} = \frac{\partial A_{2x}}{\partial y} ; \quad \frac{\partial A_{1z}}{\partial y} = \frac{\partial A_{2z}}{\partial y} \quad (13)$$

Boundary between region 2 and region 3,

$$\frac{\partial A_{2x}}{\partial x} = \frac{\partial A_{3x}}{\partial x} ; \quad \frac{\partial A_{2z}}{\partial x} = \frac{\partial A_{3z}}{\partial x} ; \quad \frac{\partial A_{2x}}{\partial y} = \frac{\partial A_{3x}}{\partial y} ; \quad \frac{\partial A_{2z}}{\partial y} = \frac{\partial A_{3z}}{\partial y} \quad (14)$$

Boundary conditions at $y = -\infty$

$$A_{3x} = 0 ; \quad A_{3z} = 0 \quad (15)$$

The magnetic vector potentials in the three regions are obtained in terms of constants C_{mn} in the six equations (6), (7), (8), (9), (10) and (11). There are 12 unknown constants C_{mn} in these equations. By substitution of these six equations into the 12 equations (12) to (15) obtained from the boundary conditions and by equating the m^{th} and n^{th} harmonic of the $\sin \gamma_m z$ and $\cos \gamma_m z$ terms separately, the unknown constants C_{mn} can be determined. The long-winded manipulation of the evaluation of the constants C_{mn} is not shown here. The final solutions for the magnetic vector potential $\bar{A}$ in the three regions are as follows:

$$A_{1x} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \frac{[\beta_{mn}(E_{3mnx} - E_{2mnx} \cot E_{1mnx} \delta)]}{E_{4mnx}} \operatorname{ch} \beta_{mn}(y-\delta) + \operatorname{sh} \beta_{mn}(y-\delta) \right\} \\ \cdot \frac{\mu_0 a_{mnx} \exp j(\alpha_n x - \omega t)}{\Delta_{mnx} \beta_{mn}} \sin \gamma_m (x-x_0) \\ - \frac{V_x a_{mnx} \{P_x(\delta) - P_x(\delta)\}}{V_x \Delta_{mnx} \Delta_{mnx} \sigma} \operatorname{ch} \beta_{mn}(y-\delta) \cdot \cos \gamma_m (x-x_0) \exp j(\alpha_n x - \omega t) \quad (16)$$

$$A_{1x} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} - \left[\frac{\beta_{mn} (E_{3mnx} - E_{2mnx} \cdot \cot E_{2mnx} \delta) \operatorname{ch} \beta_{mn} (y-\delta) + \operatorname{sh} \beta_{mn} (y-\delta)}{E_{4mnx}} \right. \\ \left. \cdot \mu_0 a_{mnx} \cdot \cos \gamma_{mx} \cdot \exp j(a_n x - \omega t) \right] \quad (17)$$

$$A_{2x} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left[\frac{P_x(y) \mu_0 a_{mnx} \cdot \sin \gamma_{mx} \cdot \exp E_{1mn} (y-\delta) \cdot \exp j(a_n x - \omega t)}{\Delta_{mnx}} \right. \\ \left. - \frac{V_x a_{mnx} \cdot \exp E_{1mn} (y-\delta)}{V_x \Delta_{mnx} \sigma} \left\{ P_x(y) - P_x(y) \left(1 - \frac{P_x(\delta) - P_x(\delta)}{\Delta_{mnx}} \right) \beta_{mn} \operatorname{sh} \beta_{mn} (y-\delta) \right. \right. \right. \\ \left. \left. \cdot \cos \gamma_{mx} \cdot \exp j(a_n x - \omega t) \right\} \right] \quad (18)$$

$$A_{2x} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left[\frac{P_x(y) \cdot \mu_0 a_{mnx} \cdot \exp E_{1mn} (y-\delta)}{\Delta_{mnx}} \cos \gamma_{mx} \cdot \exp j(a_n x - \omega t) \right] \quad (19)$$

