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AN INVESTIGATION OF MULTI-CELL BRIDGES

A THESIS

PRESENTED FOR THE DEGREE

OF

DOCTOR OF PHILOSOPHY

BY

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CHAPTER 1

INTRODUCTION

In view of its flexural stiffness and its torsional rigidity, the hollow box-girder has become today the common type of section for steel as well as reinforced and prestressed concrete bridges. In the case of long span steel bridges it has replaced the traditional type built up of main girders, cross beams and stringers as it forms a coherent entity capable of resisting forces, bending moments and torque without any bracings. Moreover, its uniform shape is more appealing from the aesthetic point of view. The box girder is suitable for use as a simple or continuous beam as well as the deck system for suspension and cable stayed bridges. In this latter case stiffening girders are not required. It is also suitable for the cross section of arch bridges.

For narrow roads a single cell box section is sufficient, on the other hand for wider bridges two and more cells are necessary.

Some designers, however, prefer separate simple cells to multicell sections. Needless to say, the height of the box girder can be varied if necessary, as in the case of the old types of plate girders and trusses.

From the point of view of prefabrication the box section is very suitable as it possesses both flexural and torsional stiffness. Consequently it can be built of relatively large segments of adequate strength to resist their own weight during transportation and erection.

The box girder is eminently suited for bridges erected by the cantilever method which has thus become the common method of erection today due to the fact that it does not impede navigation of surface traffic and does not require obstructive scaffolding.

The thesis is concerned with a theoretical and experimental investigation into the structural behaviour of box bridges. A review of existing investigations indicated that though there was a wealth of information about the elastic behaviour of such bridges, the information on post elastic behaviour up to complete collapse was somewhat limited. Moreover, the majority of investigations had been restricted to single cell bridges which were in the main simply supported.

Apart from some preliminary investigations of single cell bridges this investigation is concerned with the structural behaviour of twin cell bridges both simply supported and continuous over two spans through the elastic range and up to complete collapse. As this work formed part of a more extensive investigation of multi cell box bridges in general being carried out at The City University by Professor J.E. Gibson, a standard size for the cell dimensions of the models was chosen so as to minimise cost and time of construction.

Part of the main investigation by Professor Gibson was to determine the accuracy with which micro concrete

bridge models were able to predict the structural behaviour of the prototype. To this end a four cell box bridge continuous over two spans that had been tested by Professor Scordelis at Berkeley was selected as the prototype as the experimental results and the range of test loads were fully detailed. The width of the Scordelis bridge was 12'0" and its total span was 72'0", the model being at quarter scale was thus 3'0" in width and 18'0" in total span. The geometry of the model cross section was identical to that of the prototype and consisted of four cells with side cantilevers. Steel templates to this cross sectional shape were constructed and formed the main framing for the shuttering for the model. This standard cross section is shown in Figure

This standard shuttering was used throughout this research as it could be used to form not only four cell cross sections but also single, double or treble over any reasonable span. The reinforcement consisted of weld mesh and could therefore be varied within the range of specifications supplied by the manufacturers.

In the present work the templates and shuttering were used to construct single and double cell boxes and the investigation served in a sense as a run up to that for the four cell continuous bridge.

The theoretical investigation of the structural behaviour of single and double cell structures in the elastic range has been fully covered in the last fifteen

years by many authors. The methods employed for linear elastic behaviour consisted in the main of beam, folded plate and shell theory in the continuum methods and more recently finite element, finite strip and finite difference. Reference to these will be given later.

It was thus decided to concentrate in the main on attempting to derive upper and lower bound solutions for the collapse of the structures and to correlate these with experimental values. The upper bound being defined as that load which would cause the structure continuing excessive deformation, i.e. it would be incapable of withstanding any further load. In this field the work of Spence and Morley was adapted and extended to cover the multi cell case.

The lower bound was defined for the purpose of this investigation as the load which initiated first yielding of the steel at some point in the structure. This proved a much more elusive quantity to determine, and this will again be examined in more detail.

CHAPTER 2

LITERATURE SURVEY

Historical Review

Originally reinforced concrete box bridges were designed using simple beam and St. Venant torsion theory. However, as the width of the webs was reduced to reduce self weight, then distortional and warping effects had to be considered. An excellent account of these elastic method of analyses is given by Maisel and Roll⁽¹⁾ in which the simple beam theory is used to design a specific type of box bridge and thereafter various methods are used to investigate distortion and warping effects.

The methods discussed by these authors fall into two main groups of elastic analysis. One in which the calculations may be performed by slide rule or a desk calculator and the other which requires a digital computer. Of the former methods distortion may be examined by Knittel's method⁽²⁾ whilst distortion and distortional warping may be analysed using methods due to Richmond⁽³⁾, Kupfer⁽⁴⁾ and Steinle⁽⁵⁾. Shear lag may be examined by either a method due to Reissner⁽⁶⁾ or Moffat and Dowling⁽⁷⁾.

Of the computerised methods the grillage theory as developed by West⁽⁸⁾ allows both longitudinal bending, St. Venant torsion and distortion to be examined. However, the following computer methods allow all of the above structural effects to be analysed. The folded plate theory as originally developed by Goldberg and Leve⁽⁹⁾ was used by Lo⁽¹⁰⁾ for a program to analyse multi box sections. Again using classical elastic methods Gibson^(11,12,13)

showed that simply supported box bridges may be analysed by a computer program based on cylindrical shell theory. A single cell box bridge with side cantilevers and thus comprising six shell segments thus required the solution of only 6×8 , i.e. 48 simultaneous equations.

The discretisation methods, namely finite element and finite strip were thus being considerably developed^(14,15). Mitwally, M.H. studied simply supported box girders under different types of loading. Two analytical methods, the finite element and the finite strip have been presented and compared with the experimental results for perspex and micro concrete models.

A paper presenting an approach to the analysis of bridge decks having a cellular transverse cross section with no diaphragm was presented by F. Sawko and R.S. Cope⁽¹¹⁾. It used a modified finite element technique and the solutions were obtained by a fully automatic computer program. Only one major stress - converging modes - was presented, and a considerable saving in the number of simultaneous equations was achieved. The degree of accuracy attained was assessed by comparison with empirical values and the programs were used to investigate the effect of distortional stiffness stress.

A review of analytical methods and general computer programs which have been developed at the University of California, for the analysis of box girder bridges was presented by Scordelis^(18,19). Several solutions based

on two general methods, the direct stiffness harmonic analysis and the finite element method were described. These solutions may be used to analyse multi cell box girder bridges which are straight, skew, curved or of arbitrary general geometry and under general loading. Numerical results from three examples involving a straight two-span continuous bridge, a single span skew bridge and a single-span curved bridge were given.

Differential equations and boundary conditions, relating warping displacements and rotations to the applied torsional load were developed for uniform beams with thin-walled multi cell cross sections by S.V. Benscotti and Pasadena⁽²⁰⁾. The loading distribution and end conditions were arbitrary and a formula resembling the flexural formula was given for the normal stress in terms of warping moments.

The effect of transverse stiffness in single cell and double cell box girders have been studied by Sisodiyn, Chali and Cheung⁽²¹⁾. Solid diaphragms as well as frames of beams were introduced at different cross sections and their effect on the behaviour of boxes was studied. They concluded that end diaphragms lead to some savings of the longitudinal reinforcements, while central diaphragms had a small beneficial effect in transmitting eccentric line loads from one web to another, and additional intermediate diaphragms had a negligible effect. They recommended that these diaphragms could be replaced by frames, even though they have less rigidity.

P. Nimelyiorgskul, R.P. Pama and L.P. Sing^(22,23) made a study to develop a method of analysis of the influence of intermediate diaphragms on box girders simply supported at the ends, and subjected to concentrated loads applied at the joints. The solution - due to a unit - vertical and horizontal loads for the cases without intermediate diaphragms was obtained and used as an influence coefficient to evaluate the effect of the diaphragms.

Compared to the vast amount of research on the elastic behaviour, little work has been done on the ultimate load analysis of box girder structures.

Swan and Williams⁽²⁴⁾ investigated the ultimate strength of simply supported reinforced prestressed concrete box beams subjected to combined bending, shear and torsion. Sixteen specimens (352 mm wide x 184 mm deep) were tested; two span lengths were used and two densities of orthogonal reinforcing mesh. Two additional tests were carried out on reinforced concrete specimens under pure torsion and subsidiary tests were completed to study behaviour under bending and shear.

G. Somerville, F. Roll and J. Caldinell⁽²⁵⁾ described the building and testing of a one-twelfth scale model, made of micro concrete of a typical span of a mancunian way. Data have been obtained on the diffusion of prestress through the section; the behaviour of the structure under its design loading and under the action of point loads on the cantilevers, and

the behaviour of the structure at ultimate load. It was found that the effective flange width to be used in calculating section properties varied along the span. The behaviour of the structure under its design loading was satisfactory. At ultimate load the structure was found to have a load factor of at least 3 on full line loading.

A method of developing the mode of collapse and its associated collapse load was produced by Morris N. Fialkow and F. Sawko⁽²⁶⁾. The principle of limited analysis is used to develop collapse modes with their associated collapse loading for simply supported reinforced concrete, prismatic folded plate structures of general cross section. With the modes of collapse considered, the overall structure during collapse, consists of rigid regions of finite extent separated by narrow yield bonds, or lines along which motion is feasible in accordance with the theory of plasticity. Expression for the loading associated with each collapse mode is developed. Because by upper bound theorem of limit analysis, the values of loading obtained is greater than, or equal to, the actual load which causes collapse, the mode which has been associated with the lowest collapse load has been sought.

An automatic iterative finite element procedure for the analysis of post-tensioned box girders subjected to combined bending and torsion in the uncracked and post-cracking range is presented by D.N. Trikha and A.D. Edwards⁽²⁷⁾. The analysis includes the non-linear

behaviour of the materials, the change in the structural topology due to the initiation and propagation of cracks, and makes provision for the gradual, but progressive, breakdown of bond. The result of the analysis (single cell box girders subject to different mid-span bending), moment/torsion moment ratio, was compared with experimental models. The models were twelve simply supported, but torsionally restrained, post-tensioned single-cell box girders that were tested to failure. The girders had a span of 2.74 m. In order to investigate their effect on warping strains and to distribute the end anchorage stress, rigid diaphragms (305 mm) thick, placed centrally over the supports, were provided at each end. The girders were prestressed by two (7 mm dia.) high tensile steel wires, one in each (19 mm) duct formed in the webs of the girders. The duct profile was trapezoidal, being horizontal in the segment and sloping in the outer ones. The mid-span bending moment/torsion moment ratio was achieved by applying at mid-span a vertical downward load to one web and a vertical upward load of smaller magnitude to the other web.

R.J.S. Spence and C.T. Morley^(28,29), presented a method of predicting the collapse load of simply supported single-cell box girders of deformable cross-sections subjected to eccentric loading. Thirteen models were described of rectangular single-cell box girders of uniform cross-section and simply supported over one span

loaded by concentrated loads at midspan. Cross-section, longitudinal and transverse strength and eccentricity of loading were varied. In each case the collapse load and the deformation rates of collapse were determined. A form of ultimate load theory based on the plastic theory of structures was presented and compared with the experimental results. Some adaptations of the theory to derive a design formula for minimum transverse reinforcement were suggested. Some attempts were made to obtain a lower bound on the collapse load of rectangular box beams, by postulating equilibrium fields of stresses not violating the yield criteria.

Bouwkamp, Scordelis and Wasti^(30,31,32,33,34,35,36) made several tests on a 4 cell box structure (1 : 2.82) scale model of a typical two-lane bridge in California, having two (101.5 ft) spans with a single column bent at the centre, providing the interior support. The purpose of the paper was to investigate the response of the box girder bridge to point live loading, producing together with the dead load, nominal working stresses in the steel reinforcement. Theoretical and experimental results were compared for reactions, deflections and moments due to these point loads, both before and after the bridge has been subjected to various overload stress levels. The theoretical part was checked by the program NUPD13, FINPLAZ and IDFRAME which considered the structure to be elastic, uncracked and homogeneous. One of the spans

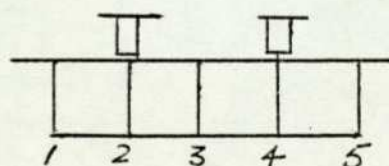
had a diaphragm while the other was kept without. Different loading patterns were applied, but only three cases were compared. The application of point load on the exterior girder of the undiaphragmed girder, another at the diaphragmed one, and a combination of both response of the structure before and after overload stress level were done. Agreement between theoretical and experimental total reactions at both ends and the central footings were reasonable. Differences between experiment and theory for the transverse distribution of the total reactions to each of the five load cells under the girder webs at the two ends of the bridge were larger than those for the total reaction. As for deflection, the theory predicts the general distribution of deflection quite well, if the theoretical values based on an uncracked section were multiplied by a factor of about 1.5 to account for cracking at this stress level. Longitudinal distributions of total moment were in good agreement between theory and experiment.

The second part was the response after overload stress level. The load was applied on the undiaphragmed span at mid-span on either of the two exterior girders. The relation between total centre end supports reactions and applied load were unchanged after all conditioning overloads. It was evident that after the working stresses conditioning load, the theory based on uncracked sections predicted the nature of the transverse distribution of

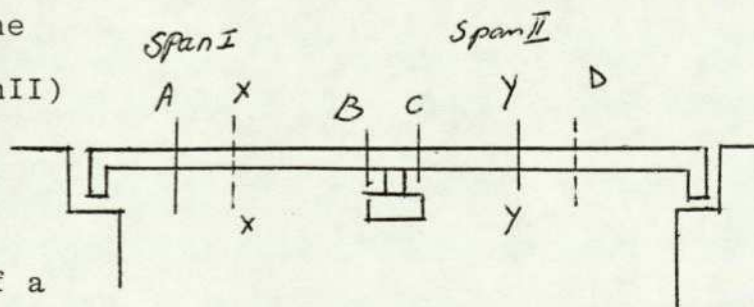
deflection between the two loadings, with the experimental values being about 60% higher than theory. But after higher and higher conditioning overloads, the theory could no longer be used to predict the transverse distribution deflections. Experimental and theoretical values were in agreement for moments based on external reactions (1% to 5%), except after a certain conditioning load where the differences were larger.

Bouwkamp, Scordelis and Wasti continued testing the model up to failure. Their particular interest was how much, if any, would the overloads change the distribution and magnitude of reactions, deflections, strains, total moment at a section, and percentage distribution of the total moment at a section to each girder; also the degree of linearity of response and the changes in stiffness of the bridge during and after each of the conditioning overloads.

The loading conditions for this latter part of the test were as shown in Figure on the two spans respectively.



Yielding of the bottom slab steel started in the undiaphragmed span (span II) at section Y. It was observed that the yield pattern took the form of a local failure on either side of girder 2 at section Y.



The girder appeared to push downward, causing a major crack in the bottom slab on the side near girder 1 that extended longitudinally about 5 ft in the direction towards the centre. In view of the local failure in span II it was decided to freeze the deflection there by the insertion at girders I, III and V of solid timber struts to maintain the deflection as it was. The load was inserted on section X of the diaphragmed span only. The load was increased until a uniform yield at mid-span section occurred. Both exterior girders showed longitudinal shear cracks at the junction of the top slab and the girders. Any further attempt to increase the load merely caused an increase in deflection and a widening of the cracks at section X.

On plotting the total load/deflection graphs for girders 1 and 5 it could be seen that the structure behaved in an almost linear elastic fashion.

SUMMARY

From all the previous work it can be seen that there exists a vast amount of research on the theoretical analysis of multi cell box girders in the elastic range. This theoretical work is partially confirmed by experimental evidence, but little work has been conducted in the non-elastic to failure range. Further, the research carried out in this non-elastic range was for single cell simply supported girders. Little information exists for

the case of multi cell simply supported and continuous box bridges.

The aim of this thesis is to investigate the collapse behaviour and failure mechanism of double-cell box bridges for the two cases of simple support and continuity over two spans.

Initial investigations of single-cell boxes were needed for comparison of previous results and theory.

CHAPTER 3

3.1 Elastic Analyses of Multi-Cell Box Bridges by Computer Methods

The methods of analysis outlined in Chapter 2 may all be suitably used in the development of computer programs. Of these, special mention should be made of the following:

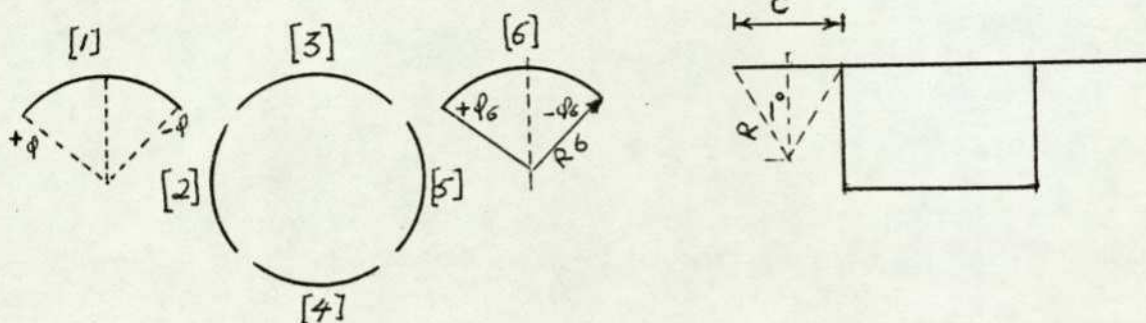
- (a) Goldberg and Leve used elastic theory to analyse prismatic folded plate structures. In this method two distinct sets of differential equations were used, namely the biharmonic derived from the examination of a membrane state of stress and the plate equation for bending. The latter is derived from consideration of the bending and shear stress resultants acting on the plate. The two differential equations were solved independently in terms of two sets of four arbitrary constants which were then determined so as to satisfy the boundary conditions of the plate problem.
- (b) This method was then extended by Lo and Scordelis to analyse single and multi cell simply supported box bridges and Lo developed a computer program for the multi cell case.
- (c) Various authors have subsequently developed similar programs again using plate methods.
- (d) Apart from the elastic methods using plate theory, both Finite Element and Finite Strip methods have been used to develop computer programs (Zien and Cheung).

- (e) As a side line to multi shell elastic theory, Gibson devised a closed multi shell program that with modification could be used to analyse box bridges.

All of the above mentioned programs are for elastic analyses and thus for elastic investigations it is immaterial which are used, the end analyses are of necessity almost identical.

In the preliminary theoretical investigations in this thesis it was decided to try out an elastic investigation for some of the single cell box bridge models and for this purpose Gibson's program was used.

The program which uses cylindrical shell theory for the closed system of shells shown in Figs. (3A and 3B) may be used to analyse the flat plate box bridge in (b)



by suitably modifying the input data for ϕ for each shell, i.e. by reducing ϕ to 1° and suitably modifying the radius R to give the required chord width for each shell in turn by using the following equation

$$C = 2R \sin 1^\circ \quad R = C/2 \sin 1^\circ = C/.0349 = \underline{28.65C}$$

This causes the shell elements to degenerate into plate elements and hence form the required bridge cross section.

Because of space shortage only a few of these analyses will be presented. It is to be observed that since this was only a comparison between different loading conditions only the first order Fourier term was used

In order that the significance of the output results may be fully appreciated, the sign convention and points of arc at which values were computed are shown in Fig. (3C).

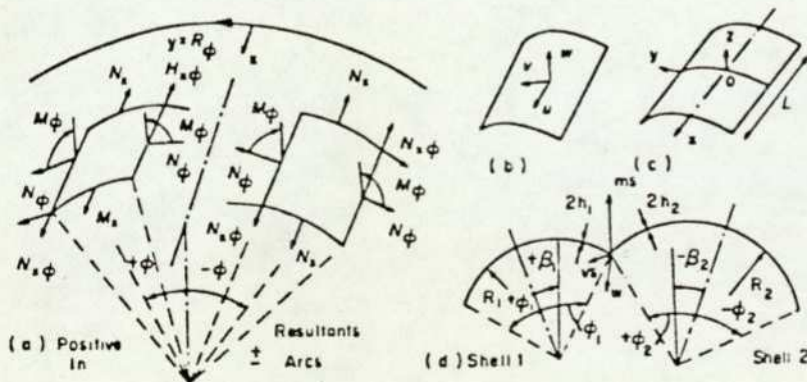


Figure (3C)

The figure indicates that each complete shell is divided into two half arcs by a centre line. The positive arc ($+\phi$) being to the left and the negative arc ($-\phi$) being to the right. Further deflections and stress resultants are given for the quarter points of each arc $+\phi$, $3/4\phi$, $+\phi/2$, $+\phi/4$ and 0. They were tabulated in the tables for the case of $+\phi$, $+\phi/2$, 0, $-\phi/2$, $-\phi$ only.

Although the program generates all deflections and stress resultants, yet certain of these only were considered critical in design. These are $M\phi$, $N\phi$, N_x , $N_x\phi$ and W .

All these stress resultants are forces per unit length, and are shown in Fig. (3D).

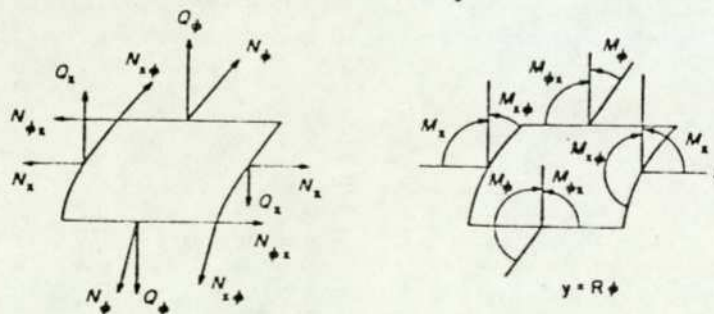


Fig. (3D)

Where:

N_x and N_ϕ are direct force resultants acting in the x and y directions respectively, and are positive when in tension as shown.

$N_{x\phi}$ are the shear resultants acting as shown in Fig. (3C).

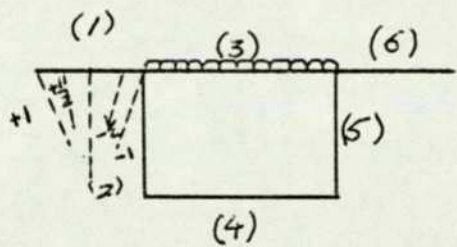
M_ϕ are moments/unit length acting about the x axis and are +ve as shown

and W is +ve when directed upwards.

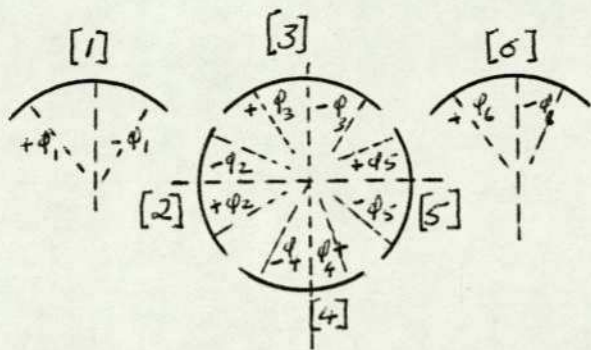
Different cases of symmetrical and unsymmetrical loadings were applied to a single cell and the results were tabulated as will be seen.

3.2.A Dimensions of Box Bridge and Loading Conditions

(I) Uniform Bending on a 120 in Box Bridge



Loading on Plate (3) only



	P	L	R	2h	ϕ	β	γ	E
(1)	0	120	86	0.5	1	45	0	1
(2)	0	120	129	0.75	1	135	0	1
(3)	10	120	215	0.5	1	45	0	1
(4)	0	120	215	0.5	1	-135	0	1
(5)	0	120	86	0.5	1	45	0	1

TABLE 3.1.A

Stress Resultants and Deflections

Element (1)

Angle	M ϕ lb in/in	N ϕ lb/in	Nx lb/in	W/E	Nx ϕ lb/in
$+\phi$	-0.0	-0.0	-2066.2	-3.3×10^6	0.0
$+\phi/2$	-0.0	0.4	-2012.1	-3.3×10^6	40.7
0	-0.0	1.6	-2084.2	-3.3×10^6	81.6
$-\phi/2$	-0.0	3.6	-2084.5	-3.3×10^6	122.6
$-\phi$	-0.1	6.4	-2079.0	-3.3×10^6	163.5

Table 3.1.B

Element (2)

Angle	M ϕ	N ϕ	Nx	W/E	Nx ϕ
$+\phi$	1.6	-0.0	4438.4	57875.6	-291.2
$+\phi/2$	-12.1	-10.3	2545.9	30136.4	-394.1
0	-25.7	-22.7	657.7	1961.0	-441.3
$-\phi/2$	-39.1	-35.7	-1229.4	-27140.2	-432.9
$-\phi$	-52.1	-44.7	-3118.6	-57650.5	-368.9

Table 3.1.C

Element (3)

Angle	M ϕ	N ϕ	Nx	W/E	Nx ϕ
$+\phi$	-52.3	-5.4	-2079.0	-3.3×10^6	-205.4
$+\phi/2$	14.4	-13.0	-2091.8	-3.31×10^6	-102.9
0	36.6	-15.5	-2096.0	-3.32×10^6	-0.0
$-\phi/2$	14.4	-13	-2091.8	-3.31×10^6	102.9
$-\phi$	-52.3	-5.4	-2079.0	-3.3×10^6	205.4

Table 3.1.D

Stress Resultants and Deflections

Element (4)

Angle	M ϕ	N ϕ	Nx	W/E	Nx ϕ
+ ϕ	1.6	11	2958.9	3.3×10^6	291.2
+ $\phi/2$	2.2	21.7	2965.2	3.3×10^6	145.7
0	2.4	25.3	2967.3	3.3×10^6	0.0
- $\phi/2$	2.2	21.7	2965.2	3.3×10^6	-145.7
- ϕ	1.6	11	2958.9	3.3×10^6	-291.2

Table 3.1.E

Element (5)

Angle	M ϕ	N ϕ	Nx	W/E	Nx ϕ
+ ϕ	-52.2	-47.7	-3118.6	-57650.8	368.9
+ $\phi/2$	-39.1	-35.7	-1229.4	-27140.5	432.9
0	-25.7	-22.7	657.7	1960.7	441.3
- $\phi/2$	-12.1	-10.3	2545.9	30136.1	394.1
- ϕ	1.6	-0.0	4438.4	57875.3	291.2

Table 3.1.F

Element (6)

Angle	M ϕ	N ϕ	Nx	W/E	Nx ϕ
+ ϕ	-0.1	6.4	-2079.0	-3.3×10^6	-163.5
+ $\phi/2$	-0.0	3.6	-2084.5	-3.3×10^6	-122.6
0	-0.0	1.6	-2084.2	-3.3×10^6	- 81.6
- $\phi/2$	-0.0	0.4	-2078.1	-3.3×10^6	- 40.7
- ϕ	-0.0	0.0	-2066.2	-3.3×10^6	0.0

Table 3.1.G

3.2.B Comments on Case I of Uniform Bending

This is a case of bending where the load is symmetrically applied only on the middle plate No. (3). It could be seen that the values of shear stresses in the tables satisfies the variation around perimeter of shear stresses at cross section by the Engineering Bending Theory due to bending. See Figure 3.2.A.

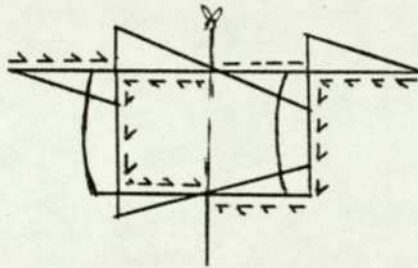


Figure 3.2.A

The values of W_x which are the longitudinal direct force resultants are dominant and have -ve values at the top flange and +ve at the bottom flange, with a variation from -ve to +ve along the two necks which satisfies the variation around perimeter for normal stresses due to that loading condition. See Figure 3.2.B.

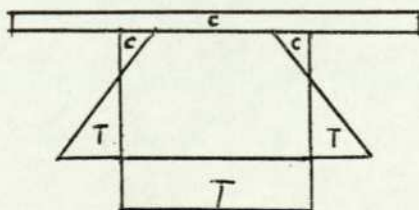
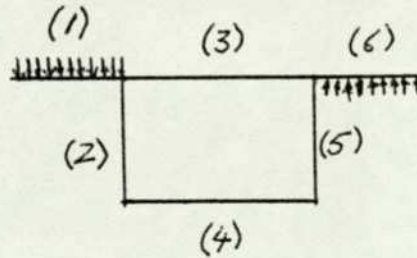


Figure 3.2.B

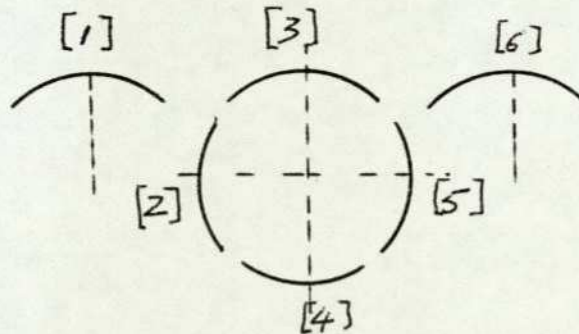
Values of vertical deflection are constant along the width of the cross-section for both top and bottom flanges. Values of $N\phi$ are small compared with the dominant values of Nx as would be expected.

3.2.C Dimensions of Box Bridge and Loading Conditions

(II) Pure Torsion on a 120 in Box Bridge



Torsion Loading



	P	L	R	2h	ϕ	β	γ	E
(1)	10	120	86	0.5	1	45	0	1
(2)	0	120	129	0.75	1	135	0	1
(3)	0	120	215	0.5	1	45	0	1
(4)	0	120	215	0.5	1	-135	0	1
(5)	0	120	129	0.75	1	-45	0	1
(6)	-10	120	86	0.5	1	45	0	1

Table 3.2.A

Stress Resultants and Deflections

Element (1)

Angle	$M\phi$	$N\phi$	Nx	W/E	$Nx\phi$
$+\phi$	0.0	-0.0	32.2	-88972.3	0.0
$+\phi/2$	-3.4	-0.0	23.6	-77142.2	-0.5
0	-14.0	-0.0	15.2	-65559.3	-0.9
$-\phi/2$	-31.8	-0.0	6.9	-54706.0	-1.1
$-\phi$	-56.7	-0.1	-1.2	-45633.6	-1.2

Table 3.2.B

Element (2)

Angle	$M\phi$	$N\phi$	Nx	W/E	$Nx\phi$
$+\phi$	-30.0	-8.8	51.2	-5191.1	-217.1
$+\phi/2$	- 6.0	-15.2	38.0	3451.7	-218.4
0	18.2	-21.6	24.8	11879.4	-219.3
$-\phi/2$	42.6	-28.1	11.5	20963.2	-219.8
$-\phi$	67.3	-34.0	-1.8	31583.1	-220.0

Table 3.2.C

Element (3)

Angle	$M\phi$	$N\phi$	Nx	W/E	$Nx\phi$
$+\phi$	10.6	21.7	-1.2	-44503.7	-221.2
$+\phi/2$	5.4	10.4	-0.7	-23157.0	-221.1
0	0.0	0.0	-0.0	0.0	-221.1
$-\phi/2$	-5.4	-10.9	0.7	23157.1	-221.1
$-\phi$	-10.6	-21.7	1.2	44503.7	-221.2

Table 3.2.D

Stress Resultants and Deflections

Element (4)

Angle	$M\phi$	$N\phi$	Nx	W/E	$Nx\phi$
$+\phi$	30.0	21.2	-34.2	-44865.1	-217.1
$+\phi/2$	15.0	10.6	-17.1	-24974.1	-215.8
0	0.0	-0.0	- 0.0	- 0.0	-215.4
$-\phi/2$	-15.0	-10.0	17.1	24974.0	215.8
$-\phi$	-30.0	-21.2	34.2	44865.1	-217.1

Table 3.2.E

Element (5)

Angle	$M\phi$	$N\phi$	Nx	W/E	$Nx\phi$
$+\phi$	-67.3	34.6	1.2	-31583.1	-220.0
$+\phi/2$	-42.6	28.1	-11.5	-20963.2	-219.8
0	-18.2	21.6	-24.8	-11879.3	-219.3
$-\phi/2$	6.0	15.2	-38.0	- 3451.7	-218.4
$-\phi$	30.0	8.8	-51.2	5191.1	-217.1

Table 3.2.F

Element (6)

Angle	$M\phi$	$N\phi$	Nx	W/E	$Nx\phi$
$+\phi$	56.7	0.1	1.2	45633.7	-1.2
$+\phi/2$	31.8	0.0	-6.9	54706.1	-1.1
0	14.0	0.0	-15.2	65599.3	-0.9
$-\phi/2$	3.4	0.0	-23.6	77142.3	-0.5
$-\phi$	-0.0	-0.0	-32.2	88972.3	-0.0

Table 3.2.G

3.2.D Comments on Case II of Pure Torsion

This is a case of pure torsion, where the box is loaded by applying two equal and opposite loads on plate (1) and plate (6) respectively.

It can be seen from the table that the values of $N_x\phi$ are constant and uniformly distributed along the edges of the plates 2, 3, 4 and 5 respectively, which are *more or less* members like those assumed by St. Venant.

See Fig. 3.2.C.

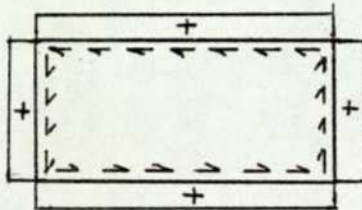


Fig. 3.2.C.

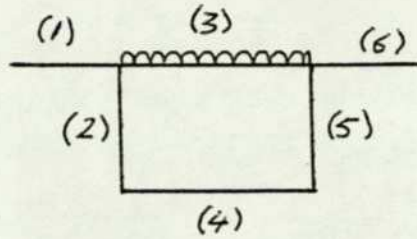
Here, values of N_x are small compared with those obtained later on for the case of a symmetrical load.

$N\phi$ are in both cases small compared with the other stresses.

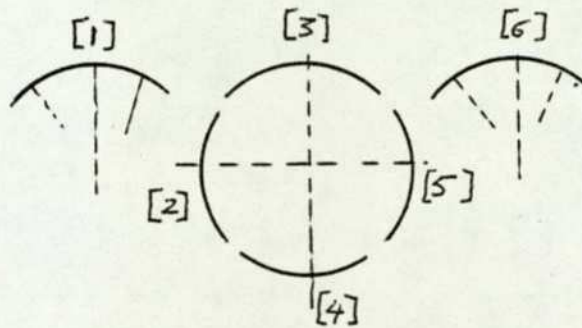
Deflections are rather small in this case of loading compared with those obtained for the symmetrical load case.

3.3.A Dimensions of Box Bridge and Loading Conditions
Span Reduced to 60 in

(III) Uniform Bending on a 60 in. long Box Bridge



Loading Condition



	P	L	R	2h	ϕ	β	γ	E
(1)	0	60	86	0.5	1	45	0	1
(2)	0	60	129	0.75	1	135	0	1
(3)	10	60	215	0.5	1	45	0	1
(4)	0	60	215	0.5	1	-135	0	1
(5)	0	60	129	0.15	1	-45	0	1
(6)	0	60	86	0.5	1	45	0	1

Table 3.3.A

Stress Resultants and Deflections

Element (1)

Angle	M ϕ	N ϕ	Nx	W/E	Nx ϕ
+ ϕ	0.0	-0.0	-514.5	211055.1	0.0
+ $\phi/2$	-0.1	0.4	-518.2	-212654.1	20.3
0	-0.2	1.6	-521.6	-214258.9	40.7
- $\phi/2$	-0.3	3.6	-524.6	-215872.3	61.3
- ϕ	-0.3	6.4	-527.4	-217499.2	81.9

Table 3.3.B

Element (2)

Angle	M ϕ	N ϕ	Nx	W/E	Nx ϕ
+ ϕ	2.2	-0.0	1126.4	3977.1	-145.0
+ $\phi/2$	-11.5	-10.2	643.8	3158.0	-197.2
0	-25.2	-22.7	165.6	1954.5	-221.0
- $\phi/2$	-38.3	-35.7	-311.6	-215.9	-216.7
- ϕ	-52.2	-47.7	-791.1	-3754.3	-184.2

Table 3.3.C

Element (3)

Angle	M ϕ	N ϕ	Nx	W/E	Nx ϕ
+ ϕ	-52.7	-6.5	-527.4	-217497.2	-102.3
+ $\phi/2$	13.6	-14.0	-519.8	-226236.5	-50.9
0	35.5	-16.5	-517.3	-231625.4	0.0
- $\phi/2$	13.6	-14.0	-519.8	-226236.5	50.9
- ϕ	-52.7	-6.5	-527.4	-217497.2	102.3

Table 3.3.D

Stress Resultants and Deflections

Element (4)

Angle	$M\phi$	$N\phi$	N_x	W/E	$N_x\phi$
$+\phi$	2.2	12.0	751.0	217320.4	145.0
$+\phi/2$	2.8	22.7	736.6	215893.5	72.0
0	3.0	26.2	731.8	215394.7	0
$-\phi/2$	2.8	22.7	736.6	215893.5	-72.0
$-\phi$	2.2	12.0	751.0	217320.4	-145.0

Table 3.3.E

Element (5)

Angle	$M\phi$	$N\phi$	N_x	W/E	$N_x\phi$
$+\phi$	-52.3	-47.7	-791.1	-3754.3	184.3
$+\phi/2$	-38.8	-35.7	-311.6	-215.9	216.7
0	-25.2	-22.7	165.6	1924.5	221.0
$-\phi/2$	-11.5	-10.2	643.8	3158.0	197.2
$-\phi$	2.2	-0.0	1126.4	3977.1	145.0

Table 3.3.F

Element (6)

Angle	$M\phi$	$N\phi$	N_x	W/E	$N_x\phi$
$+\phi$	-0.3	6.4	-527.4	-217498.7	-81.9
$+\phi/2$	-0.3	3.6	-524.6	-215871.7	-61.3
0	-0.2	1.6	-521.6	-214258.3	-40.7
$-\phi/2$	-0.1	0.4	-518.2	-212654.1	-20.3
$-\phi$	0.0	0.0	-514.5	-211054.5	-0.0

Table 3.3.G

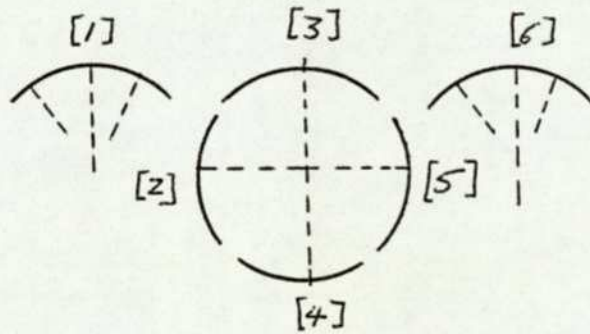
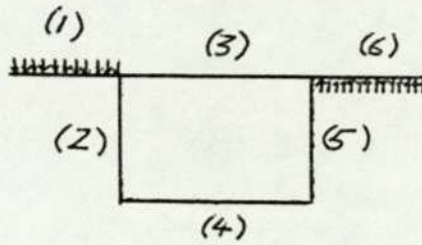
3.3.B Comments on Shorter Box due to Bending

The distribution of stress does not change from the previous longer box. However, a decrease in the values of N_x and $N_x\phi$ to approximately half the original values obtained from the previous box could be seen from the tables.

However, the values of $N\phi$ do not show that difference and remained the same. The decrease of length gave the box a strength resistance to the longitudinal stresses and decreased the effect of shear stress along its cross section.

3.3.C Dimensions of Box Bridge and Loading Conditions
Span 60 in

(IV) Pure Torson on a 60 in. Box Bridge



	P	L	R	2h	ϕ	β	γ	E
(1)	10	60	86	0.5	1	45	0	1
(2)	0	60	129	0.75	1	135	0	1
(3)	0	60	215	0.5	1	45	0	1
(4)	0	60	215	0.5	1	-135	0	1
(5)	0	60	129	0.75	1	-45	0	1
(6)	-10	60	86	0.5	1	45	0	1

Table 3.4.A

Stress Resultants and Deflections

Element (1)

Angle	M ϕ	N ϕ	Nx	W/E	Nx ϕ
$+\phi$	0.0	0.0	-2.0	-36949.1	0.0
$+\phi/2$	-3.2	0.0	-6.8	-30702.0	0.2
0	-13.6	0.0	-11.4	-24663.0	0.5
$-\phi/2$	-31.3	0.0	-15.9	-19303.2	1.1
$-\phi$	-56.2	0.1	-20.2	-15846.7	1.8

Table 3.4.B

Element (2)

Angle	M ϕ	N ϕ	Nx	W/E	Nx ϕ
$+\phi$	-28.6	-8.4	62.6	447.5	-108.2
$+\phi/2$	- 5.4	-14.9	39.3	786.4	-111.3
0	17.8	-21.5	16.2	930.5	-112.9
$-\phi/2$	41.4	-28.2	-7.0	1718.3	-113.2
$-\phi$	65.6	-34.8	-30.3	4000.6	-112.1

Table 3.4.C

Element (3)

Angle	M ϕ	N ϕ	Nx	W/E	Nx ϕ
$+\phi$	9.4	81.4	-20.2	-15697.4	-110.3
$+\phi/2$	4.7	10.7	-10.1	- 8648.8	-108.8
0	0.0	0.0	0.0	0.0	-108.3
$-\phi/2$	-4.7	-10.7	10.1	8648.8	-108.8
$-\phi$	-9.4	-21.4	20.2	15697.4	-110.3

Table 3.4.D

Stress Resultants and Deflections

Element (4)

Angle	$M\phi$	$N\phi$	Nx	W/E	$Nx\phi$
$+\phi$	28.6	20.7	-41.8	-15675.7	-108.2
$+\phi/2$	14.2	10.3	-20.8	-10242.5	-105.2
0	0.0	0.0	0.0	0.0	-104.2
$-\phi/2$	-14.2	-10.3	20.8	10242.6	-105.2
$-\phi$	-28.6	-20.7	41.8	-15675.7	-108.2

Table 3.4.E

Element (5)

Angle	$M\phi$	$N\phi$	Nx	W/E	$Nx\phi$
$+\phi$	-65.6	34.2	30.3	-4000.6	-112.2
$+\phi/2$	-41.4	28.2	7.0	-1718.3	-113.2
0	-17.8	21.5	-16.2	- 930.4	-112.9
$-\phi/2$	5.4	14.9	-39.3	- 786.4	-111.3
$-\phi$	-28.6	8.4	-62.6	- 447.4	-108.2

Table 3.4.F

Element (6)

Angle	$M\phi$	$N\phi$	Nx	W/E	$Nx\phi$
$+\phi$	56.2	-0.1	20.2	15846.7	1.8
$+\phi/2$	31.3	0.0	15.9	19393.3	1.1
0	13.6	0.0	11.4	24663.0	0.5
$-\phi/2$	3.2	0.0	6.8	30702.6	0.2
$-\phi$	0.0	0.0	2.0	36949.1	0.0

Table 3.4.G

3.3.D Comments on Shorter Box due to Pure Torsion

Again we can see from the results obtained that the values of $Nx\phi$ are more or less constant along the whole perimeter of the cross section, although its values decreased by about half of the previous results obtained from the longer box.

Values of Nx and $N\phi$ remain very small compared with the values of Nx obtained from the symmetrically loaded case.

3.4 Box Bridge with a Constant Thinner Thickness of Cross-Section

Another set of values of N_x , N_ϕ , $N_x\phi$ and M , are presented for a case of a thinner cross-section with a constant thickness for the whole cross-section.

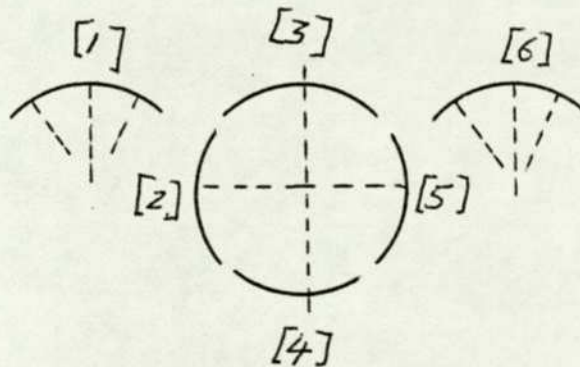
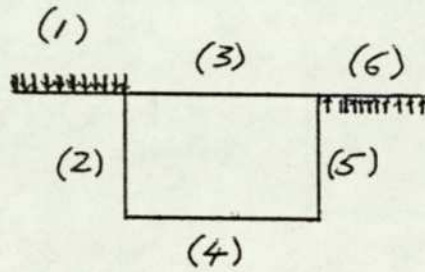
Three cases of loading have been dealt with:

- A. A case of pure torsion where the load is applied on plate (1) and (6).
- B. A case of Uniform Bending where the load is applied on plate (1), (3) and (6).
- C. A case of uniform bending where the load is applied

The results obtained have been tabulated for each Element and are presented hereafter.

3.4 Dimensions of Box Bridge and Loading Conditions with a Constant Thickenss of 1 for all Cross-sections

(I) Pure Torsion on a 120" Box Bridge



	P	L	R	2h	ϕ	β	γ	E
(1)	10	120	86	0.1	1	45	0	1
(2)	0	120	129	0.1	1	135	0	1
(3)	0	120	215	0.1	1	45	0	1
(4)	0	120	215	0.1	1	-135	0	1
(5)	0	120	129	0.1	1	-45	0	1
(6)	-10	120	86	0.1	1	45	0	1

Table 3.5.A

Stress Resultants and Deflections

Element (1)

Angle	$M\phi$	$N\phi$	Nx	W/E	$Nx\phi$
$+\phi$	0.0	0.0	-171.0	-3.9×10^6	-0.0
$+\phi/2$	-3.5	0.0	-154.8	-3.1×10^6	3.2
0	-14.2	0.1	-137.3	2.33×10^6	6.1
$-\phi/2$	-32.0	0.3	-118.9	1.65×10^6	8.6
$-\phi$	-57.1	0.5	-99.8	1.19×10^6	10.7

Table 3.5.B

Element (2)

Angle	$M\phi$	$N\phi$	Nx	W/E	$Nx\phi$
$+\phi$	-32.6	-11.1	219.2	8.5×10^5	-210.7
$+\phi/2$	-11.5	-17.4	139.4	4.79×10^5	-216.0
0	9.7	-23.8	59.5	-65790.6	-218.9
$-\phi/2$	31.2	-30.3	-20.2	-4.6×10^5	-219.5
$-\phi$	53.0	-36.1	-99.8	-3.84×10^5	-217.7

Table 3.5.C

Element (3)

Angle	$M\phi$	$N\phi$	Nx	W/E	$Nx\phi$
$+\phi$	-4.0	20.0	-99.8	-1.23×10^6	-207.0
$+\phi/2$	-1.9	9.9	-50.1	-5.77×10^5	-203.3
0	-0.0	0.0	0.0	0.0	-202.1
$-\phi/2$	1.9	-9.9	50.1	5.77×10^5	-203.3
$-\phi$	4.0	-20.0	99.8	1.23×10^6	-207.0

Table 3.5.D

Stress Resultants and Deflections

Element (4)

Angle	$M\phi$	$N\phi$	N_x	W/E	$N_x\phi$
$+\phi$	32.6	20.0	-219.2	-1.29×10^6	-210.7
$+\phi/2$	16.3	9.9	-109.9	-9.919×10^5	-202.6
0	0.0	0.0	0.0	-0.7	-199.9
$-\phi/2$	-16.3	-9.9	109.9	9.919×10^5	-202.6
$-\phi$	-32.6	-20.0	219.2	1.29×10^6	-210.7

Table 3.5.E

Element (5)

Angle	$M\phi$	$N\phi$	N_x	W/E	$N_x\phi$
$+\phi$	-53.0	36.8	99.8	3.84×10^5	-217.7
$+\phi/2$	-31.2	30.2	20.2	4.626×10^5	-219.5
0	- 9.7	23.8	-59.5	65790.5	-218.9
$-\phi/2$	11.5	17.4	139.4	-4.79×10^5	-216.0
$-\phi$	32.6	11.1	-219.2	-8.5×10^5	-210.7

Table 3.5.F

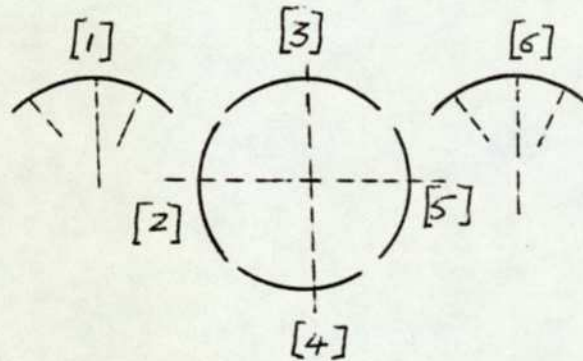
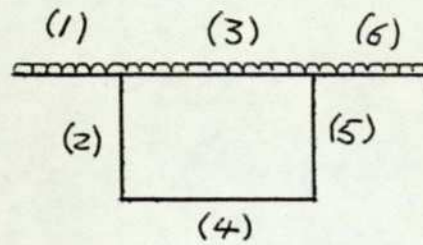
Element (6)

Angle	$M\phi$	$N\phi$	N_x	W/E	$N_x\phi$
$+\phi$	57.1	-0.5	99.8	1.19×10^6	-10.7
$+\phi/2$	32.0	-0.3	118.9	1.65×10^6	8.6
0	14.2	-0.1	137.3	2.33×10^6	6.1
$-\phi/2$	3.5	0.0	154.8	3.1×10^6	3.2
$-\phi$	0.0	0.0	171.0	3.91×10^6	0.0

Table 3.5.G

3.4.B Dimensions of Box Bridge and Loading Conditions
Constant Thickness

(II) Uniform bending on the whole breadth of the flange
on a 120" long box



	P	L	R	2h	ϕ	β	γ	E
(1)	10	120	86	0.1	1	45	0	1
(2)	0	120	129	0.1	1	135	0	1
(3)	10	120	215	0.1	1	45	0	1
(4)	0	120	215	0.1	1	-135	0	1
(5)	0	120	129	0.1	1	-45	0	1
(6)	10	120	86	0.1	1	45	0	1

Table 3.6.A

Stress Resultants and Deflections

Element (1)

Angle	M_ϕ	N_ϕ	N_x	W/E	$N_x\phi$
$+\phi$	0.0	0.0	-4177.1	-2.86×10^7	0.0
$+\phi/2$	-3.6	0.8	-4049.5	-2.88×10^7	80.8
0	-14.4	3.2	-3911.9	-2.9×10^7	159.1
$-\phi/$	-32.4	7.0	-3764.3	-2.9×10^7	234.5
$-\phi$	-57.7	12.4	-3606.3	-3.02×10^7	306.9

Table 3.6.B

Element (2)

Angle	M_ϕ	N_ϕ	N_x	W/E	$N_x\phi$
$+\phi$	-23.7	-1.5	5569.3	3.4×10^6	-548.3
$+\phi/2$	-12.6	-19.7	3271.6	1.82×10^6	-678.6
0	- 1.3	-40.8	978.2	43173.7	-741.2
$-\phi/$	10.3	-62.7	-1313.5	-1.76×10^6	-736.2
$-\phi$	22.6	-83.5	-3606.3	-3.4×10^6	-663.7

Table 3.6.C

Element (3)

Angle	M_ϕ	N_ϕ	N_x	W/E	$N_x\phi$
$+\phi$	-35.0	17.7	-3606.3	-3.04×10^7	-356.8
$+\phi/2$	32.0	4.5	-3635.4	-3.2×10^7	-178.9
0	54.3	0.1	-3645.5	-3.37×10^7	0.0
$-\phi/$	32.0	4.5	-3635.4	-3.26×10^7	178.9
$-\phi$	-35.0	17.7	-3606.3	-30.4×10^7	356.8

Table 3.6.D

Stress Resultants and Deflections

Element (4)

Angle	M ϕ	N ϕ	Nx	W/E	Nx ϕ
+ ϕ	-23.7	-10.1	5569.3	3.04x10 ⁷	548.3
+ ϕ /	-23.3	10.1	5582.7	3.19x10 ⁷	274.4
O	-23.1	16.9	5587.4	3.23x10 ⁷	0.0
- ϕ /	-23.1	10.1	5582.7	3.19x10 ⁷	-274.4
- ϕ	-23.7	-10.1	5569.3	3.04x10 ⁷	-548.3

Table 3.6.E

Element (5)

Angle	M ϕ	N ϕ	Nx	W/E	Nx ϕ
+ ϕ	-22.6	-83.5	-3606.3	-3.4x10 ⁶	663.7
+ ϕ /	10.3	-62.7	-1313.5	-1.76x10 ⁶	736.7
O	-1.3	-40.8	978.2	43177.4	741.2
- ϕ /	-12.6	-19.7	3271.6	1.82x10 ⁶	678.6
- ϕ	-23.7	- 1.5	5569.3	3.4x10 ⁶	548.3

Table 3.6.F

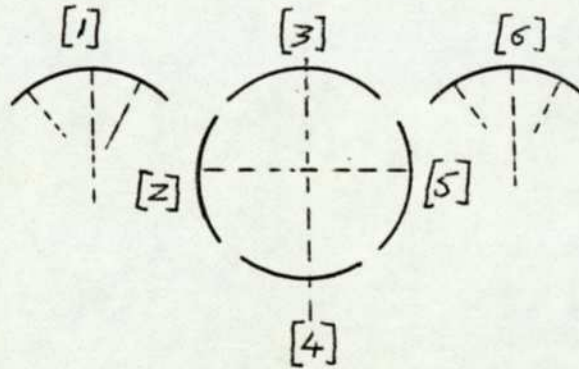
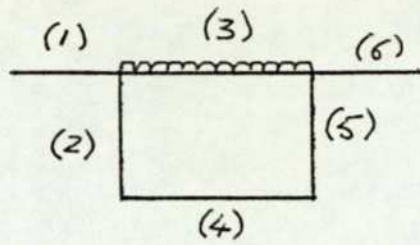
Element (6)

Angle	M ϕ	N ϕ	Nx	W/E	Nx ϕ
+ ϕ	-57.5	12.4	-3606.3	-3.02x10 ⁷	-306.0
+ ϕ /	-32.4	7.8	-3764.3	-2.96x10 ⁷	-234.5
O	-14.4	3.2	-3911.9	-2.9x10 ⁷	-159.1
- ϕ /	-3.6	0.8	-4049.5	-2.88x10 ⁷	- 80.8
- ϕ	-0.0	0.0	-4177.1	2.86x10 ⁷	0.0

Table 3.6.G

3.4.C Dimensions of Box Bridge and Loading Conditions
with Uniform Thickness

(III) Uniform Loading covering the cell of a 120" long
Box Bridge



	P	L	R	2h	ϕ	β	γ	E
(1)	0	120	86	0.1	1	45	0	1
(2)	0	120	129	0.1	1	135	0	1
(3)	10	120	215	0.1	1	45	0	1
(4)	0	120	215	0.1	1	-135	0	1
(5)	0	120	129	0.1	1	-45	0	1
(6)	0	120	86	0.1	1	45	0	1

Table 3.7.A

Stress Resultants and Deflections

Element (1)

Angle	M_ϕ	N_ϕ	N_x	W/E	$N_x\phi$
$+\phi$					
$+\phi /$	-0.1	0.5	-2367.9	-1.47×10^7	47.3
0	-0.2	1.8	-2287.4	-1.57×10^7	93.0
$-\phi /$	-0.4	4.1	-2201.7	-1.67×10^7	137.1
$-\phi$					

Table 3.7.B

Element (2)

Angle	M_ϕ	N_ϕ	N_x	W/E	$N_x\phi$
$+\phi$	-8.0	0.4	3259.6	2.0×10^6	-320.8
$+\phi /$	13.4	-10.3	1915.3	1.39×10^6	-397.1
0	-18.8	-22.7	573.4	585445.8	-433.8
$-\phi /$	-23.9	-35.5	-767.9	0.5137×10^6	-430.9
$-\phi$	-28.8	-47.7	-2110.2	1.97×10^6	-388.5

Table 3.7.C

Element (3)

Angle	M_ϕ	N_ϕ	N_x	W/E	$N_x\phi$
$+\phi$	-29.3	-3.0	-2110.2	-1.78×10^7	-209.0
$+\phi /$	37.3	-10.7	-2129.5	-2×10^7	-104.8
0	59.5	-13.3	-2136.3	-2.159×10^7	0.0
$-\phi /$	37.3	-10.7	-2129.5	-2×10^7	104.8
$-\phi$	-29.3	-3.0	-2110.2	-1.78×10^7	209.0

Table 3.7.D

Stress Resultants and Deflections

Element (4)

Angle	M_ϕ	N_ϕ	N_x	W/E	$N_{x\phi}$
$+\phi$	-8.0	4.8	3259.6	1.78×10^7	320.8
$+\phi/$	-7.5	16.0	3266.7	1.827×10^7	160.6
0	-7.3	20.6	3269.1	1.843	0.0
$-\phi/$	-7.5	16.6	3266.7	1.827×10^7	-160.5
$-\phi$	-8.0	4.8	3259.6	1.78×10^7	-320.8

Table 3.7.E

Element (5)

Angle	M_ϕ	N_ϕ	N_x	W/E	$N_{x\phi}$
$+\phi$	-28.8	-47.7	-2110.2	-1.97×10^6	388.5
$+\phi/$	-23.9	-35.5	-767.9	5.137×10^5	430.9
0	-18.8	-22.7	573.4	5.85×10^5	433.8
$-\phi/$	-13.4	-10.3	1915.3	1.39×10^6	397.1
$-\phi$	-8.0	0.4	3259.6	2.0×10^6	320.8

Table 3.7.F

Element (6)

Angle	M_ϕ	N_ϕ	N_x	W/E	$N_{x\phi}$
$+\phi$	-0.5	7.2	-2110.2	1.77×10^7	-179.5
$+\phi/$	-0.4	4.1	-2201.7	-1.67×10^7	-137.1
0	-0.2	1.8	-2287.4	-1.57×10^7	-93.0
$-\phi/$	-0.1	0.5	-2367.9	-1.47×10^7	-47.3
$-\phi$	0.0	0.0	-2443.5	1.375×10^7	0.0

Table 3.7.G

CHAPTER 4

4.1 Scope

The object of this Chapter is to deal with the tests carried out on the different groups of boxes used.

The boxes are mainly divided into three groups according to their cross-sections and support conditions.

GROUP A: Single cell box beams simply supported.

GROUP B: Double cell box beams simply as well as column supported.

GROUP C: Double cell box beams continuous over two spans.

All box beams were made of micro-concrete and tested from the elastic range to collapse. The testing of the boxes will be discussed in detail. The material used, casting techniques and the test procedure will be covered. Final test results will be obtained and plotted.

4.2 GROUP A: Single cell simply supported box beams

Three models of single cell box beams simply supported will be dealt with here. Two of these boxes (A1, A2) were 5 ft long and the third box (A3) was 10 ft long. The boxes were either symmetrically loaded by means of the application of two point loads at the mid-span section on either side of each web and acting at their central line, or unsymmetrically loaded by the application of one point load on the central line of one of the webs only.

The behaviour of each box during the elastic range and at the ultimate condition to collapse were recorded by means of strain gauges and dial gauges placed at mid-span and at quarter span. The results for each load application was recorded and plotted.

4.2.1 Material and casting

The same material and method of casting was used for the different groups mentioned above.

River washed sand from the river Thames Valley was used as an aggregate for all mixes. The sand curve lies in Zone 2 in the aggregate grading curve, BS 812.

A sample was dried and sieved. The results were plotted against standard limitations of Zone 2. The graph can be seen in Fig.(1).

An ordinary Portland cement, Blue Circle, was used in all mixes. The standard mix proportions were as follows:

Cement/aggregate ratio = 0.36

Water/cement ratio = 0.48.

All boxes were reinforced by beams of ungalvanized cold drawn high yield wire mesh of 3.18 mm (1/8") diameter and mesh spacings of 25 mm and 75 mm (1" x 3") in both directions were used.

Two stiff steel frames were made with the same shape and dimensions as the box cross-section, and were attached to the timber base. The underside of the cantilever section was attached to the metal frames to form a rigid trough, these being made of 20 mm thick plywood.

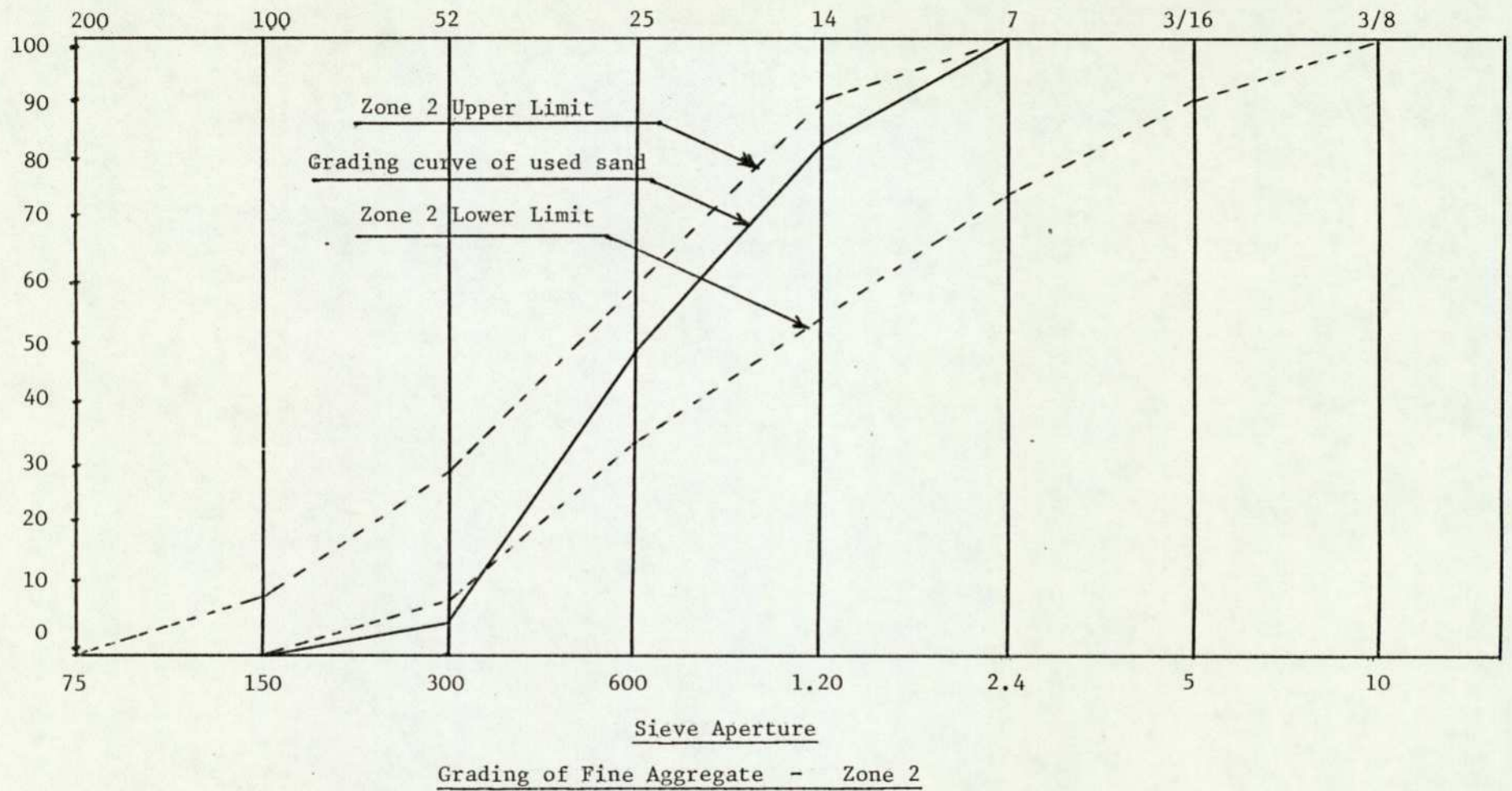
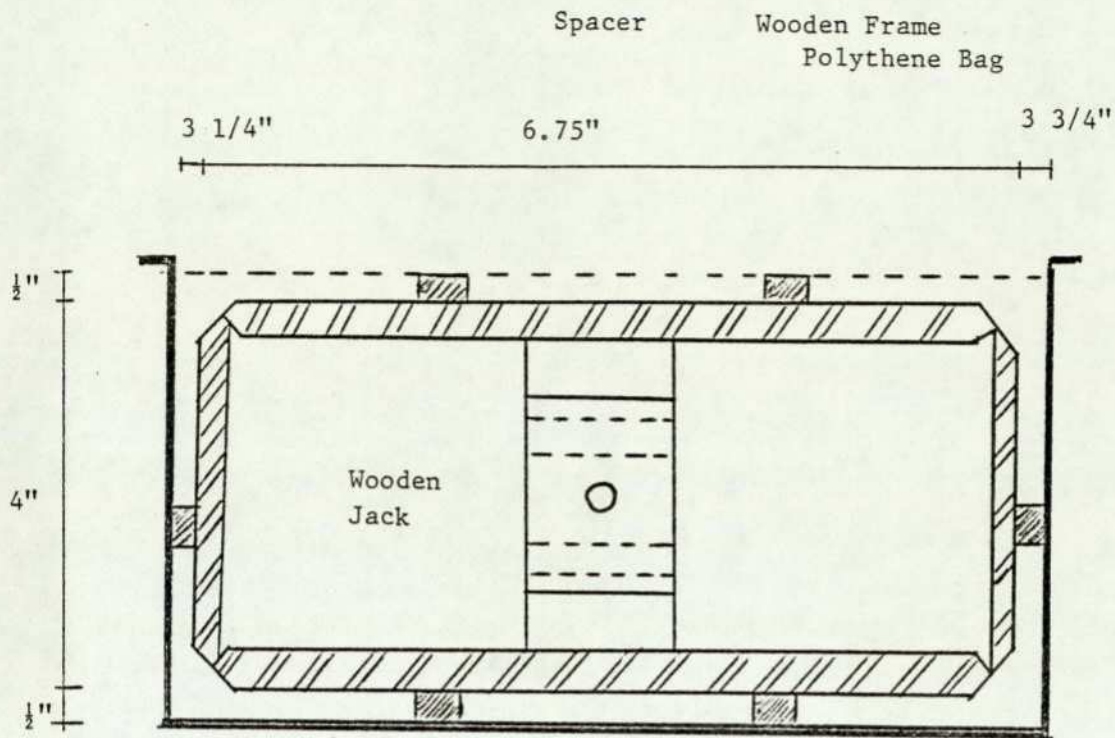


Figure 1

The ungalvanized mesh reinforcement 3.18 mm (1/8") diameter 25 x 75 mm (1" x 3") centres was bent into a U shape (7 1/2") centre to centre for the case of a single cell box beam and (15") centre to centre for the case of a double cell box. An extra mesh was placed at the centre of the (15") U shape steel for the construction of the central web in the case of the double cell box beam. Top slab reinforcement was attached, by allowing the meshed reinforcement to poke through and then bent over. All that work was done cold and then placed in the formwork.

The inside form for the formation of the cells openings was made out of four pieces of timber connected in such a way as shown in Fig.(2). They were then put in a polythene sheet which was made into a tube to allow the formwork to be stuck, and to give the concrete a good internal finish. In the case of a double cell box beam, two of these inner forms were made for each cell.

A series of timber wedges that worked on a screw jack principle were inserted inside the internal forms which pushed the forms against the steel frames and could be adjusted to give the correct wall thickness. Small steel spacers were inserted to ensure a correct wall thickness for the box and to ensure a correct cover for the reinforcement. Vibrators were adjusted to the outside of the form, therefore ensuring the vibration of the whole form. A whole setting of the



Cross-section of Mould (dimensions in inches)

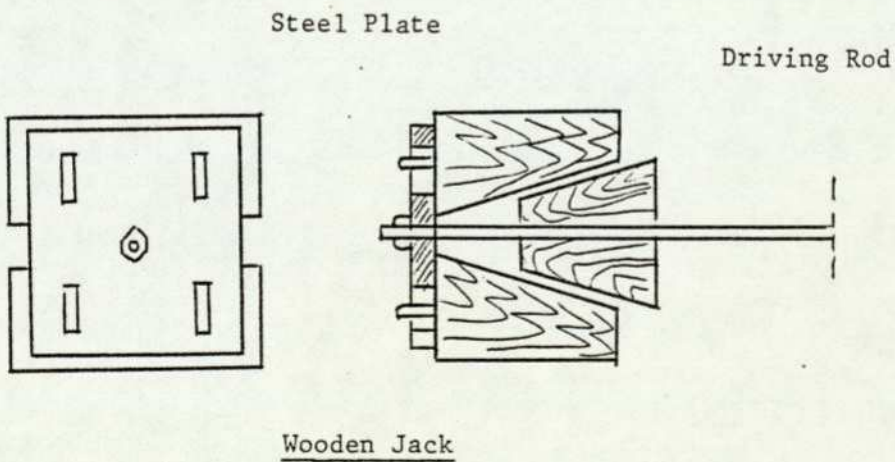


Figure 2

formwork can be seen in the photograph.

A rotating mechanical mixer was used to mix the concrete material. The concrete was then poured from one side of the section through one of the webs and was allowed to flow across the bottom flange and rise up into the other webs. Then more concrete was poured into the other webs as well in order to fill the whole section. The surface was then finished smoothly using a straight steel rod and then a piece of wood.

A series of concrete cubes 4x4x4 inches were cast as well as a cylinder 6 inches in diameter and 12 inches in height from the same mix. Some of these cubes were tested after a minimum of three weeks while the rest were tested at the same date as the testing of the boxes.

After three days the boxes were removed from the formwork and cured by covering them with a piece of wet rag for two days.

The formwork for the diaphragm was made of a rectangular box 50 mm high. It was filled with the same mix of micro-concrete and the box was put vertically upright on the vibrating table. After three days the same process was carried out for the other diaphragms.

For the case of the double cell with column support, the columns were reinforced as will be shown in a photo later, and the concrete was poured from the top

of each column. As for the continuous double cell, the middle support was first cast alone then fitted to the framework. The steel coming out of that support was bent into the box's steel cage to form the middle diaphragm reinforcement. Concrete was then poured to form the diaphragm and the rest of the box was cast as previously mentioned. No reinforcement was needed for the end diaphragms as they were very stiff in comparison to the stiffness of the flanges and web.

4.2.2 Testing Equipment

The steel rig used for all the boxes can be easily seen from most of the photographs taken. The strain gauges used were made in Japan by "Tokyo Sokki Kenkyryo Co. Ltd.", FLA-6-11, each gauge length = 6 mm, gauge resistance of 120 ± 0.3 and with a gauge factor of 2.09.

These were put on the concrete by rubbing the surface of the concrete with an old file until it produced a good smooth enough surface. If this surface was not adequate, the proposed area was filled with some 'Araldite' and rubbed down when it was hard. Then the gauges were glued into their position with Araldite. For the case of placing strain gauges on the steel reinforcement, the steel's surface was first cleaned with an emery cloth,

cleaned with Trichloroethylene(Stabilised)and then dried off, and covered with a film of super glue Cyanaocrylate and applied immediately in position on the cleaned area. It was then held under sustained pressure until the adhesive was cured. Two lead wires were attached and insulated by tapes. Each gauge was checked with a meter and covered with a film of Araldite Epoxy Resin. Then all the wires were connected in a Whinston Bridge to a Data Logger, Dynamco 6600, with 60 channels. A set of dial gauges of accuracy 0.01 mm and with a maximum range of 40 mm were placed at mid section and at quarter spans.

Demec gauges (2" and 4" gauges) were used for the double cell boxes at half and quarter span sections.

The jacks used were of 5 ton capacity and were connected to a hydraulic pump. Calibration beams for concrete were done during tests and separate tests on steel reinforcement were carried out, as will be seen.

4.2.3 Test Procedure

4.2.3A Initial Tests

Some preliminary tests had to be done on steel as well as concrete. Tests on steel were essential to know the properties of the steel mesh reinforcement that was used for all the boxes, as yielding stresses and the relation between strain gauge readings of the gauges on the steel, and the corresponding stresses due to the different loading conditions

A group of bars was taken from the original mesh of the reinforcement which was used in the construction of the boxes. Several tests were carried out on the Instrone Tensile Testing machine to get an average reading of the yielding stress. Other tests were carried out for the bars using a Clock House 5 ton tension-compression testing machine as well as a Denison Universal Testing machine Model T42 U. The bars were strain gauged to enable us to know the relation between the gauge readings on the original mesh and the corresponding stresses due to the applied loads. The loads which were tensile loads were applied on the bar in increments and the corresponding readings of the strain gauges were recorded by the use of a data logger, see table (1). The corresponding relations are plotted in Fig. (3,4,5,6)

Calibration beams of the dimensions shown in Fig. (1-1) were made, in order to have a preliminary idea

Young's Modulus of concrete and to calibrate the strain gauges on concrete. Two beams were made for each box, one of them was reinforced and the other unreinforced. Due to some problems which arose during the tests, only one beam of each kind was made. A better result could have been obtained by testing a group of beams of each kind and obtaining the mean values between them. All these tests were done at the same time as the main tests on the box beams. The results are plotted in Table (2,3), Figs. (7,8,9).

