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DESIGN OF CLOSE-RANGE PHOTOGRAMMETRY FOR
PRECISION, RELIABILITY AND SENSITIVITY

by

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*A Thesis submitted for the Degree
of Doctor of Philosophy in the
Department of Civil Engineering
at The City University*

THE CITY UNIVERSITY
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ABSTRACT

In the measurement of structural deformation by analytical close-range photogrammetry, design is a necessity. The design of precision, reliability and sensitivity for photogrammetry alone and photogrammetry with measured distances is studied. The methods which have been developed for design based on the three criteria above mentioned can be used to meet the specifications in many deformation surveys.

The problem of optimal design can be classified in terms of four interconnected problems: zero-, first-, second-, and third-order design. Zero-order design problems have been solved by adopting the free bundle adjustment method (inner constraints). As a "one-number" indicator of the precision, for sake of comparative studies, the mean variance of the object point co-ordinates has been adopted. For reliability assessment, the redundancy number lying between 0 and 1 has been discussed and shown to be advantageous over other measures especially in pre-analysis studies.

Of primary concern in the design of a network is the sensitivity of that network for the detection of point movements at some given level of statistical significance.

A new method has been developed dealing with the sensitivity of a series of formulated deformation models and successfully applied to the different simulated networks.

The main conclusions are that (1) the more cameras used, the more precise the network becomes; (2) not only the number of cameras but also the configuration of the cameras affects reliability in a significant way; (3) the higher the number of cameras used, the less sensitive the network is to hypothesised deformation; and (4) the addition of distances, measured by the conventional methods, to a large degree, does not significantly improve the results of the precision, reliability and sensitivity. This leads to the advocacy of the argument that photogrammetry alone, in deformation measurement, can be used without any survey measurements.

NOTATION

Most of the symbols and abbreviations used in this thesis are defined at their first usage in the text. Where a symbol has only a special localised meaning it is defined in the text and not included in this list.

ℓ	vector of observations
x	vector of unknown parameters
x^0	vector of approximate values of x
Δx	vector of corrections to x^0 to give x
A	design matrix
ℓ^0	vector of computed values of the observations, $\ell^0 = F(x^0)$
$C_{\ell\ell}$	covariance matrix of the observations, $C_{\ell\ell} = \sigma_0^2 W^{-1}$
σ_0^2	a priori variance factor
$\hat{\sigma}_0^2$	a posteriori variance factor
W	weight matrix of the observations
$C_{xx}^{\wedge\wedge}$	covariance matrix of the unknown parameters, $C_{xx}^{\wedge\wedge} = \sigma_0^2 Q_{xx}^{\wedge\wedge}$
$Q_{xx}^{\wedge\wedge}$	cofactor matrix of the unknown parameters, $Q_{xx}^{\wedge\wedge} = C_{xx}^{\wedge\wedge}$ if and only if $\sigma_0^2 = 1$
$\hat{x}$	least squares estimate of x
b	vector related to the observations, or vector of observational discrepancies, $b = \ell - \ell^0$
v	vector of corrections to the observations or, vector of residuals
$Q_{vv}^{\wedge\wedge}$	cofactor matrix of residuals
n_s	number of cameras or photographs
n_0	number of object points

σ^2	variance
σ	standard error
σ_m^2	mean variance of object point co-ordinates
tr	trace
H_0	null hypothesis
H_A	alternative hypothesis
α	probability of type I error
β	probability of type II error
r	redundancy number
c	sensitivity parameter
χ^2	chi-square distribution
F	Fisher distribution
$E\{x\}$	expected value of x
x^T	the transpose of x
Q^-	a generalised inverse of Q
Q^+	Moore-Penrose inverse of Q
Q^{-1}	standard Cayley inverse of Q
I	identity matrix
deg	degree
rad	radian

CHAPTER 1

INTRODUCTION

1.1. Background

The photogrammetric network is essentially formed through the processes of three-dimensional spatial resection and intersection, the resection phase being the determination of the position of the camera perspective centres and the attitude of the camera axes, and the multi-ray intersections being utilised to determine object point positions. Under the most rigorous restitution procedures, these two phases are carried out simultaneously using a method termed the bundle adjustment.

A photogrammetric network shares many common features with three-dimensional geodetic networks in Euclidean space. Therefore techniques employed in the design optimisation of monitoring networks in geodesy are applicable to photogrammetry.

Due to the complexity of the general problem of optimisation of design, it is convenient to consider it as being made up of four distinctly different problems originated by Grafarend (1974) of which the first two are especially important for this study.

(i) Zero Order Design - the datum problem, i.e. the choice of the reference system.

(ii) First Order Design - the configuration problem, i.e. the selection of the network configuration.

(iii) Second Order Design - the generalised problem, i.e. the selection of the weights of observations.

(iv) Third Order Design - the densification problem, i.e. the improvement of an existing network.

The zero order design problem involves the choice of an optimal reference system for co-ordinates, given the object points, the photogrammetric network and the precision of the observations.

The first order design problem is concerned with the search for an optimal network geometry, given both the precision of the observations and criteria for the structure of the covariance matrix of the unknown parameters.

In the second order design problem the covariance matrix of the unknown parameters $\hat{C}_{xx}$ ($\hat{C}_{xx} = \sigma_0^2 \hat{Q}_{xx}$, where σ_0^2 is the a priori variance factor, and $\hat{Q}_{xx}$ is the cofactor matrix of the unknown parameters and $\hat{Q}_{xx} = \hat{C}_{xx}$ when $\sigma_0^2 = 1$) and the design matrix A are known. What is wanted is the weight matrix, W , required for the observations in order to achieve the ideal covariance matrix (often called criterion matrix) of the unknown parameters.

The third order design problem is associated with the improvement of an existing network by alteration of either or both of A and W . For example, this type of problem includes the densification of an existing network by the addition of new stations and/or new observations.

Before seeking any solution to the design problems, the required quality of the proposed network should be fixed. The most important criteria for a monitoring network are precision, reliability, and sensitivity.

1. Precision - This is a numerical assessment of the influence of the random observational errors on the estimated parameters (e.g. co-ordinates).
2. Reliability - For a network, reliability can be defined as its ability to detect gross errors in the observations and the determination of the effect of undetected gross errors on the co-ordinates.
3. Sensitivity - As the main task of monitoring networks is the determination of movements or the proof of the non-existence of movements, the ability to test specific deformation models, given by the sensitivity of the network (first introduced by Pelzer, 1972), is of further importance. For computation of sensitivity measures, some prediction of the movements of interest must be possible.

The last criterion, namely, sensitivity, may be more important than either precision or reliability because the sensitivity analysis produces results which can easily be explained to, and understood by people unfamiliar with the methods and principles of surveying and photogrammetry.

Clearly, if a design of a network is required, then there is need for investigating the costs. However, if the optimal design is sought, a search must be made for the observation scheme which

will meet the precision, reliability and sensitivity requirements with the least cost. Obviously this cannot be undertaken unless the proposed scheme is costed. This aspect of detailed cost analysis is out of the scope of this thesis, although consideration is given to it.

There are two approaches to the solution of the aforementioned design problems. They can be solved for in an iterative (simulation) manner or by a direct (mathematical) solution. Using the simulation approach, the designer first solves for $\hat{C}_{xx}$ using a first approximation to a good solution to the problem. Then the differences between the computed and desired $\hat{C}_{xx}$ matrix are derived and the first approximation is updated. This process continues until the solution provides a $\hat{C}_{xx}$ matrix close enough to the desired one. The simulation technique has been greatly improved by the use of computer interactive graphics which allow the designer to draw a proposed network on a graphics screen and to have a visual picture of how its precision relates to the design specifications (e.g. Krakiwsky et al, 1982). Moreover, Cross and Whiting (1982) have automated the technique for level networks (Fagir, 1984). However, the main disadvantage with this method is that the optimal solution may not be achieved in practice, and these systems do not employ all the criteria necessary for the design of truly optimum monitoring networks. It is important to notice that a network or part of it may be precise without being reliable at all (Ashkenazi and Crane, 1982).

The direct approach in which the required information is solved for mathematically is intuitively more pleasing. It does, however, have several problems. The W matrix (second order design problem) that results is generally fully populated and is frequently singular. Both of these properties are in direct conflict with reality (Krakiwsky et al, 1982). To the knowledge of the writer, no mathematical (analytical) optimisation has been carried out for geodetic or photogrammetric networks, which uses reliability or sensitivity as target functions.

1.2. Objectives and Methodology

Like any engineering project, a close-range photogrammetric project should be planned and designed to produce the best solution to a problem. The recent sudden collapse of Starra Dam in Italy (July 1985) indicates the practical importance of a properly implemented monitoring survey.

There is a number of methods and instruments to detect and monitor deformation in structures. Some of these instruments, inclinometers, extensometers, strain gauges, etc., are built into the structures and provide information about their internal changes and conditions. The main disadvantages of the use of such instrumentation for monitoring deformations are:

1. Costly maintenance.
2. Difficulty of interpreting the large amounts of data supplied,
- and 3. One-dimensional information (e.g. case of extensometers) is often provided.

On the other hand, ground survey (e.g. triangulation and/or trilateration, levelling) are often used to accomplish the data gathering needed for monitoring surface structural deformations. A number of targets placed on the surface of the structure whose positions, carefully chosen, are determined by using one or more of these methods. Such a survey is repeated at predetermined intervals and in order to quantify the deformations, positions of the targets at each time are compared. At least two epochs are necessary. In general, no statement can be made about the moment of occurrence of the deformation. It should be noticed that one- or two- or three-dimensional information is the end product of such methods, however, they have their limitations especially in the following situations:

Firstly, when large number of targets on the structure have to be recorded.

Secondly, when hazardous or inaccessible positions are to be monitored.

Finally, if the rate of deformation is rapid, deformation inevitably would occur during the survey.

Consequently, close-range photogrammetry incorporated with a few spatial distances is thought to be the most economical alternative to the above mentioned methods. The choice of the distances in the object space is made due to the fact that it is more easy to measure distances than angles or height differences (e.g. in offshore structures or oil extraction platforms).

A photograph near-instantaneously records almost an infinite number of points, hence replacing hundreds of angular measurements.

In such a case, three-dimensional information about the structure as a whole can be provided and deformation is not likely to occur during the relatively short time of photography.

Close-range photogrammetry is based on the basic theoretical concept that the photograph, being a perfect plane, is a central projection of the object. Implicit in this concept is the condition of collinearity of the image point, the projection centre, and the object point. In reality, however, this collinearity condition is not exactly met because of the following reasons:

1. Lens distortion owing to the fact that when a light ray traverses from the object to its image it is deflected by the media through which it passes.
2. The image point is displaced from its theoretical position in a plane because of (a) film shrinkage or expansion, and (b) lack of film flatness.
3. Systematic and random errors in data reduction due to the instrumental errors of the photogrammetric instruments and the human factor.

To overcome such pitfalls, the camera supposed to be in use in this pre-analysis study, Zeiss (Jena) UMK 10/1318 available at The City University, is a metric camera. Most close-range cameras operate with glass plates which are known to be generally stable emulsion bases within narrow limits. Therefore, glass plates of format (130 x 180 mm) are considered. The best photogrammetric solution is attained when all systematic errors are corrected for and the random errors are minimised. Accordingly, in the analytical

method of close-range photogrammetry, the mathematical procedure followed to correct the image co-ordinates for systematic errors in image positions on the photograph is ascribed as image refinement and the co-ordinates are referred to as the refined image co-ordinates which are then used in the collinearity equations discussed in Chapter 4.

The free bundle adjustment approach (inner constraints method) has been adopted as it provides a very useful way of making a priori estimates of precision so that a particular optimum configuration of camera positions can be arrived at (Cooper, 1981) and as a solution to the zero order design problem.

Such a method has the following characteristics:

- (i) a least squares solution
- (ii) a minimum norm solution
- (iii) minimum trace of the covariance matrix of the unknown parameters.

In addition, from practical standpoint the following two points are made:

- (a) the object point co-ordinates which are of primary importance can be constrained without bothering about the camera stations.
- (b) the alternative, namely, the fixed (constrained) network adjustment is invalid due to the false assumption that the selected object points defining the co-ordinate system are stable. Especially in deformation analysis such stability is questionable.

At the present time, the detection of smaller movements has become critical for ensuring the safety of structures and in order to satisfy the increased demands placed on the monitoring of structures in a more reliable way an attempt is made to achieve optimal design of precision, reliability and sensitivity for deformation monitoring networks.

The data acquisition in close-range photogrammetry is defined by:

1. the camera in use (metric or non-metric).
2. the configuration of data acquisition, or the arrangement of the camera stations with respect to each other and with regard to the object space and it is this, coupled with the number of cameras to be used, which is of primary importance in this investigation.

The main objectives of this thesis are:

1. The development of efficient algorithms for the optimal design of precision, reliability and sensitivity of deformation monitoring networks.
2. Study of effect of incorporating survey measurements (slope distances) with photogrammetric observations in a simultaneous free bundle adjustment.
3. Investigating the relationship between the geometry of the network, i.e. its configuration and number of cameras and the three design criteria.

It is important to note that utilising a combination of both photogrammetry and survey measurements (slope distances) proved to give flexibility to the designer so as to obtain an acceptable geometry of the monitoring network for the purpose in hand. The free bundle adjustment method (inner constraints) is used in an appropriate manner in order to give rigorous a priori cofactor matrices for both the unknowns and residuals. The redundancy number is examined and shown to be a very successful tool in assessing the reliability of the networks and has a number of advantages especially in pre-analysis studies.

A new method for the design of sensitivity has been developed through a series of simulated deformation models where displacements of 10 mm are to be detected, with 95% confidence, by the simulated networks. These models are:

1. Settlement, expansion and deflexion models in the cube case study.
2. Settlement and deflexion models in the bridge case study.
3. Settlement and drift models in the dam case study.

The method proved to be very successful and gave important findings concerning the sensitivity analysis of structural deformations. These findings can be summarised as follows:

- (i) The more cameras used the less sensitive the network would be.
- (ii) The networks are found to be least sensitive to a single point movement and most sensitive to multiple point displacements.

(iii) Effect of including slope distances on the settlement model is insignificant in the cube case study while they slightly degrade the sensitivity in the bridge case study.

(iv) Incorporating slope distances has not significantly increased the sensitivity when using the deflexion and expansion models (cube case study) whereas for the deflexion model in the bridge case study the sensitivity has been slightly decreased.

Finally, the developed algorithms are successfully tested using a total of seventeen case studies.

1.3. Scope and Organisation

Chapter 2 outlines a critical review of some of the recent applications of close-range photogrammetry to engineering and structural deformation studies. Such applications cover large, medium, and small, sized structures.

Chapter 3 reports on the design criteria specified as precision, reliability and sensitivity. Among the precision criteria, the mean variance of object space co-ordinates is chosen to represent a global indicator of precision of the different configurations. To represent the reliability, the redundancy number is chosen besides other indicators.

In Chapter 4 the development of the mathematical model for the simultaneous adjustment of the photogrammetric and survey measurements is outlined. Slope distances were selected to be incorporated with photogrammetric data.

To demonstrate the suitability of the mathematical procedures developed, their application to simulated deformation analysis of a cube are addressed in Chapter 5.

The three criteria were computed from data, some of which are real and others are fictitious, acquired through application to a medium-sized bridge in Chapter 6.

Chapter 7 encompasses the results of the application of the design criteria to a typical concrete dam. Three different configurations are discussed in the photogrammetric mode only.

Chapter 8 presents the conclusions drawn from the investigation and suggestions made for future research.

The appendices are concerned with the properties of generalised matrix inverses and the development of the inner constraints approach.

CHAPTER 2

REVIEW OF SOME RECENT CLOSE-RANGE PHOTOGRAMMETRY

APPLICATIONS TO ENGINEERING

2.1. Introduction

Photogrammetry (as derived from three Greek words: photos = light; gramma = something drawn or written; metron = to measure) (Ghosh, 1979) has been defined by the American Society of Photogrammetry as "The art, science and technology of obtaining reliable information about physical objects and the environment through the process of recording, measuring and interpreting photogrammetric images and patterns of electromagnetic radiant energy and other phenomena". The photographs are commonly aerial photographs, before the advent of remote sensors, and are geometrically very close to an ideal central projection (Torlegard, 1980).

The classic and still the major application of photogrammetry is in aerial mapping wherein a sequence of stereoscopically viewed pairs of overlapping photographs taken from an aeroplane is used to generate topographic maps.

2.2. Close-range photogrammetry

Close-range photogrammetry is covered by the definition aforementioned but lies outside the specialised field of map production. Close-range means that the object to camera distance is limited. Some advocate 300 m as a maximum limit (Karara, 1985).

In close-range photogrammetry, not only central projections

(bundles and pencils of rays) are used for mathematical models of image formation, but also parallel projections, scanner generated images and others. Not only is camera film used for the sensor system but also X-rays, scanning and transmission electron microscopes, photodiode arrays with analogue or digital output are also suitable for photogrammetric measurements.

Time is the fourth dimension in close-range photogrammetry when detection of deformation of structures is required.

In any photogrammetric process, there are two major phases:-

- (a) Acquiring data from the object to be measured by taking the necessary photographs.
- (b) Reducing the photos into plans, profiles, sections or spatial co-ordinates or combinations of these.

Thus the total photogrammetric system can be subdivided into two major divisions: data acquisition and data reduction.

The data acquisition system is concerned with procuring what may be termed as the raw data or raw information, i.e. necessary and suitable photography. Whereas the data reduction system is concerned with converting the raw data or photographs into a final form suitable for the intended use of the data. The final data form may be analogue such as map or digital such as printed spatial co-ordinates.

In this research, the data acquisition tool is the metric camera (UMK 10/1318) produced by Zeiss (Jena).

2.2.1. Analogue method

The introduction of stereoscopic measurements at the turn of this century marks a significant development in photogrammetry. The application of its principle led to the development of plotting instruments. Stereometric cameras are a pair of cameras rigidly attached to a fixed base with the optical axes of the cameras being parallel to each other and both perpendicular to the base. So, the stereometric camera - plotting instruments systems were manufactured. The arrangement, then, was that the photographs taken with the stereometric camera were used in the plotting instruments for a reconstruction (or restitution) of an optical model of the object space from which measurements were taken, or plotting done, directly on the instruments.

When the optical axes of the two cameras are both perpendicular to the base, the photography obtained is referred to as the "normal case". Due to their construction, stereometric cameras are restricted to taking only "normal case" photographs (Figure 2.1).

The normal case, however, has the disadvantage that the accuracy of the co-ordinate parallel to the direction of the camera axes is three to four times less than the co-ordinate perpendicular to this direction (Marzan and Karara, 1976). It suffices for some applications but as accuracy requirements are more and more stringent, means have to be sought to at least equalise accuracies in the three dimensions. Furthermore, the traditional stereoplotting instruments are severely limited in the principal distance and size of photo-format that can be used in them (Ibid).

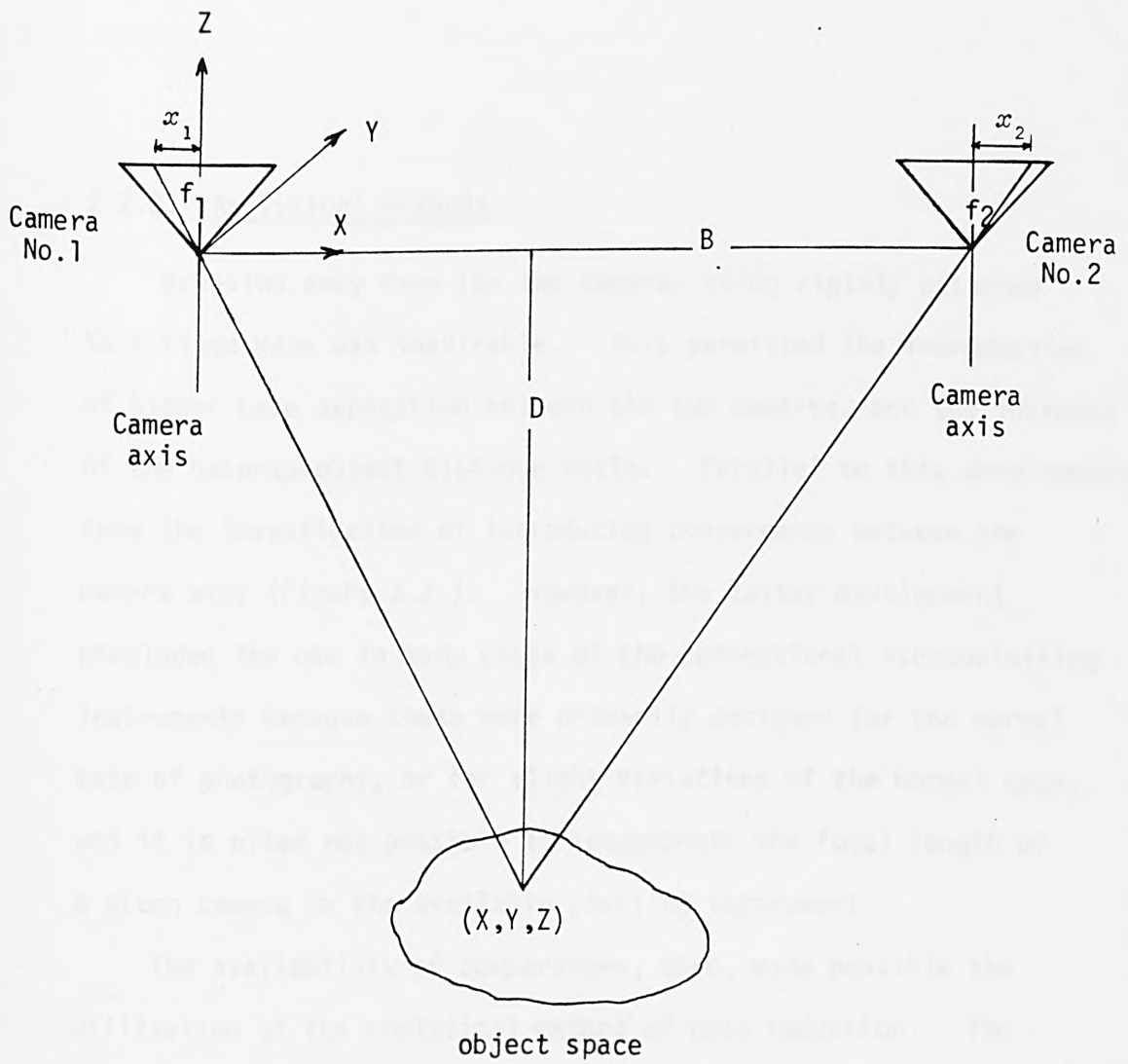


Figure 2.1. Data acquisition set-up.... the normal case of photogrammetry

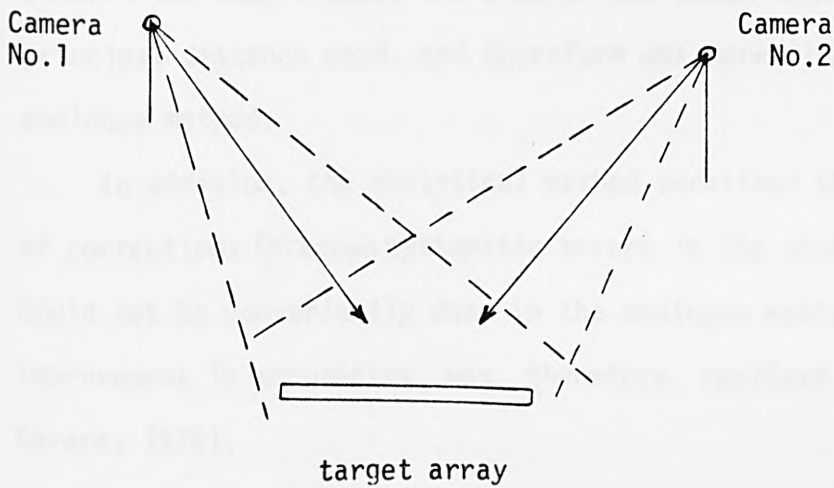


Figure 2.2. Convergent photography.

2.2.2. Analytical methods

Breaking away from the two cameras being rigidly attached to a fixed base was inevitable. This permitted the introduction of bigger base separation between the two cameras, and the increase of the base-to-object distance ratio. Parallel to this development came the investigation of introducing convergence between the camera axes (Figure 2.2.). However, the latter development precluded the use in many cases of the conventional stereoplotting instruments because these were primarily designed for the normal case of photography, or for slight variations of the normal case, and it is often not possible to accommodate the focal length of a given camera in the available plotting instrument.

The availability of comparators, then, made possible the utilisation of the analytical method of data reduction. The analytical method was not restricted by the amount of convergence between the camera axes, the size of the photo format, nor the principal distance used, and therefore was more flexible than the analogue method.

In addition, the analytical method permitted the application of corrections for known systematic errors in the photographs which could not be conveniently done in the analogue method. A great improvement in accuracies was therefore, realised (Marzan and Karara, 1976).

Usually in close-range photogrammetry the search is for great flexibility and homogeneous accuracy so the determination of dimensions of the object photographed should be an analytical

solution applying the least squares method rather than by making use of optical and mechanical devices in analogue methods. The use of photogrammetric data for further calculations, data analysis, and data banks has also promoted analytical methods(Torlegard, 1980).

2.2.2.1. Modification of methods used for air photographs

Cooper (1981) shows that when the close-range photogrammetric task resembles in several ways the normal case for topographic mapping from air photographs, it may be satisfactory to use basic functional models that have been developed primarily from aerial triangulation and to modify them to take account of the particular differences brought by close-range photography. One of the major changes from the methods of analytical aerial triangulation was the use of full terms in the rotation matrix of orientations of one camera axis in relation to the next.

2.2.2.2. Direct Linear Transformation (DLT) method

Abdel-Aziz and Karara (1971) developed an approach for data reduction not requiring the classic elements of orientation (principal point and principal distance). This approach was placed on a more rigorous foundation by Bopp and Kraus (1978). Such a method is particularly suitable for non-metric cameras.

The approach involves a direct linear transformation from comparator co-ordinates into object-space co-ordinates. Since the image co-ordinate system is not involved (on the contrary with metric camera) in this approach, fiducial marks are not needed.

DLT is a direct solution and does not involve initial approximations for the unknown parameters of inner and outer orientation of the camera.

The basic equations used in this method are (Karara, 1972):

$$x + \Delta x + \frac{\ell_1 X + \ell_2 Y + \ell_3 Z + \ell_4}{\ell_9 X + \ell_{10} Y + \ell_{11} Z + 1} = 0 \quad (2.1)$$

$$y + \Delta y + \frac{\ell_5 X + \ell_6 Y + \ell_7 Z + \ell_8}{\ell_9 X + \ell_{10} Y + \ell_{11} Z + 1} = 0 \quad (2.2)$$

where x, y - comparator co-ordinates of an image point

X, Y, Z - object-space co-ordinates of the point

$\ell_1, \ell_2 \dots \ell_{11}$ - transformation coefficients

$\Delta x, \Delta y$ - errors due to lens distortion and film deformation; the mathematical modelling of which by power series involves at least 3 coefficients (a_1, a_2, a_3).

The number of unknowns in this case is 14 ($\ell_1, \ell_2 \dots \ell_{11}, a_1, a_2, a_3$). From the knowledge of at least 7 control points and their comparator co-ordinates, the 14 unknowns can be determined.

This approach, DLT, which is originally tailored to reduce the data acquired via non-metric cameras, cannot be applied when higher precision, as in the case of deformation analysis, is required. Lenses of the non-metric cameras are designed for high resolution at the expense of high distortion which results in excluding the restitution with analogue plotters. Unlike

metric cameras, most of the non-metric cameras operate with small format film which has questionable dimensional stability particularly because no flattening mechanism is used. The parameters of orientation in DLT technique are not physical characteristics of photogrammetry, so if their estimates are sought, they have to be found out through additional computations. Finally, the non-metric cameras, confirmed by Karara (1980) cannot completely replace metric cameras in close-range photogrammetry.

2.2.2.3. Sequential Adjustment method (Fixed number of unknowns)

The two methods described above include the assumption that the co-ordinates of the control points are much more accurate than the inherent accuracy of the photogrammetry. Therefore in the least squares estimation of the parameters of the transformations, the stochastic models include the assumption that the values of the co-ordinates of the control points are fixed, error free quantities (Cooper 1981). Such an assumption is seldom justified in close-range photogrammetry of objects that are essentially three-dimensional.