$$A_{3x} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left[\frac{-E_{2mnx} \mu_0 a_{mnx} \cdot \sin \gamma_{mx} \cdot \exp(\beta_{mn} y - E_{1mn} \delta) \cdot \exp j(a_n x - \omega t)}{E_{4mnx} \Delta_{mnx} \cdot \sin E_{2mnx}} \right. \\ \left. - \frac{V_x a_{mnx}}{\sigma V_x \Delta_{mnx}} \left\{ \frac{-E_{2mnx}}{\delta \cdot E_{4mnx} \cdot \sin E_{2mnx}} - \frac{E_{2mnx}}{\delta \cdot E_{4mnx} \cdot \sin E_{2mnx}} \right. \right. \right. \\ \left. \left. \left(1 - \frac{P_x(\delta) - P_x(\delta)}{\Delta_{mnx}} \right) \beta_{mn} \operatorname{sh} \beta_{mn} (y-\delta) \right\} \operatorname{ch} \gamma_{mx} \cdot \exp \beta_{mn} y \cdot \exp j(a_n x - \omega t) \right] \quad (20)$$

$$A_{3x} = \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left[\frac{E_{2mnx} \mu_0 a_{mnx} \cdot \cos \gamma_{mx} \cdot \exp(\beta_{mn} y - E_{1mn} \delta) \cdot \exp j(a_n x - \omega t)}{\delta \cdot E_{4mnx} \Delta_{mnx} \cdot \sin E_{2mnx}} \right] \quad (21)$$

The electromagnetic force of the machine is found by using numerical surface integration of Maxwell's second stress tensor. The surface of the integration is chosen to be the same as the surface of the conductor, thus the exact force density can be found. The three axis forces are:

$$\vec{F} = \int_{(S_1 + S_2)} \vec{T} \cdot d\vec{S}$$

and the three rotational couples are:

$$\vec{M} = \int_{(S_1 + S_2)} (\vec{r} \times \vec{T}) \cdot d\vec{S}$$

Where S_1 and S_2 are the top and bottom surfaces of the conductor.

ELECTROMECHANICAL STIFFNESS AND DAMPING COEFFICIENTS

The mechanical equation of motion for six degrees of freedom^{1,2} are:

$$[M] \ddot{[R]} + [C] \dot{[R]} + [K] [R] = [F_e]$$

where $[M] = \text{dia } [m, m, m, I_x, I_y, I_z]$

$$[R] = [x, y, z, \theta_x, \theta_y, \theta_z]$$

$[C] = C_{ij}$, a 6×6 damping coefficient matrix.

$[K] = K_{ij}$, a 6×6 stiffness coefficient matrix

$[F_e]$ = external forces or torque applied.

and m is the mass of the stator.

I_x, I_y and I_z are the moment of inertia of the stator along the three axes,

$C_{ij} = \frac{\partial F_i}{\partial R_j}$, i.e. the rate of change of force or torque in the i^{th} direction with respect to the change of velocity in the j^{th} direction,

$$K_{ij} = \frac{\partial F_i}{\partial \dot{R}_j}$$

$[K][R]$ are forces or torques generated due to the changes in the position of the stator, and $[C][\dot{R}]$ are the forces or torques generated due to the changes in velocity or angular velocity of the stator in six degrees of freedom.

In order to analogue the three axis forces and the three rotational couples to the mechanical system in six degrees of freedom, all the forces and rotational couples have to be evaluated for different stator positions and different velocities.