Test 1	
Load lbs.	m.v.
0	0
100	+80
200	+150
300	224
400	297.5
500	395
600	532
700	750
800	
850	

Test 2	
Load lbs.	m.v.
0	0
100	88
200	172
300	243
400	314
500	402
600	516
700	645
800	819
850	1005
900	

Test 3	
Load lbs.	m.v.
0	0
100	87
200	154
300	219
400	281
500	346
550	383
600	428
650	476
700	257
750	585
800	654
850	752
900	1017

Test 4	
Load lbs.	m.v.
0	0
50	51
100	90
150	128
200	167
250	205
300	242
350	277.5
400	315.5
450	362
500	418
550	483
600	553
650	628
700	734
750	927
800	1331
850	
900	

Table 1

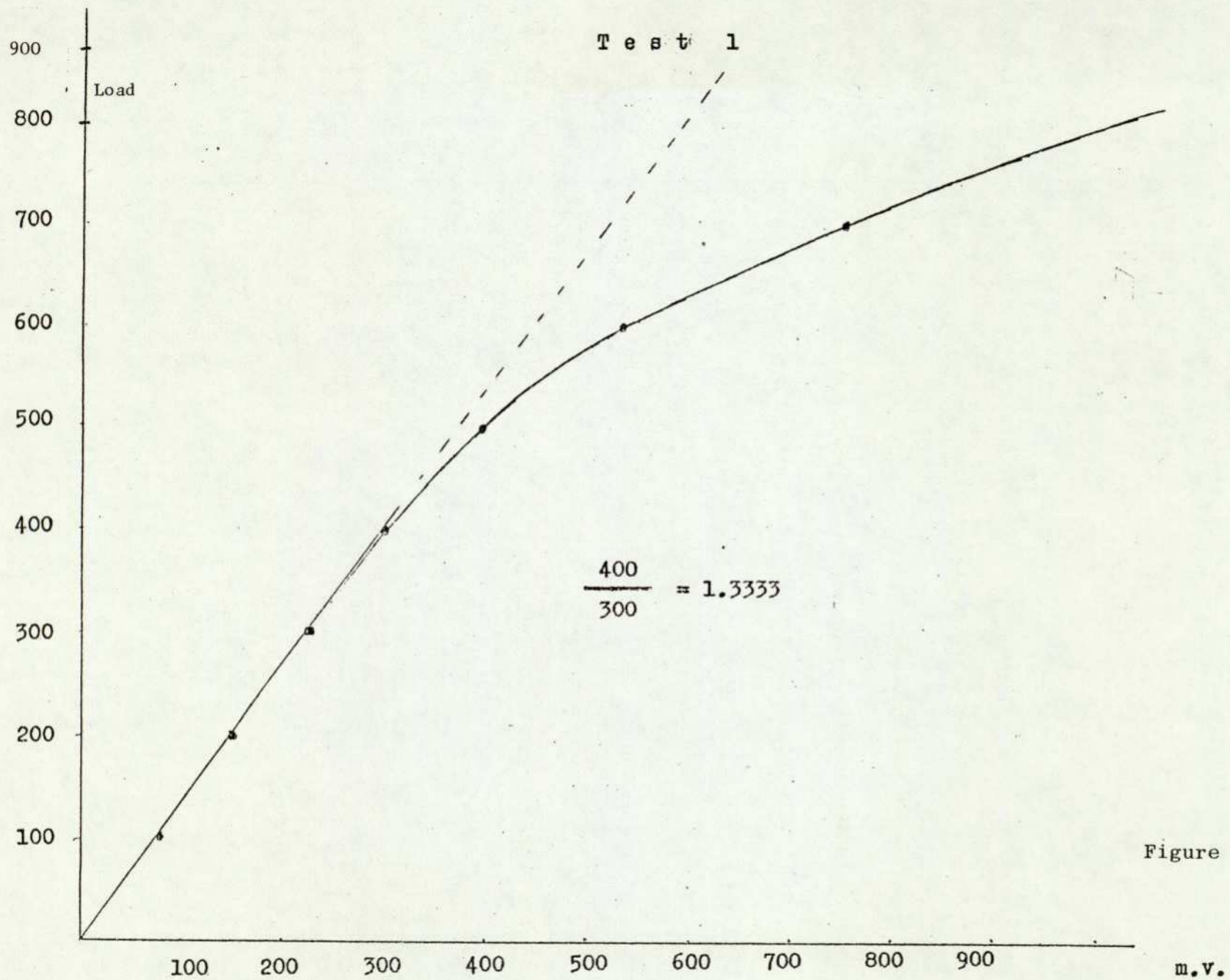


Figure 3

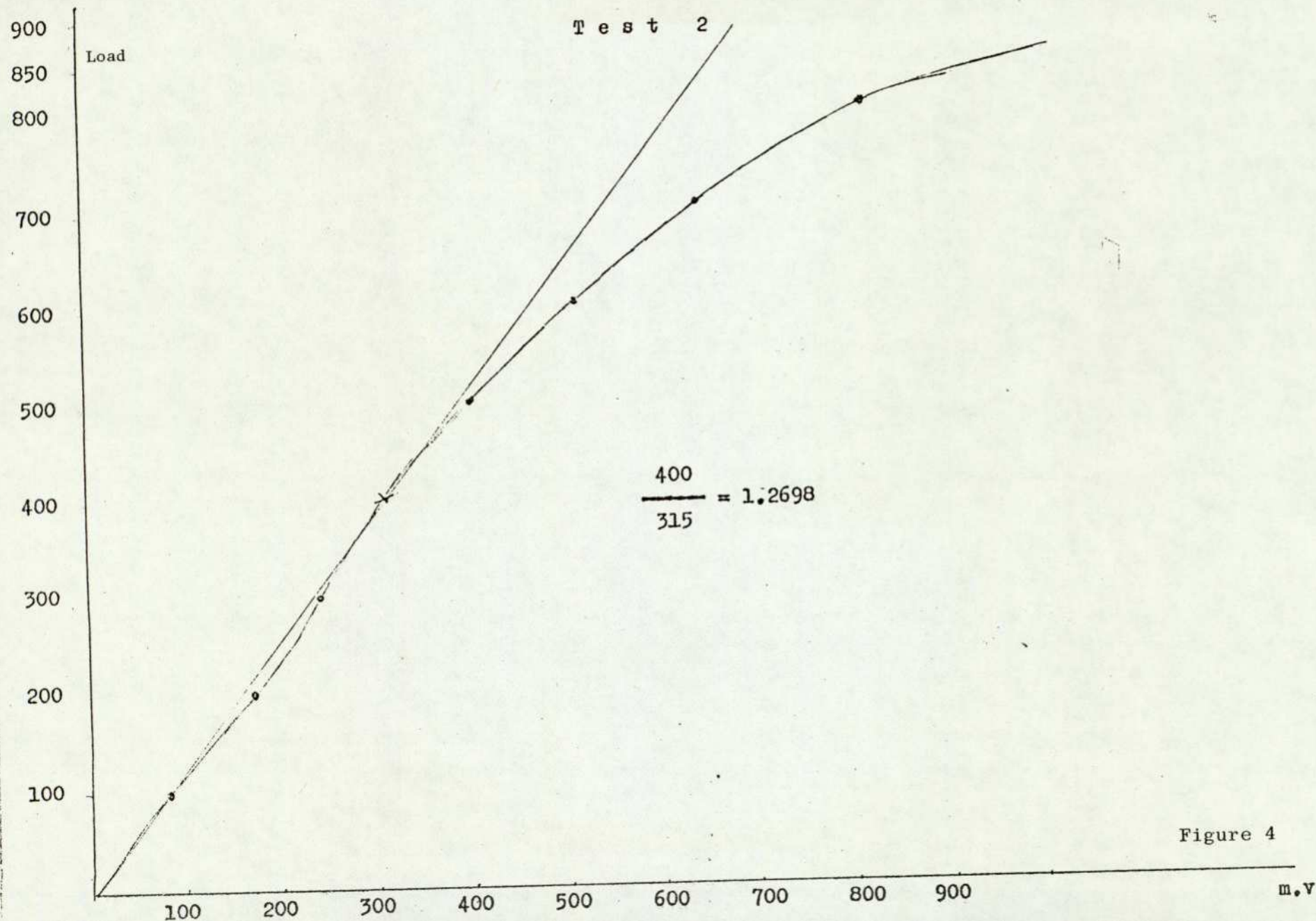


Figure 4

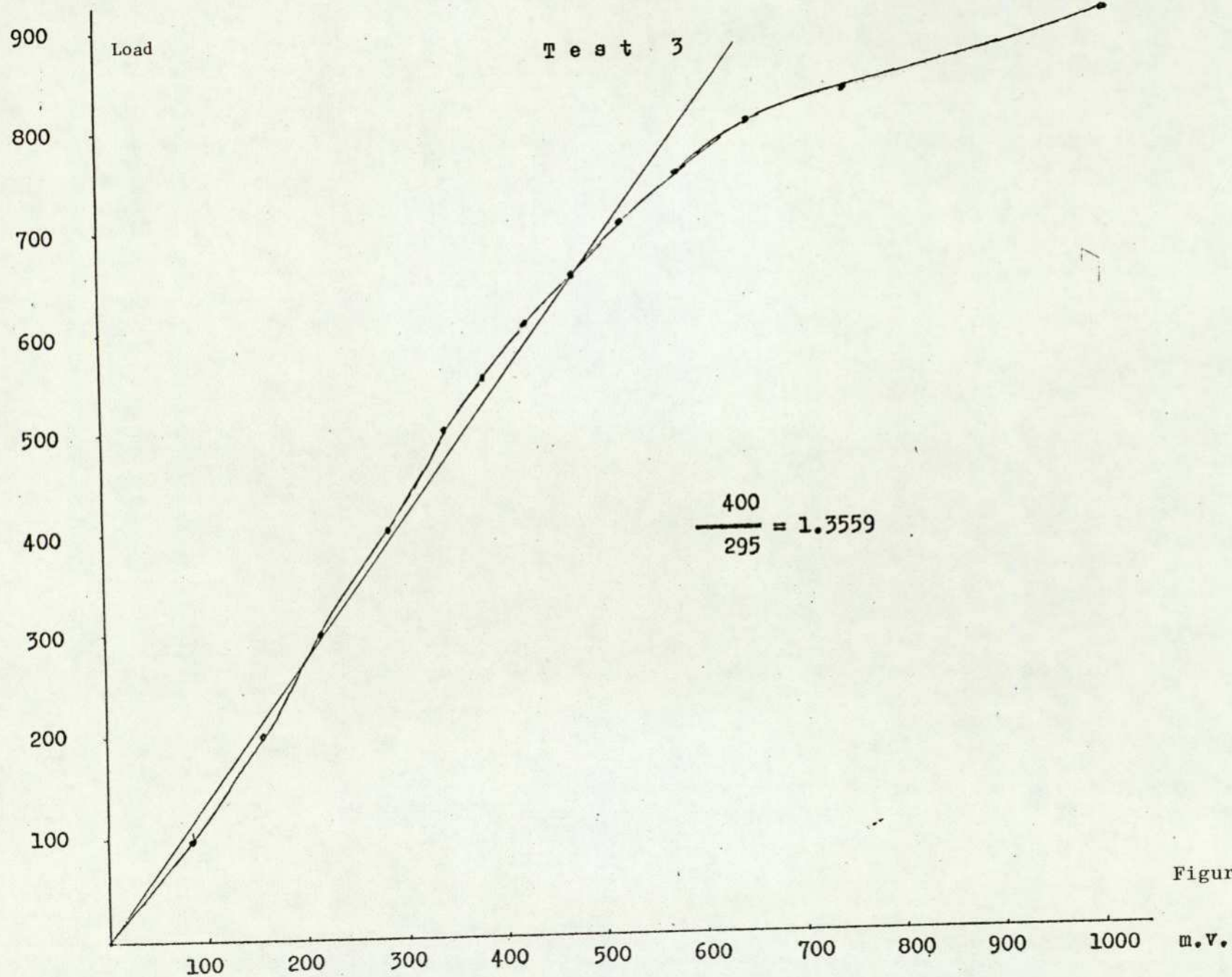


Figure 5

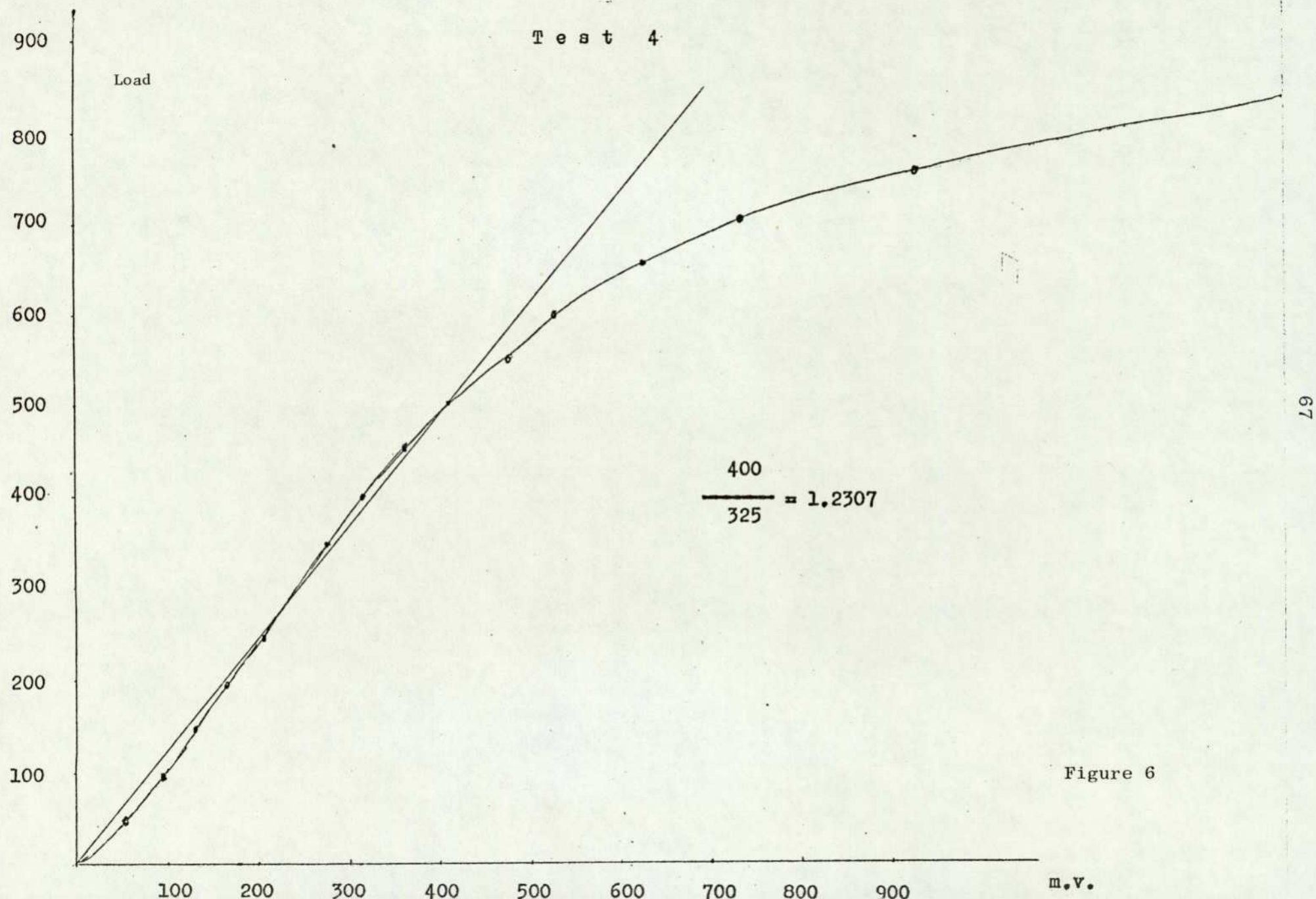


Figure 6

From the drawings:

$$\frac{P}{m.v.} = 1.3333, \quad 1.2698, \quad 1.3559, \quad 1.2307$$

$$\text{Average} = 1.297$$

$$\text{Stress} = \frac{P}{A} \propto m.v.$$

$\frac{\delta 1}{m.v_1}$	$\frac{\delta 2}{m.v_2}$	$\frac{\delta 3}{m.v_3}$	$\frac{\delta 4}{m.v_4}$
108.604,	103.4317,	110.445,	100.2460

$$\text{Stress calibration} \approx 105.68$$

Calibration Beam 2 Reinforced

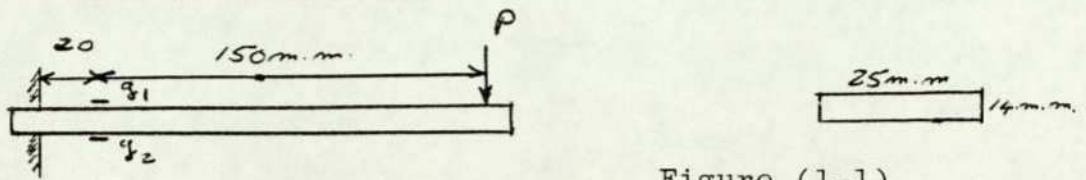
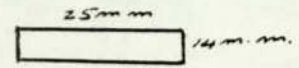
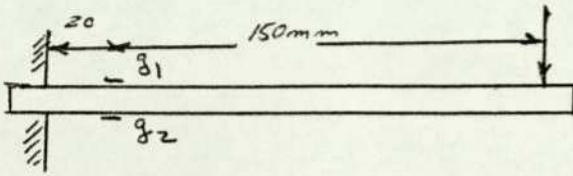


Figure (1-1)

P (N)	g ₁	g ₂	mm	Net g ₁	Net g ₂	Near gptg ₂	Net
0	- 0010	- 0186	16.29				
4	- 0007	- 0190	16.275	+0003	-0004	3.5	.015
8	- 0004	- 0194	16.245	+0006	-0008	7.0	.045
12	+ 0000	- 0198	16.165	+0010	-0012	11.0	0.125
16	+ 0004	- 0202	16.115	+0014	-0016	15.0	0.175
20	+ 0007	- 206	16.06	+0017	-0020	18.5	0.23
24	+ 0012	- 0210	16.04	+0022	-0024	23.0	0.25
28	+ 0016	- 0215	15.97	+0026	-0029	27.5	0.32
32	+ 0021	- 220	15.930	+0031	-0034	32.5	0.36
36	+ 0053	- 0235	15.865	+0064	-0049	56.5	0.425
40	+ 0103	- 0244	15.75	+0113	-0058	85.5	0.54
Broke 44							

Table (2)

Table 2

Calibration Beam 3Not Reinforced

	P (N)	g_1	g_2		$N_{et}g_1$	$N_{et}g_2$	Mean Value	
	0	-0339	-0155	15.10				
	4	-0343	-0152	15.095	-0004	+0003	3.5	.005
	8	-0348	-0148	15.065	-0009	+0007	8.0	.035
	12	-0353	-0143	15.005	-0014	+0012	13.0	.095
	16	-0357	-0140	14.955	-0018	+0015	16.5	0.145
	20	-0362	-0135	14.90	-0023	+0020	21.5	0.2
	24	-0367	-0131	14.865	-0028	+0024	26.0	0.235
	28	-0373	-0128	14.815	-0034	+0027	30.5	0.285
	32	-0378	-0124	14.75	-0039	+0031	35.0	0.35
Broke	36	-0337	-0399	14.68				0.42
	40	-0335	-0162					
	44	-0335	-0160					

Table 3

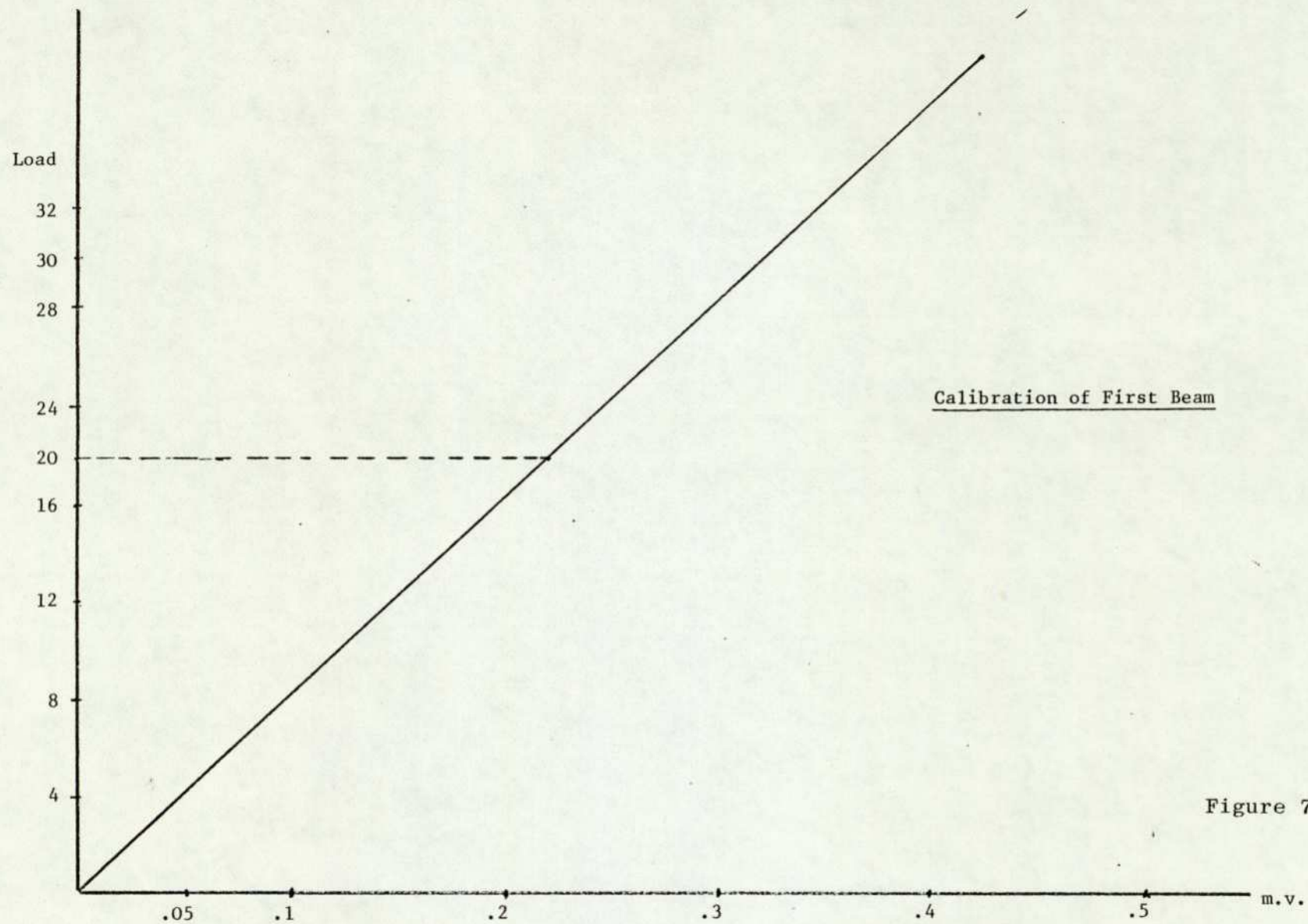
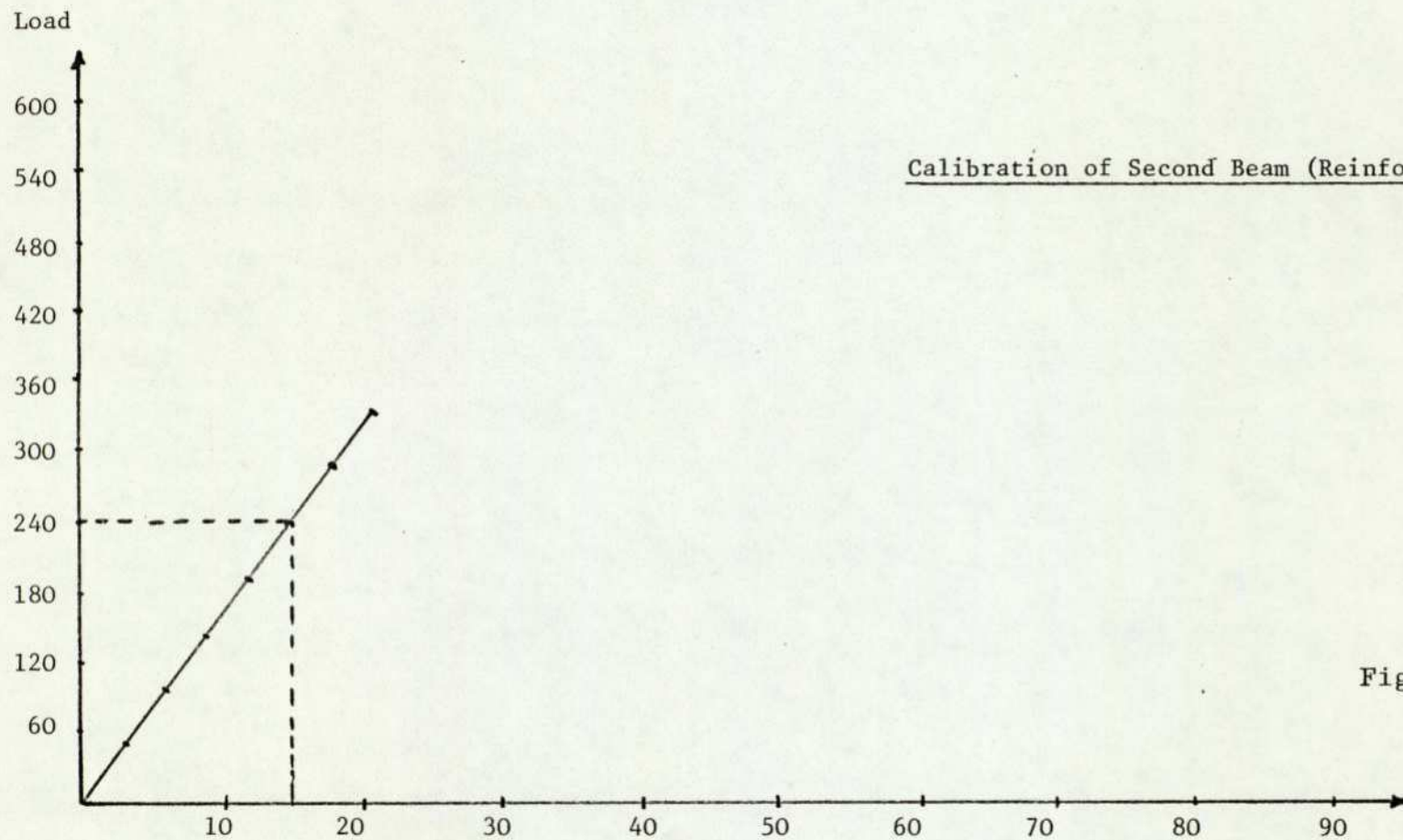


Figure 7



Calibration of Second Beam (Reinforced)

Figure 8

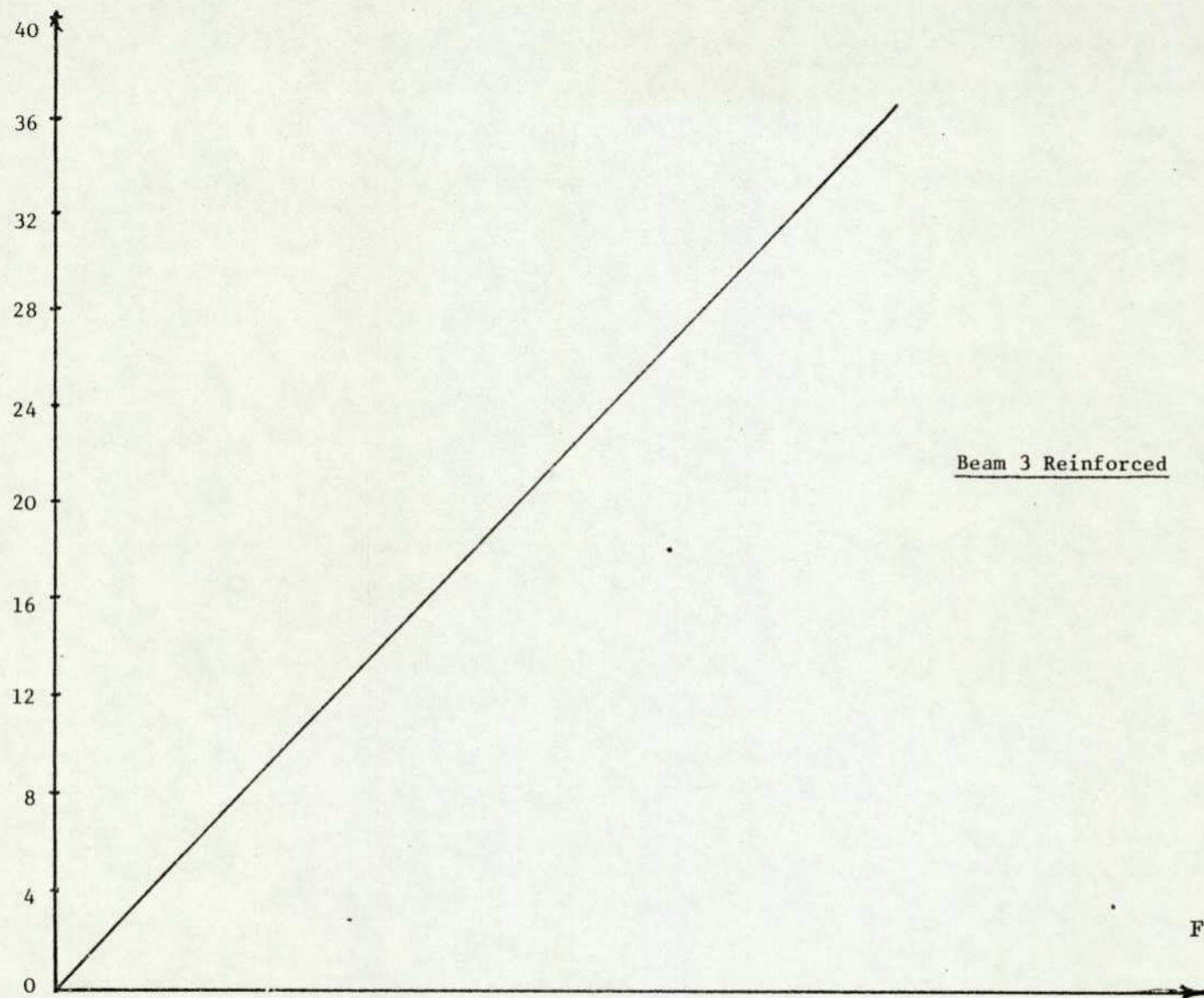


Figure 9

4.2.3B Main Test

Two box beams were tested (A1, A2).

Box beam A1 was symmetrically loaded, while beam A2 was unsymmetrically loaded. The loads were applied by means of hydraulic jacks connected to a pressure gauge, where the pressure was obtained from an electrically operated hydraulic pump. The loads were applied at mid-span of the box, at centre lines of each web, for the symmetrically loaded case and at the centre line of one web for the unsymmetrically loaded case. An initial load was obtained by applying increments of load equal to 200 lb until a load equal to approximately one third of the estimated failure load was reached. It was then unloaded back to zero and the load was repeated and increased until collapse occurred.

After each increment of load the readings of the strain gauges as well as those of the dial gauges were taken. Observation of cracks were made during the tests and marked on the beams. The beams were cut into two parts at failure section and observations were made for the accuracy of casting and examining the thickness and locations of reinforcement steel for the webs and flanges. A typical cross section for these boxes can be seen in Fig. (A-1) as well as the elevation side view Fig. (A-2).

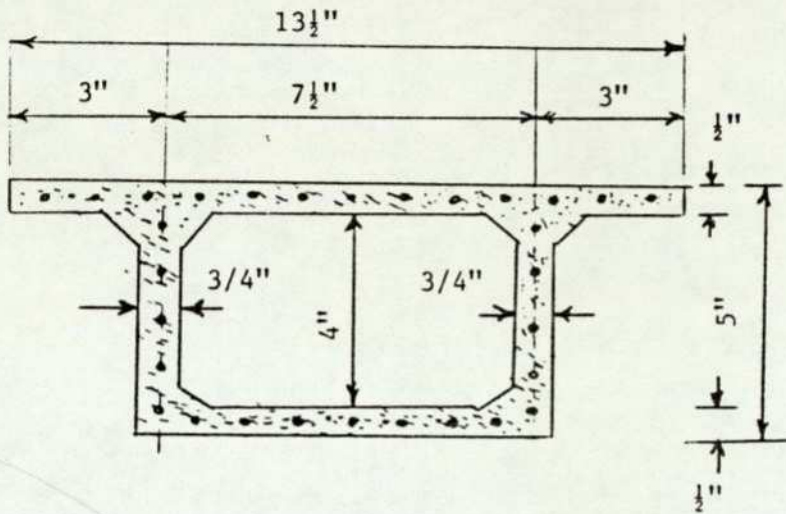
Dial gauges were used to measure the deflections during the tests. Four of these gauges were placed at the mid-span of the beam, Fig. (A-3), and another four were placed a quarter span, Fig.(A-4). The ninth dial gauge was placed at one of the box's ends to ensure that no longitudinal movements of the box were made during the tests, Fig. (A-5). Horizontal displacements as well as vertical displacements were recorded for each box at each load increment.

Twenty two strain gauges were used for A1 and seventeen strain gauges for A2. Two of these strain gauges in each case were placed on the mid bar of the top and bottom reinforcement of each flange. The location of these gauges can be seen from Fig. (A-6) and Fig. (A-7). The readings of the gauges were taken automatically by a Dynamco 6600 Data logger. The reason for not placing all the strain gauges of the top flange for A1 was that they would interfere with the location of the ball bearing which was cemented at that point where the loads from the jacks would rest upon for the application of the pressure.

Relations between load/deflection and load/stress were tabulated and plotted, Figs. (A-8 - A-13), Tables (A-1, A-2, A-3).

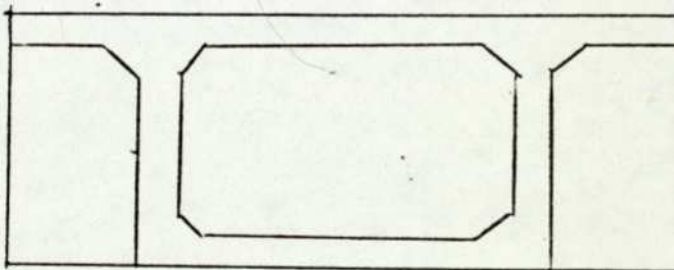
Dimensions of cross-section

For Box Bridges A1, A2



Reinforcement standard $1" \times 3" \times \frac{1}{8}"$ mesh

Figure (A-1)



5 cms Diaphragm

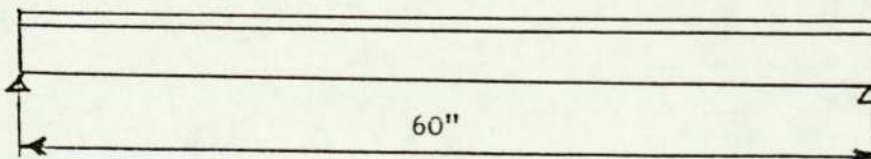


Figure (A-2)

Location of Dial Gauges for
Beam A1 and Beam B2

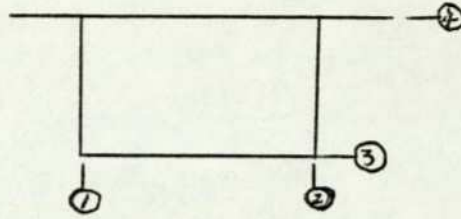


Figure (A-3)

Mid-span Section

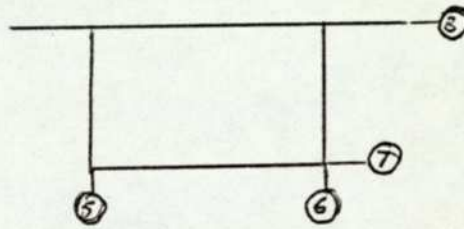


Figure (A-4)

Quarter-span Section

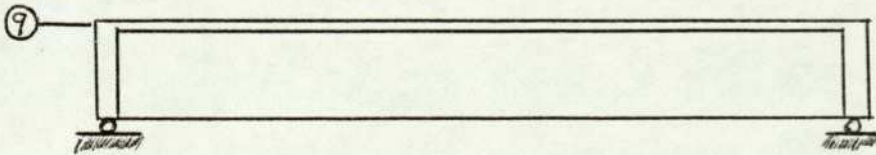
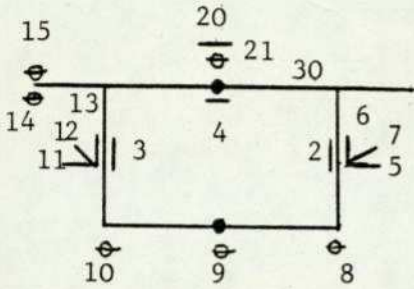


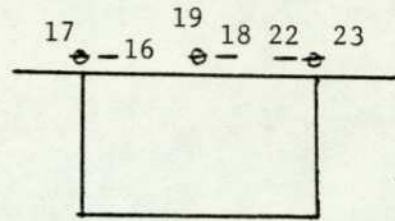
Figure (A-5)

Horizontal Location of Dial Gauges at Support's End

Location of Strain Gauges



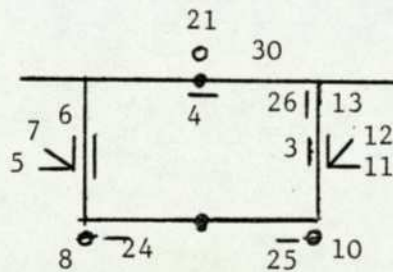
At mid-span



at 35 mm away from mid-span

Fig. (A-6)

FOR BEAM A1



mid-span

FOR BEAM A2

Fig. (A-7)

- Strain gauge on steel
- ⊖ Longitudinal strain gauge on concrete
- Transverse strain gauge on concrete

Test Results for Symmetrically Loaded Case BOX BEAM A1

Gauge No Load	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0
200	-1	0	0	0	-1	-1	+1	+1	+2	+1	0	-2	+1
400	0	0	0	0	-	-3	+4	+4	+5	-1	-2	-4	-1
600	-4	-2	-1	+1	-6	-3	+11	+9	+9	-1	-2	-7	-2
800	-9	-6	0	+8	-9	-2	+17	+13	+14	+3	-2	-10	0
900	-5	-1	0	+14	-8	+4	-23	+44	-25	-6	-7	-12	+11
1000	-5	-2	0	+15	-8	+5	-24	+41	-26	-6	-8	-12	+12
1400	-4	-8	+1	+18	-11	+9	-31	+38	-30	+22	+19	-13	+16
1800	-37	-14	+5	+11	-32	-8	-54	+38	-42	+127	+131	-18	+27
2200	-46	-20	+7	+9	-34	-8	-64	+40	-50	+241	+222	-20	+38
2600	-51	-27	+9	+5	-34	-14	-78	+41	-58	-369	+206	-16	+58
3000	-106	-35	+8	0	-16	-24	-101	+45	-67	-	-	+43	+245

Readings of Strain Gauges

Table (A-1)

Test Results for Symmetrically Loaded Case BOX BEAM A1

Gauge No. Load	15	16	17	18	19	20	21	22	23	30	31
0	0	0	0	0	0	0	0	0	0	0	0
200	-1	-1	-1	+1	-1		-2	0	-2	-2	+1
400	-4	0	-5	0	-5		-7	0	-6	-6	+4
600	-8	+1	-8	-1	-10		-12	+1	-9	-9	+8
800	-11	+3	-11	0	-16		-17	+3	-13	-14	+14
900	-23	+6	-30	+3	-28		-33	+6	-34	-18	+54
1000	-31	+10	-37	+4	-40		-41	+6	-40	-25	+101
1400	-46	+13	-54	+6	-60		-40	+8	-56	-37	+167
1800	-64	+17	-73	+10	-76		-85	+11	-75	-49	+238
2200	-79	+23	-86	+17	-90		-107	+15	-91	-58	+320
2600	-104	+28	-106	+22	-114		-143	+18	-115	-71	+526
3000	-160	+37	-129	+31	-157		-196	+20	-156	-87	+526

Readings of Strain Gauges

Table (A-1)

Test Results for Symmetrically Loaded Case BEAM A1

Gauge No. Load	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
200	0.01	0.08	0.025	0.065	0.01	0.09	0	0.06
400	0.09	0.13	0.025	0.055	0.03	0.13	0	0.06
600	0.19	0.25	0.015	0.045	0.10	0.2	0	0.06
800	0.42	0.45	0.02	0.040	0.24	0.34	0	0.06
900	0.84	0.89	.025	0.040	0.48	0.58	0	0.07
1000	0.96	0.02	.03	0.050	0.56	0.66	0	0.07
1400	0.58	0.65	.025	0.05	0.93	0.04	0	0.08
1800	3.02	3.11	.04	0.095	1.98	2.12	0.01	0.15
2200	4.13	4.21	.02	0.065	2.75	2.9	0.01	0.11
2600	5.63	5.70	.03	0.03	3.66	3.85	0.01	0.11
3000	7.62	7.84	.07	-	5.00	5.15	0.03	0.11

Dial Gauge Readings for Beam A1

All deflections are in mm.

Table (A-2)

Test Results for Unsymmetrically Loaded Beam BOX BEAM A2

Gauge No. Load	2	3	4	5	6	7	8	10	11	12	13	24	25	26	21	30	31
0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	
200	-2	0	+1	-2	-1	-1	+1	0	+1	+1	-2	-1	+1		-3	-2	
400	-2	0	+3	-4	-1	0	+2	+1	+2	+2	-4	-2	+1		-8	-6	
600	-4	0	+6	-7	-2	0	+3	+1	+3	+3	-6	-3	+2		-10	-9	
800	+98	+2	+1	-26	-25	-14	-8	-6	-3	-3	-20	+3	+12		-22	-13	
1000	+194	+2	-1	-30	-28	-17	-10	-9	-4	-4	-22	+1	+13		-27	-17	
1200	+211	+4	-5	-40	-36	-19	-13	-22	-7	-7	-28	0	+17		-34	-19	
1400	+235	+4	-6	-45	-41	-21	-9	-25	-7	-7	-31	-1	+23		-43	-22	
1600	+219	+4	-4	-55	-46	-20	+19	-28	-7	-7	-33	0	+32		-51	-25	
1800	+245	+4	0	-62	-46	-18	+112	-30	-8	-8	-35	-2	+38		-60	-26	
2000	+282	+5	-3	-76	-52	-14	+250	-36	-17	-17	-39	-2	+45		-71	-25	
2200	+284	+4	0	-78	-50	-14	+287	-38	-25	-25	-45	-2	+48		-78	-28	
2400	+318	+3	+3	-81	-47	-9	+359	-35	-45	-45	-56	-3	+61		-90	-26	
2600	+315	+4	+4	-83	-46	-8	+409	+22	+48	+48	-58	-4	+73		-100	-28	
2800	+291	+4	+5	-85	-45	-4	-	-	-	-	-58	-5	+230		-115	-26	
3000	+503	+5	+2	-89	-42	+41	-	-	-	-	-56	-8	-		-151	-18	

Readings of Strain Gauges

Table (A-3)

Test Results for Unsymmetrical Loading Case BEAM A2

Average Number

Gauge No. Load								
	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
200	-0.13	0.2	0.04	0.23	-0.05	0.26	0.04	0.22
400	-0.04	0.32	0.05	0.24	-0.01	0.34	0.04	0.23
600	0.05	0.45	0.05	0.26	0.05	0.43	0.04	0.24
800	1.08	0.64	0.11	0.35	0.76	1.20	0.06	0.29
1000	1.55	1.16	0.11	0.38	1.07	1.54	0.07	0.32
1200	2.05	1.85	0.12	0.48	1.39	1.99	0.10	0.41
1400	2.34	2.53	0.16	0.69	1.48	2.41	0.18	0.65
1600	2.69	3.26	0.21	0.9	1.66	2.87	0.23	0.84
1800	3.05	3.97	0.24	1.05	1.99	3.43	0.26	0.98
2000	3.69	5.03	0.31	1.20	2.35	4.08	0.28	1.17
2200	3.92	5.47	0.34	1.28	2.55	4.42	0.28	1.23
2400	4.41	6.38	0.61	1.44	2.86	4.97	0.35	1.33
2600	4.76	6.99	0.78	1.53	3.09	5.34	0.33	1.41
2800	5.44	8.07	1.12	1.65	3.54	5.96	0.38	1.52
3000	6.55	9.78	1.64	1.80	4.23	6.89	0.41	1.66

Dial Gauge Readings for Beam A2
All deflections are in mm

Table (A-4)

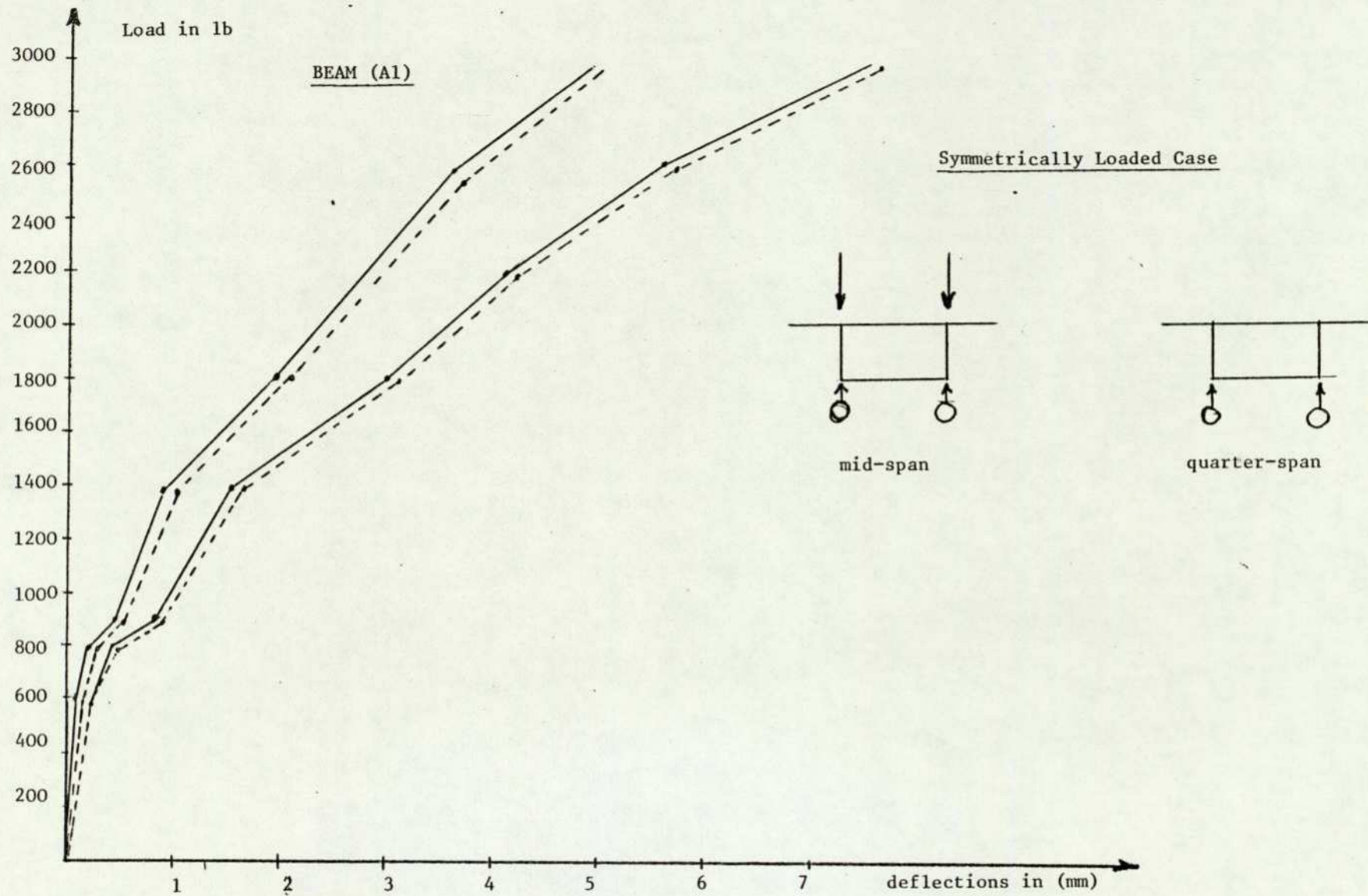


Figure (A-8)

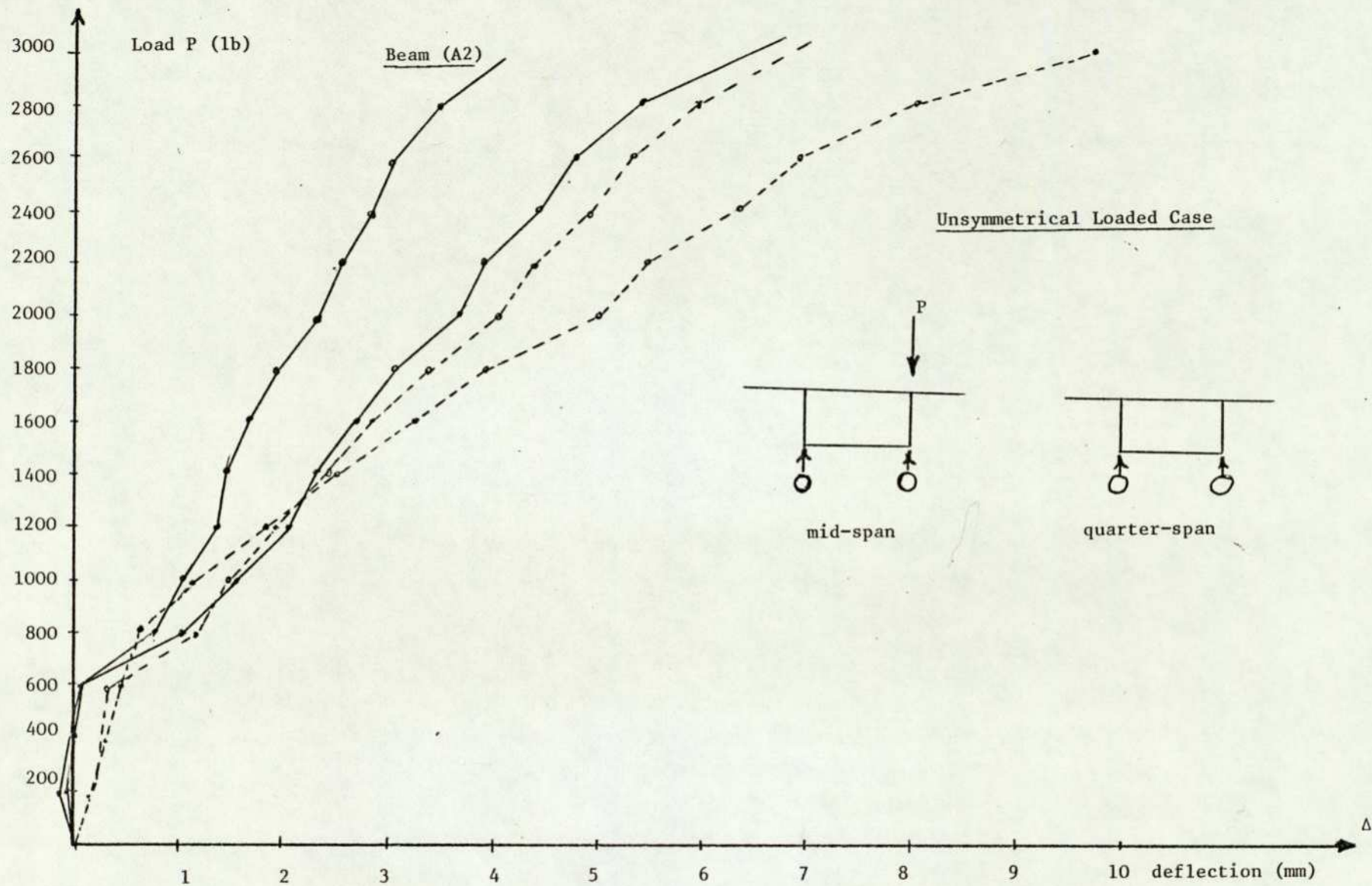


Figure (A-9)

Readings of Longitudinal Strain Gauges of
Beam (A1)

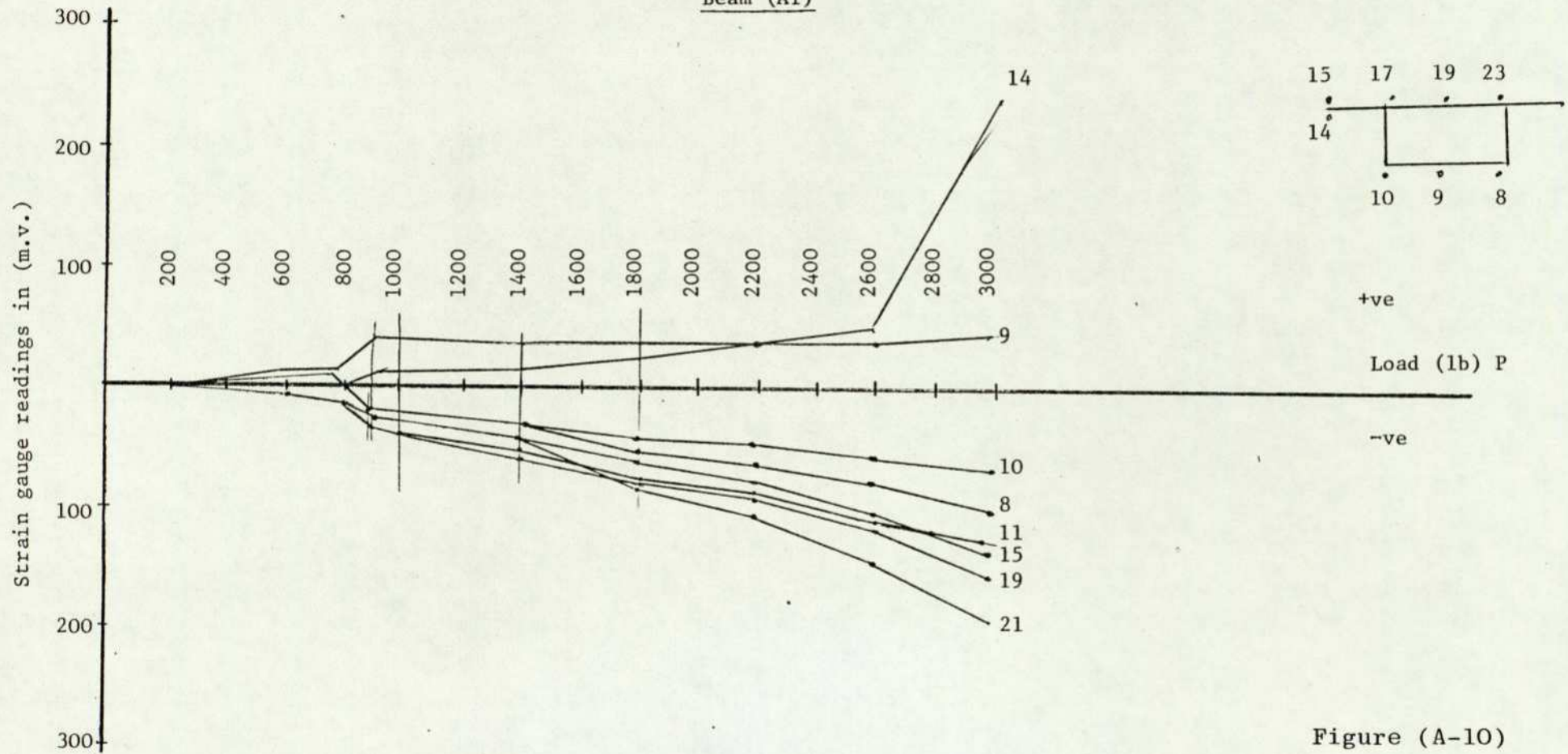


Figure (A-10)

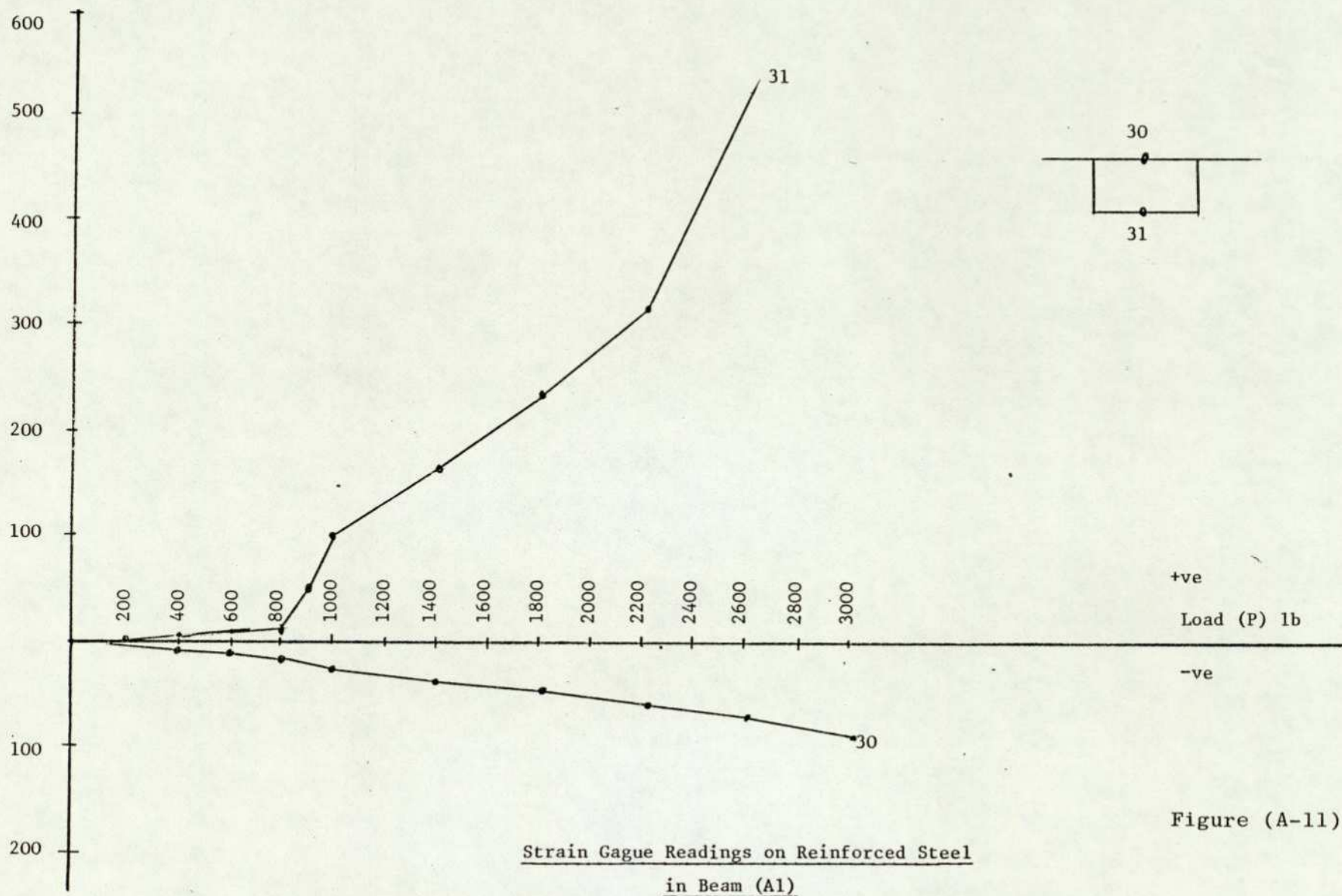


Figure (A-11)

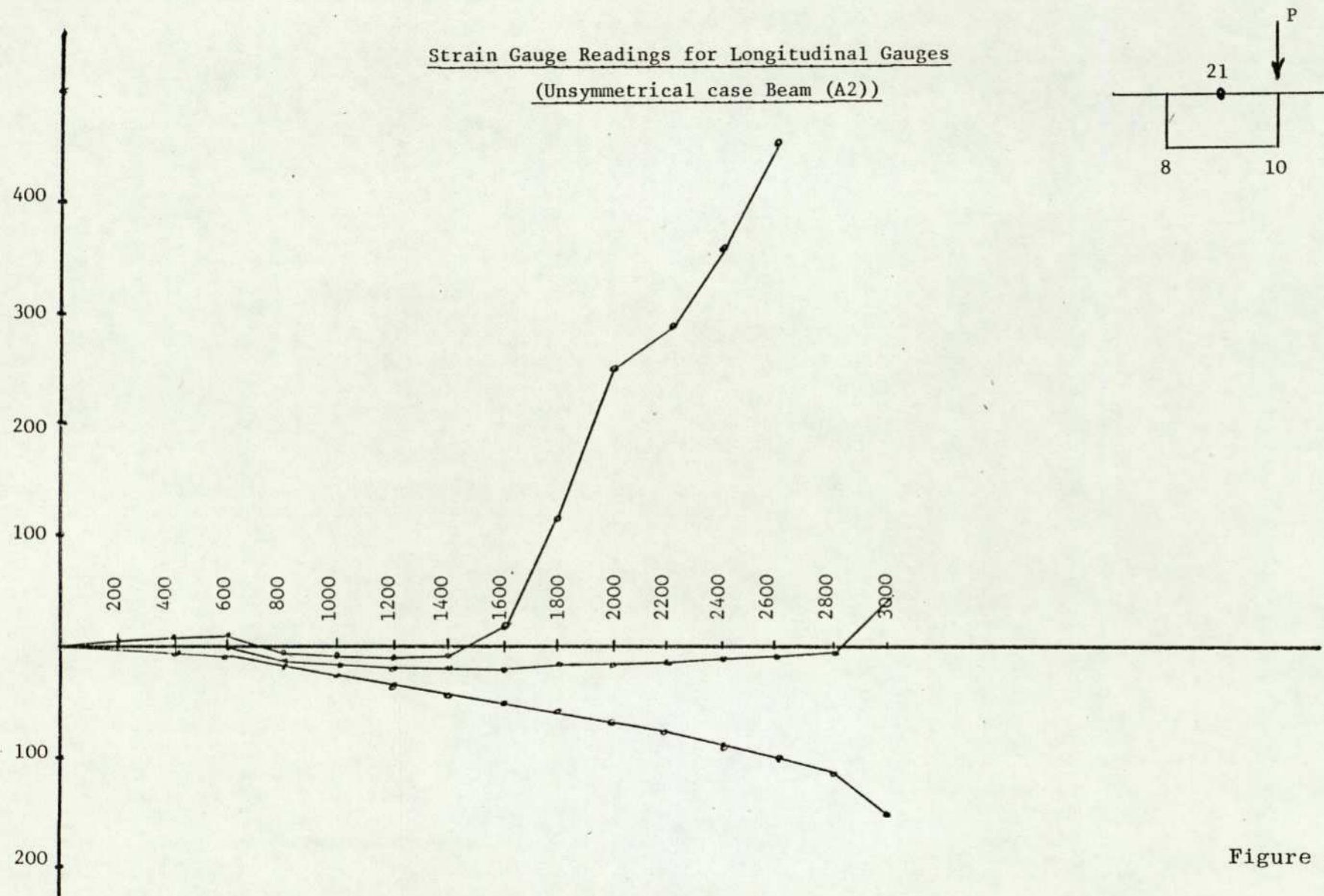


Figure (A-12)

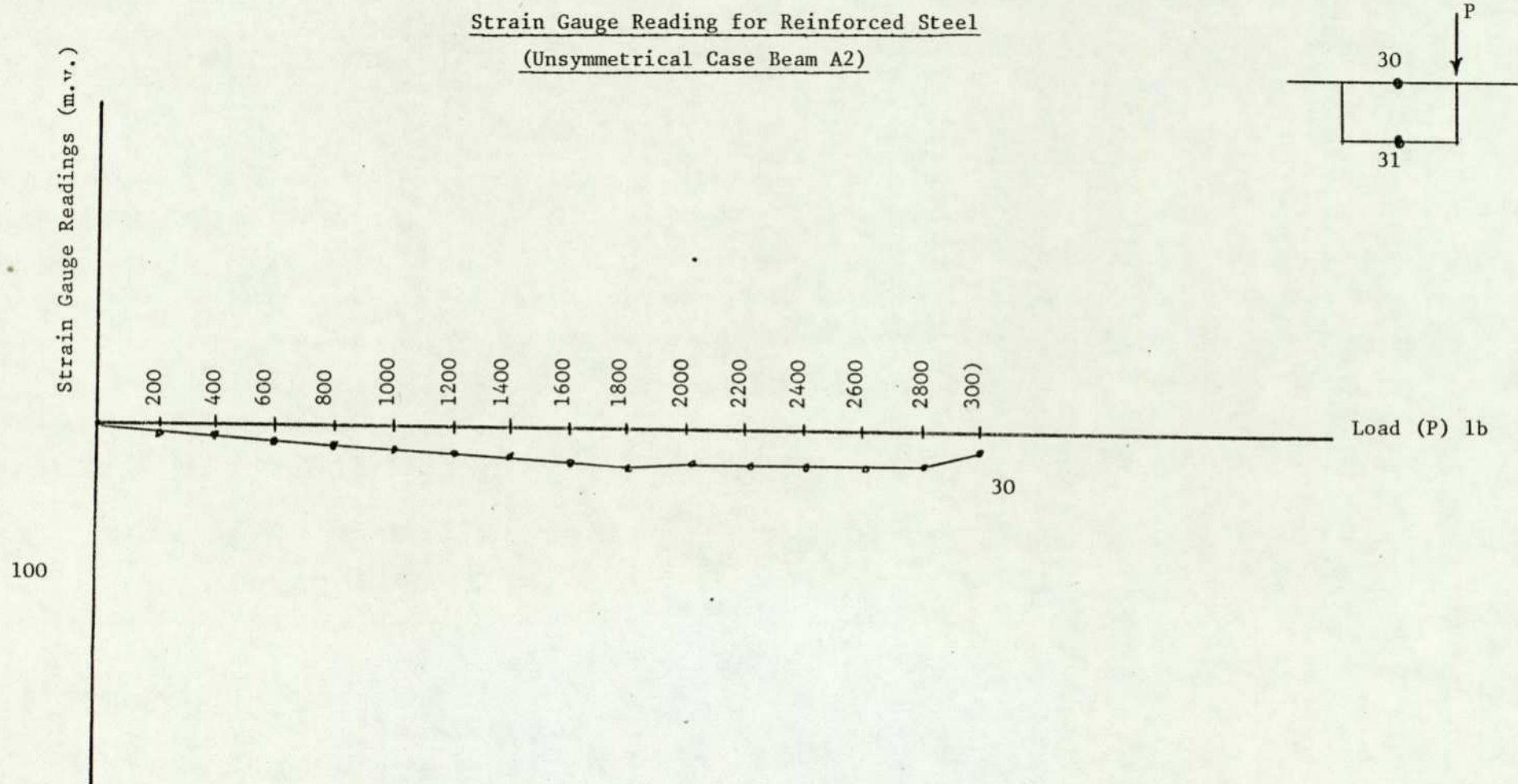


Figure (A-13)

4.2.4 Additional test

A third test was carried out for A3 which belongs to Group A with the same cross-section as A1 and A2, but with a longer span of about twice the length of those previously done (10 ft). The load was applied symmetrically, i.e. two equal point loads on the centre line of each web respectively. The load was increased in increments to collapse. Readings of strain gauges and dial gauges were recorded after each increment. The locations of the dial gauges were exactly as previously done for A1 and A2, see Fig. (A-16). However, a more detailed strain gauge distribution was done for gauges on the reinforcement steel. Seventeen gauges were placed on the steel at the mid section instead of two as done previously. This was done as more information was needed about the changes of stresses along the width of the flange as well as along the height of the web during the applied loads to collapse. Twenty four gauges were located on the outer surface of the concrete. Some were longitudinal, others horizontal, see Fig. (A-17). All the gauges were placed at the mid-section of the box. Load/deflection and load/stress relations were tabulated and plotted Fig. (A-10,A-19,A-20), Tables (A-5,A-6,A-7).

In this case (A3) loads were applied by means of two proving rings attached by means of one horizontal bar to ensure that they were on the same level. These proving rings were used instead of hydraulic jacks as the latter did not give accurate pressure at the

Dimensions of cross-section

For Box Bridge A3

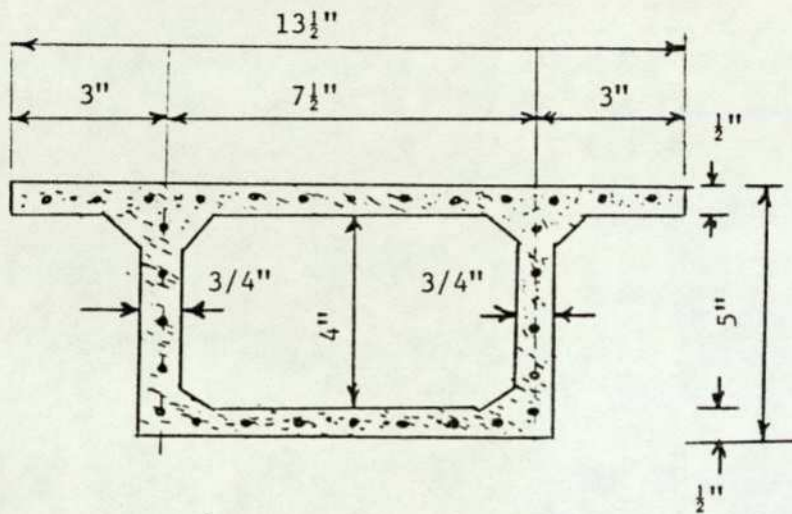
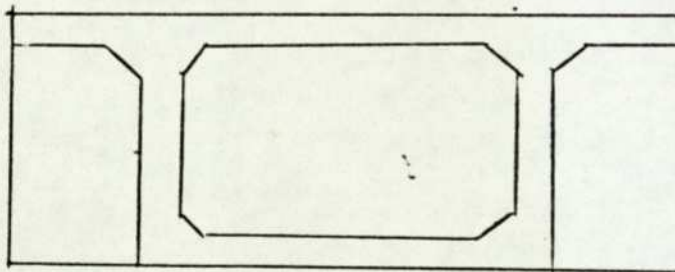


Fig. (A-14)

Reinforcement standard $1" \times 3" \times \frac{1}{8}"$ mesh



5 cms Diaphragm

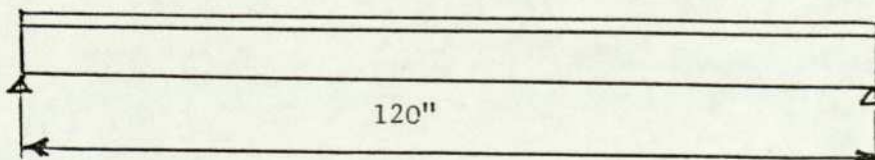


Fig. (A-15)

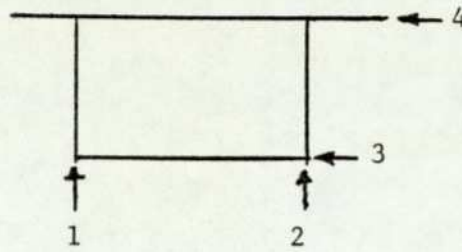
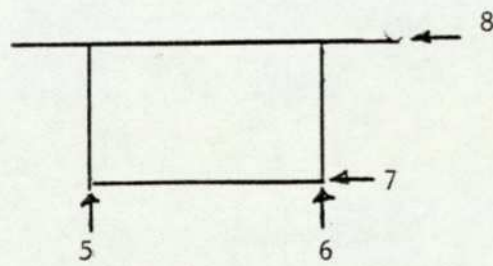
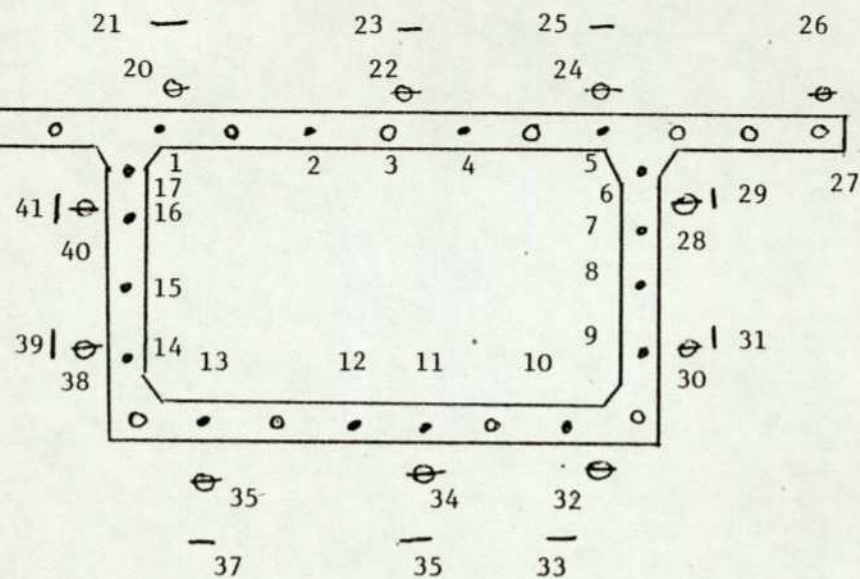
Location of Dial Gauges for A3Mid-span SectionQuarter-span Section

Figure (A-16)

SIDE 1



- Gauges on steel
- ⊙ Longitudinal G on concrete
- Transveral G on concrete

Location of Strain Gauges

Figure (A-17)

Net Reading of Dial Gauges for Box Bridge A3

Dial Gauge No.	Load	70	100	140	200	240	300	340	400	440	500	540
1		.92	1.4	2.98	7.05	8.9	11.34	13.1	15.85	18.1	20	
2		.-	1.38	2.92	7.05	8.9	11.35	13.1	15.9	18.15	20.4	
3		0	0	0	-0.1	-.1	-.1	-.15	-.15	-.15		
4		.60	.92	1.83	4.91	6.2	7.9	9.05	11	12.4	14.2	
5		.60	.90	1.83	4.98	6.22	7.95	9.15	11.1	12.7	14.4	
6		0	0	0	0	0	0.05	0.05	0.05	0.05	0.15	

Table (A-5)

Net Reading for Strain Gauges on Steel for BOX BRIDGE A3

Load Gauge No.	70	100	140	200	240	300	340	400	440	500	540
1	-10	-14	-21	-31	-37	-46	-51	-61	-70	-82	-98
2	-9	-14	-19	-29	-35	-44	-49	-58	-65	-60	
3	-9	-14	-20	-31	-38	-47	-52	-60	-67	-78	-87
4	-8	-12	-17	-26	-31	-39	-44	-53	-59	-69	-80
5	-14	-19	-27	-37	-43	-51	-56	-64	-70	-79	-86
6	-11	-15	-20	-25	-26	-27	-27	-24	-20	-11	-10
7	0	1	+3	+14	+23	+40	+53	+73	+92	+125	+196
8	+9	+14	+25	+45	+62	+94	+114	+152	+184	+243	+350
9	+19	+29	+46	+73	+89	+125	+155	+193	+206	+161	+113
10	+22	+33	+57	+123	+157	+220	+259	+318	+368	+438	+501

Table (A-6)

Net Reading for Strain Gauges on Steel for BOX BRIDGE A3

Load Gauge No.											
	70	100	140	200	240	300	340	400	440	500	540
11	+24	+36	+63	+128	+160	+211	+245	+306	+355	+410	+443
12	+22	+33	+58	+121	+154	+205	+236	+285	+326	+364	+397
13	+24	+38	+70	+133	+167	+226	+269	+330	+385	+438	+462
14	+19	+28	+50	+94	+120	+168	+195	+245	+293	+368	+453
15	+8	+14	+28	+59	+77	+105	+125	+157	+189	+243	+336
16	0	+1	+6	+19	+27	+41	+52	+68	+84	+116	+178
17	+10	+15	+20	+27	+30	+36	+39	+39	+42	+40	+26

Table (A-6)

Net Reading of Strain Gauges for Concrete for BOX BRIDGE A3

Load Gauge No.	70	100	140	200	240	300	340	400	440	500	540
18	+2	+3	+4	+5	+6	+7	+9	+10	+11	+13	+16
19	-9	-14	-20	-33	-41	-55	-65	-81	-96	-117	-148
20	0	-1	+1	+2	+2	+4	+4	+5	+7	+7	+12
21	-8	-12	-18	-27	-32	-44	-49	-63	-75	-92	-119

Table (A-7)

Net Readings of Strain Gauges on Concrete for BOX BRIDGE A3

Gauge No.	Load										
	70	100	140	200	240	300	340	400	440	500	540
22	-2	-2	-4	-4	-5	-5	-6	-4	-3	+1	+14
23	-8	-13	-19	-31	-39	-53	-60	-75	-88	-110	-142
24	-4	-5	-7	-6	-6	-5	-6	+6	+6	+10	+25
25	-11	-15	-23	-35	-42	-53	-62	-75	-86	-102	-131
26	0	0	0	-2	-2	-3	-1	0	+2	+3	+6
27	-9	-13	-19	-28	-33	-45	-53	-64	-75	-90	-114
28	+1	+2	+1	-3	-7	-5	+9	+18	+21	+27	+149
29	-1	-1	0	+2	+1	-2	-4	-5	-6	-9	-9
30	+4	+6	+7	+9	+17	+50	+60	+65	+72	+80	+96
31	-2	-3	-6	-11	-13	-6	-16	-18	-16	-8	+12
32	+2	+2	-2	-5	-5	-3	+5	+191	+227	+230	+202

Table (A-7)

Net Readings of Strain Gauges for Concrete for BOX BRIDGE A3

Load Gauge No.	70	100	140	200	240	300	340	400	440	500	540
33	-1	-2	-1	+1	+3	+7	+9	+2	+3	+5	+7
34	0	-2	-1	-12	-9	-4	-10	-8	+5	+21	+1105
35	+3	+5	+8	+6	+4	+1	-1	-2	-5	-10	-14
36	0	+1	-6	-18	-22	-23	-22	-21	-19	-17	-13
37	-2	-1	+4	+12	+16	+21	+22	+20	+19	+18	+19
38	+4	+5	+5	+4	+2	+1	+1	-1	-2	-2	-3
39	-2	-3	-3	-4	-3	-3	-3	-2	-2	-3	-1
40	+2	+2	+1	0	-1	-1	-2	-3	-3	-4	-7
41	-1	-2	-3	-4	-6	-8	-8	-10	-12	-5	-15

Table (A-7)

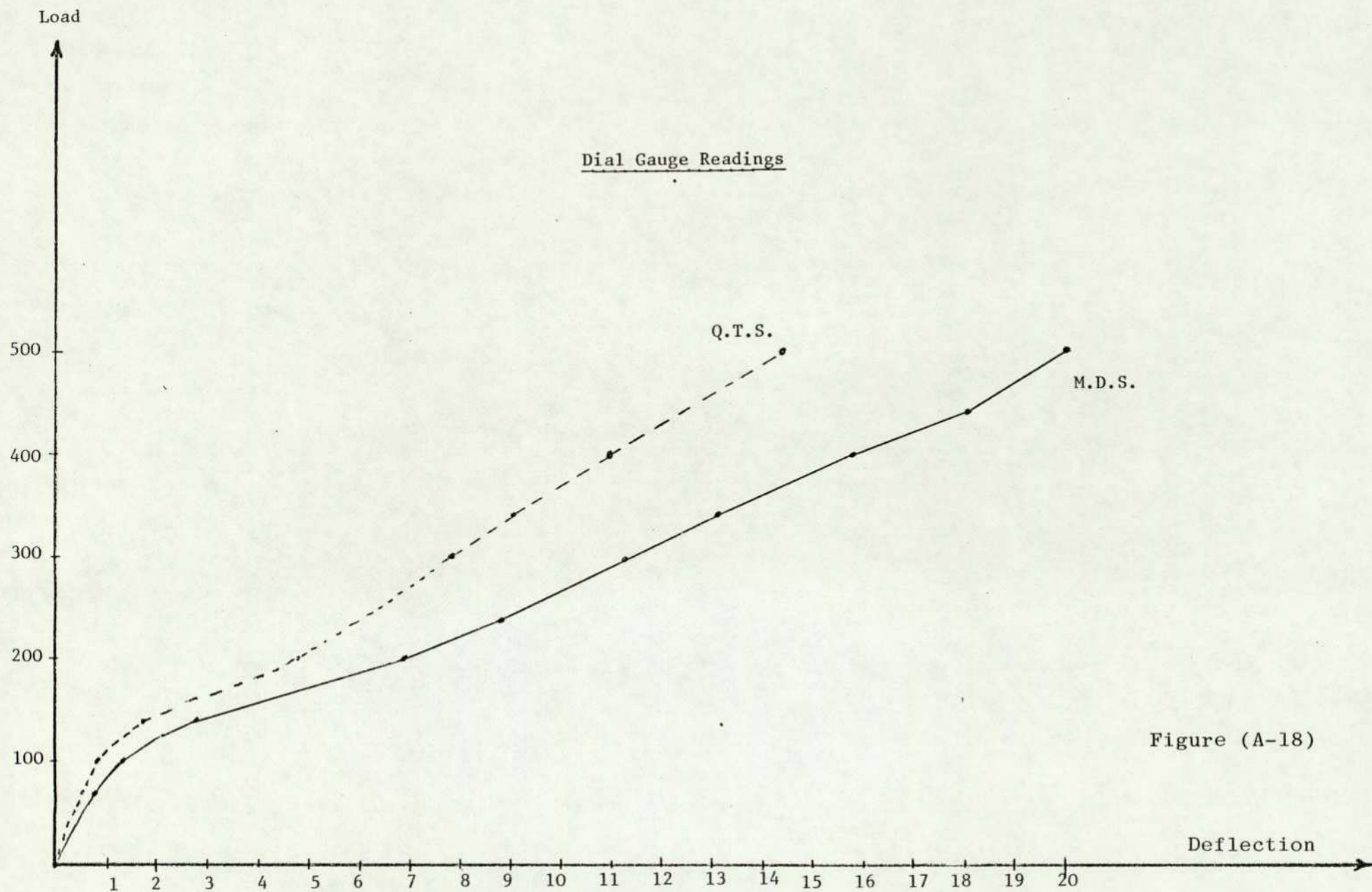


Figure (A-18)

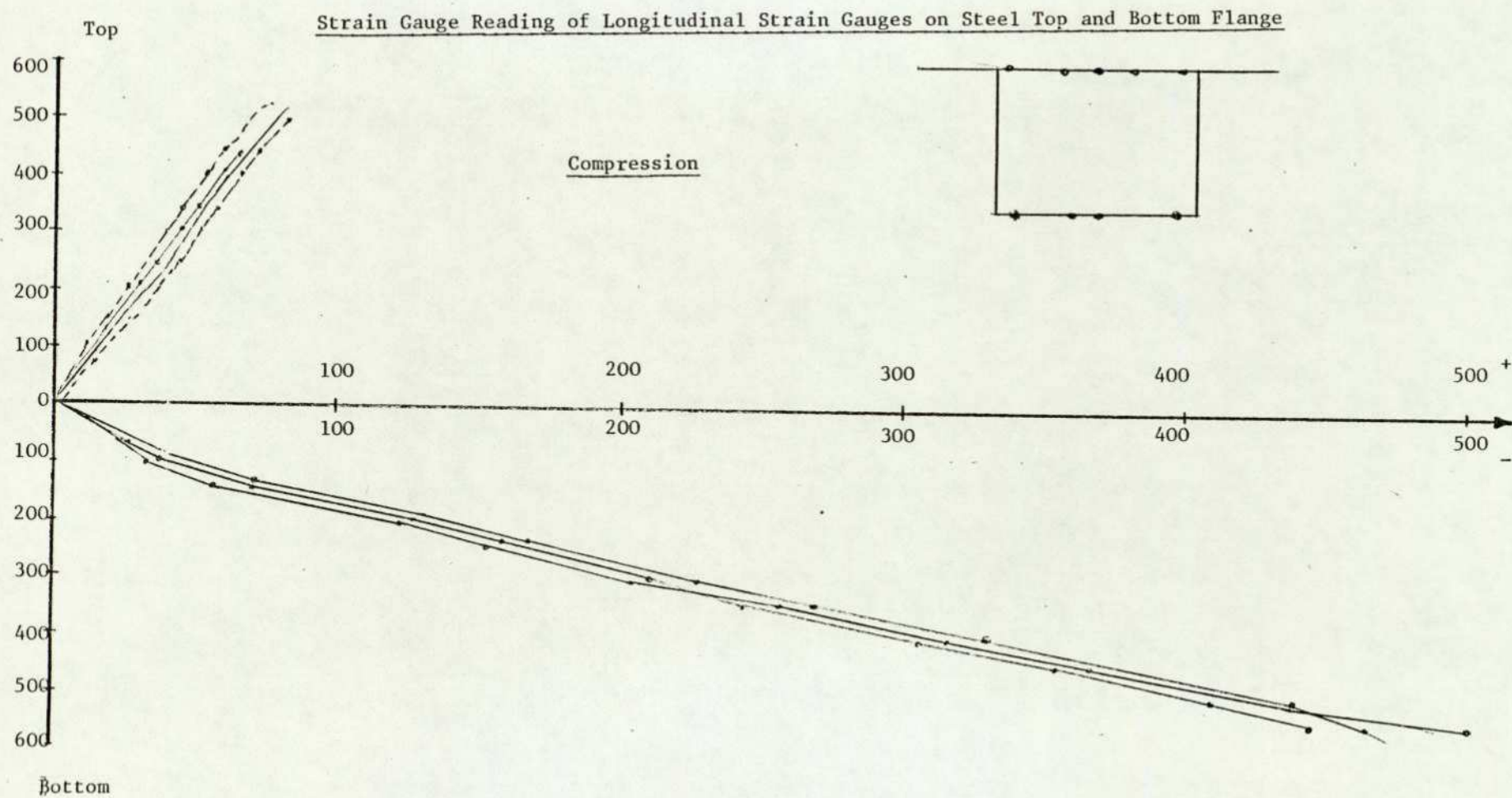


Figure (A-19)

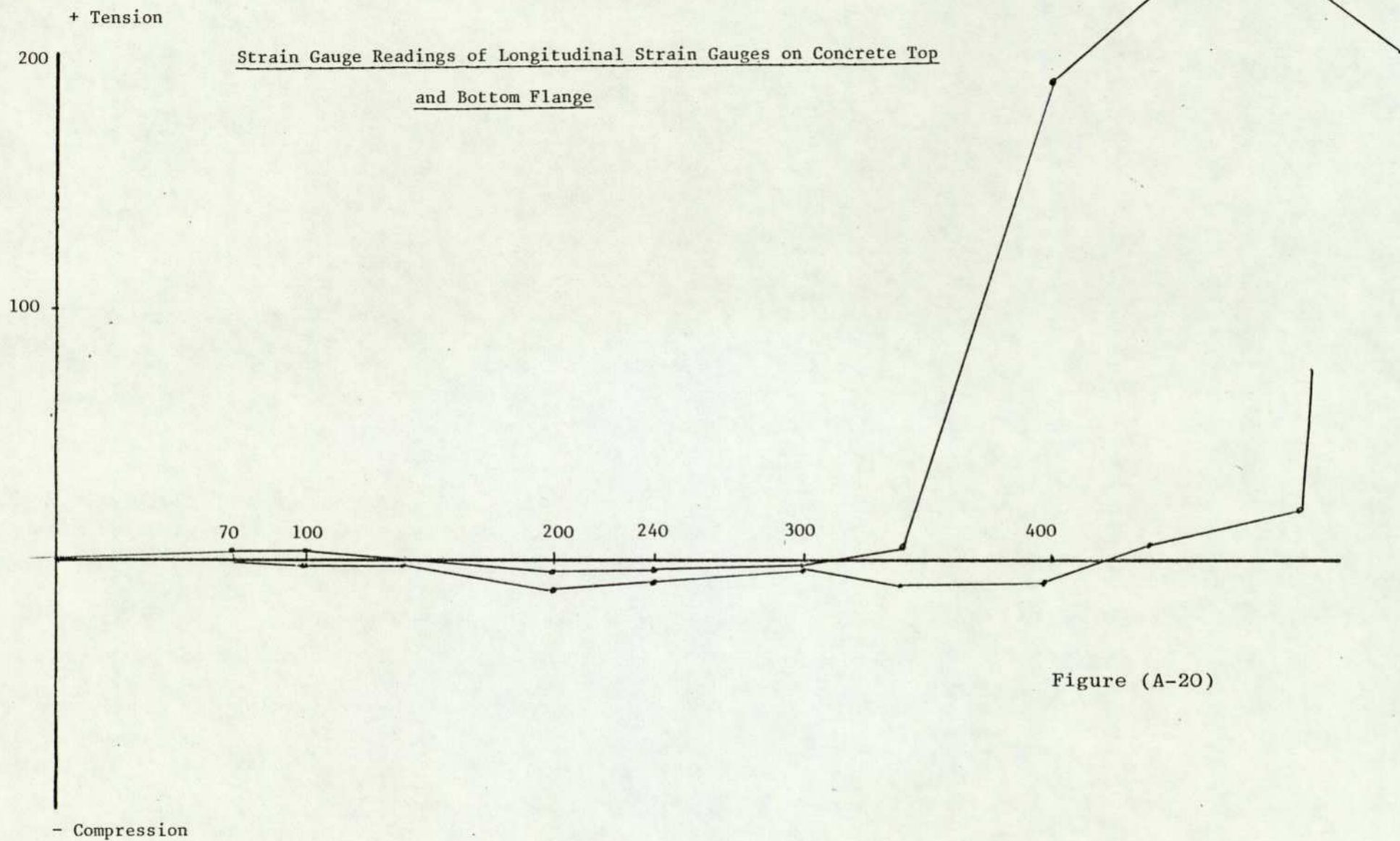


Figure (A-20)

primary loads and that might have been due to the losses of pressure in the jacks themselves.