One important advantage of sequential adjustment is that the observer can, after a few measurements, examine the solution and then take more measurements to improve the results if necessary.

Following Cooper (1981), suppose that the first sequence is to estimate Δ from p observation equations and associated weight matrix:

$$A_p \Delta = V_p + b_p \quad : W_p \quad (2.3)$$

The least squares solution is:

$$\hat{\Delta}_1 = (A_p^T W_p A_p)^{-1} A_p^T W_p b_p \quad (2.4)$$

and the covariance matrix of $\hat{\Delta}_1$ is $(A_p^T W_p A_p)^{-1}$.

The solution vector $\hat{\Delta}_1$ and its covariance matrix are regarded as measurements to be incorporated in the second sequence with q additional measurements:

$$\begin{bmatrix} I \\ A_q \end{bmatrix} \Delta = \begin{bmatrix} v_p \\ v_q \end{bmatrix} + \begin{bmatrix} \hat{\Delta}_1 \\ b_q \end{bmatrix} : \begin{bmatrix} A_p^T W_p A_p & 0 \\ 0 & W_q \end{bmatrix} \quad (2.5)$$

The least squares solution to the second sequence is:

$$\hat{\Delta}_2 = \left(\begin{bmatrix} I \\ A_q \end{bmatrix}^T \begin{bmatrix} A_p^T W_p A_p & 0 \\ 0 & W_q \end{bmatrix} \begin{bmatrix} I \\ A_q \end{bmatrix} \right)^{-1} \begin{bmatrix} I \\ A_q \end{bmatrix}^T \begin{bmatrix} A_p^T W_p A_p & 0 \\ 0 & W_q \end{bmatrix} \begin{bmatrix} \hat{\Delta}_1 \\ b_q \end{bmatrix} \quad (2.6)$$

Equation (2.6) after reduction and substitution of $\hat{\Delta}_1$ from (2.4) becomes:

$$\hat{\Delta}_2 = (A_p^T W_p A_p + A_q^T W_q A_q)^{-1} (A_p^T W_p b_p + A_q^T W_q b_q) \quad (2.7)$$

Now, if the solution for Δ is obtained from $(p+q)$ observation equations in one sequence, the equations, then, are:

$$\begin{bmatrix} A_p \\ A_q \end{bmatrix} \Delta = \begin{bmatrix} v_p \\ v_q \end{bmatrix} + \begin{bmatrix} b_p \\ b_q \end{bmatrix} : \begin{bmatrix} W_p & 0 \\ 0 & W_q \end{bmatrix} \quad (2.8)$$

The least squares solution to the second sequence is:

$$\hat{\Delta} = \left(\begin{bmatrix} A_p \\ A_q \end{bmatrix}^T \begin{bmatrix} W_p & 0 \\ 0 & W_q \end{bmatrix} \begin{bmatrix} A_p \\ A_q \end{bmatrix} \right)^{-1} \begin{bmatrix} A_p \\ A_q \end{bmatrix}^T \begin{bmatrix} W_p & 0 \\ 0 & W_q \end{bmatrix} \begin{bmatrix} b_p \\ b_q \end{bmatrix} \quad (2.9)$$

Equation (2.9) after reduction becomes:

$$\hat{\Delta} = (A_p^T W_p A_p + A_q^T W_q A_q)^{-1} (A_p^T W_p b_p + A_q^T W_q b_q) \quad (2.10)$$

Equations (2.7) and (2.10) are identical. Hence, the result of making an adjustment in two sequences is the same as that which would be obtained from a simultaneous adjustment.

2.2.2.4. Bundle Adjustment method

The photogrammetric network is essentially formed through the processes of three-dimensional spatial resection and intersection. The resection phase being the determination of the position of the camera perspective centres and the attitude of the camera axes, and the multi-ray intersections being utilised to determine the object point positions. Under the most rigorous restitution procedures, these two phases are carried out simultaneously using a method termed the bundle adjustment. System self-calibration is also afforded using this technique when the model is extended to include inner orientation parameters.

By virtue of the bundle adjustment method, one can reconstruct what is, in effect, a three-dimensional model of an object from

measurements made on two or more photographs of the object. The difference between the self-calibration technique and other methods is that self-calibration attempts this modelling without requiring any additional observations to be made specifically for the purpose of systematic error compensation.

The bundle method is based on the collinearity of the object point, the perspective centre and the imaged point.

Granshaw (1980) suggests that convergent multi-station photography (bundle method) may permit the recovery of the inner orientation elements, even if there are no control points, because of much lower correlation between the interior and exterior orientation elements. He concludes that the bundle adjustment is a powerful computational technique which provides the necessary flexibility which is essential in the various situations that may be encountered when co-ordinating engineering and industrial structures by photogrammetric methods.

2.2.2.5. Free adjustment method

The concept of inner accuracy introduced by Meissl (1962) allows the effect of an arbitrary set of parameters to be filtered out of a covariance matrix. If the filter parameters are selected to be rotational, translational and scaling parameters, then the inner accuracy refers to the precision of a free network adjustment (Granshaw, 1980).

Consider a bundle adjustment where the shape of an engineering structure is sought, the filter parameters are seven (in three-dimensional space). The shape is determined by purely photogrammetric

measurements without control. In a free network adjustment an attempt is made to determine a solution vector and a covariance matrix when the system of normal equations is singular due to rank defect. Any solution vector will be biased, statistically, but as only the shape defined by the co-ordinates, and not the co-ordinates themselves, is important, this is not of significance.

In a free network adjustment a solution where the trace of the covariance matrix is a minimum is chosen. The geometrical interpretation of the minimum trace is that there should be no overall translational, rotational or scaling changes from the approximate values. Thus precision estimates are referred not to particular (arbitrary) points, but to the network of points as a whole.

Despite the fact that one of the earliest applications of a free network adjustment was to the relative orientation of a photogrammetric model (Meissl, 1965), the method has received most attention in connection with geodetic networks, although one notes the wider concept of inner accuracy has been used by Ebner (1974) and Gruen (1976) in a photogrammetric context (Granshaw, 1980). The results can be determined by transformations on arbitrary covariance matrices (Meissl, 1962, 1964), generalised inverses (Mittermayer, 1972), and zero eigenvalue concepts (Mittermayer, 1973) amongst others. Other useful summaries are given by Ashkenazi (1973) and Welsch (1979). Details of generalised inverses are given in Appendix A.

2.2.3. Semi-Analytical Method

This approach is similar to the semi-analytical approach in aerotriangulation. A stereoscopic pair of photographs is mounted in a stereoplotter, and relative orientation is performed to construct a three-dimensional optical model of the object being mapped. The spatial co-ordinates of any discrete point in the model can be measured with respect to the co-ordinate system of the stereoplotter.

Linking of the overlapping models and absolute orientation to an object space co-ordinate system can be performed analytically using the appropriate analytical technique.

Since a stereoplotter is used for the construction of any optical model, the type of cameras to be used and the geometric configuration of the photography must be restricted by the mechanical limitation of the stereoplotter (Wong, 1975).

2.4. Merits and drawbacks of close-range photogrammetry

In comparison with other measuring techniques in general and to surveying in particular, close-range photogrammetry has many advantages:

- (i) the object is untouched during measurement (non-contact)
- (ii) the data acquisition is rapid
- (iii) the photographs are a permanent record
- (iv) not only rigid and fixed objects but also deformation and movement can be measured, especially inaccessible or hazardous objects.

- (v) time dependent parameters such as velocity, acceleration and frequency can be determined
- (vi) photography and evaluation are flexible and can be optimised to the project requirements as, for example, precision and reliability
- (vii) analytical methods provide a means of integration with succeeding calculations and data handling.

In order to make photogrammetry more effective and better suited to new fields of application, there are some drawbacks to be overcome.

- (i) the results of the measurement are not immediately at hand, because time is needed for processing the photographs and for evaluation
- (ii) it must be possible to photograph the object
- (iii) errors during photography and development of the film/plate can ruin the whole measuring project, so expertise is required.

2.5. Representative examples of applications

An attempt has been made to categorise the applications according to the size of the structure. Cheffins and Chisholm (1980) chose an arbitrary subdivision which classifies structures of 20 m to over 200 m as large, 2 m to 20 m as medium-sized and 0.2 m to 2 m as small.

2.5.1. Large structures

Examples of photogrammetric measurements of structures in the range 20 m to 200 m or larger are generally to be found in the Civil Engineering branches of industry, although there are notable exceptions such as in the shipbuilding industry. Very often, the engineer chooses to use those methods and equipment with which he is familiar, unless the alternative offers distinct advantages. If photogrammetry is just as good as, but not better than, an established technique, it is unlikely to be adopted because it requires expertise and equipment which are relatively unusual.

(a) Rockfill Dams

1. The Building Research Station and Hunting Surveys Ltd. were jointly responsible for the photogrammetric study of the constructional displacements of the rockfill dam at Llyn Brianne in mid-Wales (Atkinson, 1976). The dam, when completed, was 300 m wide and 90 m high (Cheffins and Chisholm, 1980). A Wild P30 phototheodolite (principal distance 163.65 mm) was used to provide stereoscopic coverage at eight different stages of construction, of both upstream and downstream shoulders. Control points were fixed on the valley sides by theodolite observations. Eighty targetted points at various levels of the dam as the construction progressed were established. Three-dimensional displacements were determined to an average accuracy of 0.05 m according to Moore (1973).

2. Brandenberger (1974) used six camera stations to obtain 16 photos of the downstream face of a rockfilled dam (at Outardes, Quebec, Canada). The photographs were taken not to provide conventional stereomodels by normal case photography but to give adequate coverage of the area of interest, given the limitation of the topography. Geodetically surveyed control points (including the camera positions) were used as constraints in the estimation of co-ordinates of the photogrammetric points. The standard error of the positions of photogrammetrically determined points was found to be of the order of 28 mm (vector) over a dam length of about 240 m (or 1/8600), relative to the control.

(b) Buildings and Structural models

1. Cooper and Shortis (1980) used stereophotogrammetry with relative and absolute orientation for measuring two large structures; elevation of south transept of St. Paul's Cathedral, London, and a tower crane under load. They concluded that the deformation was predominantly in the XZ plane, where the X-axis was roughly parallel to the facade of the building and the Z-axis was vertically upwards. Added to that, the accuracy can be improved by: first, the disposition of the cameras so as to get stronger geometrical configuration. Secondly, the case study of the crane, the crane has to be closer to a background where control can be better distributed.

2. Nooshin and Butterworth (1974) describe experimental investigation of a 1:50 scale model of a prestressed cable roof (Atkinson, 1976). The roof consists of a number of suspension cables which are connected

at one end to a reinforced concrete spatial beam and at the other end to a flexible cable which is in turn supported by two pin-ended steel pylons. A family of prestressing cables is arranged orthogonally to the suspension cables. The object of the investigation was to check the preliminary design and to provide information for the final design of the cable roof. Stereometric photography of the model and five premarked control points was taken with an Officine Galileo camera mounted on the laboratory ceiling above the model. The co-ordinates of all points of cable intersection were determined with a standard deviation of ± 0.1 mm in plan and ± 0.3 mm in elevation.

(c) Cooling Towers and Storage Tank Calibration

1. Chisholm (1977) used a Wild RC5 A aerial camera (nominal principal distance 152 mm) to check for any distortion in the shape of three cooling towers, owned by Imperial Chemical Industries Ltd. (ICI) in north-east England, from their original design shape. The use of a large format (230 mm x 230 mm) allowed fairly large scale photography of the complete extent of each tower, the tallest of which is approximately 115 m high. For each of the first two towers seven overlapping stereopairs were taken. However, for the third one 14 overlapping stereopairs had to be taken at distances varying from 25 m to 40 m. Classical triangulation methods, using a theodolite from a series of baselines around each tower, established the co-ordinates of at least six identifiable points on each stereoscopic overlap in a local origin Cartesian system. Contours defining the shape of the towers were plotted;

these were compared with the theoretical design contours. The maximum deviations were of the order of 0.3 m. The subsequent strength analysis using these data from comparisons showed that one tower would have to be demolished, one would survive with appropriate strengthening, while the third tower was relatively free from deformation.

2. Papo and Perelmutter (1980) used free network analysis of storage tank calibration. A control network of 12 points symmetrically located with respect to the tank bottom, together with the bottom centre was established. These points were used as camera stations as well. At height 14 m, 160 points were targetted to be photographed. They came to conclusion that the residuals of a free network adjustment can best disclose the existence of certain unmodelled systematic effects.

(d) Shipbuilding

Because of the need to make very large vessels, they are sometimes built in two separate parts owing to the limited size of existing berths. Cheffins and Chisholm (1980) have reported that Newton (1974) used a Galileo Santoni (nominal focal length 150 mm) with a workable format area of 110 mm x 160 mm to assess the quality control in shipbuilding using photogrammetry. A 50 m x 30 m cross section of the vessel was photographed with two overlapping photographs, each comprising 13 exposures, for each section of the ship. 27 premarked points were used as control established by traditional triangulation from a measured baseline.

Three-dimensional photogrammetric measurement was carried out in a Wild A7 stereoplotter. Some of the photogrammetric measurements were repeated using a stereocomparator and the accuracy in co-ordinate position within the plane of the mating surface was ± 3 mm.

2.5.2. Medium-sized structures

(a) Bridges

1. Christensen (1980) used stereophotogrammetry for observing displacements of a bridge loaded to failure. Vertical displacements were checked by comparison with similar results achieved by levelling to a series of rods hung up under the bridge. There was accordance between the two methods. The results of strain were inconsistent (Christensen, 1980) because the camera stations and control points were lying almost on the same circle.
2. Veress (1980) used photogrammetry for dimensional control of a curved segmental concrete bridge. He used control survey, both distances and angles, and adjusted a horizontal network by trilateration. He reached a conclusion that one of the most influential factors on achievable accuracy is the precision of a control network.
3. Cheffins and Chisholm (1980) used a Zeiss (Jena) phototheodolite (nominal focal length 162 mm) and format of 180 mm x 130 mm to photograph a low skew brick built arch carrying a commuter railway by an armoured lining which would be prepared so that it closely followed the distorted contour. 16 premarked reference points were established in the abutments at the springing of the arch and

signalised with white paint. Stereoscopic photogrammetry achieved by setting up 4 camera stations (2 at each end of the arch).

Complete profiles, which include the arch, the abutments and the roadway, were plotted photogrammetrically at a scale of 1:12.

(b) Marine propeller

A study conducted by Cooper (1979) to see how far photogrammetry could be used to determine how the profiles of the finished propeller blade surfaces deviate from the design dimensions. The propeller had six blades and was about 7.5 m in diameter. The camera used was Zeiss (Jena) UMK (nominal focal length 100 mm) and usable format size of approximately 120 mm x 165 mm. Stereoscopic photography was taken of both sides of the object, which was contained within a single overlap on each view. Four external markers were placed on the floor surrounding the propeller, and these were co-ordinated by steel band and precise level to an estimated accuracy better than 1 mm. 16 premarked control points had been defined on the aft face of the propeller and were measured with a cylindrical polar co-ordinate measuring device to an estimated accuracy of 2 mm to 3 mm (r.m.s.) (Cooper, 1979).

(c) Measurement of Car body

Cooper (1979) studied the potential of applying photogrammetry to the measurement of a car body. He used 39 premarked control points on the car body, their three-dimensional co-ordinates were measured by the manufacturer using a conventional mechanical probe with readings taken from steel scales. A strip of 6 photographs was obtained by setting the camera (UMK 10/1318) successively at

about 0.75 m intervals along a line roughly parallel to and about 1.5 m from the side of the car. Block adjustment was carried out using only 13 of the 39 measured points. The other 26 were regarded as check points. The r.m.s. in the longitudinal direction was 0.77mm, 0.34 mm in the height, and 1.88 mm in the transverse direction. He attributed the unexpected figures of accuracy to inherent random errors present in the measurements of the control which were greater than the internal random errors in the photogrammetry.

2.5.3. Small structures

The models of chemical plants are of crucial importance in both design and maintenance. These plants in general consist of pipelines of different dimensions and are mainly of complicated structure. A survey according to conventional methods often is impossible or possible only with extreme difficulties (Jaensch, 1979).

ICI used a 1000 mm x 750 mm x 500 mm model which represents the pipe system they need (Cheffins and Chisholm, 1980). The model is assembled in 24 sections which could be separated to gain access to the inside. Stereoscopic photography was taken, using a specially made Galileo Santoni camera of focal length 75 mm, from distances as close as 500 mm and complete cover of all sections of the model might need 50 to 100 stereopairs. The pair of photographs were set in the plotting instrument Galileo Santoni Stereosimplex IIB to obtain an oriented scaled model after a small adjustment. Design drawings of pipe layouts in both plan and elevation were

plotted simultaneously at 1:16 scale.

All the foregoing applications are typical examples representing the use of close-range photogrammetry in engineering and monitoring of large structures' deformations. The methods used in these examples differ from each other in the way of formulation of the adjustment technique. On the other hand, such methods are similar in having no proper or rigorous a priori planning. For most of the applications mentioned before, if a prior design criteria mostly represented by precision, reliability, and sensitivity had been performed, better results could have been attained in more economical ways.

Therefore, such criteria, addressed in detail in the next Chapter (Chapter 3) are taken into consideration as grounds for this investigation.

Rather than using the stereophotogrammetry or nearly vertical photography techniques, the free bundle adjustment procedure is adopted as it leads to more homogeneous and most precise specifications which are to be met in monitoring deformation of structures.

CHAPTER 3

OPTIMAL DESIGN OF NETWORKS

3.1. Network Design

By network optimisation it is meant the design or reconnaissance of such networks subject to criteria derived from and determined by the purpose of these networks (Baarda, 1977).

In planning an optimal multi-station photogrammetric network for some special purpose, such as for monitoring structural deformation or for determining the precise shape characteristics of an object, due attention must be paid to the quality of the network design.

Quality is usually expressed in terms of the precision and reliability of the photogrammetric network (Fraser, 1984) but it also may include aspects of economy and testability (Dodson, 1983).

Alberda (1980) hints that in the design of a network, although powerful mathematical techniques are available, it does not seem possible so far to optimise all aspects in a unified model. Baarda's criterion theory for precision gave for the first time a consistent framework for decisions to be made by geodesists when planning a network (Molenaar; personal communication).

Following the widely accepted classification scheme of Grafarend (1974) which may be thought of in terms of the fixed and free parameters of the least squares adjustment to be carried out, the interconnected problems of network design can be identified as:

- (i) Zero Order Design : the datum problem.
- (ii) First Order Design : the configuration problem.
- (iii) Second Order Design : the weight problem.
- (iv) Third Order Design : the densification problem.

3.1.1. Zero Order Design

The datum problem involves the choice of an optimal reference system for the object space co-ordinates, given the photogrammetric network design and the precision of the observations. That is, for fixed A , the design matrix, and W , the weight matrix, one usually seeks, through the selection of an appropriate datum, an optimum form of the cofactor matrix of the unknown parameters, $Q_{xx}^{\wedge\wedge}$.

It is usually required to express the spatial position of object points within a three-dimensional XYZ Cartesian co-ordinate system, however the observations (photo co-ordinates, and possibly object space distances, and angles) do not contain any information about the datum of this co-ordinate system, other than perhaps its scale if spatial distances are observed. Thus, a datum must be defined by the imposition of constraints which establish the origin, orientation and scale of the XYZ reference co-ordinate system.

Parameters of the shape of network, namely distance ratios and space angles, are determined solely as a function of the observations, and they are invariant with respect to changes in the datum, or zero-variance computational base. On the other hand, object space co-ordinates relate to the datum, and thus when the minimal constraints (Appendix B) are changed so one can expect the

solution vector and the cofactor matrix of the unknown parameters to be altered.

In situations where the datum is arbitrarily assigned as in the case of relative deformation networks in which all points are assumed unstable, zero order design can be thought of as being the process of establishing a particular zero-variance computational base. Such a base for a given network geometry yields a cofactor matrix $Q_{\hat{x}\hat{x}}$ of the unknown parameters (exterior orientation, object space co-ordinates and additional parameters, if any) which is "best" in some sense (Fraser, 1984).

The solution for x which is optimal in the sense of minimising the mean variance σ_m^2 of the object point co-ordinates (which are of main concern) is provided by a free network adjustment. Such an approach, either using the Moore-Penrose inverse technique or the method of inner constraints (Blaha, 1971) yields a minimum Euclidean norm of the object point co-ordinate corrections (and hence for the co-ordinates) as well. The latter technique need not apply to all object points, and the imposition of this implicit minimal constraint may simply refer to a chosen subset of the target array.

According to Fraser (1984), the common, computationally simpler approach of "fixing" object point co-ordinates to remove the seven network defects of translation (three), rotation (three), and scale (one) will yield a mean variance σ_m^2 for the object points, which is larger in magnitude than that obtained from the inner constraints adjustment. Difference in the precision of functions of unknown parameters (e.g. distances) derived from $\hat{x}_2$ a subset of $\hat{x}$ corresponding

to object point unknown parameters, may, however, be insignificant from a practical point of view.

An S-transformation (Baarda, 1973) could be used (Fraser, 1985) to transform both $\hat{x}_2$ and its corresponding cofactor matrix $Q_{\hat{x}\hat{x}}^{(2)}$ relating to one zero-variance computational base into their corresponding values for any other zero computational base including minimal constraints. For example, after the cofactor matrix of object point XYZ co-ordinates is computed for a datum of seven explicitly fixed co-ordinate values, the corresponding solution for $Q_{\hat{x}\hat{x}}^{(2)}$ is obtained simply by applying an S-transformation.

In close-range photogrammetric networks with dense target arrays, where a full cofactor matrix may not be sought, it is often computationally more practical to re-adjust the network with a different datum rather than applying an S-transformation as the latter (Strang Van Hees, 1982) does necessitate the computation of a full $Q_{\hat{x}\hat{x}}^{(2)}$ matrix.

3.1.2. First Order Design

The configuration problem is concerned with the search for an optimal network geometry, given both the precision of the observations and criteria for the structure of the covariance matrix of the unknown parameters. In other words, this procedure entails the finding of an optimal design matrix A given weight matrix W subject in certain cases to satisfying criteria imposed by an ideal covariance matrix. For example, a criterion may be that $Q_{\hat{x}\hat{x}}^{(2)}$ has a structure which is both homogeneous and isotropic, i.e. all point error ellipsoids are spheres of equal radius (Fraser, 1984).

The more flexible and seemingly the simplest part of network design, first order design is the most difficult problem. There are constraints on the choice of the positions of the points as these are largely dictated by topography, geology, accessibility, and aesthetics (Murnane, 1984). The locations of the target array points are determined by engineering requirements.

Although its principal component is imaging geometry, first order design also embraces such aspects as camera and target point locations and camera selection.

The most common approach to first order design is through network simulation (Fraser, 1984) and such a process is adopted in this investigation as pre-analysis is undertaken.

(a) Imaging geometry

Simulations conducted by Granshaw (1980) have clearly indicated the significant overall accuracy enhancement that can be anticipated through the use of convergent rather than "normal" photography (Chapter 2).

In numerous practical applications, the site of a photogrammetric survey can impose restraints on the selection of an ideal imaging geometry, especially in industrial photogrammetry.

(b) Number of Camera Stations

Depending on the imaging geometry adopted, the use of additional camera stations can be expected not only to improve precision, but also significantly to enhance the network's reliability. Additional imaging rays increase the redundancy in a spatial intersection, and also alter the intersection geometry.

Fraser (1984) suggests that more attention should be paid to the imaging geometry, rather than concentrating too much on obtaining a coverage of a certain arbitrary number of photographs.

(c) Image scale and focal length

According to Fraser (1984) there is a linear relationship between object point precision and imaging scale. Concerning the focal length of the taking camera, as it increases, so the geometry of multi-ray intersections tends to become more homogeneous, thus leading to a reduction in the range of object point standard errors. Kenefick (1971) proposes that long focal length cameras are less subject to the critical influence of film unflatness, however, the choice of focal length is most often limited by both camera availability and the physical layout of the survey site.

3.1.3. Second Order Design

The second order design is defined as the problem of finding optimal weights for the observations which are projected in a geodetic or photogrammetric network with given point positions (Schmitt, 1978). This problem is characterised by an unknown W and fixed A and $Q_{\hat{x}\hat{x}}$, i.e. by a solution to the matrix equation:

$$(A^T W A)^- = Q_{\hat{x}\hat{x}} \quad (3.1)$$

The generalised inverse $()^-$ becomes either $()^+$, i.e. Moore-Penrose inverse for a minimum norm solution of a free network or $()^{-1}$, i.e. standard Cayley inverse for a constrained network (Schmitt, 1980). In the area of geodetic networks, considerable research attention

has recently been directed to the analytical solution of equation (3.1) to mention but few; Cooper and Leahy (1978), Cross and Fagir (1982), Fagir (1984), Grafarend (1977), and Schaffrin (1977).

3.1.4. Third Order Design

The densification problem concerns the question how best to improve an existing network by additional stations and observations. Regarding the photogrammetric network optimisation, Fraser (1984) considers the densification problem is solved at the first order design stage due to the fact that object point precision is largely independent of target array density in networks with "strong" geometries. This problem can be considered as a special case of the first and second orders.

3.2. Quality of networks

The photogrammetric network is essentially formed through the process of three-dimensional spatial resection and intersection, the resection phase being the determination of the position of the camera perspective centres and the attitude of the camera axes, and the multi-ray intersections being utilised to determine object point positions. These two phases are carried out simultaneously in the case of bundle adjustment.

The quality of a geodetic or photogrammetric network is characterised by quantitative assessment of the influence of observational errors on the estimated co-ordinates and derived functions. If these observational errors are of known distribution (often normally distributed), then quality is appraised by the size

of errors that can be anticipated with a certain probability.

The errors in quantitative measurements can be broadly classified into the following four types (Slama, 1980):

- (i) blunders
- (ii) constant errors
- (iii) systematic errors, and
- (iv) random errors.

Therefore quality could be assumed with regard to the following:

(i) Precision

This is a numerical assessment of the effect of the random observational errors on the final results, e.g. co-ordinates.

(ii) Reliability

A definition may be given to reliability as the ability of a network to detect gross errors in the observations and the determination of the effect of undetected gross errors on the co-ordinates.

(iii) Systematic error compensation

Systematic errors in observations can be of tremendous importance in monitoring networks. The reason for this is that, if undetected, they may be mistaken for a systematic deformation (Dodson, 1983). While there is no a priori knowledge about blunders, neither about their location nor about their size, we have a fairly well documented knowledge about possible systematic errors that are likely to occur in the data. This allows us to model those anticipated systematic errors as additional parameters in the estimation model. This procedure is called self calibration.

A widely accepted strategic approach in the treatment of additional parameters is to include a fairly large set in order to be sure to cover all possible errors. Because this creates the danger of overparameterization, a procedure for the deletion of non determinable additional parameters must be incorporated (Gruen, 1985). Alternatively prior calibration is required.

(iv) Sensitivity

If a three-dimensional network is subjected to deformation the sensitivity of that network is expressed in terms of the minimum level of detectable object point movement (Fraser, 1982b). Prior information on quantity and direction of expected deformations gives important hints on network configuration and observation scheme (Welsch, 1982).

It should be pointed out that the problems of control of systematic errors is not considered here and what follows pertains only to precision, reliability and sensitivity as criteria for quality.

3.2.1. Precision

The precision of a network can be defined as its ability to propagate random errors. Information about precision is derived from the covariance matrix of the unknown parameters.

The Gauss-Markov model is the estimation model most widely used in photogrammetric linear or linearised estimation problems. An observation vector l of dimension $n \times 1$ is functionally related to a $u \times 1$ unknown parameter vector x ($x = x^0 + \Delta x$) through:

$$A \Delta x = b + v \quad (3.2)$$

where A is the design matrix, Δx is a vector of corrections to x^0 (approximate values of x) to give x , b is a vector related to the observations, and v is a vector of corrections to the observations. If the observations have an associated weight matrix W , then the normal equations are given by:

$$A^T W A \Delta x = A^T W b \quad (3.3)$$

and the covariance matrix is given as (Cross, 1983; Bomford, 1980): (assuming the variance factor $\sigma_0^2 = 1$)

$$\begin{aligned} \sigma_0^2 Q_{xx}^{\wedge\wedge} &= (A^T W A)^{-1} \\ Q_{xx}^{\wedge\wedge} &= (A^T W A)^{-1} \end{aligned} \quad (3.4)$$

where $Q_{xx}^{\wedge\wedge}$ is the cofactor matrix of the unknown parameters.