TEST AND RESULTS

A linear machine was built and tested under static conditions. The machine details are:

Stator length	0.53 m	Stator width	0.08 m
Pole pitch	0.112 m	Synchronous speed	79 m/s
Length of overhangs	0.045 m	Slot width	0.013 m
Slot pitch	0.019 m	Thickness of conductor	0.0032 m
Conductor width	0.24 m	Conductivity of conductor	3.5×10^7 S/m
Supply frequency	350 Hz	Current sheet density	24500 A/m

Fig. 5 shows the forces and couples under different stator positions, i.e. F_i versus R_j . The corresponding electromechanical stiffness coefficients (experimental) matrix is,

$$\begin{bmatrix} 0 & 88.7 & 9.32 & 2.3 & 2.5 & 0 \\ 0 & 863 & 10.0 & 1.6 & 0 & 12.3 \\ 0 & 0 & -14.6 & 2.2 & 0 & 0 \\ 0 & 0 & 6.28 & 0.55 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

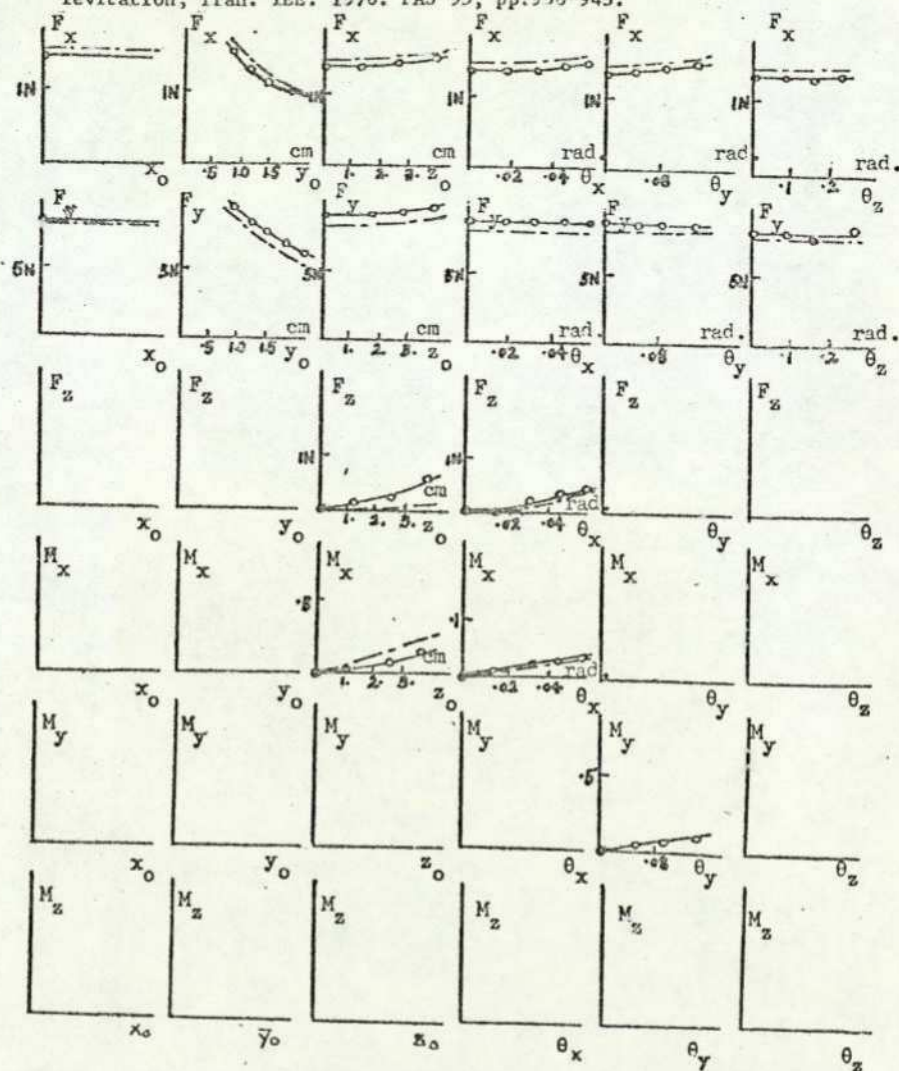
PRINCIPAL SYMBOLS

$\bar{A}$	magnetic vector potential, Wb/m
B_x, B_y, B_z	x, y and z components of magnetic flux density, Tesla
$[C]$	damping coefficient matrix
$\bar{F}$	force vector, N
$[K]$	stiffness coefficient matrix