The rings were calibrated in order to have a correct reading of the pressure on the box.

4.2.5 Observations during tests

During the tests on A1, when applying the increments of loads cracks were heard and signs of them could be seen at about the 800 lb load. Then on further application of the load more new cracks were formed and the previous cracks were gradually increased in length and width across the breadth of the bottom flange and along the height of the web.

Most of the cracks were formed at the bottom of the flange and extended along the whole breadth on increasing the load. Some of these cracks extended along the height of the web, and on further increase of load more cracks were seen along the web's height and some of them extended till the level of the top flange. Most of the cracks were found in the middle third of the box beam's length, with the maximum width of the crack at the middle and decreasing along both sides to the ends of the middle third. The box collapsed at 3000 lb and was divided into approximately two equal halves

with a wide cracked open section of about 4.8 mm in width. On further applying the load until about 3150 lb the whole structure collapsed completely.

During the tests on A2 cracking was heard at a load of about 600 lb and visible hair cracks could just be seen at the bottom flange at mid-span. On increasing the load these hair cracks increased in size and number and some of them extended along the whole breadth of the flange. All these hair cracks were at right angles to the horizontal direction. On further increase of load, cracks were seen along the height of the loaded web. At 3000 lb load, the beam collapsed and was divided into approximately two equal parts with the formation of one large cracked mid section as in the case of A1. Two new cracks could be seen at the top and bottom flange. At the bottom, the crack was in the longitudinal direction extending a short distance on both sides of the mid section. At the top flange, the cracks had the form of an arc shape around the point of application of the load which also extended a short distance along both ends of the centre line.

As for tests on A3, the observations during the test exactly resembled those on A1 with the exception of the collapse load as A3 collapsed under a load of 1500 lb.

All loading conditions can be seen in the photographs and their corresponding collapse form.

4.3 GROUP B

This part of the chapter deals with the double cell box beam models. Some of these beams were simply supported while others were column supported. Various tests under different loading conditions were carried out for the different box beams.

Three box beams, B1, B2 and B3 were fully investigated as an example of this group. B1 was symmetrically loaded to failure, while B2 was unsymmetrically loaded to failure. Both B1 and B2 were simply supported while B3 was column supported and symmetrically loaded to failure too.

The behaviour of each box during the elastic range and the ultimate conditions to collapse were recorded.

Tables and graphs showing the different behaviour of the boxed under their applied loads were tabulated and plotted.

4.3.1 Material, casting and testing equipment

The material and methods of casting were exactly as previously mentioned for Group A, since all boxes were constructed by the same mix and reinforced by the same steel mesh. The casting of the column support for the case of B3 can be seen in the photograph. A typical shape of the cross section could be seen in Figs. (B-1).

Loading during tests were applied by means of a pressure bag or by means of a hydraulic jack, the latter being the same as previously mentioned. As for the pressure bag, it was made out of polythene and attached underneath a reaction wooden box by means of double sided sellotape. A valve was fitted to the air bag at its mid section and connected to 120 p.s.i. air compressor by means of a plastic tube.

Pressure was measured by means of a pressure gauge and a mercury manometer. The whole air bag with the reaction wooden box was strapped by means of 3 steel frames at each third of the box's length and connected to six proving rings respectively. These proving rings helped in calibrating the pressure from the mercury manometer and to ensure the distribution of pressure along the box. The location of these proving rings can easily be seen from the general layout of B1 (Fig. B-2) and from the photographs shown later on.

Electric strain gauges were located on the steel mesh of the steel reinforcement only for this group of boxes as the previous readings for the strain gauges on the concrete surface for the previous loads did not reveal satisfactory results due to some primary micro-concrete cracks. The number and location of these strain gauges can be seen in Fig. (B-3).

For B2, the unsymmetrically loaded box, the load was applied by means of a bogey load acting on one of its cells only. Each of the two jacks was resting on a steel box channel, and each channel was supported on two 1/2" smooth rollers resting on a V grooved plate 8" long. This arrangement can easily be seen from the layout of B2 (Fig. B-13) and from the different photographs shown at the end of Group B section.

Strain gauges, dial gauges and Demec gauges were used to investigate the different changes in the boxes during the different loading conditions Figs. (B-4), (B-14)

4.3.2 Test Procedure

Three double cell box beams were tested, B1, B2 and B3.

The box beam B1 was symmetrically loaded by the application of a U.D.L. along its whole length and covering its two cells by means of a pressure bag. The pressure was applied in increments by compressing air into the bag. A pressure gauge, a mercury manometer and six pressure rings were used to determine the exact pressure applied on the box. The load was then increased to collapse.

All the readings were recorded for the different applied pressure increments. All cracks were located and marked and compared with each other. Readings for different relations were recorded and plotted, see Fig. (B-5 to B-12), Tables(B-1, B-2).

The second box, B2, was unsymmetrically loaded by the application of a load on one of the cells only. At the beginning the load was applied by means of a pressure bag that covered the whole length of one cell only. The load was increased within the elastic range only, since the pressure required for collapse could not be resisted by the material of the bag.

Another method of loading was used, by the application of a bogey load on one of the box's cell as previously mentioned.

The pressure was applied from a pressure hand pump and the whole system was connected to the two jacks in order to obtain the same value of pressure in each jack for each loading case. The load was then increased to collapse. The readings of all the corresponding gauges were recorded and plotted in Fig. (B-15 - B-20), Tables (B-3, B-4, B-5).

Both boxes, B1, B2, were simply supported.

The third box beam B3 of the series of tests carried out for Group B was column supported at both ends, see Fig. (B-21). The box was symmetrically loaded by means of a pressure bag covering its two cells and its whole length. The load was increased within the elastic range. Then that pressure bag was removed and a live load was applied at mid-section across the two cells, centre to centre with the central line of the two outside ends. The load was increased to collapse and graphs were plotted showing the different changes due to increase of load, Fig. (B-22 - B26).

4.3.3 Observations during tests

For B1 the symmetrically loaded box, the failure occurred at mid span with the box beam dividing into two equal parts. Yielding of the bottom steel reinforcement of the bottom flange and the corresponding yield of the longitudinal steel in the web at that failure section could be seen. Location of the steel mesh in all the plates was investigated and its correct position checked. Cracks along the bottom flange perpendicular to the longitudinal direction occurred along its whole breadth, and, with the increase in load, they propagated into the three webs. Failure was due to the yielding of the longitudinal steel at that section. Failure occurred at a pressure of 4.87 p.s.i.

For B2 the unsymmetrically loaded box, the failure occurred near the point of application of one of the plates nearer to the centre line of the box, and hence divided the box beam completely into two parts. The failure was due to the yielding of the longitudinal steel by the formation of a wide crack at that section along the whole breadth of the bottom flange and into the three webs. No appreciable effect due to the unsymmetrical loading case could be seen, except some increase of deflection of the loaded cell compared with that of the unloaded one. Failure occurred at total load (5400 lb).

The third test, B3, had the same failure features as that of B1. Failure occurred at the mid-section under maximum B.M. effect.

All diagrams of the different loadings, different support conditions and different failure patterns are shown in Figs.

Cross Section View for the Three Different Boxes

(B1, B2, B3)

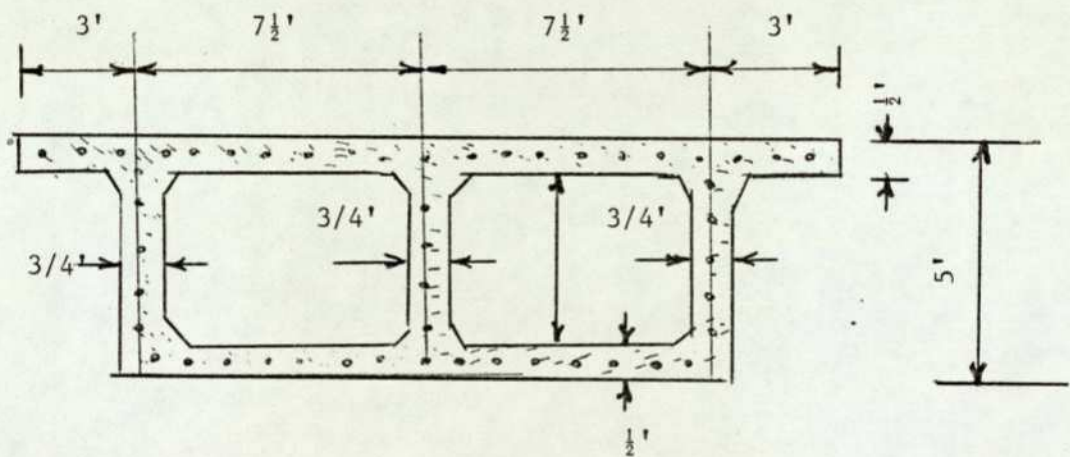
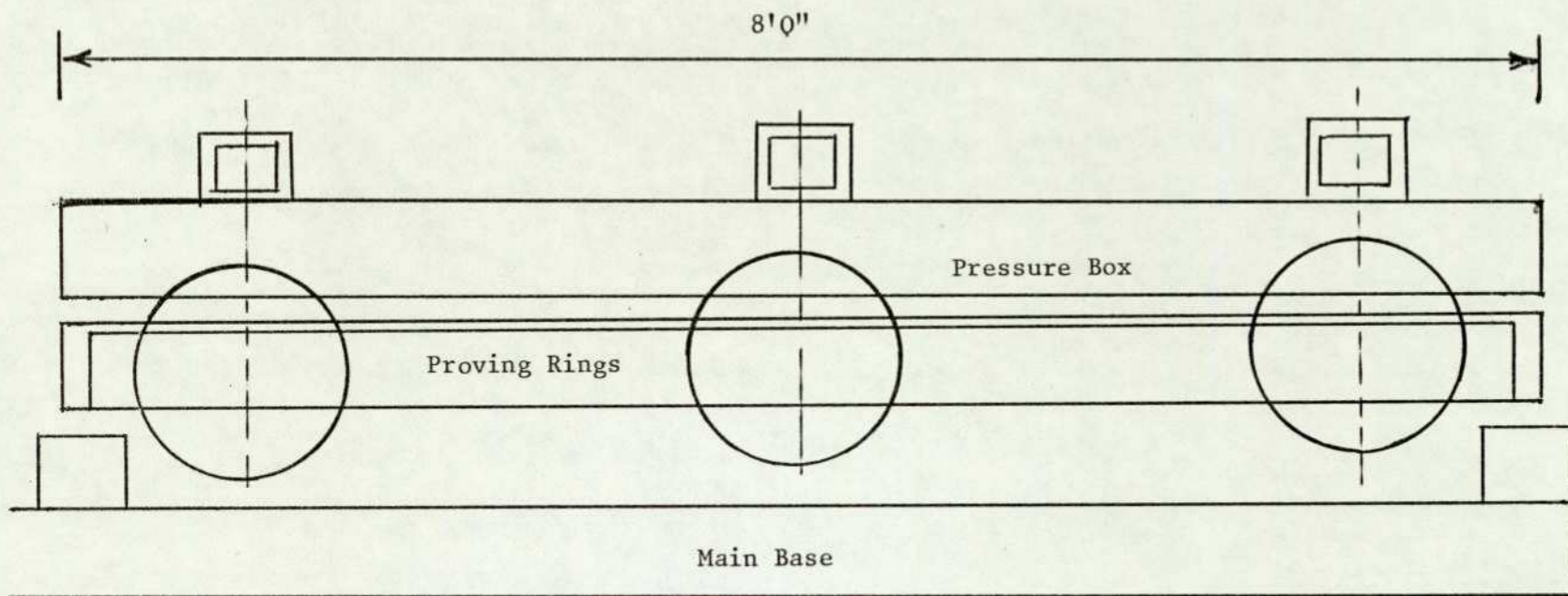


Figure (B-1)



GENERAL LAYOUT OF BOX BEAM B1

Figure B-2

Location of Strain Gauges

For three Box Beams (B1, B2, B3)

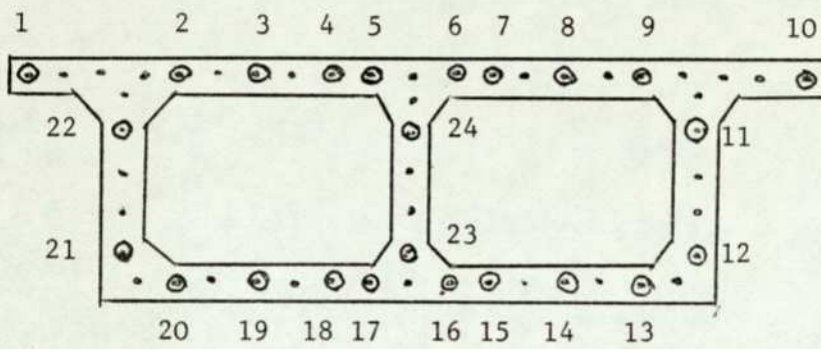


Figure B-3

Location of Dial Gauges

For three different Boxes (B1, B2, B3)

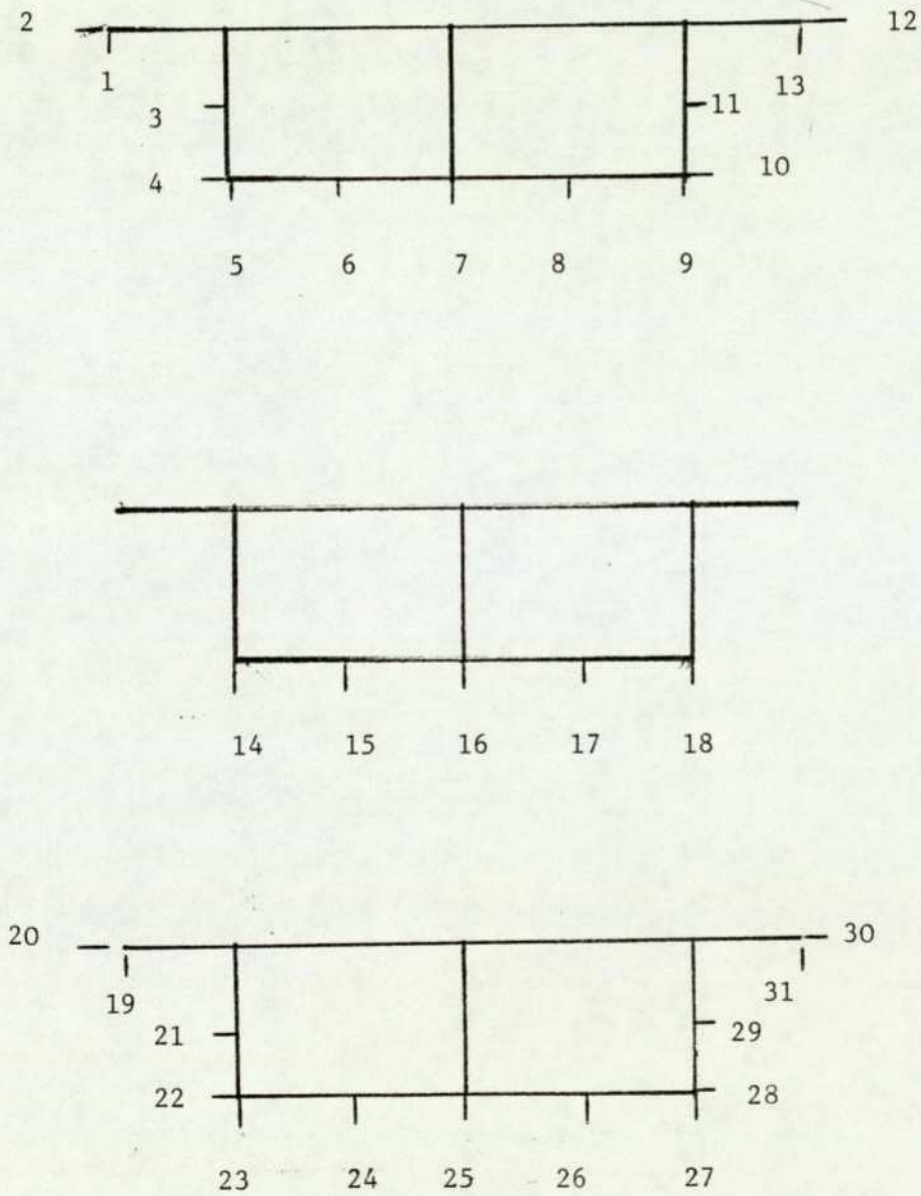


Figure B-4

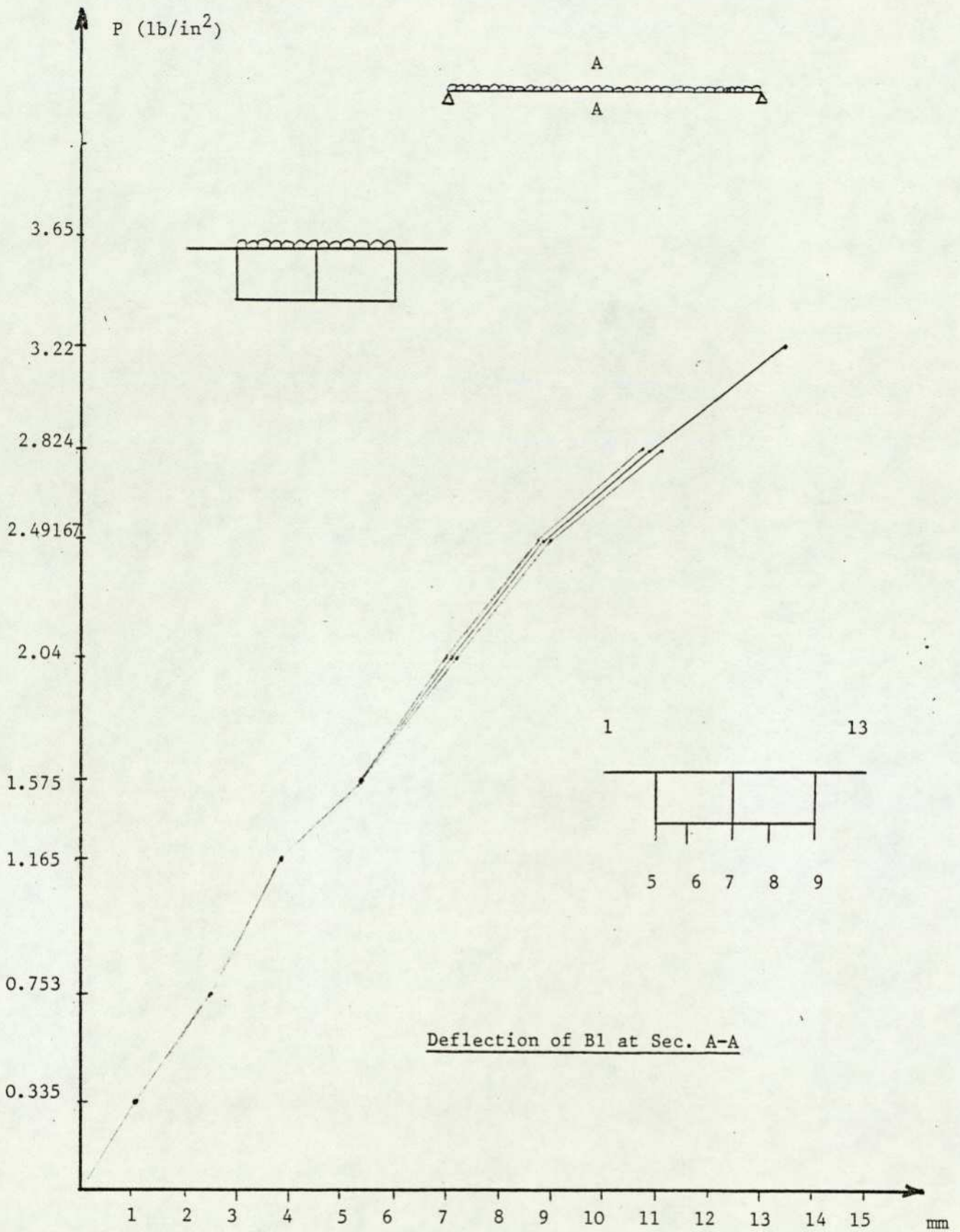


Figure B-5

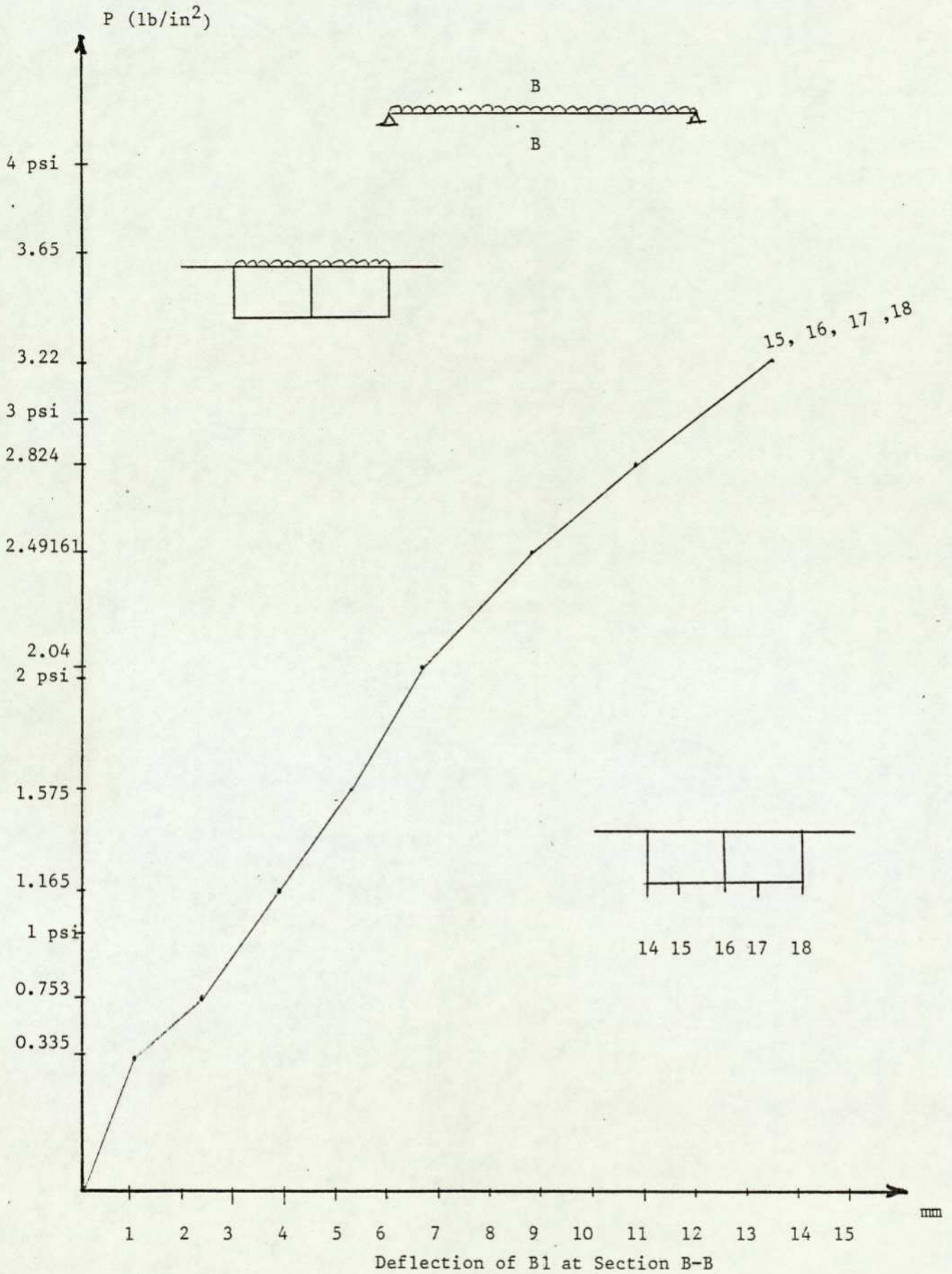
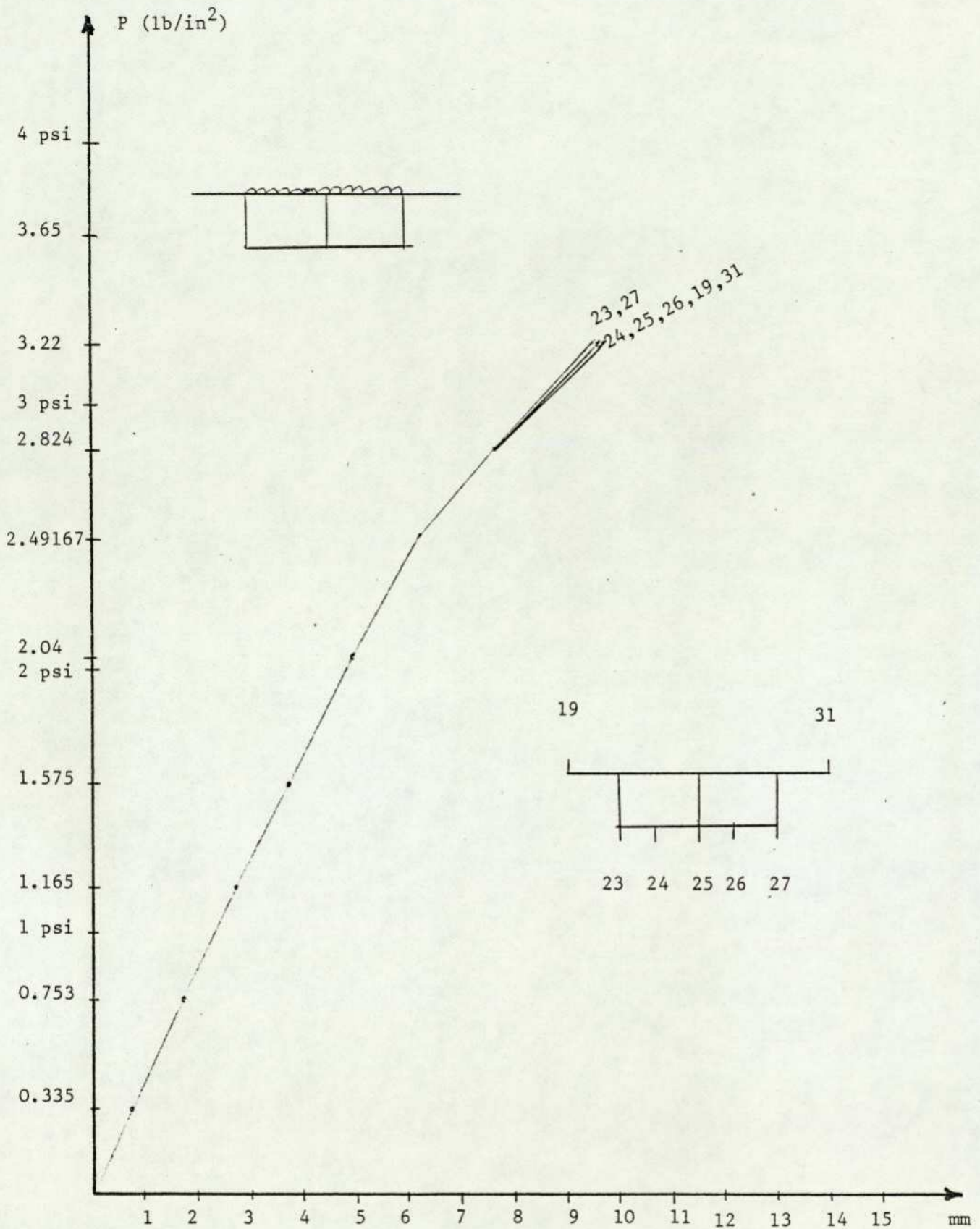


Figure B-6



Deflection of $B1$ at Quarter Span

Figure B-7

Gauge No.	Load p.s.i.	0.5	1	1.5	2	2.5	3	3.5	4
1		1.07	2.5	3.88	5.34	7.05	8.75	10.77	
2		0.07	0.16	0.2	0.3	0.35	0.43	0.6	
3		0.11	0.06	0.11	.01	0.09	0.15	0.24	
4		0	.04	+.07	+.07	+0.06	0.09	0.21	
5		1.08	2.52	3.91	5.39	7.11	8.83	10.85	13.46
6		1.07	2.52	3.92	5.39	7.1	8.86	10.9	13.5
7		1.07	2.52	3.93	5.41	7.15	9.89	10.94	13.58
8		1.05	2.51	3.95	5.43	7.17	8.93	11.0	13.6
9		1.05	2.54	3.94	5.44	7.18	8.93	10.99	13.63
10		0.12	2.54	3.94	5.44	7.18	8.93	10.99	13.63
11		0.02	0.06	0.06	0.07	0.11	0.13	0.3	

Net Reading of Dial Gauges for BOX B1

Table (B-1)

Gauge No.	Load p.s.i.	0.5	1	1.5	2	2.5	3	3.5	4
12		0.02	0	0	0.01	0.01	0.02	-.06	
13		1.04	2.49	3.9	5.39	7.18	8.91	11.06	
14			2.44	3.85	5.29	6.99	8.73	10.81	13.43
15		1.03	2.35	3.8	5.26	6.98	8.72	10.81	13.37
16		1.03	2.45	3.84	5.29	6.99	8.73	10.84	13.44
17		1.03	2.43	3.86	5.31	7.01	8.71	10.91	13.49
18		1.01	2.45	3.89	5.34	7.06	8.61	10.91	13.59
19		0.15	1.75	2.67	3.69	4.9	6.19	9.66	
20		0	0	-.08	-.04	-.05	-.05	-.06	
21		.01	.04	0.06	0.08	0.08	0.08	0.09	

Net Readings of Dial Gauges for BOX B₁

Table (B-1)

Gauge No.	Load p.s.i.								
		0.5	1	1.5	2	2.5	3	3.5	4
22		0.03	0.02	0.05	0.04	0.05	0.07	0.09	
23		0.77	1.76	2.71	3.71	4.92	5.23	7.69	9.57
24		0.76	1.76	2.72	3.73	4.94	6.74	7.7	9.6
25		0.77	1.77	2.75	3.77	5.0	6.33	7.79	9.7
26		0.77	1.77	2.75	3.78	5.0	6.31	7.78	9.71
27		0.61	1.6	2.62	3.63	4.9	6.27	7.69	9.58
28		0	0	-.02	-.02	-.04	-.03	-.04	
29		0	0	0	0	.01	.02	.02	
30		-.02	-.03	-.03	-.03	-.05	-.07	-.09	
31		0.74	1.74	2.69	3.67	4.88	6.24	7.75	

Net Readings of Dial Gauges for BOX B₁

Table (B-1)

Load p.s.i. Gauge No.	0.5	1	1.5	2	2.5	3	3.5	4	4.5
1	-9	-17	-31	-42	-54	-64	-75	-89	-102
2	-7	-17	-26	-35	-45	-54	-63	-75	-86
3	-9	-18	-26	-35	-46	-55	-63	-75	-88
4	-9	-21	-24	-34	-45	-53	-64	-75	-85
5	-6	-15	-25	-33	-43	-51	-61	-72	-81
6	-8	-16	-25	-34	-44	-52	-61	-72	-81
7	-6	-15	-24	-32	-42	-50	-58	-70	-79
8	-5	-13	-28	-34	-45	-51	-63	-75	-86
9	-9	-17	-28	-37	-48	-57	-69	-82	-92
10	-11	-18	-29	-39	-51	-62	-72	-84	-96
11	-3	-5	-8	-9	-12	-2	-13	-2	-10
12	+25	+58	+88	+124	+162	+201	+248	+315	+395

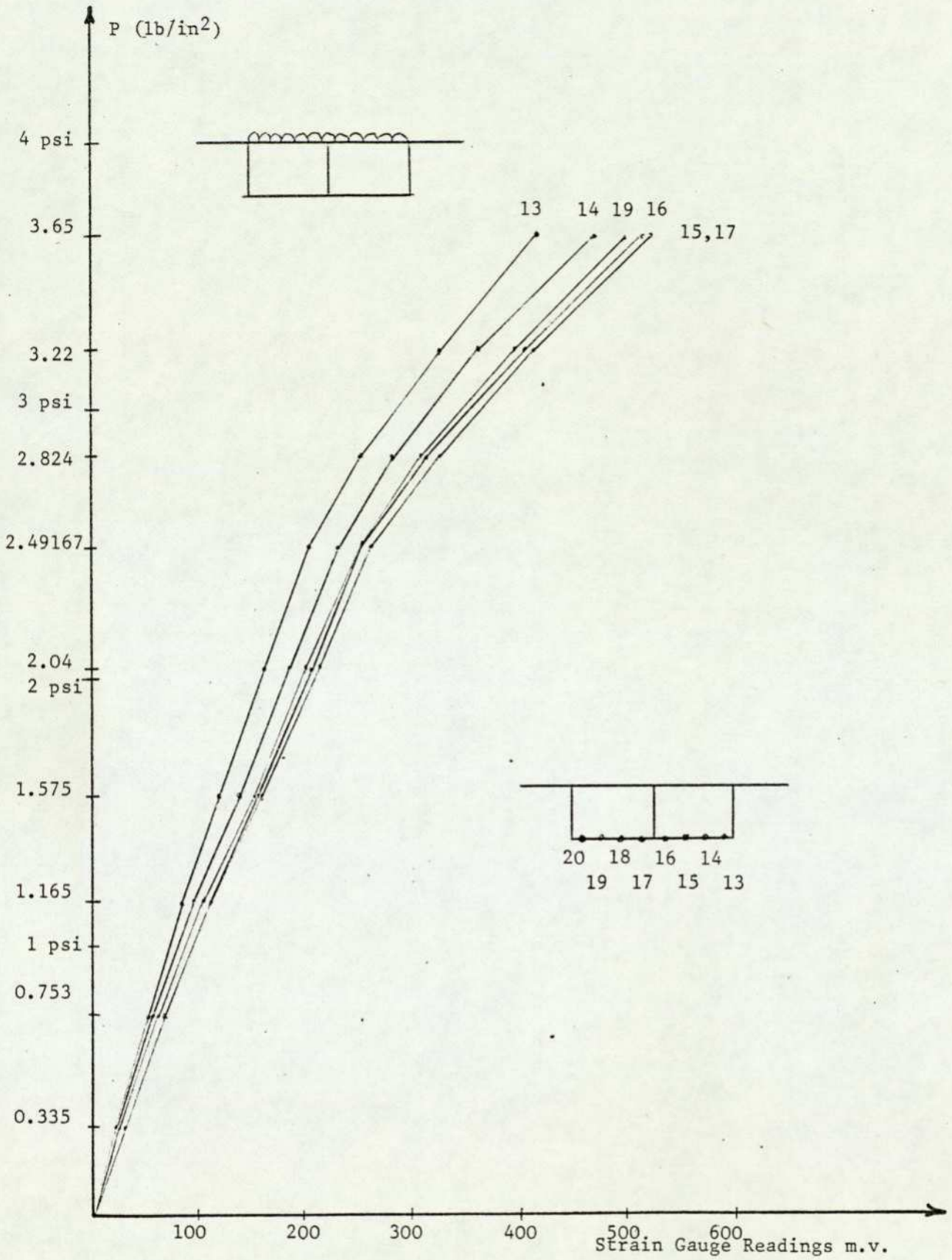
Net Readings of Strain Gauges for BOX B1

Table (B-2)

Load p.s.i. Gauge No.	0.5	1	1.5	2	2.5	3	3.5	4	4.5
13	+2	+55	+86	+120	+164	+205	+254	+326	+416
14	+27	+59	+99	+124	+189	+232	+283	+362	+474
15	+31	+72	+114	+161	+215	+264	+327	+415	+523
16	+29	+66	+107	+154	+202	+252	+314	+405	+518
17	+29	+73	+119	+164	+217	+268	+324	+416	+519
19	+29	+73	+114	+158	+200	+254	+310	+396	+498
20	+31	+71	+112	+154	+203	+249	+303	+337	+474
21	+25	+56	+92	+127	+167	+201	+243	+304	+379
22	-5	-8	-9	-11	-17	-18	-19	-19	-15
23	+23	+54	+91	+122	+158	+192	+236	+301	+384
24	+4	+4	+18	+24	+23	+42	+51	+70	-180

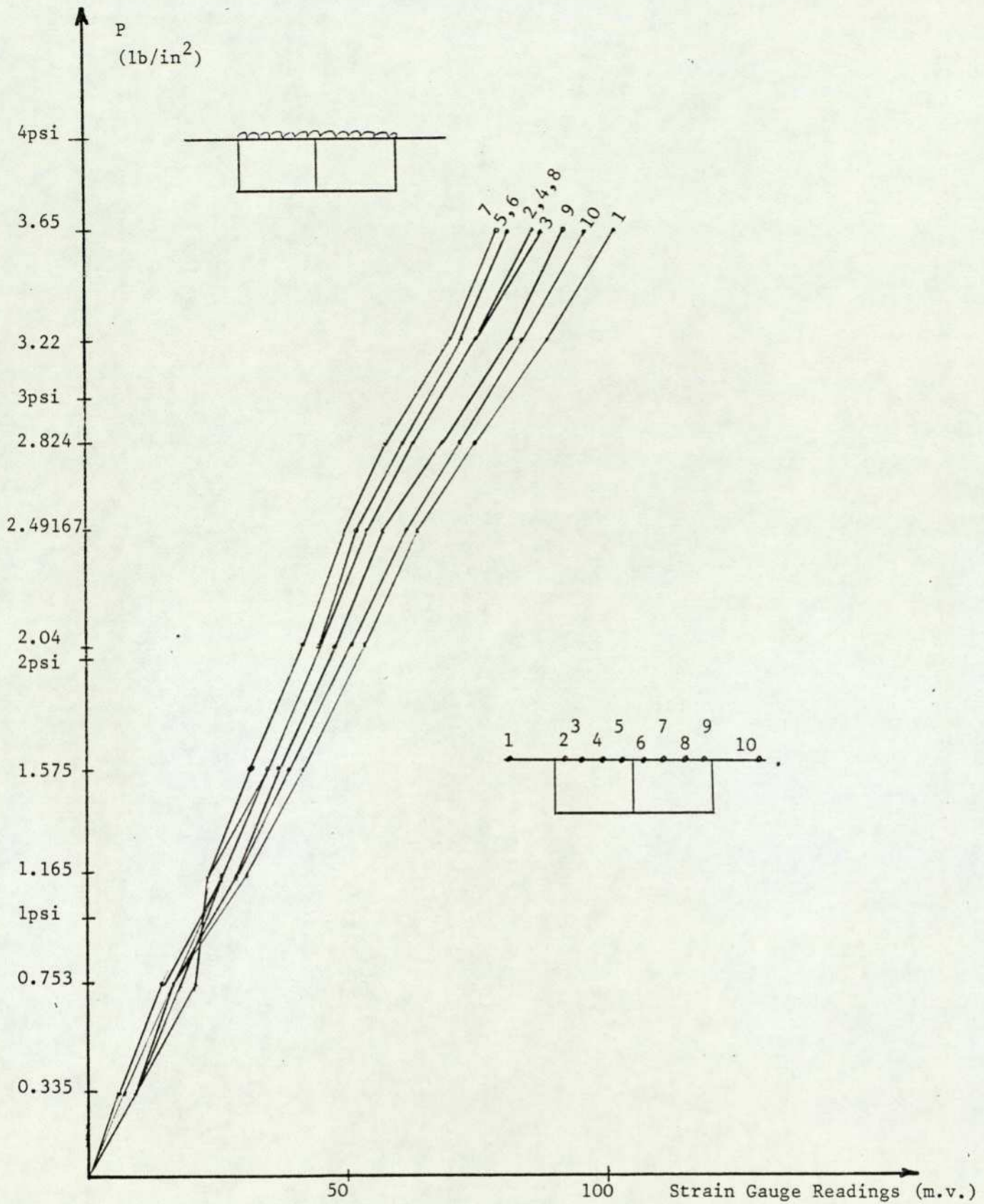
Net Readings of Strain Gauges for BOX B1

Table (B- 2)



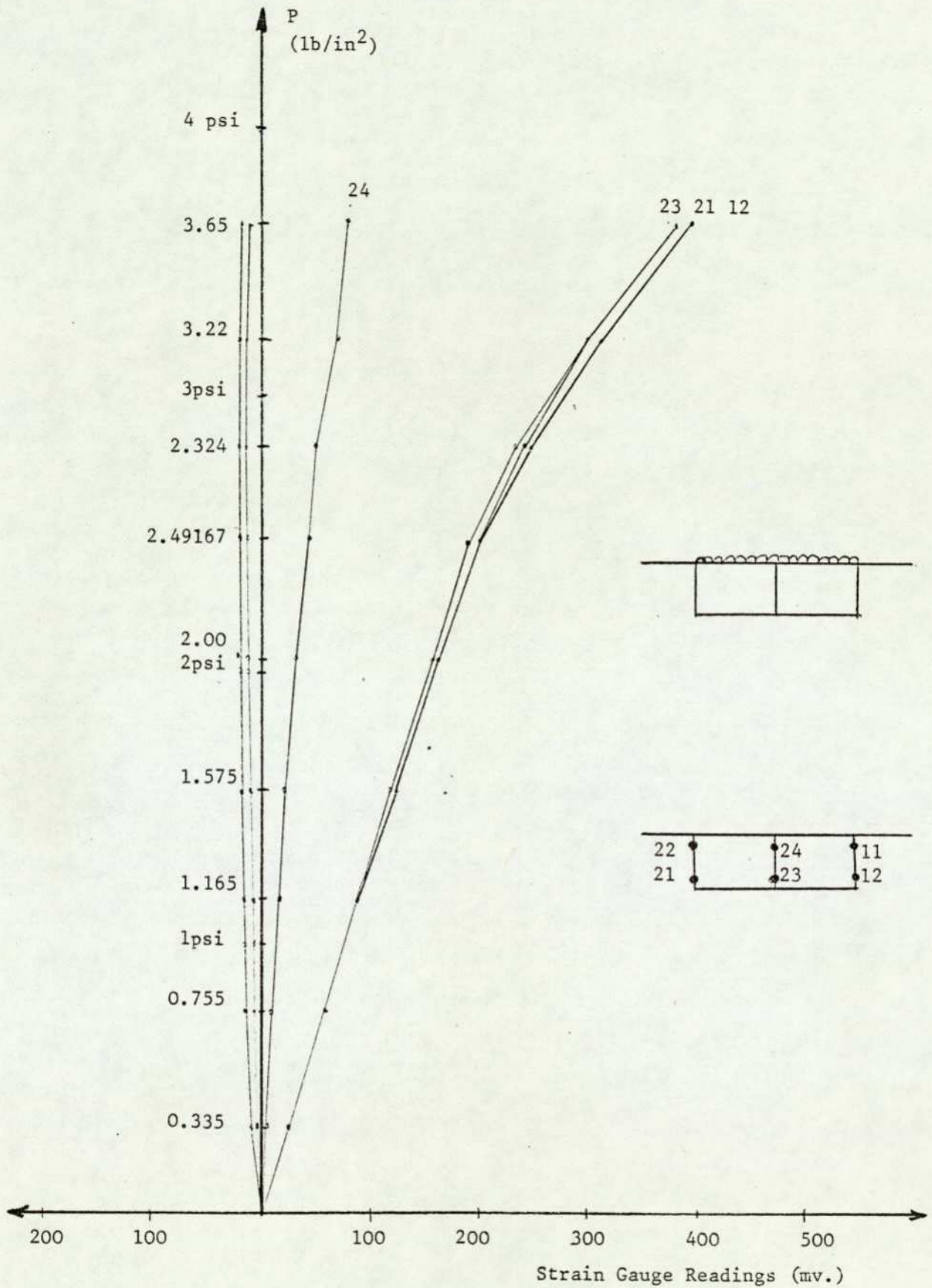
Bottom Readings of Strain Gauges on Steel for B1

Figure (B-8)



Top Readings of Strain Gauges on Steel for B1

Figure (B-9)

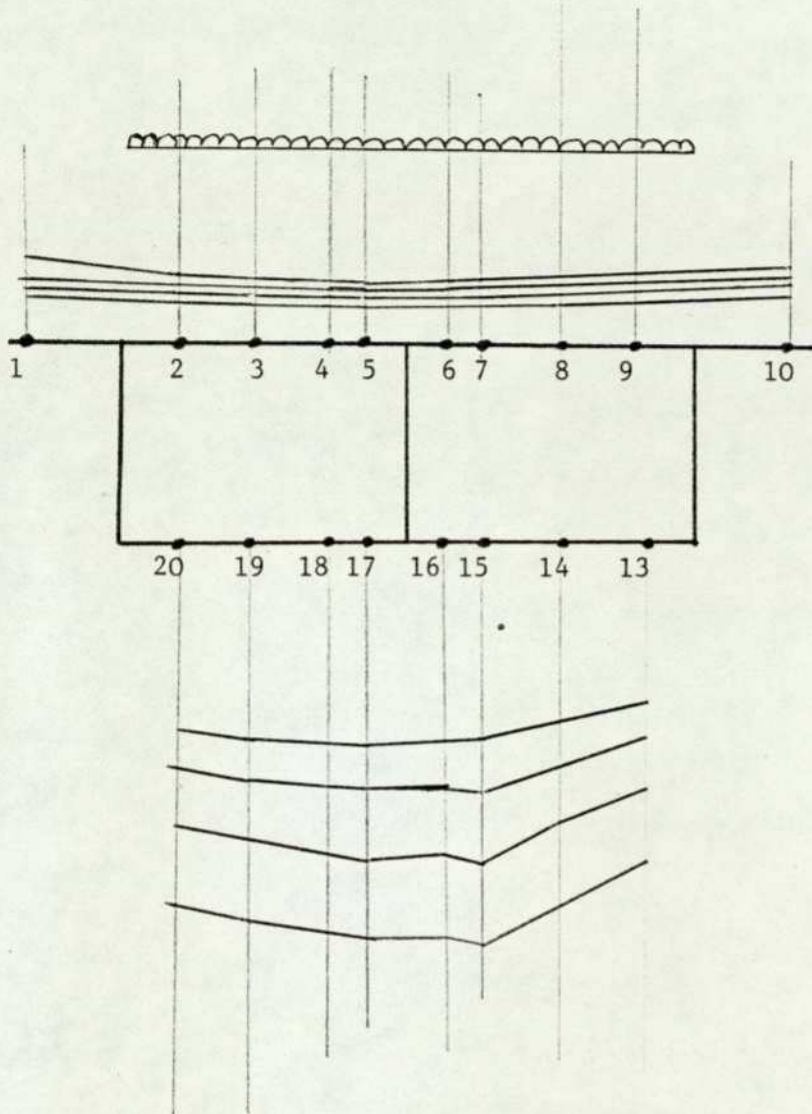


Readings of Strain Gauges Located on Webs for B1

Figure B-10

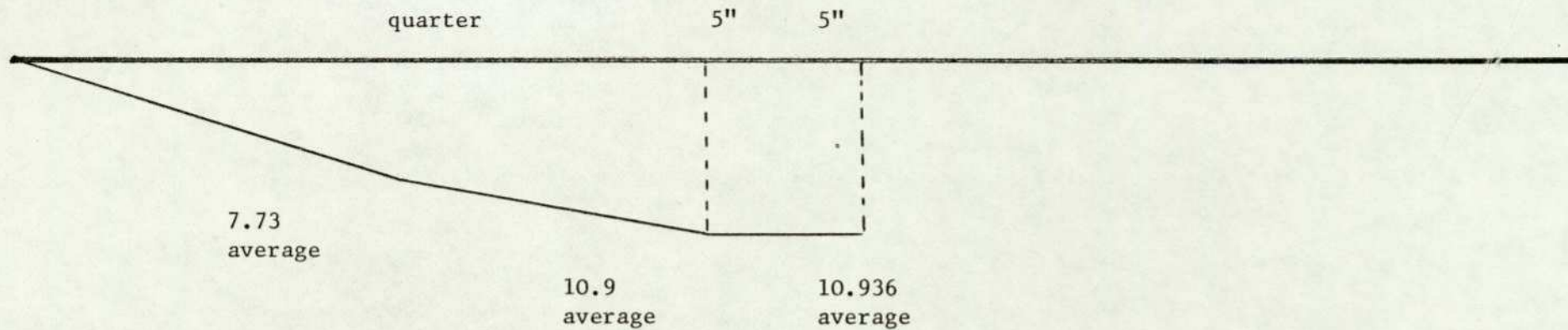
Profile of Stresses

(For Symmetrical Case B1)



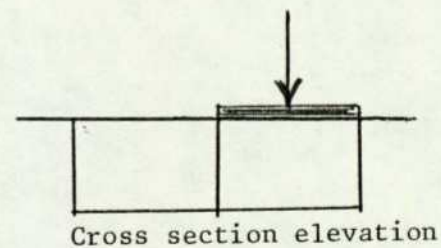
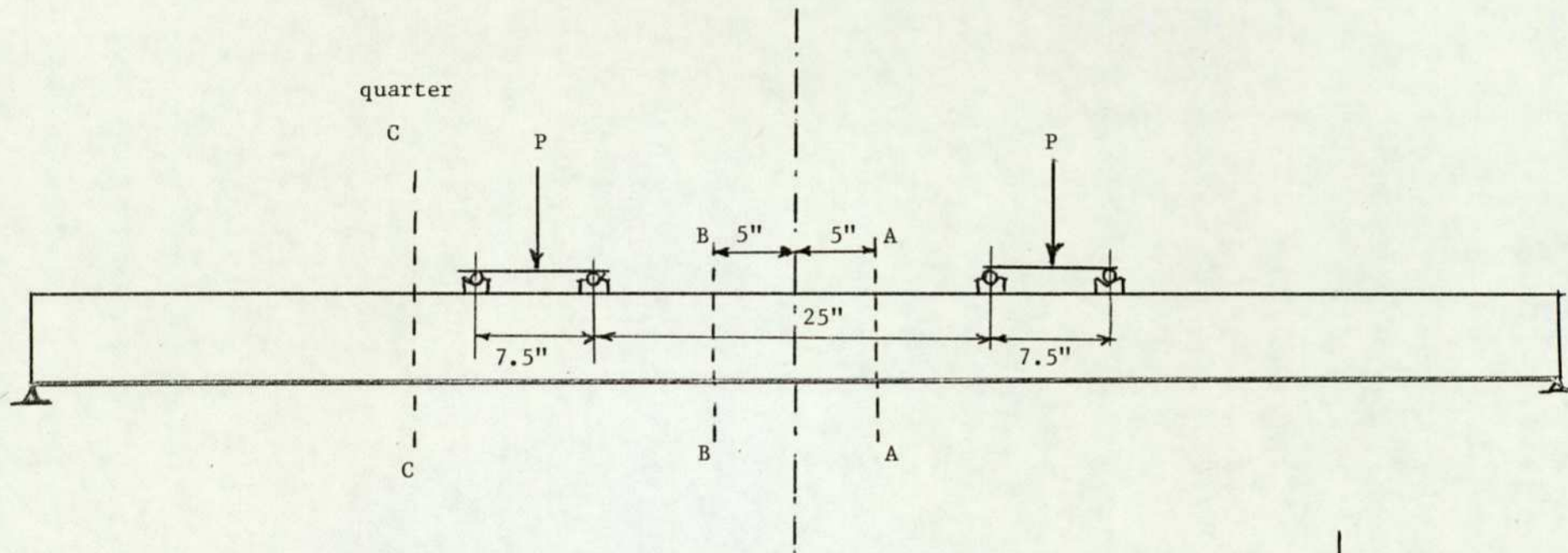
13	14	15	16	17	19	20
205	232	264	252	268	254	249
254	283	327	314	324	310	302
326	362	415	405	416	396	377
416	474	523	518	519	498	474

Figure B-11



Longitudinal Deflection for a Stage of Loading for R1

Figure B-12



LONGIDUTINAL SIDE VIEW OF BOX B2

Figure B-13

Load Increments

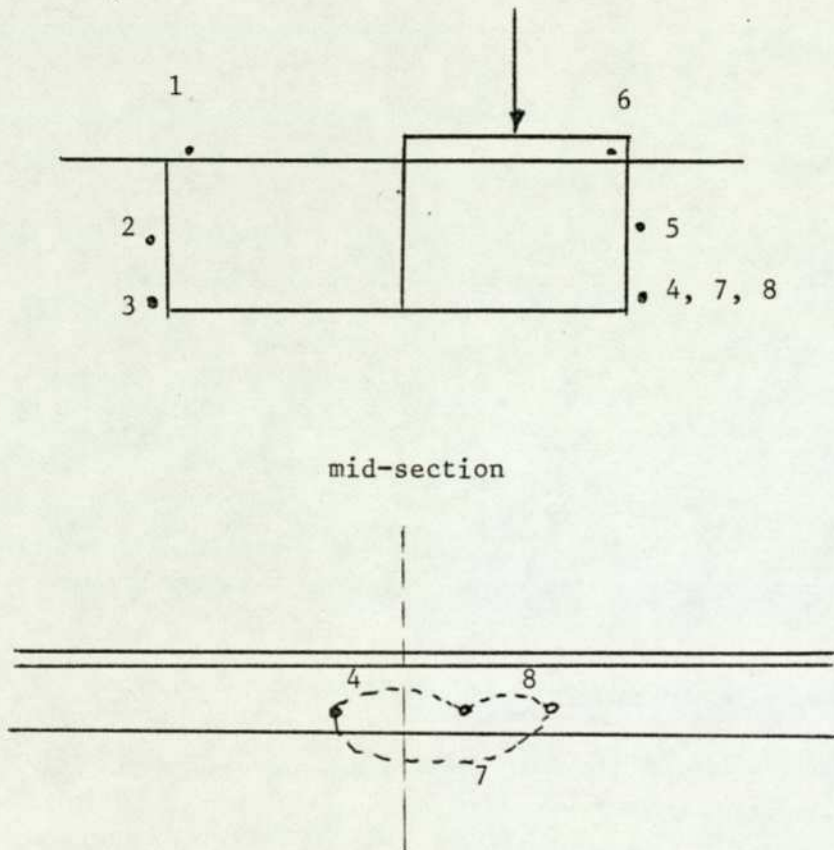
Surface area = 720 in²

	<u>lb</u>		
Load	360	180 (on each jack) 180	0.5 p.s.i.
	1440	720 (on each jack) 720	2 p.s.i.
	2160	1080 (on each jack) 1080	3 p.s.i.
	2880	1440 (on each jack) 1440	4 p.s.i.
	3240	1620 (on each jack) 1620	4.5 p.s.i.
	3600		5 p.s.i.
	3960		5.5 p.s.i.
	4320		6 p.s.i.
	4680		6.5 p.s.i.
	5040		7 p.s.i.
	5400		7.5 p.s.i.

Box failed at 7.5 p.s.i.

	2700
5400	2700

Location of Demec Gauges
For Beam B2



1,2,3,4,5,6,8 2" Demec
 7 4" Demec

Calibration/division

$$2" = 2.47 \times 10^{-5}$$

$$4" = 2.00 \times 10^{-5}$$

Figure B-14

Net Reading for Demec Gauge for BOX B2

$\begin{array}{c} L \\ G \end{array}$	1	2	3	4	4.5	5	5.5	6	6.5	7
1	-2	-5	-10	-13	-15	-16	-19	-21	-24	-28
2	+1	+1	+1	+2						
3	+12	+28	+42	+44	+46	+45	+48	+124		
4	0	0	+1	0	0	0	+1	+3	+5	
5	-1	+1	+1	+1	+2	+2	+1	+1	0	0
6	-3	-7	-9	-13	-14	-17	-19	-20	-24	-27
7	-14	+13	+19	+27	+33	+40	+50	+161	+84	+121
8	+9	+22	+30	+44	+53	+62	+77	+97	+234	+184

Table B-3

1, 2, 3, 4, 5, 6, 8 2" Demec

7 4" Demec

Calibration

$$2'' = 2.47 \times 10^{-5}$$

$$4'' = 2.00 \times 10^{-5}$$

Net Readings of Dial Gauges for BOX B2

Load Gauge No.	1	2	3	4	4.5	5	5.5	6
1	1.94	4.15	5.99	7.93	9.27	10.66		
2	.03	0.05	0.17	0.31	0.47			
3	.01	0.01	-0.02	0.1	0.09	.05		
4	0.16	0.06	0.04	0.09	0.12	0.07		
5	2.02	4.24	6.12	8.07	9.42	10.91	12.57	14.49
6	2.08	4.35	6.25	8.47	9.68	11.24	12.93	14.34
7	2.14	4.45	6.4	8.47	9.91	11.52	13.23	16.32
8	2.22	4.56	6.52	8.59	10.06	11.71	13.51	3.6
9	2.27	4.67	6.65	8.78	10.28	11.97	13.81	
10	0.14	0.31	0.35	0.44	0.45	0.48		
11	0.11	0.26	0.31	0.37	0.4	0.44		

Table B-4

Net Readings of Dial Gauges BOX B2

Load Gauge No.	1	2	3	4	4.5	5	5.5	6
12	0.26	0.7	0.67	0.42	0.02	0.04		
13	2.31	4.58	6.55	8.7	10.16	11.82		
14	1.95	4.11	5.98	7.93	9.23	10.78	12.33	
15	2.05	4.26	6.11	8.12	9.47	10.59		
16	2.11	4.34	6.24	8.28	9.64	11.31		
17	2.18	4.48	6.39	8.49	9.89	11.58		
18	2.27	4.57	7.54	8.65	10.09	11.81		
19	1.3	2.71	4.01	5.37	6.34	7.41		
20	0	0.07	0.14	0.15	0.88			
21	0	0	0	0	0	0		
22	-.02	0	-0.05	.03	0.06	0.13		

Table B-4

Net Readings of Dial Gauges BOX B2

Load Gauge No.	1	2	3	4	4.5	5	5.5	6
23	1.34	1.81	4.08	5.44	6.39	7.4	8.49	9.82
24	1.4	2.91	4.2	5.6	6.58	7.64	8.8	10.18
25	1.45	3.01	4.32	5.75	6.75	7.85	9.04	10.5
26	1.49	3.09	4.44	5.89	6.93	8.07	9.31	10.8
27	1.58	3.23	4.53	6.0	7.04	8.29	9.43	11.03
28	0.09	0.14	0.17	0.21	0.24	0.26		
29	0.12	0.2	0.22	0.3	0.34	0.4		
30	0.1	0.31	0.4	0.53	0.6	0.36		
31	1.65	3.26	4.62	6.06	7.06	8.14		

Table B-4

Net Readings of Strain Gauges BOX B2

Load Gauge No.											
	1	2	3	4	4.5	5	5.5	6	6.5	7	7.5
1	-38	-66	-89	-106	-115	-127	-136	-147	-158	-172	-189
2	-22	-49	-69	-87	-98	-109	-118	-129	-143	-157	-177
3	-25	-50	-68	-85	-94	-107	-114	-125	-139	-153	-170
4	-20	-44	-61	-79	-88	-98	-107	-118	-129	-141	-152
5	-13	-30	-44	-56	-62	-69	-76	-84	-91	-97	-105
6	-21	-43	-58	-72	-82	-90	-98	-107	-118	-129	-141
7	-24	-54	-70	-87	-95	104	-113	-123	-135	-148	-166
8	-22	-49	-68	-85	-97	-107	-117	-130	-144	-162	-184
9	-20	-43	-60	-76	-84	-93	-102	-112	-125	-141	-161
10	-25	-45	-61	-76	-84	-92	-101	-110	-120	-132	-145
11	-12	-19	-26	-32	-34	-37	-38	-39	-37	-32	-25
12	+48	+103	+150	+195	+224	+257	+291	+337	+406	+503	+606
13	+43	+92	+131	+172	+192	+213	+233	+256	+277	+335	+398

Table B-5

Net Readings of Strain Gauges BOX B2

Gauge No.	Load										
	1	2	3	4	4.5	5	5.5	6	6.5	7	7.5
14	+62	+133	+196	+255	+290	+329	+371	+425	+494		
15	+63	+138	+201	+263	+301	+344	+396	+465	+548	+666	+840
16	+60	+130	+191	+248	+283	+324	+370	+435	+519	+609	+716
17	+55	+124	+182	+235	+266	+302	+345	+402	+481	+591	+741
18	+58	+125	+181	+237	+273	+316	+368	+443	+568	+784	
19	+131	+189	+244	+279	+312	+363	+443	+581	+837	+1390	
20	+109	+161	+214	+247	+285	+331	+400	+521	+746	+1178	
21	+42		+136	+183	+212	+247	+286	+343	+425	+596	+1056
22	-12	-23	-31	-35	-36	-36	-35	-33	-25	-6	+55
23	+39	+85	+119	+161	+188	+219	+254	+307	+391	+664	+896
24	+5	+10	+17	+24	+31	+40	+51	+69	+99	+137	+261

Table B-5

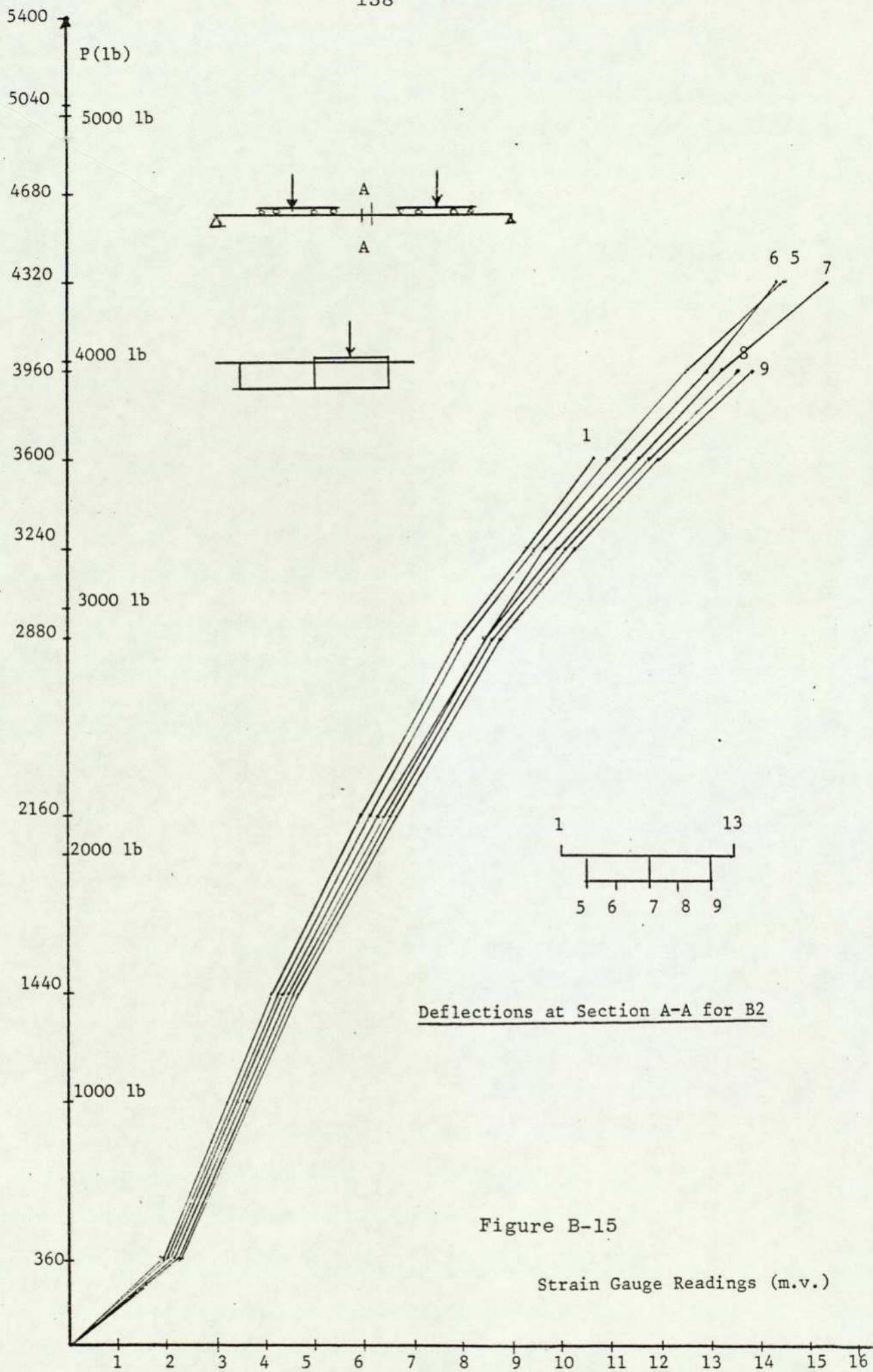
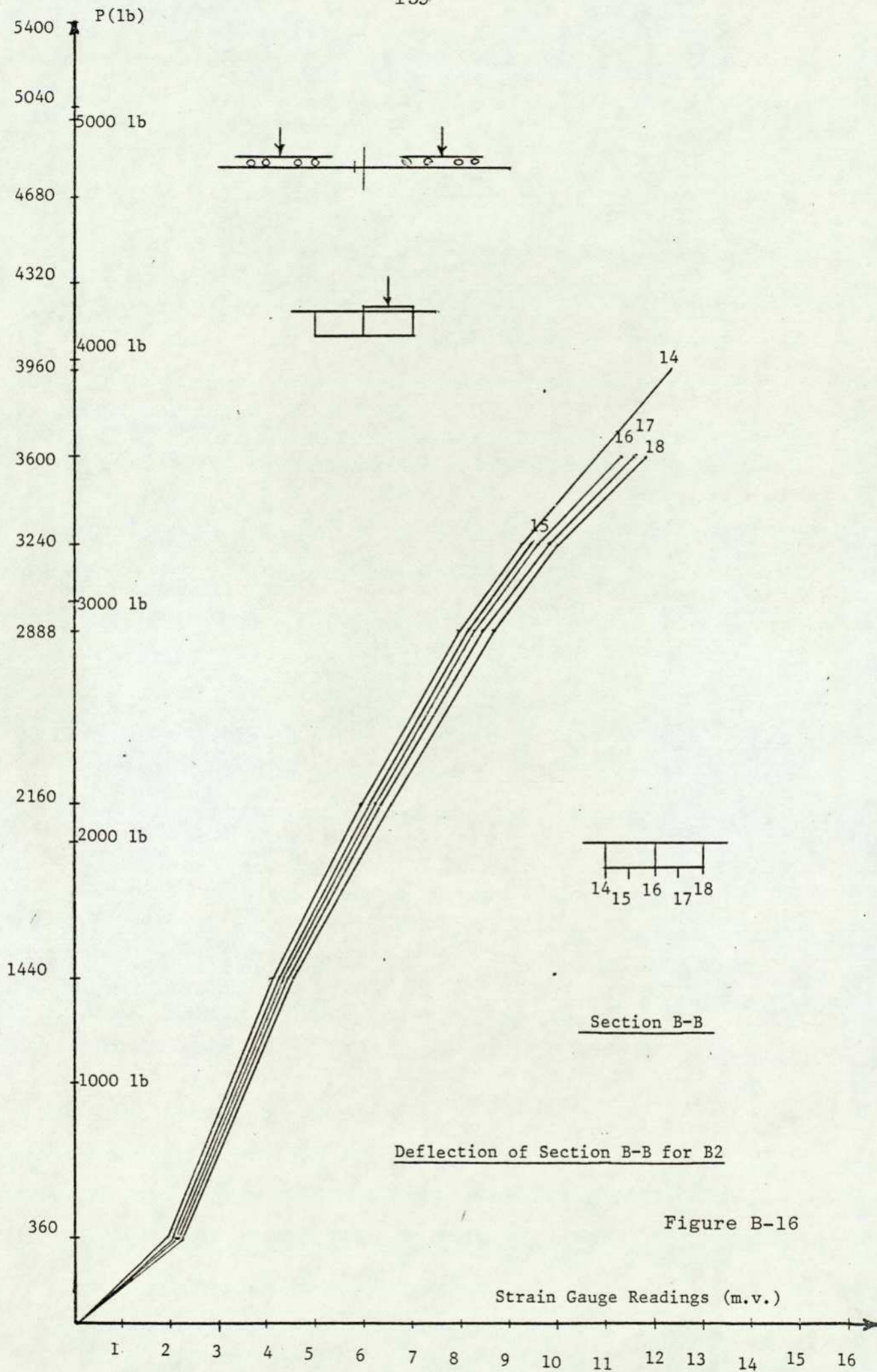
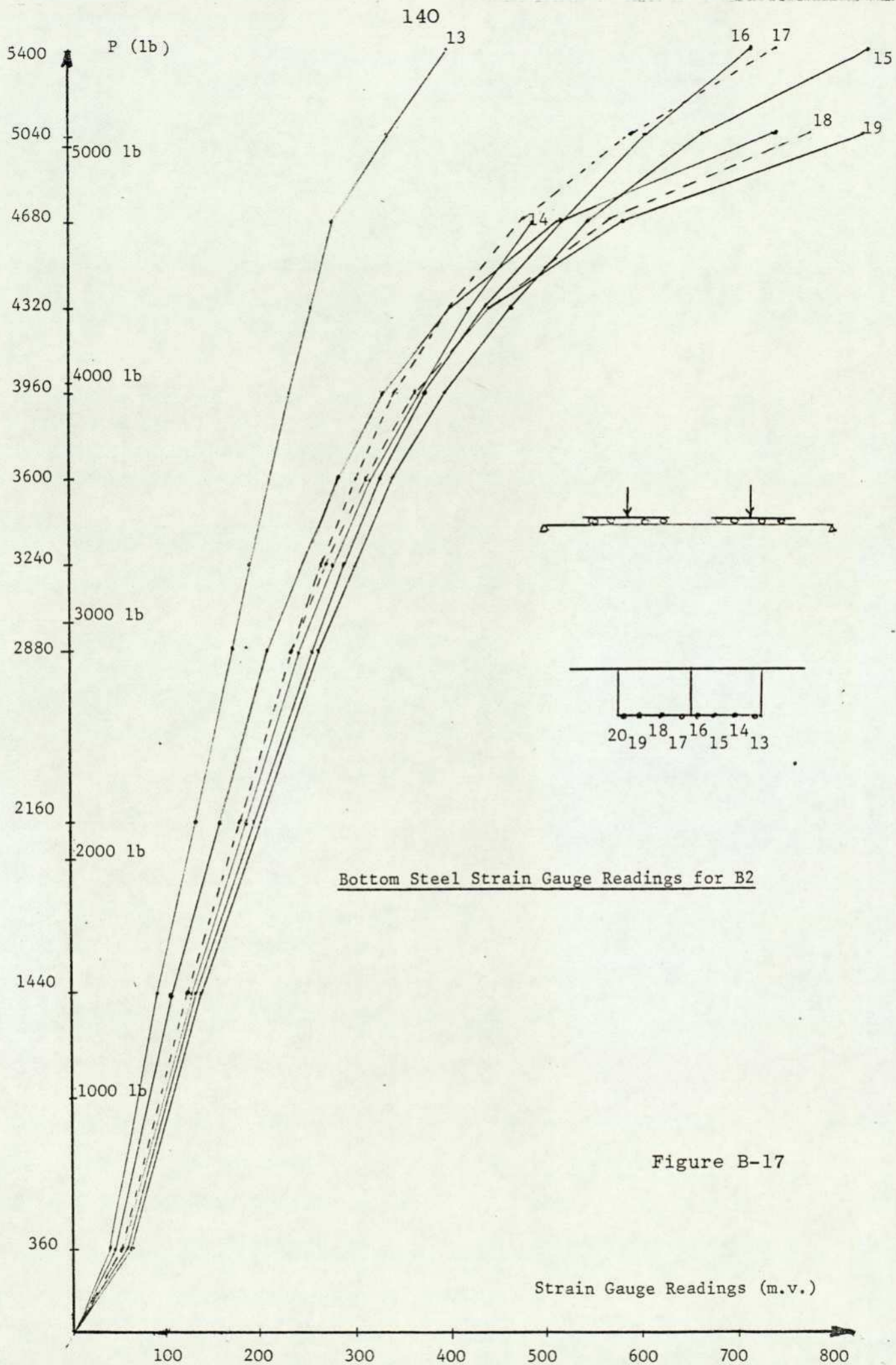
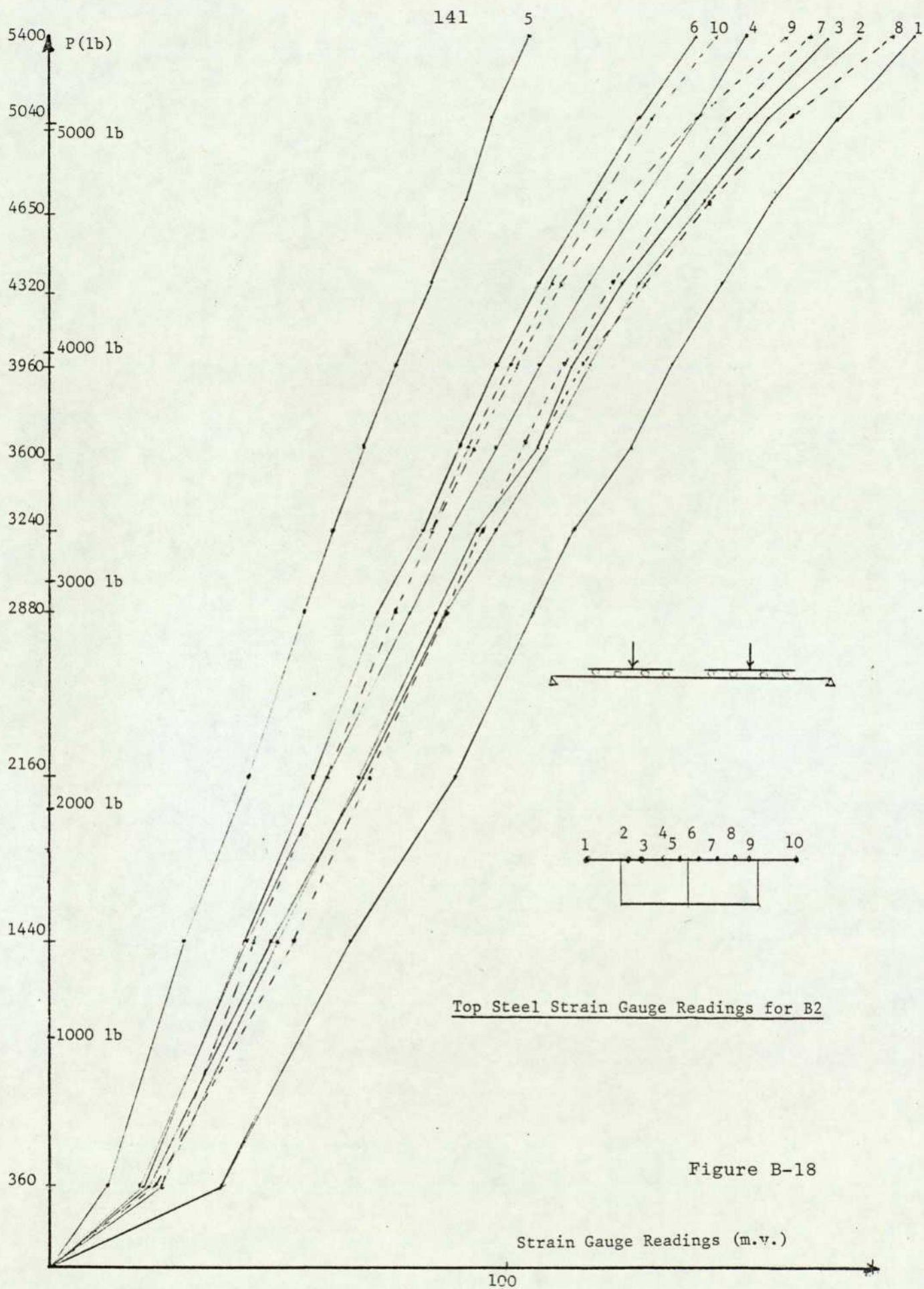


Figure B-15

Strain Gauge Readings (m.v.)