If it happened that matrix $(A^T W A)$ is singular the standard Cayley inverse is to be replaced by a particular generalised inverse, the Moore-Penrose inverse. Mittermayer(1972) has shown that when using the latter inverse for solving equation (3.4) it leads to a covariance matrix of minimum trace.

An insight into the covariance matrix shows that precision depends on the geometry of the network characterised by the design matrix, the quality of the observations expressed by the weight matrix and the sort of inverse used.

It is notable that the covariance matrix, parallel to what was mentioned in the preceding paragraph, depends on the choice of the reference system or the so-called datum parameters. In other words how the measurements had been adjusted.

In practice, there are two alternatives: constrained (or fixed network) adjustment and a free (network) adjustment.

3.2.1.1. Constrained Network Adjustment

In the constrained adjustment two of the observation stations are assumed to be fixed (or their co-ordinates are given known variances), or fixing one point, an azimuth and a distance in the two-dimensional network. If we change the datum parameters, different solutions for equations (3.4) are produced. Such different solutions are related to each other by Baarda (1973) S-transformations.

The main disadvantage of such a network is the probably false postulation of an umoving (stable) station(s).

3.2.1.2. Free Network

Literally, free network means a network with which adjustment is made free from any kind of constraint such as given earlier. In this method of adjustment the corrections to the approximate co-ordinates are derived by selecting the "best" co-ordinate system (Blaha, 1971). In this sense "best" is interpreted as resulting in the smallest trace of the covariance matrix for the unknowns, namely, the aforementioned corrections. In other words, neither the length of the solution vector nor the sum of the corresponding

variances can be decreased any further by a change in location of fixed point, observed azimuth or measured distance. It can be shown (Mittermayer, 1972) that, in a two-dimensional network, the position is fixed by the centre of gravity (centroid) of the network and the scale and orientation are determined by the average distance and bearing from that centre to the points.

From the estimation viewpoint, it can be shown that the solution for free network adjustment characterised by singular normal equations is biased (Cooper, 1980). This means that the expected values of computed co-ordinates are different from their true values (Appendix B). In the meantime, whereas the co-ordinates and their variances depend on the arbitrarily chosen reference system, the variances of a distance or bearing, if any provided, are invariant. Equivalence of both unbiased estimable quantities and invariant quantities has been proved by Grafarend and Schaffrin (1976) (Fagir, 1984). Bearing in mind one of the objectives of this study of deformation monitoring it should be noticed that the co-ordinates themselves are of no concern but their difference is of utmost importance. Such a difference is datum independent if the same network was used in two or more epochs.

It is why the precision of estimable quantities which are invariant with respect to datum transformations should be considered as criteria to define the quality of networks.

3.2.1.3. Common Precision Criteria

(i) Standard error of co-ordinates

The standard errors of co-ordinates in the direction of the co-ordinate system are obtained by taking the square root of the corresponding diagonal elements of the covariance matrix of the unknown parameters. As mentioned before, the covariance matrix is dependent on the chosen reference system, these quantities are datum dependent. Therefore they are not valid representations of the precision of the network.

(ii) Absolute error ellipses and ellipsoid

Error ellipse and ellipsoid are used to evaluate the precision of geodetic or photogrammetric determination of position in two- and three- dimensional spaces respectively (Slama, 1980). The absolute ellipses are informative, to some extent, but for some monitoring purposes may be misleading due to the fact that their size is dependent on their distance from the points selected as datum points. The closer to the datum the lesser the size (higher precision) of the ellipse and vice versa.

Let the covariance matrix (Σ) of a point j which has the co-ordinates X_j , Y_j and Z_j be defined as follows (dropping the index j):

$$\Sigma = \sigma_0^2 Q_{xx}^{-1} = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} \\ \sigma_{XY} & \sigma_Y^2 & \sigma_{YZ} \\ \sigma_{XZ} & \sigma_{YZ} & \sigma_Z^2 \end{bmatrix} \quad (3.5)$$

Following Mikhail (1976), the ellipsoid of constant probability is then given by the following quadratic form:

$$X^T \Sigma^{-1} X = [X \ Y \ Z] \Sigma^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = k^2 \quad (3.6)$$

when $k = 1$, it is called the standard ellipsoid. The semi-axes of the ellipsoid (a,b,c) are determined by diagonalising Σ by writing:

$$\begin{bmatrix} \sigma_u^2 & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & \sigma_w^2 \end{bmatrix} = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = T^T \Sigma T \quad (3.7)$$

where

T is an orthogonal matrix whose columns are the normalised eigenvectors of Σ ;

$\lambda_1, \lambda_2, \lambda_3$ are the eigenvalues of Σ ; and

u, v, w is a rotated co-ordinate system such that the random variables in the directions of its axes are uncorrelated.

In regard to the statistical significance of the error ellipsoid, the possibility of a point falling inside or on the ellipsoid defined by $a = k\sigma_u$, $b = k\sigma_v$, $c = k\sigma_w$ is expressed as:

$$P\left[\left(\frac{u^2}{\sigma_u^2} + \frac{v^2}{\sigma_v^2} + \frac{w^2}{\sigma_w^2}\right) < k^2\right] = P[\chi_3^2 < k^2] = 1 - \alpha \quad (3.8)$$

For the standard ellipsoid $(1 - \alpha) = 0.199$ which is obtained from χ^2 with three degrees of freedom. Confidence regions are established through selecting the significance level α and then the multiplier k is to be computed. For example, for $\alpha = 0.05$

$$P\left[\chi_3^2 < \chi_{0.05,3}^2\right] = P[\chi_3^2 < 7.81] = 0.95$$

and

$$k = \sqrt{7.81} = 2.79.$$

Thus for the ellipsoid whose semi-axes are

$$a = 2.79\sigma_u, \quad b = 2.79\sigma_v, \quad c = 2.79\sigma_w$$

there is 95 percent probability that the computed position of point j would fall within that ellipsoid or simply there is 95 percent confidence region.

The eigenvalues (the squares of the semi-axes of the ellipsoid) are the roots of the following characteristic equation:

$$\begin{aligned} \lambda^3 - (\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2)\lambda^2 + (\sigma_X^2\sigma_Y^2 + \sigma_Y^2\sigma_Z^2 + \sigma_X^2\sigma_Z^2 - \sigma_{XY}^2 - \sigma_{YZ}^2 - \sigma_{XZ}^2)\lambda \\ - \sigma_X^2\sigma_Y^2\sigma_Z^2 - 2\sigma_{XY}\sigma_{YZ}\sigma_{XZ} + \sigma_X^2\sigma_{YZ}^2 + \sigma_Y^2\sigma_{XZ}^2 + \sigma_Z^2\sigma_{XY}^2 = 0 \end{aligned} \quad (3.9)$$

Having determined the eigenvalues, the eigenvectors are determined and normalised to obtain the T-matrix. Then, the error ellipsoid can be plotted by automatic plotting devices (Veress et al, 1979).

Because of the involved computation, it is neither practical nor necessary to perform analysis using error ellipsoids for all points in the photogrammetric solution (Slama, 1980). However, for many applications, the two-dimensional equivalent, error ellipse, is enough to determine the precision of the system. The formulae

for the error ellipse are the same as for the error ellipsoid, except that terms which belong to two selected dimensions are retained and the third dimension terms are neglected.

For the X,Y selected directions, equation (3.6) will read:

$$[X \quad Y] \Sigma^{-1} \begin{bmatrix} X \\ Y \end{bmatrix} = k^2 \quad (3.10)$$

where

$$\Sigma = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{bmatrix} \quad (3.11)$$

Then the semi major and semi minor axes a,b may be computed from (Mikhail, 1976; Bomford, 1980; Cross, 1983):

$$a^2 = \frac{1}{2}(\sigma_X^2 + \sigma_Y^2) + \sqrt{\frac{1}{4}(\sigma_X^2 + \sigma_Y^2)^2 - \sigma_{XY}^2} \quad (3.12)$$

$$b^2 = \frac{1}{2}(\sigma_X^2 + \sigma_Y^2) - \sqrt{\frac{1}{4}(\sigma_X^2 + \sigma_Y^2)^2 - \sigma_{XY}^2}$$

and the angle θ between the semi major axis of the ellipse and the X axis is obtained from:

$$\tan 2\theta = \frac{2\sigma_{XY}}{\sigma_X^2 - \sigma_Y^2} \quad (3.13)$$

It is clear that a and b are the square roots of the eigenvalues of the characteristic equation :

$$\lambda^2 - (\sigma_X^2 + \sigma_Y^2)\lambda + \sigma_X^2\sigma_Y^2 - \sigma_{XY}^2 = 0 \quad (3.14)$$

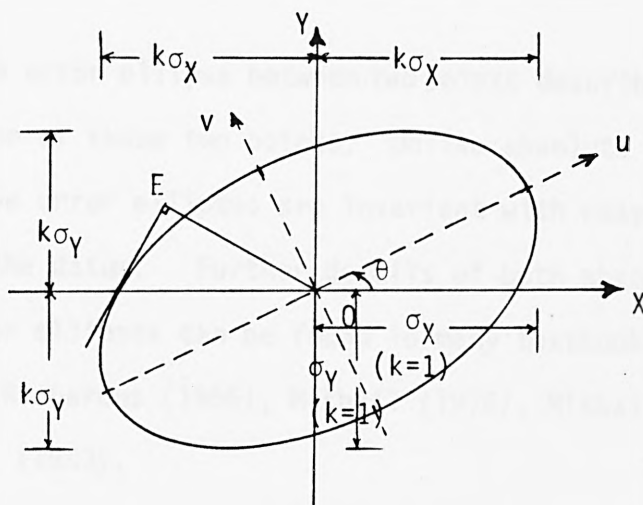


Figure 3.1. The standard Ellipse (after Mikhail, 1976)

Unlike the standard errors of co-ordinates which define the precision in the directions of the co-ordinates system chosen only, the standard error ellipse can be used to determine the standard error of any point (e.g. point E) at any direction (e.g. OE) by drawing the foot point curve or the pedal curve. However, it is the error ellipse rather than its pedal curve that is generally most useful in practice (Cross, 1983).

Statistically, if the observational errors are normally distributed, then there is 39.4 percent confidence region (Cross, 1983; Schofield, 1984).

(iii) Relative measures of Precision

The standard errors of quantities derived from the co-ordinates (e.g. co-ordinate differences, distances, angles) are often used as criteria of precision for networks due to the reason that they are not only invariant but they also related to the purpose of the network.

The relative error ellipse between two points describes the relative precision of those two points. Unlike absolute error ellipses, relative error ellipses are invariant with respect to translations of the datum. Further details of both absolute and relative error ellipses can be found in many textbooks to mention but few: Richardus (1966), Mikhail (1976), Mikhail et al (1981), and Cross (1983).

3.2.1.4. Global measures of Precision

To compare alternative network configurations, global measures which describe the precision of a network as a whole are desirable. The following are single numbers which could be used for this task.

(i) Mean Variance of Object Points

The most widely accepted indicator of statistical quality is simply the covariance matrix ($\sigma_0^2 Q_{xx}^{\wedge}$) of co-ordinates X,Y,Z. Near homogeneous precision is often desired for the X,Y,Z co-ordinates and a single estimator σ_m^2 can be employed to express the mean variance of the n_0 object point co-ordinates (assuming $\sigma_0^2 = 1$):

$$\sigma_m^2 = \frac{1}{3n_0} \text{tr} Q_{xx}^{\wedge} \quad (3.15)$$

The magnitude of σ_m^2 can be expected to vary dramatically with different imaging geometries and also with changes in minimal and redundant control configurations.

It is this criterion which was chosen to be used in the assessment of global precision in this investigation.

(ii) Average Standard Error of Derived Quantities

Distances and directions are examples of such derived quantities. It is important to point out that the choice of the quantities would be a function of the purpose of the network. Ashkenazi and Cross (1972) have used the average standard of errors of selected distances and directions to analyse the scale and orientation of the U.K. network (Fagir, 1984)

(iii) Maximum Eigenvalue of the Covariance Matrix

The largest eigenvalue of the covariance matrix can be considered as a viable global scalar measure of precision. Its square root is the largest semi-axis of the hypersphere associated with the covariance matrix. Use of the largest eigenvalue as a criterion is therefore tantamount to saying that the best determination of a point is the one that produces the standard ellipse whose major axis is smallest. However, the main drawback of using the maximum eigenvalue as criterion is that the eigenvalues are dependent on the reference base.

3.2.2. Reliability of Networks

Reliability is concerned with the control of quality of conformance of an observed network to its design, i.e. to see if the assumptions made in the design are not invalidated by disturbances (Alberda, 1976). It should describe the qualities of the network with respect to the possibility of detecting gross errors in the observations and the influence of undetected gross errors on the co-ordinates.

Due to the increasing use of automatic data acquisition and the great amount of data captured, it is essential to have an objective measure to recognise the possible presence of gross errors or outliers. Hawkins (1980) defines an outlier as an observation which deviates so much from other observations as to arouse suspicions that it was generated by a different mechanism.

Baarda (1967, 68, 73, 76) developed an extensive reliability theory where the problem of gross error detection is treated on statistical bases (Molenaar; personal communication).

3.2.2.1. Internal Reliability

Internal reliability may be defined as the ability of a network to detect gross observational errors. Baarda (1968), Pelzer (1977), Niemier et al (1982), Ashkenazi and Crane (1982) and Cross (1983) have developed different criteria for internal reliability based on statistical tests examining the residuals or their cofactor matrix. The latter can be derived as follows:

Restating equation (3.3):

$$A^T W A \Delta \hat{x} = A^T W b \quad (3.16)$$

then

$$\Delta \hat{x} = (A^T W A)^{-1} A^T W b \quad (3.17)$$

and for the residuals v we obtain from equation (3.2):

$$\hat{v} = A \Delta \hat{x} - b \quad (3.18)$$

$$= A (A^T W A)^{-1} A^T W b - b \quad (3.19)$$

$$= [A (A^T W A)^{-1} A^T W - I] b \quad (3.20)$$

Applying the Gaussian propagation of error laws, the cofactor matrix of the residuals $\hat{Q}_{\hat{V}\hat{V}}$ can be found from:

$$\hat{Q}_{\hat{V}\hat{V}} = [A(A^T W A)^{-1} A^T W - I] C_{\ell\ell} [A(A^T W A)^{-1} A^T W - I]^T \quad (3.21)$$

$$= \sigma_0^2 [A(A^T W A)^{-1} A^T W - I] W^{-1} [W A (A^T W A)^{-1} A^T - I] \quad (3.22)$$

(with $C_b = C_{\ell\ell} = \sigma_0^2 W^{-1}$; $C_{\ell\ell}$ is the covariance matrix of the observations, $\sigma_0^2 = 1$)

Hence;

$$\hat{Q}_{\hat{V}\hat{V}} = W^{-1} - A(A^T W A)^{-1} A^T \quad (3.23)$$

The product $\hat{Q}_{\hat{V}\hat{V}} W$ is an idempotent matrix (Latin: idem = same, potent = power), and singular as the only idempotent matrices that are nonsingular are identity matrices (Searle, 1982).

From equation (3.23) we get:

$$\hat{V} = - \hat{Q}_{\hat{V}\hat{V}} W b \quad (3.24)$$

A popular method of checking for gross errors after initial adjustment(post adjustment) is to compare the weighted residuals with the a posteriori variance factor $\hat{\sigma}_0^2$. For instance, if

$$|\hat{v}_i W_{ii}^{1/2}| > 3 \hat{\sigma}_0 \quad (3.25)$$

we suspect a gross error in the i th observation. However, such a technique assumes that all the residuals can be represented by a common a posteriori variance factor which is not the case and there is, additionally, no account taken of the design matrix A .

Gruen (1978,1979) has suggested the adoption of the data snooping technique for gross error detection in photogrammetry. The method is based upon $Q_{\hat{V}\hat{V}}$ given in equation (3.23) and requires not all the elements of $Q_{\hat{V}\hat{V}}$ but the diagonal elements only. Such a technique requires a considerable amount of computation, although the problem is not so severe in engineering or large structures monitoring applications as it is with the large blocks of photography encountered in aerial triangulation.

If the j th observation, ℓ_j , is suspected of containing a gross error Δ_j and all other observations have only random, normally distributed errors ϵ_j , since the systematic errors are supposed to be eliminated or compensated for, we can set the following test:

$$H_0 : \ell_j = \bar{\ell}_j + \epsilon_j \quad (3.26)$$

$$H_A : \ell_j = \bar{\ell}_j + \epsilon_j + \Delta_j$$

where $\bar{\ell}_j$ is the observation vector without errors.

Baarda (1968) introduced a test statistic

$$\hat{\omega}_j = \frac{\hat{d}_j}{\sigma_{\hat{d}_j}} \quad (3.27)$$

where

$$\hat{d}_j = \ell_j - \hat{\ell}_j^* \quad (3.28)$$

in which $\hat{\ell}_j^*$ is the j th observed quantity computed from the parameters derived from least squares computation of all observations except ℓ_j (Cross, 1983).

Following the notions of Pelzer (1979), Fagir (1984) proved that equation (3.27) could be reduced to the so-called standardised residuals test given as:

$$\hat{\omega}_j = \hat{v}_j / \sigma \hat{v}_j \quad (3.29)$$

As we are in pre-analysis stage, $\hat{v}$ is not at hand so we will apply the approach proposed by Cross (1983) for assigning the test statistics for reliability appraisal. Such an approach presents itself as an ideal candidate for such kind of study.

Now, if H_0 is true, $\hat{\omega}_j \sim N(0,1)$ but under H_A the normal distribution will have a mean not zero but δ_j as given by:

$$\delta_j = \Delta_j / \sigma \hat{d}_j \quad (3.30)$$

Due to the fact that Δ_j is not known a choice should be made so that the probability (α) of type I error (rejecting H_0 although it is true) and the probability (β) of type II error (accepting H_0 although it is false) are as small as possible and hence determine an upper bound δ_0 (Figure 3.2) on δ_j . Therefore the largest gross error which will remain undetected is given by:

$$\Delta_0 = \delta_0 \cdot \sigma \hat{d}_j \quad (3.31)$$

and

$$\text{(Fagir, 1984)} \quad \sigma \hat{d}_j = 1 / (c_j^T W Q \hat{v} \hat{v}^T W c_j)^{1/2} \quad (3.32)$$

where c_j^T is a null vector but for the j th element which is unity, i.e.

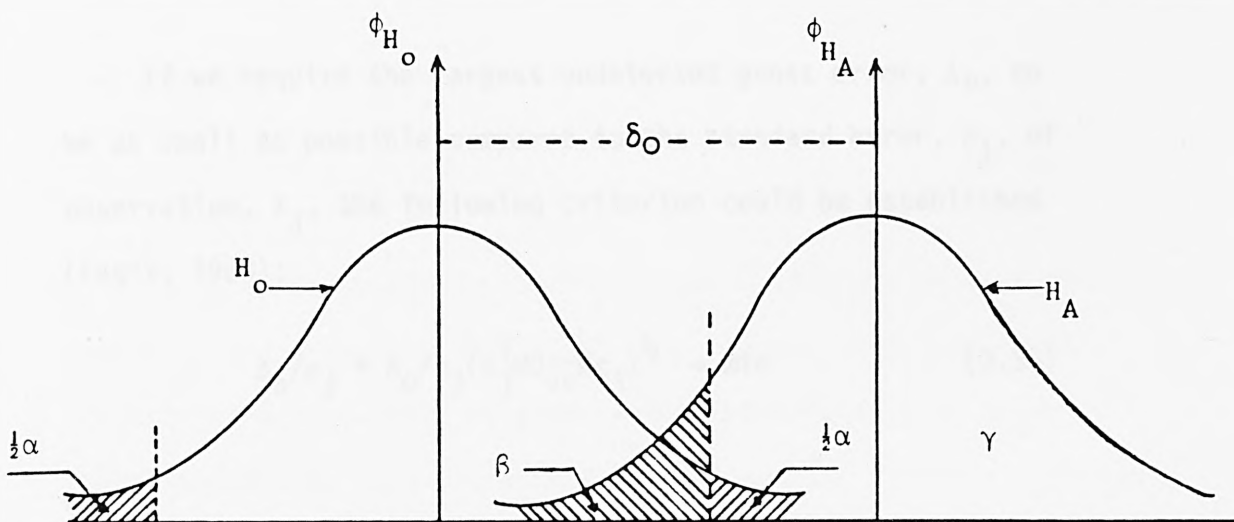


Figure 3.2. Probability density function of statistic ω_j .

$$c_j = [0 \ 0 \ \dots \ 1 \ 0]^T \quad (3.33)$$

Substituting (3.32) into (3.31) we get:

$$\Delta_o = \delta_o / (c_j^T W Q \hat{V} W c_j)^{1/2} \quad (3.34)$$

If we require the largest undetected gross error, Δ_o , to be as small as possible compared to the standard error, σ_j , of observation, l_j , the following criterion could be established (Fagir, 1984):

$$\Delta_o / \sigma_j = \delta_o / \sigma_j (c_j^T W Q \hat{V} W c_j)^{1/2} \rightarrow \min \quad (3.35)$$

where

$$\sigma_j = (c_j^T W^{-1} c_j)^{1/2} \quad (3.36)$$

Substituting (3.36) into (3.35) yields:

$$\Delta_o / \sigma_j = \delta_o / (c_j^T W^{-1} c_j c_j^T W Q \hat{V} W c_j)^{1/2} \quad (3.37)$$

or

$$\Delta_o / \sigma_j = \delta_o \tau_j \quad (3.38)$$

Hence:

$$\Delta_o = \delta_o \sigma_j \tau_j \quad (3.39)$$

Since δ_o depends only on the chosen probabilities α and β , it could be regarded as constant and the criterion (3.35) reduced to:

$$\tau_j^2 \rightarrow \text{minimum} \quad (3.40)$$

It is extremely important that for uncorrelated observations τ_j^2 is given as:

$$\tau_j^2 = \frac{\sigma_j^2}{\sigma^2 \hat{v}_j} \quad (3.41)$$

From equation (3.41) the limits of τ_j^2 are:

$$1 \leq \tau_j^2 \leq \infty \quad (3.42)$$

It is worth mentioning that equation (3.41) was used in this thesis as criterion for the reliability but one important point against its use is that, as seen from equation (3.42) its upper bound is infinity so τ could take any value up to infinity. However, it is very useful tool for screening the maximum undetected gross error in any observation (equation 3.39) which could be used as a global measure for reliability, i.e.

$$\Delta_0 \rightarrow \text{a minimum} \quad (3.43)$$

Further, τ is independent of both $\hat{v}$ and the reference system.

An alternative was sought to circumvent the loose upper bound of τ . The redundancy number technique proved to be very efficient criterion to assess reliability of different configurations of networks simulated in this study.

Redundancy Number Technique

Recall equation (3.24) which reads as follows:

$$\hat{v} = - Q_{\hat{v}\hat{v}}^{-1} W_b$$

The effect of a gross error Δl_j on the residual $\hat{v}_j$ of an observation l_j is therefore:

$$\Delta \hat{v}_j = -r_j \Delta l_j \quad (3.44)$$

where

r_j is the j th diagonal element of the matrix $Q_{\hat{V}\hat{V}}W$. and is called redundancy number (Förstner, 1979; El Hakim, 1981). The redundancy number indicates the reliability of the adjustment of a particular observation. Zero redundancy means no reliability while increasing redundancy indicates increasing reliability.

So the limits for r_j are as follows:

$$0 \leq r_j \leq 1 \quad (3.45)$$

Thus if r_j is large, the gross error Δl_j is well revealed in the residual $\hat{v}$ ($\hat{v}$ is the visible part of the true error ϵ (Pope, 1975) and can easily be detected.

As has been demonstrated earlier $Q_{\hat{V}\hat{V}}W$ is an idempotent matrix with rank r (r = redundancy). Hence it yields:

$$t_r(Q_{\hat{V}\hat{V}}W) = \text{rank}(Q_{\hat{V}\hat{V}}W) = \text{rank}(Q_{\hat{V}\hat{V}}) = r \quad (3.46)$$

That is, the trace of $Q_{\hat{V}\hat{V}}W$ equals the redundancy of the system and r_j can be interpreted as the contribution of the observation l_j to the total redundancy r .

It is the author's point of view that for the evaluation of reliability, the local redundancy indicated by r_j is more important than the total redundancy r .

However for the sake of comparison between different systems the following criterion can serve as a global reliability measure (Gruen, 1978, 1979):

$$R = \frac{\text{tr}(\hat{Q}_{VV}W)}{m} = \frac{r}{m} \quad (3.47)$$

where m denotes number of observations.

The main restraint on using (3.47) is that such measure presumes that the redundancy is distributed homogeneously on the observations.

For the optimisation standpoint another global criterion has been introduced to indicate the overall reliability of different networks' configurations. This criterion is the maximum redundancy number of network. That is:

$$r(x) = r_{j_x} \rightarrow \text{a maximum} \quad (3.48)$$

and

$$r(y) = r_{j_y} \rightarrow \text{a maximum}$$

where x, y are the observed photographic co-ordinates and

$$r(\text{measurement}) = r_j \rightarrow \text{a maximum} \quad (3.49)$$

for survey measurements.

The local redundancies again are mainly influenced by parameters as:

- (i) number of rays determining an object point.
- (ii) type of an object point (control or non-control point).
- (iii) number and distribution of image points.

(iv) number and distribution of control points.

(v) focal length of the taking camera.

The parameters mentioned in (iii), (iv), (v) may play a certain role, which however is not expected to be as significant as the influence of the first two parameters (Gruen, 1979).

In this thesis, the central point is the adoption of free network approach, i.e. there is no use of control points. So, the parameter No.(i) (above) will only be considered.

3.2.2.2. External Reliability

External reliability indicates the influence of undetected gross errors on an arbitrary function of the co-ordinates. Such a function can be: the co-ordinate itself, an angle, a distance, a difference of co-ordinates or an area computed with the adjusted co-ordinates.

Consider a quantity F , with least squares estimate $\hat{F}$, computed from the parameters and let $\Delta\hat{F}_j$ be the effect of a gross error of size Δ_j^u in the j th observation on $\hat{F}$. Then it can be shown that for uncorrelated observations:

$$(\text{Cross, 1983}) \quad \Delta\hat{F}_j \leq \delta_j^u \gamma_j \sigma_{\hat{F}} \quad (3.50)$$

where

$$\gamma_j = \hat{\sigma}_j / \sigma_{\hat{F}} \quad (3.51)$$

Inspection of equation (3.50) leads to the fact that γ_j could be used as an indicator of the external reliability since by its multiplication by the standard error of the wanted function of the parameters it gives the maximum effect on that function. It is important to point out that for the evaluation of external reliability one must consider the precision of the co-ordinates and their functions respectively.

We could have an appraisal for external reliability which is independent of the precision through soliciting the relationship between τ_j and γ_j .

Recalling equation (3.41) and (3.51) we formulate:

$$\tau_j^2 - \gamma_j^2 = (\sigma_j^2 - \hat{\sigma}_j^2) / \sigma_{\hat{v}_j}^2$$

with

$$\sigma_{\hat{v}_j}^2 = \sigma_j^2 - \hat{\sigma}_j^2$$

Thence:

$$\tau_j^2 - \gamma_j^2 = 1$$

Finally,

$$\gamma_j^2 = \tau_j^2 - 1 \quad (3.52)$$

Hence the computation of τ_j automatically leads to γ_j .

As it can be noticed from (3.52) that for an observation, the higher its internal reliability the higher is its external reliability.

$\gamma_{\max}$ could be proposed to serve as global external reliability indicator as follows:

$$\gamma_{\max} \rightarrow \text{a minimum} \quad (3.53)$$

Such a criterion will ensure that the influence on the parameters of undetected gross error of a designated size is minimum.

As has been mentioned before with regard to τ_j , γ_j is independent of the selection of reference system.