$\overline{T}$	Maxwell's second stress tensor
$\overline{V}$	velocity vector, m/s
$\overline{\omega}$	angular velocity vector, rad/s
μ	permeability

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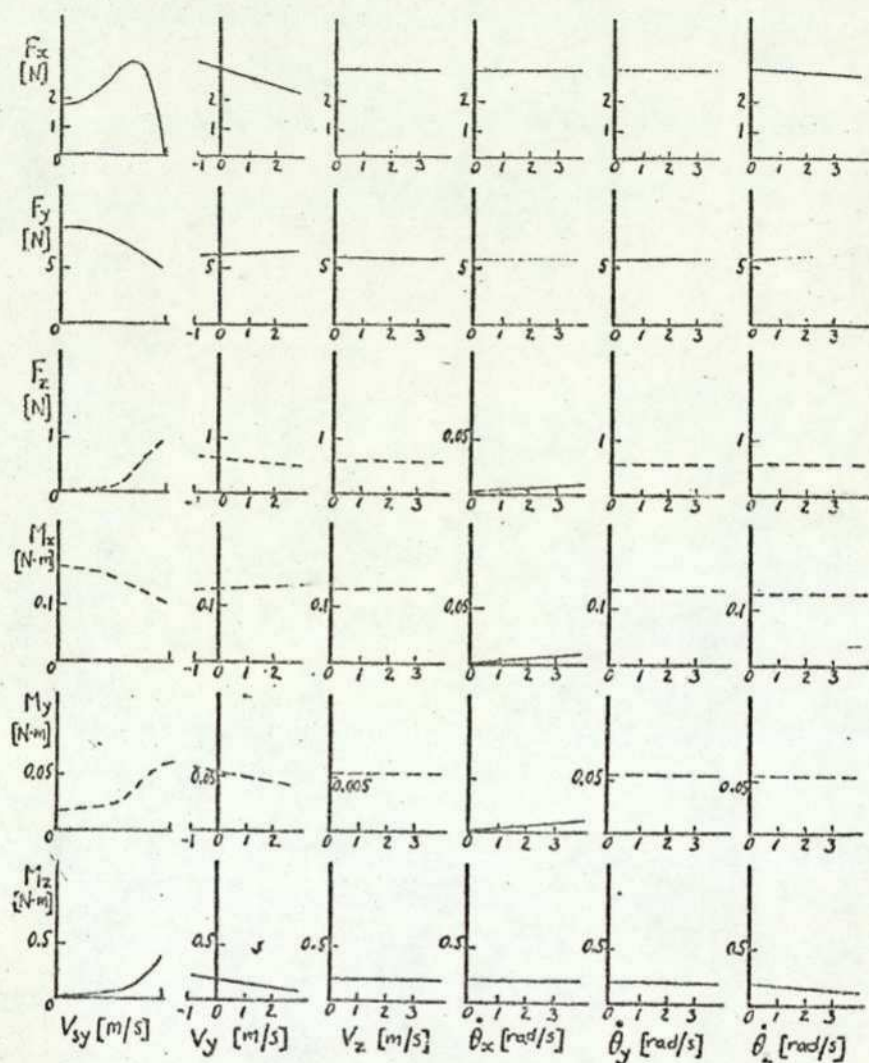


FIG. 6

Fig. 5. Stiffness. Variation of forces and couples as a function of linear and angular displacement. Experimental results shown thus.

Fig. 6. Damping. Variation of forces and couples as a function of linear and rotational velocities for a slip of 0.2. Broken lines indicate moments or forces over half the conductor plate.