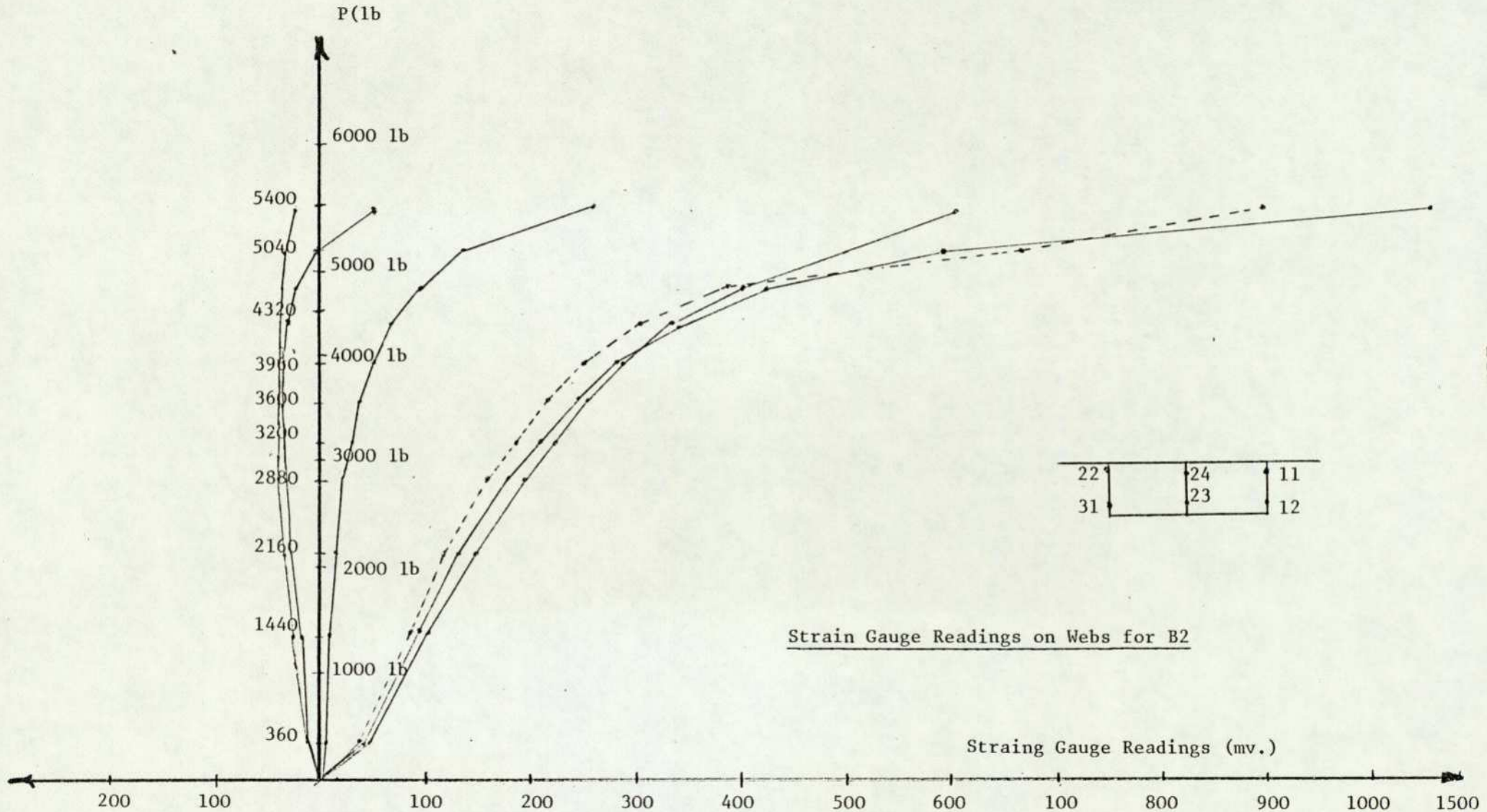
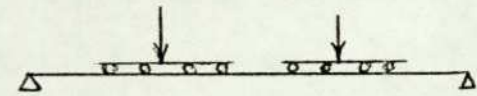


Figure B-19

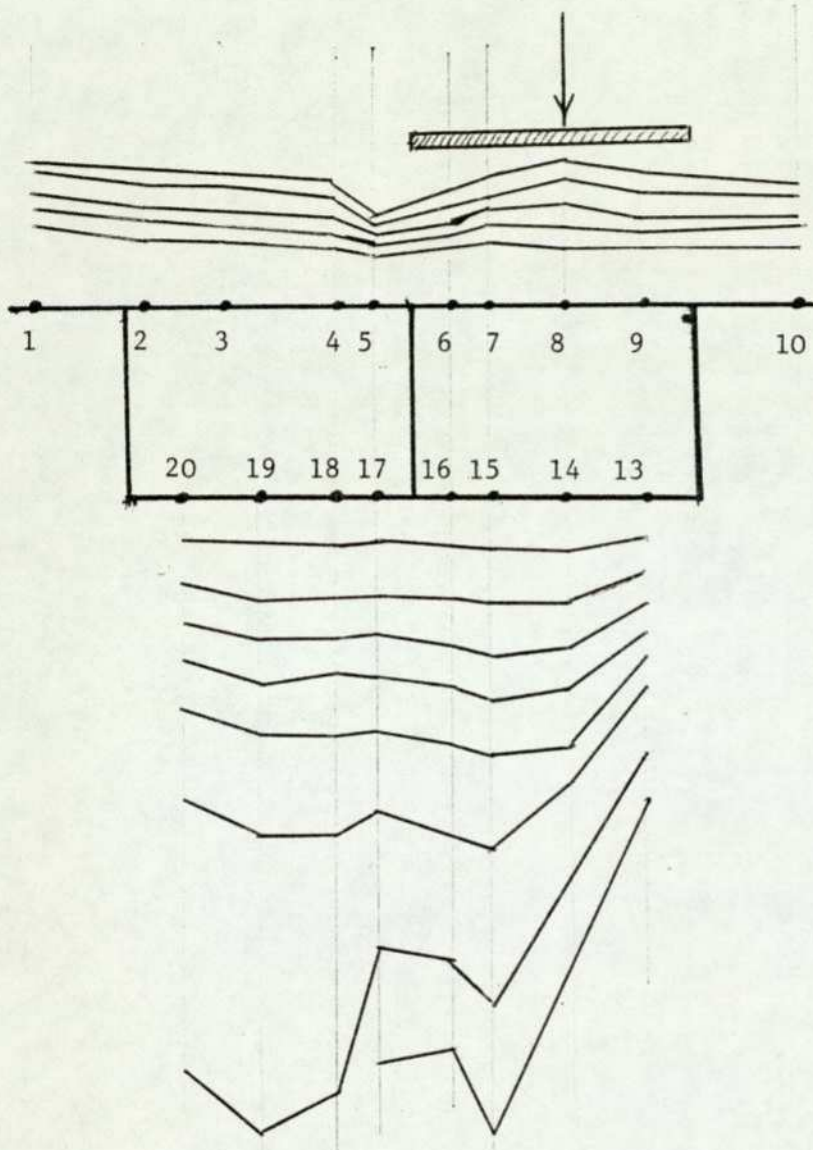
Profile of StressesFor Unsymmetrical Case B2

Figure B-20

Layout of Box B₃

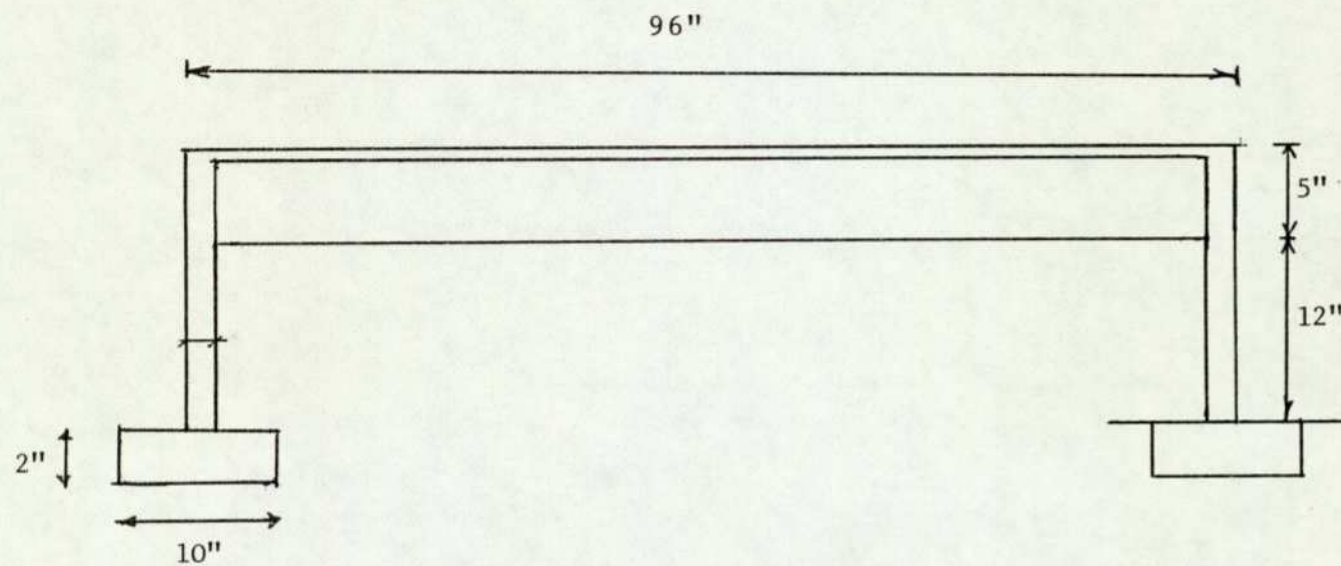
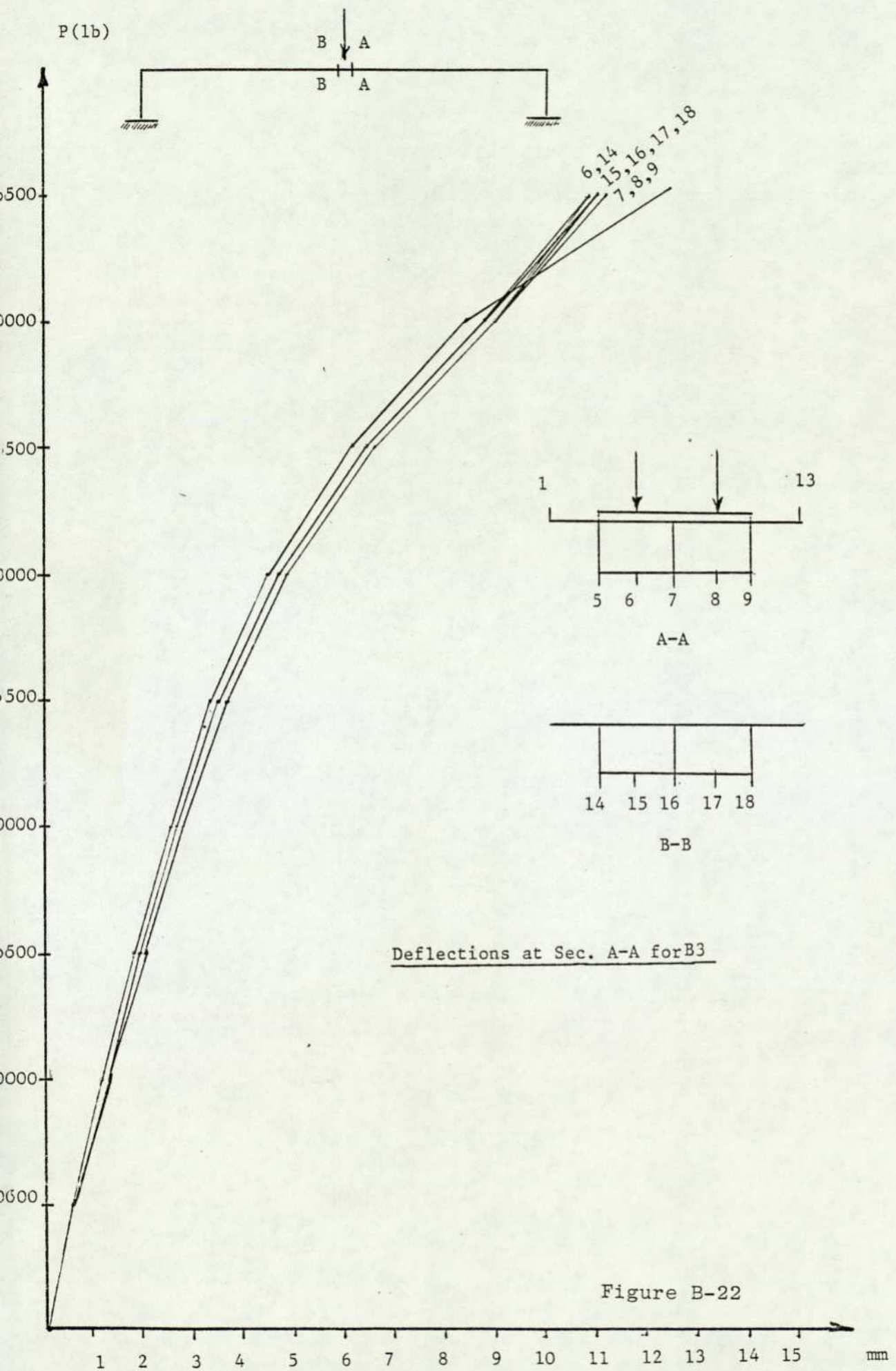
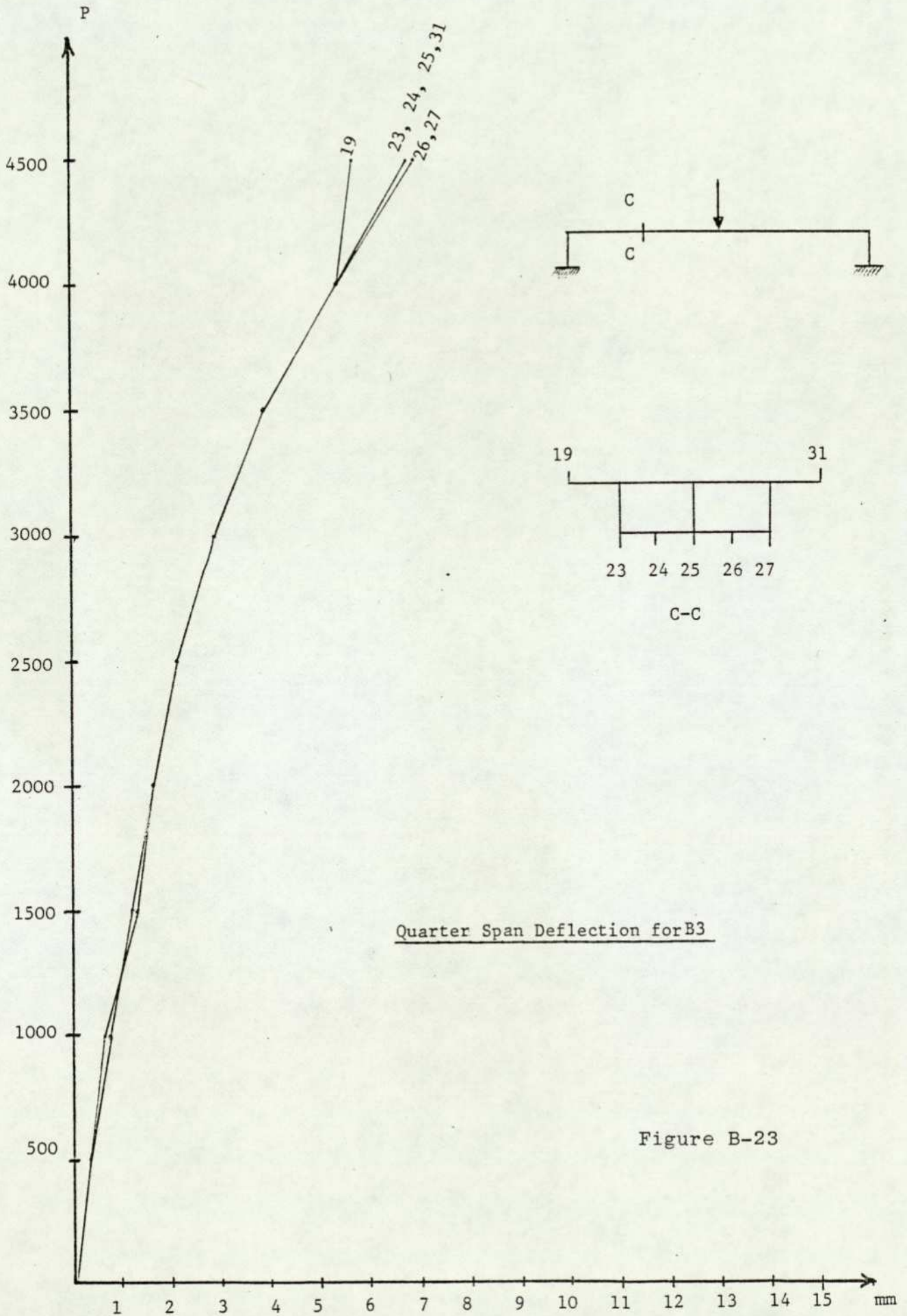


Figure B-21





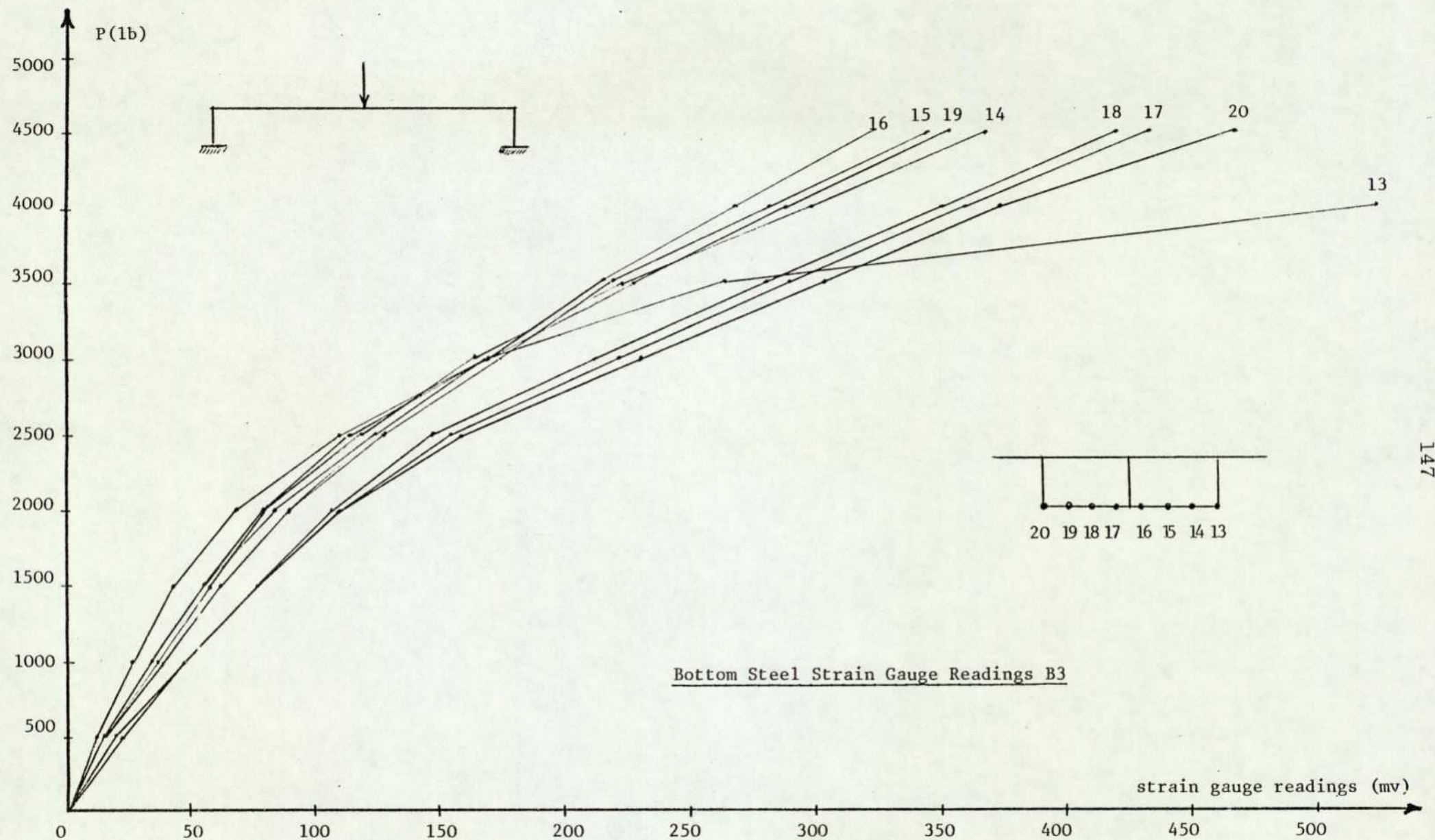


Figure B-24

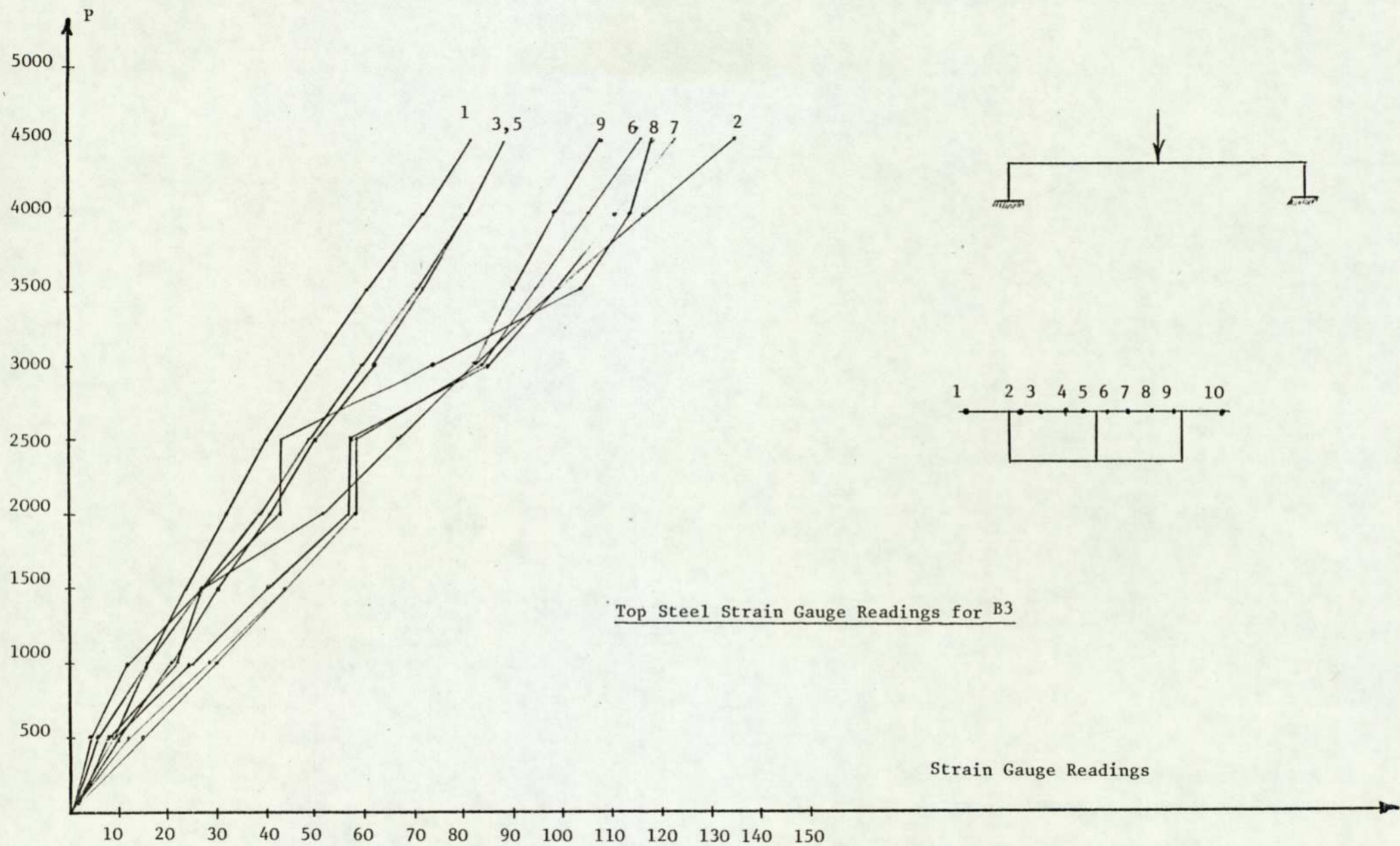


Figure B-25

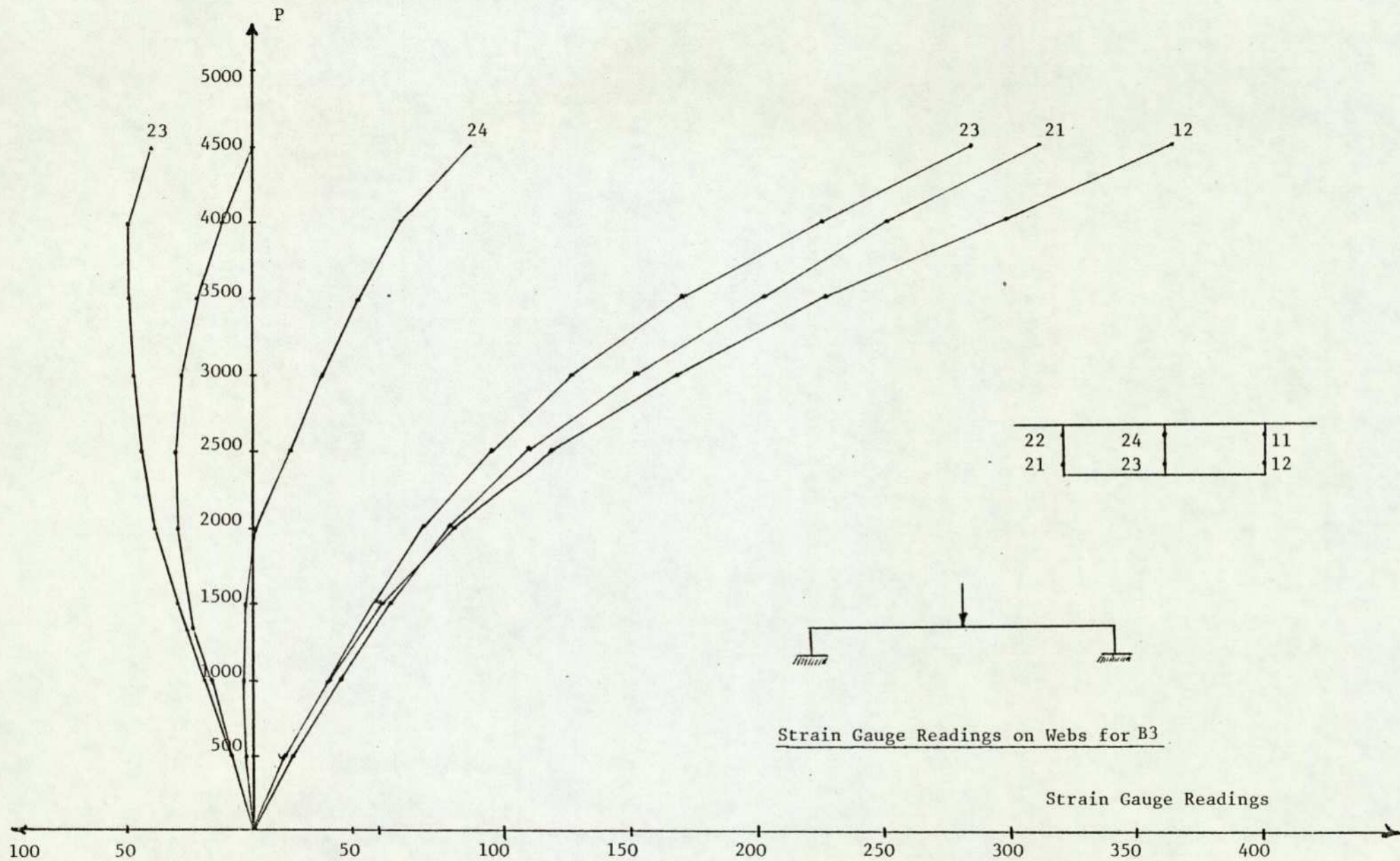
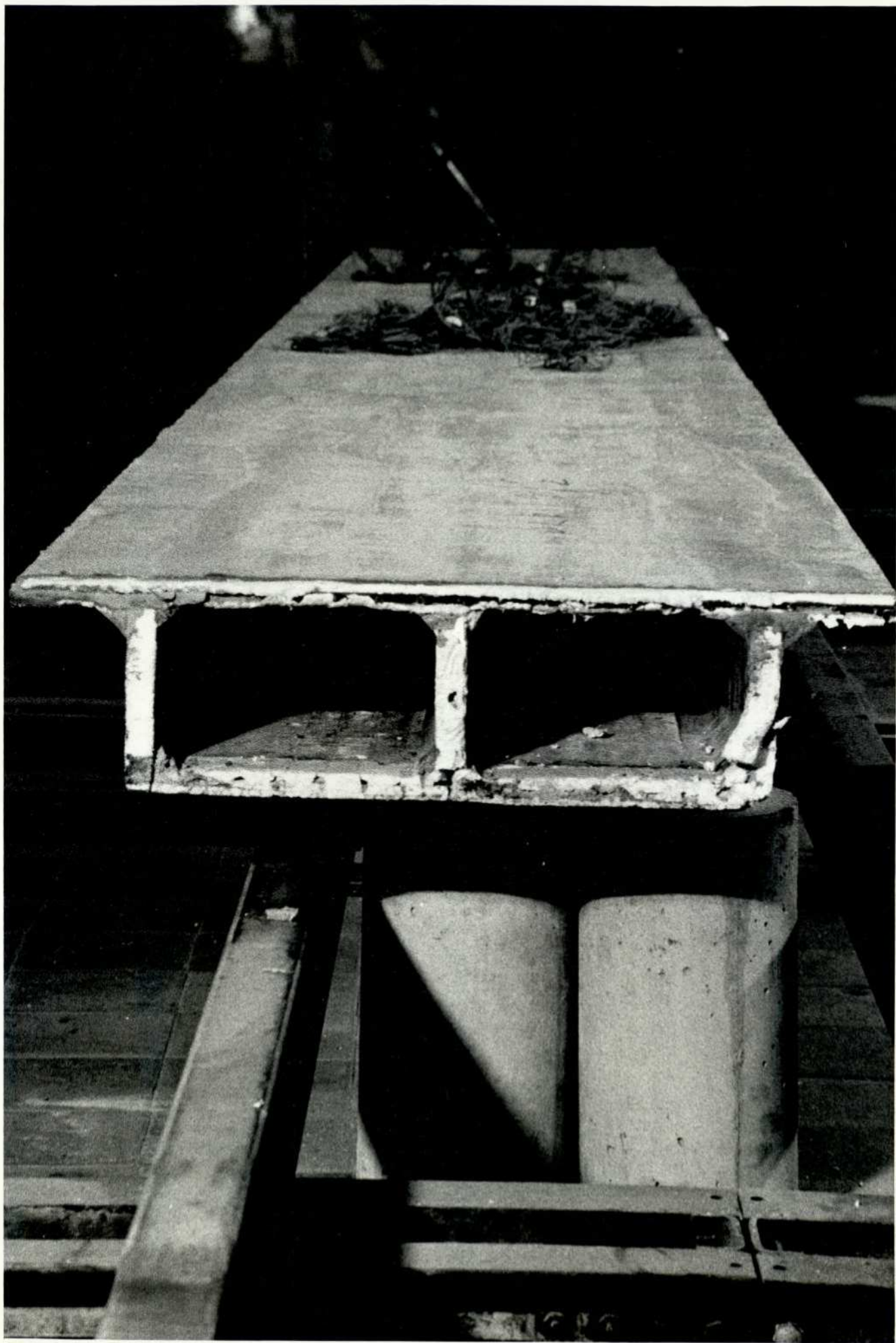
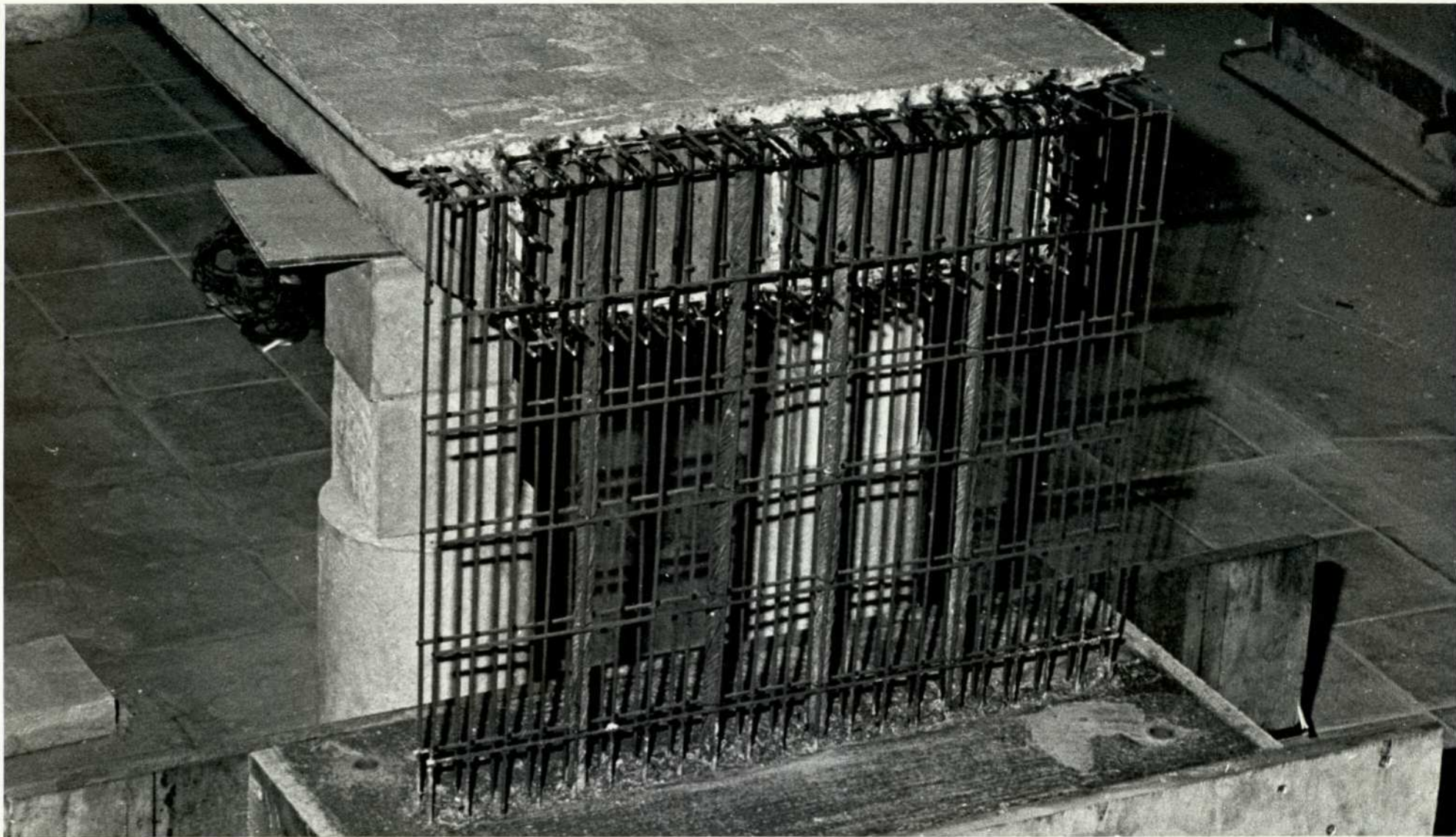


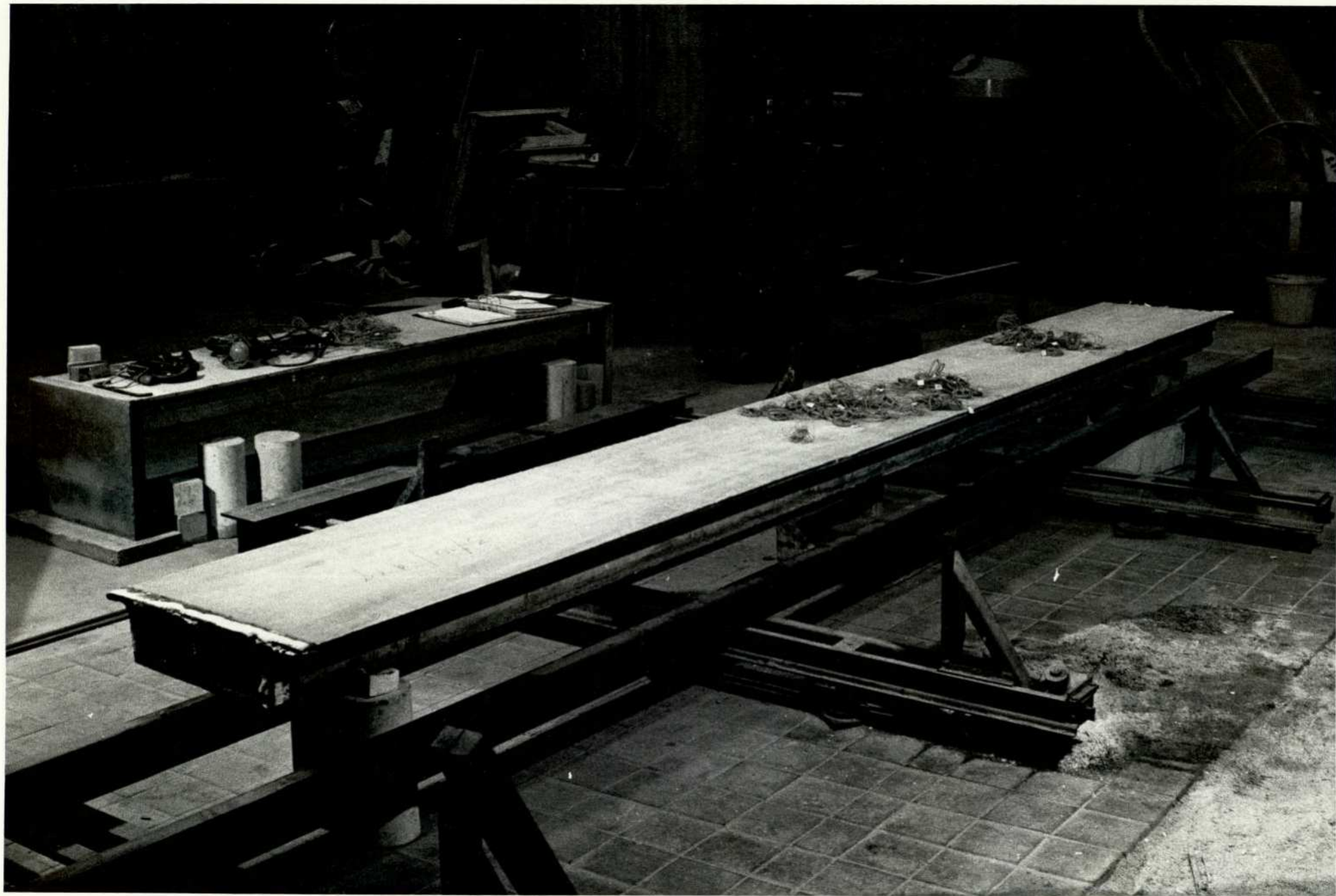
Figure B-26



VIEW OF TYPICAL 2 CELL
CROSS-SECTION

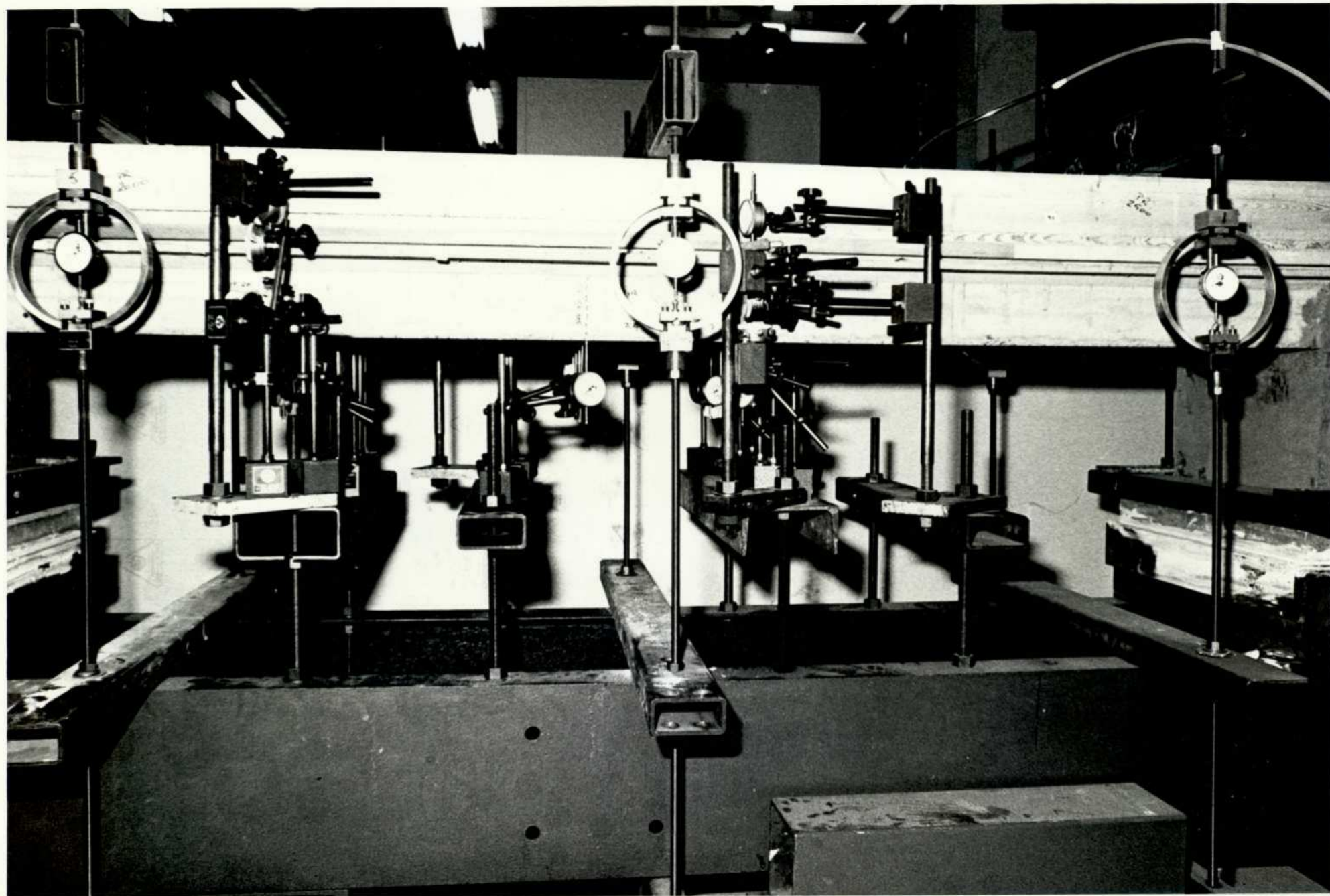


DETAIL OF COLUMN
REINFORCEMENT

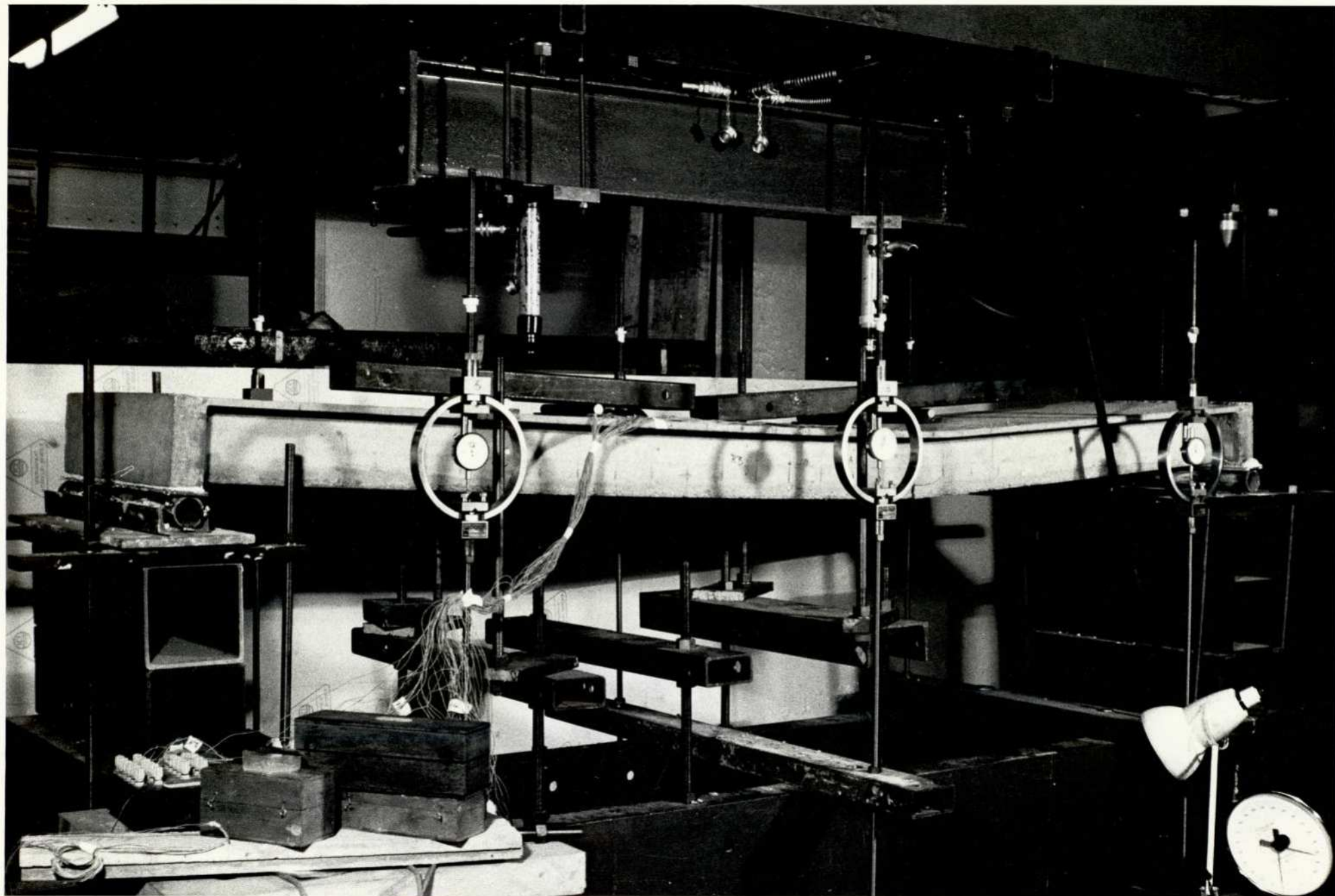


GENERAL LAYOUT

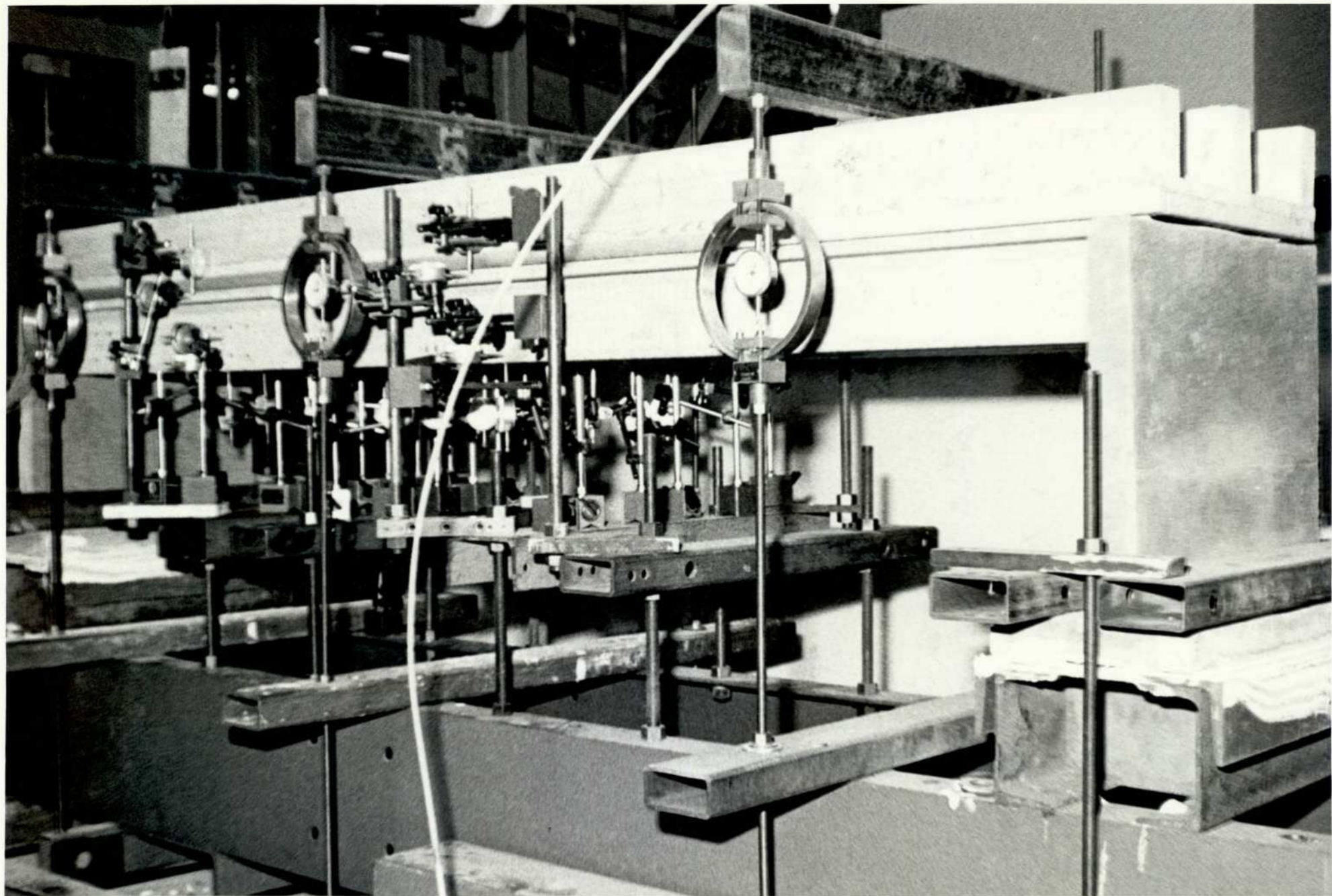
12

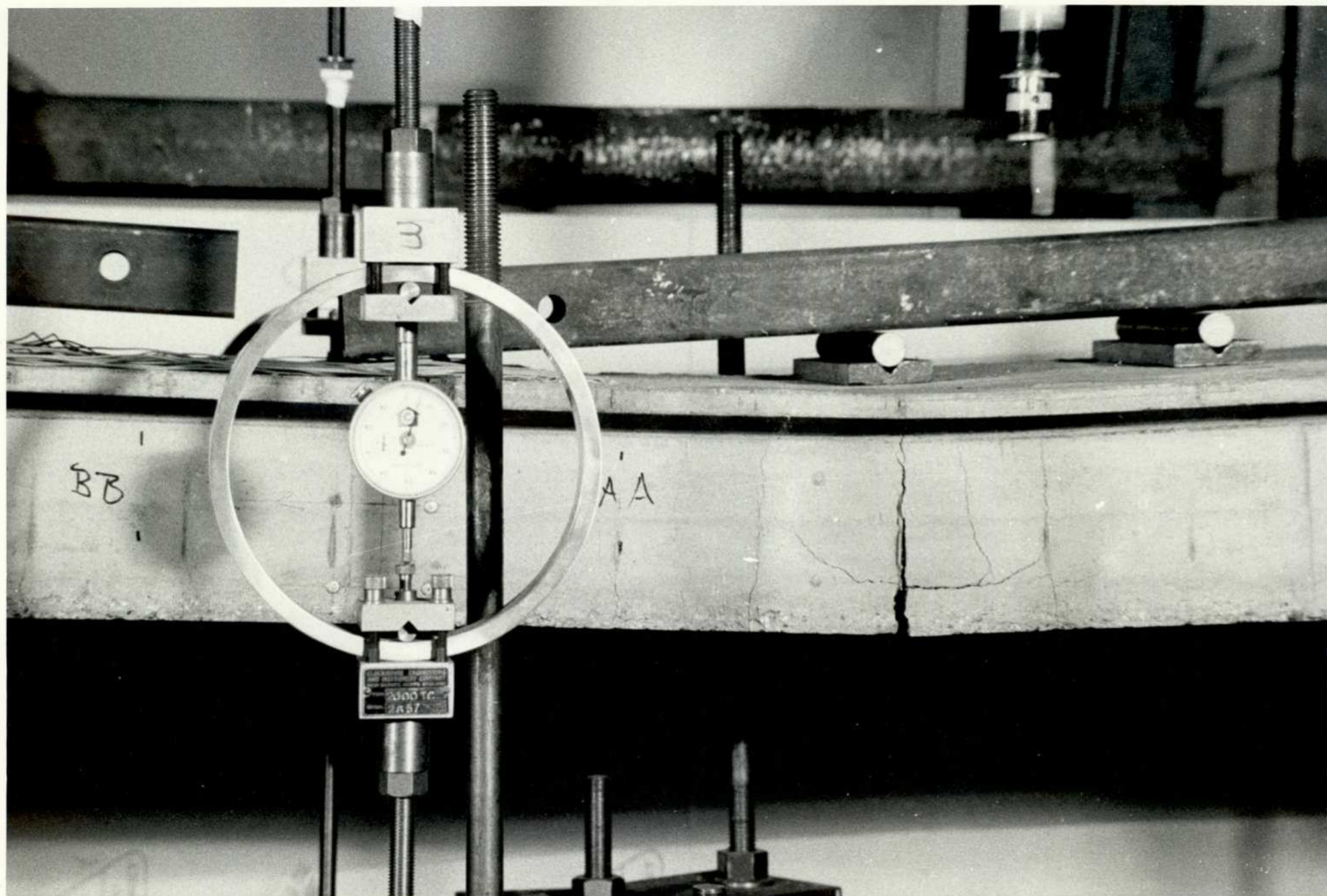


LOADING OF B1

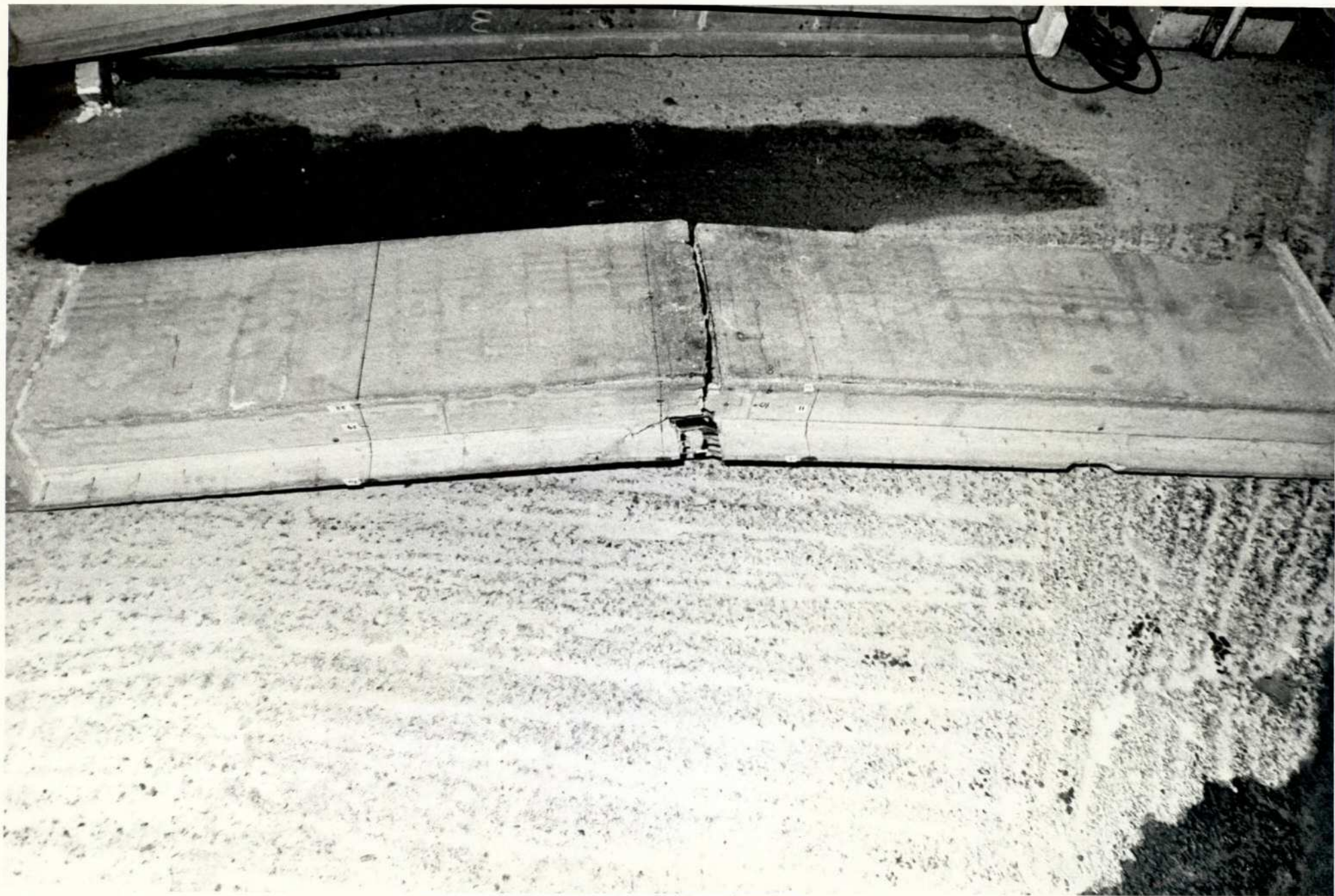


LOADING OF R2



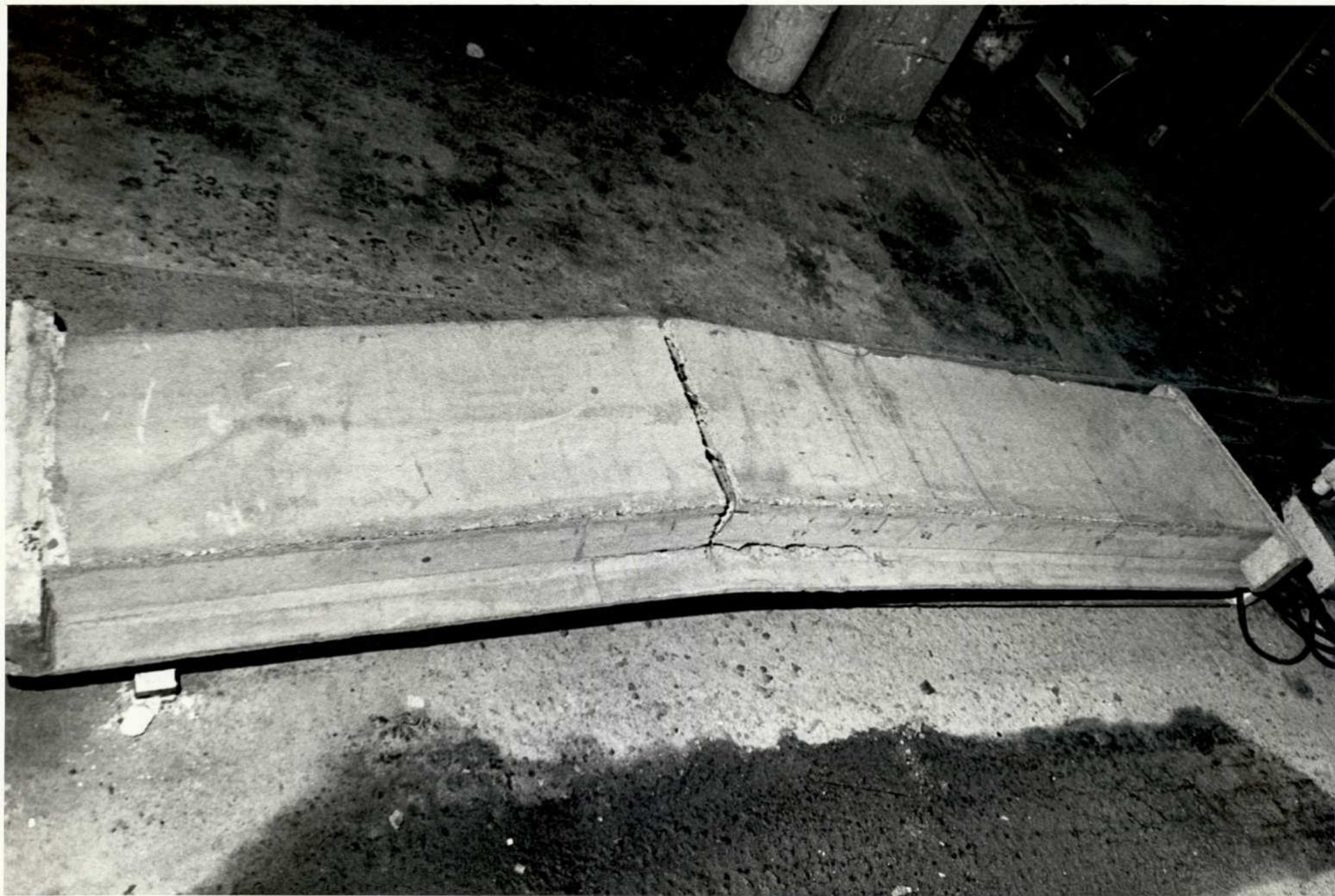


FAILURE OF B2



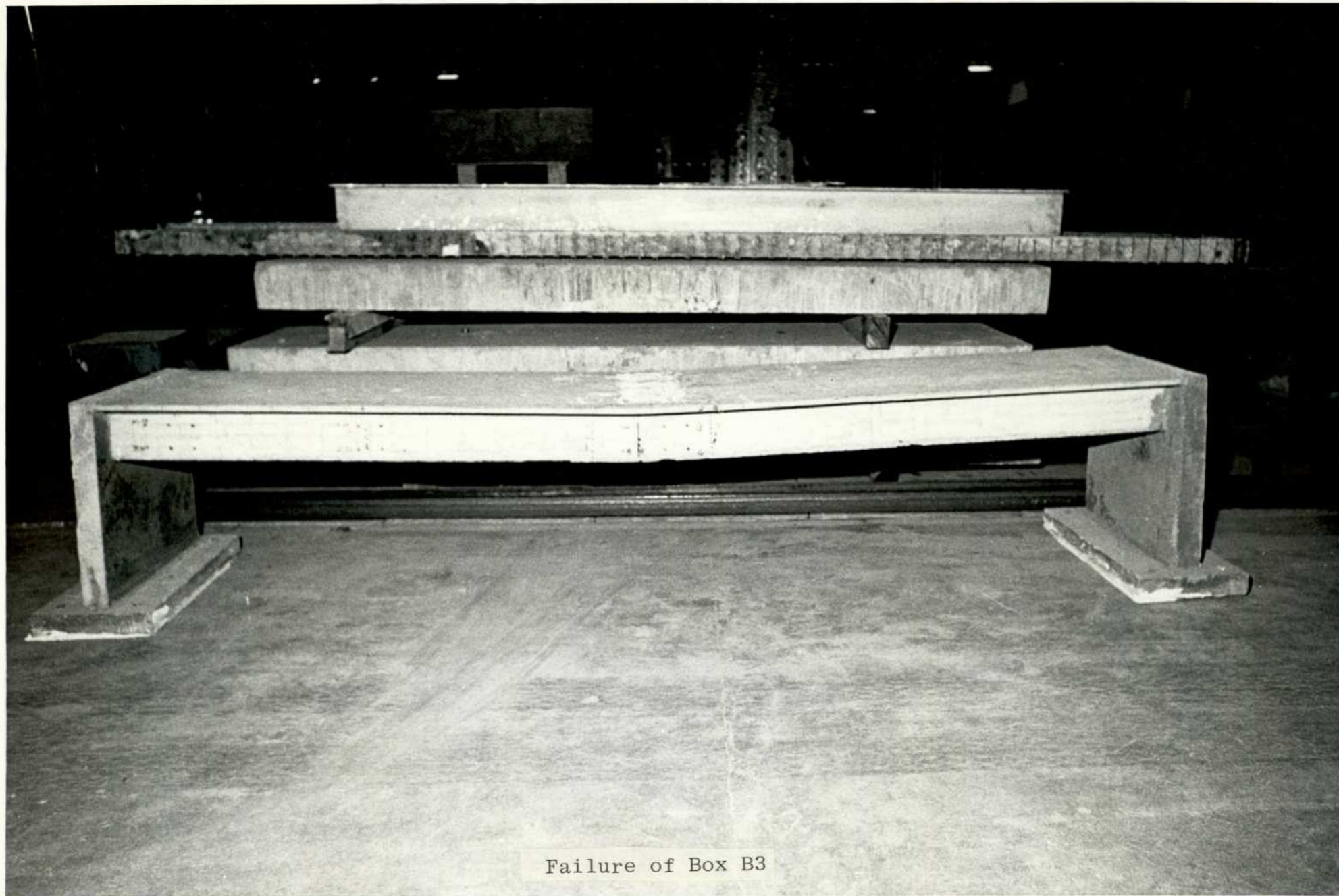
FAILURE OF R1

17



FAILURE OF B2

18



Failure of Box B3

19



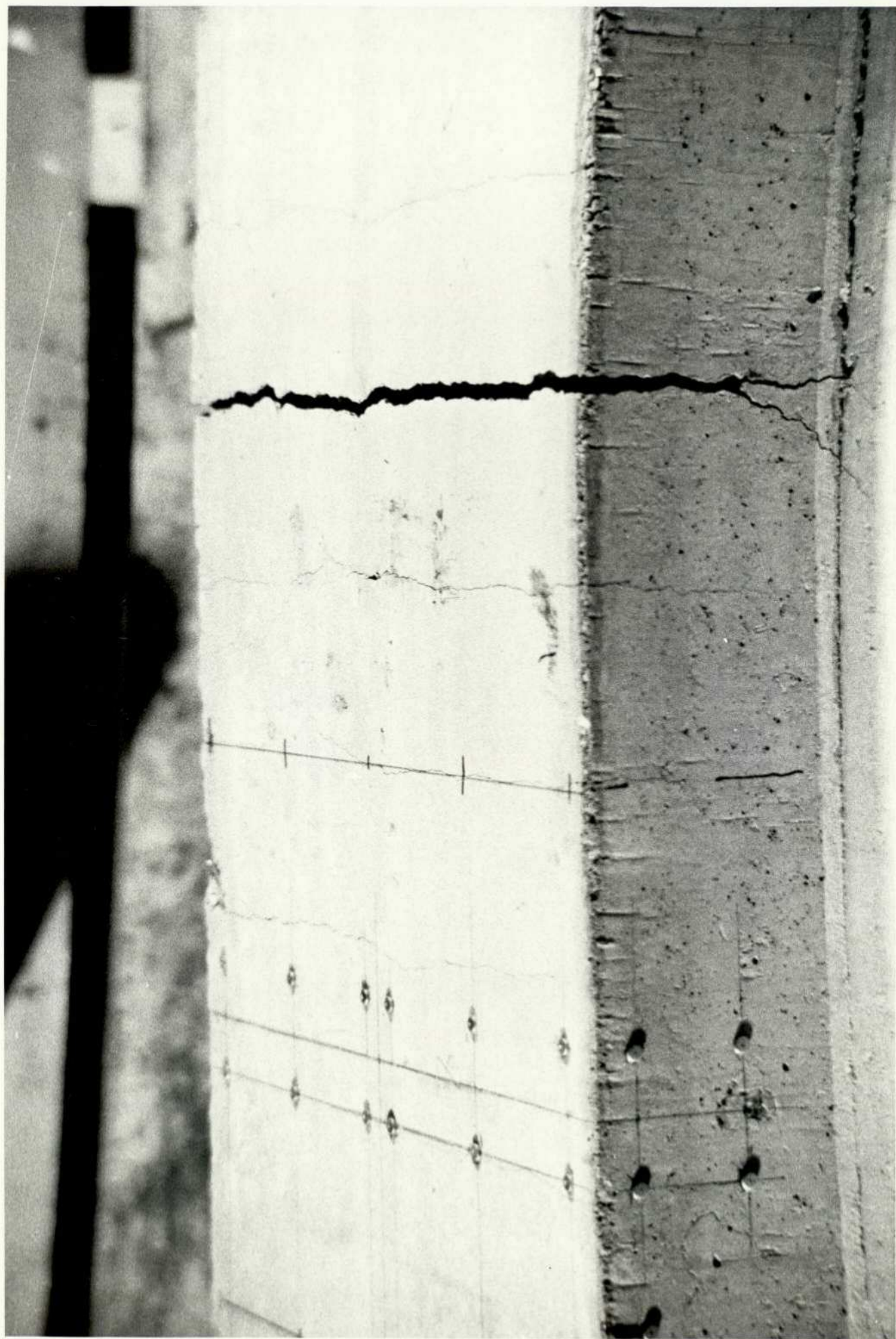
CRACK PATTERN

2^o



CRACK PATTERN

21



Cracked Section

22

4.4 GROUP C

This section deals with double cell box girder bridges continuous over two spans, and having a column support at the middle.

A Compulog was used in recording different results of gauges, which enabled a large range of loading cases to be applied to the box and tested within the elastic limit.

By comparing these different cases of loading, the critical load condition was applied to the box and that load was increased into the non-linear range to collapse.

Behaviour of the box beam (C) within the elastic range under different loading conditions was recorded and plotted as well as its behaviour to collapse.

4.4.1 Material, casting and testing equipment

The material used for this group was the same mix of micro-concrete used for the previous tests. The steel mesh used was exactly the same as the mesh used before. Nevertheless, initial tests for the different material properties and calibration factors were carried out to ensure correct results from the tests and as a check for the previous results.

In box bridge C an extra column was fitted to the mid-section of the box. The dimensions of the column can be seen in Fig. (C-3). This column was first placed in the centre of the formwork by drilling a hole in the bottom surface of the formwork. Then the cage reinforcement was placed and the pouring of the concrete was done as mentioned before. Additional longitudinal steel bars from the same mesh were welded to the top flange reinforcement as a negative bending moment diagram would occur in that case with a -ve value at the top flange in that part.

Strain gauges were placed on the steel meshes before casting at different sections and their locations can be seen in Fig. (C-4).

A set of dial gauges to record various different deflections at different loading conditions were placed as shown in Fig. (C-5). The two cross sections A and B where strain gauges were fixed were about 5 1/2" - 6" apart. The third section C was at a quarter length of

the whole length, i.e. the centre of one of the spans of the bridge.

The dial gauges were located at the mid section of each span and dial gauges were put near the mid support arc on either side of the column, as can be seen from the various diagrams.

Demec gauges were located on the top surface of the upper flange.

A set of six gauges were placed near the mid section of each span. These positions can be seen from Fig. (C-6). Loads were applied by means of jacks fixed to a movable frame that enabled a quick and accurate movement of the jacks from one point to another. The whole frame in which the box bridge was tested and the different measuring instruments can be seen in the various photographs taken during the test.

A Compulog was used for this test which enabled a varying range of readings to be taken under different loading conditions. Again the Compulog can be seen in the different photographs.

4.4.2 Test Procedure

Different loading conditions were applied on the two span continuous box of total span 16 ft. Each span

was 8 ft long. The loads applied were usually at the mid section of each span.

At first a whole set of line loads were applied. Some cases were symmetrically loaded, such as applying a line load on each mid section covering the width of that section from centre line to centre line of the two outside webs. Other cases were unsymmetrically loaded by applying line loads covering centre line to centre line of the mid web and one of the outside webs.

These unsymmetrical loads were then applied symmetrically with respect to the longitudinal centre line or unsymmetrically with respect to that centre line. All these loads were applied within the elastic limits.

In each case readings of strain gauges, dial gauges and Demec gauges were recorded. Tables and diagrams (Figs. (C-7)-(C-19) and Tables (C-1, C-2, C-3)

Another set of loading was done by the application of point loads at the centre line of the webs at the mid section of each span. Again a different pattern of symmetrically loading and unsymmetrically loading was carried out for these loads within the elastic range. Readings of the different strain gauges, dial gauges and Demec gauges were recorded at each stage and can be seen from the Tables and Figs. (C-20 - C-38) and T(C-4)-(C-11).

After all these loading conditions were carried out a whole of the behaviour of the box was examined.

A set of two point loads on each of the outside webs, one on each span as shown in Fig. (C-1)

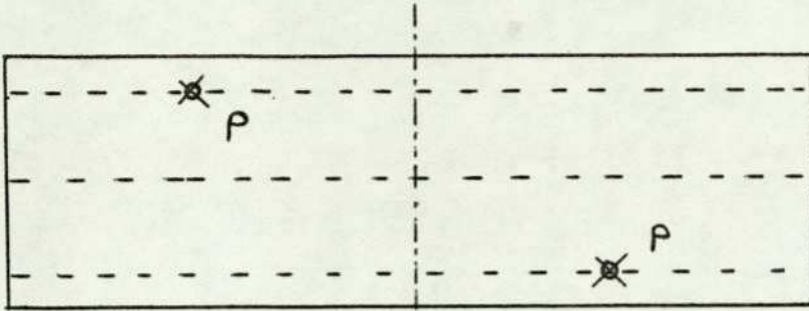


Figure (C-1)

was applied and the load was increased until some cracks could be seen in the two spans.

All results from the measuring equipment were recorded and plotted.

Another loading condition was applied by the application of two live loads on each mid section of each span and the loads were increased to failure as shown in Fig.(C-2).