It is worth mentioning that γ_j computed as in equation (3.52) was used in this study but here, of crucial importance, the term reliability is chosen to be more strongly connected with the detection than with the effect of gross errors. Therefore a brief mention has been given on the topic of external reliability.

3.2.3. Sensitivity of Networks

3.2.3.1. Introduction

The concept of sensitivity analysis can be outlined in terms of a deformation analysis which employs tests for departures from congruency (Fraser, 1983) in other words, the proof of existence or non-existence of point movements. Since neither "true" co-ordinates nor "true" deformations can be obtained through a physical measuring process it is necessary to turn to statistical or "estimated" deformations in order to establish whether or not significant movements have occurred between two measuring epochs.

Essentially, the congruence of the two networks is examined within the tolerance implied by their respective covariance matrices. It is therefore not surprising that the covariance matrix of object points co-ordinates $\sigma_{\hat{Q}_{xx}}^2$ (2) plays a key role in assessing how sensitive a network is to the detection of systematic point

displacements which are likely to occur under formulated deformation models. Such deformation models are formulated under the assumption that there is prior information regarding the magnitude, direction and extent of expected deformations provided by other disciplines (e.g. civil engineering, geology, etc.)

3.2.3.2. The Global Congruency Test

Fundamentally, this test examines the null hypothesis that the object target point array is stable over all measuring epochs. For the case of a two-epoch analysis, the null hypothesis H_0 can be written as:

$$H_0 : E\{\hat{x}_2^{(i+1)}\} - E\{\hat{x}_2^{(i)}\} = E\{d\} = 0 \quad (3.54)$$

in which $\hat{x}_2^{(i)}$ is the vector of XYZ object point co-ordinates at epoch i , and d is the vector of co-ordinate differences.

Pelzer (1971) introduced the following test statistic which determines whether or not H_0 will be rejected.

$$\bar{F} = \frac{d^T Q_d^+ d}{h\sigma_0^2} \quad (3.55)$$

where

$$Q_d = Q_{\hat{x}\hat{x}}^{(2)} + Q_{\hat{x}\hat{x}}^{(1)} \quad (3.56)$$

The cofactor matrix Q_d of co-ordinate differences has a rank h ; in photogrammetric context $\text{rank}(Q_d) = 3n_0 - 7$ where n_0 is the number of object points. The Moore-Penrose inverse in equation (3.55) is necessitated because of the rank defect of Q_d .

If the null hypothesis is accepted, i.e. no point displacement occur, $\bar{F}$ follows the central Fisher distribution. Should the null hypothesis is rejected, the point movements are implied and an alternative hypothesis is to be specified. It could be given as follows:

$$H_A = E \{d\} = \tilde{d} \neq 0 \quad (3.57)$$

Now $\bar{F}$ is distributed according to the non-central F-distribution with non-centrality parameter.

$$\omega = \frac{\tilde{d}^T Q_d^+ \tilde{d}}{\sigma_0^2} \quad (3.58)$$

The probability $\gamma = 1 - \beta$ that $\tilde{d}$ will lead to the rejection of the null hypothesis at significance level α is termed the power of the test with respect to the alternative hypothesis. For a specified probabilities α and β a critical value for ω (say ω^u) can be computed as a function of α, β and rank (Q_d)

Nomograms for ω^u , for three power values ($\gamma = 0.70, 0.80, 0.90$) are given in Baarda (1968). For this investigation values of $\alpha = 0.05$ and $\gamma = 0.80$ were selected.

If $\omega > \omega^u$ for a given $\tilde{d}$ then it can be concluded that $\tilde{d}$ represents a detectable movement at the assigned probability levels α and β .

It should be noted that the global congruency test only tests the magnitude of the movement on the bearing designated by the vector $\tilde{d}$.

An alternative approach for sensitivity analysis is given by Niemier et al (1982) in which the use of external reliability measures in sensitivity analysis has been discussed (Fraser, 1983).

3.2.3.3. Sensitivity Measure

The cofactor matrices $Q_{\hat{x}\hat{x}}^{(1)}$, $Q_{\hat{x}\hat{x}}^{(2)}$ given in equation (3.56) can be determined from simulation procedures and for sensitivity analysis are generally assumed to be equal. Hence, the test statistic given in equation (3.58) can be calculated a priori from the knowledge of the proposed measuring system and the movements required to be detected.

The importance of the test of ω given in equation (3.58) in terms of sensitivity analysis is that any so-called form vector $\tilde{d}$ of modelled point displacements can be assumed in order to ascertain a just-detectable deformation.

Let c be a scalar value, called sensitivity parameter, then if:

$$\begin{aligned}\omega^u &= c^2 \omega \\ &= \frac{(\tilde{c}\tilde{d})^T Q_d^{-1} (\tilde{c}\tilde{d})}{\sigma_0^2}\end{aligned}\tag{3.59}$$

then $\tilde{c}\tilde{d}$ represents a critical amount of movement, i.e. a just-detectable deformation.

For a sensitivity analysis of a single network which is remeasured at another epoch $\tilde{c}\tilde{d}$ can be considered as measure of the sensitivity of the network.

Finally, it is worth mentioning that the criterion given in equation (3,59) was used coupled with the use of the nomograms of Baarda (1968) in this research to assign the sensitivity of different simulated networks.

CHAPTER 4

MATHEMATICAL DEVELOPMENT

4.1. Introduction

In technical practice, as well as in all experimental sciences, one is faced with the following problem: evaluate quantitatively parameters describing properties, features, relations, or behaviour of various physical objects. The parameters can be evaluated usually only on the basis of the results of some measurements or observations.

The problem gets more complicated as the system whose parameters we are trying to determine gets more complex. The problem to be treated has to be first translated into the language of mathematics. That is the problem has to be first mathematically formulated.

The mathematical formulation of the problem would really be the mathematical formulation of the relation between the observed quantities and the wanted quantities. This relationship is called the mathematical model.

4.2. The Mathematical Model

The mathematical model links observable reality with the mechanism generating the observations (Morrison, 1976). In this investigation, the mathematical model denotes the relationships between observations (measurements), for instance, image co-ordinates and slope distances; and the three-dimensional co-ordinates of the object points.

Mathematical models may be divided into two general parts: (1) functional model (deterministic), and (2) stochastic. The functional model should represent, as far as possible, the physical relationship between measurements and three-dimensional co-ordinates. Morrison (1976) suggests that the functional model should be sufficiently tractable to permit the sort of mathematical manipulations required for the estimation of its parameters and other inferences about its nature. The stochastic model describes the properties of random errors inherent in the observations and how these errors will propagate through the used mathematical model and result in certain errors in the derived quantities of the three-dimensional co-ordinates of the object points.

4.2.1. The Functional Model

It is proposed that slope distances between object points are to be incorporated with the photogrammetric observations in an attempt to investigate their effect on the different adopted design criteria discussed in the preceding chapter. Therefore, in addition to the model of the photogrammetric measurements the model for slope distances is to be addressed in this section.

4.2.1.1. Functional model for a photograph

Assuming that light rays travel in straight lines, that all rays entering a camera lens system pass through a single point and that the lens system is distortionless, then a projective relationship exists between the photographic co-ordinates of the

image points and the object space co-ordinates of the corresponding object points as illustrated in Figure 4.1.

This projective relationship can be represented as follows:

$$\begin{bmatrix} x_j - x_i^c \\ y_j - y_i^c \\ z_j - z_i^c \end{bmatrix} = \lambda \begin{bmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{23} & r_{33} \end{bmatrix}_i \begin{bmatrix} x_{ij} - x_{p_i} \\ y_{ij} - y_{p_i} \\ -f_i \end{bmatrix} \quad (4.1)$$

By manipulation of matrices

$$\begin{bmatrix} x_{ij} - x_{p_i} \\ y_{ij} - y_{p_i} \\ -f_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}_i \begin{bmatrix} x_j - x_i^c \\ y_j - y_i^c \\ z_j - z_i^c \end{bmatrix} \quad (4.2)$$

where subscript i refers to the photo i ($i = 1, 2, \dots, m$) and subscript j refers to the object point A ($j = 1, 2, \dots, n$). Here x, y are image co-ordinates. x_p and y_p are the photo co-ordinates of the principal point (p). f is the focal length of the camera. λ is a scale factor and r 's are elements of R , a $(3,3)$ orthogonal matrix, functions of three independent rotation parameters (ω, ϕ, κ) of the camera. The latter represent the rotations of the image space co-ordinate system (x, y, z) with respect of the object space co-ordinate system (X, Y, Z) . x_j, y_j and z_j are the spatial co-ordinates of the object point A . Superscript c refers to the camera, so x^c, y^c and z^c are the spatial co-ordinates of the

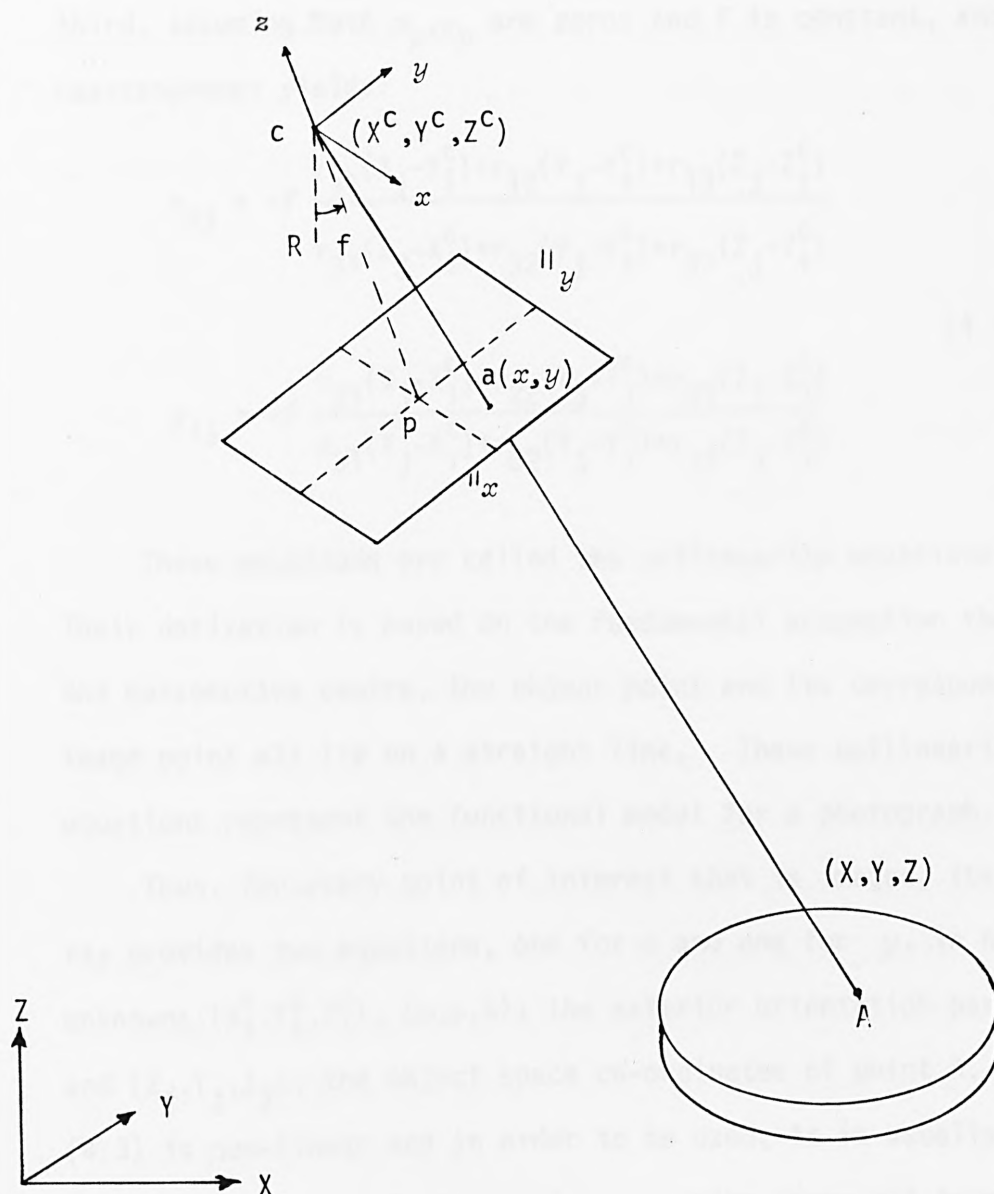


Figure 4.1. The geometrical basis of analytical photogrammetry

perspective centre in the object space co-ordinate system.

Dividing the first two equations in system (4.2) by the third, assuming both x_p, y_p are zeros and f is constant, and rearrangement yields:

$$\begin{aligned} x_{ij} &= -f \frac{r_{11}(x_j - x_i^C) + r_{12}(y_j - y_i^C) + r_{13}(z_j - z_i^C)}{r_{31}(x_j - x_i^C) + r_{32}(y_j - y_i^C) + r_{33}(z_j - z_i^C)} \\ y_{ij} &= -f \frac{r_{21}(x_j - x_i^C) + r_{22}(y_j - y_i^C) + r_{23}(z_j - z_i^C)}{r_{31}(x_j - x_i^C) + r_{32}(y_j - y_i^C) + r_{33}(z_j - z_i^C)} \end{aligned} \quad (4.3)$$

These equations are called the collinearity equations. Their derivation is based on the fundamental assumption that the perspective centre, the object point and its corresponding image point all lie on a straight line. These collinearity equations represent the functional model for a photograph.

Thus, for every point of interest that is imaged, its light ray provides two equations, one for x and one for y , in nine unknowns: (x_i^C, y_i^C, z_i^C) , (ω, ϕ, k) ; the exterior orientation parameters and (x_j, y_j, z_j) , the object space co-ordinates of point A. System (4.3) is non-linear and in order to be used, it is usually reduced to a linear form by Taylor series expansion where all terms above and including the second order are dropped.

On the other hand, as is the case in practice, several photographs are taken from different locations of the same object so an over-determined model is available and hence, least squares is applied to get a unique solution vector and its covariance matrix.

4.2.1.2. Functional model for a Slope distance

The relationship between the measured slope distance (ℓ_{dh}) between points D,H and their three-dimensional co-ordinates (X_d, Y_d, Z_d) and (X_h, Y_h, Z_h) depicted in Figure 4.2 can be expressed as follows:

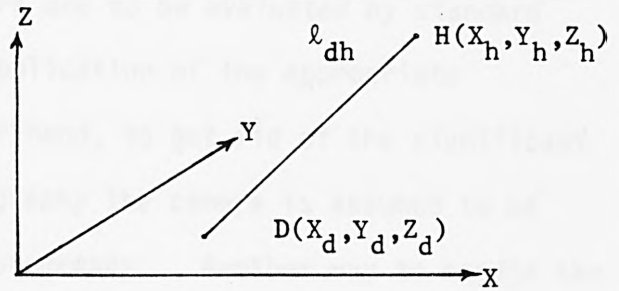


Figure 4.2. Typical slope distance

$$\ell_{dh} = [(X_h - X_d)^2 + (Y_h - Y_d)^2 + (Z_h - Z_d)^2]^{\frac{1}{2}} \quad (4.4)$$

4.2.1.3. Notes on the functional models

The aforementioned functional model for a photograph as a perspective projection described by the collinearity equations (4.3) is rather theoretical. Due to some non-perspective projection parameters the photograph departs from such a projection. The following can be considered as non-perspective projection parameters:

- i) the photographic co-ordinates of the principal point (p) are probably not (0,0).
- ii) the axes of the photograph defined by the lines connecting the fiducial marks might be non-orthogonal.
- iii) the radial and decentring lens distortion.
- iv) deformation of the emulsion carrier (film or glass).
- v) the atmospheric refraction.

So, these factors affecting the functional model must be eliminated or corrected for in the functional model for the photograph. For the functional model for distances no mention of the systematic errors which would occur has been given. It is assumed that these errors are to be evaluated by standard calibration methods, and application of the appropriate corrections. On the other hand, to get rid of the significant systematic errors of photography the camera is assumed to be calibrated prior to the photography. Another way to handle the problem of systematic errors of photography, as has been mentioned in section (3.2), is to use an additional parameter set to take account of these errors (Gruen, 1978).

It has been decided that no additional parameters be adopted, thus avoiding instability of the solution which might occur if additional parameters were introduced (Cooper, 1983). Moreover, inclusion of additional parameters might lead to the singularity of the augmented normal matrix given in section (4.2.3.2). Such instability and singularity are due to significant correlations between the additional parameters and the exterior orientation elements.

The general form of the functional model including both photogrammetric and slope distance observations can be written as:

$$l = F(x, c) \quad (4.5)$$

where

- l is the vector of observations;
- x is the vector of unknown parameters; and
- c is the vector of constants.

4.2.2. The Stochastic Model

In this investigation, as mentioned in section (4.2.1), the observations are photographic co-ordinates and slope distances. The stochastic model is the means by which the random errors are dealt with. Therefore equation (4.5) expresses the non-linear relationship between the vector of observations namely, the photographic co-ordinates and the slope distances, and the vector of unknown parameters namely, the exterior orientation elements and the object point co-ordinates.

The random or residual errors designated as v are introduced to give the stochastic model based on two assumptions. Firstly, $E\{v\} = 0$. This yields the linearised observation equations:

$$A\Delta x = b + v \quad (4.6)$$

where

A is the design matrix, $A = \left(\frac{\partial F}{\partial x} \right)_{x=x^0}$

Δx is the vector of corrections to x^0 (the approximate co-ordinates of x) to give x via $x = x^0 + \Delta x$.

b is the vector related to the observations or, vector of observational discrepancies, $b = \ell - \ell^0$, $\ell^0 = F(x^0)$,

and v is the vector of residuals or, vector of corrections to the observations.

Secondly, the covariance matrix of the observations ℓ is symmetric positive-definite matrix $C_{\ell\ell} = \sigma_0^2 W^{-1}$, in which σ_0^2 is the a priori variance factor and W is the weight matrix of the observations.

The weight matrices are proportional to the inverse of the estimated covariance matrix of the observations. For example, the covariance matrix of the image co-ordinates x and y takes the form:

$$C_{xy} = \sigma_0^2 \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix} \quad (4.7)$$

in which σ_x^2 and σ_y^2 are the variances of x and y respectively and σ_{xy} is the covariance of the two measurements. From equation (4.7) the weight matrix of these measurements will be:

$$W_{xy} = \sigma_0^{-2} C_{xy}^{-1} = \sigma_0^{-2} \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}^{-1} \quad (4.8)$$

Although the co-ordinates for a given point may be correlated it is assumed that co-ordinates for different points are uncorrelated. Further assumptions have been made on the weight matrix in this study. That is x and y co-ordinates of a given point are also uncorrelated. Although such an assumption may be invalid it is accepted because it is not possible to account for such a correlation in the covariance matrix of the observations. As a consequence, the covariance matrix for the image co-ordinates x and y and the corresponding weight matrix W have a diagonal form:

$$W_{xy} = \sigma_0^{-2} C_{xy}^{-1} = \sigma_0^{-2} \begin{bmatrix} \frac{1}{\sigma_x^2} & 0 \\ 0 & \frac{1}{\sigma_y^2} \end{bmatrix} \quad (4.9)$$

Applying the same assumptions stated above, the weight matrix for the other observed quantities can be formulated.

In other words both $C_{\ell\ell}$ and W are diagonal matrices. Consequently, the entire weight matrix can be written as:

$$W = \begin{bmatrix} W_p & 0 \\ 0 & W_g \end{bmatrix} \quad (4.10)$$

where W_p is the diagonal weight matrix of the photogrammetric observations, and

W_g is the diagonal weight matrix of the slope distance observations.

4.2.3. The Least Squares estimation and Datum definition

It is relevant to mention at the outset that, briefly, the adjustment process determines the unknown parameters based on the information contained in the discrepancies between the measured values and those computed from the assumed model.

The operator that relates the corrections in the unknown parameters to these discrepancies is the design matrix of the problem. In the usual case where there are redundant observations available, the row space of the design matrix has a dimension larger than that of its column space. If its columns are linearly independent, then the rank is equal to their number, the dimension of the column space, and the problem will have a unique solution. In the event that two or more of its columns are linearly dependent, the design matrix is rank deficient, its deficiency determined by the number of interdependent columns.

We can now, in light of the above discussion, start as follows: Recalling equations (4.6) and (4.10) the problem is to estimate Δx from A and W using the principle of least squares which minimises the quadratic form:

$$\Omega = \hat{v}^T W \hat{v} \quad (4.11)$$

If A is of full column rank, the Best Linear Unbiased Estimate (BLUE) $\hat{\Delta x}$ can be found (e.g. Mikhail, 1976; Welsch, 1979) by solving the normal equations:

$$(A^T W A) \hat{\Delta x} = A^T W b \quad (4.12)$$

$$\text{then} \quad \hat{\Delta x} = (A^T W A)^{-1} A^T W b \quad (4.13)$$

and the covariance matrix of the least squares estimate is given as:

$$\hat{C}_{\hat{\Delta x} \hat{\Delta x}} = \hat{C}_{\Delta x \Delta x} = \hat{\sigma}_0^2 (A^T W A)^{-1} \quad (4.14)$$

in which $\hat{\sigma}_0^2 = \hat{v}^T W \hat{v} / v$: the a posteriori variance factor and v is the number of degrees of freedom. The latter represents the number of observations in excess of the minimum required for a unique solution.

Because the functional model includes co-ordinates whose reference system has not been defined in the observation equations (4.6) nor in the normal equations (4.12), the matrix $(A^T W A)$ will be singular.

The datum definition is referred to as the Zero Order design problem which has been discussed in Chapter 3. It has been

mentioned that rank deficiency of the design matrix as well as the normal equations matrix can be as high as seven: position of the origin of the co-ordinates (three elements); direction of co-ordinate axes (three elements); and the scale (one element). Therefore such elements have to be defined in order to define the datum. Conventionally these elements can be defined by assigning fixed values to all three co-ordinates of two selected points and to one co-ordinate of a third, non-collinear point. In deformation analysis, however, such a procedure has the main disadvantage that it is obligatory at the second epoch to relocate the same datum points which were chosen at the first epoch. These points, themselves, are likely to undergo deformation. In other words there is no guarantee they are stable.

To overcome the problem of dependency on stability of any points in the network, the free network technique (inner constraints method) was used to define the datum.

From the computational viewpoint, there are two approaches to get the solution and the covariance matrix of the unknown parameters in a free network adjustment.

4.2.3.1. The Moore-Penrose inverse approach

Appendix A demonstrates some of the details concerning the definition and characteristics of such an inverse.

Now, let $N = A^T W A$, $u = A^T W b$

then the normal equations(4.12) can be expressed as:

$$N \hat{\Delta x} = u \quad (4.15)$$

As matrix N is singular, the Moore-Penrose inverse N^+ can be applied to replace the standard Cayley inverse given in equation (4.14). However, the solution in such a case will be Best Linear Biased Estimate (BLBE) (Welsch, 1979).

Then

$$\Delta \hat{x} = N^+ u \quad (4.16)$$

The following properties are associated with the above solution:

- (i) it is a least square solution (i.e. $\hat{v}^T W \hat{v}$ is a minimum)
- (ii) it is a minimum norm solution (i.e. $\Delta x^T \Delta x$ is a minimum),
- and (iii) its covariance matrix and hence that of $\hat{x}$ via $\hat{x} = \hat{x}^0 + \Delta \hat{x}$ has a minimum trace (i.e. $\text{tr} N^+ = \text{minimum}$).

4.2.3.2. The minimal set of constraints approach

An alternative method to achieve the minimum trace covariance matrix (Granshaw, 1980) is to apply an appropriate set of constraints $G^T \Delta x = 0$ to the observation equations. Such set is a subset of minimal constraints and is called inner constraints (Appendix B).

$$A \Delta x = b + v$$

$$G^T \Delta x = 0 \quad (4.17)$$

where

$$G \text{ is of order } (6n_s + 3n_o) \times 7 : \begin{bmatrix} n_s = \text{no. of cameras used} \\ n_o = \text{no. of object points} \end{bmatrix}$$

and satisfies the following two conditions:

(i) the columns of G are linearly independent, between themselves and the columns of A .

(ii) $G^T \Delta x = 0$

Denoting $\hat{k}$ as the estimator for the vector of Lagrangian multipliers of order (7×1) the least squares solution of the system (4.17) is given by:

$$\begin{bmatrix} A^T W A & G \\ G^T & 0 \end{bmatrix} \begin{bmatrix} \Delta \hat{x} \\ \hat{k} \end{bmatrix} = \begin{bmatrix} A^T W b \\ 0 \end{bmatrix} \quad (4.18)$$

The matrix of coefficients is now regular and the solution $\Delta \hat{x}$ can be found in the conventional way.

It should be noted that what we are primarily concerned about, are the object point co-ordinates and not the exterior orientation parameters of the cameras. Therefore it is reasonable to apply inner constraints not to minimise the trace of the covariance matrix of the full solution but to minimise only the trace of the covariance matrix of the object point co-ordinates.

Let us partition Δx as $\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$ corresponding to $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ where

Δx_1 represents the vector of corrections to the approximate orientation elements to give x_1 and Δx_2 represents the vector of corrections to the approximate co-ordinates of the n_0 object points to give x_2 and G as $\begin{bmatrix} 0 \\ G \end{bmatrix}$. The linearised observation equations with no constraints will read:

$$\begin{bmatrix} A_1 & A_2 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = b + v \quad (4.19)$$

Then the photogrammetric normal equations are given by

$$\begin{bmatrix} A_1^T W A_1 & A_1^T W A_2 \\ A_2^T W A_1 & A_2^T W A_2 \end{bmatrix} \begin{bmatrix} \hat{\Delta x}_1 \\ \hat{\Delta x}_2 \end{bmatrix} = \begin{bmatrix} A_1^T W b \\ A_2^T W b \end{bmatrix} \quad (4.20)$$

Bordering (4.20) with $G = \begin{bmatrix} 0 \\ G \end{bmatrix}$ yields the augmented photogrammetric normal equations:

$$\begin{bmatrix} A_1^T W A_1 & A_1^T W A_2 & 0 \\ A_2^T W A_1 & A_2^T W A_2 & G \\ 0 & G^T & 0 \end{bmatrix} \begin{bmatrix} \hat{\Delta x}_1 \\ \hat{\Delta x}_2 \\ \hat{k} \end{bmatrix} = \begin{bmatrix} A_1^T W b \\ A_2^T W b \\ 0 \end{bmatrix} \quad (4.21)$$

The standard Cayley inverse of the coefficient matrix of the augmented photogrammetric normal equations gives the minimum trace solution for $\hat{\Delta x}_2$ (and hence for $\hat{x}_2$) if the following transformation matrix G is applied (see Appendix B).

$$G_j = \begin{bmatrix} 1 & 0 & 0 & 0 & Z_j^0 & -Y_j^0 & X_j^0 \\ 0 & 1 & 0 & -Z_j^0 & 0 & X_j^0 & Y_j^0 \\ 0 & 0 & 1 & Y_j^0 & -X_j^0 & 0 & Z_j^0 \end{bmatrix} \quad (4.22)$$

in which the subscript j denotes the j th object point with approximate co-ordinates (X_j^0, Y_j^0, Z_j^0) . The first three columns of G_j define the origin of the datum, the columns 4, 5 and 6 define the direction of the axes and the last column (7th column) defines the scale.

It should be noticed that if, in the network used, one or more slope distances are measured, then the 7th column is omitted from G. Moreover, if height differences are measured in the object space, then the 4th, 5th, and the 7th columns are to be dropped.

Investigation of the product $G^T \Delta x$ shows that the inner constraints are equivalent to the following equations:

$$(i) \quad \sum_{i=1}^{n_0} \delta X_i = \sum_{i=1}^{n_0} \delta Y_i = \sum_{i=1}^{n_0} \delta Z_i = 0$$

$$\text{or} \quad \sum_{i=1}^{n_0} X_i = \sum_{i=1}^{n_0} X_i^0$$

(i.e. the co-ordinate system's origin is the centroid of the approximate co-ordinates x_i^0).

$$(ii) \quad \sum_{i=1}^{n_0} (Y_i^0 \delta Z_i - Z_i^0 \delta Y_i) = \sum_{i=1}^{n_0} (Z_i^0 \delta X_i - X_i^0 \delta Z_i) = \sum_{i=1}^{n_0} (X_i^0 \delta Y_i - Y_i^0 \delta X_i) = 0$$

(i.e the mean orientation of the system of points will not change after adjustment).

$$(iii) \quad \sum_{i=1}^{n_0} (X_i^0 \delta X_i + Y_i^0 \delta Y_i + Z_i^0 \delta Z_i) = 0 \text{ (i.e. the mean scale of the network will be held fixed).}$$

Thus the zero variance datum is implicitly defined in terms of the initial values of the co-ordinates (approximate co-ordinates) of the object points. Hence, the derived covariance matrices of the object points are related to this datum. It is notable that the co-ordinates are, in this way, datum dependent whereas some

of their derived functions (e.g. adjusted measurements) are not.