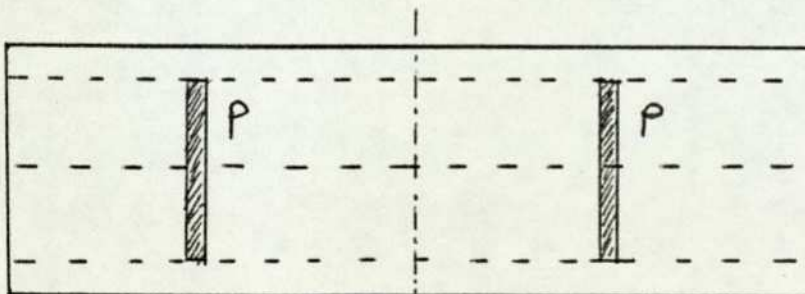
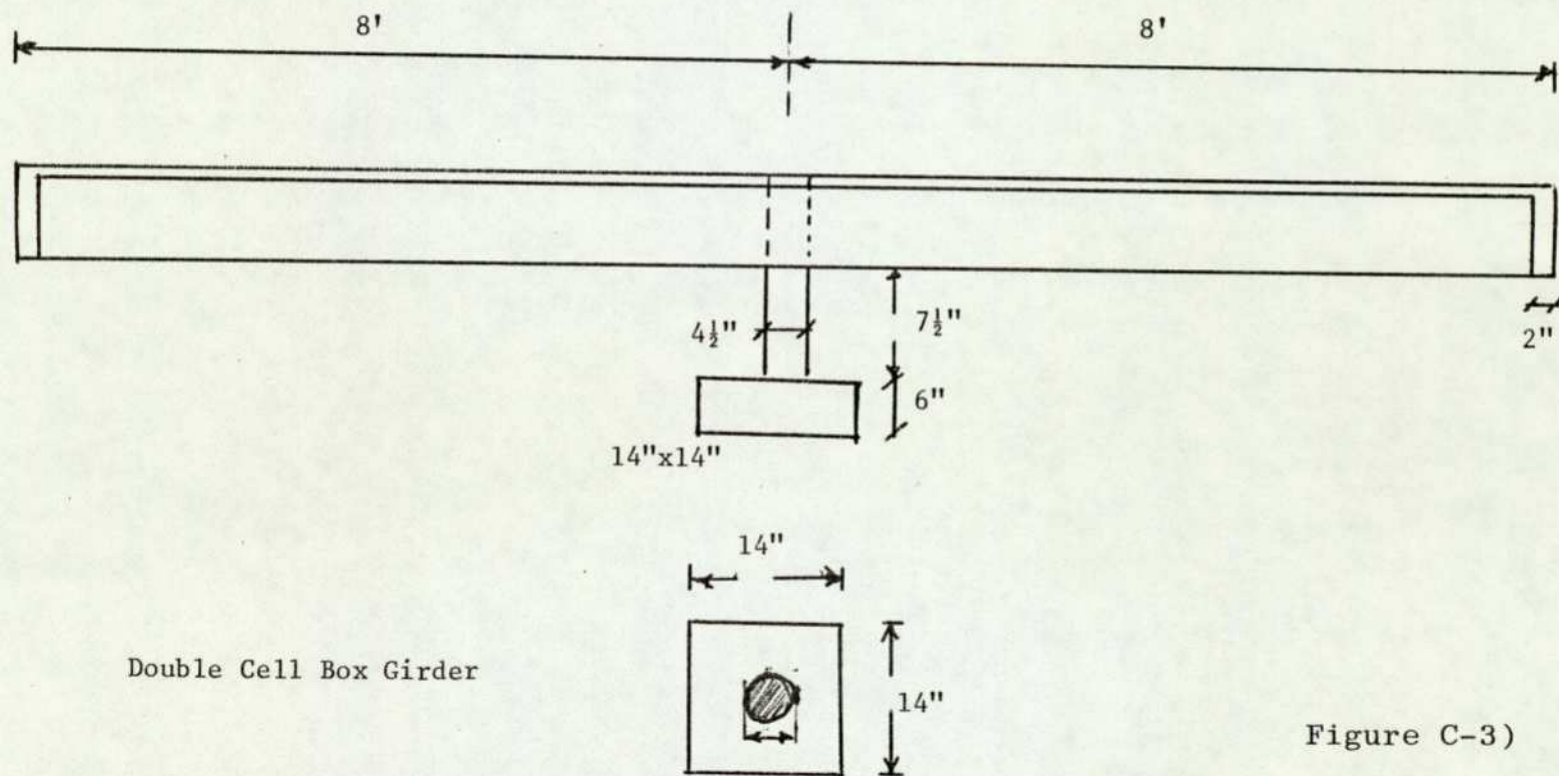


Figure (C-2)

Readings of the different strain gauges and dial gauges were recorded as shown in Fig. T(C-12)-(C-17).

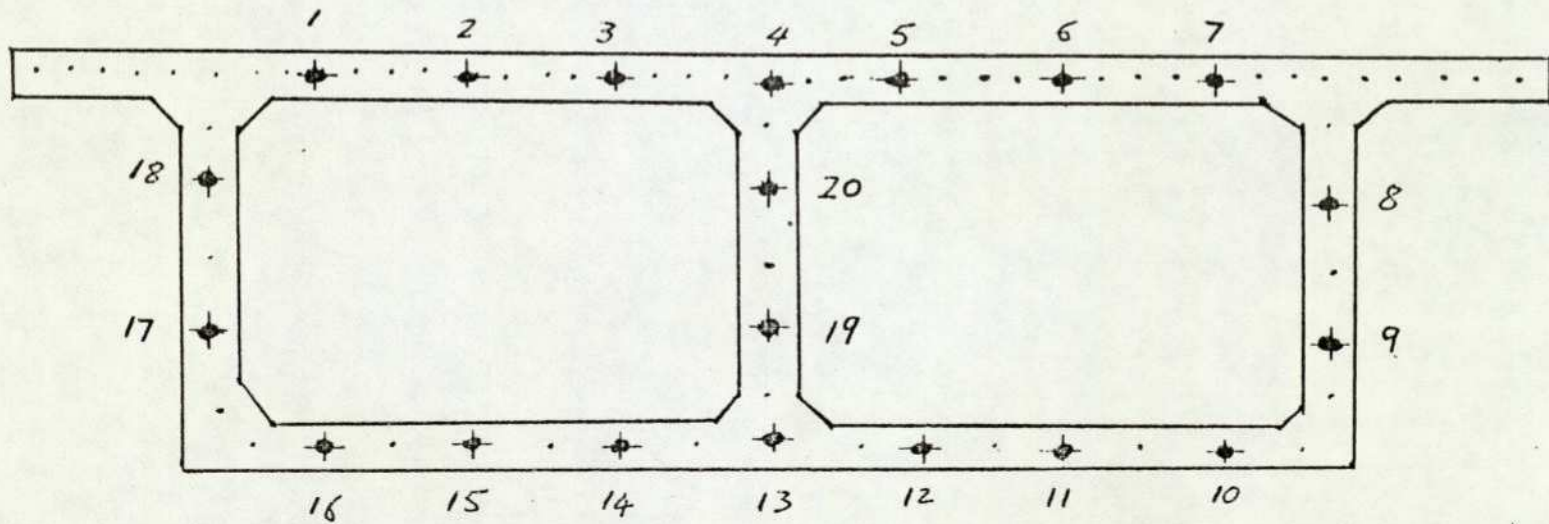
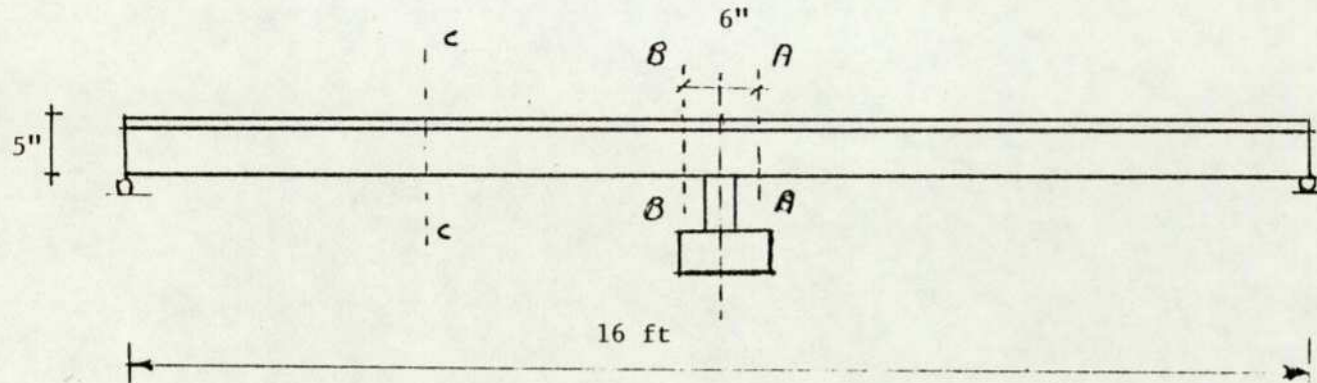
Layout of Box C



Double Cell Box Girder

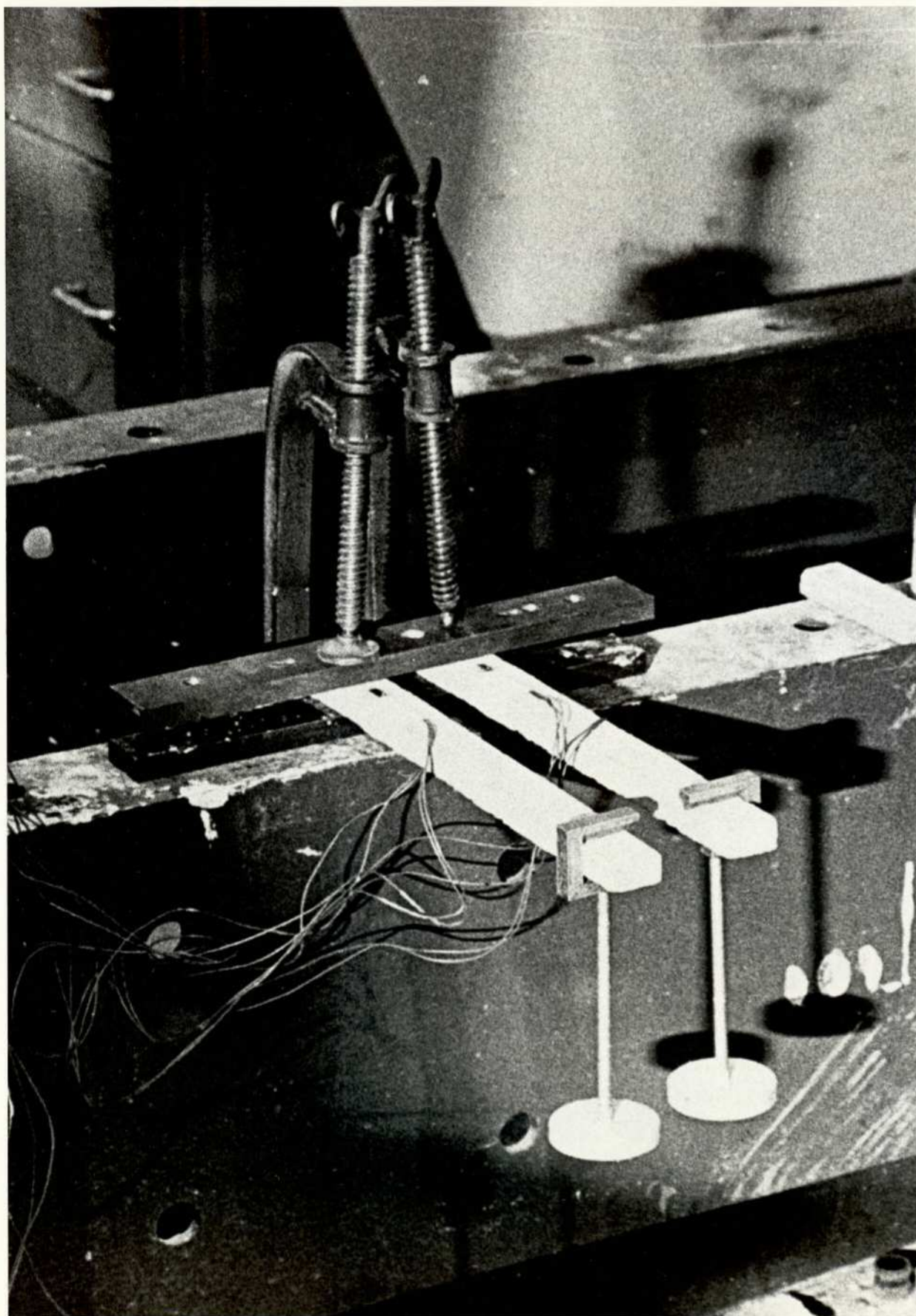
Figure C-3)

Location of Strain Gauges on Steel



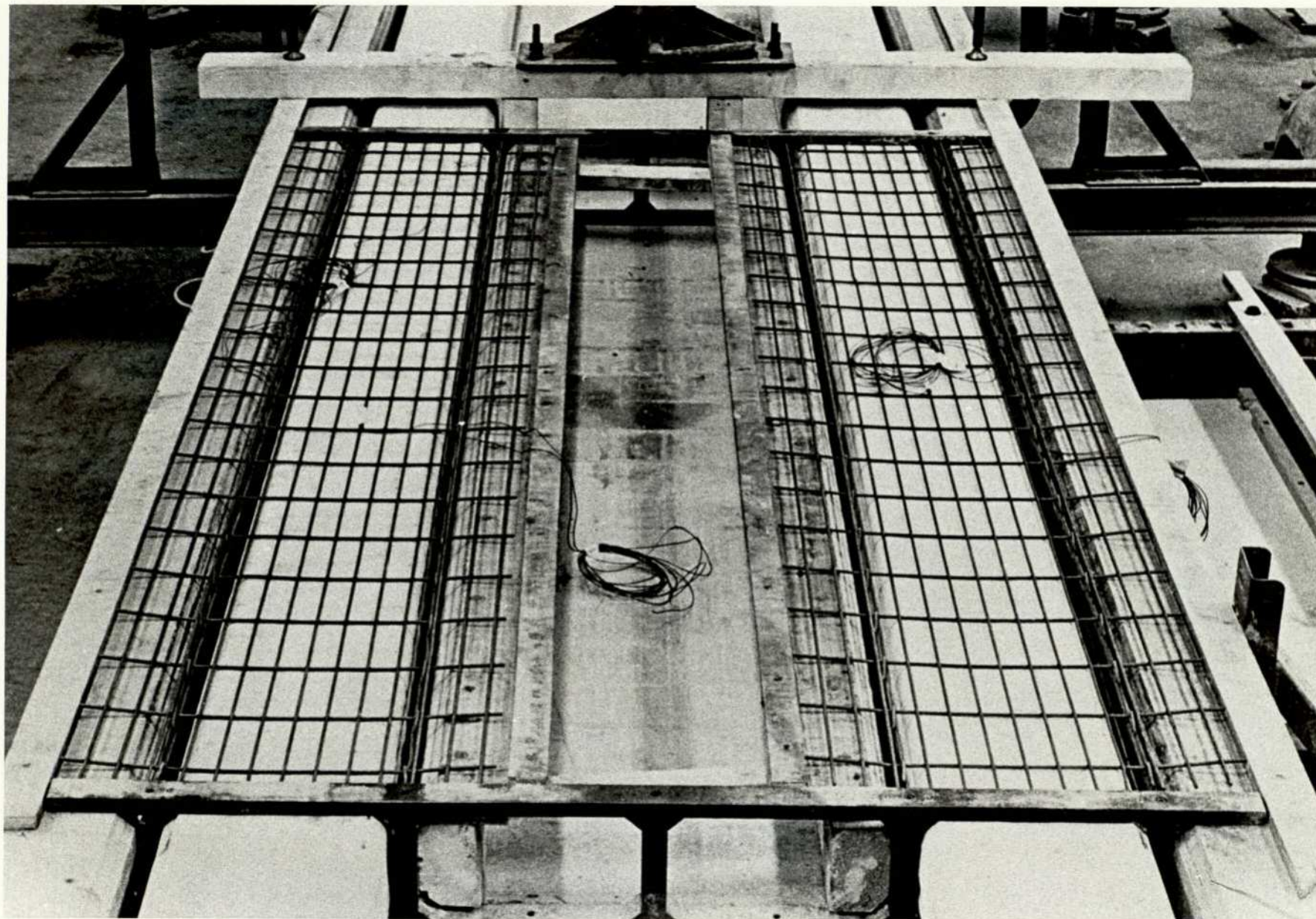
SECTION A-A

Figure C-4

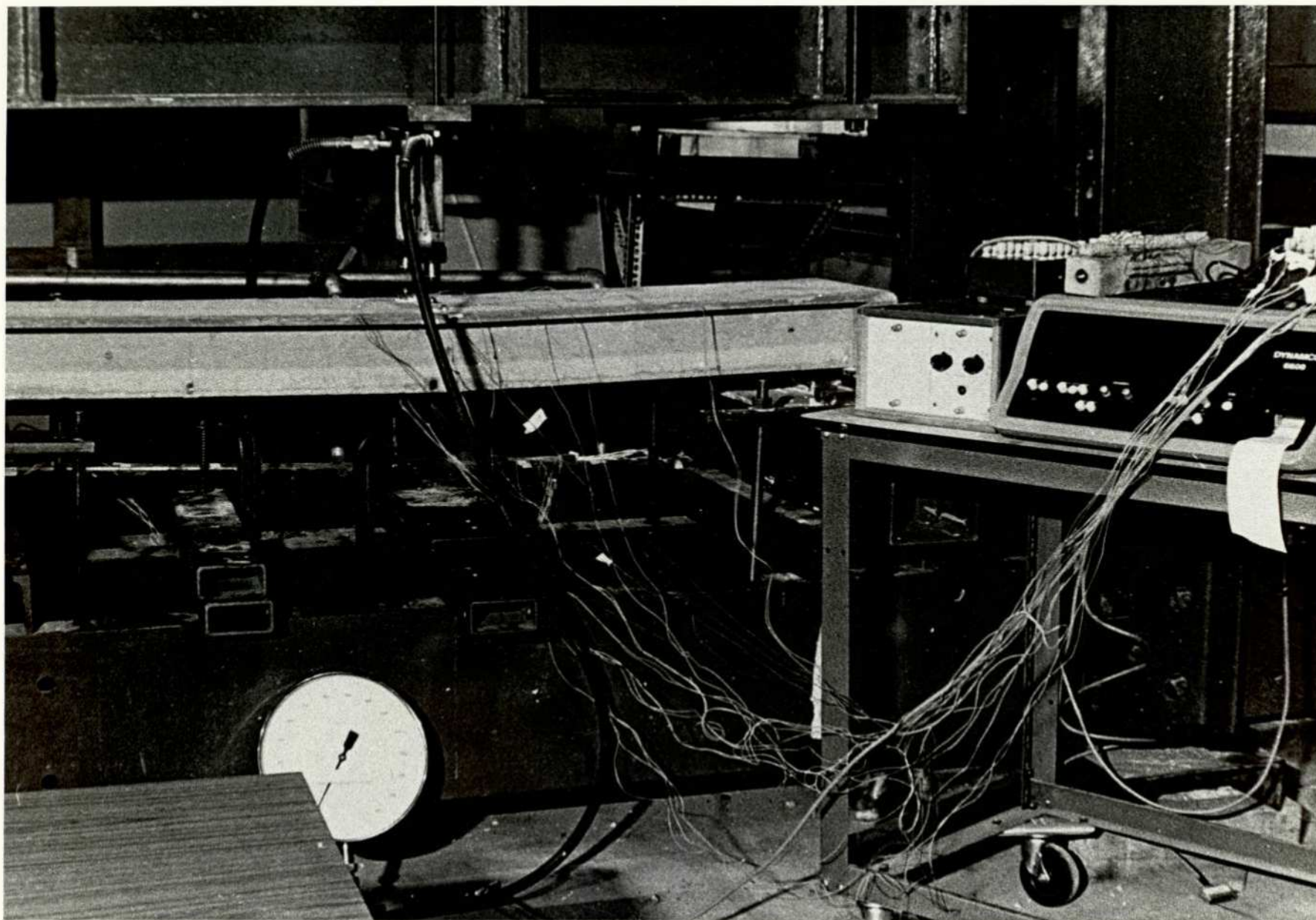


CALIBRATION BEAM



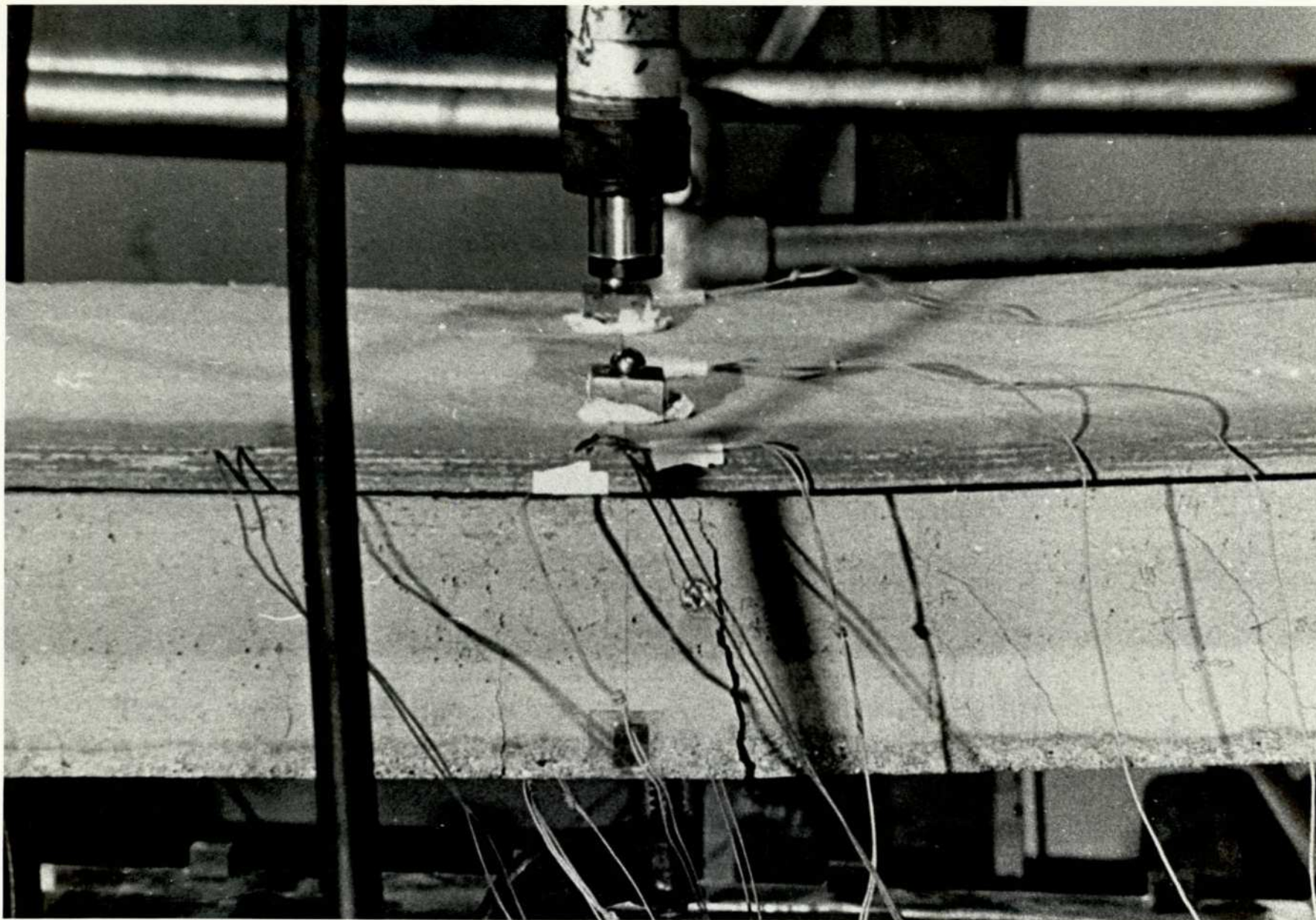


FORMWORK FORM



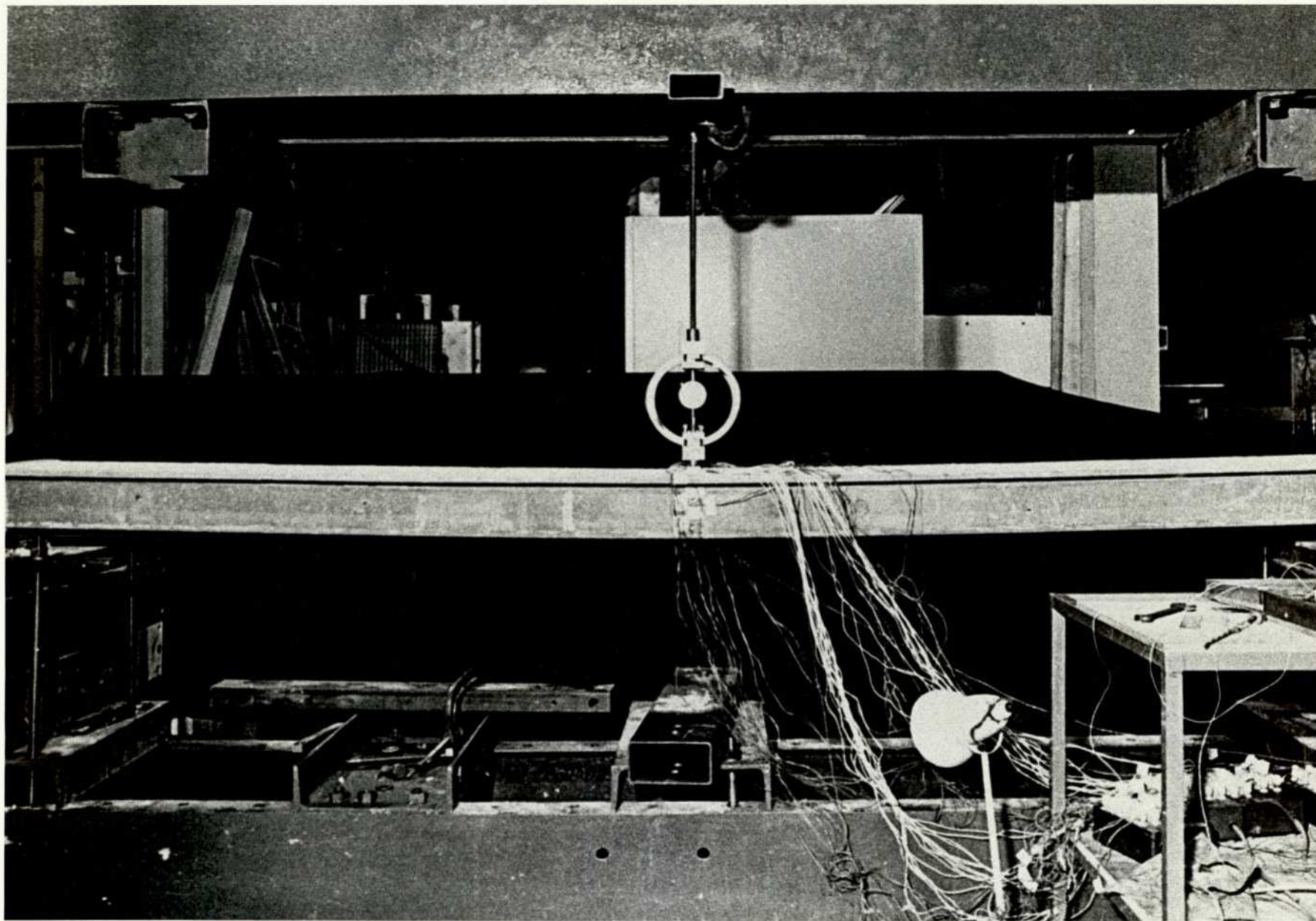
LOADING OF BOX BRIDGE A1

3



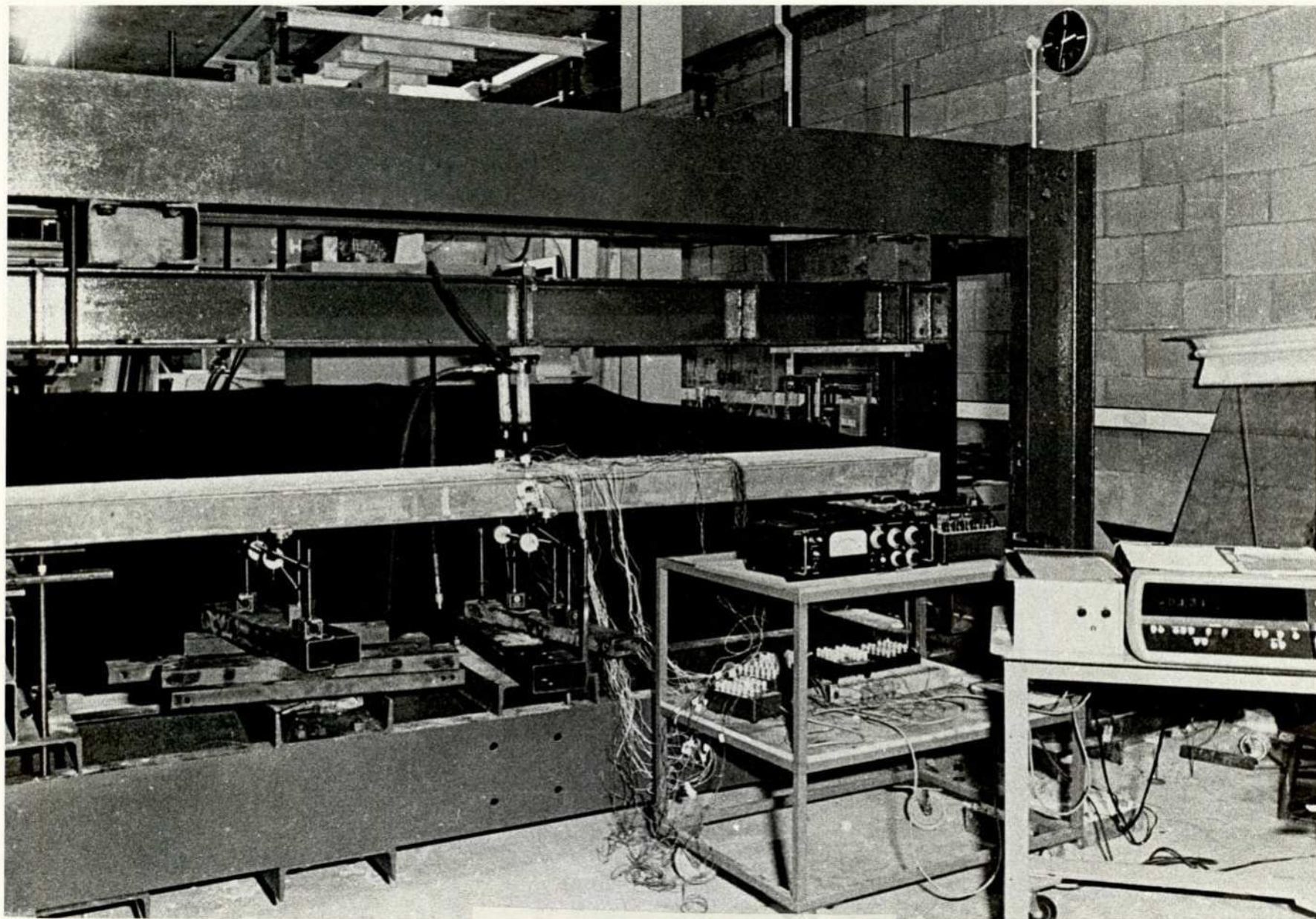
LOADING OF BOX BRIDGE A2

4

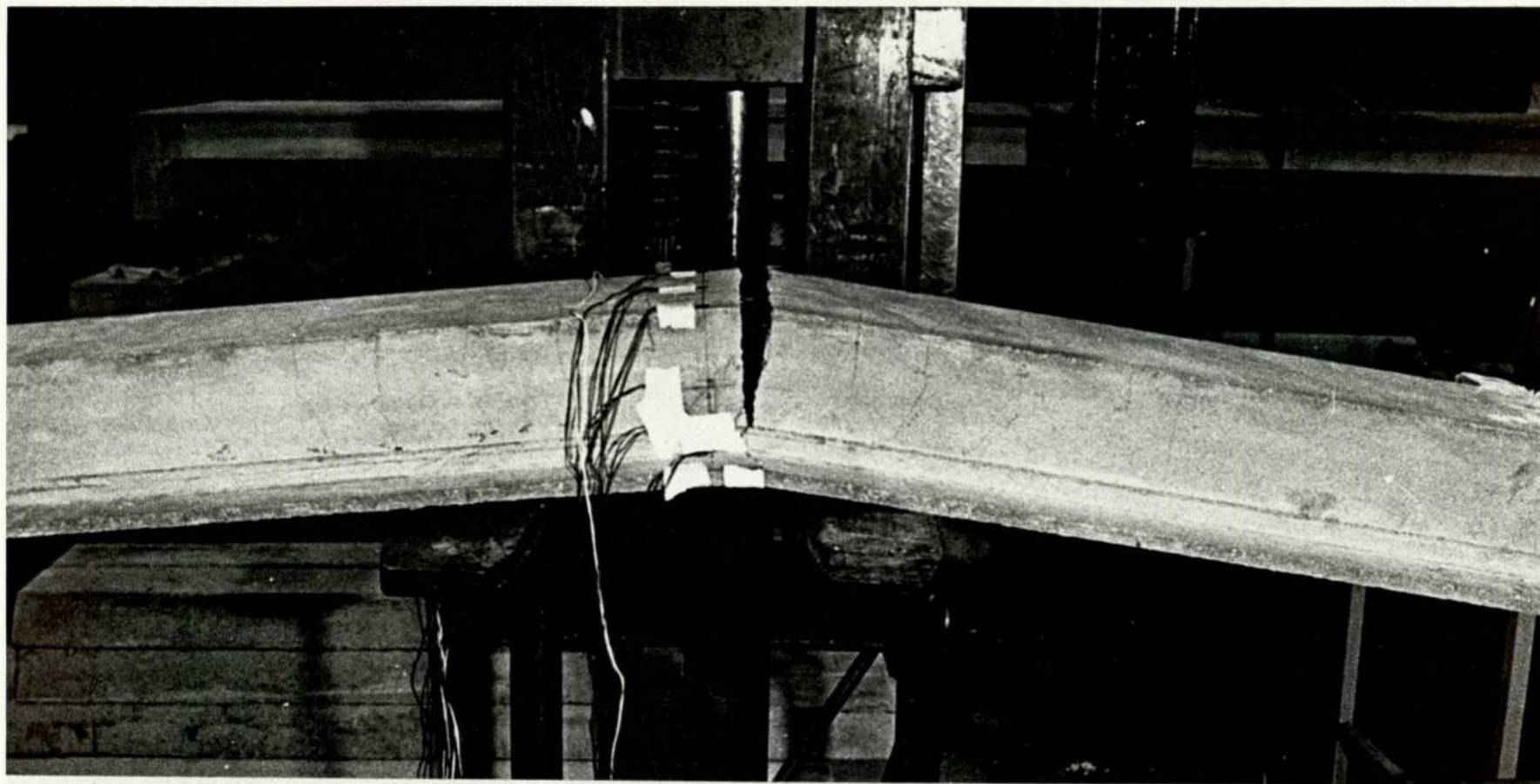


LOADING OF BOX BRIDGE A3

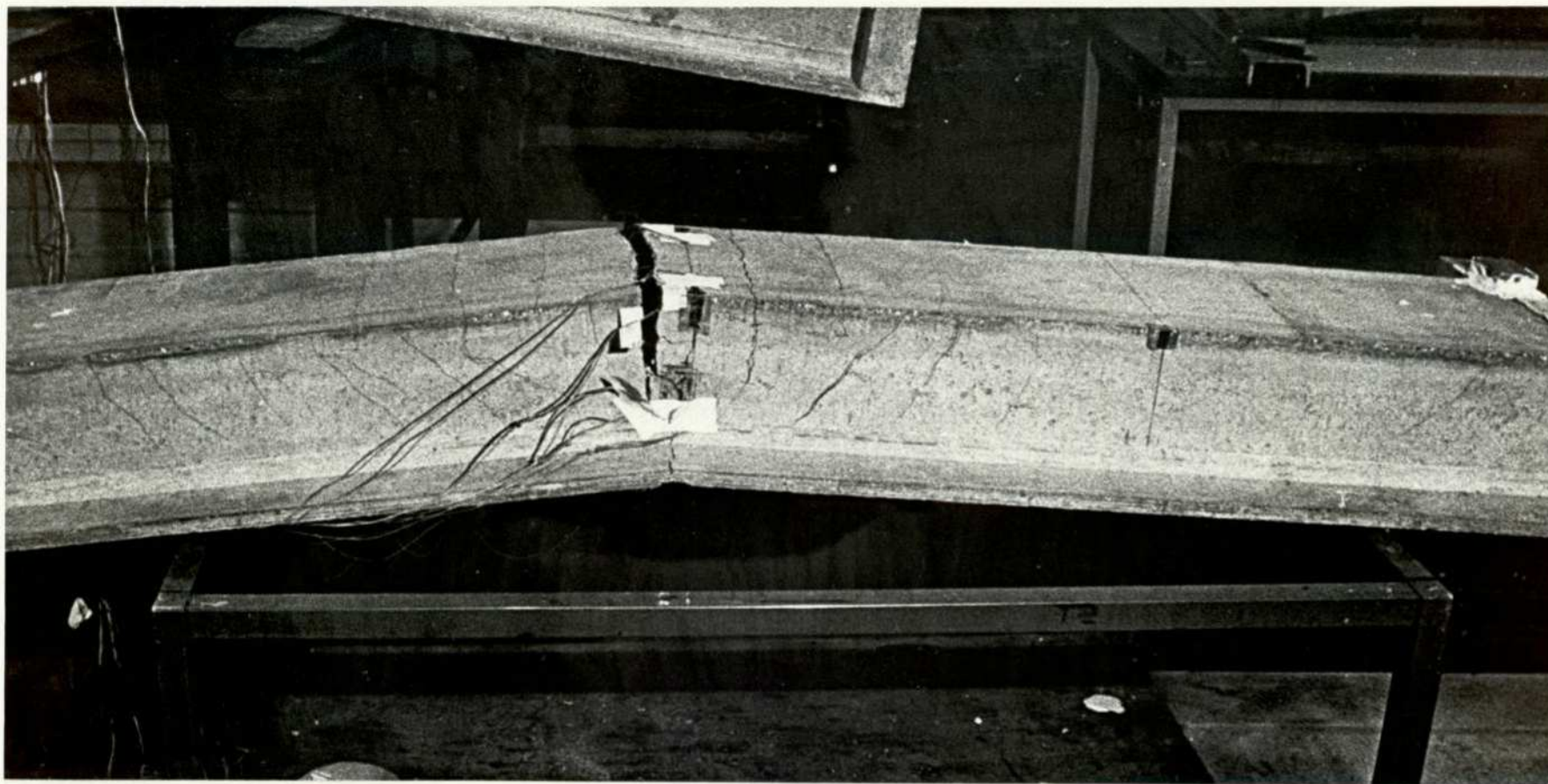
5



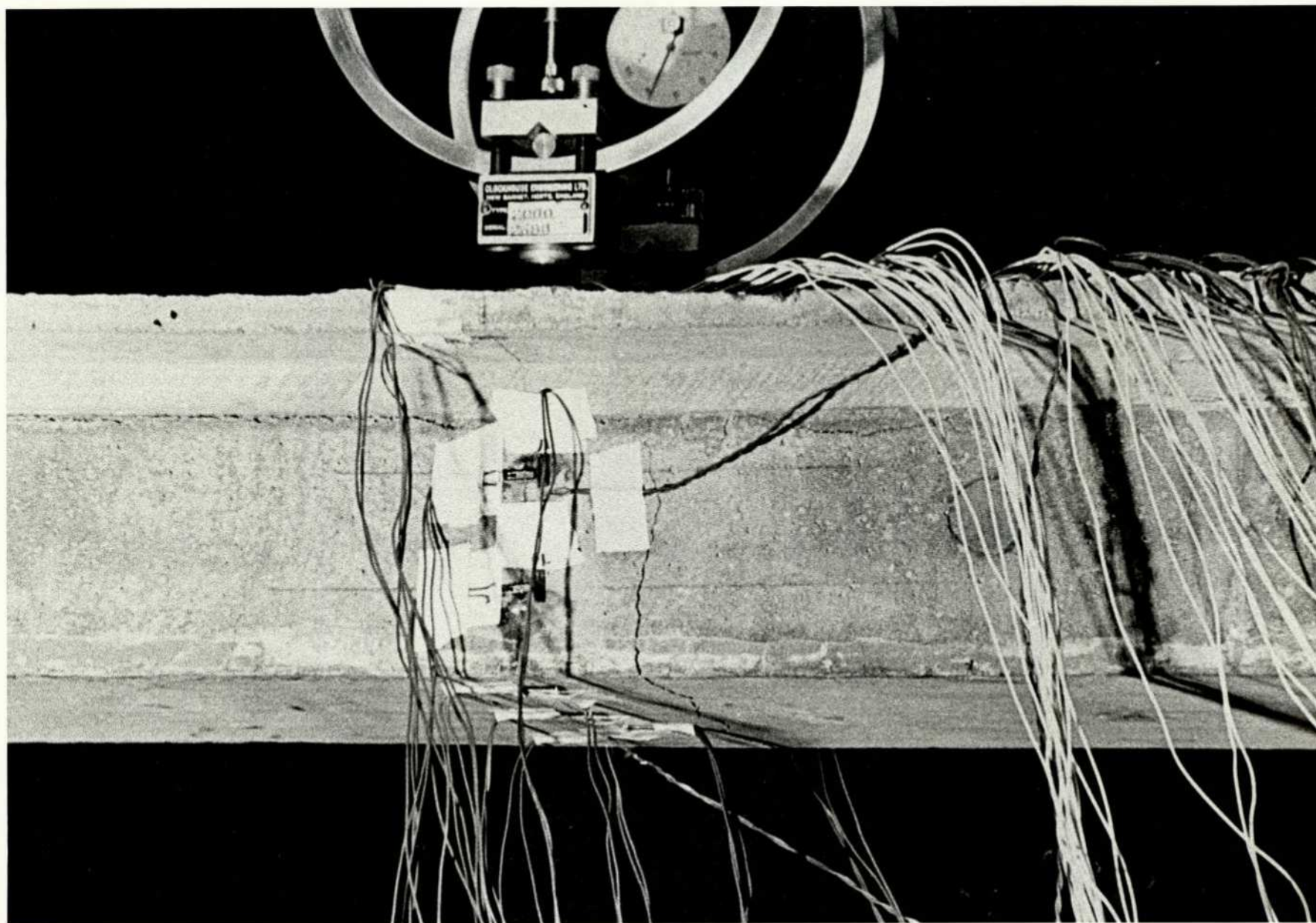
LOADING OF BOX BRIDGE A3



FAILURE OF BOX BRIDGE A1



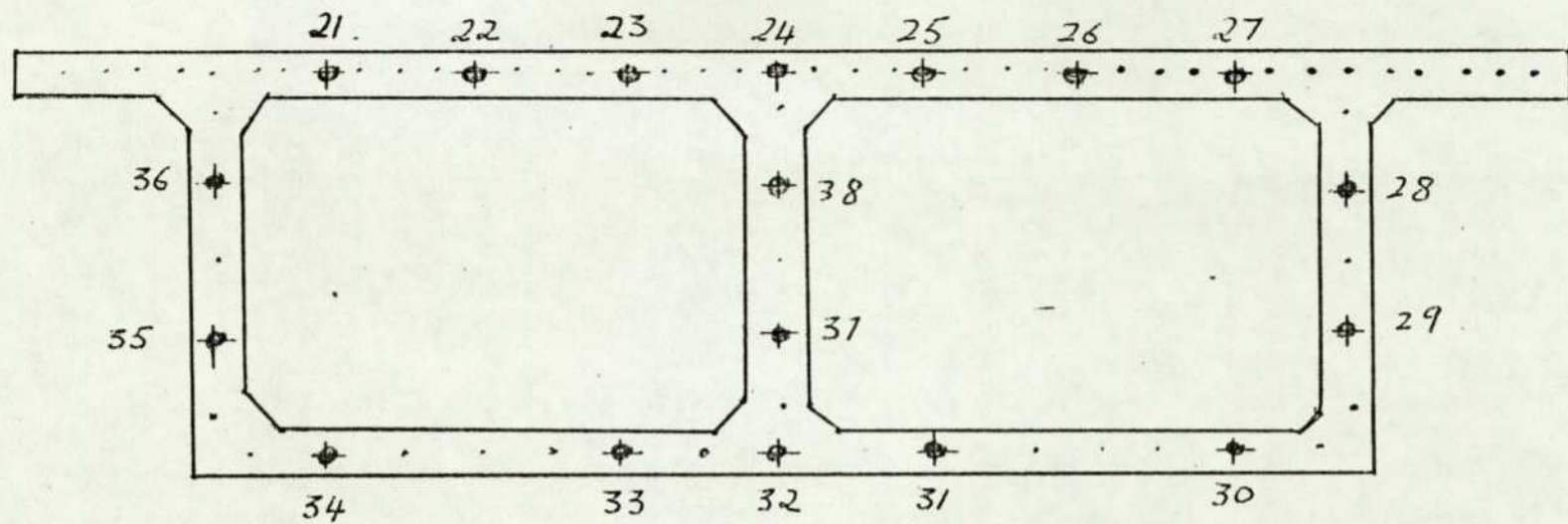
FAILURE OF BOX BRIDGE A2



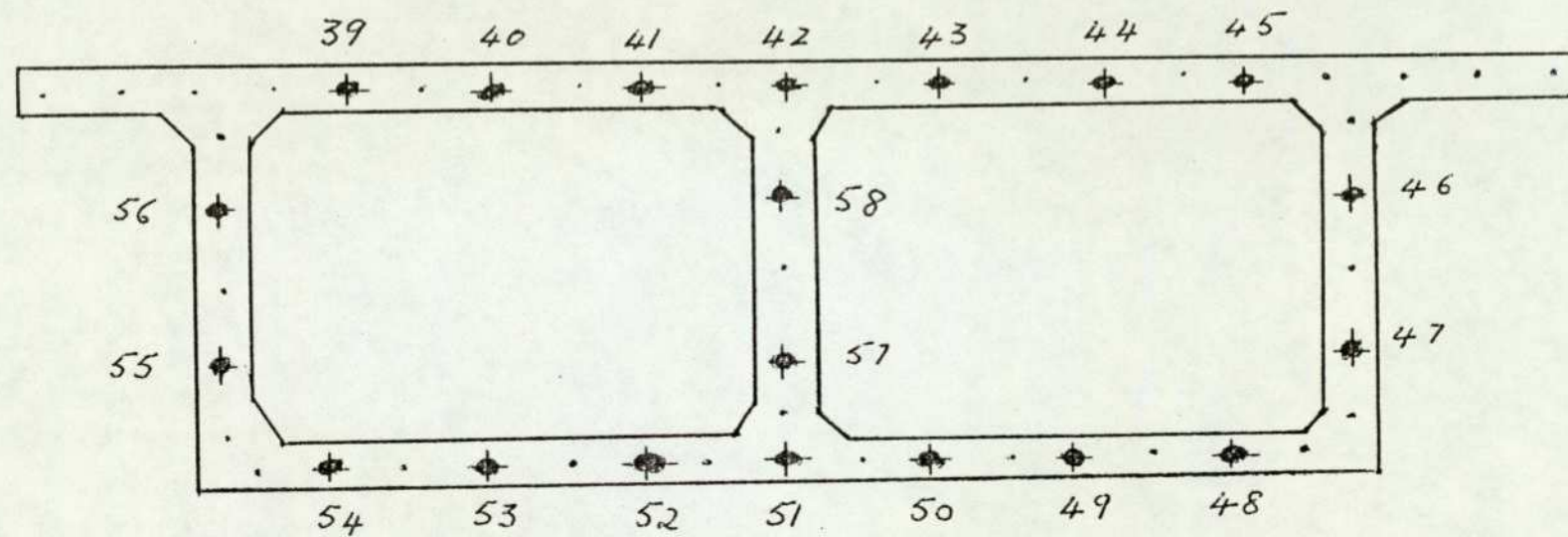
FAILURE OF BOX BRIDGE A3

9

Location of Strain Gauges on Steel

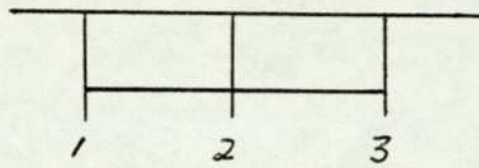
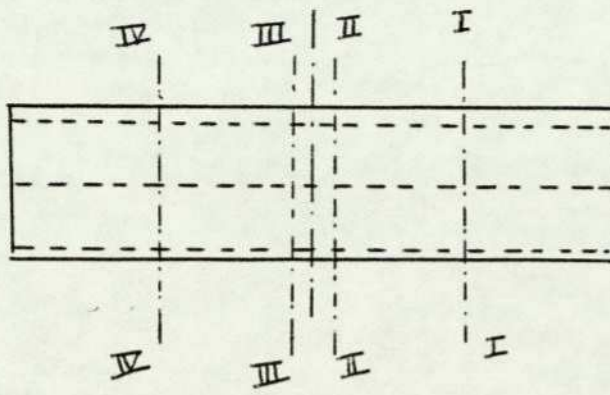


SECTION B-B

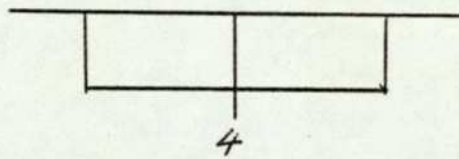


SECTION C-C

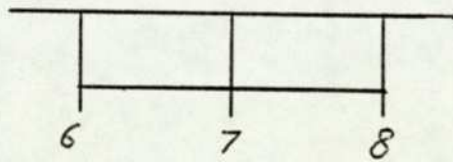
Figure C-4

Location of Dial Gauges for Box C

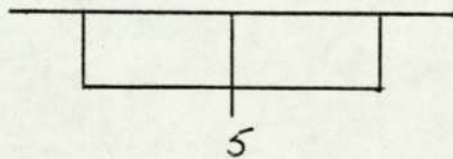
SECTION I-I



SECTION II-II



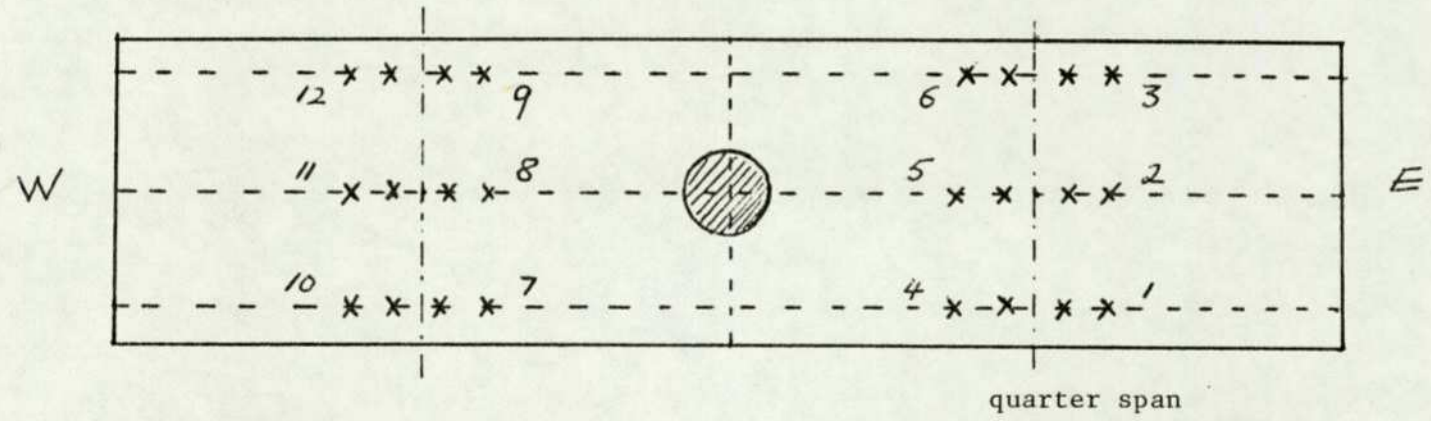
SECTION III-III



SECTION IV-IV

Figure C-5

Location of Demec Gauges for Box C

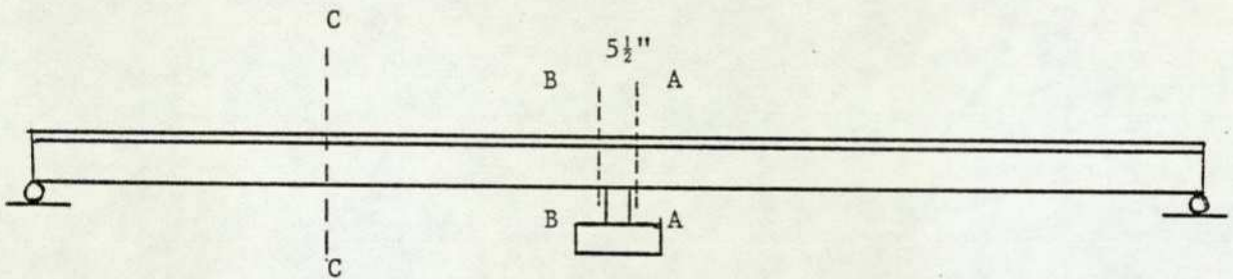


4" Demec gauges
 One dimension represents a strain of 2.00×10^{-5}

Figure C-6

Stress distribution due to the application of different
cases of Line Loads

Results of Tests 1, 2, 3, 4, 5, 6, 7



Location of cross-section

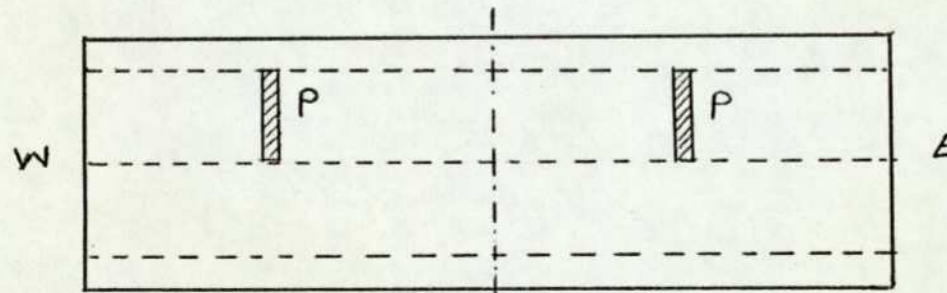
Figure C-7

TEST RESULTS FOR THE CASE OF LINE LOADS

Deflections for Tests, 1, 2, 3
(Line Load)

Test Number	Test 1		Test 2		Test 3		
Load	0	100	0	100	0	100	0
Gauge No.							
1	0	41	0	41	0	39	0
2	0	59	0	53	1	52	1
3	0	67	1	66	1	66	1
4	0	2	0	7	2	7	2
5	0	8	1	8	1	7	1
6	0	30	0	28	0	28	0
7	0	50	3	50	4	50	4
8	0	65	5	65	4	66	5

Table C-1

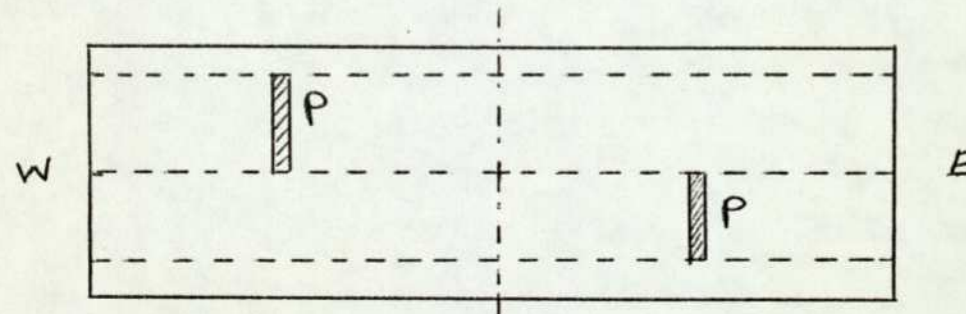


Location of Load

Deflections for Tests, 4, 5
(Line Load)

Test Number	Test 4		Test 5		
Load	0 100		0 100		0
Gauge No.					
1	0	63	2	63	2
2	0	61	2	60	2
3	0	52	2	53	1
4	0	5	0	6	6
5	0	5	1	5	0
6	0	38	0	38	0
7	0	45	1	45	1
8	6	53	4	57	4

Table C-2

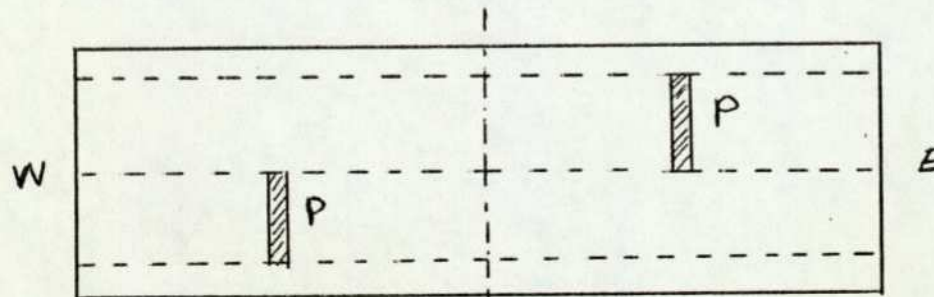


Location of the Load

Deflections of Tests 6, 7
(Line Loads)

Test Number	Test 6		Test 7		
Load Gauge No.	0	100	0	100	0
1	0	50	2	50	2
2	0	59	1	59	1
3	0	66	2	66	2
4	0	5	1	5	0
5	0	5	1	5	0
6	0	52	2	52	2
7	0	45	1	45	1
8	0	40	2	40	2

Table C-3



Location of the Load

Location of Line Load for Test 1, Test 2
and Test 3

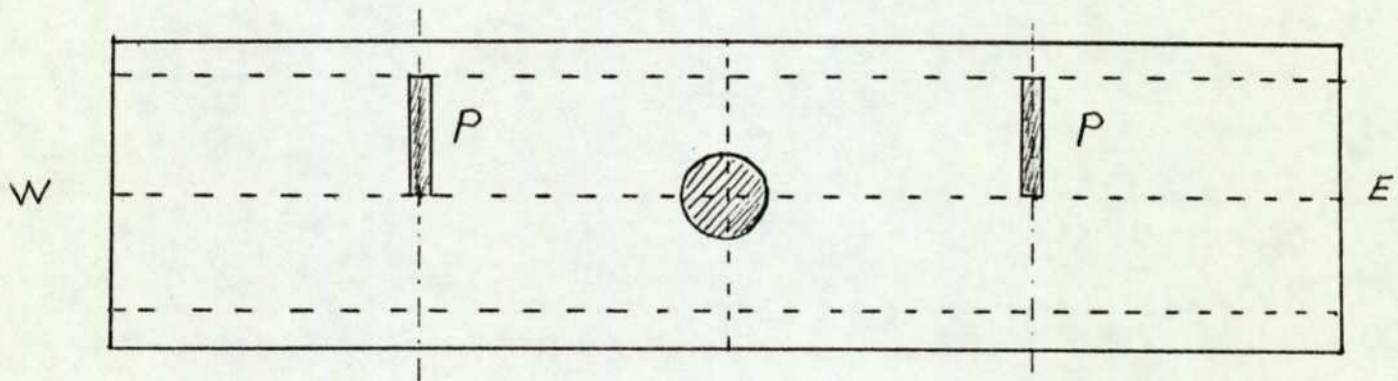


Figure C-8

SECTION A-A

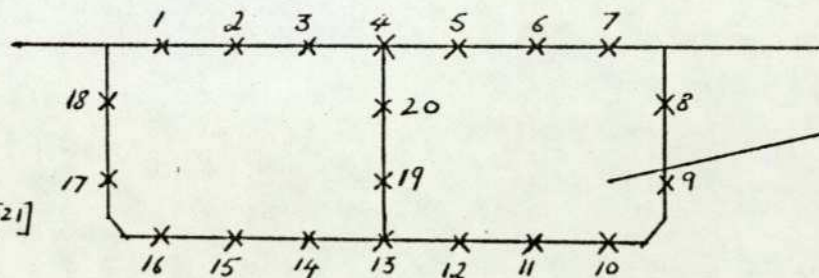
Test 1

Test 2 Nos. between ()

Test 3 Nos. between []

54.5 - (52) - [52.5]

21.5 - (22) - [21]



37 - (37) - [35.5]

16.5 - (15) - [16]

22.5 - (21) - [20.5]

28 - (2.5) - [26.5]

38.5 - (37) - [36.5]

49 - (48) - [47]

43 - (40) - [40]

61.5 - (60) - [59]

64.5 - (58.5) - [57]

77 - (74.5) - [74]

63 - (62.5) - [62]

38 - (38.5) - [38]

(+)

(-)

Figure C-9

SECTION B-B

Test 1

Test 2 Nos. between ()

Test 3 Nos. between []

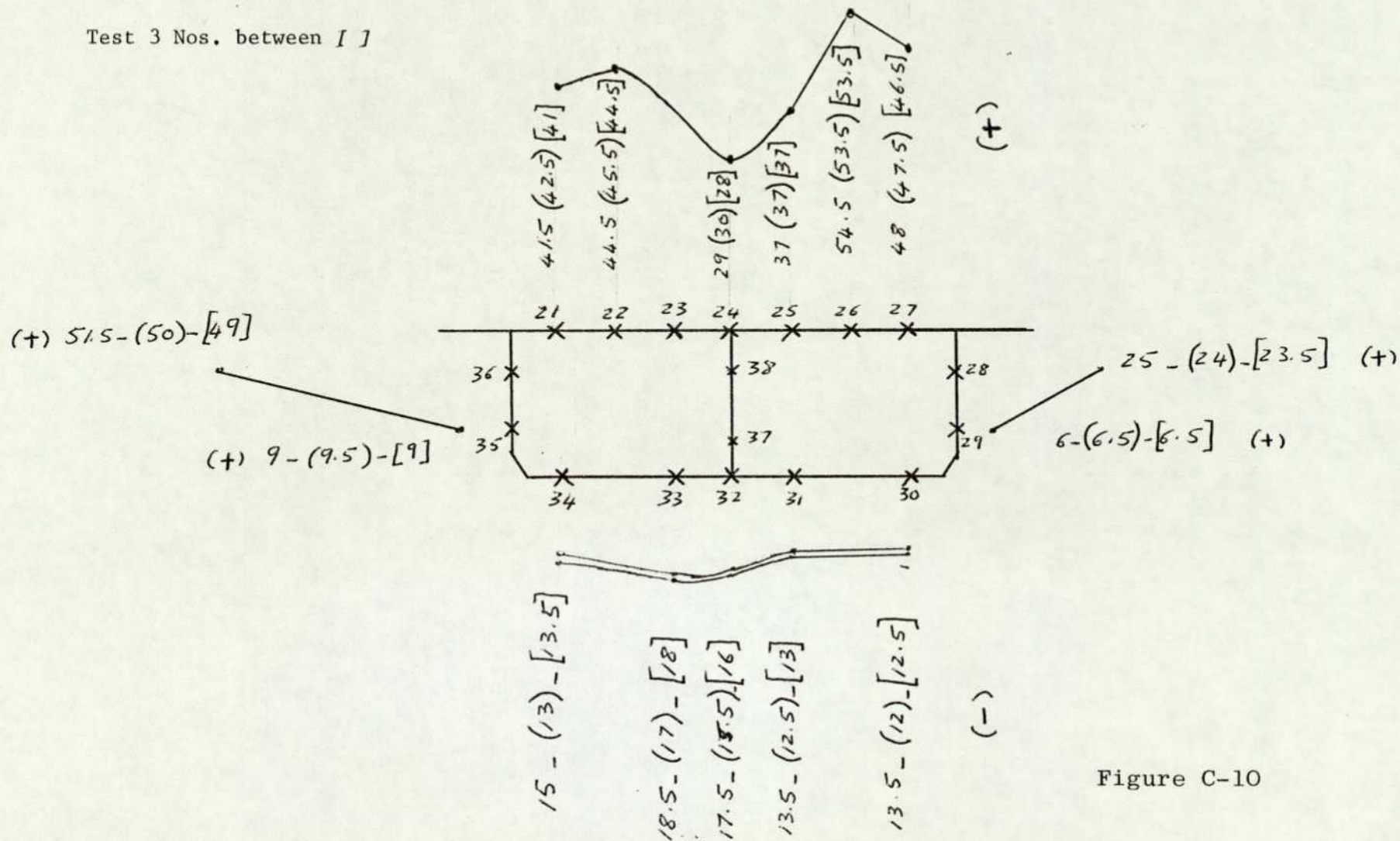


Figure C-10

Location of Line Load for Test 4 and Test 5

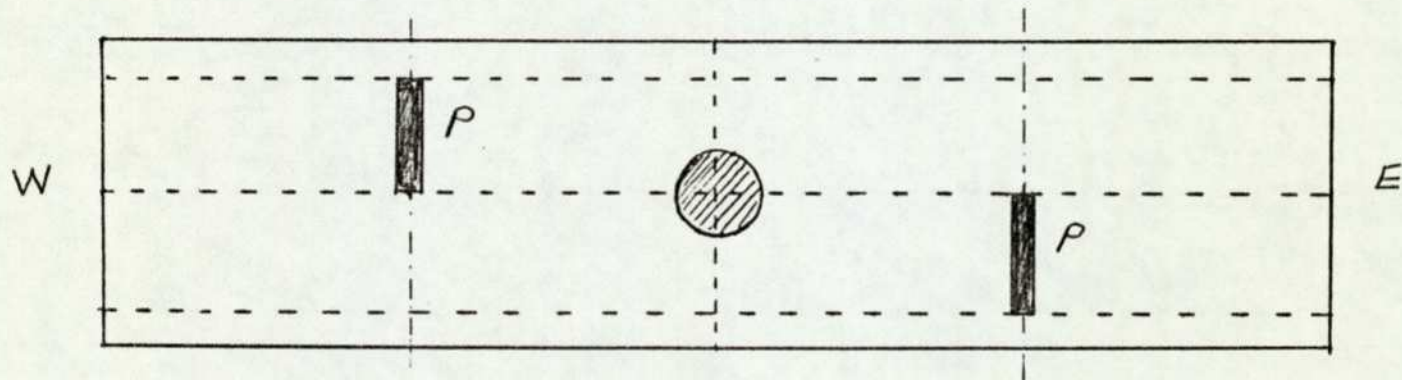


Figure C-12

SECTION A-A

Test 4

Test 5 Nos. between ()

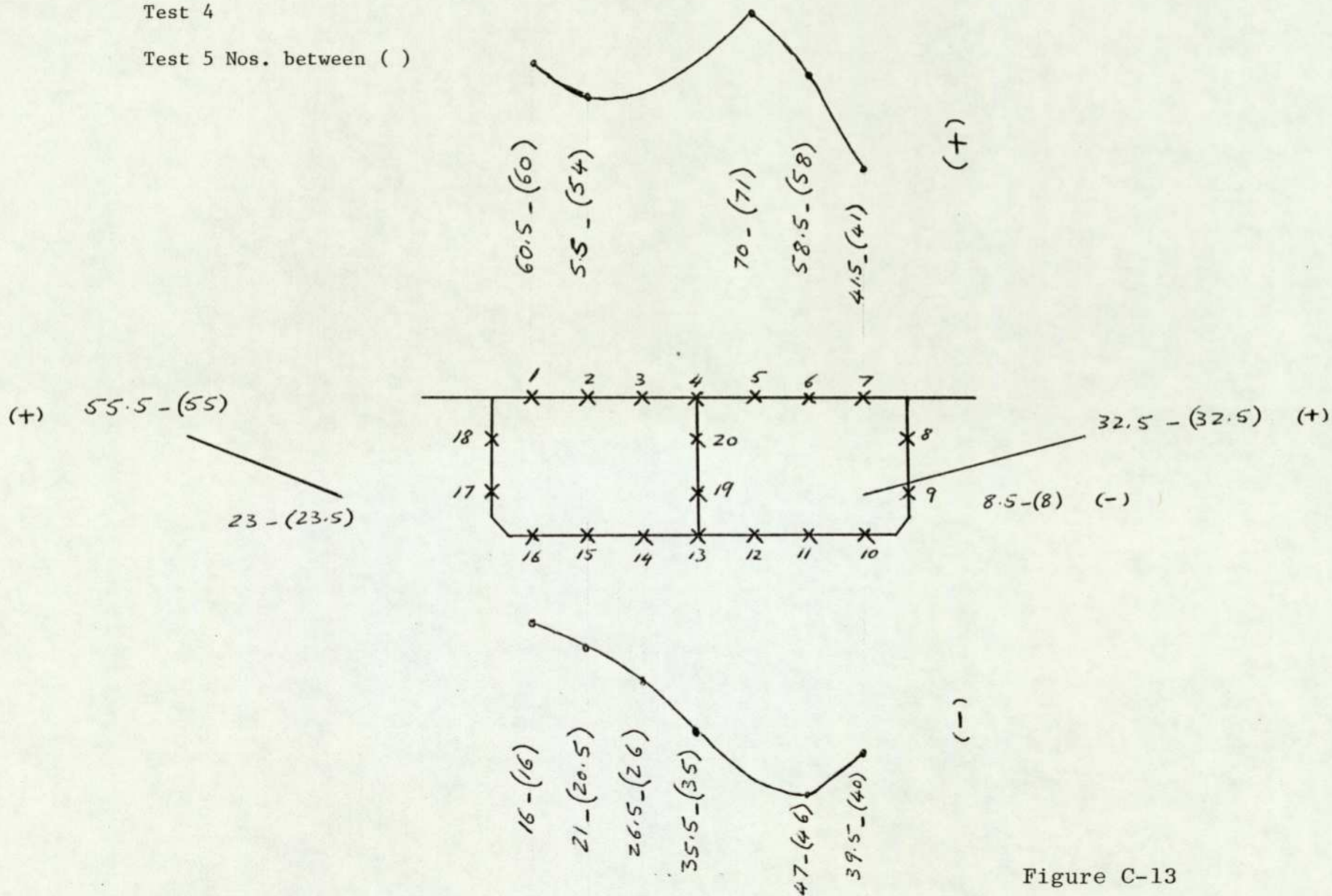


Figure C-13

SECTION B-B

Test 4

Test 5 Nos. between ()

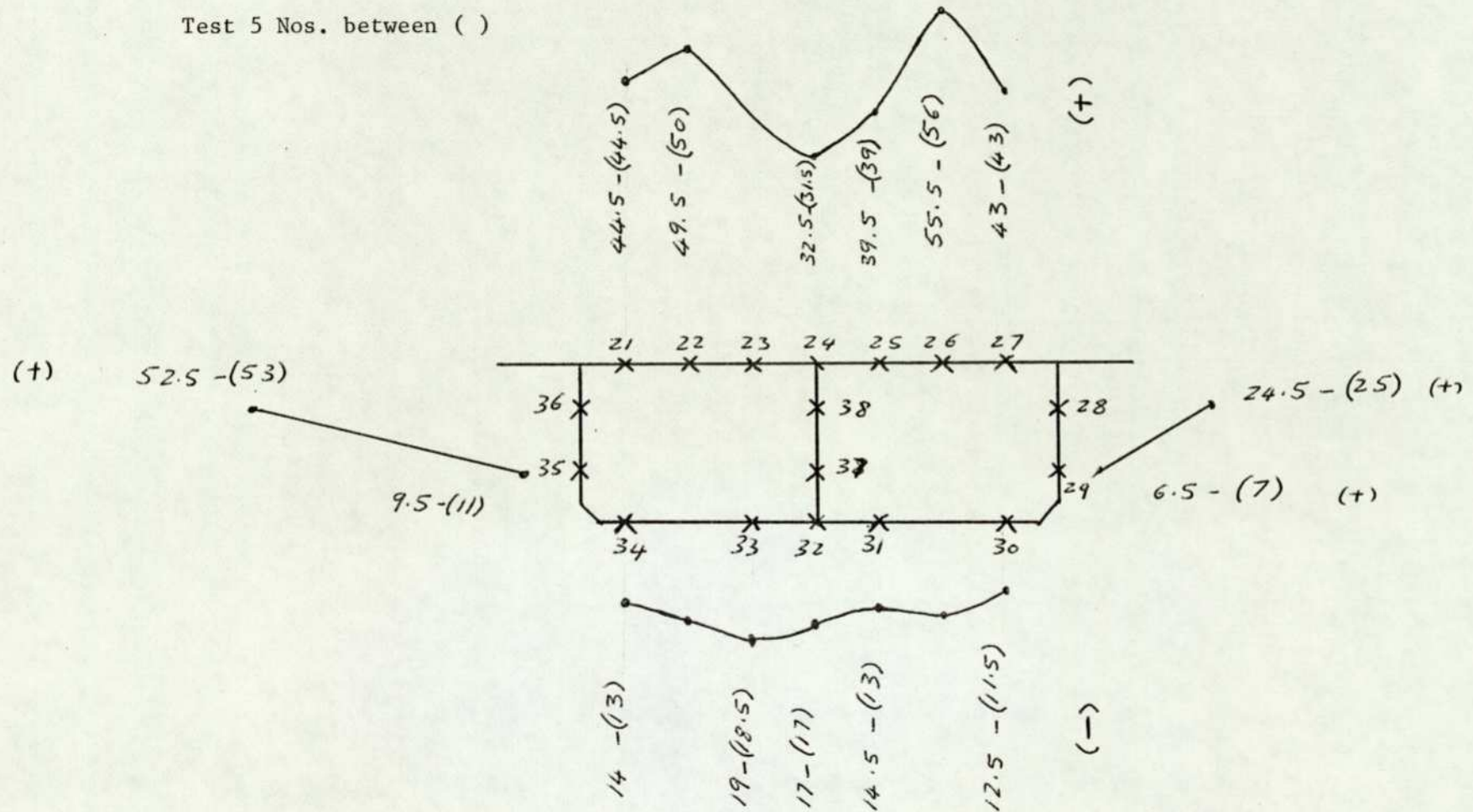


Figure C-14

SECTION C-C

Test 5

Test 5 Nos. between ()

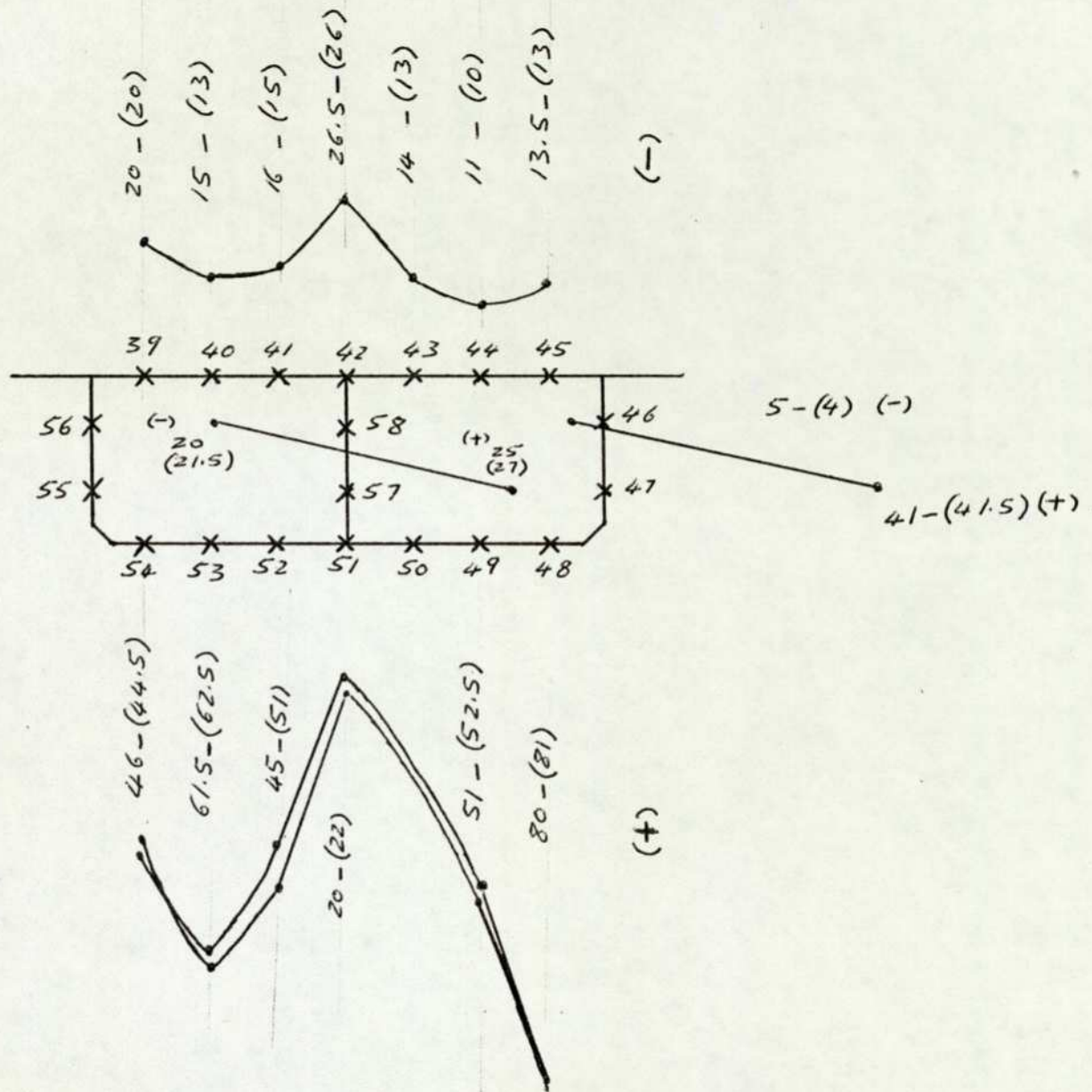


Figure C-15

Location of Line Load for Test 6 and Test 7

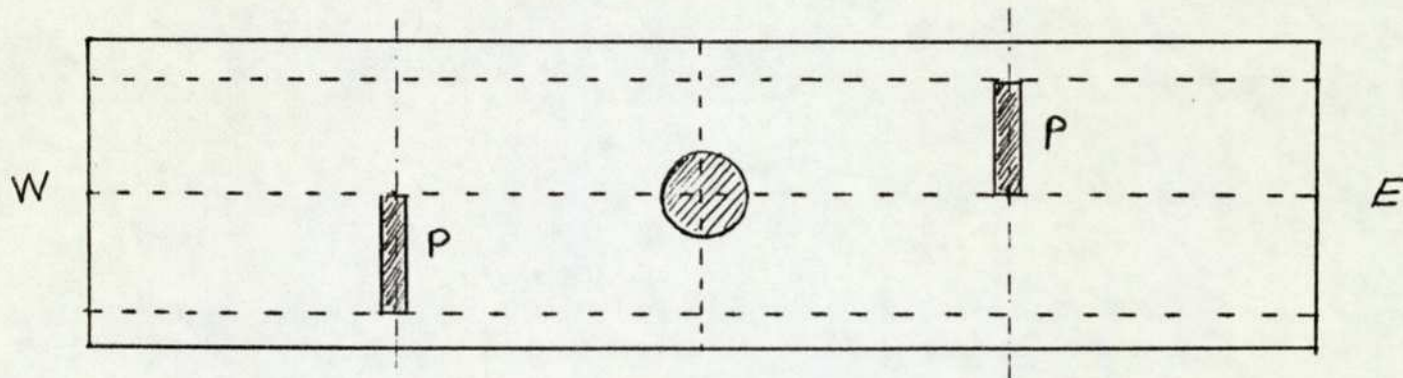


Figure C-16

SECTION A-A

Test 6

Test 7 Nos. between ()

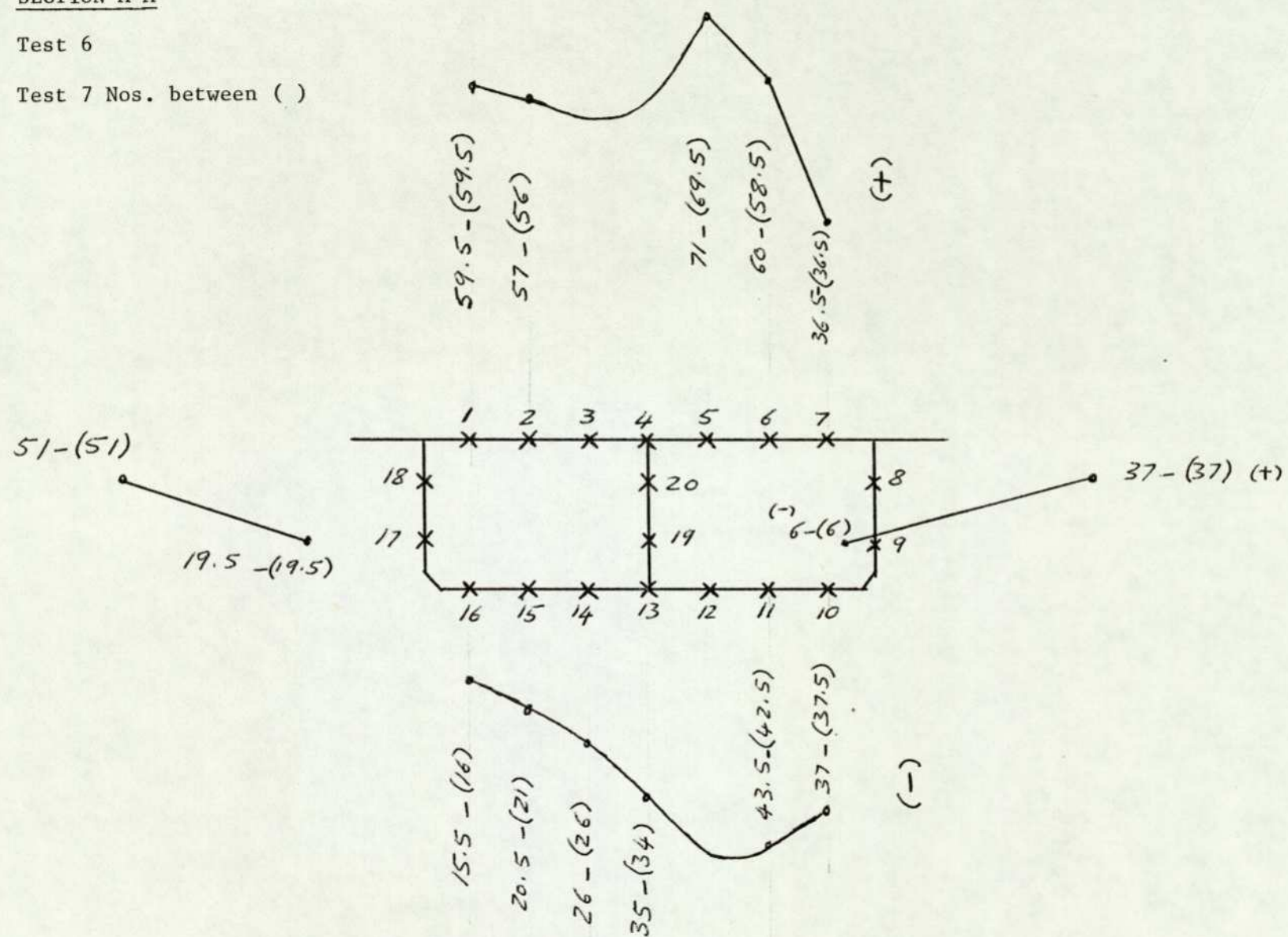


Figure C-17

SECTION B-B

Test 6

Test 7 Nos. between ()

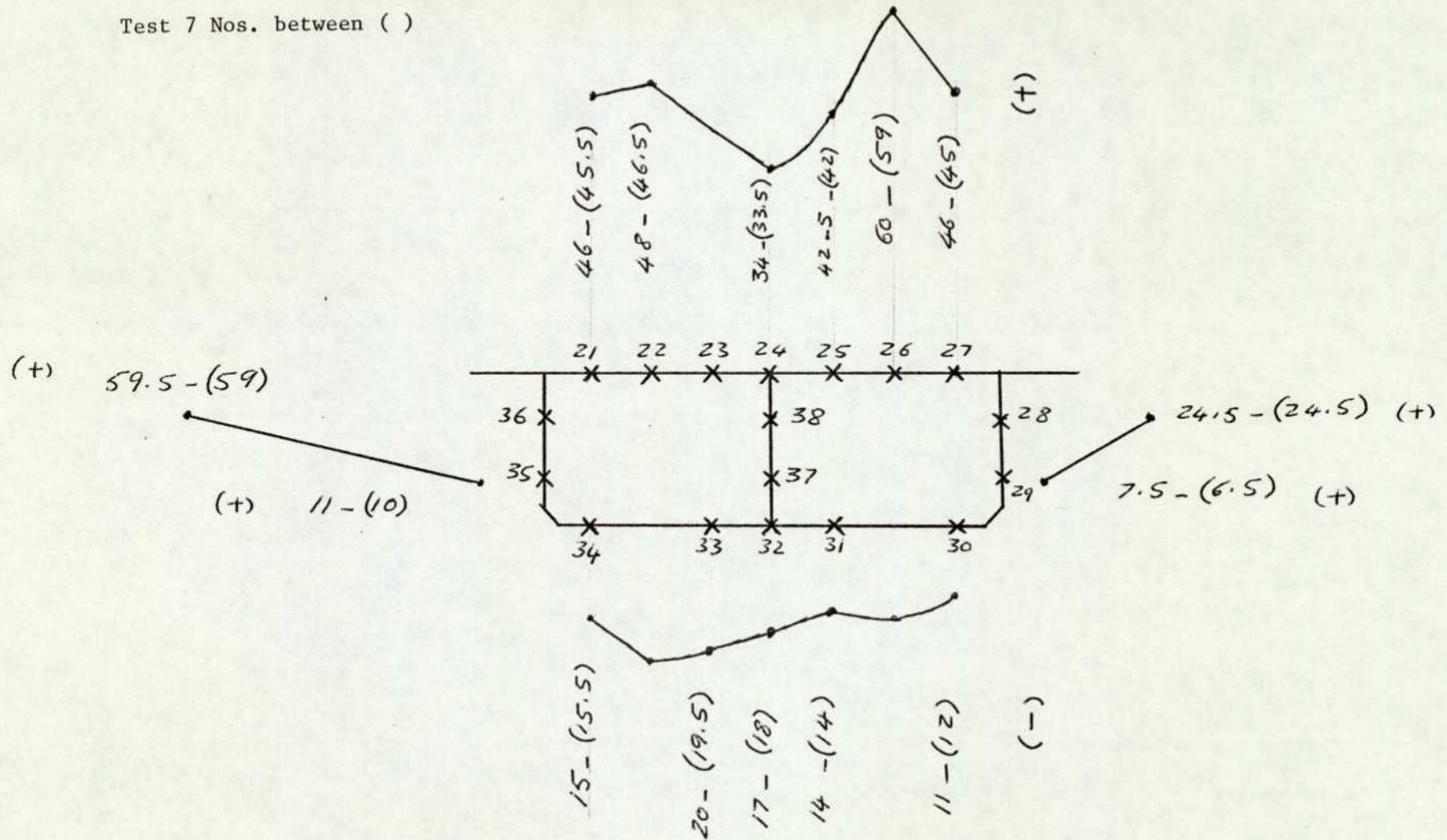


Figure C-18

SECTION C-C

Test 6

Test 7 Nos. between ()

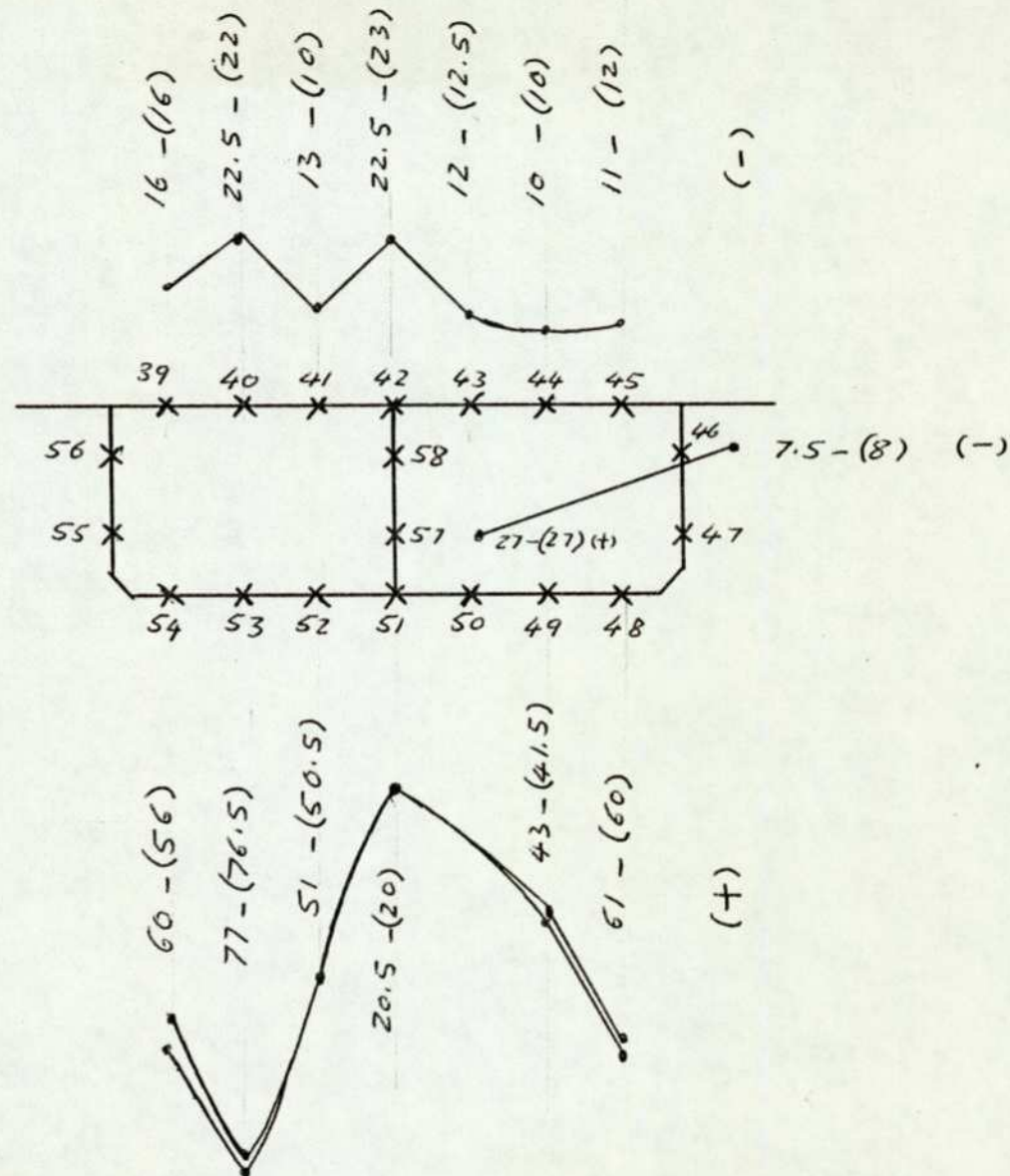
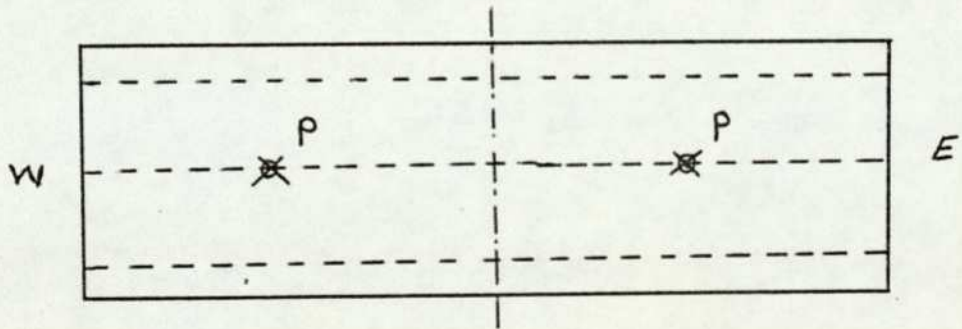


Figure C-19

TEST RESULTS FOR THE CASE OF POINT LOADS

Deflection results for Tests 1, 2
(Point Load)

Test Number	Test 1		Test 2	
Load Gauge No.	0	100	0	100
1	0	57	2	54
2	0	64	2	63
3	0	60	2	59
4	0	5	2	7
5	0	7	0	6
6	0	48	3	50
7	0	50	3	53
8	0	48	3	52
9	0	3	-2	1
10	0	7	0	5



Location of Load for Tests 1, 2

Table C-4

Demec Gauge Readings for Test 1, 2(Point Load)

Test Number	Test 1		Test 2	
Load	0	100	0	100
Gauge No.				
1	661	657	661	657
2	651	656	661	656
3	631	647	651	647
4	657	656	658	655
5	662	660	661.5	660
6	655	656.5	658	656
7	655	653	655	655
8	660	658	660	658
9	641	645	647	646
10	662.5	661	663	661
11	655	654	655	654
12	645	645.5	643	643

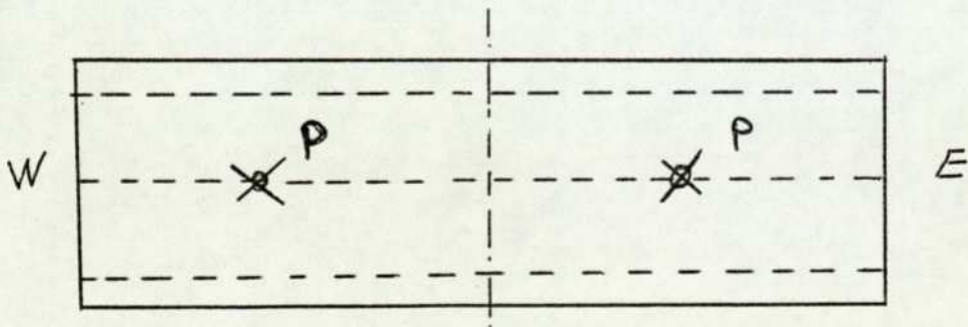
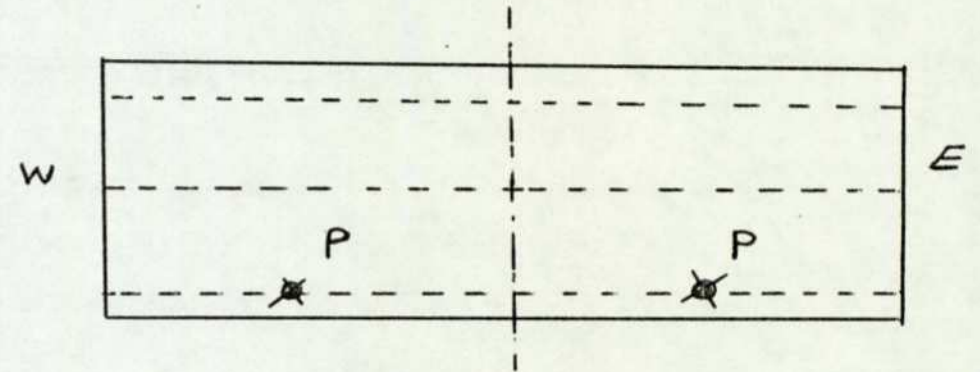
Location of Load for Test 1, 2

Table C-5

Deflections for Test 3

Test Number	Test 3		
Load	0	600	0
Gauge No.			
1	2	65	3
2	2	45	2
3	2	30	1.5
4	2	2	2
5	0	3	0
6	3	58	5
7	3	37	3
8	3	22	2
9	-4	10.5	-3
10	-2	-7	-4

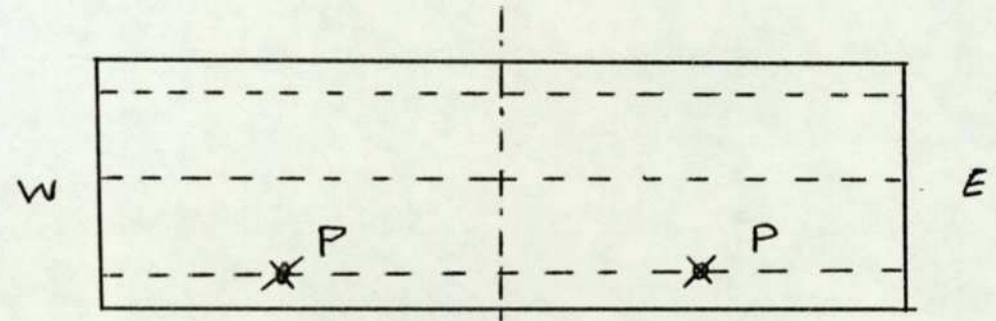


Location of Load

Table C-6

Demec Gauge Readings for Test 3

Test Number	Test 3		
Load	0	600	0
Gauge No.			
1	661	657	661
2	661	657	661
3	651	648	651
4	658	655	658
5	661	660	661.5
6	658	657	658
7	655	653	655
8	660	659	659
9	648	647	648
10	663	661	663
11	655	654	655
12	645	643	645



Location of Loading

Table (C-7)

Deflections and Demec Gauge Readings for Test 5

Test Number	Test 5		
Load	0	600	0
Gauge No.			
1	2	37	2
2	1	45	2
3	0	56	2
4	1	5	2
5	0	5	0
6	5	54	5
7	3	40	3
8	2	32	2
9	-2	2	-2
10	-2	2	-2
Demec Gauges			
1	661	659	661
2	661	657	661
3	651	647	651
4	658	657	658
5	661.5	660	661.5
6	658	657	658
7	655	653	655
8	659	658	660
9	648	647	648
10	663	661	663
11	655	655	655
12	645	645	644

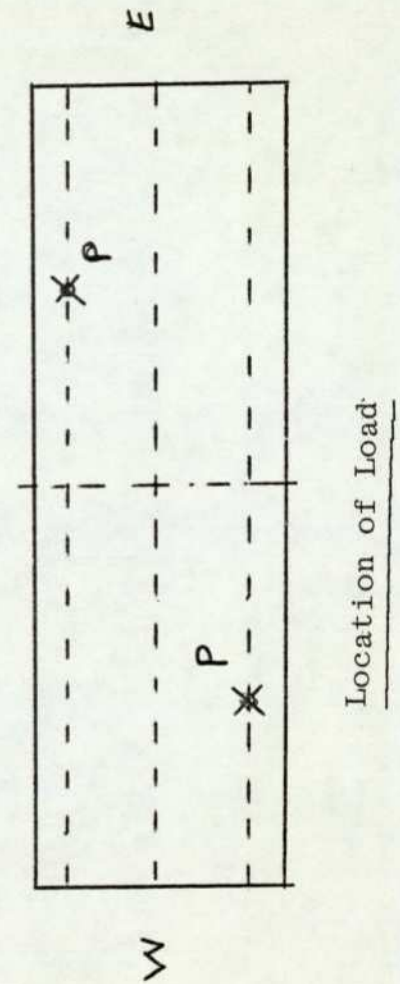


Table (C-8)

Table (C-9)

Deflections and Demec Gauge Readings for Test 7

Test Number	Test 7		
Load	0	600	0
Gauge No.			
1	0	15	0
2	0	35	2
3	0	61	3
4	0	3	2
5	0	6.5	0
6	0	25	0
7	0	50	2
8	0	80	3
9	0	-11	-2.5
10	0	25	0
Demec Gauges			
1	661	658	661
2	661	658	661
3	656	647	650
4	659	656	659
5	661	660	661
6	658	657	658
7	655	652	655
8	659	656	659
9	648	646	648
10	662	660	662
11	655	653	655
12	645	650	645

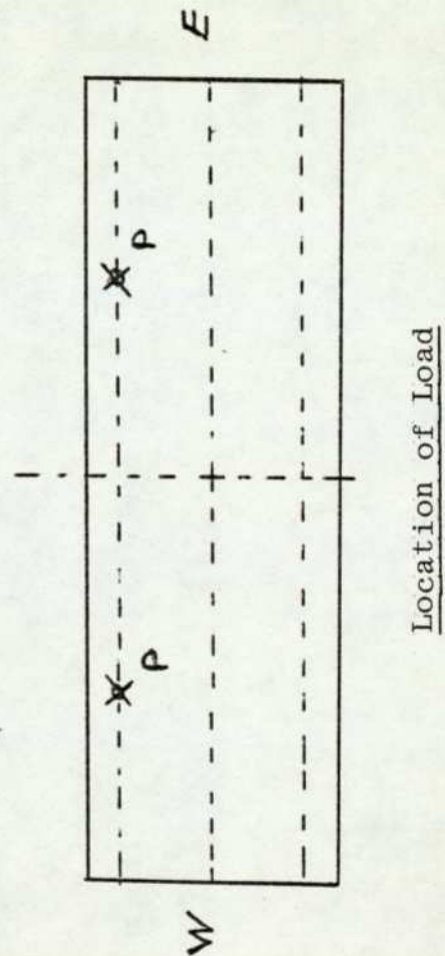
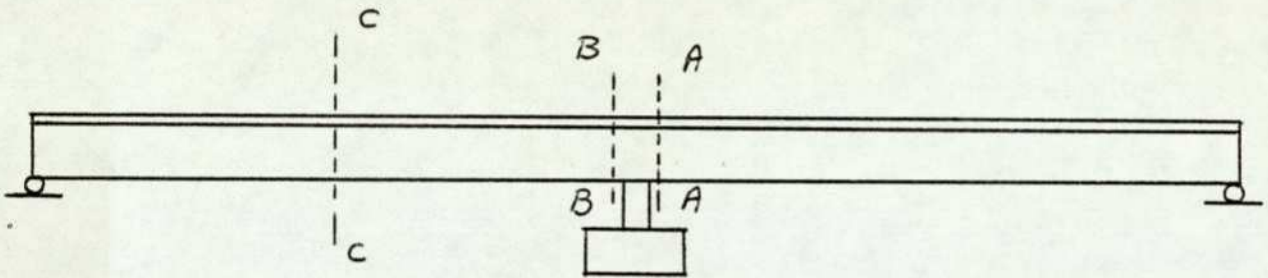


Table C-10

Table C-11

Stress distribution due to the application of
different cases of Point Loads

Tests 1, 2, 3, 4, 5 and 6



Location of cross-section

Figure C-20

Location of Point Loads for Test 1 and Test 2

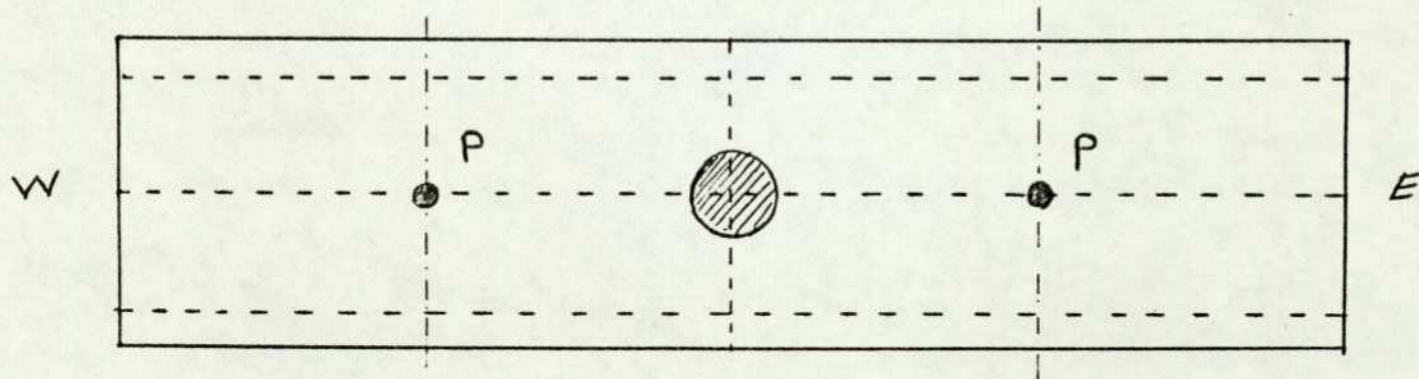


Figure C-21

SECTION A-A

Test 2

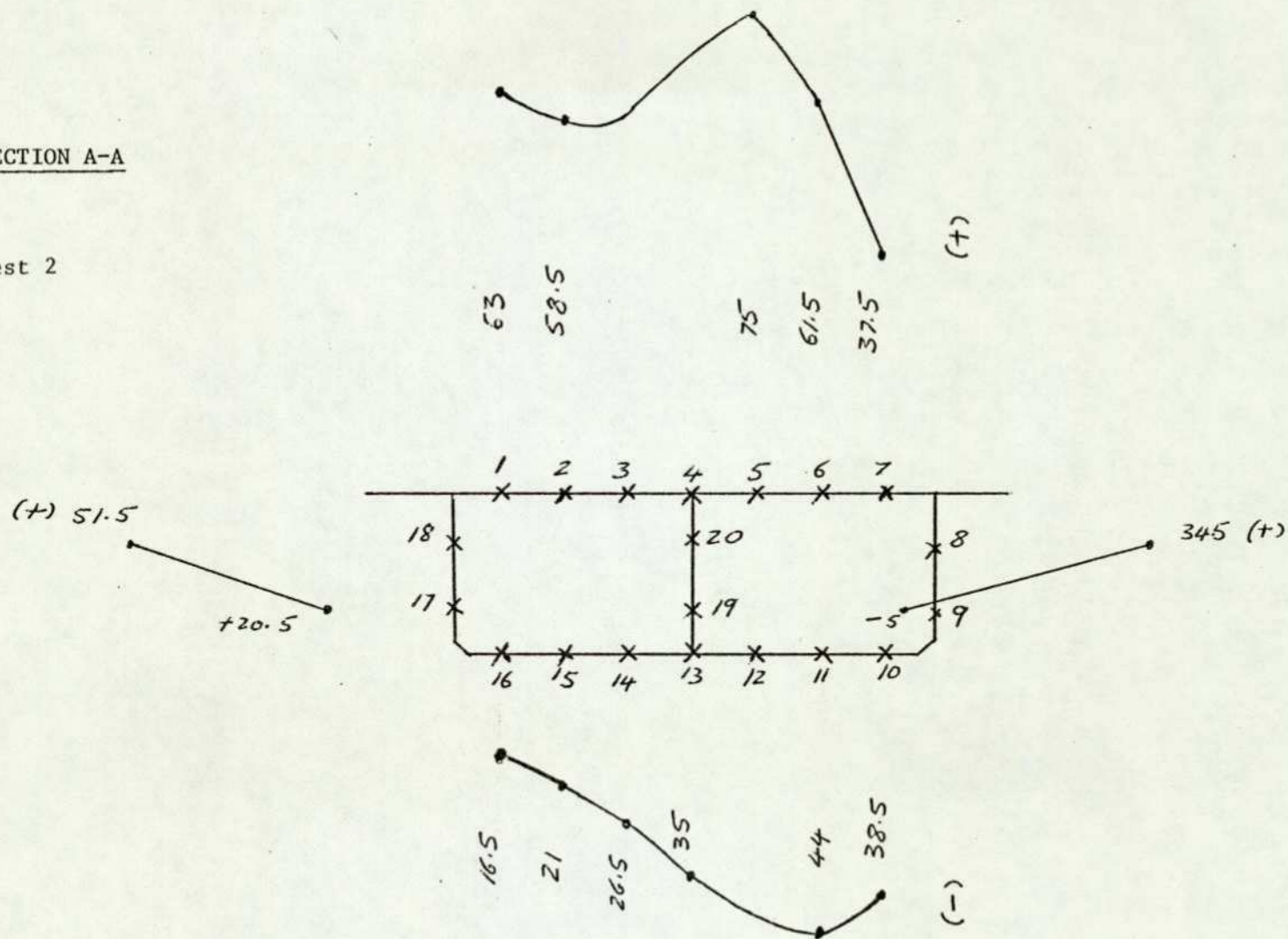


Figure C-22

Test 2

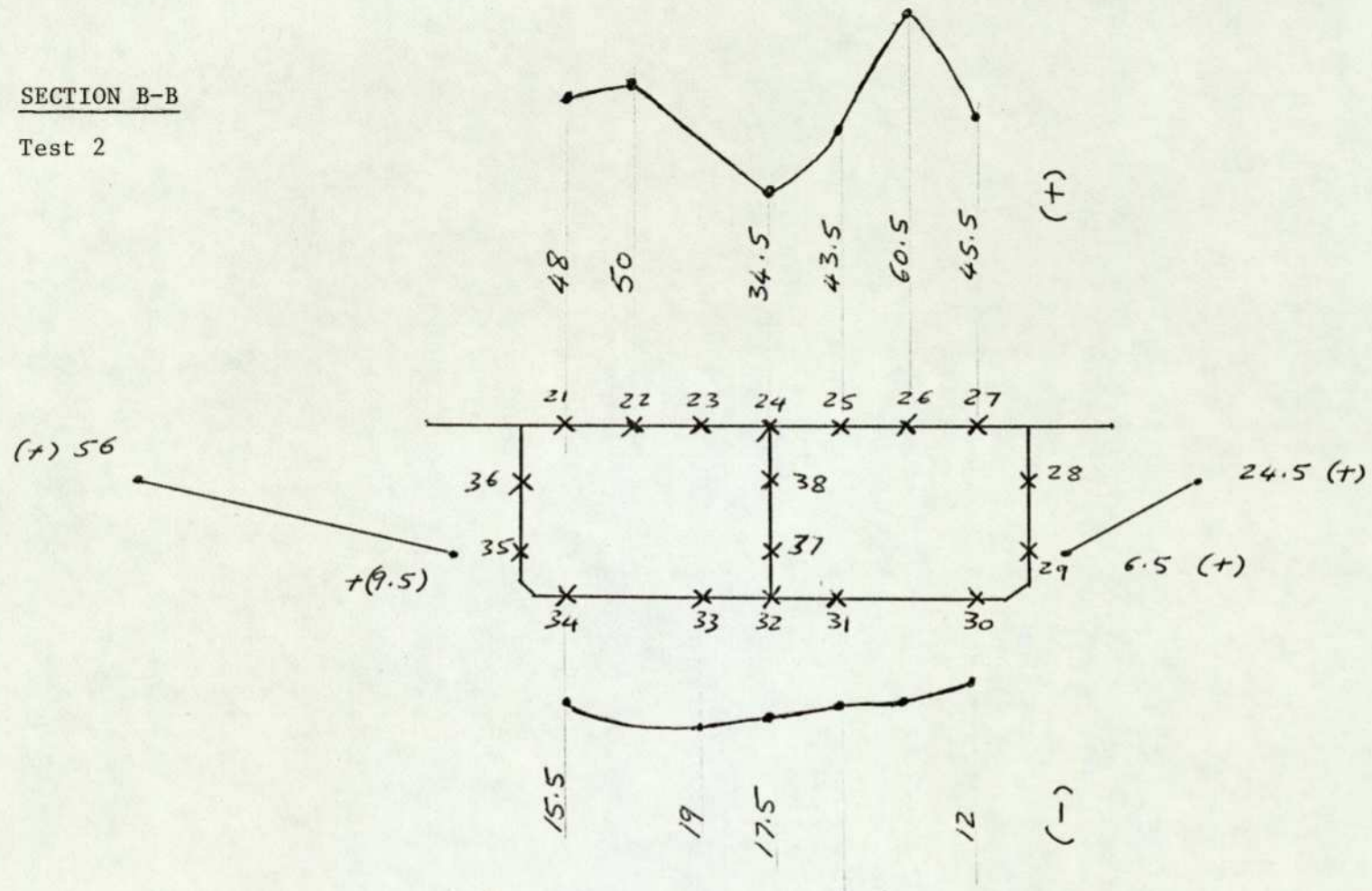


Figure C-23

SECTION C-C

Test 2

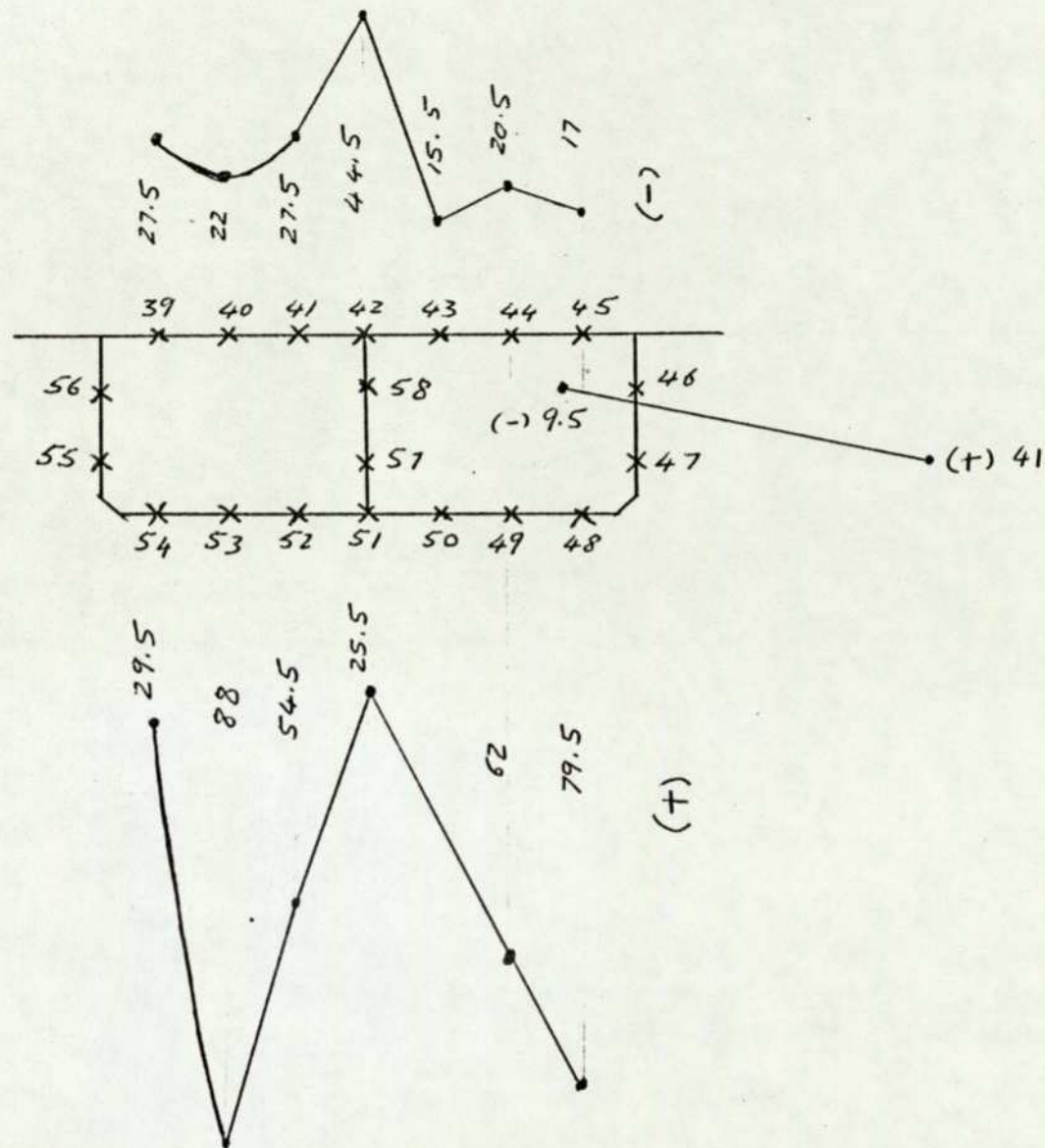


Figure C-24

Location of Point Load for Test 3 and Test 4

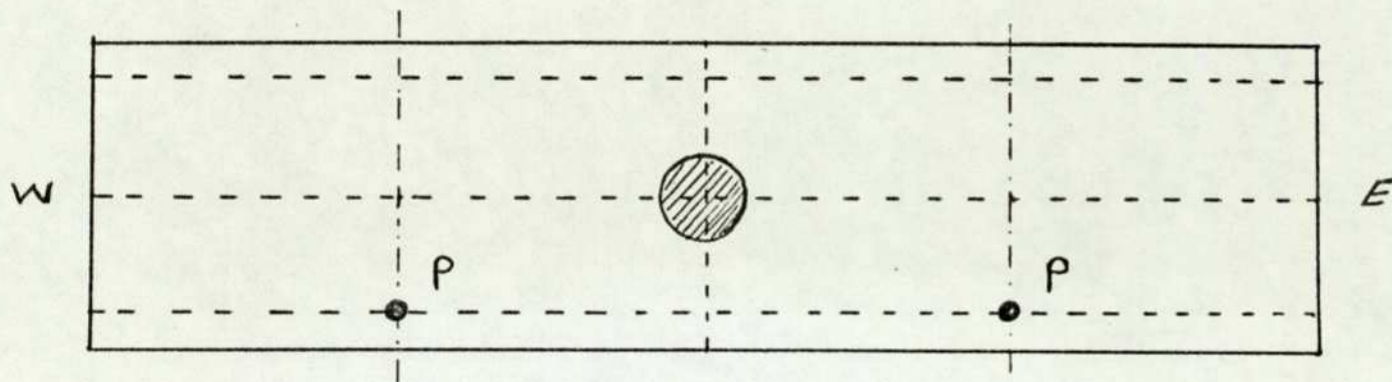


Figure C-25

SECTION A-A

Test 3

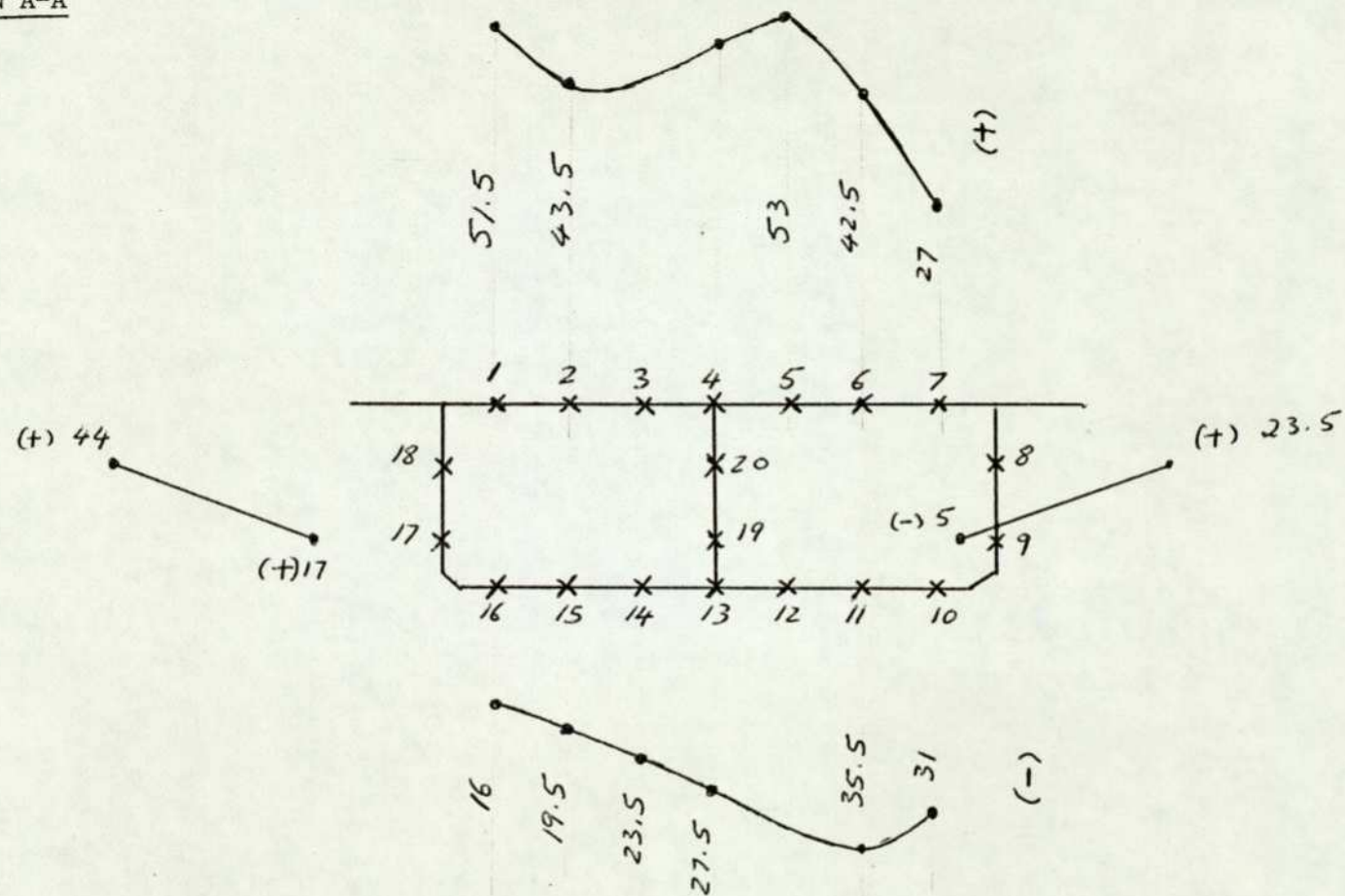


Figure C-26

SECTION B-B

Test 3

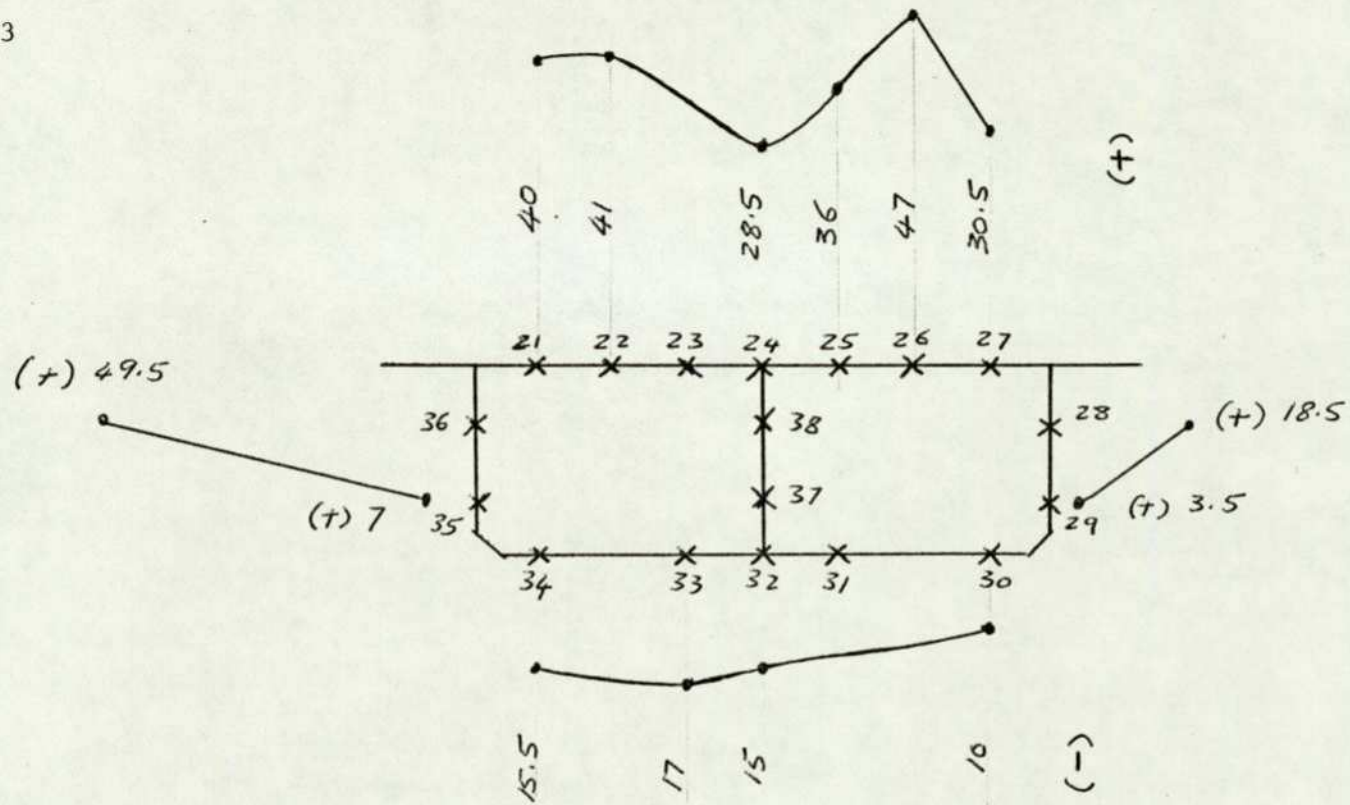


Figure C-27

SECTION C-C

Test 3

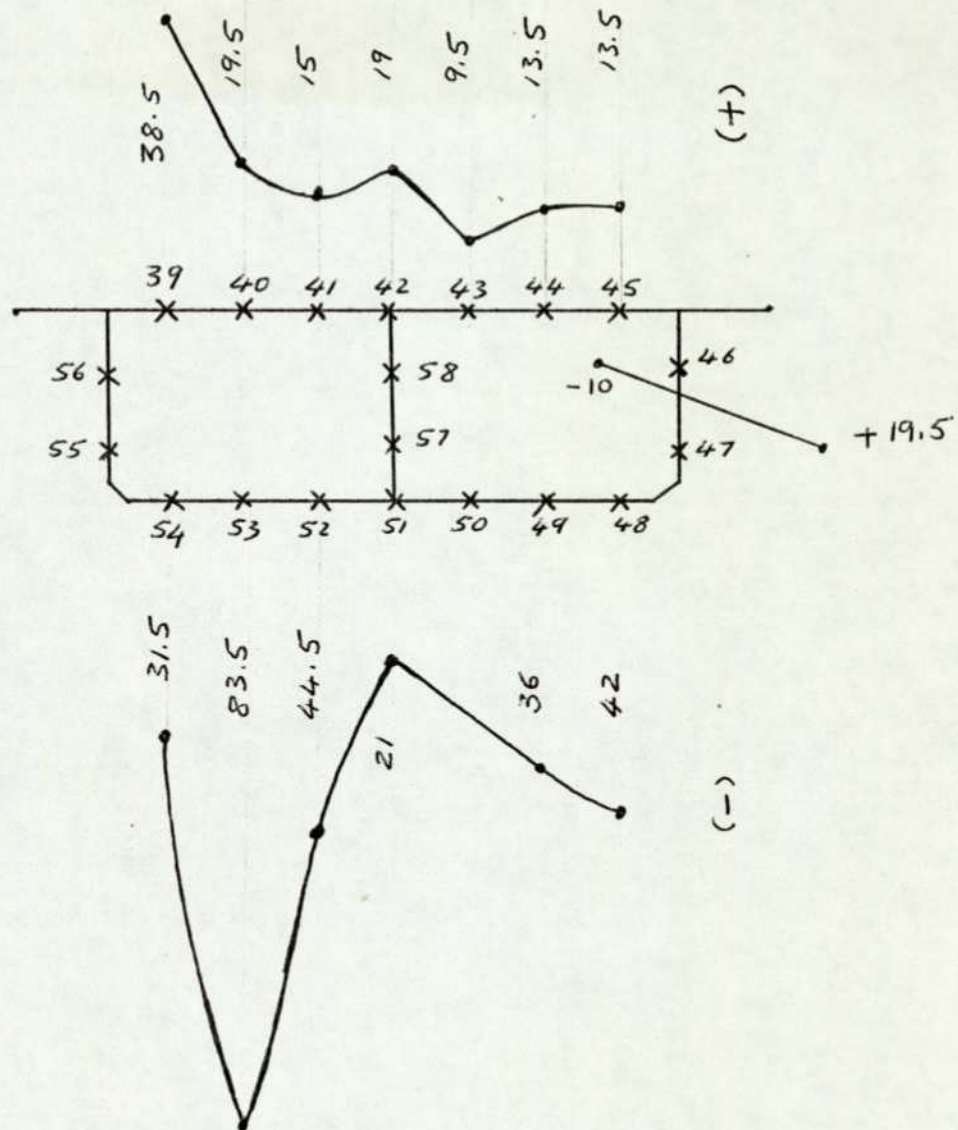


Figure C-28

SECTION A-A

Test 4

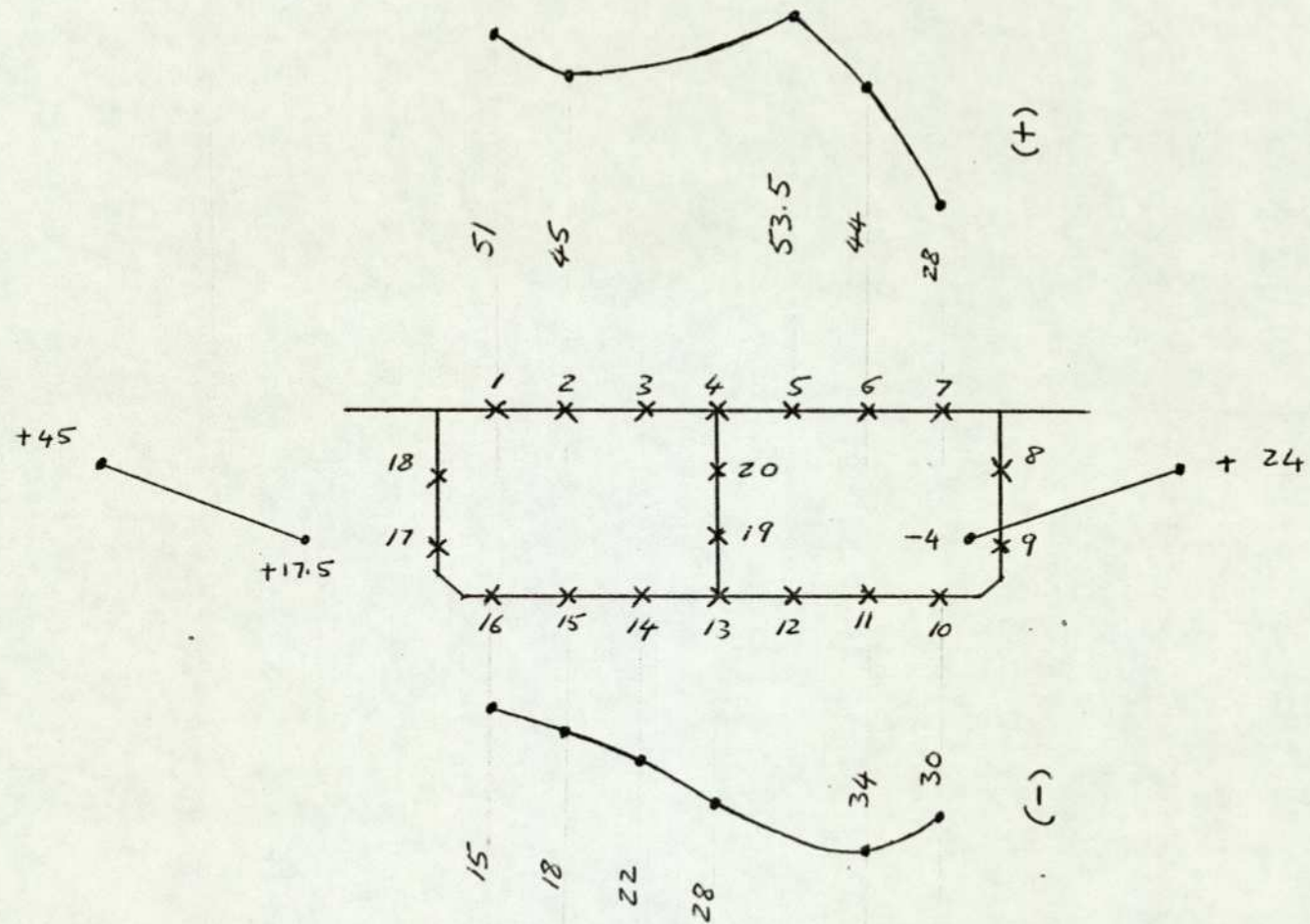


Figure C-29

SECTION B-B

Test 4

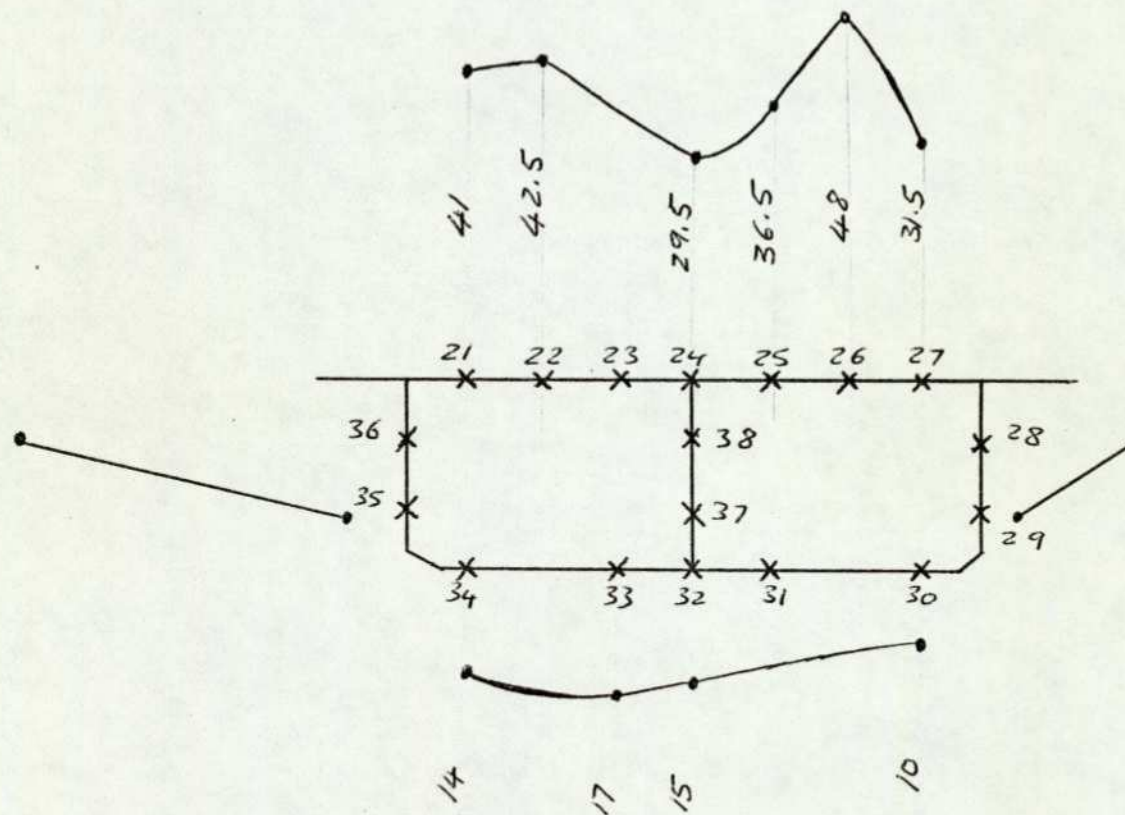


Figure C-30

SECTION C-C

Test 4

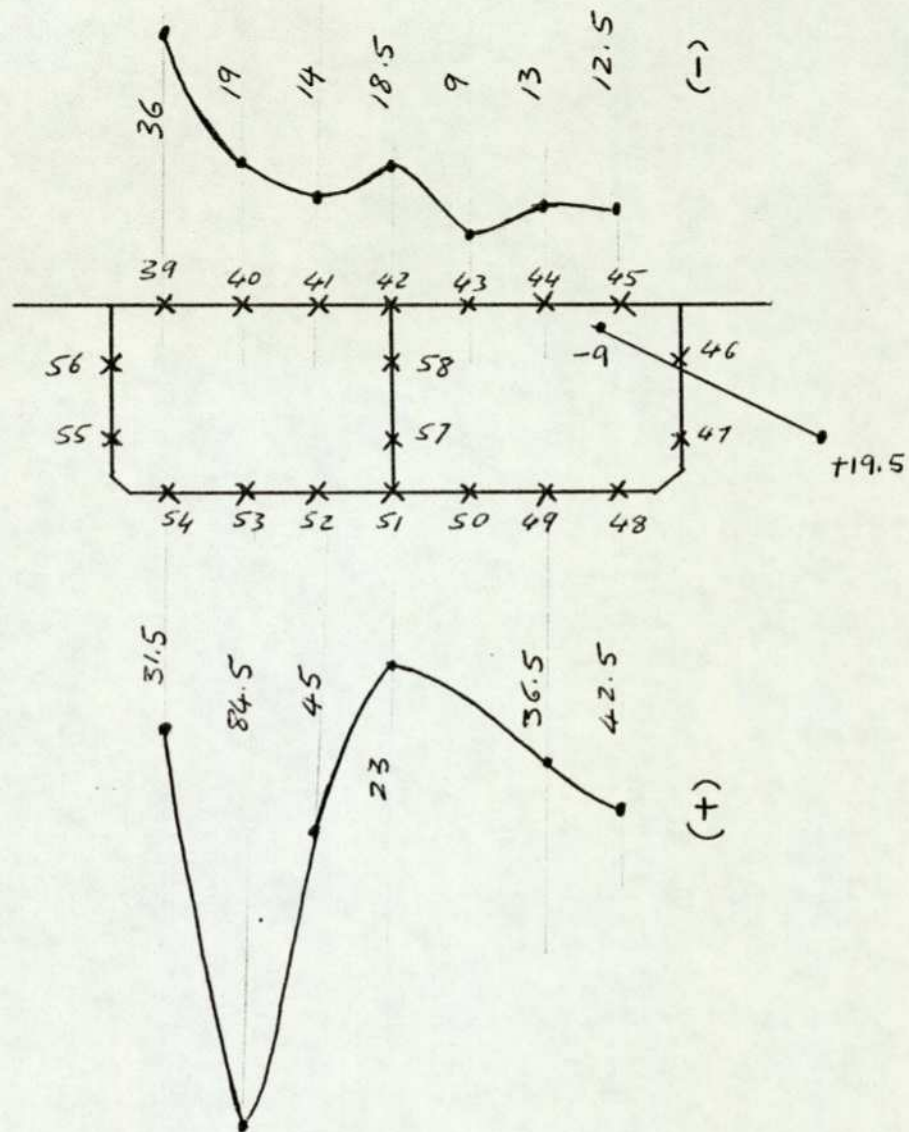


Figure C-31

Location of Point Load for Test 5 and Test 6

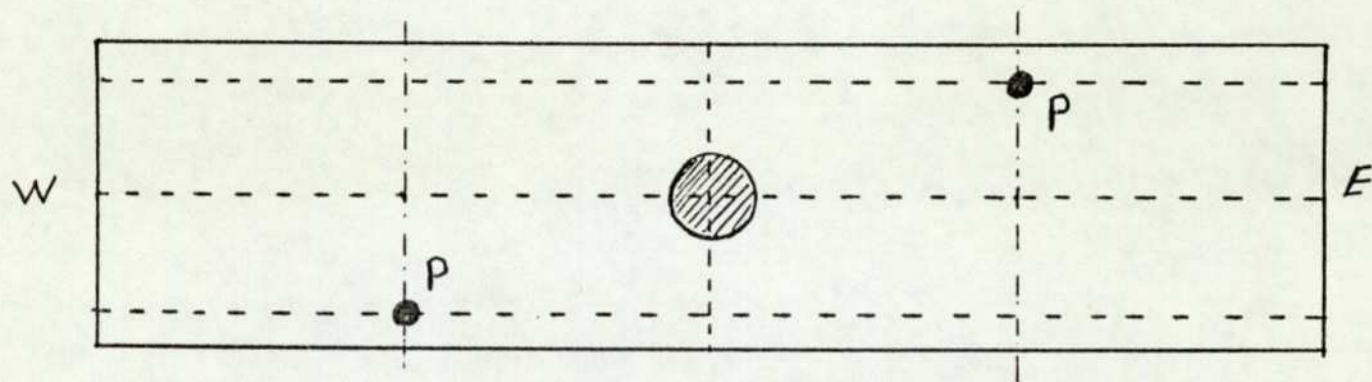


Figure C-32

SECTION A-A

Test 5

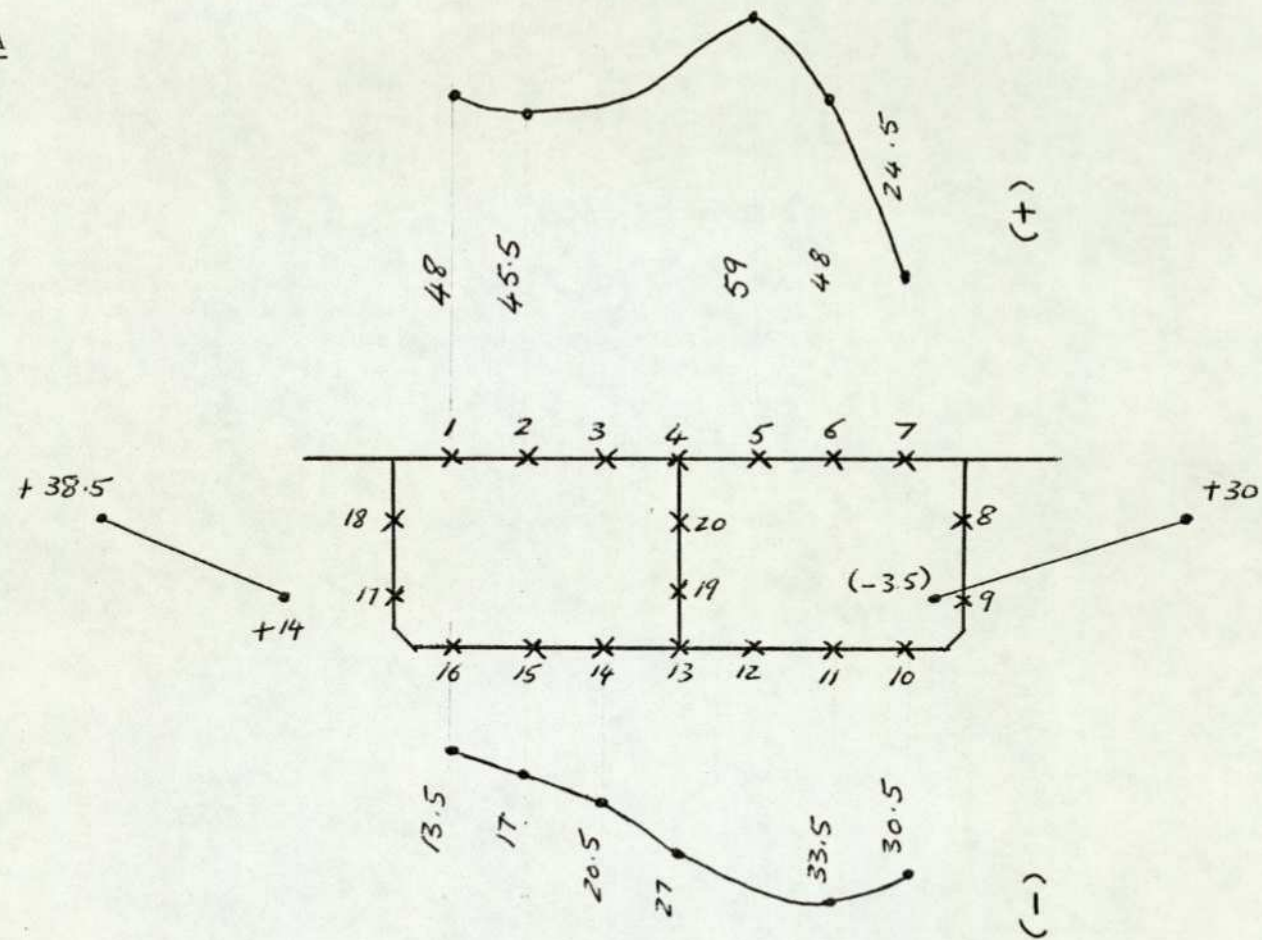


Figure C-33

SECTION B-B

Test 5

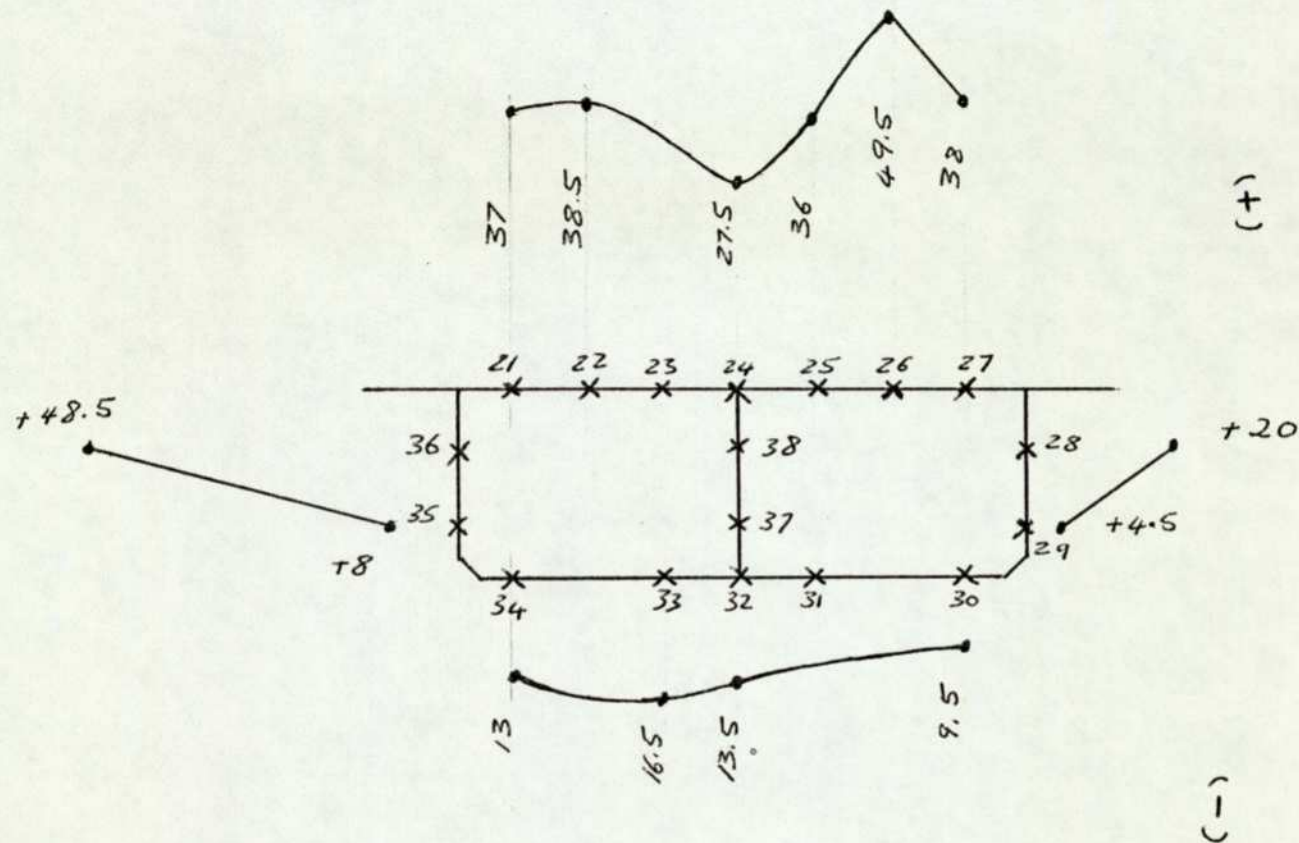


Figure C-34

SECTION C-C

Test 5

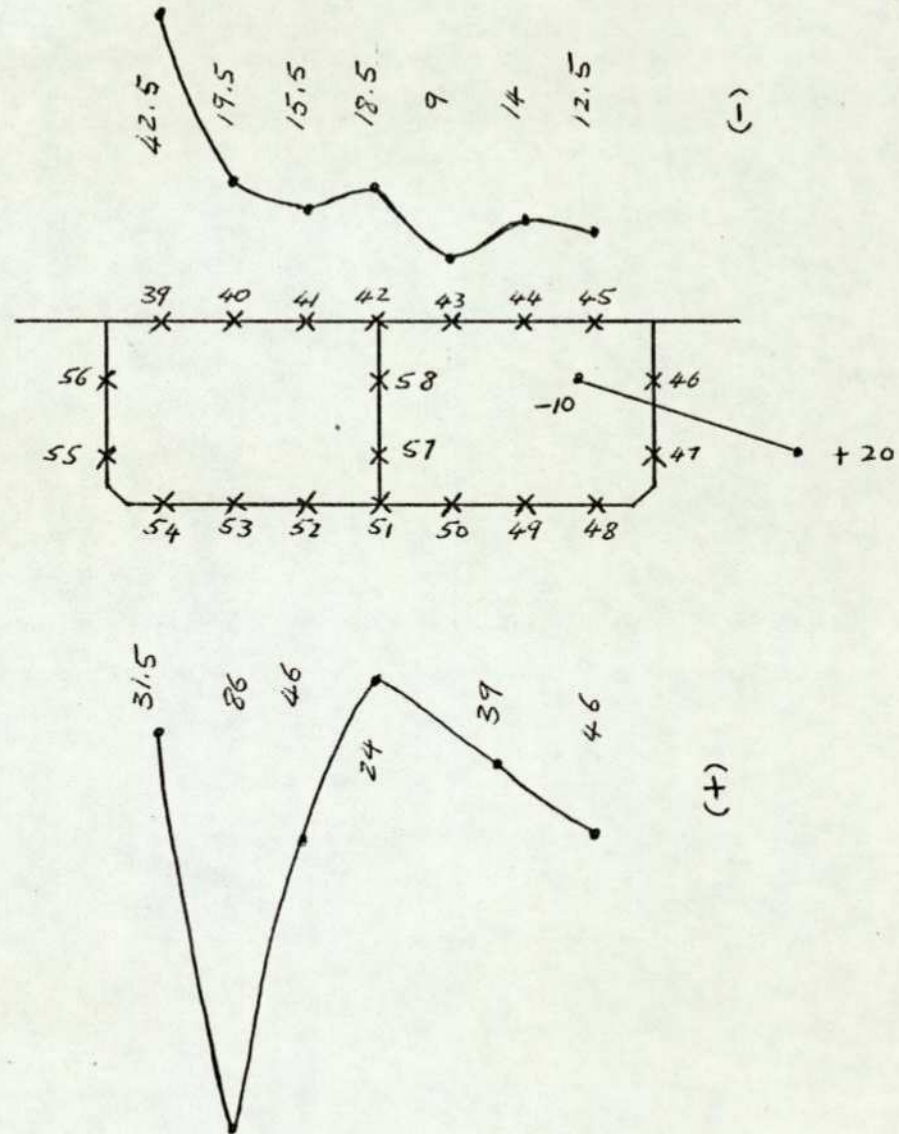


Figure C-35

SECTION A-A

Test 6

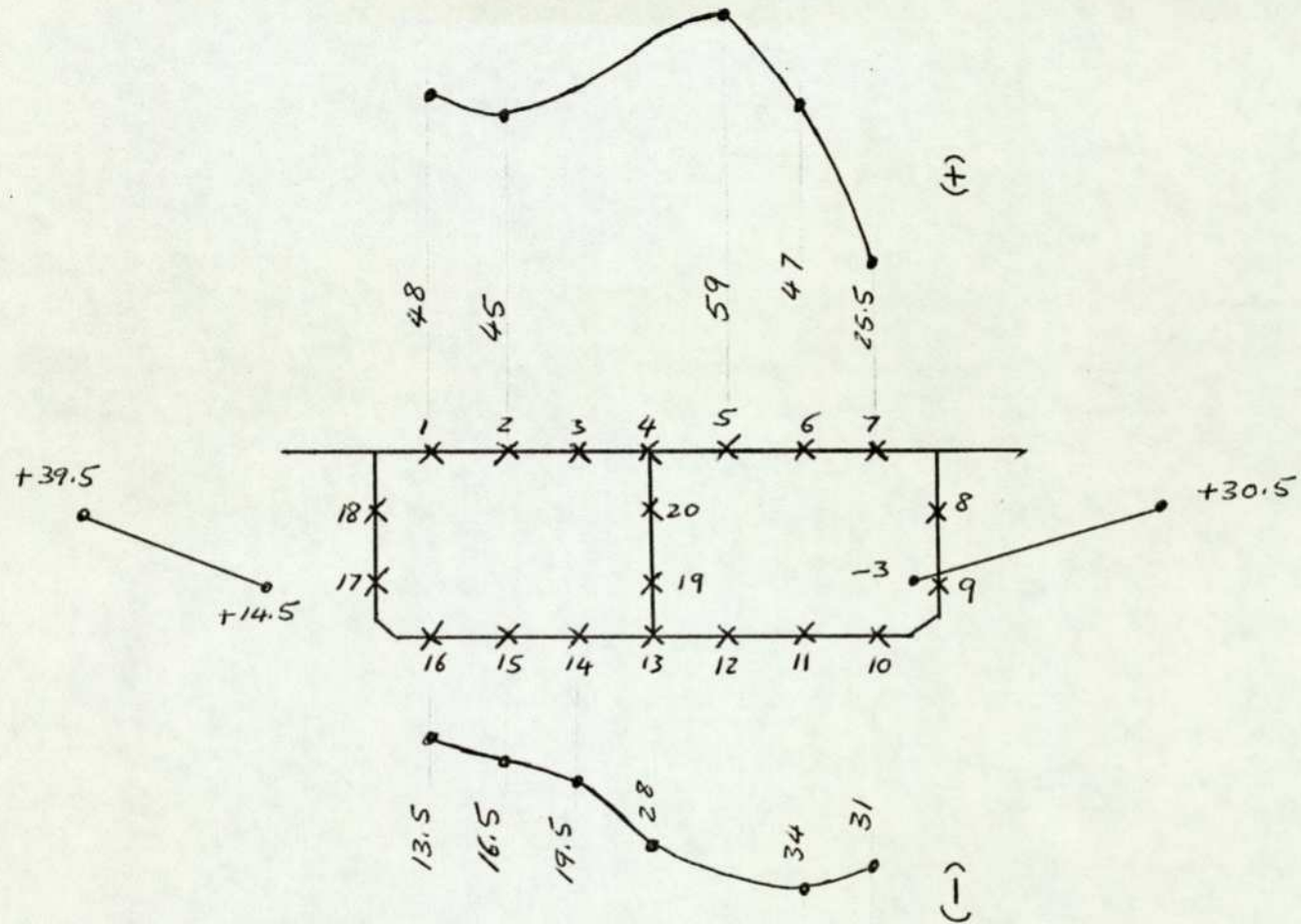


Figure C-36

SECTION B-B

Test 6

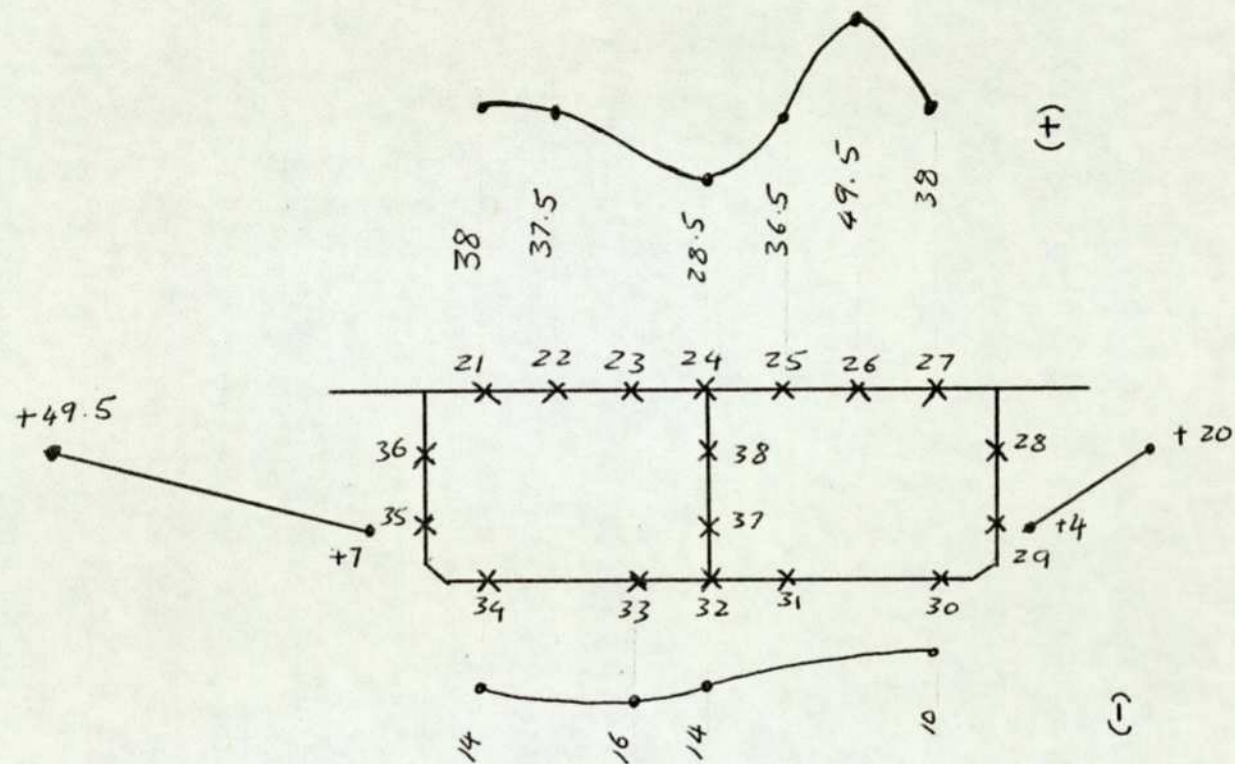


Figure C-37

SECTION C-C

Test 6

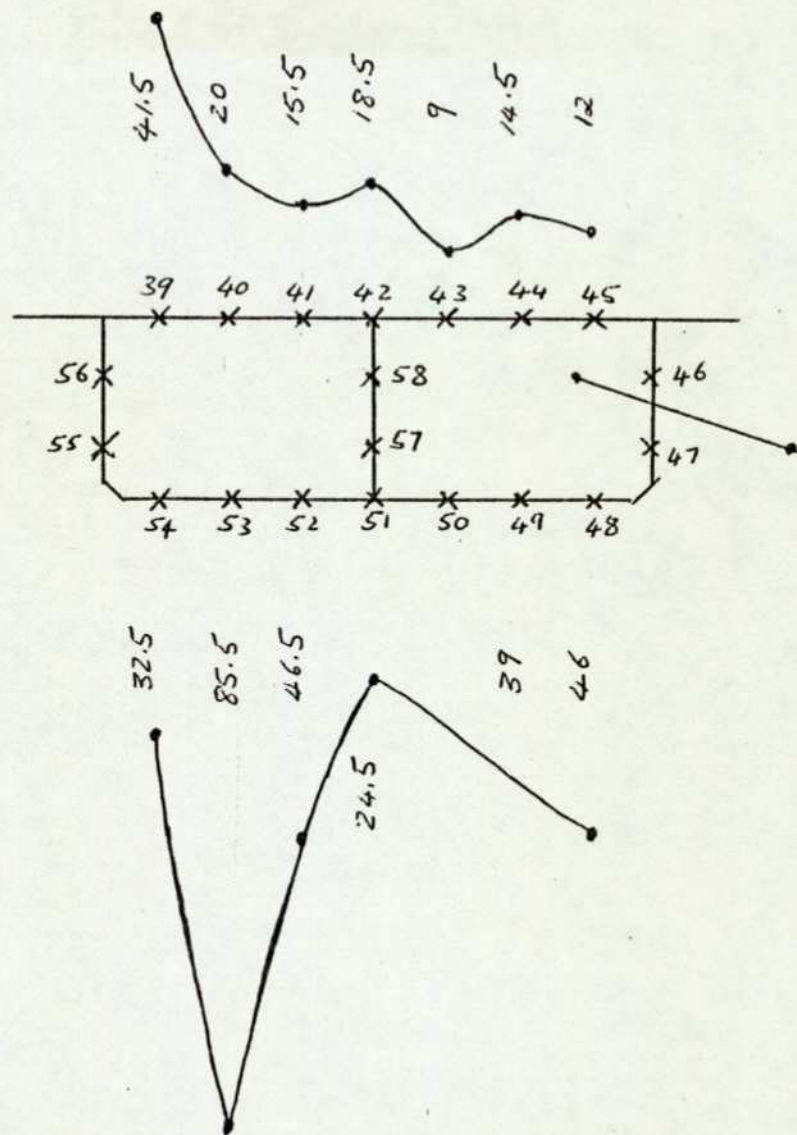


Figure C-38

Stress Results for line load at mid section of the two spans respectively

At 1000 lb on each span									
G.No.	δ	G.No.	δ	G.No.	δ	G.No.	δ	G.No.	δ
1	-94	2	-110	3	0	4	0	5	-112
6	-78	7	-69	8	-81	9	-6	10	62
11	67	12	-234	13	48	14	37	15	28
16	20	17	-53	18	-87	19	-30	20	-82
21	-87	22	-80	23	-721	24	-72	25	-67
26	-90	27	-90	28	-64	29	-39	30	15
31	20	32	24	33	32	34	21	35	-32
36	-114	37	-37	38	-70	39	62	40	150
41	95	42	88	43	34	44	45	45	18
46	16	47	-27	48	-24	49	-47	50	81
51	-83	52	-80	53	-84	54	-41	55	0
56	24	57	-24	58	61	59	38	60	23
61	-19	62	5	63	5	64	-3	65	6
66	20								