To conclude, in this research, equations (4.10), (4.21) and (4.22) are of crucial importance to the simulated cases discussed in the coming chapters when assessing the design criteria.

CHAPTER 5

SIMULATED NETWORKS FOR DEFORMATION ANALYSIS (CUBE)

5.1. Introduction

To design and plan a project before it is carried out is a common engineering practice. Pre-analysis is the simulation of the propagation of uncertainties in observations to uncertainties in results. In this way it is possible during the design of a photogrammetric network to predict the accuracies of the results and compare them with the desired accuracies. If the predicted results are too accurate or not accurate enough the design can be changed before any expensive work is carried out.

In this Chapter the simulation procedures used to demonstrate the concepts of precision, reliability and sensitivity analysis are described.

5.2. Network Configurations

Suppose a solid cube (Figure 5.1) of 4 m side comprises 26 regularly distributed points on its six faces. Such points represent the target array. Eight convergent photographs shown (each camera axis being directed towards one corner and the centre of the cube) will image the object with considerable redundancy. Fictitious provisional co-ordinates were assumed for the targets and the camera stations based on an origin at the centre of the cube. The taking camera is supposed to be the Zeiss (Jena) UMK 10/1318 with nominal principal distance 100 mm. Table (5.1) shows the

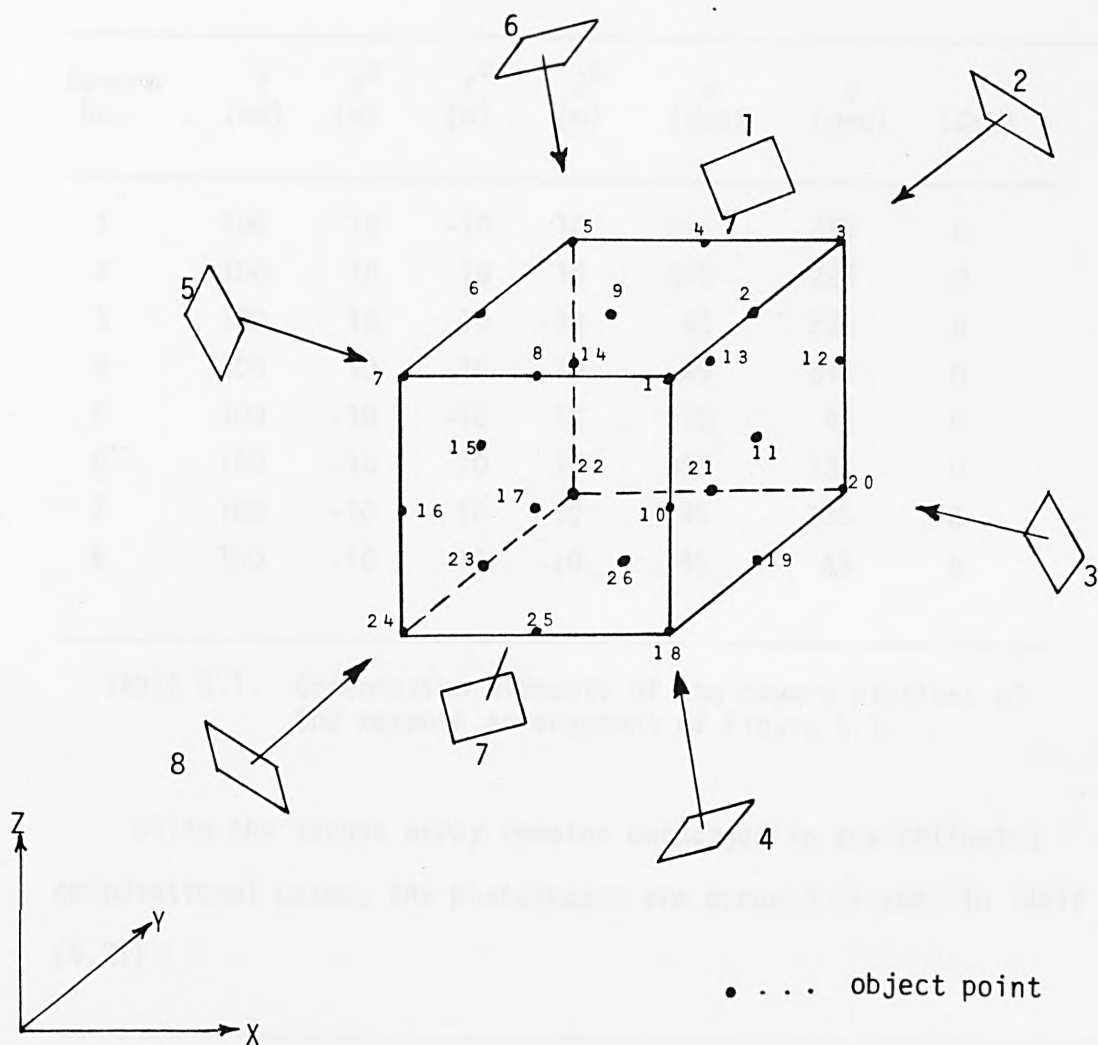


Figure 5.1. Object target array and Camera station configuration.

orientation elements of the individual photographs.

Camera No.	f (mm)	x^C (m)	y^C (m)	z^C (m)	ω (deg)	ϕ (deg)	κ (deg)
1	100	10	-10	10	315	315	0
2	100	10	10	10	315	225	0
3	100	10	10	-10	45	225	0
4	100	10	-10	-10	45	315	0
5	100	-10	-10	10	315	45	0
6	100	-10	10	10	315	135	0
7	100	-10	10	-10	45	135	0
8	100	-10	-10	-10	45	45	0

TABLE 5.1. Orientation elements of the camera stations of the network arrangement of Figure 5.1.

While the target array remains unchanged in the following computational cases, the photographs are grouped as seen in Table (5.2.)

Case	Photo-arrangement	Total number of object points	Number of slope distances
1	1-2-3-4	25	-
2	Case 1 + slope dist.	25	8
3	1-2-3-4-5	25	-
4	Case 3 + slope dist.	25	8
5	1-2-3-4-5-6	26	-
6	Case 5 + slope dist.	26	8
7	1-2-3-4-5-6-7	26	-
8	Case 7 + slope dist.	26	8
9	1-2-3-4-5-6-7-8	26	-
10	Case 9 + slope dist.	26	8

TABLE 5.2 Photo-arrangements, number of targets and slope distances.

Observation No.	Target Object Points	
	From	To
1	1	20
2	3	18
3	7	18
4	1	24
5	5	24
6	7	22
7	3	22
8	5	20

TABLE 5.3. Simulated slope distances.

5.3. The Observation equations

5.3.1. The Observation Equations of Photogrammetric Measurements

The observation equations are confined to the image co-ordinates. As the camera can only be used to recreate directions, the determination of object point co-ordinates must be achieved by intersection from at least two spatially separated camera stations.

As has been mentioned in Chapter 4, the collinearity condition equations(4.3) constitute the functional model for a photograph. Therefore, for any object point imaged and measured on a photograph, two equations can be written. One equation relates the x-co-ordinate of the photogrammetric measurement to the six elements of exterior orientation ($X_i^C, Y_i^C, Z_i^C, \omega_i, \phi_i, \kappa_i$) of the i th camera and to the three object point co-ordinates of the point (X_j, Y_j, Z_j), and other equation relates the y-co-ordinate of the photogrammetric measurement to the same items.

Linearisation of equations(4.3) using Taylor series expansion and neglecting second and higher order terms yields the following observation equations for photogrammetric measurements based on approximate values x^0 for x .

$$A_p \Delta x = V_p + b_p \quad (5.1)$$

Adopting the partitioning forms: $A_p = [A_{p1}, A_{p2}]$, $\Delta x = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$, $b_p = \begin{bmatrix} b_{p1} \\ b_{p2} \end{bmatrix}$, equation (5.1) could be put in the form:

$$A_{p1} \Delta x_1 + A_{p2} \Delta x_2 = V_p + b_p \quad (5.1a)$$

where: Δx_1 is the vector of correctionsto photo orientation elements

Δx_2 is the vector of corrections to object point co-ordinates

b_p is the vector of photo observational discrepancies

V_p is the vector of photo co-ordinate residuals,

and A_{p1}, A_{p2} are the design matrices.

For one object point imaged on one photograph:

$$A_{p1} = \begin{bmatrix} \frac{\partial x}{\partial X^C} & \frac{\partial x}{\partial Y^C} & \frac{\partial x}{\partial Z^C} & \frac{\partial x}{\partial \omega} & \frac{\partial x}{\partial \phi} & \frac{\partial x}{\partial \kappa} \\ \frac{\partial y}{\partial X^C} & \frac{\partial y}{\partial Y^C} & \frac{\partial y}{\partial Z^C} & \frac{\partial y}{\partial \omega} & \frac{\partial y}{\partial \phi} & \frac{\partial y}{\partial \kappa} \end{bmatrix}; \quad A_{p2} = \begin{bmatrix} \frac{\partial x}{\partial X} & \frac{\partial x}{\partial Y} & \frac{\partial x}{\partial Z} \\ \frac{\partial y}{\partial X} & \frac{\partial y}{\partial Y} & \frac{\partial y}{\partial Z} \end{bmatrix}$$

(2,6) (2,3)

$$\Delta x_1 = \begin{bmatrix} \delta X^C \\ \delta Y^C \\ \delta Z^C \\ \delta \omega \\ \delta \phi \\ \delta \kappa \end{bmatrix}; \quad \Delta x_2 = \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix}; \quad V_p = \begin{bmatrix} v_x \\ v_y \end{bmatrix}; \quad b_p = \begin{bmatrix} b_x \\ b_y \end{bmatrix}$$

(6,1) (3,1) (2,1) (2,1)

If there are n_s camera stations and n_o object points, the design matrix, A , is of order $2n_i \times (6n_s + 3n_o)$ in which n_i is the number of image points measured.

5.3.2. The Observation Equations of Survey measurements

Observation equations were developed for the slope distances in Chapter 4. It should be noted that in order to combine photogrammetric and survey measurements in a simultaneous adjustment, all the observation equations must be expressed in the same reference co-ordinate system. The utilisation of survey data will require the development of a solution which can combine both photogrammetric and survey measurements in a simultaneous solution.

In matrix form, the observation equations for slope distances based on approximate values x^0 for x can be:

$$A_g \Delta x_2 = v_g + b_g \quad (5.2)$$

where

b_g is the vector of survey observational discrepancies

v_g is the vector of survey observation residuals,

and A_g is the design matrix.

For slope distance, ℓ_{ij} ,

$$A_g = \begin{bmatrix} \frac{\partial \ell_{ij}}{\partial X_j} & \frac{\partial \ell_{ij}}{\partial Y_j} & \frac{\partial \ell_{ij}}{\partial Z_j} & \frac{\partial \ell_{ij}}{\partial X_i} & \frac{\partial \ell_{ij}}{\partial Y_i} & \frac{\partial \ell_{ij}}{\partial Z_i} \end{bmatrix} \quad (1,6)$$

$$\text{and } v_g = [v_{\ell_{ij}}] \quad (1,1) \quad ; \quad b_g = [b_{ij}] \quad (1,1)$$

Equation (5.1a) coupled with equation (5.2) constitute the complete extended observation equations in the simultaneous bundle adjustment model.

$$\begin{bmatrix} A_{p_1} & A_{p_2} \\ (2mn, 6m) & (2mn, 3n) \\ 0 & A_g \\ (k, 6m) & (k, 3n) \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ (6m, 1) \\ \Delta X_2 \\ (3n, 1) \end{bmatrix} = \begin{bmatrix} v_p \\ (2mn, 1) \\ v_g \\ (k, 1) \end{bmatrix} + \begin{bmatrix} b_p \\ (2mn, 1) \\ b_g \\ (k, 1) \end{bmatrix} \quad (5.3)$$

where

m, n, k represent orders of the above matrices as follows:-

m = number of photographs

n = number of object points imaged on m (or less) photographs,

k = number of survey measurements (slope distances) measured between targets.

5.4. Formation of the Normal Equations

5.4.1. Structure of the Normal equations matrix (N) with strictly photogrammetric measurements

From the linearised observations equations (5.1a) and their corresponding weights we can form the normal equations. The partitioned normal equations can be given by:

$$\begin{bmatrix} A_1^T W_p A_1 & A_1^T W_p A_2 \\ A_2^T W_p A_1 & A_2^T W_p A_2 \end{bmatrix} \begin{bmatrix} \Delta \hat{x}_1 \\ \Delta \hat{x}_2 \end{bmatrix} = \begin{bmatrix} A_1^T W_p b \\ A_2^T W_p b \end{bmatrix} \quad (5.4)$$

in which W_p is the image co-ordinate weight matrix. With $N_s = A_1^T W_p A_1$, $N_o = A_2^T W_p A_2$ and $N_{so} = A_1^T W_p A_2$, equation (5.4) could be written in the following simplified form:

$$\begin{bmatrix} N_s & N_{so} \\ N_{so}^T & N_o \end{bmatrix} \begin{bmatrix} \hat{\Delta x}_1 \\ \hat{\Delta x}_2 \end{bmatrix} = \begin{bmatrix} A_1^T W_p b \\ A_2^T W_p b \end{bmatrix} \quad (5.4a)$$

where the suffix o indicates object point co-ordinates and the suffix s indicates the camera parameters.

If there are n_s camera stations; each photograph will produce a 6 x 6 entry in N_s . In the meantime, if we have n_o object points; N_o will be a series of 3 x 3 diagonal blocks, with a block for each object point. N_{so} represents the interaction between an object point and the photographs on which it is imaged. Consequently, the entire coefficient matrix of normal equations is of order $(6n_s + 3n_o) \times (6n_s + 3n_o)$. Typical structures of normal equations are represented in Figures (5.2) through (5.6). It is notable that such structures show the property of being sparse banded-bordered matrices as only a few of the total n_o are imaged on each photograph.

It should be noted that if an object point does not image on a particular photograph, the corresponding 6 x 3 submatrix of N_{so} is null and the structure of N_{so} is not regular.

Due to the lack of datum definition, the normal equations matrix (5.4a) is singular and the observation equations (5.1a) have column rank deficiency of 7 (3 translations, 3 rotations and one scale) as discussed in Chapter 4.

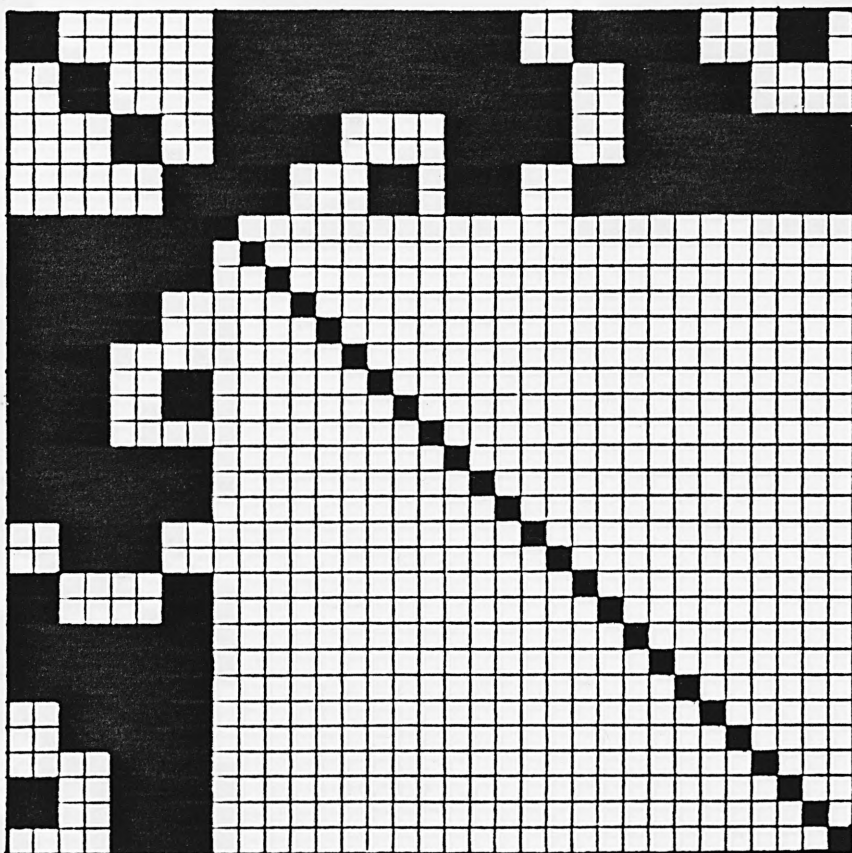


Figure 5.2. Structure of the coefficient matrix of the normal equations for 4 photos and 25 object points.

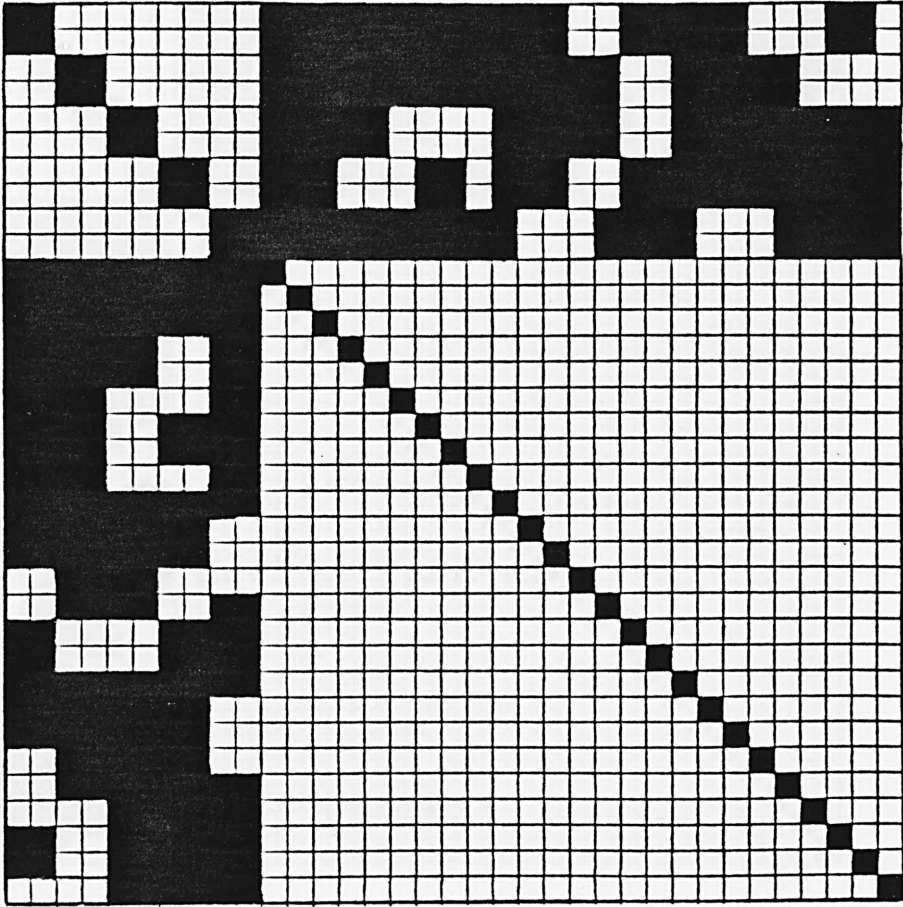


Figure 5.3. Structure of the coefficient matrix of the normal equations for 5 photos and 25 object points.

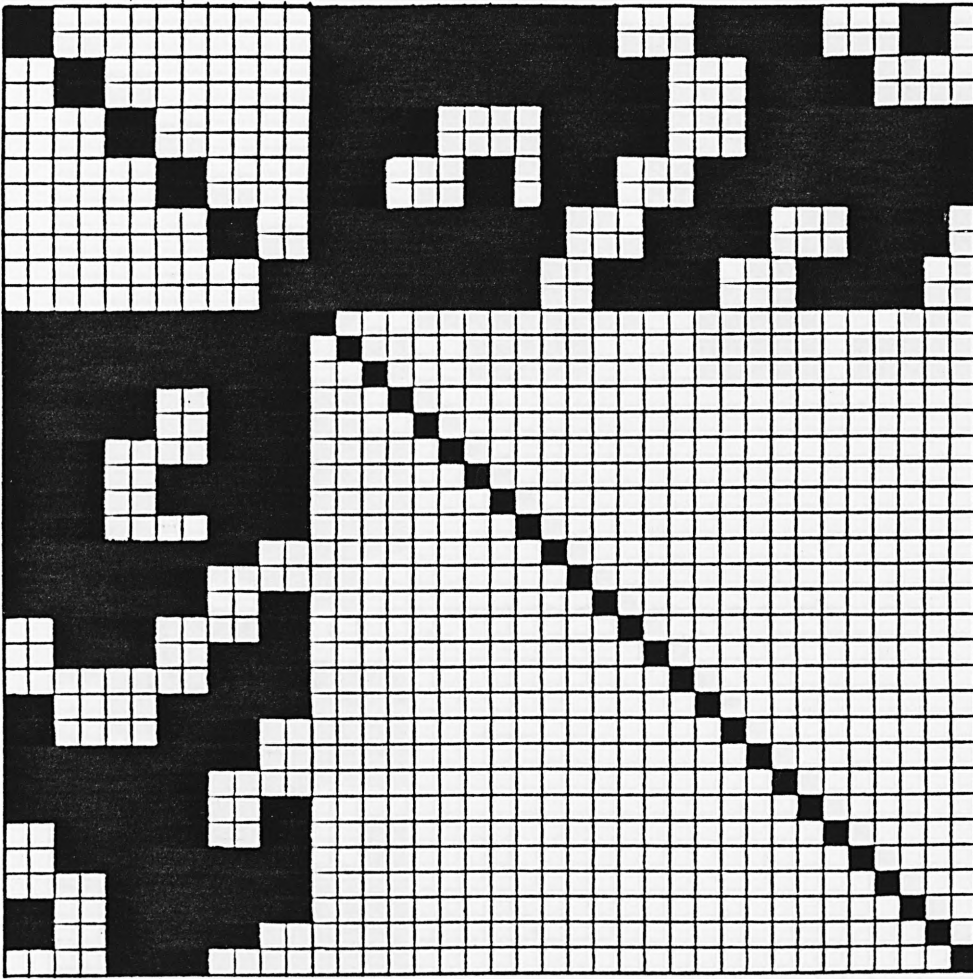


Figure 5.4. Structure of the coefficient matrix of the normal equations for 6 photos and 26 object points.

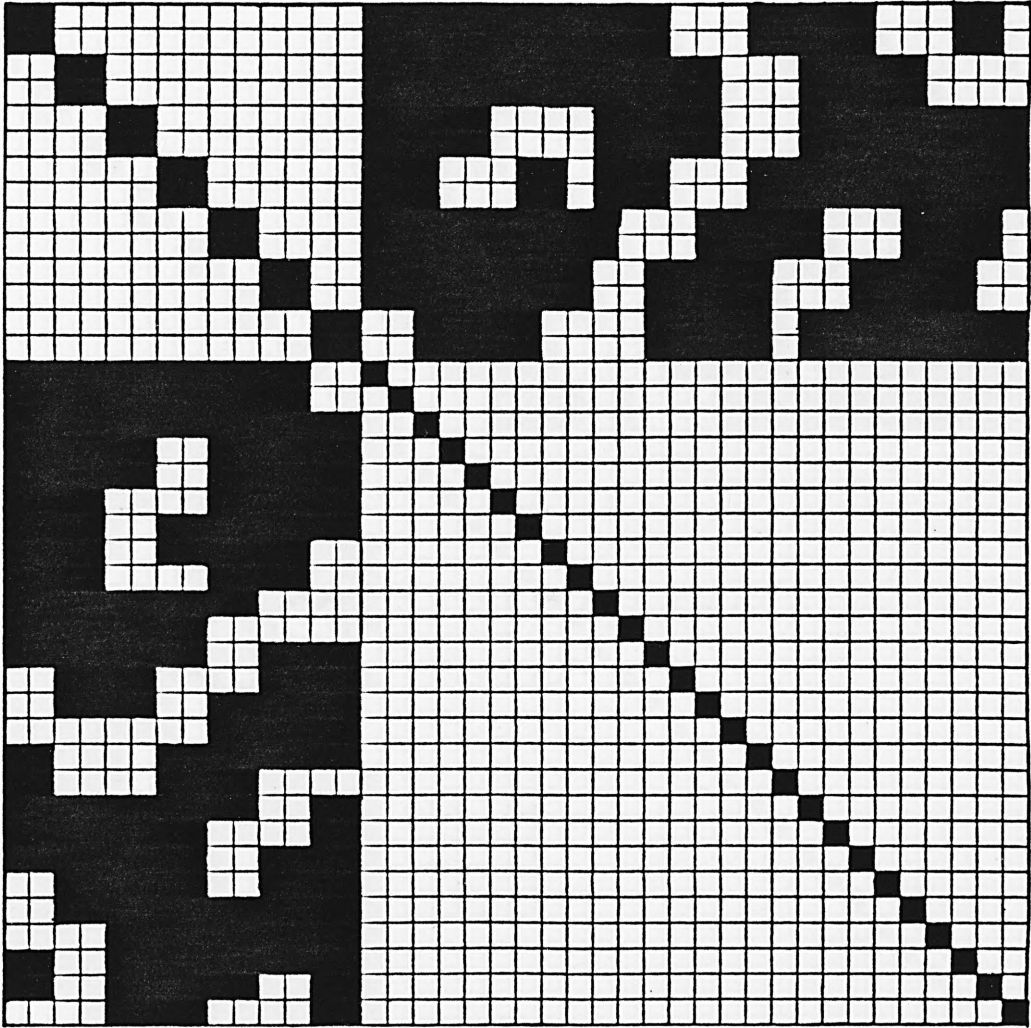


Figure 5.5. Structure of the coefficient matrix of the normal equations for 7 photos and 26 object points.