Table C-12

Stress Results for Line Load at Mid section of the two spans respectively

At 2000 lb on each span									
G No.	δ	G.No.	δ	G.No.	δ	G.No.	δ	G.No.	δ
1	250	2	-278	3	0	4	0	5	-256
6	-204	7	-180	8	-207	9	-52	10	99
11	112	12	162	13	86	14	79	15	56
16	39	17	-147	18	-207	19	-114	20	-221
21	-215	22	-213	23	-465	24	-189	25	-183
26	-230	27	-225	28	-169	29	-120	30	31
31	43	32	54	33	71	34	47	35	-96
36	-301	37	-111	38	-190	39	159	40	289
41	211	42	200	43	94	44	99	45	57
46	26	47	-121	48	-109	49	-53	50	290
51	108	52	240	53	268	54	-54	55	0
56	53	57	-107	58	141	59	71	60	50
61	-41	62	5	63	2	64	1	65	7
66	-54								

Table C-13

Stress Results for Line Load at Mis Section of the two Spans respectively

At 3000 lb for each span									
G.No.	δ	G.No.	δ	G.No.	δ	G.No.	δ	G.No.	δ
1	-396	2	-460	3	0	4	0	5	-380
6	-324	7	-283	8	-318	9	-92	10	119
11	137	12	523	13	112	14	116	15	79
16	53	17	-241	18	-322	19	-215	20	-369
21	-330	22	-335	23	-236	24	-312	25	-300
26	-352	27	-342	28	-261	29	-184	30	48
31	66	32	80	33	106	34	70	35	-163
36	-467	37	-195	38	-314	39	228	40	334
41	296	42	302	43	168	44	164	45	91
46	30	47	-206	48	-208	49	-100	50	456
51	-214	52	-422	53	-380	54	-135	55	0
56	78	57	-246	58	186	59	81	60	74
61	-53	62	2	63	-11	64	3	65	5
66	-80								

Table C-14

Stress Results for Line Load at Mid Section of the two spans respectively

At 4000 lb on each span									
G.No.	δ	G.No.	δ	G.No.	δ	G.No.	δ	G.No.	δ
1	-536	2	-655	3	0	4	0	5	-509
6	-461	7	-393	8	-455	9	-148	10	127
11	150	12	570	13	126	14	150	15	100
16	68	17	-347	18	-457	19	-319	20	-323
21	-475	22	-475	23	-209	24	-472	25	-441
26	-497	27	-483	28	-370	29	-269	30	58
31	82	32	104	33	141	34	96	35	-210
36	-598	37	-300	38	-461	39	299	40	466
41	429	42	388	43	284	44	326	45	101
46	34	47	-352	48	-387	49	-160	50	598
51	-307	52	-633	53	-576	54	-287	55	0
56	106	57	-375	58	317	59	58	60	96
61	-59	62	-3	63	-31	64	-95	65	-30
66	-79								

Table C-15

Stress Results for Line Load at Mid Section of the Two Spans Respectively

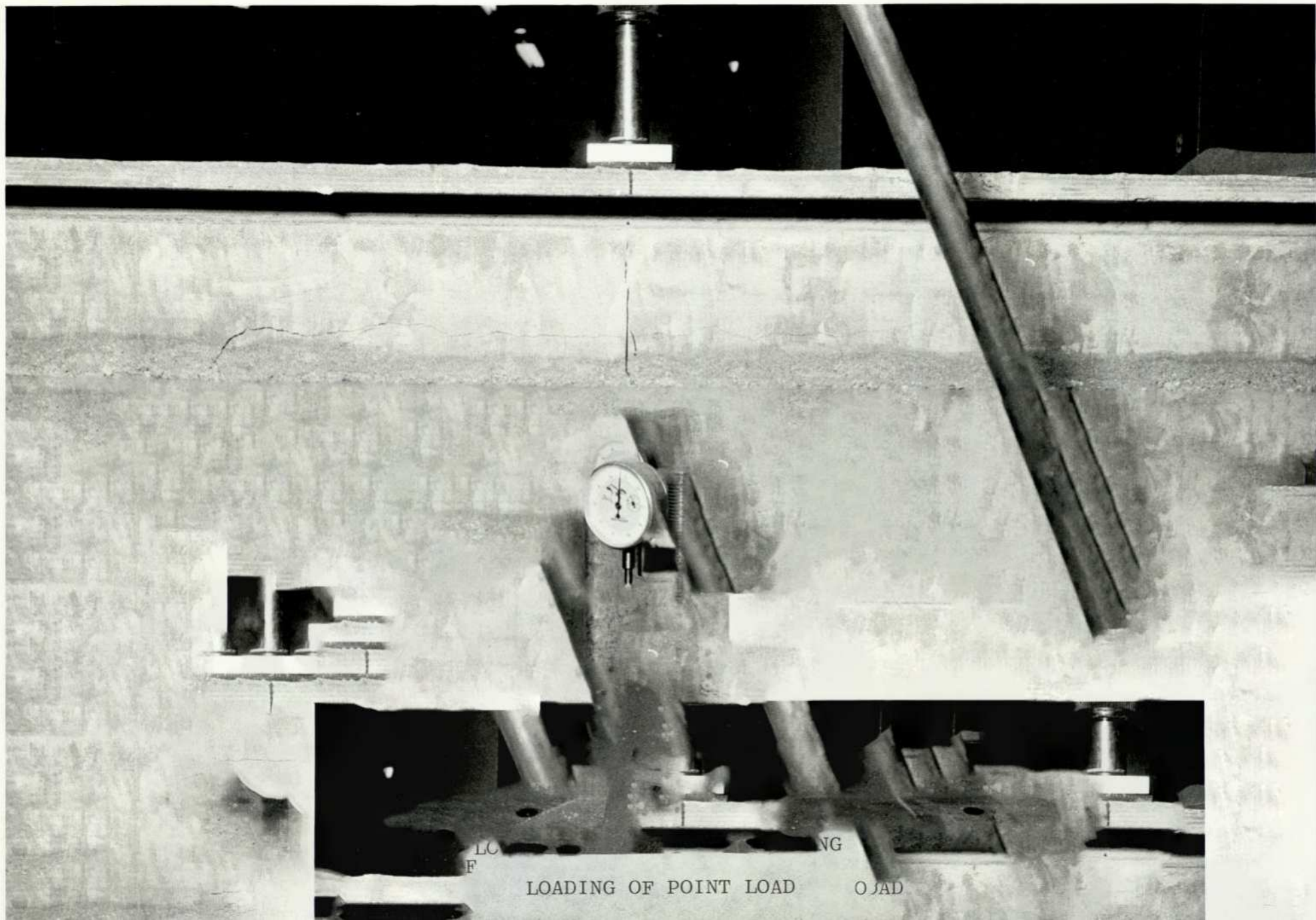
At 4500 lb on each span									
G.No.	δ	G.No.	δ	G.No.	δ	G.No.	δ	G.No.	δ
1	-576	2	-734	3	0	4	0	5	-556
6	-514	7	-430	8	-507	9	-279	10	98
11	138	12	570	13	130	14	169	15	114
16	85	17	-377	18	-500	19	-376	20	-586
21	-489	22	-498	23	-35	24	-470	25	-451
26	-508	27	-539	28	-406	29	-305	30	45
31	75	32	106	33	155	34	118	35	-236
36	-682	37	-329	38	-488	39	668	40	1150
41	808	42	363	43	1147	44	924	45	185
46	-17	47	-829	48	-883	49	-406	50	-94
51	-852	52	-914	53	-671	54	-812	55	0
56	121	57	-238	58	2523	59	47	60	92
61	-67	62	-6	63	-38	64	-90	65	4
66	-130								

Table C-16

Deflection Readings for Line Load at Mid Section of the two spans respectively
Collapse Test

Gauge No.	0	1000	2000	3000	4000	5000
Load						
1	0	95	215	473	645	705
2	0	105	277	495	859	982
3	0	115	260	445	655	932
4	0	9	11	20	30	49
5	0	10	29	37	55	49
6	0	105	310	510	795	1335
7	0	111	335	549	895	978
8	0	95	205	395	642	1405
9	0	6	15	26	39	53
10	0	9	22	24	31	9

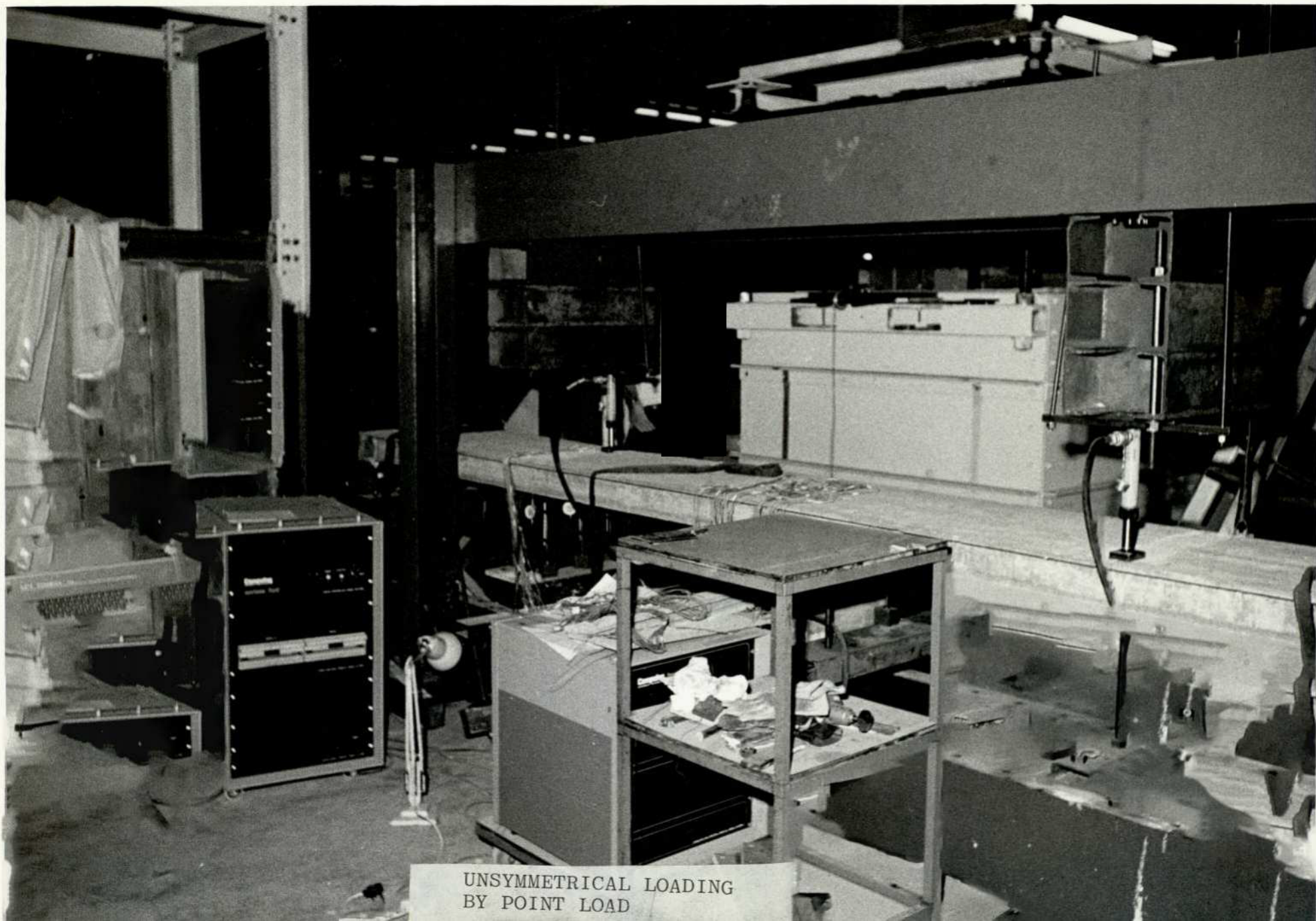
Table C-17



LOADING OF POINT LOAD

O JAD

23



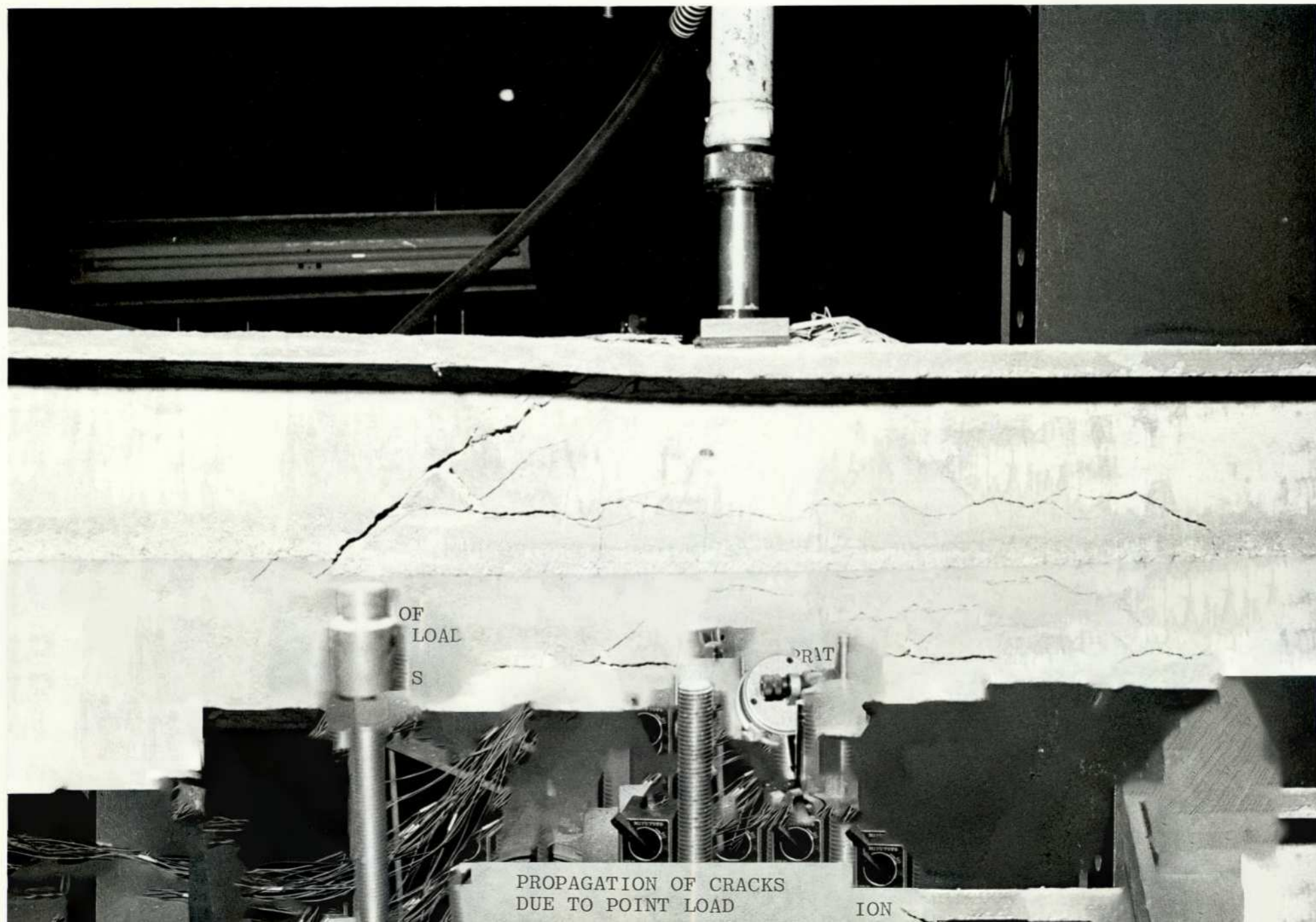
UNSYMMETRICAL LOADING
BY POINT LOAD

24



LOCAL CRACKING

25



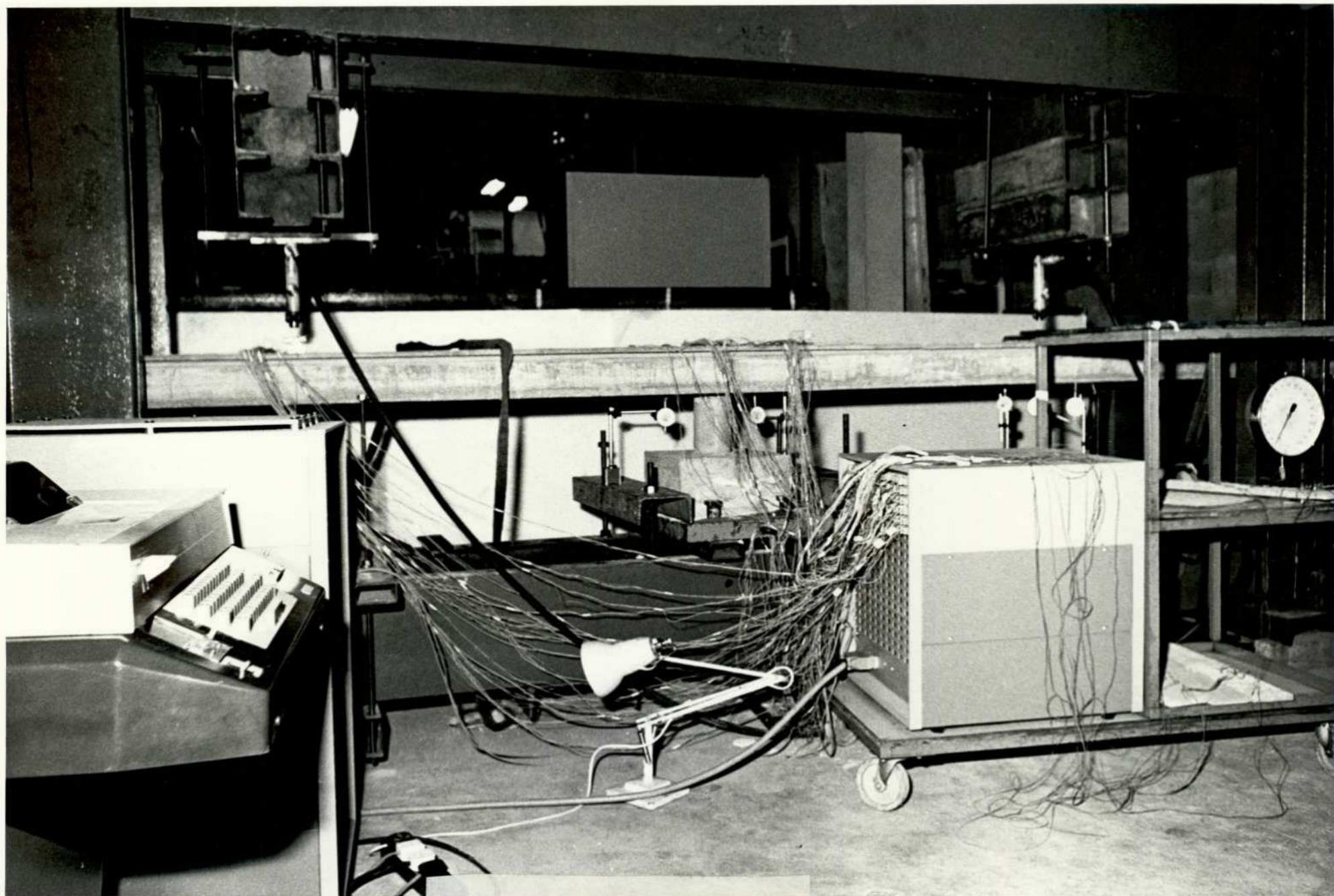
OF
LOAD
S

PRAT

PROPAGATION OF CRACKS
DUE TO POINT LOAD

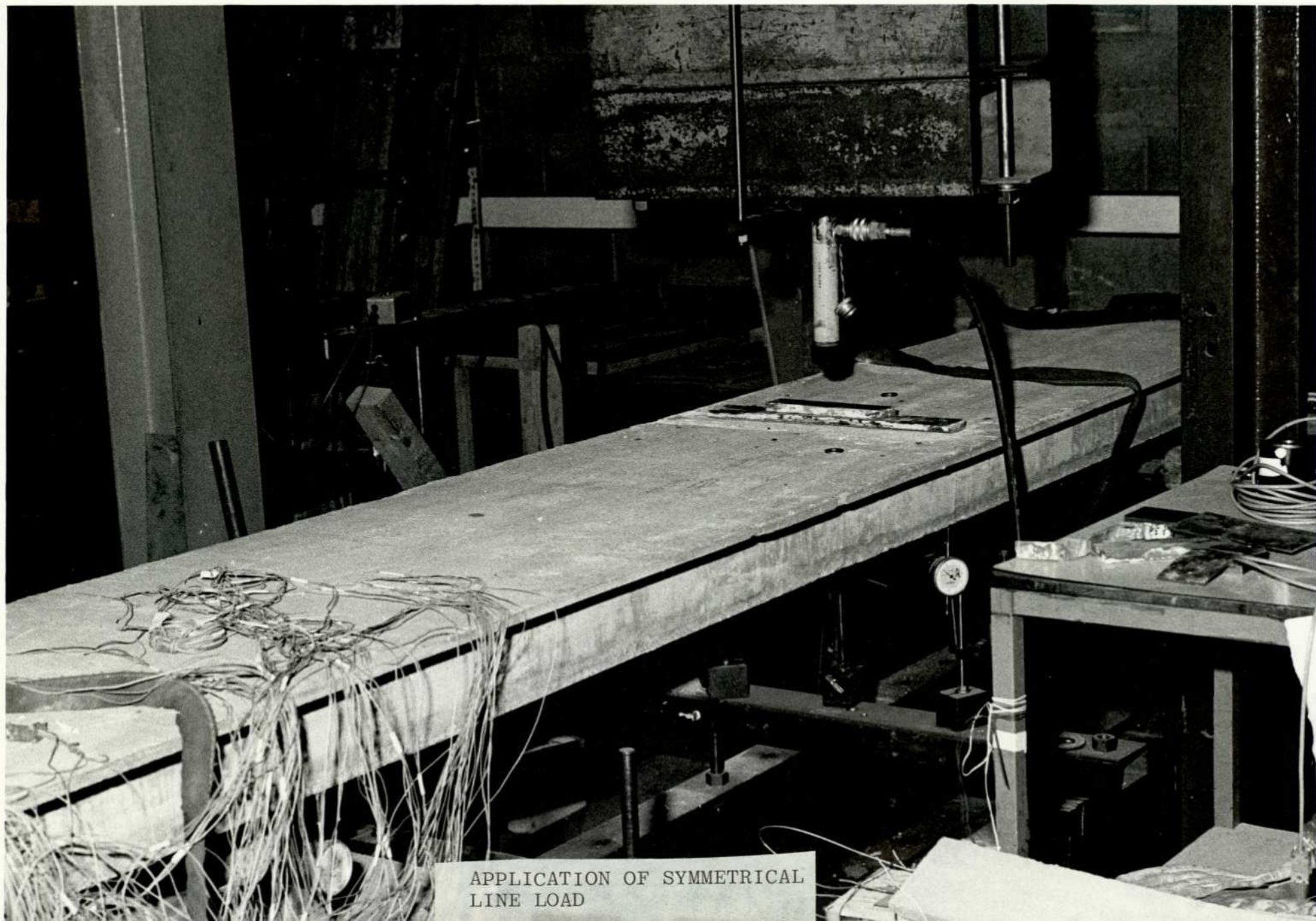
ION

26



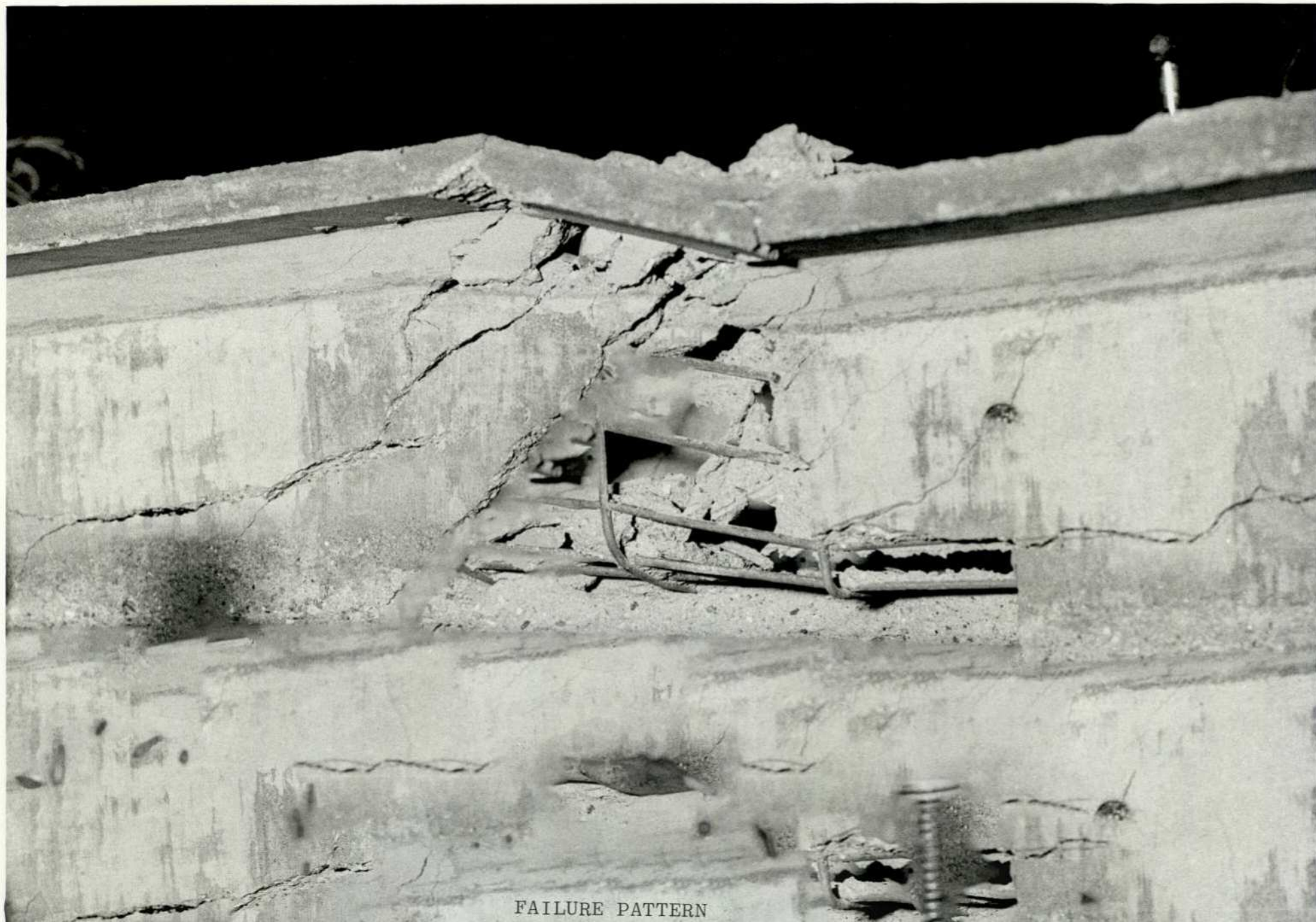
APPLICATION OF UNSYMMETRICAL
LINE LOAD

27



APPLICATION OF SYMMETRICAL
LINE LOAD

28



FAILURE PATTERN

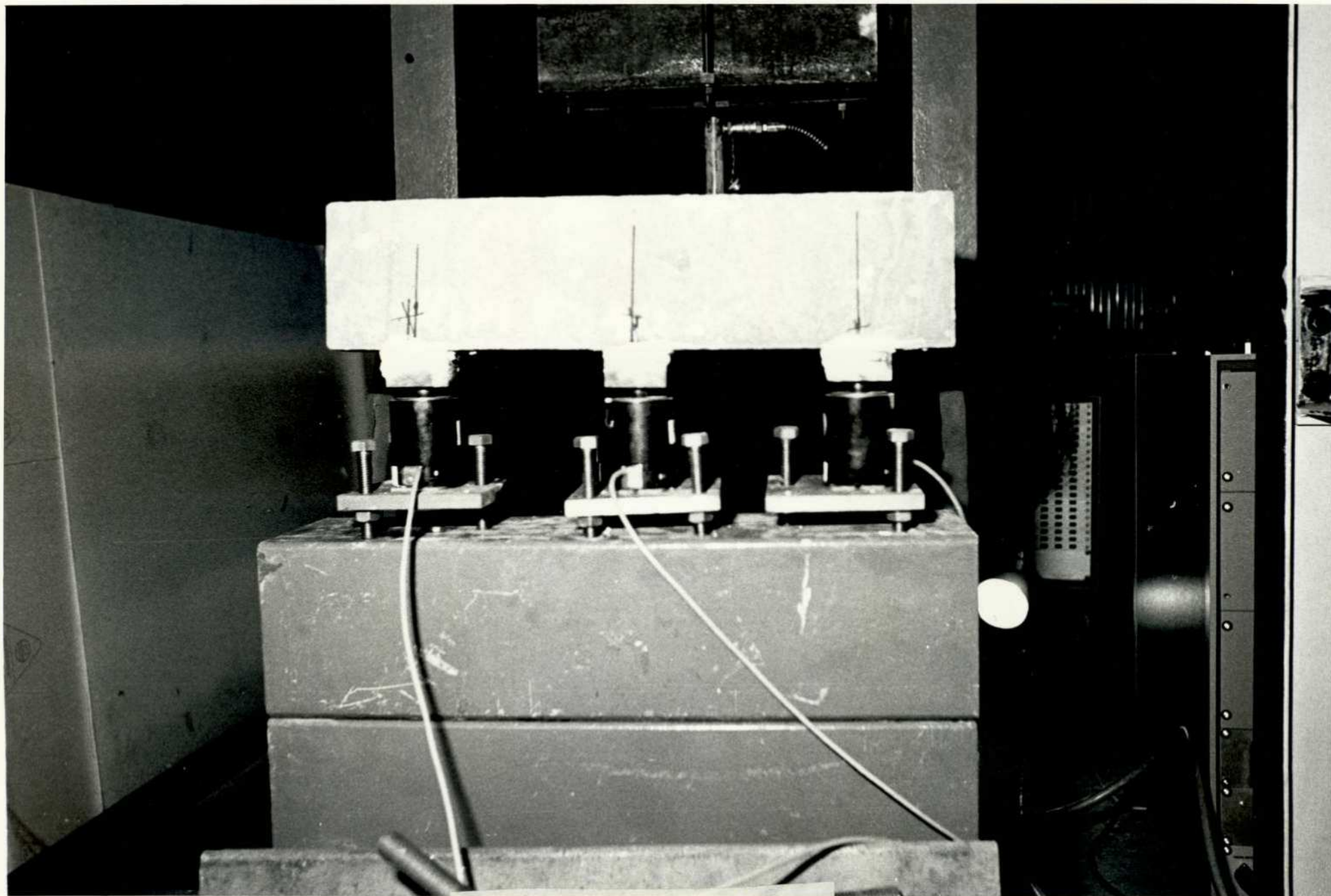
2⁹



CRACK PROPAGATION



PLAN VIEW AFTER FAILURE



LOCATION OF LOAD CELLS

CHAPTER 5

5.1 Introduction

The elastic analysis previously described for single cell box bridges have been extended to the analysis of multi-box bridges and are fully described in the literature. If elastic plate theory is used following Goldber and Love then equilibrium and compatibility of displacement of plate members meeting at the line joints provide a sufficient number of equations for an analytical solution. Similarly if a full shell analysis using a degenerate technique is applied then again equilibrium and displacement continuity at the line joints provides a sufficient number of equations. In this latter method the complementary function for each plate contains eight unknown arbitrary constants and thus in the case of the double cell box beam with side cantilevers shown in Fig. (1), since there are 9 plates

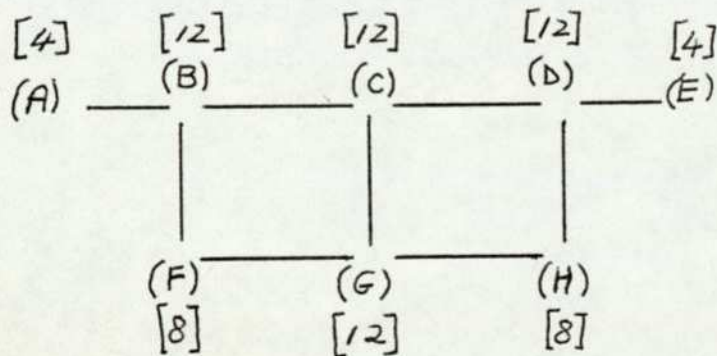


Fig. 1

there are 9×8 i.e. 72 arbitrary constants requiring 72 equations to complete the analysis. At each free edge (A) and (E) four equilibrium equations may be determined

indicated by (4) whilst at the joints at which three plates meet, namely (B), (C), (D) and (G) there are two sets of four displacement conditions and one set of four equilibrium conditions providing at such joints 12 equations in all, indicated by (12) as shown.

At the two remaining joints (F) and (H) at which only two plates meet, there are at each such joints four equations of displacement compatibility and four of equilibrium, providing in all 8 equations indicated by (8) in Fig. (1). Thus, in all, the total number of equations is $2 \times 4 + 4 \times 12 + 2 \times 8$, i.e. 72 equations, thus allowing the arbitrary constants to be uniquely determined and hence the required deflections and stress resultants. Solutions of this nature may of course only be achieved by computer programs.

If, on the other hand, a finite element method of attack is used and if, say, 8×4 rectangular elements are used per plate since at each node there are 6 D.O.F. i.e. approximately $8 \times 4 \times 6$ i.e. 192 D.O.F. are required per plate, demanding for nine plates something of the order of 1800 D.O.F. or equations to be solved. Obviously this requires a very much longer computer time run and a much greater expense is involved.

The finite strip method would of course considerably reduce the number of equations, but still the method is complicated and requires a computer program.

The above methods are of course for elastic analyses, namely for uncracked sections and in recent years more interest has been shown in the development of non computerised and less complicated theories which do, however, attempt to involve the actual structural behaviour of reinforced concrete boxes as observed experimentally. Such theories allow the practising engineer relatively quickly to obtain approximate results which help at least in a preliminary proportioning of the section. Methods for the prediction of collapse mechanism, as well as collapse load, will be shown for double cell box girders. Moreover, the method proposed therein for the lower bound limit theory can be used with a certain factor of safety for a quick elastic design of the box girder.

On the other hand, the proposed method of determining the upper bound theory again with an approximate factor of safety could be adopted for a plastic design of the problem.

The two methods of analysis are based on elastic theory which according to Calladine may be stated as follows.

For the upper bound theorem: "If an estimate of the plastic collapse load of a body is made by equating internal rate of dissipation of energy to the rate at which external forces do work in any postulated mechanism

of deformation of the body, the estimate will be either high, or correct."

On the other hand, for the lower bound theorem statement: "If any stress distribution throughout the structure can be found which is everywhere in equilibrium internally and balances certain external loads, and at the same time does not violate the yield condition, those loads will be carried safely by the structure".

The application of those two theorems to the experimental investigation of twin cell boxes will be given later.

Reference has been made() for the applications of these theorems to single cell simply supported box girders, and a different proposed mechanism of failure for eccentrically loaded single cell boxes has been shown.

A full application of these theorems to double cell box girders has been developed together with calculations of the stresses occurring in each web due to the application of an eccentric load on one of the cells only.

5.2 Upper Bound Solution

In this part, the actual collapse mechanism has been obtained for the different tests under different cases of loading. A work equation has been developed for each case. The mechanisms of collapse were based on the experimental observations during the tests.

5.2.1 Symmetrically Loaded Boxes

For a symmetrically loaded single cell box under the case of two point loads acting at centre line of the two webs at mid-span, and for a twin cell box, symmetrically loaded by a uniform distributed load covering its two cells, the mechanisms of collapse were formed by a full division of the boxes at mid-section into two equal parts for both cases respectively. See Fig.(2)

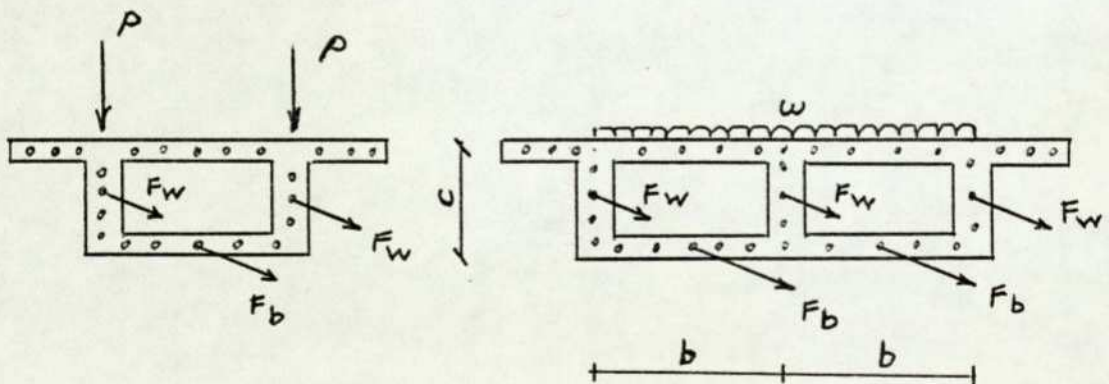


Fig. 2

By considering one or two mechanisms shown in Fig. (3A) and Fig. (3B) for a single cell box and a twin cell box respectively, each beam divides into two parts at mid-span forming a hinge at that section.

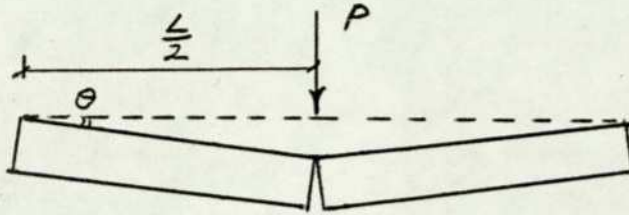


Fig.(3A)

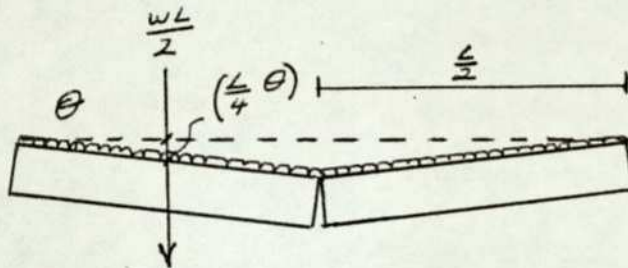


Fig.(3B)

Fig. 3

5.2.1A Single Cell Box

The two halves rotate as rigid bodies, rotating by an angle θ about the support for each case.

The rate of work done for the external load P in the case of a single cell would be:

$$W_{es} = \frac{PL}{2} \theta \quad 1(A)$$

where W_{es} = rate of work done by external load P in the case of the single cell box

The internal rate of dissipation of work can be considered by the hinge condition. If it is assumed (Spence) that the concrete has an infinite compressive strength, then the two halves of the beam will rotate and open about the top surface of the top flange as shown in Fig.(4)

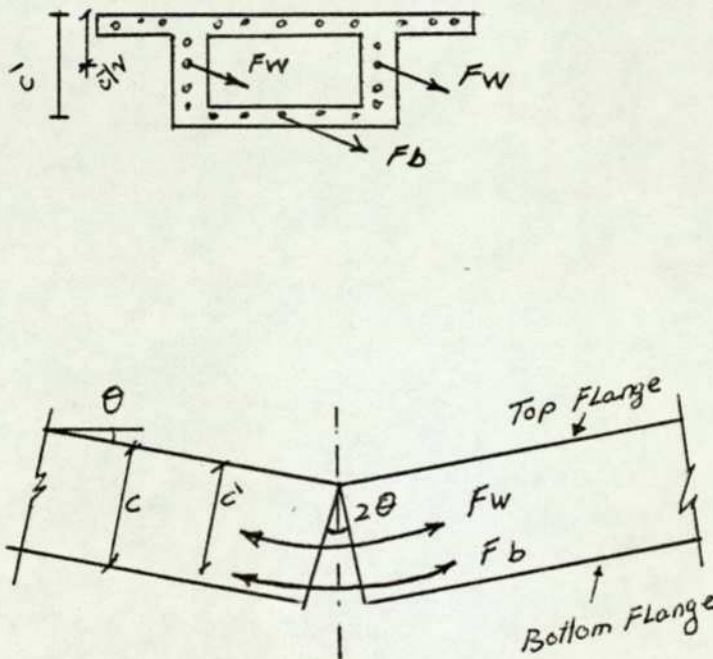


Fig. 4

The rate of work done by the steel bars would be:

$$W_{is} = (2F_w \cdot c' / 2 + F_b \cdot c') \cdot 2\theta = 2(F_w c' + F_b c') \theta \quad 2(A)$$

where: F_w = force in steel in one web at yield
(uniformly distributed on a vertical section)

F_b = force in bottom steel at yield

c' = depth to bottom steel

It is assumed that F_w acts at $c'/2$ where W is the rate of internal work for the case of the single cell.

By equating equations 1(A) and 2(A)

$$\frac{PL}{2} \cdot \theta = 2(F_w c' + F_b c') \theta \quad 3(A)$$

$$\text{whence } P = \frac{4}{L} (F_w + F_b) c \quad 4(A)$$

where P is the load on the single cell box.

But as crushing of concrete was found on the top flange of some boxes at collapse, it was found more conservative to assume that the rotation of the two halves in each case would be around the top steel reinforcement imbedded in the top flange as shown in Fig. (5) i.e. c' is now replaced by c'' the distance between top and bottom flange steel.

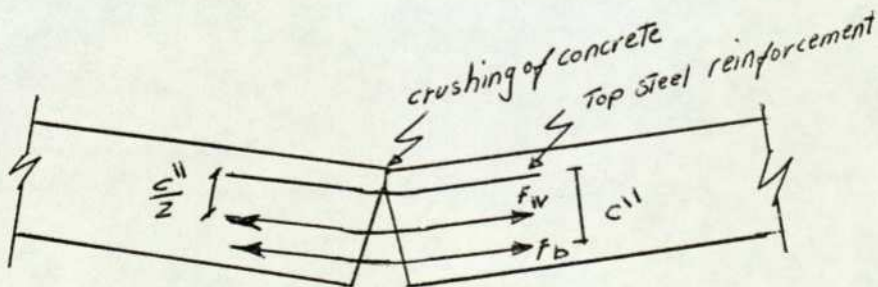


Fig. 5

Equation 4(A) would be adjusted to

$$P = \frac{4}{L} (F_w + F_b)c''$$

where P is the final collapse load for the case of the single cell.

5.2.1B Double Cell Box

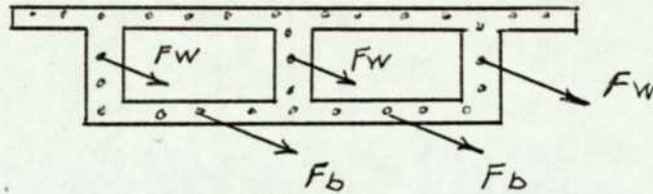


Fig. 6

In the case of the double cell box shown in Fig.(6) again a central line hinge will form as shown in Fig.(3) and the rate of work done by the U.D.L. will be

$$W_{ed} = 2\left(\frac{wL}{2} \cdot \frac{L}{4} \cdot \theta\right) = \frac{wL^2\theta}{4} \quad 1(B)$$

where W_{ed} = rate of work done by external load

w = the U.D.L. across the complete width $2b$.

The rate of work done by steel

$$W_{id} = (3F_w \cdot c' / 2 + F_b \cdot c') 2\theta = (3F_w + 2F_b) c' \theta \quad 2(B)$$

where F_w is the force in the steel in one web at yield

F_b is the force in the bottom steel at yield

c' = depth of bottom steel

and again it is assumed that F_w acts at $c'/2$.

Equating 1(B) and 2(B)

$$w \frac{L^2}{4} \cdot \theta = (3F_w + 2F_b) c' \theta$$

$$w = \frac{4}{L^2} (3F_w + 2F_b) c' \quad 3(B)$$

or if w is the total load on bridge, i.e. $W = wL$

$$W = \frac{4}{L} (3F_w + 2F_b) c' \quad 4(B)$$

Again if c'' is the distance between the top and bottom flange steel

$$W = \frac{4}{L} (3F_w + 2F_b) c'' \quad 5(B)$$

5.2.2 Unsymmetrically Loaded Boxes

For the unsymmetrical case of loading, a collapse mechanism will be considered for each type of box. According to the tests applied to the boxes under the different cases of unsymmetric loading, the boxes failed by being divided into two parts, with a complete yielding of the bottom steel at that failure section. However, there was a difference in the recorded deflection between the loaded and unloaded side.

If we consider the two diagrams in Fig. (7).

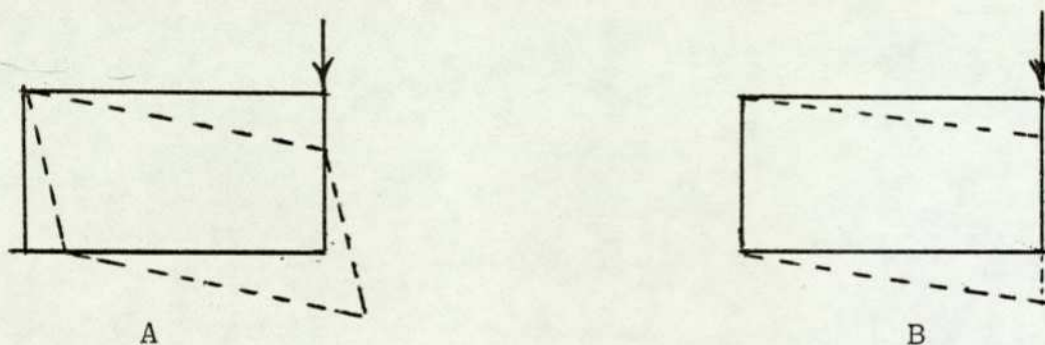


Fig. 7

The results obtained from the two mechanisms (A) and (B) would give two different values of the collapse load. Case A obviously overestimates the upper limit, by exaggerating the work done during deformations. This is mainly due to the assumed lateral movements. However, the measurements taken during the actual test did not reveal appreciable side deformation. It follows that the second mechanism is a better prediction of the

actual behaviour of the box girder under the eccentric load. The same criteria were observed for double cell boxes as no horizontal movements were observed.

By considering the mechanism shown in Fig.(8A)

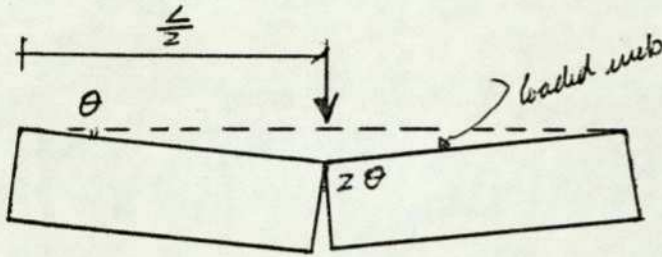


Fig. (8A)

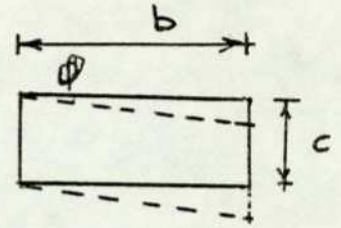


Fig. (8B)

Fig. 8

Fig.(B) shows the cross-section of the box at mid-span. Hinges are formed at the four corners of the cross-section with an angle ϕ formed at each corner. A plastic hinge is going to be formed at the top flange at mid-span, and loaded web will rotate with an angle θ as shown in Fig.(8A). No lateral movements of top or bottom flanges occur.

If θ is the angle of rotation of the two halves about the end diagrams and ϕ is the out of plane angle of the rotation of the flanges at mid-span, it can be shown:

$$\frac{L\theta}{2} = b\phi, \quad \theta = \frac{2b}{L} \phi.$$

The hinges rotate at an angle ϕ at each corner at mid-span and reduce linearly to zero at support.

The rate of work done by the external load P is shown in Fig. (9)

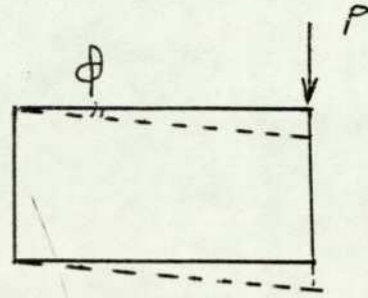


Fig. 9

$$w_e = \frac{PL}{2} \theta = Pb\phi - 1(C)$$

The internal rate of dissipation of work is taken by the mid-span hinge, the corner hinge and the twisting of flanges, web and diaphragms. The latter condition is very small and hence can easily be neglected.

5.2.2.1 Effect of mid-span hinge

The mean relative displacement = $c'\theta$ where c' is the depth to the main reinforcement.

The work done would be:

$$w_1' = (F_w + F_b)c'\theta = (F_w + F_b)c' \frac{2b}{L} \phi \quad 2(C)$$

where F_w and F_b is the force in one web and in the bottom flange at yield respectively.

A refinement is made to equation 2(C) by taking the effective depth to be the distance between the two reinforcements at the top and bottom flanges respectively.

That amendment is carried out due to the appearance of crushing of the concrete which could occur at the top flange of the box.

Hence:

$$w_1 = (F_w + F_b)c'' \frac{2b}{L} \phi \quad 3(C)$$

where c'' is the new depth.

5.2.2.2 Effect of Corner Hinges

The work done by these hinges is:

$$w_2 = 4m_c L \phi \quad 4(C)$$

where m_c is the plastic moment per unit length at each corner, and ϕ is the angle of rotation of the corners.

By equating equation 1(C) with equations 3(C) and 4(C) i.e. ($w_{ex} = w_{1N}$)

$$P \left(1 + \frac{2c''}{L} (F_w + F_b) + \frac{4L}{b} m_c \right) \quad 5(C)$$

where P is the eccentric load applied at centre line of one web at mid section at failure.

By applying more or less the same line of thought an equation could be adopted for the collapse load in the case of a double cell box.

Two cases of eccentric loading are going to be dealt with here in order to verify the experimental tests carried out.

By considering the two cases of loading shown in Fig.(11)

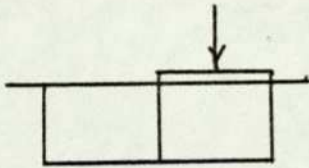
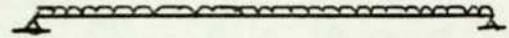
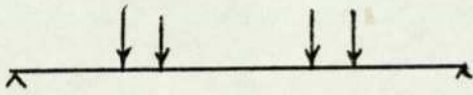


Fig.(10A)

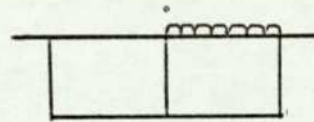
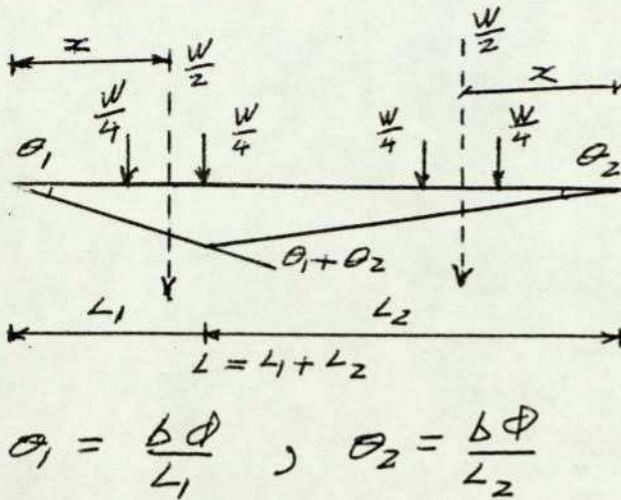


Fig.(10B)

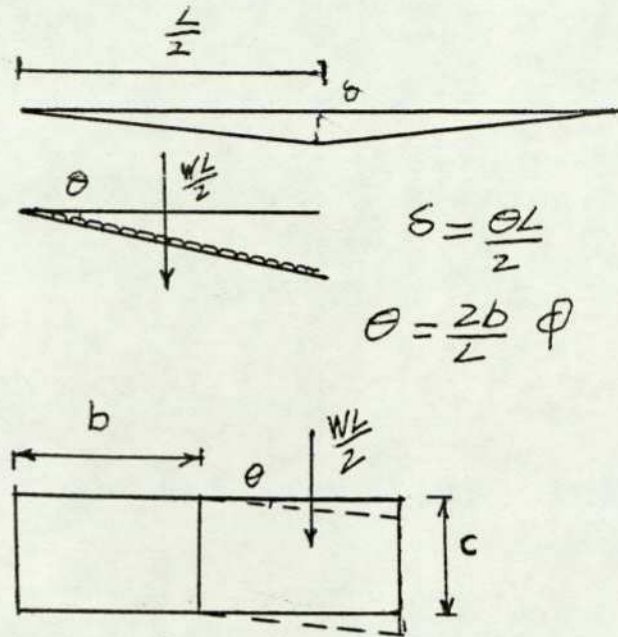
Fig. 10

The case of four line loads on one cell and the case of a uniformly distributed load on one cell only is represented in Fig. 10A and Fig. 10B respectively. The two associated forms of collapse mechanism are shown in Fig. (11)



CASE A

Fig.(11A)



CASE B

Fig.(11B)

Fig. 11

For case A of loading, as shown in Fig.(11A) the rate of work done by the external load would be in the form

$$w_{ex} = \frac{w}{4} \cdot x \cdot b \phi \left(\frac{1}{L_1} + \frac{1}{L_2} \right) \quad 6(C)$$

The rate of work dissipated by the steel through the mid-span hinge:

$$w_1 = Fw + \frac{Fb}{2} (\theta_1 + \theta_2) \cdot c \cdot \frac{1}{2} \quad 7(C)$$

where Fw and Fb are as mentioned before. The rate of work dissipated by the corner hinge

$$w_2 = 2 m_c L \phi \quad 8(C)$$

By equating equation 6(C) with the sum of the two equations 7(C), 8(C)

$$\frac{w}{4} \cdot x \cdot b \left(\frac{1}{L_1} + \frac{1}{L_2} \right) = Fw + \frac{Fb}{2} \left(\frac{1}{L_1} + \frac{1}{L_2} \right) \frac{bc}{2} + 2m_c L$$

9(C)

where w is the collapse load.

On the other hand, for case B of loading as shown in Fig. (11B), the case of a uniformly distributed load on one cell only, the rate of work done by the external load would be:

$$w_{ex} = \frac{wLB}{4} \phi$$

10(C)

The internal rate of dissipation of work is taken by the mid-span hinge and by the corner hinges.

Hence the effect of the mid-span hinge:

$$w_1 = \left(Fw + \frac{Fb}{2} \right) \frac{2bc}{L} \phi$$

11(C)

where Fb is the total force at bottom flange at yielding for the whole breadth.

The second effect, that of the corner hinge would be:

$$w_2 = 2m_c L$$

12(C)

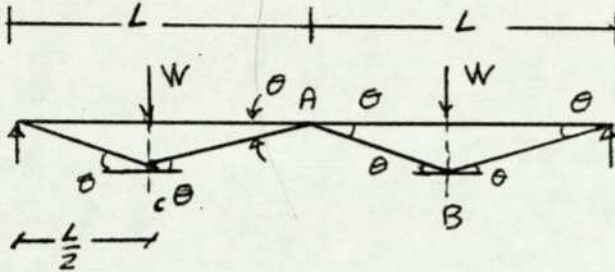
By equating equation 10(C) with the sum of

$$\frac{wLb}{4} = \frac{2bc}{L} \left(Fw + \frac{Fb}{2} \right) + 2 m_c L \quad 13(C)$$

where w is the uniform distributed load on one cell at collapse.

5.3 Double continuous Cell Box

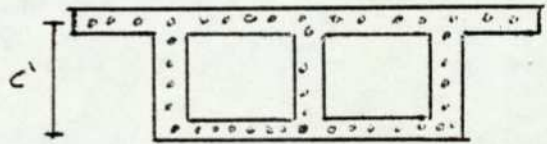
Following the same line of thought and from the observations done during the test. Failure was a pure flexural collapse as shown in Fig (12A). An upper



Fig(12A)

Bound solution could be applied to that group of Boxes to predict the collapse load.

Considering a Box with a cross-section shown in Fig (12B) which is a typical Double cell cross-section used for all Boxes.



Fig(12B)

The work done at section C:

$$W_c = [F_{bc} \cdot c' + 3 \cdot F_{wc} \cdot \frac{c'}{2}] 2\theta = [F_{bc} + 1.5 \cdot F_{wc}] c' \cdot 2\theta$$

The work done at section A:

$$W_A = [F_{tA} \cdot c' + 3 \cdot F_{wA} \cdot \frac{c'}{2}] 2\theta = [F_{tA} + 1.5 \cdot F_{wA}] c' \cdot 2\theta$$

$$\therefore 2 \left(\frac{L \cdot W \cdot \theta}{2} \right) = 2W_c + W_A$$

$$\therefore W = \frac{2 \cdot c'}{L} [2(F_{Dc} + 1.5 F_{wc}) + (F_{tA} + 1.5 F_{wA})]$$

5.3 Lower Bound Limit Approach

The lower bound limit is defined as the maximum load for which the associated stresses nowhere violate the yield condition.

In the following analysis, a simply supported two cell box girder is investigated for the case of a uniformly distributed load of intensity ($w = w'b$) covering one cell only. In this case, (w) is the intensity of loading per unit area, as shown in Figure (13A).

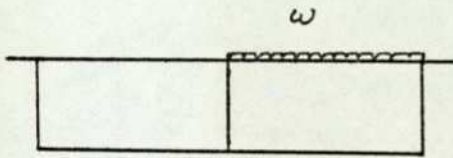


Figure (13A)

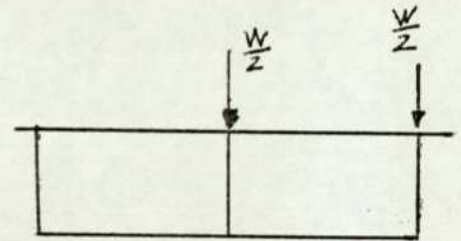


Figure (13B)

The uniformly distributed load w will for simplicity be replaced by two line loads $w/2$ and $w/2$ acting in the plane of the webs as shown in Figure (13B). This is reasonably valid provided that the uniformly distributed load does not cause excessive bending stresses in the top flange.

These two line loads can be replaced by the following uniformly distributed loads $w/4$, $w/2$, $w/4$ on the webs and the uniformly distributed line torque

$\frac{w}{4} \cdot 2b \cdot 1 \cdot wb/2$ as shown in Figure 14A.

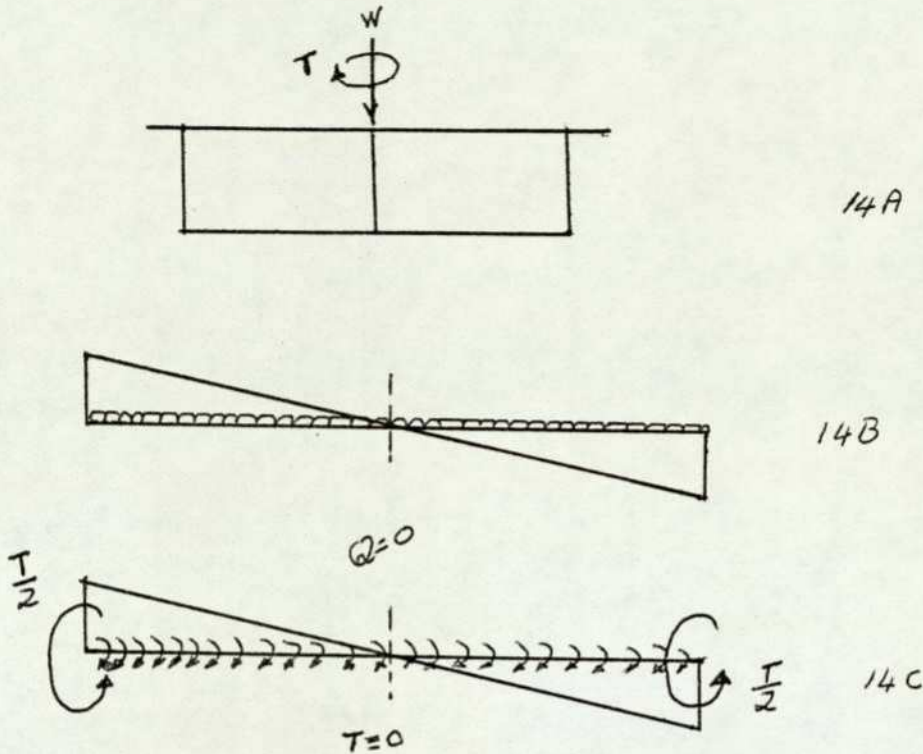


Figure (14)

Consequently there will be neither shear nor torque at the mid-span - see Figure (14B, 14C). In this way the analysis of stresses is simplified.

In search of an equilibrium system of stresses which supports the applied loads and remains within the yield limit everywhere, the two cell box girder is split up into three webs and four flanges.

These are all flat plates connected by forces and couples along their edges. Such forces and couples are nothing else but the internal equal and opposite forces and couples which become external forces and couples acting

on either side of the separation. The equilibrium of every plate support thus means the equilibrium of all external forces and couples acting along its edges. The direct loads $w/2$ and $w/2$ act in the plane of the webs of the loaded cell and thus affect their equilibrium.

All throughout the analysis it is assumed that the concrete cannot resist any tensile stresses while its strength in compression is unlimited. In this way the object of the calculation is to ensure that the stresses in the steel reinforcement do not exceed yield stresses (δy) for steel. Further it is taken for granted that if the stresses due to the edge forces and couples do not violate the yield condition, then all stresses induced in the different parts are not likely to violate the same conditions. Shear perpendicular to flanges and webs is left out. Also, in plane shear in flanges due to torque is neglected. Consequently the torque is assumed to be taken mainly by shear in the outside webs. This assumption has been justified by the fact that no appreciable side movements were noticed during the experimental tests.

Due to the fact that both shear and torque are equal to zero at mid-span, there would be no shearing forces in the three webs at that section. Also, no twisting couples occur in the webs and flanges. In this way the central mid-span section provides a suitable starting point for the study of the equilibrium

ov the box.

Following the investigation outlined by Spence and Morley for a single cell box their method may be extended to the analysis of a twin cell box as follows.

By considering the box shown in Figure 15, which is a half section of the entire box and dividing its flanges and webs into separate plates and considering

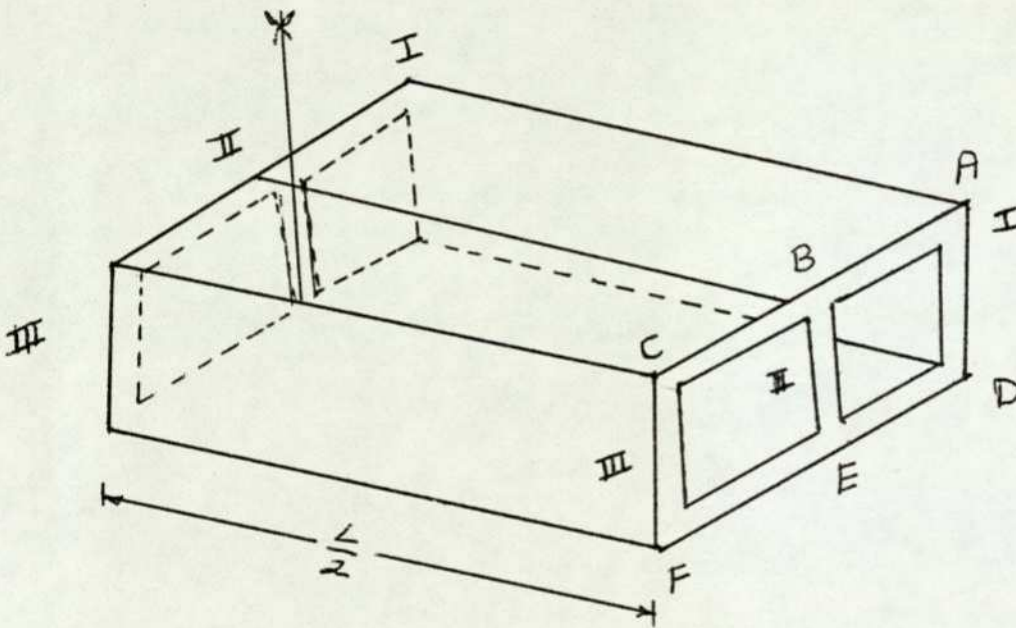


Figure 15

their equilibrium conditions as will be shown. An equation is developed for the lower bound limit by which the collapse load can be calculated.

N.B. In the following diagrams all forces and moments which do not enter the equilibrium conditions have not been shown in figures.

Equilibrium of Web I

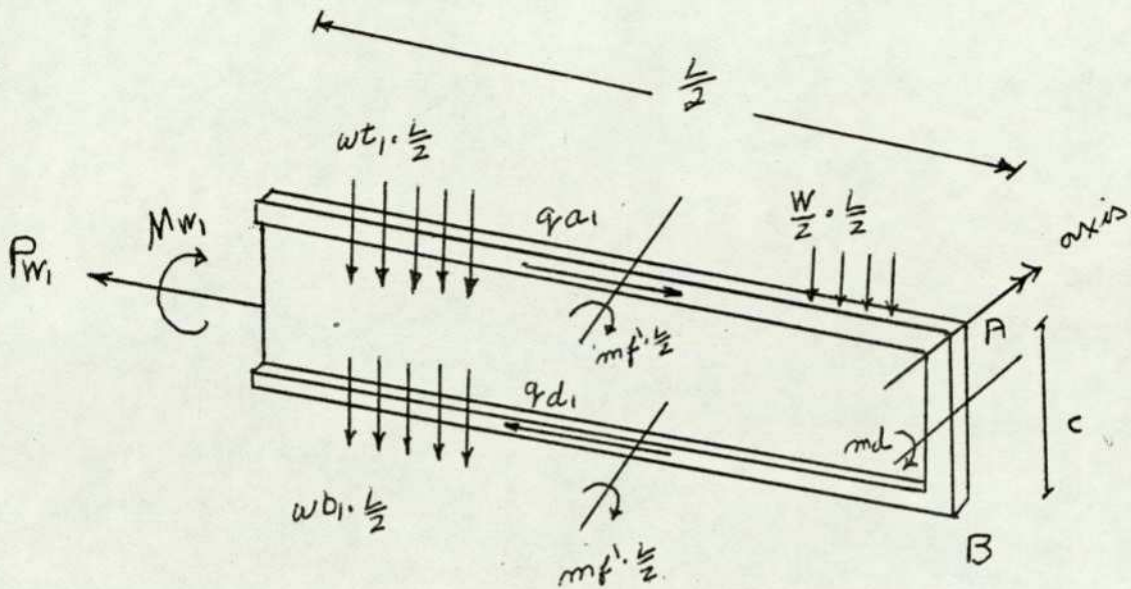


Figure 16

wt_1 transverse shear/unit length at top flange

wb_1 transverse shear/unit length at bottom flange

$w/2$ is external load/unit length

By considering the Web I shown in Figure 16, if moments about the axis at A are taken for the given load the equation of equilibrium would be

$$(Pw_1 \cdot \frac{c}{2}) + Mw_1 + (mf' \cdot \frac{L}{2})2 + md' \cdot c + qd_1 \cdot c - (wt + wb_1 + \frac{w}{2}) \cdot \frac{L^2}{8} = 0 \quad (1)$$

where Pw_1 and Mw_1 are the stress resultants. Moments mf' , and md' are moments/unit length in the top flange and diaphragms as shown in Figure these are assumed to be constant along the whole length.

Let,

$$(Pw_1 \cdot \frac{c}{2} + Mw_1) = FW_1 \cdot \frac{c}{2}$$

where FW_1 is the assumed total force at the centre of the plate, thus equation (11) reduces to

$$FW_1 \cdot \frac{c}{2} + (mf' \frac{L}{2})^2 + md'c + qd_1c - (wt + wb_1 + \frac{w}{2}) \frac{L^2}{8} = 0.$$

Equilibrium of Bottom Flange I

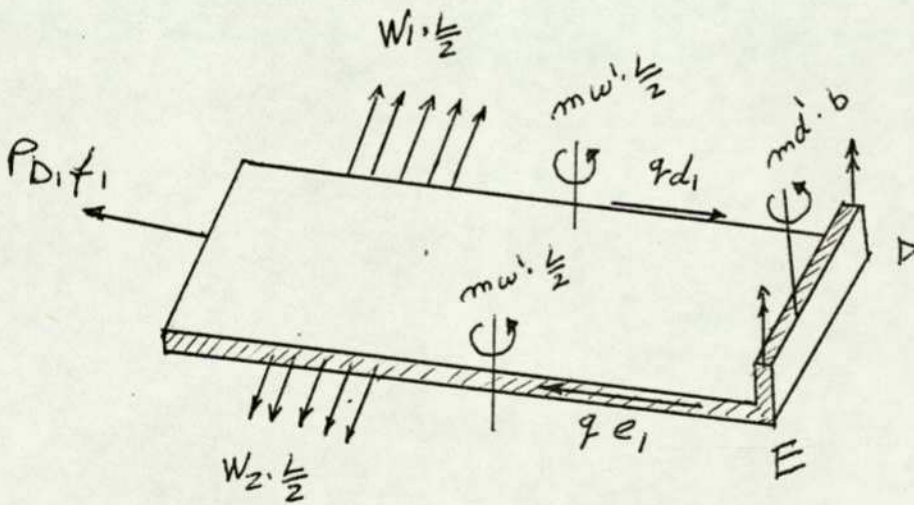


Figure (17)

The Figure 17 shows a flange which is cut by a vertical plane just inside each web and at mid-span and

by a horizontal plane at the end diaphragm.

By taking moments about axis passing through E, the equilibrium would be:

$$Pbf_1 \cdot \frac{b}{2} + (m'w \cdot \frac{L}{2}) \times 2 + m'd \cdot b + w_2 \cdot \frac{L^2}{8} - w \cdot \frac{L^2}{8} - qd_1 \cdot b = 0 \quad (2)$$

By taking moments about a second axis passing through D:

$$Pbf_1 \cdot \frac{b}{2} + w_1 \cdot \frac{L^2}{8} - w_2 \cdot \frac{L^2}{8} - qe_1 \cdot b - (m'w \cdot \frac{L}{2}) \times 2 - md' \cdot b = 0 \quad (3)$$

where w_1 and w_2 are normal forces/unit length.

mw' and $m'd$ are moments/unit length along the plate edges as shown in the diagram.

Equilibrium of Web II (middle web)

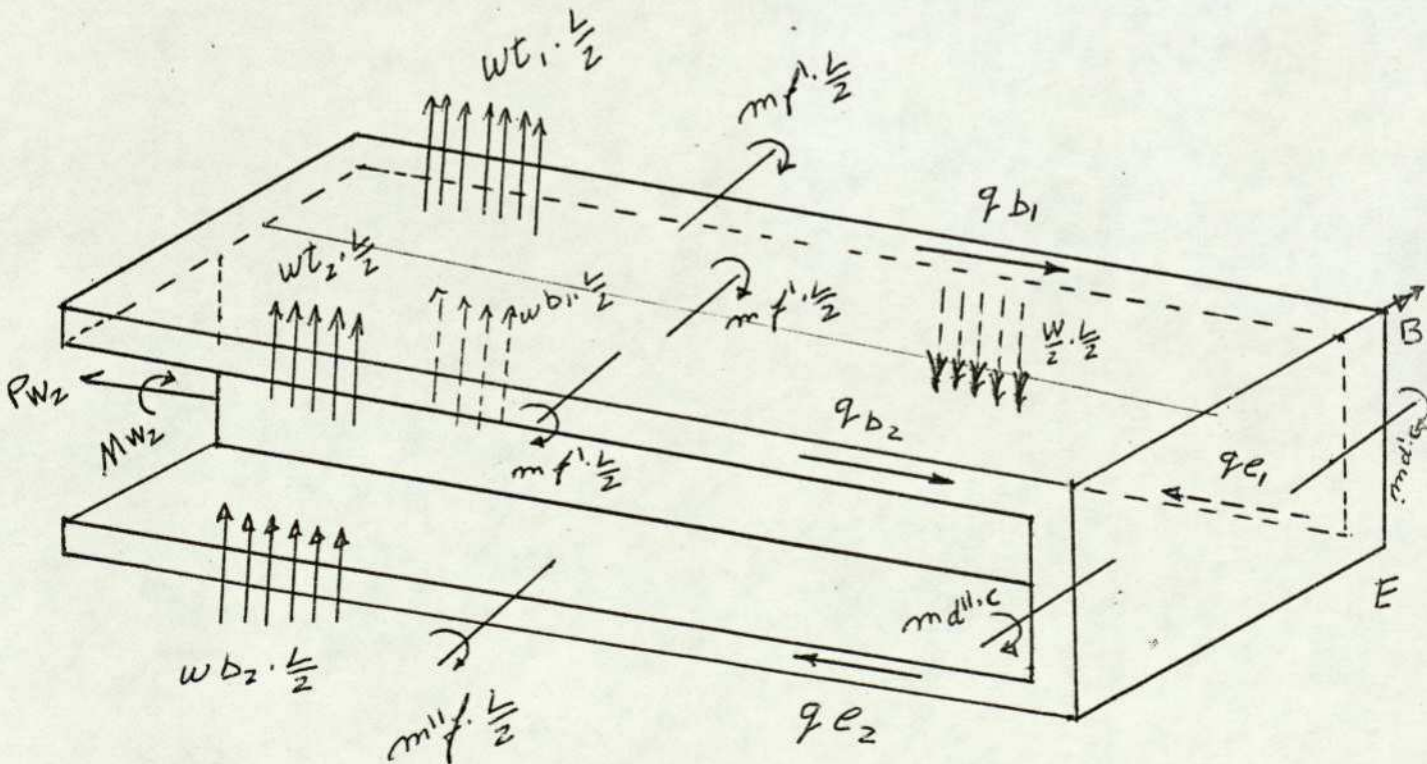


Figure 18

Considering the diagram shown in Figure 18, which represents the mid-web (web II) and taking moments about the axis passing through B:

$$\begin{aligned} (Pw_2 \cdot \frac{c}{2} + Mw_2) + \frac{L^2}{8} \cdot (wt_1 + wt_2 + wb_1 + wb_2 - \frac{w}{2}) + (m'f \cdot \frac{L}{2}) \times 2 \\ + (m''f \cdot \frac{L}{2}) \times 1 + m'd \cdot c + md'' \cdot c + (qe_1 + qe_2) \cdot c = 0 \quad (4) \end{aligned}$$

Pw_2 and Mw_2 are the stress resultants, $\frac{w}{2}$ is the effect of the uniformly distributed load.

Let,

$$(Pw_2 \cdot \frac{c}{2} + Mw_2) = Fw_2 \cdot \frac{c}{2}$$

where Fw_2 would be the assumed total longitudinal force at the centre of the web.