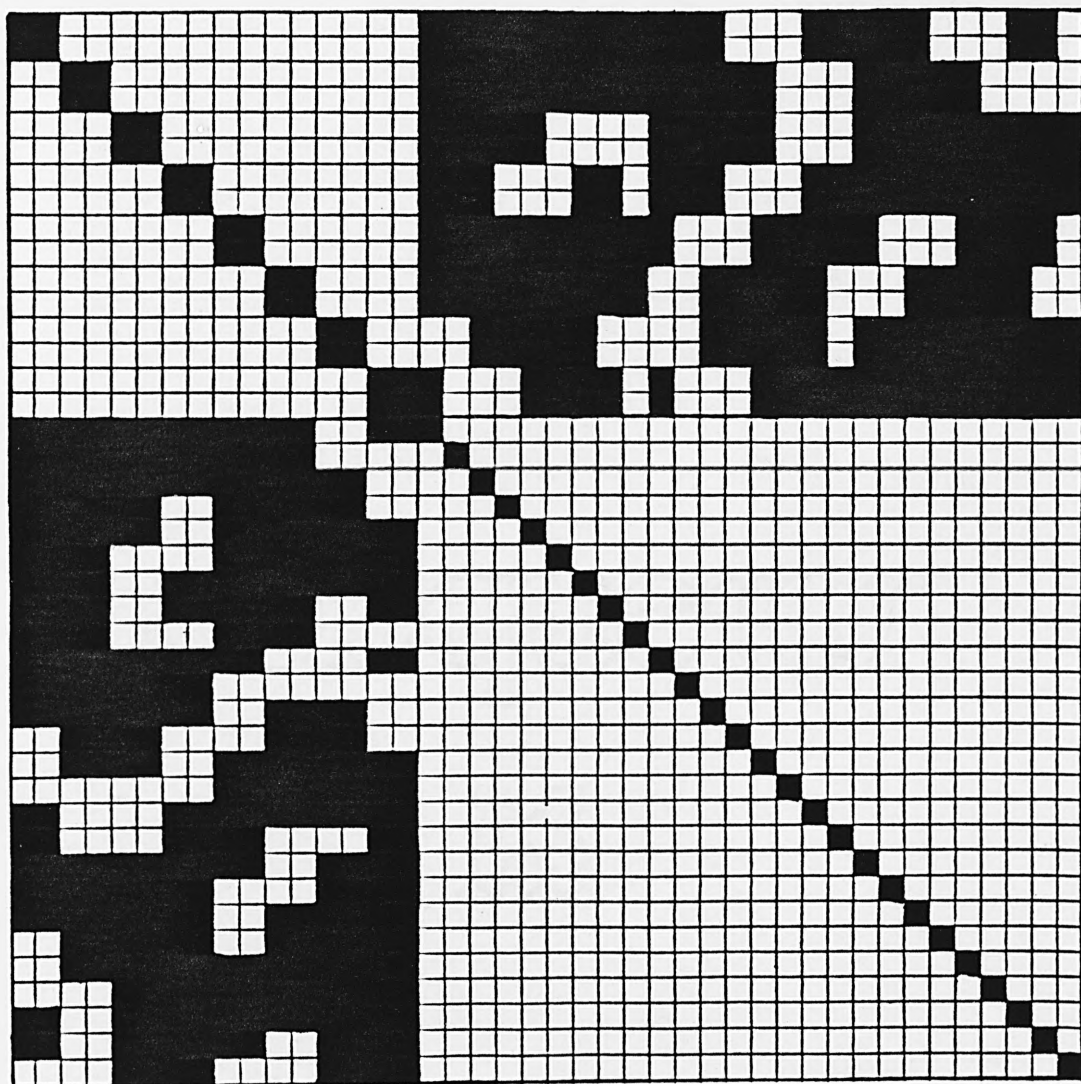


Figure 5.6. Structure of the coefficient matrix of the normal equations for 8 photos and 26 object points.

5.4.2. Effect of Survey measurements on the structure of N

The survey measurements are confined to the targets in this study. The slope distances between object points represent the survey measurements subject to investigation. Hence, their contribution to the normal equations will be in the submatrix N_o keeping the order of the normal equations constant for each two successive cases of the same number of cameras.

The structure of the normal equations with only photogrammetric observations as discussed in Section (5.4.1) can be exploited efficiently by reducing the size of the normal equations when seeking their solution. Such reduction can be performed through successive inversion of n_o (3×3) submatrices or n_s (6×6) ones depending on the size of n_o and n_s . However, the inclusion of n_d distances between object points, given in Table (5.3) necessitates the inversion of n_d (6×6) submatrices in addition to the usual inversion of a series of one of the two categories mentioned before.

Such an inclusion of survey observations in object space has a devastating effect upon the matrix N in terms of computer storage as the quality of sparsity is no longer retained. Further, the correlation between targets of object points increases the bandwidth of N_o significantly.

With the presence of survey measurements (slope distances) the normal equations (5.4) will be modified to count for the contribution of these survey measurements and could be written as:-

$$\begin{bmatrix} A_1^T W_p A_1 & A_1^T W_p A_2 \\ A_2^T W_p A_1 & A_2^T W_p A_2 + A_g^T W_g A_g \end{bmatrix} \begin{bmatrix} \Delta \hat{x}_1 \\ \Delta \hat{x}_2 \end{bmatrix} = \begin{bmatrix} A_1^T W_p b \\ A_2^T W_p b + A_g^T W_g b_g \end{bmatrix} \quad (5.5)$$

in which subscripts p and g designate photogrammetric and survey respectively, and W_g denotes the survey measurements weight matrix. Of prime computational importance is the fact that the normal equations matrix is created here as a point-wise accumulation without actually forming the observation equations matrix A. This means that the different observation equations need never be formed and considerable savings in computing and storage requirements is attained.

5.5. Inversion Algorithms and Covariance matrix of Co-ordinates

It has been mentioned, Section (4.2.2) that the common assumption that the a priori covariance matrix of the photogrammetric observations is a scalar matrix is in use; however it is almost certainly fallacious, yet it has the merit of being common to all the results which we wish to compare. As one of the primary goals of this investigation is to compare different network configurations to assess precision, reliability and sensitivity criteria for different designs, the most universal estimate of precision is the covariance matrix derived from the inverse of the coefficient matrix of the normal equations as discussed in Section (4.2.3).

Owing to the column rank deficiency of (5.4a) and (5.5), there is no standard Cayley inverse as has been addressed in the above-mentioned sections, but a generalised inverse, namely the Moore-Penrose

can be used. Such an inverse yields minimum trace covariance matrix of the unknowns. Through this investigation, F01BLF subroutine from the National Algorithm Group (NAG) Library was used to determine the Moore-Penrose inverse of the normal equations in cases 1, 2, 3 and 4. Unfortunately, its use was inefficient as it is rather expensive in terms of execution time and computer storage.

Alternatively, the likely most straight-forward approach is the method of inner constraints as mentioned in Section (4.2.3.2) and discussed, in detail, in Appendix B. It involves the bordering of the submatrix corresponding to the object point co-ordinates of the singular matrix, equation (5.5) with a transformation matrix, G , to minimise only the trace of their cofactor matrix.

G satisfies the condition $G^T \Delta x_2 = 0$. For an object point, j , in a network with a datum defect of seven, the appropriate 3×7 matrix G_j is given as follows (see Appendix B):

$$G_j = \begin{bmatrix} 1 & 0 & 0 & 0 & Z_j^0 & -Y_j^0 & X_j^0 \\ 0 & 1 & 0 & -Z_j^0 & 0 & X_j^0 & Y_j^0 \\ 0 & 0 & 1 & Y_j^0 & -X_j^0 & 0 & Z_j^0 \end{bmatrix} \quad (5.6)$$

Under the scheme of inner constraints, the augmented normal equations matrix (non-singular) can be written as:

$$\begin{bmatrix} A_1^T W_p A_1 & A_1^T W_p A_2 & 0 \\ A_2^T W_p A_1 & A_2^T W_p A_2 + A_g^T W_g A_g & G \\ 0 & G^T & 0 \end{bmatrix} \begin{bmatrix} \Delta \hat{X}_1 \\ \Delta \hat{X}_2 \\ \hat{K} \end{bmatrix} = \begin{bmatrix} A_1^T W_p b \\ A_2^T W_p b + A_g^T W_g b_g \\ 0 \end{bmatrix} \quad (5.7)$$

in which $\hat{K}$ is an estimator for the vector of Lagrangian multipliers.

With the relatively small networks encountered in deformation monitoring, a direct solution can be considered for equation (5.7) using the standard Cayley inverse. The Cholesky factorisation algorithm best suited symmetric matrices cannot be used for the solution of equation (5.7) due to the introduction of a null principal submatrix which causes the matrix to become indefinite. Thus, an alternative to the Cholesky factorisation must be found.

At first sight, Gaussian elimination with partial or complete pivoting may appear to represent a useful alternative solution technique. In applying complete pivoting, the first trailing submatrix produced will generally no longer be symmetric. However, to preserve the symmetry property, the choice of pivots is restricted to the diagonal elements, yet the augmented matrix has a null principal submatrix which will give rise to zero pivots.

Further, the recursive partitioning technique, which could have been used to reduce the size of the normal equations matrix, cannot be applied as it necessitates the presence of the inverse of one of the main block diagonal submatrices which is not the case.

An efficient inversion subroutine, available at the London University Computer Centre: Scientific Subroutine Package (SSP), was used. Such a subroutine was tested and found to give very accurate results when double precision arithmetic was used. The computer programs were coded in Fortran 77 Language and were run on the Honeywell, Amdahl and Cray 1S systems.

Now, if the standard Cayley inverse of equation(5.7) is expressed as:

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ & M_{22} & M_{23} \\ \text{Symmetric} & & M_{33} \end{bmatrix}$$

then M_{22} is identical to the minimum trace cofactor matrix $Q_{xx}^{(2)}$ obtained when using the Moore-Penrose inverse.

As has been mentioned in Section (4.2.3.2) the incorporation of slope distances, the seventh column of G in equation (5.6) is to be suppressed.

The inner constraints can be imposed through G related in some cases to all object points and in others to a sub-set of targets. However, the former situation can be advantageous especially in sensitivity analysis stage.

5.6. Simulated Networks

5.6.1. Description

In order to examine the numerical behaviour of the different criteria for design, namely, precision, reliability and sensitivity, an experiment was conducted in which data were obtained through simulation. Of primary concern was examination of the magnitude of the change of the mean variance σ_m^2 of the object point co-ordinates, the maximum internal reliability $r_{\max}$ of the observations and the indicative parameter of sensitivity c which accompany addition of slope distance measurements and/or changes in the number of cameras used.

Ten distinct cases(Table (5.2)) were considered, 5 μ m standard deviation has been assumed for the photogrammetric co-ordinates. Photography on glass plates with format 130 x 180 mm was assumed. The odd-numbered cases comprise pure photogrammetric observations whereas the rest (even-numbered) include in addition to the photogrammetric observations, 8 diagonal distances, with presupposed 0.5 mm standard deviation on the four upright faces (Tables (5.2), (5.3)) of the cube. The object target array comprises 26 points as mentioned earlier in Section (5.2).

The least number of photographs to cover the cube was found to be 4, otherwise some of the object points would appear only on one photograph, in which case its co-ordinates and their variances would be indeterminate. Therefore the start arrangement was to have 4 photographs as shown in Figure (5.1).

Density of object points was such that, on average, 19 image points would appear on each photograph. Examination of Table (5.2) shows that the number of object points in the first four cases is 25. There is one missing point (point 15) which does not appear in cases 1, 2, however it appears in cases 3,4 on photograph 5. The computer programs are coded in a way to override any point which appears only on one photograph. This assumption helps comparison between two identical sets of object points. On the other hand, addition of photographs 6,7, 8 (Figure (5.1)) leads to imaging point 15 on 2,3, 4 photographs respectively. So the number of targetted object points will be 26, instead of 25, in cases 5-10.

5.6.2. Results and Analysis

It is relevant to mention at the outset that due to the bulk of the results, it is not possible to include all of them in the text but a sample is to be included. Results of cases 9 and 10 were chosen to be typical representations as they give comprehensive illustration for the concepts to be discussed herein.

5.6.2.1. Mean Variance of Object Point co-ordinates

An indication of the effect of number of cameras employed in cases 1-10 is perhaps best gained by equation (3.15) which reads

$$\sigma_m^2 = \frac{1}{3n_0} \text{tr } Q_{\hat{x}\hat{x}}^{(2)} \quad (5.8)$$

in which n_0 is the number of object points and $Q_{\hat{x}\hat{x}}^{(2)}$ is their cofactor matrix. Because the geometry of the target array is the same for each case, the derived a priori precision of the object point co-ordinates depends almost solely on the number of cameras.

The variances of the estimated object point co-ordinates for each case have been computed using the free network bundle adjustment procedure. Figures (5.7) and (5.8) illustrate the variation in the mean variance σ_m^2 with changes in the configuration of camera stations in both pure and combined cases, which comprise slope distances, respectively. It is obvious from the two figures that precision increases with the increase of the number of cameras used. As expected, the incorporation of the 8 slope distances does improve the precision. However, such an improvement is marginal "within"

POINT		VARIANCES		
NO	X (mm ²)	Y (mm ²)	Z (mm ²)	
1	0.141903720 00	0.147686720 00	0.147686720 00	
2	0.154790300 00	0.163509650 00	0.162730120 00	
3	0.141903720 00	0.147686720 00	0.147686720 00	
4	0.164473660 00	0.164535120 00	0.164535120 00	
5	0.141903720 00	0.147686720 00	0.147686720 00	
6	0.154790300 00	0.163509650 00	0.162730120 00	
7	0.141903720 00	0.147686720 00	0.147686720 00	
8	0.164473660 00	0.164535120 00	0.164535120 00	
9	0.244125940 00	0.243007220 00	0.209525730 00	
10	0.154790800 00	0.162730120 00	0.163509650 00	
11	0.186263390 00	0.232673890 00	0.232673890 00	
12	0.154790800 00	0.162730120 00	0.163509650 00	
13	0.244125940 00	0.209525730 00	0.243007220 00	
14	0.154790300 00	0.162730120 00	0.163509650 00	
15	0.186263390 00	0.232673890 00	0.232673890 00	
16	0.154790800 00	0.162730120 00	0.163509650 00	
17	0.244125940 00	0.209525730 00	0.243007220 00	
18	0.141903720 00	0.147686720 00	0.147686720 00	
19	0.154790300 00	0.163509650 00	0.162730120 00	
20	0.141903720 00	0.147686720 00	0.147686720 00	
21	0.164473660 00	0.164535120 00	0.164535120 00	
22	0.141903720 00	0.147686720 00	0.147686720 00	
23	0.154790800 00	0.163509650 00	0.162730120 00	
24	0.141903720 00	0.147686720 00	0.147686720 00	
25	0.164473660 00	0.164535120 00	0.164535120 00	
26	0.244125940 00	0.243007220 00	0.209525730 00	

TABLE 5.4. Estimates of the variances of the 26 object point co-ordinates for case 9.

POINT		VARIANCES		
NO	X (mm ²)	Y (mm ²)	Z (mm ²)	
1	0.127320230 00	0.131226350 00	0.110959510 00	
2	0.168089700 00	0.163200100 00	0.173010270 00	
3	0.127320230 00	0.131226350 00	0.110959510 00	
4	0.164214630 00	0.178338090 00	0.174980070 00	
5	0.127320230 00	0.131226350 00	0.110959510 00	
6	0.168089700 00	0.163200100 00	0.173010270 00	
7	0.127320230 00	0.131226350 00	0.110959510 00	
8	0.164214630 00	0.178338090 00	0.174980070 00	
9	0.243850610 00	0.242769000 00	0.222686680 00	
10	0.167912560 00	0.176114230 00	0.163121180 00	
11	0.201693660 00	0.232652130 00	0.232403890 00	
12	0.167912560 00	0.176114230 00	0.163121180 00	
13	0.244100360 00	0.225901930 00	0.242776290 00	
14	0.167912560 00	0.176114230 00	0.163121180 00	
15	0.201693660 00	0.232652130 00	0.232403890 00	
16	0.167912560 00	0.176114230 00	0.163121180 00	
17	0.244100360 00	0.225901930 00	0.242776290 00	
18	0.127320230 00	0.131226350 00	0.110959510 00	
19	0.168089700 00	0.163200100 00	0.173010270 00	
20	0.127320230 00	0.131226350 00	0.110959510 00	
21	0.164214630 00	0.178338090 00	0.174980070 00	
22	0.127320230 00	0.131226350 00	0.110959510 00	
23	0.168089700 00	0.163200100 00	0.173010270 00	
24	0.127320230 00	0.131226350 00	0.110959510 00	
25	0.164214630 00	0.178338090 00	0.174980070 00	
26	0.243850610 00	0.242769000 00	0.222686680 00	

TABLE 5.5. Estimates for the variances of the 26 object point co-ordinates for case 10.

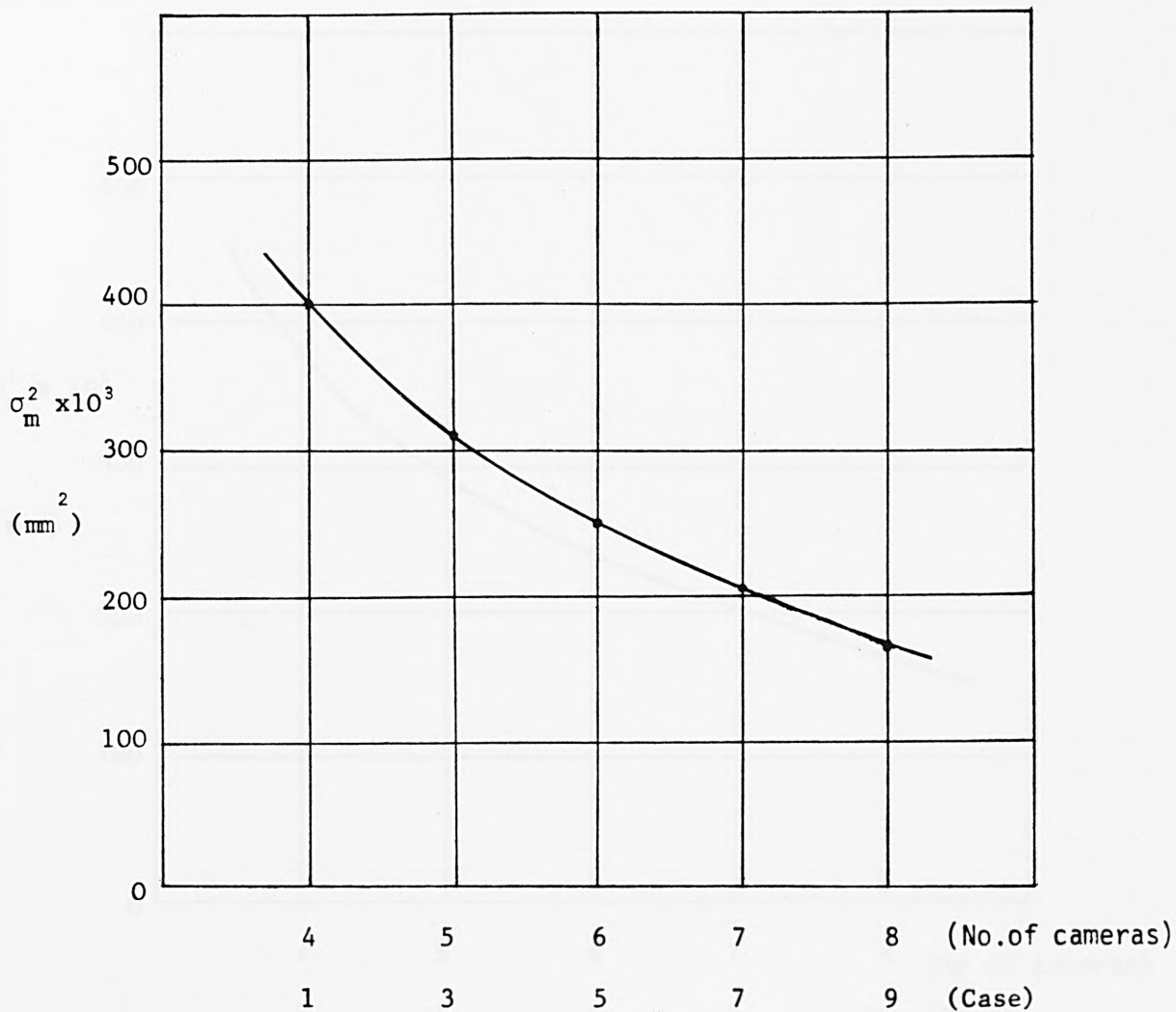


Figure 5.7. Relation between no.of cameras and precision
(Photogrammetry)

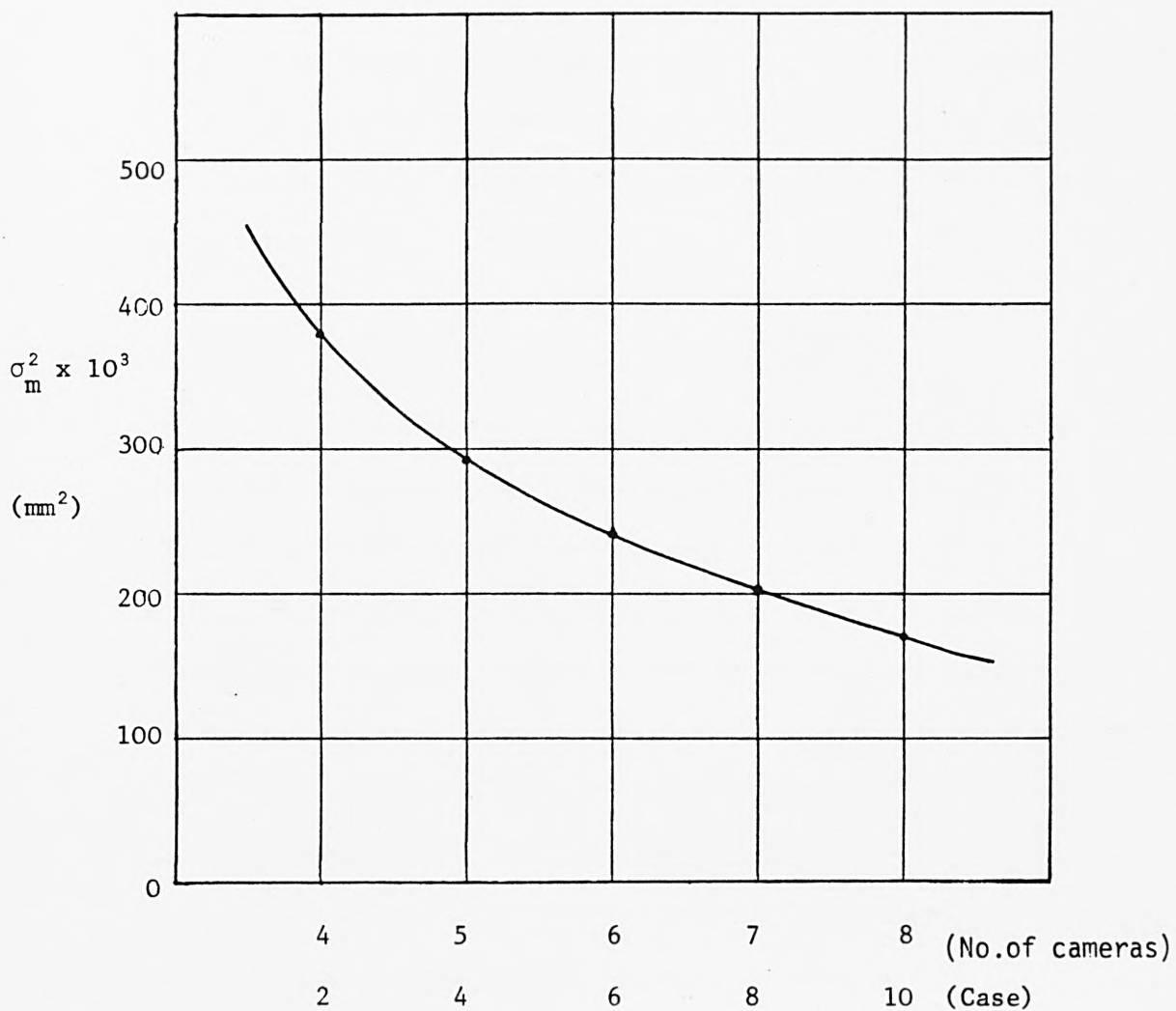


Figure 5.8. Relation between no.of cameras and precision
(Photogrammetry + 8 distances)

the same number of camera cases. On the other hand, the improvement in precision "between" different number of camera cases is more significant. On the average there is a 24% and 23% improvement in precision should the number of cameras be increased to be 8 instead of 4 for the pure cases and combined cases respectively. The highest precision level is exhibited by cases 9 and 10.

5.6.2.2. $r_{\max}$ and other reliability indicators

In the design phase of a monitoring network, reliability values are computed to ensure that the network geometry and planned observation scheme exhibit a high degree of self-checking. Reliability values were attained by applying the concept of "redundancy numbers" (r_i), for each observation, computed from the diagonal elements ($q_{\hat{v}_i \hat{v}_i}$) of the cofactor matrix of the residuals $Q_{\hat{v}\hat{v}}$. Equation (5.9) represents such redundancy numbers. It can be argued that by computing the redundancy numbers in simulated tests, it is possible to discover weak situations in advance and avoid them while planning the photogrammetric project.

The global reliability of the x- and y- co-ordinate observations are represented by $r_{\max}(x)$ and $r_{\max}(y)$ respectively. Equation (5.9) can be given as:

$$r_i = q_{\hat{v}_i \hat{v}_i} \times w_{\ell_i} \quad (5.9)$$

where w_{ℓ_i} is the a priori weight of observation ℓ_i .

PHOTO	POINT	$r(x)$	$r(y)$
1	1	0.440251680 00	0.438486600 00
	2	0.547907600 00	0.571427720 00
	3	0.608310040 00	0.629910640 00
	4	0.648297680 00	0.670342120 00
	5	0.633745720 00	0.621203600 00
	6	0.687692840 00	0.671920760 00
	7	0.670607200 00	0.639901680 00
	8	0.597263240 00	0.555515320 00
	9	0.557774720 00	0.580557320 00
	10	0.549432200 00	0.569903120 00
	11	0.588433480 00	0.547137920 00
	12	0.648541320 00	0.649282280 00
	16	0.686560640 00	0.673052960 00
	17	0.554353000 00	0.583979040 00
	18	0.608663960 00	0.629556760 00
	19	0.649227720 00	0.648595880 00
	20	0.581180240 00	0.612393840 00
	24	0.634347320 00	0.620602000 00
	25	0.647486400 00	0.671153440 00
2	1	0.608663960 00	0.629556760 00
	2	0.549432200 00	0.569903120 00
	3	0.440251680 00	0.438486600 00
	4	0.597263240 00	0.555515320 00
	5	0.670607200 00	0.639901680 00
	6	0.686560640 00	0.673052960 00
	7	0.634347320 00	0.620602000 00
	8	0.647486400 00	0.671153440 00
	9	0.554353000 00	0.583979040 00
	10	0.649227720 00	0.648595880 00
	11	0.588433480 00	0.547137920 00
	12	0.547907600 00	0.571427720 00
	13	0.557774720 00	0.580557320 00
	14	0.687692840 00	0.671920760 00
	18	0.581180240 00	0.612393840 00
	19	0.648541320 00	0.649282280 00
	20	0.608310040 00	0.629910640 00
	21	0.648297680 00	0.670342120 00
	22	0.633745720 00	0.621203600 00

TABLE 5.6. Computed values of redundancy numbers for case 9.