Equilibrium of Flange II

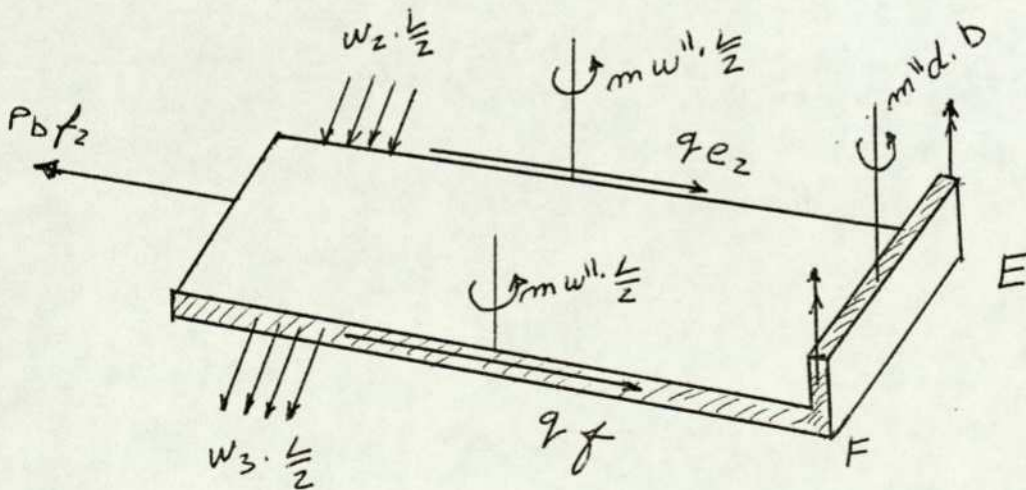


Figure 19

The figure shown represents the flange II. By taking two equations for the moment, one around the axis passing through point F, while the other around the axis passing through E respectively.

$$Pbf_2 \cdot \frac{b}{2} + (w_2 + w_3) \frac{L^2}{8} + (mw'' \cdot \frac{L}{2}) \times 2 + md'' \cdot b - qe_2 \cdot b = 0 \quad (5)$$

$$Pbf_2 \cdot \frac{b}{2} - (w_2 + w_3) \frac{L^2}{8} - qf \cdot c - (mw'' \cdot \frac{L}{2}) \times 2 - md'' \cdot b = 0 \quad (6)$$

Equilibrium of web III (unloaded web)

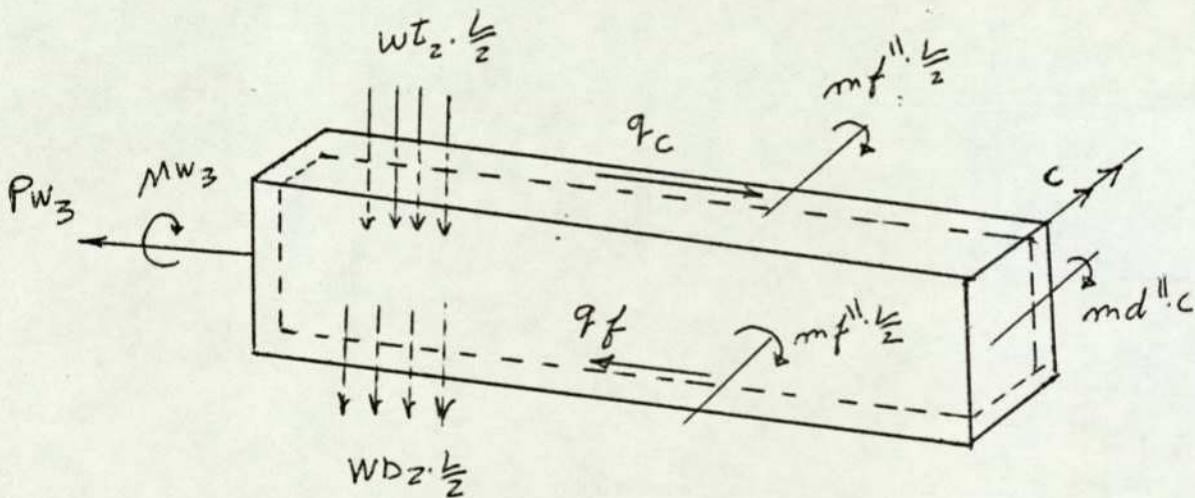


Figure 20

Considering the diagram shown in Figure 20 which represents the unloaded web (web III), and taking moments about the axis passing through point c, the equilibrium equation would be:

$$(Pw_3 \cdot \frac{c}{2} + Mw_3) + (mf'' \cdot \frac{L}{2}) \times 2 + m''d \cdot c + qf \cdot c - (wt_2 + wb_2) \cdot \frac{L^2}{8} = 0 \quad (7)$$

Equilibrium of transverse splice

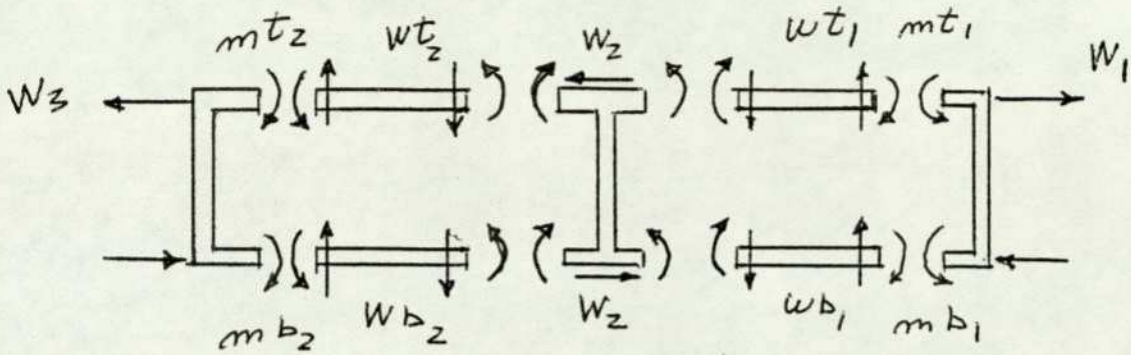


Figure 21

Considering the diagram of the transverse splice shown in Figure 21, we can obtain the following relations:

$$wt_1 = \frac{2mt_1}{b}, \quad wt_2 = \frac{2mt_2}{b}, \quad wb_1 = \frac{2mb_1}{b}, \quad wb_2 = \frac{2mb_2}{b}$$

$$w_1 = \frac{mt_1 + mb_1}{c}, \quad w_3 = \frac{mt_2 + mb_2}{c}$$

$$2mc_1 = mt_1 + mb_1$$

$$2mc_2 = mt_2 + mb_2.$$

According to the different experiments carried out on the 2 cell box girders, it was found that almost no lateral movements occurred and no effect due to thrust was seen. Hence, it is possible to ignore the effect of the moments mf' , mf'' , md' , md'' , mw' and mw'' from the previous equilibrium equation.

By omitting those moments, the new form of the equations would be:

$$Fw_1 \cdot \frac{c}{2} + qd_1 \cdot c - (wt_1 + wb_1 + \frac{w}{2}) \cdot \frac{L^2}{8} = 0 \quad (8)$$

$$Fb_1 \cdot \frac{b}{2} + \frac{L^2}{8} (w_1 - w_2) - qd_1 \cdot b = 0 \quad (9)$$

$$Fb_1 \cdot \frac{b}{2} + \frac{L^2}{8} (w_1 - w_2) - qe_1 \cdot b = 0 \quad (10)$$

$$Fw_2 \cdot \frac{c}{2} + \frac{L^2}{8} (wt_1 + wt_2 + wb_1 + wb_2 - \frac{w}{2}) + (qe_1 + qe_2) c = 0 \quad (11)$$

$$Fb_2 \cdot \frac{b}{2} + \frac{L^2}{8} (w_2 + w_3) - qe_2 \cdot b = 0 \quad (12)$$

$$Fb_2 \cdot \frac{b}{2} - (w_2 + w_3) \frac{L^2}{8} - qf \cdot b = 0 \quad (13)$$

$$Fw_3 \cdot \frac{c}{2} - (wt_2 + wb_2) \cdot \frac{L^2}{8} + qf \cdot c = 0 \quad (14)$$

In order to obtain a relation between the values Fw_1 , Fw_2 , Fw_3 , Fb_1 and Fb_2 , these seven equations may be rearranged as follows.

By adding equation (8) $\times \frac{1}{c}$ to equation (9) $\times \frac{1}{b}$ and substituting wt_1 , wt_2 , wb_1 , wb_2 , w_1 and w_2 for values of mc_1 and mc_2

$$\frac{Fw_1}{2} + qd_1 - (wt_1 + wb_1 + \frac{w}{2})\frac{L^2}{8c} + \frac{Fb_1}{2}(w_2 - w_1) - qd_1 = 0$$

$$\frac{Fw_1 + Fb_1}{2} - (wt_1 + wb_1)\frac{L^2}{8c} + (w_2 - w_1)\frac{L^2}{8b} - \frac{WL^2}{16c} = 0$$

$$\begin{aligned} \frac{Fw_1 + Fb_1}{2} - (2mt_1 + 2mb_1)\frac{L^2}{8bc} + (mt_2 + mb_2 - mt_1 \\ - mb_1 - mt_2 - mb_1)\frac{L^2}{8bc} - \frac{WL^2}{16c} = 0 \end{aligned}$$

We obtain:

$$\frac{WL^2}{16c} = \frac{Fw_1 + Fb_1}{2} + \frac{L^2}{4bc} (mc_2 - 4mc_1) \quad (15)$$

By adding equation (13) $\times \frac{1}{b}$ to equation (14) $\times \frac{1}{c}$

$$\frac{Fb_2}{2} - (w_2 + w_3)\frac{L^2}{8b} - qf + \frac{Fw_3}{2} - (wt_2 + wb_2)\frac{L^2}{8c} + qf = 0$$

$$\begin{aligned} \frac{Fw_3 + Fb_2}{2} - (mt_2 + mb_2 - mt_1 - mb_1 + mt_2 + mb_2)\frac{L^2}{8bc} \\ - (2mt_2 + 2mb_2)\frac{L^2}{8bc} = 0 \end{aligned}$$

$$\frac{Fw_3 + Fb_2}{2} + (2mc_1 - 8mc_2)\frac{L^2}{8bc} = 0$$

$$\frac{Fw_3 + Fb_2}{2} = (4mc_2 - mc_1)\frac{L^2}{4bc} \quad (16)$$

By adding equation (11) $\times \frac{1}{c}$ to equation (12) $\times \frac{1}{b}$
to equation (10) $\times \frac{1}{b}$

$$\begin{aligned} \frac{Fw_2}{2} + \frac{L^2}{8c} (wt_1 + wt_2 + wb_1 + wb_2 - \frac{w}{2}) + (qe_1 + qe_2) + \frac{Fb_1}{2} + \frac{L^2}{8b}(w_1 - w_2) \\ - qe_1 + \frac{Fb_2}{2} + \frac{L^2}{8b}(w_2 + w_3) - qe_2 = 0 \\ Fb_1 + Fw_2 + Fb_2 + \frac{L^2}{2bc} (3mc_1 + 3mc_2) = \frac{WL^2}{8c} \end{aligned} \quad (17)$$

By dividing equation (17) by 2 and subtracting
from equation (15):

$$\frac{Fw_1 + Fb_2}{2} + \frac{L^2}{4bc} (mc_2 - 4mc_1) - \frac{Fb_1 + Fw_2 + Fb_2}{2} - \frac{3L^2}{4bc} (mc_1 + mc_2) = 0$$

$$\frac{Fw_1 - Fw_2 - Fb_2}{2} = \frac{L^2}{4bc} (2mc_2 + 7mc_1) \quad (18)$$

Adding equation (18) to equation (16)

$$\frac{Fw_1 + Fw_3 - Fw_2}{2} = \frac{L^2}{4bc} (6mc_2 + 6mc_1) \quad (19)$$

Adding equation (19) to equation (17)

$$\frac{WL^2}{8c} = \frac{Fw_1 + Fw_2 + Fw_3}{2} + (Fb_1 + Fb_2) \quad (20)$$

This equation shows that the maximum binding moment at the centre of the span for a uniform distributed load on one cell only will be resisted by steel reinforcement in the three webs as if they were concentrated at mid-height and by the steel reinforcement of the two bottom flanges acting at the centre of this flange. This direct issue leads to the determination of the lower bound limit.

The stress distribution would be in the form shown in Figure 22.

Another equation for the case of 4 line loads which represents the second unsymmetrical load applied to the tested boxes could be calculated:

$$\frac{W}{2} \frac{31.75}{c} = \frac{Fw_1 + Fw_2 + Fw_3}{2} + Fb_1 + Fb_2 \quad (21)$$

where W is the external load.

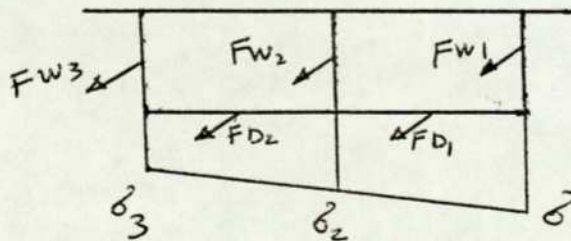


Figure 22

It remains to calculate the values of Fw_1 , Fw_2 , Fw_3 , Fb_1 and Fb_2 , which depends on the stress distribution along the cross section at that section, and hence on the share of loads taken by each web due to the application of an unsymmetric load on one of its cells.

The aim of the forthcoming part of the chapter is to find those stresses of the applied load taken by each web and hence the two equations (20) and (21) could be easily calculated by applying the limit bound conditions.

5-3-1 The Two Cell Box Girder as a Beam on Three Elastic Supports

In order to disclose the behaviour of the deformable box girder under the effect of a more or less general case of eccentric loading.

The system is replaced by a structural model which is claimed to give a fair idea of the actual phenomena. The proposed structural model consists of three elastic main girders carrying a number of elastic cross beams. In this way the problem is referred to that of a simple beam on these elastic supports. This is perhaps the simplest way of representation.

If it is desired to have a better adaptation to the box girder, the cross beams in the proposed simple model could be fixed to the three main girders, thus forming an orthogonal gridwork capable of taking bending and torque. In this way the structural behaviour between the different box girder sections will be fairly represented.

In the following investigation the simple proposed structural model will be treated.

Taking the same length of the main girder to be (L) and the same moment of inertia for every one of them (I) , a constant elastic coefficient (K) for all elastic supports is used.

For the given case of eccentric loading shown in Figure 23 the relation between the elastic supports

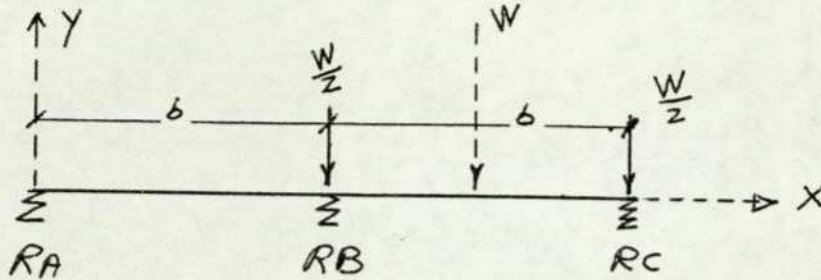


Fig. 23

R_A , R_B , and R_C could be calculated respectively.

Two equilibrium conditions could be attained from Figure . By taking equilibrium condition in the y axis direction, the first equation would be

$$R_A + R_B + R_C = \frac{W}{2}$$

where R_A , R_B and R_C are the three reactions of the three supports A, B and C respectively; (W) being the external load.

The second equilibrium condition is attained by taking moments about support C

$$2R_A + R_B \cdot b = \frac{W}{2} \cdot b$$

where b is the distance between any two adjacent supports.

A third condition needs to be obtained in order to attain the three required relations.

By assuming the removal of support (B), the portion $\frac{W}{2}$ of the eccentric load, acting at support (C) will produce a direct reaction of $\frac{W}{2}$ at that support. The second portion $\frac{W}{2}$ acting at support (B) will produce B.M of maximum value at point (B) and two reactions $\frac{W}{4}$ at ends A and B respectively as shown in Figure(24A) where $(R_{C1} = 3W/4)$, $(R_{A1} = W/4)$.

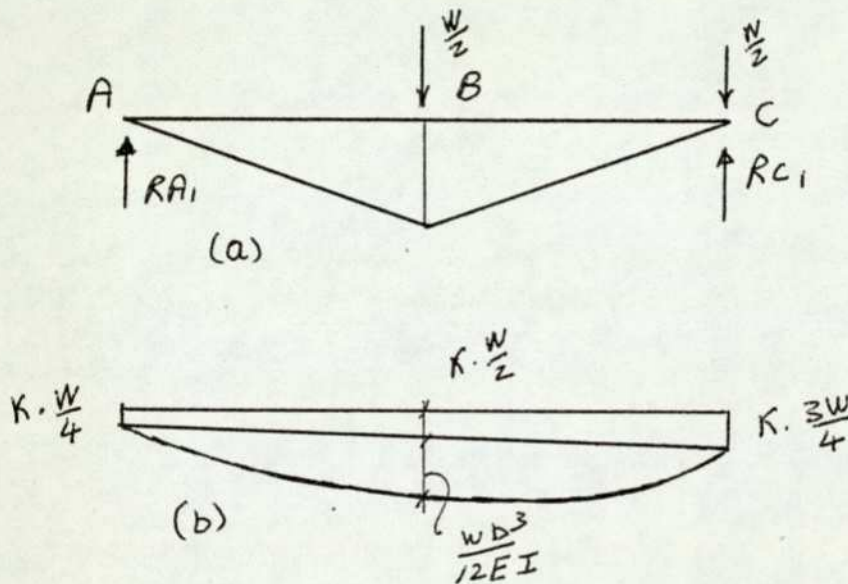


Figure 24

Hence the corresponding deflection produced at supports A and C would be $K \cdot \frac{W}{4}$ and $K \cdot \frac{3W}{4}$ respectively, with a mid value of $K \cdot \frac{W}{2}$ at point (B). An additional deflection at point (B), due to the application of $\frac{W}{2}$ will attain a value of $\frac{Wb^2}{12EI}$, producing a total deflection at point (b) equal to $K \cdot \frac{W}{2} + \frac{Wb^3}{12EI}$, as shown in Figure (24B)

By applying a unit load at point (B) as shown in Figure (25A) two reactions, R_{A2} and R_{C2} will be obtained

with values of $(\frac{1}{2})$, producing a uniform deflection of $(k.\frac{1}{2})$ for the whole beam.

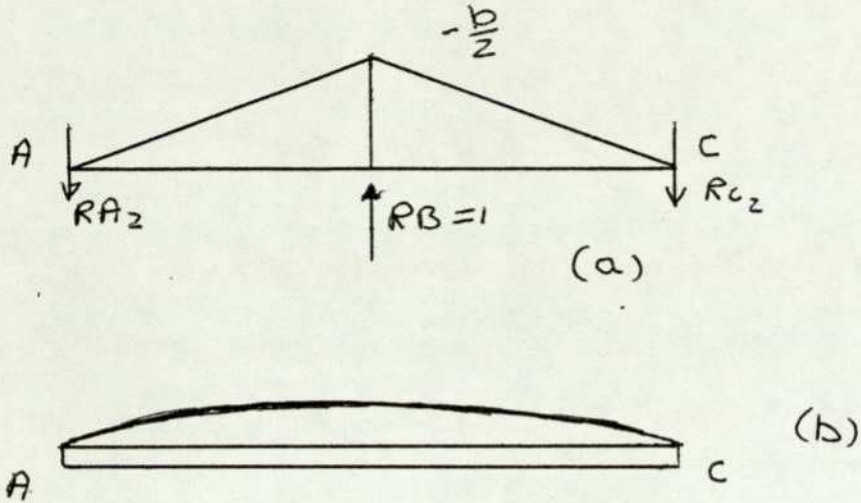


Figure 25

An additional deflection due to the unit load at point B is produced as shown in Figure(25B) . Total deflection at that point would be equal to $\frac{Kw}{2} + \frac{b^3}{6EI}$.

The third required equation for equilibrium at support B:

$$K.\frac{w}{2} + \frac{wb^3}{12EI} - \frac{1}{2}K + \frac{b^3}{6EI} RB = K.RB.$$

By solving those three equations, the values of the three reactions would be in the form:

$$\begin{aligned} RA &= \frac{3KEI}{9KEI + b^3} \cdot \frac{w}{4} \\ RB &= \frac{6KEI + b^3}{9KEI + b^3} \cdot \frac{w}{2} \\ RC &= \frac{21KEI + 2b^3}{9KEI + b^3} \cdot \frac{w}{4} \end{aligned}$$

Remembering that ($b = 7.5'$) and ($L = 96'$) in the actual model, the values RA , RB and RL will be slightly less than ($1/12$), slightly more than ($1/3$) and slightly less than ($7/12$) of the total load respectively.

It is interesting to notice that the values $\frac{W}{12}$, $\frac{W}{3}$ and $\frac{7W}{12}$ which are not very far from the values found from the elastically supported beam, coincide with the values found for the assumption of a rigid cross section of the box girder as in Figure (26)

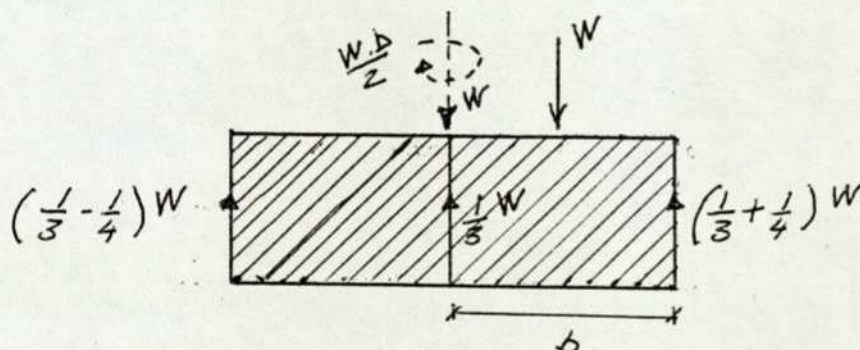


Figure 26

They show that the central load is taken equally by the three supports, while the torque $w \cdot \frac{b}{2}$ is taken by the two end supports only, which are thus:

$$RA = \frac{1}{3}W - \frac{1}{4}W = \frac{W}{12} \quad , \quad RB = \frac{W}{3} \quad , \quad RC = \frac{W}{3} + \frac{W}{4} = \frac{7}{12}W$$

In other words, the twisting moment due to the excessive load, will be taken by the two outside webs only. This means that the upper and lower flanges will take no shear from the twisting moment of the eccentric load. This justifies the assumption which has been made on the theoretical investigation.

Applying these results to the box girder, the horizontal position of the upper and lower flanges will deflect to an inclined elevation, corresponding to the values of the reactions R_A , R_B , and R_C respectively.

At the same time, the vertical web will tilt slightly in a clockwise direction. Remembering that the whole width (15.75") and depth, centre to centre of flanges (4.5"), the horizontal movements of the corners will be very small. Further, the fact that the compression in the upper flange over the support C gives bigger strains than over the support A as shown in Figure (27)

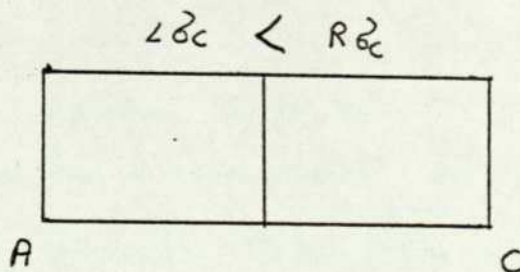


Figure 27

This flange will deflect laterally towards the left, thus reducing the side-sway movements.

A similar phenomena applies to the lower flange which receives bigger tension strain towards (C) than at (A), so it tends to deflect slightly towards the right as shown in Figure (28)

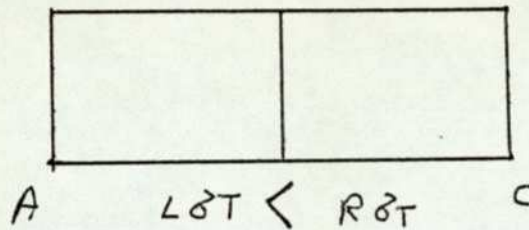


Figure (28)

As both horizontal flanges have a relatively large height, such movements remain very small as observed during the test. Further, the absolute values of shear in the two cells are equal, and represent at the same time a small portion of the total loading. Consequently, the upper and lower flanges in the two cells will take a slightly curved (S) form, falling slightly away from the inclined datum of deflection - Figure (29)

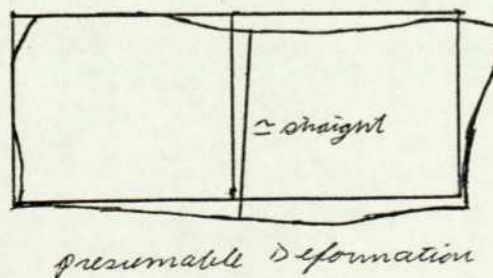


Figure 29

The shear in every cell will be presumably taken equally by the two flanges, giving four equal moments m_c at their corners.

Further, since the signs on either side of the central web are +ve and -ve respectively, the corresponding corner moments (m_c) values will have equal and opposite values - see Figure (30)

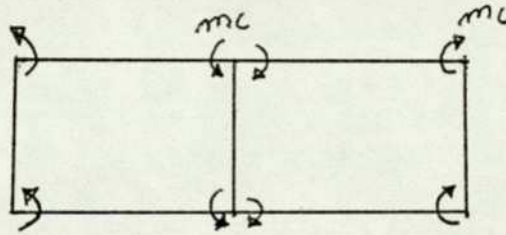


Figure 30

The corresponding points of contraflexure are at the mid points of flanges and outside webs. The middle web receives no bending moment and thus remains straight.

The corner points of the two cell girders do not fall on the inclined datum of deflection mentioned before for the approximate values $\frac{W}{12}$, $\frac{W}{3}$ and $\frac{7W}{12}$ respectively. The slight deviations are deflections due to the differences between these approximate values and the real values of the reactions of the elastic supports.

Following this line of thought, the corner movements are assumed equal and opposite for the two cells of the multi cell girder in the previous theoretical investigation. Also, the horizontal deflections are left out as they are very small.

Further, the same concept had lead to the assumed mechanism of failure for the calculation of the upper limit analysis. These were based on the assumption that the two webs of the single cell girders move downwards, almost parallel to one another, with hinges at their corners. In this way the mechanism which allows for a horizontal movement of the lower flange is disregarded.

In the case of the two cell box, the mechanism of failure was assumed, introducing four hinges at the corners of the loaded cell, leaving the unloaded half intact. Furthermore, the analogy of the beam on three elastic supports has shown that the two webs of the loaded cell take practically all the eccentric load leaving a slight shear for the third web.

Refinement for assumption of beams on three elastic supports

It must be remarked that the values (1:4:7) adapted for the reactions on the elastic supports of the analogous beam are only approximate values. A

correction is hereafter made to suit the values obtained earlier for these reactions.

The values of the stiffness (K) and the moments of inertia (I_t) of the cross beams are needed. As for stiffness (K), it is calculated from the deflection of a single longitudinal beam having $-\frac{1}{3}$ of the total moment of inertia of the two cell section and carrying a single load ($w = 1$) at mid span, as shown in Figure 31

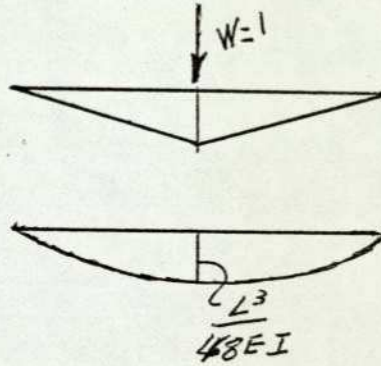


Figure 31

Thus $K = \frac{WL^3}{48EI}$ where $W = 1$, $L = 96''$

$$I_L = \frac{1}{3} I_B.$$

Further, with regard to the moment of inertia of the analogous beam, reference is made to the analogous system of three main girders connected by a number of cross girders of equal stiffness.

If a single load W is applied at the mid-point, a certain load distribution will take place due to the presence of the cross girders. However, the stiffness

of the central cross girder may be increased to account for the effect of all cross girders. The calculation of the grid with this ideal cross girder is rendered simple, but gives a satisfactory approximation.

If the same cross-section is adopted for all cross girders, the effect of each cross girder on the load distribution would depend only on its relative position in the system. This effect, however, may be assumed proportional to the corresponding deflections of the main girder.

Thus, the moment of inertia (I_{ix}) of an ideal cross girder at mid span, representing a certain cross girder at a distance (X) from the support, is given by the relation $I_{ix} = I_t \cdot \frac{\delta x}{\delta m}$, summing

for all cross girders: $I_i = \frac{I_t}{\delta m} \sum \delta x$.

This is the moment of inertia of the ideal girder at mid-span, which is likely to give nearly the same load distribution as all cross girders in the grid. (See Appendix 1).

A better refinement of the previous results is given hereafter, taking unequal stiffness of the main girders.

By assuming half of the flange to be acting with one girder, while the other half is acting with the next girder, the two cell box section will be split up into three girders whose moments of inertia are

$$(I_1, I_2, I_3 = I_1).$$

These have the ratios $(0.7l_B : I_B : 0.7l_B)$

giving stiffness factors $c, c' = 0.7c$ and c respectively.

By introducing these new values in the previous

calculations of the reactions of the analogous beam

on the elastic supports, we attain those three equations

$$RA + RB + RC = W$$

$$2RA \cdot b + RB \cdot b = \frac{W \cdot b}{2}$$

$$\frac{Wc}{2} + \frac{Wb^3}{12EI_t} - \frac{1}{2}c + \frac{b^3}{GEI_t} \quad RB = 0.7c \quad RB.$$

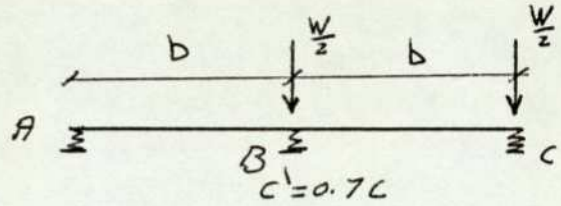


Fig. (32)

By solving these equations we get the values of the reactions as follows:

$$RA = \frac{1.2 \, CEI_t}{7.2 \, CEI_t + b^3} \cdot \frac{W}{4}$$

$$RB = \frac{6C \, EI_t + b^3}{7.2c \, EI_t + b^3} \cdot \frac{W}{2}$$

$$RC = \frac{7.8 \, CEI_t + b^3}{2(7.2C \, EI_t + b^3)} \cdot W$$

A study of the new elastic reactions shows that by accounting for a bigger stiffness of the central girder, the end reactions of the unloaded web is reduced from

$\frac{W}{12}$ to $\frac{W}{24}$, the central reaction, however, is increased from $\frac{W}{3}$ to $\frac{W}{24}$ while the other end reaction on the loaded side is reduced from $\frac{7}{12}W$ to $\frac{1}{1.85}W$.

In other words, the previous ratios of 1:4:7 become now nearly 1:10:13. Consequently, this refinement shows that the two loaded girders carry not only 91.7% but even 95.8% of the total load. Further, the refinement reduces the shears in the two cells to $+\frac{W}{24}$ instead of $+\frac{W}{12}$ as found for equal stiffness of the three main girders. This shows that the two cell section will twist as a rigid rectangular and then deforms elastically to a much smaller degree than predicted before.

5.3.2 The two cell Box Girder as a Rigid Tubular Section

Preliminary to any further discussion, the two cell box girder is considered to be a hollow coherent tube closed at both ends by two diaphragms and subjected to the two uniformly distributed loads $\frac{W}{2}$ and $\frac{W}{2}$ in the planes of the central and outside webs respectively, as in Figure (33)

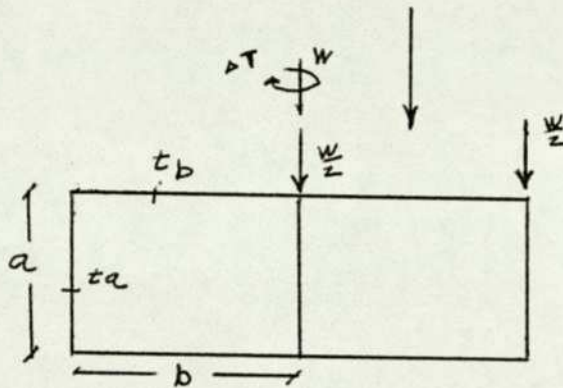


Fig. (33)

Here ($W = w' \cdot b$) and the total loading ($W = w \cdot L$)

over the total length of the girder. Consider further the loads to be represented by a uniformly distributed load W acting in the middle axis together with a uniformly distributed couple

$$\Delta T = \frac{W}{2} \cdot b$$

as shown in Figure 34

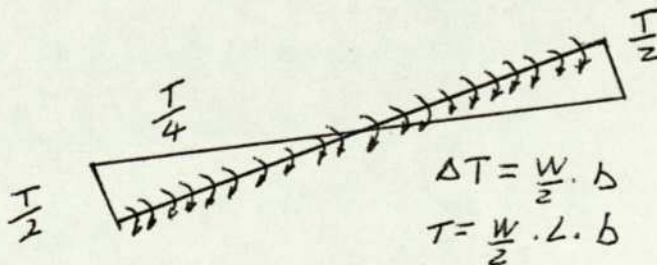


Fig. 4

Assuming the central loading w to be taken equally by the three webs, the corresponding deflection at mid span would be in the form

$$\delta_b = \frac{5}{384} w' \cdot b \cdot \frac{L^4}{EI_t}$$

where (L) is the box length, (I_t) inertia of the cross section.

Further, assuming the uniformly distributed twisting moment (ΔT) shown in Figure , to be resisted by the two cell section as a monolithic entity, the corresponding angle of twist at the mid point of the span would be:

$$\theta = \theta \cdot \frac{L}{2} = \frac{T}{4} \cdot \frac{L}{2} \cdot \frac{\frac{a}{t_a} + \frac{2b}{t_b}}{8a^2b^2G}$$

$$\text{where } \frac{T}{4} = \frac{w' \cdot b}{2} \cdot b \cdot \frac{L}{4} = \frac{w \cdot b}{8} \cdot$$

See Figure (35)

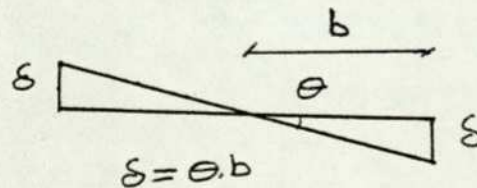


Fig. 35

The corresponding deflection of the loaded web becomes:

$$\delta t = \theta \cdot b \cdot$$

The ratio between the deflection due to twisting and flexure $\frac{\delta t}{\delta b} = \text{very small}$.

In other words, there will be almost no twisting at all, leaving the vertical load to be taken nearly uniformly by the whole box section.

This may take place at the early stages of loading i.e. while the concrete is capable of resisting the shear stresses. But sooner or later, the concrete ceases to take up the shear stresses and whatever tensile stresses are associated with them. Consequently, the section starts to lose its originally considerable torsional rigidity and stops gradually to act as a coherent entity. In fact, had it not been for the mesh reinforcement with its longitudinal, as well as transversal, steel bars, the structure would have deteriorated at an earlier stage of the loading. However, the embedded cage of orthogonal steel wires takes over the job of resisting tension as well as shear stresses.

The foregoing analysis is in agreement with the measured deflections which shows clearly that the cross section tilts first slightly, then more and more as the eccentric loads increase. This phenomena, however, is not shared to the same extent by the measured stresses in the steel reinforcement, which do not show such big changes.

The difference between the two recorded observations of deflections and stresses is attributed to

the difference in the behaviour of the box girder in bending and in twisting. In the case of bending the vertical load is carried by the whole section, giving more or less equal shares to the three webs. The corresponding deflections will therefore be more or less the same. This applies also to the associated stresses in the longitudinal steel reinforcement.

On the other hand, the twisting moment will produce opposite shears in the outside webs, leaving the central one unaffected. These associated shear deflections are likely to change the deflection's line in the transverse direction rather than the stresses in the longitudinal steel reinforcement.

In addition, it must be mentioned that the behaviour in bending is based on the assumption that parallel plane sections remain plane but not paralel, after bending - see Figure(36) . It is this bending of parallel sections which leads to the simple formula of bending stresses, and the calculation of the deflections due to bending. The latter are much bigger than the deflections due to shear.

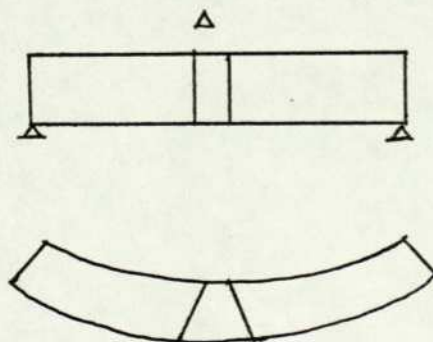


Fig. 36

On the other hand, the behaviour of the girder in torsion is based on the assumption that parallel plane sections remain plane and parallel after twisting. Consequently, there will be shear stresses and associated small shear deflections only. Any other stresses or strains due to warping, for example, will be resisted and balanced by the box girder as a short hollow tube in the transverse direction. Obviously, any longitudinal slice through the box girder is a closed rectangular frame formed by the flanges and the end diaphragms.

Finally, as the load approaches the ultimate limit, the two webs of the loaded cell take the greater part of the eccentric load, leaving a lesser share for the unloaded web. This observation which is due to the gradually increased lack of coherence in the model, led to the mechanism of failure proposed for the prediction of the upper bound limit. It has also favoured the reference to the analogous beam on elastic supports in studying the lower bound. However, this assumption is rather conservative as it exaggerates the influence of the eccentric loading on the loaded outside web.

5-3-3 Alternative approaches and comparison of previous assumptions

By looking at the two previous approaches, the calculation of the shares of load taken by the three webs, due to the application of an eccentric load on the box may be deduced. The first approach considers the box to be composed of three separate girders connected together by a number of cross beams, thus forming a grid. The cross beams have been replaced by a single ideal cross girder. This assumption neglects obviously the torsional rigidity of the box section. It gives different shares of loading on the three webs, as the twisting moment is here entirely taken by upward and downward loading of the outside webs. Hence, it is too far on the safe side. On the other hand, in the second approach, the box is considered to act as a rigid hollow tube, capable of resisting the entire twisting moment. In this way, only torsional shear stresses in the flanges and the outside webs are produced, due to this moment. No direct stresses, however, occur in the cross section. Consequently, all webs will take equal shares of the total loading giving equal flexural deflections. The effect of torque is limited here to small differential shear deflections. This assumption obviously gives too large a value for the critical load.

If a designer considers the box girder to be a rigid tubular section, any central or eccentric loading is taken equally by the three webs giving more or less equal flexural deflections. The effect of a twisting moment is confined to torsional shear stresses and slight differential shear deflections without producing direct bending stresses or flexural deflections. On the other hand, if the designer follows the second line of thought, splitting up the two cell box girder into three separate girders, neglecting the torsional rigidity of the tubular section altogether, such a design gives heavier sections and is thus less competitive but involves a higher degree of safety.

It can be seen that the box girder acts partly as a rigid tubular section and partly as separate girders connected by cross beams. This is clear from the results of the ad hoc tests which reveal bigger deflections of the loaded web and smaller deflections of the unloaded one. The differences, however, are more than is expected in the case of a rigid and less than is expected in the case of separate girders with stiff cross beams.

An average of the two cases can be suggested as a middle rule, in order to reduce the cross-section and still allow for an appropriate margin of safety. In this way, half the torque is assumed to be taken by

the box girder as a torsional couple, while the other half is assumed to be taken by equal and opposite flexure of the outside web.

However, the results of the tests carried out on single cell box sections show that the box behaves as a rigid tubular section. The experiments carried out for the two symmetrical half loads on two webs, and for a full load, on one web only, gave almost the same value for the total critical load.

The two cell box girder possesses a bigger torsional rigidity. At the same time the torsional couple due to the one cell loading is relatively smaller than for the case of the single cell girder. Consequently, it may be predicted that the two cell box girder will behave more like a rigid hollow tube than as a grid of three separate girders connected by cross beams. The experiments carried out strengthened this point of view.

Obviously, an average solution for the ratio of torque taken in bending and in torsion is not very correct.

Alternatively, a proposed approach for calculating that ratio is shown in the next paragraph.

5.3.4 An assumed strain energy approach to obtain the ratio of torsion taken in torque and that taken in bending

A better prediction of the unsymmetrical load for the two cell box girder can be obtained by the strain energy method.

Here again the eccentric loading is decomposed into a symmetrical central loading (W) and a twisting couple ($\frac{W \cdot b}{2} = 3.75W$). This obviously applies to the three cases shown in Figure (37)

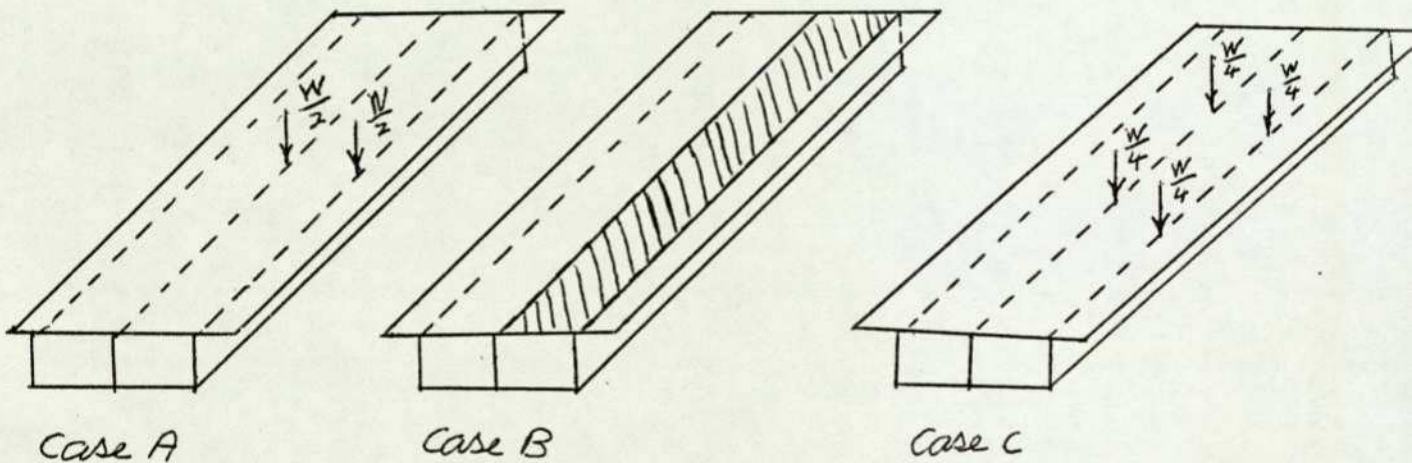


Fig. 37

Case A The case of two half loads acting at the mid points of the girders I and II.

Case B The case of uniformly distributed loads of intensity $\frac{W}{2} = \frac{W}{2} L$ covering the entire length of the webs I and II.

Case C The case of four point loading $\frac{W}{4}$ at a distance of (31.75") from each end of the two beams I and II.

This latter case assimilates the actual four strip loadings carried out in the test. In principle, the three cases will be subjected to the same total load (W) and the total torsional moment $\frac{w.b}{2}$ as in Figure (38)

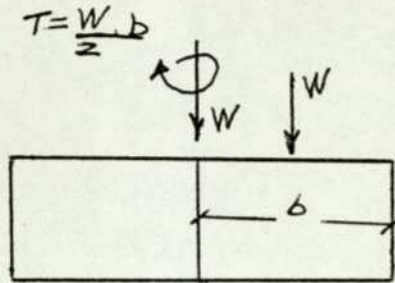


Fig. 38

Yet one application of this total load and total couple differ from case to case. The effect of this difference is clear in the treatment of the three cases illustrated hereafter, and in Appendix (2A, 2B, 2C).

As for the central load (W), this is taken equally by the three girders I, II and III, giving an equal share of $\frac{W}{3}$ for every girder. The twisting couple ($T = \frac{1}{2}.w.b$), however, will be split up into a torsional couple (T_t) and a bending couple (T_b) as in Figure 39 where ($T = T_t + T_b$). Needless to say, that T_t will be taken by the whole section of the box girder on torsion giving a shear stress $\tau = \frac{T_t}{2At_{f,w}}$ in the two flanges and the two outside webs only.

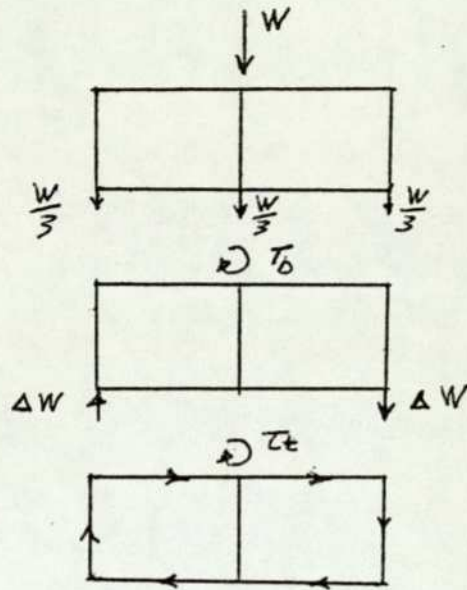


Figure (39)

The central web of the symmetrical two cell box is not affected by this torsional couple. At the same time (T_t) causes an angular twist per unit length

$$\theta = \frac{T_t}{4A_m^2 G} \frac{ds}{t} .$$

Due to this angular twist, a certain deflection of the outside girder I and II is imposed upon the girder. As already explained at an earlier stage, such deflections are very small with regard to the deflections due to bending, caused by the central load (W) and the bending couple (T_b) as shown in Figure 40 .

For this reason the effect of the torsional couple (T_t) on the deflections of the outside webs is neglected, leaving only the angular twist (θ). The corresponding work done ($WT = \frac{1}{2} \cdot Tt_x \theta t_x$) integrated over the whole length of the box girder. (Tt_x) and (θ_x) refer to the couple and angle of twist at any section of the span. This work done represents, at the same time, the internal strain energy accumulated in the box girder due to (T_t).

Further, the bending couple (T_b) will produce two equal and opposite total additional loadings of the two outside beams I and II equal to $\Delta W = \pm \frac{Tb}{2b}$ as shown in Figure . Consequently, the total loads imposed upon the three beams I, II and III due to (W) and (Tb) are $\frac{W}{3} + \frac{Tb}{2b}$, $\frac{W}{3}$ and $\frac{W}{3} - \frac{Tb}{2b}$ on the I, II and III respectively.

The corresponding strain energy accumulated in the three beams due to (W) and (Tb) will not be the same. On the contrary, different moments will be produced in the three girders due to one additional (+ve and -ve) loading of the outside girders.

Here again, the strain energy in each beam is not calculated from the respective bending moments, but from the work done by the imposed loads during the deflections. In other words, the strain energy girder

will be calculated from the integration of $(\frac{1}{2} \cdot W_x \cdot \delta_x)$ over the length of each girder. In this way the total strain energy accumulated in the two cell girder becomes:

$$\epsilon = W_{1N} = W_T + W_{b_{1,2,3}}$$

where (W_T) is the work done by the torsional couple (T_v) during the twist (θ) , and (W_b) is equal $(W_{b_1} + W_{b_2} + W_{b_3})$, the work done by the corresponding loads (W_1) , (W_2) and (W_3) found earlier on the three beams I, II and III due to (W) and (T_b) .

It remains now to determine the work done by the external loading $\frac{W}{2}$ and $\frac{W}{2}$ on the girders I and II. This is simply found by integrating the values $(\frac{1}{2} \cdot W_{1,2} \cdot \delta_{1,2})$ of respective beams I and II.

The values δ_1 and δ_2 are the deflections of the end beam I and the central beam II under the loads $\frac{W}{3} + \frac{T_b}{2b}$ and $\frac{W}{3}$ respectively.

$$\text{Thus, } W_{e_x} = \frac{1}{2} \cdot \frac{W}{2} \cdot \delta_1 + \delta_2 \cdot$$

Equating the expressions of external and internal work for the three beams of the two cell girder gives an equation in (T_t) and (T_b) . Another equation in the two terms $\frac{W \cdot b}{2} = T_t + T_b$. The two equations give finally the respective values of the two couples (T_b) and (T_t) in the different cases of loading A, B and C. These represent the shares of the torsional couple

$T = W \cdot \frac{b}{2}$ taken by the two cell box girder on torque bending respectively.

Hence the share of every girder in bending (i.e. $\frac{W}{3} + \Delta W$, $\frac{W}{3}$ and $\frac{W}{3} - \Delta W$) can be calculated.

Finally, the critical load of the two cell box girder can be obtained from the three corresponding shares of the three components, the Beams I, II and III.

CHAPTER 6

COMPARISON

A better understanding of the previously proposed methods of analysis for the calculation of the collapse load of box girder bridges could be obtained by comparing them with some of the results recorded from the experimental tests.

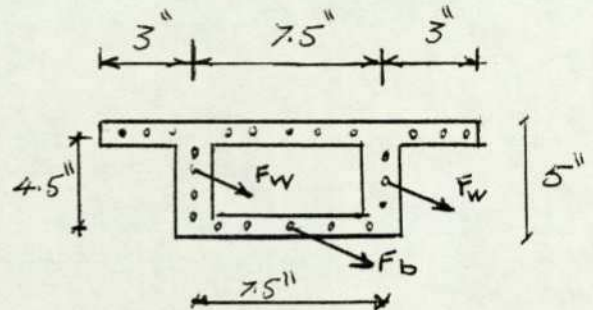
In the following, examples of the tests carried out for the different groups mentioned in Chapter 4 will be compared with the proposed theoretical methods.

Group A

This group deals with single cell box girders. The cross section of a typical box can be seen from the diagrams shown in Chapter 4. For simplicity, some of the forthcoming diagrams will be drawn in the form of centre lines of web and flanges respectively.

Upper Bound Approach

Consider the Figure shown
and using the previous
notations.



$$c' = 4.5'', b = 7.5''$$

$$\delta y = 68452.71 \text{ lb/in}^2$$

$$\text{area of bar} = 0.0122 \text{ in}^2$$

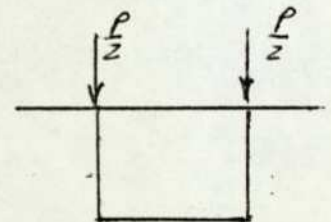
$$F_b = \text{Force in bottom steel at yield} = (\delta)(A)(\text{No. of Bar}) \\ = 6680.98 \text{ lb}$$

$$F_w = \text{Force in steel in one web} = (\delta)(A)(\text{No. of Bar}) \\ = 3340.49 \text{ lb}$$

I Case of Symmetrical Load

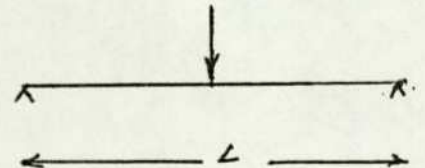
$$P = \frac{4}{L} (F_w + F_b) c'$$

$$P = 3006.441 \text{ lb}$$



The experimental result $P_{ex} = 3000 \text{ lb}$

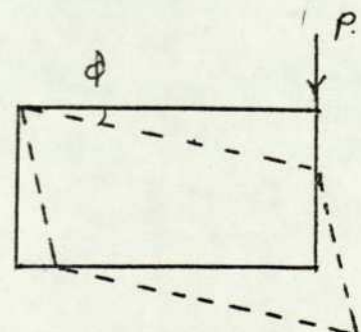
$$\frac{P_{ex}}{P} = \frac{3000}{3006.41} = 0.9918$$



II Case of Unsymmetrical Load

Mechanism of Failure A

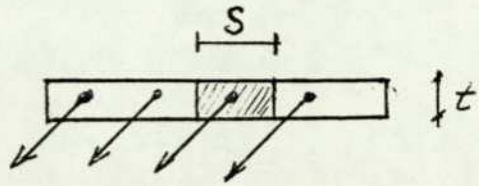
$$P_A = \frac{2c'}{L} (F_w + F_b) + \frac{4L}{b} mc$$



$$mc = (\delta y)(A)(\text{spacings})\left(\frac{t}{2}\right)$$

$$= 69.59 \text{ lb in/in}$$

$$P_A = 3730.1005 \text{ lb}$$

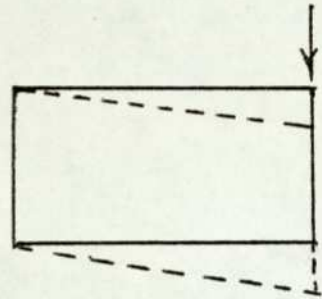


Mechanism of Failure B

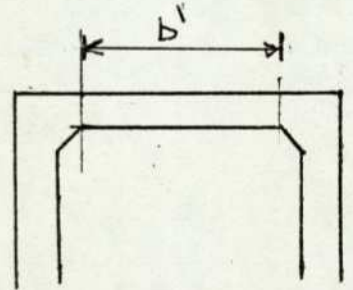
This was the mechanism observed during the test, as there was no lateral movement in the box.

$$P_B = \frac{2c}{L} (F_w + F_b) + \frac{2L}{b} mc$$

$$P = 2616.6605 \text{ lb}$$



A better adjustment for the position of the corner hinges as mentioned before could be as shown in the diagram. As the flanges are thinner and weaker compared to that of the web, hence the hinges would *occur* by the end of the *haunches* in the flange. Hence,



$$P_{B'} = \frac{2c}{L} (F_w + F_b) + \frac{2c}{b'} mc$$

$$P_{B'} = 2895.02 \text{ lb.}$$

P_{ex} = load obtained from the experimental results
 = 3000 lb.

$$\frac{P_{ex}}{P_A} = \frac{3000}{3730} = 0.8$$

$$\frac{P_{ex}}{P_B} = \frac{3000}{2895.02} = 1.03$$

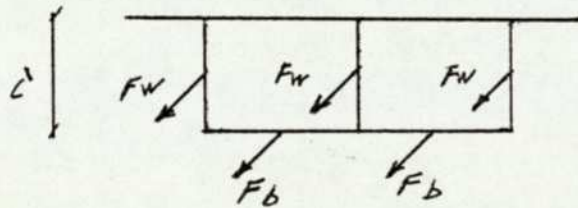
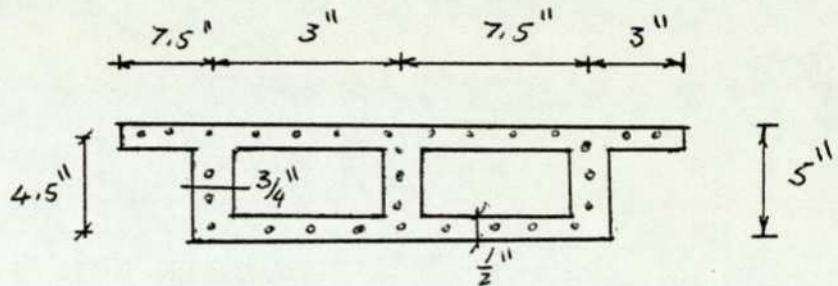
It can be seen from the two results obtained from Mechanism A and that from Mechanism B, that the first exaggerates the work done by the hinges, while the latter coincides with the results obtained from the test.

The experimental results obtained from the case of a symmetrical load and that of an unsymmetrical load are the same. This shows that the failure due to bending (yield of longitudinal steel) is dominant and that the section is quite rigid. The results obtained by the theoretical upper bound approach are very near those obtained from the experimental results, which strengthens the idea of predicting collapse loads by this method for single cell box girders.

Group B

This group deals with double cell box girders and was thoroughly explained in Chapter 4. (Boxes B1, B2 and B3).

Full details of their cross sections can be seen in Figure

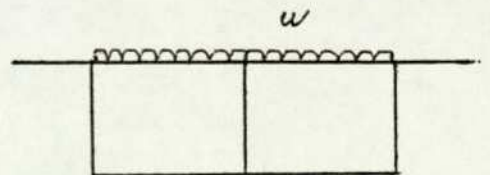


Upper Bound Approach

I Case of Symmetrically Applied Load

$$W = \frac{4}{L^2} c' (2F_b + 3F_w)$$

$$= 77.143645 \text{ lb/in}$$



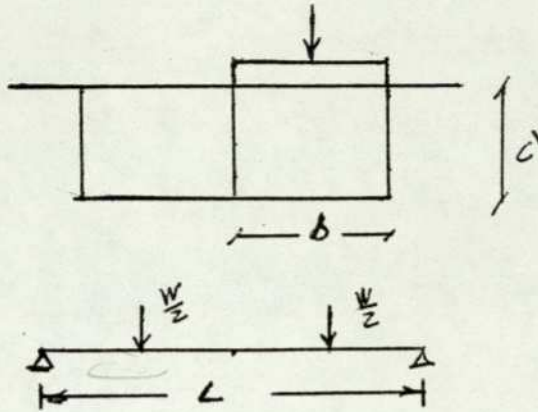
$$w_{\text{pressure}} = 4.89 \text{ p.s.i.}$$

$$W_{\text{ex}} = \text{experimental result obtained} = 4.87 \text{ p.s.i.}$$

$$\frac{W_{\text{ex}}}{w} = \frac{4.87}{4.89} = 0.99591$$

II Case of Unsymmetrical Load

This was done by applying a ^{body} ~~bay~~ of load on one of the cells of the double cell box as shown.



$$\frac{W}{4} \cdot x \cdot b \left(\frac{1}{L_1} + \frac{1}{L_2} \right) = \frac{F_w + F_b}{2} \left(\frac{1}{L_1} + \frac{1}{L_2} \right) \frac{bc}{2} + 2 mc \cdot L$$

$$\text{where } L_1 = 35.5'' \quad c = 4.5''$$

$$L_2 = 60.5'' \quad b = 5.75''$$

$$x = 31.75 \quad F_w = 3340.49$$

$$\frac{F_b}{2} = 5845.8614$$

$$W = 701.795 \text{ lb.}$$

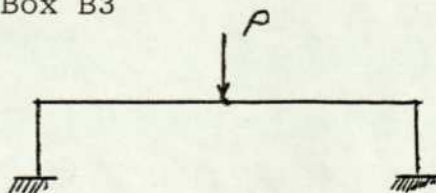
$$W_{\text{ex}} = \text{experimental results obtained} = 5400 \text{ lb}$$

$$\frac{W_{\text{ex}}}{W} = \frac{5400}{701.785} = .77$$

III Case of Column Support Box B3

$$P_u = 7175.8783 \text{ lb}$$

$$P_{\text{ex}} = 6200 \text{ lb}$$

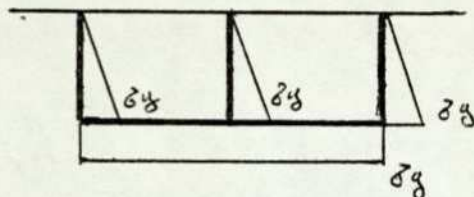


$$\frac{P_{\text{ex}}}{P_u} = \frac{6200}{7175.8183} = 0.864$$

Lower Bound Approach

A given lower bound for each of the previous double cell box girders could be calculated by assuming a complete yield of the bottom steel and a yield of the bottom steel of the web decreasing linearity to zero and the top (R) of the top flange.

The stress distribution along the box would take the form shown in the diagram.



$$\text{L.B for Box B1} = 4.1164$$

$$\text{L.B for Box B2} = 4780.04430 \text{ lb}$$

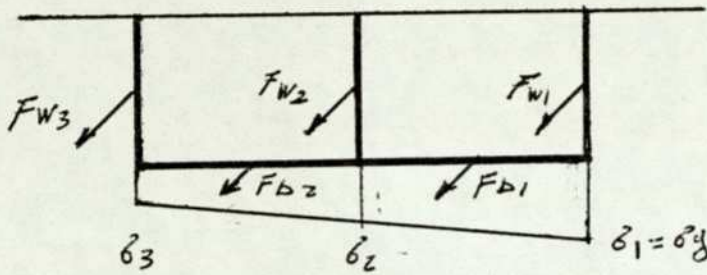
$$\text{L.B. for Box B3} = 4571.7289 \text{ lb}$$

$$\text{Ratio B1} = \frac{4.1164}{4.87} = 84.53$$

$$\text{Ratio B2} = \frac{4780.044}{5400} = 88.52$$

$$\text{Ratio B3} = \frac{4571.7289}{6200} = 0.7374.$$

Using the lower bound described in Chapter 5 and the calculated ratio of stress along the cross section of the box, the stress distribution would take the form shown in Figure



$$Fb_1 = (0.9864273) \delta y \cdot Ast \ fb$$

$$Fb_2 = (0.9592819) \delta y \cdot Ast \ fb$$

$$\left(\frac{W}{2}\right) \frac{(31.65)}{c} = \frac{(Fw_1 + Fw_2 + Fw_3)}{2} + Fb_1 + Fb_2$$

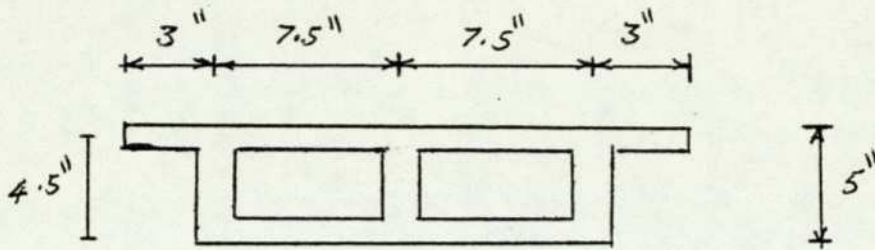
$$W = 3915.1293$$

$$W_{ex} = \text{experimental results} = 5400$$

$$\frac{W}{W_{ex}} = \frac{3915.1293}{5400} = 0.7250$$

Group C

This group deals with a continuous double cell box over spans, the cross section of which was as explained in Chapter 4.



$$W = \frac{c'}{L} 2 \quad 2(Fb_c + 1.5Fw_c) + (Ft_A + Fw_A)$$

$$W = 6960 \text{ lb}$$

$$W_{ex} = 4750$$

$$\frac{W_{ex}}{W} = \frac{4750}{6960} = 0.68.$$

CHAPTER 7

CONCLUSION

It can be seen from the previous work done in Chapter 4 which dealt with the experimental work carried out, and Chapter 5 which dealt with the assumed theoretical approaches, that the upper bound limit approach which is based on the final failure mechanism of the structure is in good agreement with the experimental one.

The small discrepancies which might occur in some results, mainly the higher values of the theoretical upper bound results rather than those of the experimental results are mainly due to the exaggeration of the proposed work done from the corner hinges and the calculation of the corresponding corner moment which depends on the exact location of the steel reinforcement within the plate thickness, which is something rather difficult to check.

The method of loading which was either in the form of a uniform load or, by the use of a standard boggy load, was done to represent the actual type of load which is used in design codes loading which is more representative of the real loads on bridges. As the application of single point loads would be rather unrealistic and in any case would cause local failure, these in the main have been avoided in this present work.

The different tests carried out on single cell boxes and double cell boxes under different loading conditions showed that the value of the critical load did not differ in the case of applying a symmetrical load to that of applying an unsymmetrical one for these boxes. That was due to the high torsional rigidity of the boxes, mainly perhaps due to the fact that the models were rather thick. But on the other hand, these thicknesses were of the order of those which are not uncommon in practice.

However, in the case of the continuous twin cell bridge the column head only supported the central portion of the transverse bridge deck and thus torsionally overall the bridge was less stiff. Under these circumstances an eccentric point load did cause premature local failure.

The proposed method of lower bound approach is much lower than the experimental ones and are smaller by (25% - 30%). This is due to the different assumptions used during the calculations and mainly due to the assumed area where the first signs of yield occur.

It would seem that for the majority of twin cell boxes the mechanisms proposed for the upper bound would be sufficient. However, the following further lines of research would need to be pursued before this claim could be completely substantiated.

(1) Different span/width ratios should be examined.

(2) Experimental investigations of the structural behaviour of three and four cell cross sections should be carried out.

(3) Further investigations of continuous bridges with different types of intermediate supports should be carried out.

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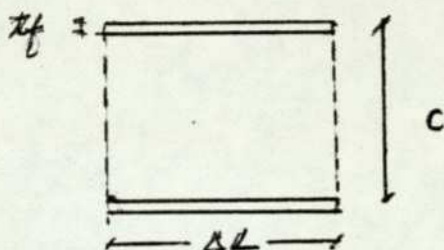
APPENDIX

APPENDIX I

Calculation of moment of inertia for ideal cross girders.

For a large number of cross girders, each of which
(ΔL)

$$I_t = \frac{1}{6} \cdot \Delta L \cdot (tf)^3$$



Hence, moment of inertia of ideal cross girder

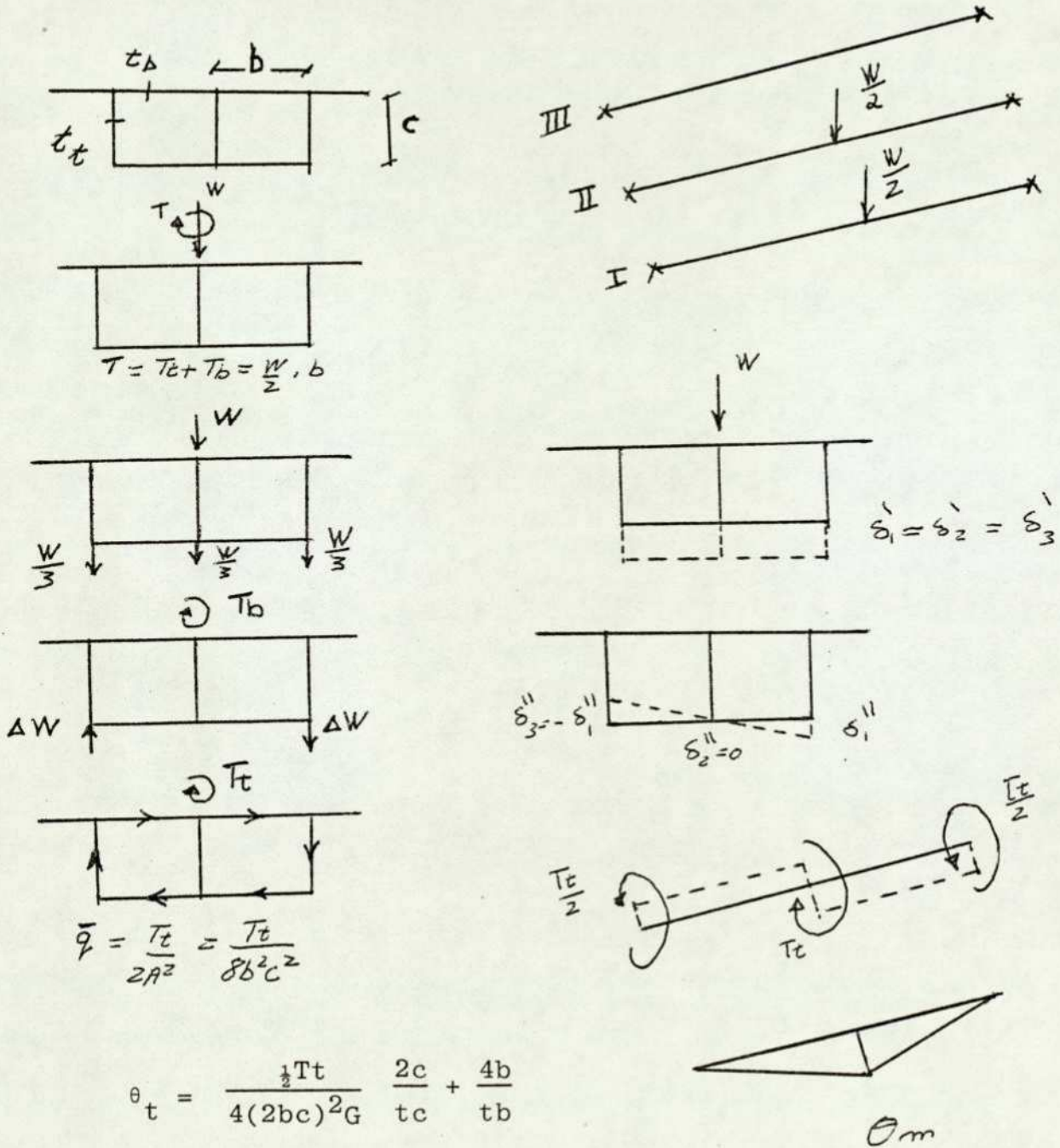
$$\begin{aligned} I_i &= \frac{I_t}{m} \sum \delta_x \\ &= \frac{(f)^3}{6 \delta_m} \int_0^L \delta_x \cdot dx \end{aligned}$$

$$\delta_x = \frac{x}{4EI} \left[\frac{L^2}{4} - \frac{x^2}{3} \right]$$

$$\delta_m = \frac{L^3}{48EI}$$

APPENDIX 2A

Assuming two point loads acting on beam I and beam II as shown in Figure and cross section dimensions of box shown in Figure



$$\theta_t = \frac{\frac{1}{2} T t}{4(2bc)^2 G} \frac{2c}{t_c} + \frac{4b}{t_b}$$

$$\theta_{r/l} = \theta_t \cdot \frac{L}{2}$$

$$\delta_1' = \delta_2' = \delta_3' = \frac{WL^3}{48EI_{\text{total}}}$$

$$\delta_1'' = \frac{(\Delta W)L^3}{48E \frac{1}{3}}, \quad \Delta W = \frac{Tb}{zb}$$

$$\delta_1 = \delta_1' + \delta_1''$$

$$\delta_2 = \delta_2'$$

$$\delta_3 = \delta_3' - \delta_3''$$

External work done

$$W_{ex} = \frac{1}{2} \cdot \frac{W}{2} \delta_1 + \delta_2$$

Internal work done

$$W_{IN} = W_t + W_b$$

$$W_t = \frac{1}{2} \cdot Tt \cdot \theta_m$$

$$W_b = \frac{1}{2} \frac{W}{3} + \frac{Tb}{2b} \delta_1 + \frac{1}{2} \frac{W}{3} \delta_2 + \frac{1}{2} \frac{W}{3} - \frac{Tb}{2b} \delta_3$$

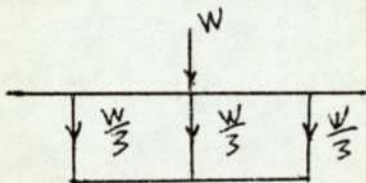
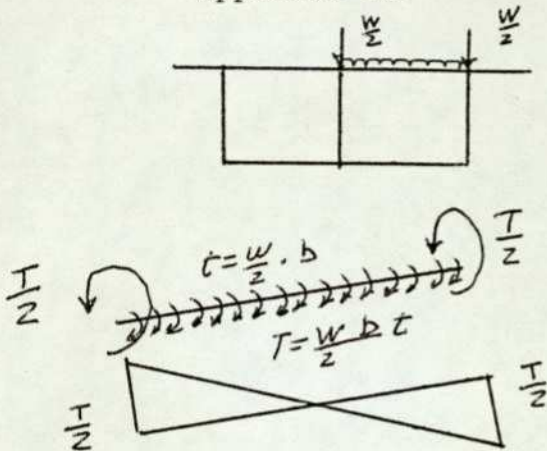
$$W_{ex} = W_{in} \quad (1)$$

$$T = Tt + Tb = \frac{Wb}{2} \quad (2)$$

By solving these two equations the ratio of $Tt : Tb$ could be found.

APPENDIX 2B

Considering the assumed system of loading shown in figures with the same cross section as shown in Appendix 2A.

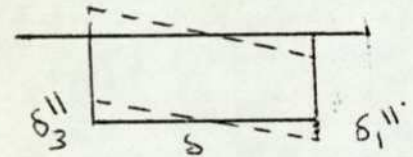
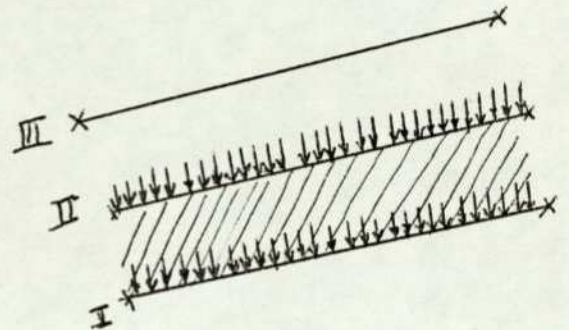


External Work

$$W_{ex} = W_{ix} + W_{2x}$$

$$W_{1x} = \frac{W}{2\pi} \cdot \frac{5}{384} \cdot \frac{\frac{W}{3} + \frac{Tb}{2b} \cdot L^3}{\frac{EI}{3}}$$

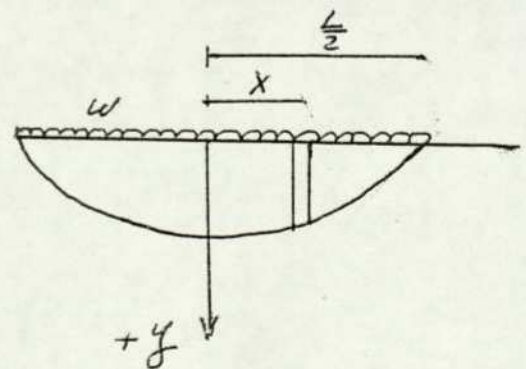
$$W_{2x} = \frac{W}{2\pi} \cdot \frac{5}{384} \cdot \frac{\frac{W}{3} \cdot L^3}{\frac{EI}{3}}$$



$$Tb = \Delta W \cdot L2b$$

$$\Delta W = \frac{Tb}{2bL} / \text{unit length}$$

$$\delta_1'' = \frac{5}{256} \cdot \frac{Tb}{b} \cdot \frac{L^3}{EIb}$$



General Work Done

$$\Delta w_x = \frac{1}{2} \cdot w \cdot dx \cdot \delta_x$$

$$= \frac{1}{2} \cdot w \delta_m \cos \frac{\pi x}{L} \cdot dx$$

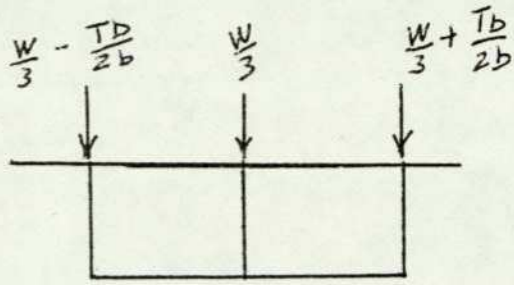
$$w_x = \frac{1}{2} \cdot w \cdot \delta_m \cos \frac{\pi x}{L} \cdot dx$$

$$w_x = \frac{WL}{\pi} \delta_m$$

$$= \frac{W}{\pi} \delta_m$$

Internal Work

$$W_{in} = w_B + w_T$$



$$w_B = \left(\frac{W}{3} + \frac{Tb}{2b}\right)^2 \frac{5}{384} \frac{L^3}{EI/3} \frac{1}{\pi} + \left(\frac{W}{3}\right)^2 \frac{5}{384} \frac{L^3}{EI/3} \frac{1}{\pi} + \left(\frac{W}{3} - \frac{Tb}{2b}\right)^2 \frac{5}{384} \frac{L^3}{EI/3} \frac{1}{\pi}$$

$$\Delta\theta = \frac{Tt \left(\frac{1}{2} - \frac{x}{L}\right)}{4(2bc)^2 \cdot G} \left(\frac{2c}{tc} + \frac{4b}{tb}\right) dx$$

$$\theta_x = \int_0^x \Delta\theta$$

$$\theta_m \text{ at } x = \frac{L}{2}$$

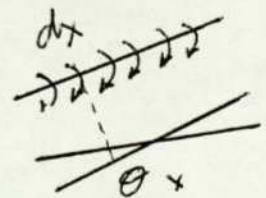
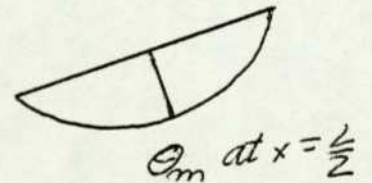
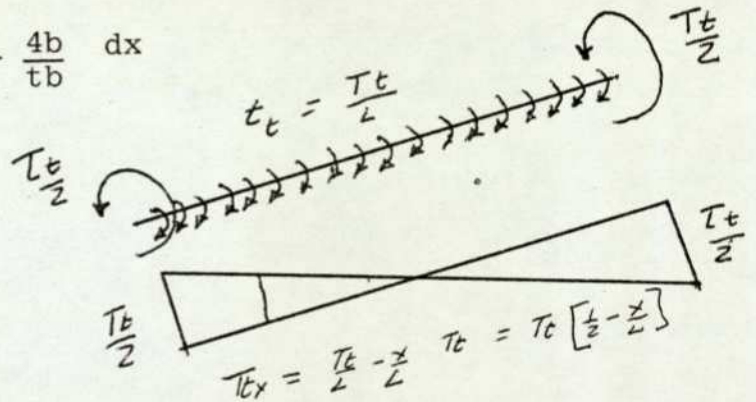
$$\Delta w_T = \frac{1}{2} (t_t) dx \theta_x$$

$$= \frac{1}{2} \left(\frac{Tt}{L}\right) dx \theta_x$$

$$w_T = \int_0^L \delta w_T$$

$$W_{in} = W_{ex} \quad (1)$$

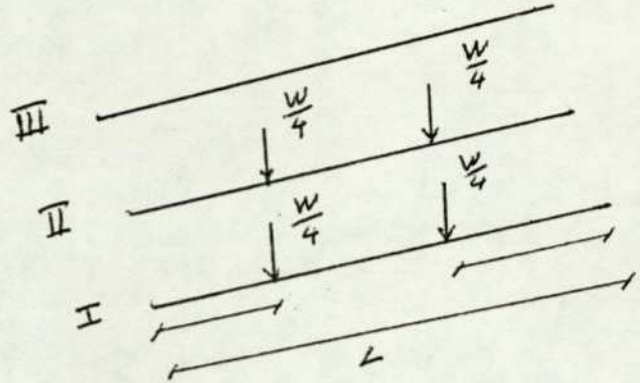
$$T = Tb + Tt = \frac{wb}{2} \quad (2)$$



By solving these two equations we obtain the values of Tb and Tt .

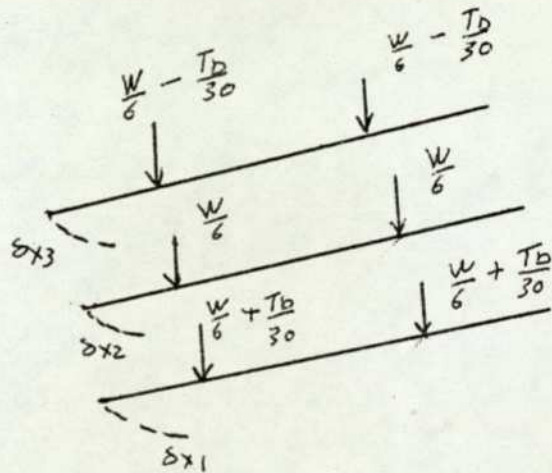
APPENDIX 2C

Considering the case of loading shown in Figure



Effect of W and T_v as shown in Figure

$$\delta x = (0.0001232)W'$$



External Work done

$$W_{ex} = \frac{1}{2} \times 2 \times \frac{W}{4} (0.0001232) \left(\frac{W}{3} + \frac{T_b}{15} \right) + \frac{1}{2} \times 2 \times \frac{W}{4} (0.0001232) \frac{W}{3} .$$

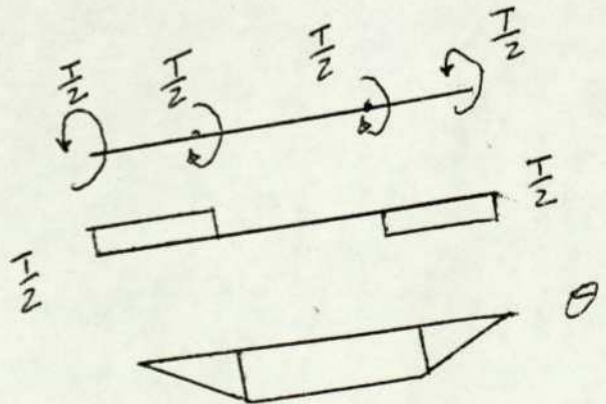
Effect of T_t

$$\theta_o = \frac{T}{4A_m^2 G} \frac{ds}{t}$$

$$Q_x = 31.75 \theta_o .$$

Work done to T_t

$$W_T = \frac{1}{2} . T_t \theta_x .$$



Work done due to bending

$$WB = (0.0015486)W^2 + (0.0000412) Tb^2$$

$$\text{Internal work done} = W_{in} = WB + WT$$

$$W_{ex} + W_{in}$$

$$T = Tt + Tb = 3.75W$$

By solving those two equations a relation between Tt and Tb could be obtained.