PHOTO	POINT	$r(x)$	$r(y)$
3	1	0.581180240 00	0.612393840 00
	2	0.648541320 00	0.649282280 00
	3	0.608310040 00	0.629910640 00
	4	0.648297680 00	0.670342120 00
	5	0.633745720 00	0.621203600 00
	10	0.649227720 00	0.648595880 00
	11	0.588433480 00	0.547137920 00
	12	0.547907600 00	0.571427720 00
	13	0.557774720 00	0.580557320 00
	14	0.687692840 00	0.671920760 00
	18	0.608663960 00	0.629556760 00
	19	0.549432200 00	0.569903120 00
	20	0.440251680 00	0.438486600 00
	21	0.597263240 00	0.555515320 00
	22	0.670607200 00	0.639901680 00
	23	0.636560640 00	0.673052960 00
	24	0.634347320 00	0.620602000 00
	25	0.647486400 00	0.671153440 00
	26	0.554353000 00	0.583979040 00
4	1	0.608663960 00	0.629556760 00
	2	0.649227720 00	0.648595880 00
	3	0.581180240 00	0.612393840 00
	7	0.634347320 00	0.620602000 00
	8	0.647486400 00	0.671153440 00
	10	0.549432200 00	0.569903120 00
	11	0.588433480 00	0.547137920 00
	12	0.648541320 00	0.649282280 00
	16	0.636560640 00	0.673052960 00
	17	0.554353000 00	0.583979040 00
	18	0.440251680 00	0.438486600 00
	19	0.547907600 00	0.571427720 00
	20	0.608310040 00	0.629910640 00
	21	0.648297680 00	0.670342120 00
	22	0.633745720 00	0.621203600 00
	23	0.687692840 00	0.671920760 00
	24	0.670607200 00	0.639901680 00
	25	0.597263240 00	0.555515320 00
	26	0.557774720 00	0.580557320 00

TABLE 5.6. (Continued)

PHOTO	POINT	r(x)		r(y)	
5	1	0.670607200	00	0.639901680	00
	2	0.686560640	00	0.673052960	00
	3	0.634347320	00	0.620602000	00
	4	0.647486400	00	0.671153440	00
	5	0.608663960	00	0.629556760	00
	6	0.549432200	00	0.569903120	00
	7	0.440251680	00	0.438486600	00
	8	0.597263240	00	0.555515320	00
	9	0.554353000	00	0.583979040	00
	10	0.687692840	00	0.671920760	00
	14	0.649227720	00	0.648595880	00
	15	0.588433480	00	0.547137920	00
	16	0.547907600	00	0.571427720	00
	17	0.557774720	00	0.530557320	00
	18	0.633745720	00	0.621203600	00
	22	0.581180240	00	0.612393840	00
	23	0.648541320	00	0.649282280	00
	24	0.608310040	00	0.629910640	00
	25	0.648297680	00	0.670342120	00
6	1	0.634347320	00	0.620602000	00
	2	0.686560640	00	0.673052960	00
	3	0.670607200	00	0.639901680	00
	4	0.597263240	00	0.555515320	00
	5	0.440251680	00	0.438486600	00
	6	0.549432200	00	0.569903120	00
	7	0.608663960	00	0.629556760	00
	8	0.647486400	00	0.671153440	00
	9	0.554353000	00	0.583979040	00
	12	0.687692840	00	0.671920760	00
	13	0.557774720	00	0.530557320	00
	14	0.547907600	00	0.571427720	00
	15	0.588433480	00	0.547137920	00
	16	0.649227720	00	0.648595880	00
	20	0.633745720	00	0.621203600	00
	21	0.648297680	00	0.670342120	00
	22	0.608310040	00	0.629910640	00
	23	0.648541320	00	0.649282280	00
	24	0.581180240	00	0.612393840	00

TABLE 5.6. (Continued)

PHOTO	POINT	r(x)	r(y)
7	3	0.634347320 00	0.620602000 00
	4	0.647486400 00	0.671153440 00
	5	0.608663960 00	0.629556760 00
	6	0.649227720 00	0.648595880 00
	7	0.581180240 00	0.612393840 00
	12	0.686560640 00	0.673052960 00
	13	0.554353000 00	0.583979040 00
	14	0.549432200 00	0.569903120 00
	15	0.588433480 00	0.547137920 00
	16	0.648541320 00	0.649282280 00
	18	0.633745720 00	0.621203600 00
	19	0.687692840 00	0.671920760 00
	20	0.670607200 00	0.639701680 00
	21	0.597263240 00	0.555515320 00
	22	0.440251680 00	0.438486600 00
	23	0.547907600 00	0.571427720 00
	24	0.608310040 00	0.629910640 00
	25	0.648297680 00	0.670342120 00
	26	0.557774720 00	0.580557320 00
8	1	0.634347320 00	0.620602000 00
	5	0.581180240 00	0.612393840 00
	6	0.649227720 00	0.648595880 00
	7	0.608663960 00	0.629556760 00
	8	0.647486400 00	0.671153440 00
	10	0.686560640 00	0.673052960 00
	14	0.648541320 00	0.649282280 00
	15	0.588433480 00	0.547137920 00
	16	0.549432200 00	0.569903120 00
	17	0.554353000 00	0.583979040 00
	18	0.670607200 00	0.639701680 00
	19	0.687692840 00	0.671920760 00
	20	0.633745720 00	0.621203600 00
	21	0.648297680 00	0.670342120 00
	22	0.608310040 00	0.629910640 00
	23	0.547907600 00	0.571427720 00
	24	0.440251680 00	0.438486600 00
	25	0.597263240 00	0.555515320 00
	26	0.557774720 00	0.580557320 00

TABLE 5.6. (Continued)

PHOTO	POINT	$r(x)$		$r(y)$	
1	1	0.453066240	00	0.445887600	00
	2	0.550596760	00	0.573003920	00
	3	0.616604720	00	0.691825000	00
	4	0.648993640	00	0.671484160	00
	5	0.672857920	00	0.662692080	00
	6	0.689236320	00	0.672440960	00
	7	0.717712560	00	0.657477240	00
	8	0.600204040	00	0.556824000	00
	9	0.558588080	00	0.580928800	00
	10	0.551802920	00	0.571699320	00
	11	0.589341520	00	0.547410520	00
	12	0.650302200	00	0.650095920	00
	16	0.637775720	00	0.674709360	00
	17	0.554749320	00	0.534301200	00
	18	0.655712840	00	0.678880040	00
	19	0.650502760	00	0.648787640	00
	20	0.616136240	00	0.627122480	00
	24	0.637050840	00	0.667242040	00
	25	0.647834320	00	0.672322400	00
2	1	0.617424280	00	0.691005440	00
	2	0.552328360	00	0.571272320	00
	3	0.453851600	00	0.445102280	00
	4	0.600379720	00	0.556648360	00
	5	0.715057320	00	0.660134440	00
	6	0.638092600	00	0.673584720	00
	7	0.673175280	00	0.662374720	00
	8	0.648236480	00	0.672241280	00
	9	0.555229720	00	0.534287160	00
	10	0.651016480	00	0.649381640	00
	11	0.589340960	00	0.547411080	00
	12	0.550341280	00	0.573160960	00
	13	0.558201760	00	0.531348760	00
	14	0.638964680	00	0.673520360	00
	18	0.613267380	00	0.629990840	00
	19	0.649821400	00	0.649469000	00
	20	0.654987200	00	0.679605680	00
	21	0.648692800	00	0.671463960	00
	22	0.636090200	00	0.668212680	00

TABLE 5.7. Computed values of redundancy numbers for case 10.

PHOTO	POINT	r(x)		r(y)	
3	1	0.613267880	00	0.629290840	00
	2	0.649821400	00	0.649469000	00
	3	0.654987200	00	0.679605680	00
	4	0.648692800	00	0.671463960	00
	5	0.636080200	00	0.668212680	00
	10	0.651016480	00	0.649381640	00
	11	0.589340960	00	0.547411080	00
	12	0.550341280	00	0.573160960	00
	13	0.558201760	00	0.581348760	00
	14	0.688964680	00	0.673520360	00
	18	0.617424280	00	0.691005440	00
	19	0.552328360	00	0.571272320	00
	20	0.453851600	00	0.445102280	00
	21	0.600379720	00	0.556648360	00
	22	0.715057320	00	0.660134440	00
	23	0.688092600	00	0.673584720	00
	24	0.673175280	00	0.662374720	00
	25	0.648236480	00	0.672241280	00
	26	0.555229720	00	0.584287160	00
4	1	0.655712840	00	0.678380040	00
	2	0.650502760	00	0.648787640	00
	3	0.616136240	00	0.627122480	00
	7	0.637050840	00	0.667242040	00
	8	0.647834320	00	0.672322400	00
	10	0.551802920	00	0.571699320	00
	11	0.589341520	00	0.547410520	00
	12	0.650302200	00	0.650095920	00
	16	0.687775720	00	0.674709360	00
	17	0.554749320	00	0.584801200	00
	18	0.453066240	00	0.445887600	00
	19	0.550596760	00	0.573003920	00
	20	0.616604720	00	0.691825000	00
	21	0.648993640	00	0.671484160	00
	22	0.672857920	00	0.662692080	00
	23	0.689236320	00	0.672440960	00
	24	0.717712560	00	0.657479240	00
	25	0.600204040	00	0.556824000	00
	26	0.558588080	00	0.580928800	00

TABLE 5.7. (Continued)

PHOTO	POINT	$r(x)$		$r(y)$	
5	1	0.715057320	00	0.660134440	00
	2	0.683092600	00	0.673584720	00
	3	0.673175280	00	0.662374720	00
	4	0.648236480	00	0.672241280	00
	5	0.617424280	00	0.691005440	00
	6	0.552328360	00	0.571272320	00
	7	0.453851600	00	0.445102280	00
	8	0.600379720	00	0.556648360	00
	9	0.555229720	00	0.584287160	00
	10	0.638964680	00	0.673520360	00
	14	0.651016480	00	0.649381640	00
	15	0.589340960	00	0.547411080	00
	16	0.550341280	00	0.573160960	00
	17	0.558201760	00	0.581348760	00
	18	0.636080200	00	0.668212680	00
	22	0.613267880	00	0.629990840	00
	23	0.649821400	00	0.649469000	00
	24	0.654987200	00	0.679605680	00
	25	0.643692800	00	0.671463960	00
6	1	0.673175280	00	0.662374720	00
	2	0.683092600	00	0.673584720	00
	3	0.715057320	00	0.660134440	00
	4	0.600379720	00	0.556648360	00
	5	0.453851600	00	0.445102280	00
	6	0.552328360	00	0.571272320	00
	7	0.617424280	00	0.691005440	00
	8	0.648236480	00	0.672241280	00
	9	0.555229720	00	0.584287160	00
	12	0.638964680	00	0.673520360	00
	13	0.558201760	00	0.581348760	00
	14	0.550341280	00	0.573160960	00
	15	0.589340960	00	0.547411080	00
	16	0.651016480	00	0.649381640	00
	20	0.636080200	00	0.668212680	00
	21	0.648692800	00	0.671463960	00
	22	0.654987200	00	0.679605680	00
	23	0.649821400	00	0.649469000	00
	24	0.613267880	00	0.629990840	00

TABLE 5.7.(Continued)

PHOTO	POINT	r(x)		r(y)	
7	3	0.637050840	00	0.667242040	00
	4	0.647834320	00	0.672322400	00
	5	0.655712840	00	0.678880040	00
	6	0.650502760	00	0.648787640	00
	7	0.616136240	00	0.627122480	00
	12	0.637775720	00	0.674709360	00
	13	0.554749320	00	0.584801200	00
	14	0.551802920	00	0.571699320	00
	15	0.589341520	00	0.547410520	00
	16	0.650302200	00	0.650095920	00
	18	0.672857920	00	0.662692080	00
	19	0.689236320	00	0.672440960	00
	20	0.717712560	00	0.657479240	00
	21	0.600204040	00	0.556824000	00
	22	0.453066240	00	0.445887600	00
	23	0.550596760	00	0.573003920	00
	24	0.616604720	00	0.691825000	00
	25	0.648993640	00	0.671484160	00
	26	0.558588080	00	0.580928800	00
8	1	0.637050840	00	0.667242040	00
	5	0.616136240	00	0.627122480	00
	6	0.650502760	00	0.648787640	00
	7	0.655712840	00	0.678880040	00
	8	0.647834320	00	0.672322400	00
	10	0.687775720	00	0.674709360	00
	14	0.650302200	00	0.650095920	00
	15	0.589341520	00	0.547410520	00
	16	0.551802920	00	0.571699320	00
	17	0.554749320	00	0.584801200	00
	18	0.717712560	00	0.657479240	00
	19	0.689236320	00	0.672440960	00
	20	0.672857920	00	0.662692080	00
	21	0.648993640	00	0.671484160	00
	22	0.616604720	00	0.691825000	00
	23	0.550596760	00	0.573003920	00
	24	0.453066240	00	0.445887600	00
	25	0.600204040	00	0.556824000	00
	26	0.558588080	00	0.580928800	00

TABLE 5.7. (Continued)

Figure (5.9) illustrates the relation between number of cameras and the reliability of the x-co-ordinate photogrammetric observations whereas Figure (5.10) shows that with respect to the y-co-ordinate. Illustrated in Figures (5.11) and (5.12) are the reliability of the x- and y- co-ordinate combined observations respectively.

The reliability of the x- and y- co-ordinates of cases 1,2 is sufficient (Case 1: $r(x) = 0.583$, $r(y) = 0.540$; Case 2: $r(x) = 0.586$, $r(y) = 0.610$). On the other end of the scale, cases 9,10 display the highest reliability (Case 9: $r(x) = 0.688$, $r(y) = 0.673$; Case 10: $r(x) = 0.718$, $r(y) = 0.692$). One plausible explanation for such a situation is due to the symmetric network arrangement. The latter cases are the optimum configuration in reliability sense.

Again in reliability analysis of the x- and y- co-ordinate, as in precision, there is slight improvement "within" the same number of camera cases.

With regard to the "between" percentage of improvement for purely photogrammetric cases, it can be noticed that there is some similarity in the trend of the rate of improvement in x- and y- co-ordinate observations.

For the combined different number of camera cases x- and y- co-ordinate reliability improvements, it seems that the improvement rate is insignificant especially after the 6 camera case.

The reliability of the slope distances are plotted against the number of cameras in Figure (5.13). As can be seen from this figure, with the number of slope distances kept constant, the more

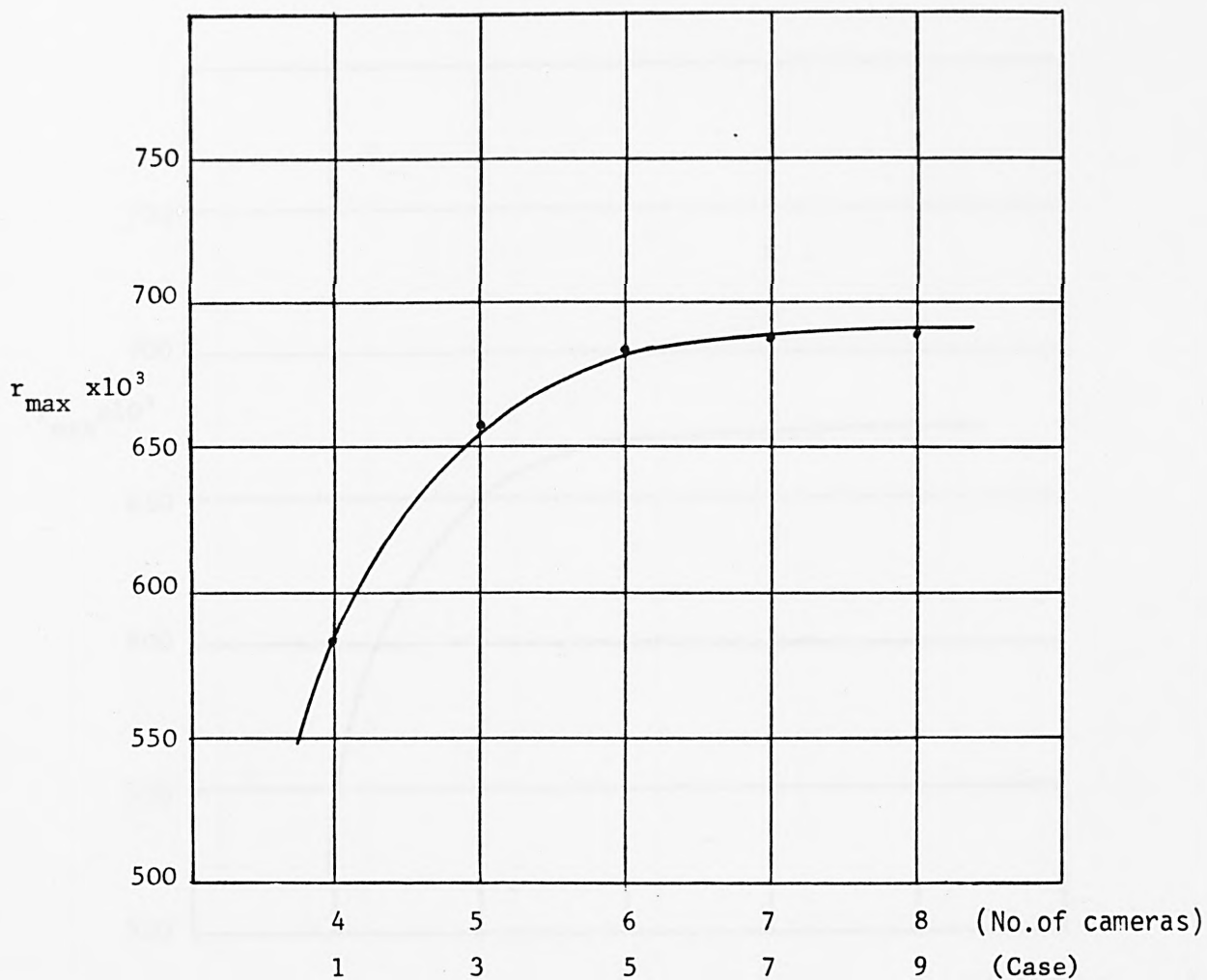


Figure 5.9. No. of cameras against Reliability (internal) of photo x-co-ordinates.
(Photogrammetry)

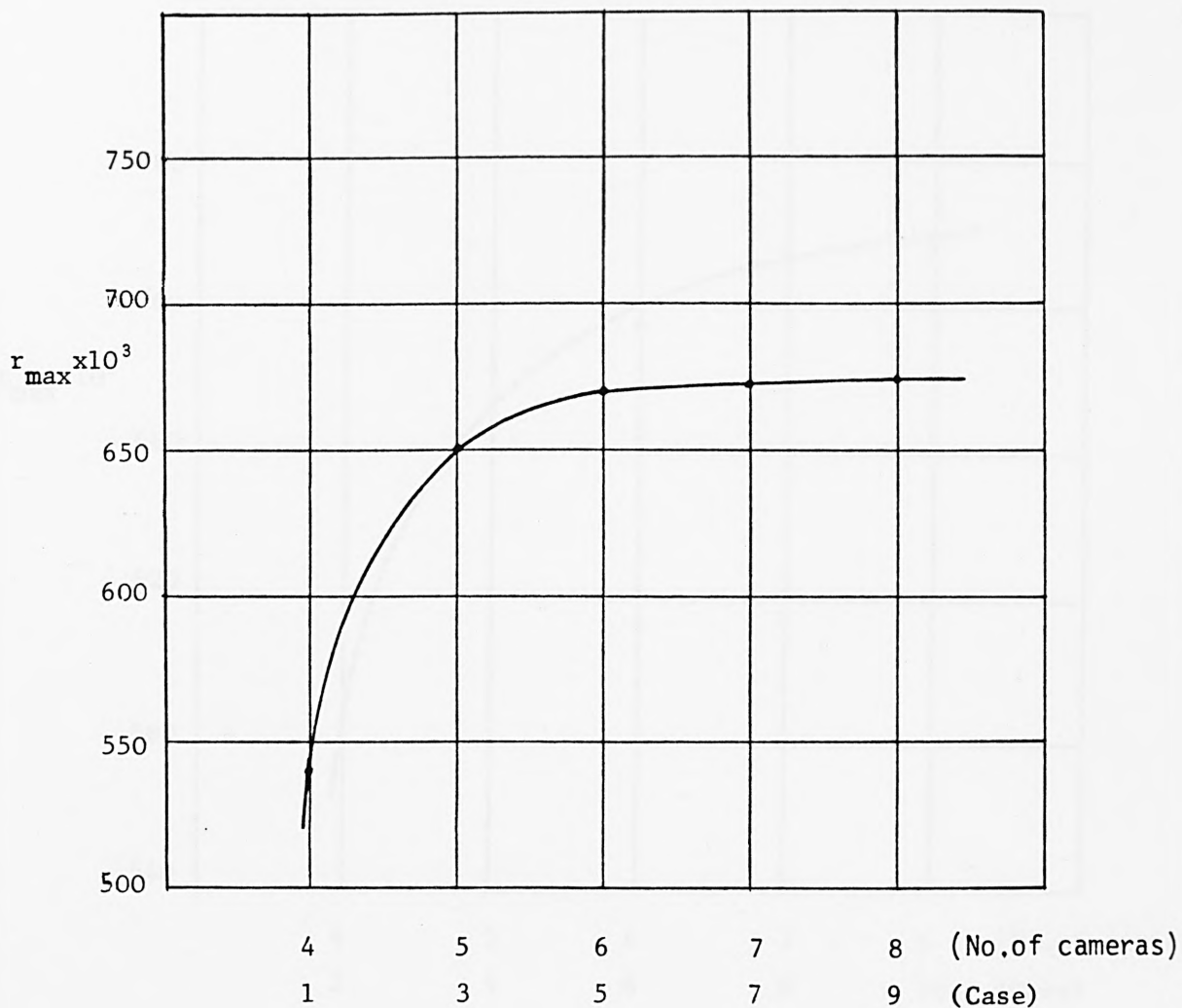


Figure 5.10. No.of cameras against Reliability (internal)
of photo y-co-ordinates.
(Photogrammetry)

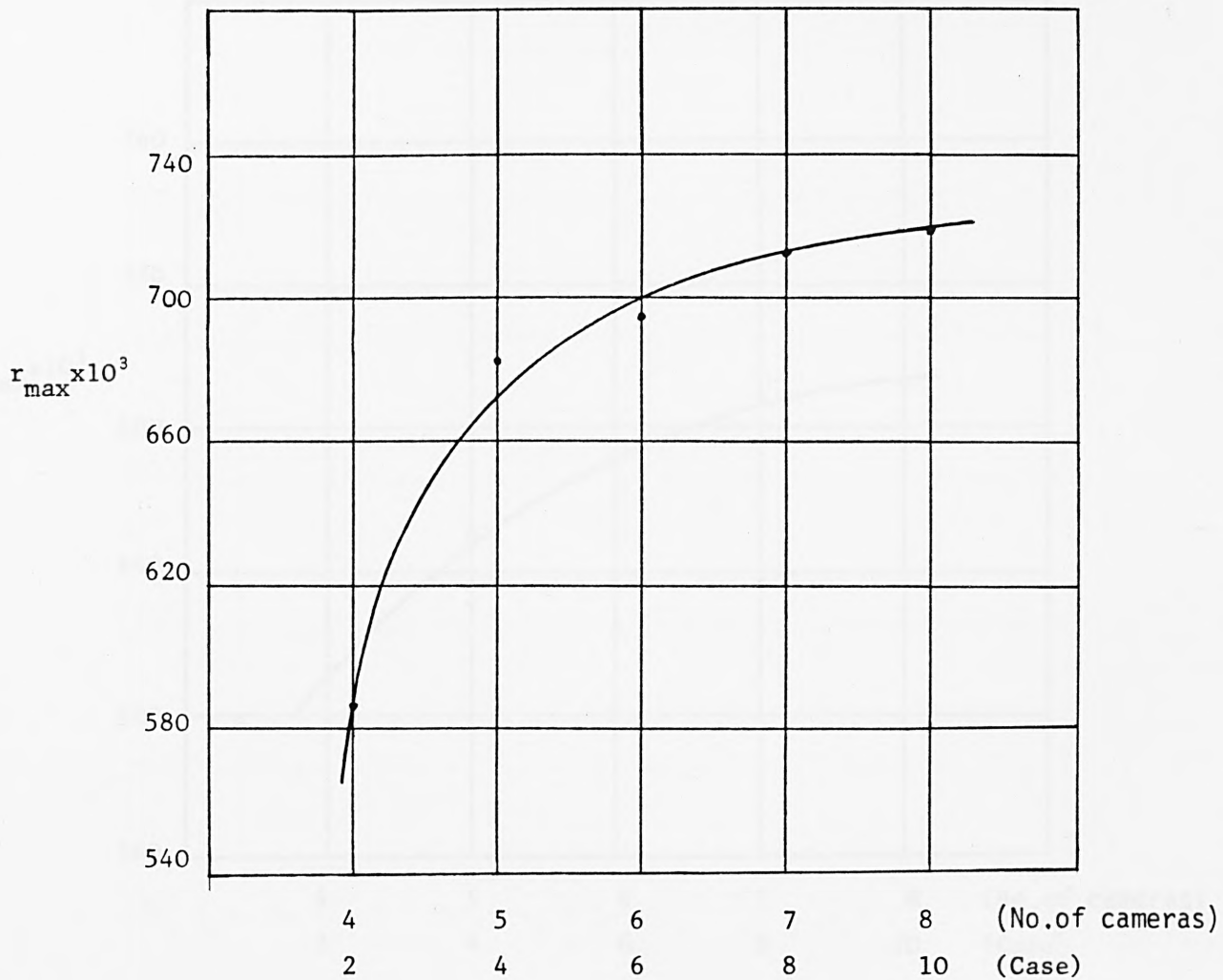


Figure 5.11. No.of cameras against Reliability (internal) of photo x-co-ordinates.
(Photogrammetry + 8 distances)

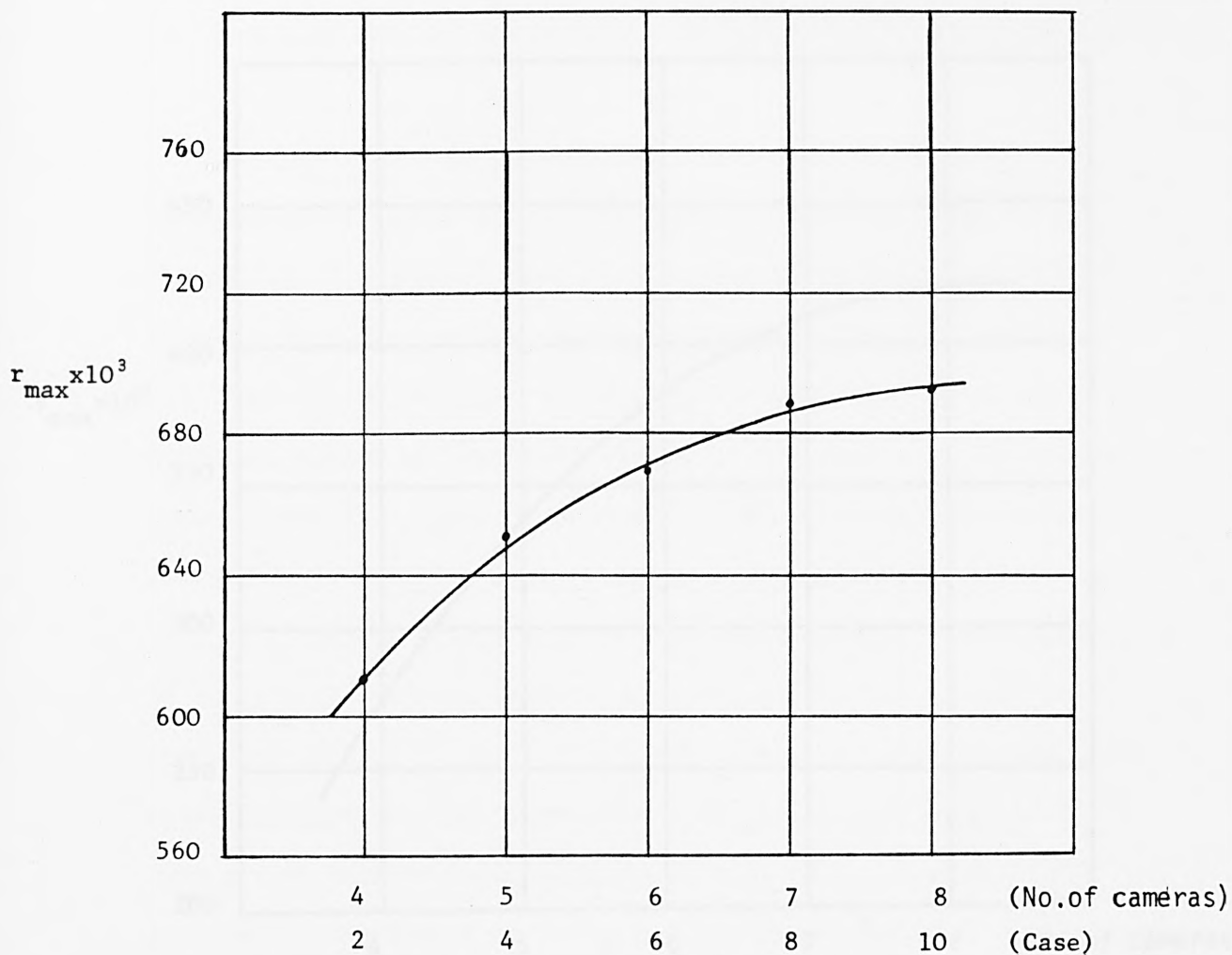


Figure 5.12. No. of cameras against Reliability (internal) of photo y-co-ordinates.
(Photogrammetry + 8 distances)

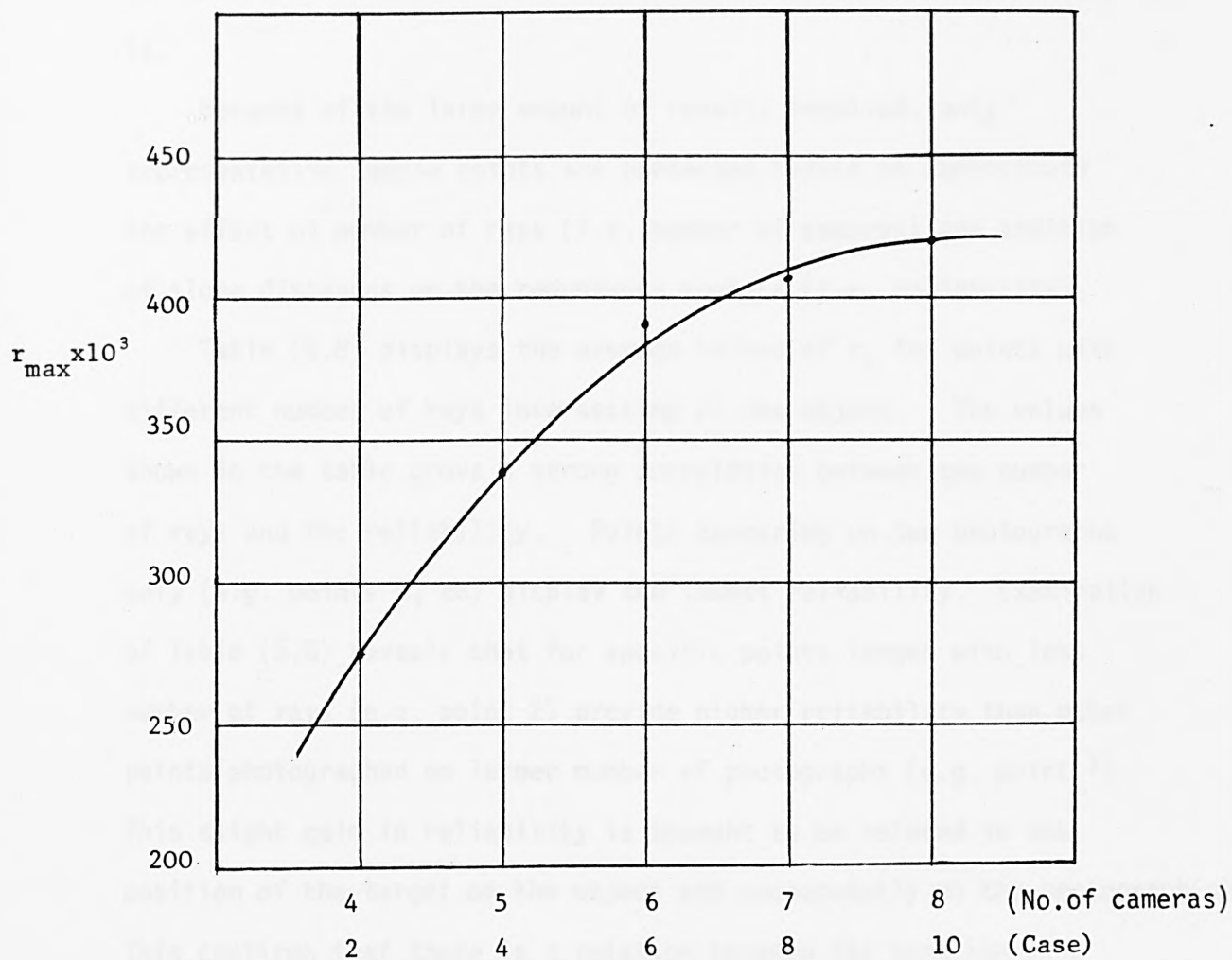


Figure 5.13. No. of cameras against Reliability (internal) of survey measurements (8 distances).

the number of cameras the more the reliability of slope distances is.

Because of the large amount of results involved, only representative sample points are presented herein to demonstrate the effect of number of rays (i.e. number of cameras) and addition of slope distances on the redundancy numbers (i.e. reliability).

Table (5.8) displays the average values of r_i for points with different number of rays intersecting at the object. The values shown in the table prove a strong correlation between the number of rays and the reliability. Points appearing on two photographs only (e.g. points 9, 26) display the lowest reliability. Examination of Table (5.8) reveals that for specific points imaged with less number of rays (e.g. point 2) provide higher reliability than other points photographed on larger number of photographs (e.g. point 1). This slight gain in reliability is thought to be related to the position of the target on the object and consequently on the photograph(s). This confirms that there is a relation between the location of a point on the photograph and the reliability.

When two measured distances originate from a point, the redundancy number increases substantially. Table (5.9) lists the values of redundancy numbers with and without adding distances and the percentage improvement. The latter varies between 14 to 4 for the x-co-ordinate whereas it ranges from 18 to 5 for the y-co-ordinate. It is also noted that the improvement in redundancy numbers slows down considerably after 6 rays. Therefore, it is not economically desirable to try to improve the redundancy if the object points

already have 6 rays and display redundancy numbers of the order of 0.6 or more.

Object Point Number	Number of Rays	Redundancy numbers	
		$r(x)$	$r(y)$
1	4	0.450	0.431
	5	0.522	0.525
	6	0.564	0.563
	7	0.597	0.599
2	4	0.516	0.488
	5	0.579	0.573
	6	0.623	0.626
9	2	0.314	0.139
	3	0.440	0.464
	4	0.552	0.581
26	2	0.314	0.139
	3	0.436	0.474
	4	0.557	0.581

TABLE (5.8) Relation between number of rays and redundancy numbers.

Object Point Number	Number of Rays	r (without distances)		r (with distances)		Improvement %	
		r(x)	r(y)	r(x)	r(y)	r(x)	r(y)
1	4	0.450	0.431	0.490	0.508	8.9	17.9
	5	0.522	0.525	0.561	0.576	7.5	9.7
	6	0.564	0.563	0.596	0.604	5.7	7.3
	7	0.597	0.599	0.624	0.634	4.5	5.8
3	4	0.451	0.430	0.493	0.506	9.3	17.7
	5	0.511	0.499	0.546	0.555	6.9	11.2
	6	0.564	0.563	0.597	0.603	5.9	7.1
	7	0.593	0.594	0.619	0.629	4.4	5.9
5	4	0.478	0.479	0.547	0.567	14.4	18.4
	5	0.516	0.518	0.563	0.574	9.1	10.8
	6	0.567	0.575	0.602	0.615	6.2	7.0
	7	0.597	0.599	0.624	0.633	4.5	5.7
7	4	0.421	0.453	0.502	0.515	19.2	13.7
	5	0.515	0.518	0.564	0.573	9.5	10.6
	6	0.556	0.559	0.593	0.604	6.7	8.1
	7	0.597	0.599	0.624	0.633	4.5	5.7

TABLE (5.9) Effect of distances between object points and their image co-ordinate reliability (Sample of points having 4, 5, 6, 7 rays).

On the other hand, for object points which have no distances, their image co-ordinates reliabilities are insignificantly improved. Table (5.10) represents the improvement gained by the inclusion of slope distances in the photogrammetric network as a whole. It should be noted that points with 2 or 3 rays can be seen to have had no significant improvement compared with that of "distance-connected" points.

Object Point Number	Number of Rays	r (without distances)		r (with distances)		Improvement %	
		$r(x)$	$r(y)$	$r(x)$	$r(y)$	$r(x)$	$r(y)$
2	4	0.516	0.488	0.523	0.491	1.4	0.6
	5	0.579	0.573	0.583	0.575	0.7	0.4
	6	0.623	0.626	0.626	0.628	0.5	0.3
4	4	0.524	0.518	0.526	0.520	0.4	0.4
	5	0.584	0.581	0.586	0.584	0.4	0.5
	6	0.629	0.630	0.631	0.631	0.4	0.2
6	4	0.502	0.527	0.506	0.529	0.8	0.4
	5	0.577	0.591	0.580	0.592	0.5	0.2
	6	0.629	0.630	0.631	0.631	0.4	0.2
8	4	0.500	0.520	0.509	0.522	1.8	0.4
	5	0.583	0.582	0.585	0.584	0.4	0.4
	6	0.631	0.633	0.632	0.634	0.2	0.2
9	2	0.314	0.139	0.314	0.139	0.0	0.0
	3	0.440	0.464	0.441	0.465	0.2	0.2
	4	0.552	0.581	0.554	0.581	0.4	0.0
26	2	0.314	0.139	0.314	0.139	0.0	0.0
	3	0.436	0.474	0.437	0.474	0.2	0.0
	4	0.557	0.581	0.558	0.582	0.2	0.2

TABLE (5.10) Effect of slope distances on redundancy numbers.

Table (5.11) summarises the precision and reliability values ($r_{i \max}$) of the photogrammetric measurements.

Case	Precision σ_m^2 (mm ²)	Reliability	
		max $r(x)$	max $r(y)$
1	0.40027546	0.583039	0.540271
2	0.38408995	0.585583	0.610033
3	0.30379145	0.657194	0.650357
4	0.29312554	0.681672	0.651317
5	0.25444611	0.682046	0.669123
6	0.24798812	0.694132	0.670347
7	0.20352679	0.685850	0.671341
8	0.19979033	0.712184	0.688901
9	0.17192943	0.687693	0.673053
10	0.16986719	0.717713	0.691825

TABLE (5.11) Summary of precision and reliability ($r_{\max}$) of photogrammetric measurements.

Naturally the global indicators don't provide in each case for sufficient precision/or reliability of all individual object points/and observations, e.g. if some observations have been cancelled or if points cannot be observed from certain camera stations. Such a situation is encountered in the computer programs, when dealing with object point (no.15) in cases 1-4, and observations from or to that point. So, in addition to the computation of the exact redundancy numbers other individual indicators, namely Tau (internal reliability), Gam (external reliability) and Delta (max. undetected gross error) were computed according to the equations presented in

PHO PT.		TAU				GAM			
NO NO		x		y		x		y	
1	1	0.150712570	01	0.151015610	01	0.112757620	01	0.113162330	01
	2	0.135097200	01	0.132287660	01	0.908364060	00	0.866026910	00
	3	0.123214610	01	0.125997090	01	0.802432900	00	0.766502940	00
	4	0.124197470	01	0.122138260	01	0.736546860	00	0.701267120	00
	5	0.125615280	01	0.126877030	01	0.760210450	00	0.780882950	00
	6	0.120587630	01	0.121994700	01	0.673897300	00	0.698763720	00
	7	0.122114120	01	0.125009600	01	0.700846560	00	0.750160030	00
	8	0.129394880	01	0.134168940	01	0.821159920	00	0.894500070	00
	9	0.133896920	01	0.131243390	01	0.890414800	00	0.849989880	00
	10	0.134909630	01	0.132464490	01	0.905572070	00	0.868725620	00
	11	0.130362090	01	0.135192190	01	0.836317780	00	0.909776200	00
	12	0.124174140	01	0.124103270	01	0.736153380	00	0.734957250	00
	16	0.120687020	01	0.121892050	01	0.675674170	00	0.596970000	00
	17	0.134209520	01	0.130858330	01	0.896607350	00	0.844032120	00
	18	0.128177330	01	0.126032500	01	0.801837070	00	0.767084820	00
	19	0.124108480	01	0.124168920	01	0.735045300	00	0.735065280	00
	20	0.131173040	01	0.127786390	01	0.848903180	00	0.795572800	00
	24	0.125555700	01	0.126938520	01	0.759225570	00	0.781881490	00
	25	0.124275260	01	0.122064420	01	0.737857700	00	0.699980180	00
2	1	0.128177330	01	0.126032500	01	0.801837070	00	0.767084820	00
	2	0.134909630	01	0.132464490	01	0.905572070	00	0.868725620	00
	3	0.150712570	01	0.151015610	01	0.112757620	01	0.113162330	01
	4	0.129394880	01	0.134168940	01	0.821159920	00	0.894500070	00
	5	0.122114120	01	0.125009600	01	0.700846560	00	0.750160030	00
	6	0.120687020	01	0.121892050	01	0.675674170	00	0.596970000	00
	7	0.125555700	01	0.126938520	01	0.759225570	00	0.781881490	00
	8	0.124275260	01	0.122064420	01	0.737857700	00	0.699980180	00
	9	0.134309520	01	0.130858330	01	0.896607350	00	0.844032120	00
	10	0.124108480	01	0.124168920	01	0.735045300	00	0.736065280	00
	11	0.130362090	01	0.135192190	01	0.836317780	00	0.909776200	00
	12	0.135097200	01	0.132287660	01	0.908364060	00	0.866026910	00
	13	0.133896920	01	0.131243390	01	0.890414800	00	0.849989880	00
	14	0.120587630	01	0.121994700	01	0.673897300	00	0.698763720	00
	18	0.131173040	01	0.127786390	01	0.848903180	00	0.795572800	00
	19	0.124174140	01	0.124103270	01	0.736153380	00	0.734957250	00
	20	0.123214610	01	0.125997090	01	0.802432900	00	0.766502940	00
	21	0.124197470	01	0.122138260	01	0.736546860	00	0.701267120	00
	22	0.125615280	01	0.126877030	01	0.760210450	00	0.780882950	00
3	1	0.131173040	01	0.127786390	01	0.848903180	00	0.795572800	00
	2	0.124174140	01	0.124103270	01	0.736153380	00	0.734957250	00
	3	0.123214610	01	0.125997090	01	0.802432900	00	0.766502940	00
	4	0.124197470	01	0.122138260	01	0.736546860	00	0.701267120	00
	5	0.125615280	01	0.126877030	01	0.760210450	00	0.780882950	00
	10	0.124108480	01	0.124168920	01	0.735045300	00	0.736065280	00
	11	0.130362090	01	0.135192190	01	0.836317780	00	0.909776200	00
	12	0.135097200	01	0.132287660	01	0.908364060	00	0.866026910	00
	13	0.133896920	01	0.131243390	01	0.890414800	00	0.849989880	00
	14	0.120587630	01	0.121994700	01	0.673897300	00	0.698763720	00
	18	0.128177330	01	0.126032500	01	0.801837070	00	0.767084820	00
	19	0.134909630	01	0.132464490	01	0.905572070	00	0.868725620	00
	20	0.150712570	01	0.151015610	01	0.112757620	01	0.113162330	01
	21	0.129394880	01	0.134168940	01	0.821159920	00	0.894500070	00
	22	0.122114120	01	0.125009600	01	0.700846560	00	0.750160030	00
	23	0.120687020	01	0.121892050	01	0.675674170	00	0.596970000	00
	24	0.125555700	01	0.126938520	01	0.759225570	00	0.781881490	00
	25	0.124275260	01	0.122064420	01	0.737857700	00	0.699980180	00
	26	0.134309520	01	0.130858330	01	0.896607350	00	0.844032120	00

TABLE 5.12. Values of Tau and Gam for Case 9
(at $\alpha = 0.05$ and $\gamma = 0.8$)

PHO PT.		TAU		GAM	
NO	NO	X	Y	X	Y
4	1	0.128177330 01	0.126032500 01	0.801837070 00	0.767084820 00
	2	0.124108480 01	0.124166920 01	0.735045300 00	0.736005280 00
	3	0.131173040 01	0.127786390 01	0.848903180 00	0.795572800 00
	7	0.125555700 01	0.126938520 01	0.759225570 00	0.781881490 00
	8	0.124275260 01	0.122006420 01	0.737857700 00	0.599980180 00
	10	0.1174909630 01	0.132464490 01	0.705572070 00	0.868725620 00
	11	0.130362090 01	0.135192190 01	0.836317780 00	0.909776200 00
	12	0.124174140 01	0.124103270 01	0.736153380 00	0.734957250 00
	16	0.120697070 01	0.121892050 01	0.675674170 00	0.696970000 00
	17	0.134309520 01	0.130358330 01	0.896607350 00	0.844032120 00
	18	0.150712570 01	0.151015610 01	0.112757620 01	0.113162330 01
	19	0.135097200 01	0.132287660 01	0.908364060 00	0.866026910 00
	20	0.128214610 01	0.125997090 01	0.802432900 00	0.766502940 00
	21	0.124197470 01	0.122138260 01	0.736546860 00	0.701267120 00
	22	0.125615280 01	0.126377030 01	0.760210450 00	0.780882950 00
	23	0.120587630 01	0.121994700 01	0.673897300 00	0.698763720 00
	24	0.122114120 01	0.125009600 01	0.700846560 00	0.750160030 00
	25	0.129394880 01	0.134168940 01	0.821159920 00	0.894500070 00
	26	0.133896920 01	0.131243390 01	0.890414800 00	0.849989880 00
5	1	0.122114120 01	0.125009600 01	0.700846560 00	0.750160030 00
	2	0.120637020 01	0.121892050 01	0.675674170 00	0.696970000 00
	3	0.125555700 01	0.126938520 01	0.737857700 00	0.781881490 00
	4	0.124275260 01	0.122006420 01	0.759225570 00	0.599980180 00
	5	0.123177330 01	0.126032500 01	0.801837070 00	0.767084820 00
	6	0.134909630 01	0.132464490 01	0.705572070 00	0.868725620 00
	7	0.150712570 01	0.151015610 01	0.112757620 01	0.113162330 01
	8	0.129394880 01	0.134168940 01	0.821159920 00	0.894500070 00
	9	0.134309520 01	0.130858330 01	0.896607350 00	0.844032120 00
	10	0.120587630 01	0.121994700 01	0.673897300 00	0.698763720 00
	14	0.124103480 01	0.124168920 01	0.735045300 00	0.736005280 00
	15	0.130362090 01	0.135192190 01	0.836317780 00	0.909776200 00
	16	0.135097200 01	0.132287660 01	0.908364060 00	0.866026910 00
	17	0.133896920 01	0.131243390 01	0.890414800 00	0.849989880 00
	18	0.125615280 01	0.126377030 01	0.760210450 00	0.780882950 00
	22	0.131173040 01	0.127786390 01	0.848903180 00	0.795572800 00
	23	0.124174140 01	0.124103270 01	0.736153380 00	0.734957250 00
	24	0.129214610 01	0.125997090 01	0.802432900 00	0.766502940 00
	25	0.124197470 01	0.122138260 01	0.736546860 00	0.701267120 00
6	1	0.125555700 01	0.126938520 01	0.759225570 00	0.781881490 00
	2	0.120687020 01	0.121892050 01	0.675674170 00	0.696970000 00
	3	0.122114120 01	0.125009600 01	0.700846560 00	0.750160030 00
	4	0.129394880 01	0.134168940 01	0.821159920 00	0.894500070 00
	5	0.150712570 01	0.151015610 01	0.112757620 01	0.113162330 01
	6	0.134909630 01	0.132464490 01	0.705572070 00	0.868725620 00
	7	0.123177330 01	0.126032500 01	0.801837070 00	0.767084820 00
	8	0.124275260 01	0.122006420 01	0.737857700 00	0.599980180 00
	9	0.134309520 01	0.130858330 01	0.896607350 00	0.844032120 00
	12	0.120587630 01	0.121994700 01	0.673897300 00	0.698763720 00
	13	0.133896920 01	0.131243390 01	0.890414800 00	0.849989880 00
	14	0.135097200 01	0.132287660 01	0.908364060 00	0.866026910 00
	15	0.130362090 01	0.135192190 01	0.836317780 00	0.909776200 00
	16	0.124103480 01	0.124168920 01	0.735045300 00	0.736005280 00
	20	0.125615280 01	0.126377030 01	0.760210450 00	0.780882950 00
	21	0.124197470 01	0.122138260 01	0.736546860 00	0.701267120 00
	22	0.129214610 01	0.125997090 01	0.802432900 00	0.766502940 00
	23	0.124174140 01	0.124103270 01	0.736153380 00	0.734957250 00
	24	0.131173040 01	0.127786390 01	0.848903180 00	0.795572800 00

TABLE 5.12 (Continued)

PHO PT.	TAU		GAM	
NO NO	x	y	x	y
7	0.125555700 01	0.126938520 01	0.759225570 00	0.781881490 00
3	0.124272260 01	0.122064420 01	0.737857700 00	0.699980180 00
4	0.124272260 01	0.122064420 01	0.801837070 00	0.767084820 00
5	0.120172330 01	0.126032500 01	0.735045300 00	0.736065280 00
6	0.124101480 01	0.124160920 01	0.948903180 00	0.795572800 00
7	0.131173040 01	0.127786390 01	0.675674170 00	0.696970000 00
12	0.120637020 01	0.121892050 01	0.896607350 00	0.844032120 00
13	0.134309520 01	0.133858330 01	0.905572070 00	0.868725620 00
14	0.134909630 01	0.134646490 01	0.836317780 00	0.909776200 00
15	0.130362090 01	0.135192190 01	0.736153380 00	0.734957250 00
16	0.124174140 01	0.124103270 01	0.760210450 00	0.780882950 00
18	0.125615280 01	0.126877030 01	0.673897300 00	0.698763720 00
19	0.129587630 01	0.121994700 01	0.700846360 00	0.750160030 00
20	0.122104120 01	0.124009600 01	0.521159920 00	0.894500070 00
21	0.129392480 01	0.134168940 01	0.708465600 00	0.894500070 00
22	0.150712370 01	0.151015610 01	0.112757620 01	0.113162330 01
23	0.135092200 01	0.133287660 01	0.708364060 00	0.866025910 00
24	0.129214610 01	0.129977090 01	0.802432900 00	0.766502940 00
25	0.124197470 01	0.122158260 01	0.736546860 00	0.701767120 00
26	0.133896020 01	0.131243390 01	0.890414800 00	0.849989880 00
8	0.125555700 01	0.126938520 01	0.759225570 00	0.781881490 00
1	0.131173040 01	0.127786390 01	0.848903180 00	0.795572800 00
5	0.124109480 01	0.124160920 01	0.735045300 00	0.736065280 00
6	0.120172330 01	0.126032500 01	0.801837070 00	0.767084820 00
7	0.124272260 01	0.122064420 01	0.737857700 00	0.699980180 00
8	0.120587630 01	0.121892050 01	0.675674170 00	0.696970000 00
10	0.120587630 01	0.121892050 01	0.675674170 00	0.696970000 00
14	0.124174140 01	0.124103270 01	0.736153380 00	0.734957250 00
15	0.130362090 01	0.135192190 01	0.836317780 00	0.909776200 00
16	0.134909630 01	0.132464490 01	0.905572070 00	0.868725620 00
17	0.134309520 01	0.130858330 01	0.896607350 00	0.844032120 00
18	0.122114120 01	0.125009600 01	0.700846360 00	0.750160030 00
19	0.120587630 01	0.121994700 01	0.673897300 00	0.698763720 00
20	0.125615280 01	0.126877030 01	0.760210450 00	0.780882950 00
21	0.124197470 01	0.122158260 01	0.736546860 00	0.701767120 00
22	0.123214610 01	0.125997090 01	0.802432900 00	0.766502940 00
23	0.135092200 01	0.133287660 01	0.908364060 00	0.866025910 00
24	0.150712570 01	0.151015610 01	0.112757620 01	0.113162330 01
25	0.129394880 01	0.134168940 01	0.821159920 00	0.894500070 00
26	0.133896020 01	0.131243390 01	0.890414800 00	0.849989880 00

TABLE 5.12 (Continued)

PHO PT.		TAU				GAM			
NO	NO	x		y		x		y	
1	1	0.148565900	01	0.149757060	01	0.109871870	01	0.111477250	01
	2	0.134766380	01	0.132105590	01	0.903444070	00	0.863243160	00
	3	0.127349300	01	0.120226960	01	0.788533150	00	0.667422090	00
	4	0.124130360	01	0.122034360	01	0.735423110	00	0.699455790	00
	5	0.121909710	01	0.122841220	01	0.697278900	00	0.713439880	00
	6	0.120452530	01	0.121947510	01	0.671476860	00	0.697939400	00
	7	0.118038780	01	0.123327230	01	0.627148670	00	0.721775980	00
	8	0.129077500	01	0.134011180	01	0.816149550	00	0.892132050	00
	9	0.133799400	01	0.131201420	01	0.888947670	00	0.849341710	00
	10	0.134619510	01	0.132256240	01	0.901244270	00	0.865546780	00
	11	0.130261620	01	0.135158520	01	0.834750870	00	0.909275850	00
	12	0.124005910	01	0.124025580	01	0.733312080	00	0.733644700	00
	16	0.120570360	01	0.121742340	01	0.673767280	00	0.694348360	00
	17	0.134761540	01	0.130766310	01	0.895888390	00	0.842604780	00
	18	0.123493230	01	0.121367800	01	0.724608740	00	0.687760340	00
	19	0.123986790	01	0.124150570	01	0.732988740	00	0.735755660	00
	20	0.127397710	01	0.126276870	01	0.789314680	00	0.771093280	00
	24	0.125289000	01	0.122421670	01	0.754806880	00	0.706191560	00
	25	0.124741880	01	0.121958260	01	0.737295430	00	0.698127240	00
2	1	0.127264760	01	0.120298240	01	0.787166950	00	0.668705170	00
	2	0.134555460	01	0.132305660	01	0.900287290	00	0.866301710	00
	3	0.148437310	01	0.149889110	01	0.109697920	01	0.111654580	01
	4	0.129058610	01	0.134032320	01	0.815850820	00	0.892449590	00
	5	0.118257740	01	0.123078960	01	0.631260080	00	0.717525570	00
	6	0.120552590	01	0.121843930	01	0.673270210	00	0.696128040	00
	7	0.121890980	01	0.122870640	01	0.696776300	00	0.713946390	00
	8	0.124203340	01	0.121965620	01	0.736645710	00	0.698255780	00
	9	0.134203440	01	0.130823820	01	0.895017500	00	0.843497010	00
	10	0.123937960	01	0.124093780	01	0.732160790	00	0.734795910	00
	11	0.130261680	01	0.135158450	01	0.834751830	00	0.909274830	00
	12	0.134793160	01	0.132087490	01	0.903910570	00	0.862966160	00
	13	0.133845690	01	0.131154030	01	0.889644280	00	0.848609360	00
	14	0.120476270	01	0.121849750	01	0.671902680	00	0.696229930	00
	18	0.127695290	01	0.125989070	01	0.794108810	00	0.766371100	00
	19	0.124051780	01	0.124085430	01	0.734087440	00	0.734655950	00
	20	0.123561620	01	0.121302990	01	0.725773650	00	0.686615980	00
	21	0.124159640	01	0.122036190	01	0.735908780	00	0.699487820	00
	22	0.125384560	01	0.122332720	01	0.756391960	00	0.704648510	00
3	1	0.127695290	01	0.125989070	01	0.794108810	00	0.766371100	00
	2	0.124051780	01	0.124085430	01	0.734087440	00	0.734655950	00
	3	0.123561620	01	0.121302990	01	0.725773650	00	0.686615980	00
	4	0.124159640	01	0.122036190	01	0.735908780	00	0.699487820	00
	5	0.125384560	01	0.122332720	01	0.756391960	00	0.704648510	00
	10	0.123937960	01	0.124093780	01	0.732160790	00	0.734795910	00
	11	0.130261680	01	0.135158450	01	0.834751830	00	0.909274830	00
	12	0.134793160	01	0.132087490	01	0.903910570	00	0.862966160	00
	13	0.133845690	01	0.131154030	01	0.889644280	00	0.848609360	00
	14	0.120476270	01	0.121849750	01	0.671902680	00	0.696229930	00
	18	0.127695290	01	0.125989070	01	0.794108810	00	0.766371100	00
	19	0.124051780	01	0.124085430	01	0.734087440	00	0.734655950	00
	20	0.123561620	01	0.121302990	01	0.725773650	00	0.686615980	00
	21	0.124159640	01	0.122036190	01	0.735908780	00	0.699487820	00
	22	0.125384560	01	0.122332720	01	0.756391960	00	0.704648510	00
	23	0.120552590	01	0.121843930	01	0.673270210	00	0.696128040	00
	24	0.121890980	01	0.122870640	01	0.696776300	00	0.713946390	00
	25	0.124203340	01	0.121965620	01	0.736645710	00	0.698255780	00
	26	0.134203440	01	0.130823820	01	0.895017500	00	0.843497010	00

TABLE 5.13. Values of Tau and Gam for Case 10
(at $\alpha = 0.05$ and $\gamma = 0.8$)

PHO P1.		TAU				GAM			
		x		y		x		y	
NO	NO								
4	1	0.123493230	01	0.121367800	01	0.724608740	00	0.687760340	00
2	2	0.1239986790	01	0.124150570	01	0.732988740	00	0.735755560	00
3	3	0.127397710	01	0.126276870	01	0.769314680	00	0.771090380	00
7	7	0.125289000	01	0.124242167	01	0.754806880	00	0.706191560	00
8	8	0.124241880	01	0.121958260	01	0.737295430	00	0.698127240	00
10	10	0.134619510	01	0.132256240	01	0.901244270	00	0.865545780	00
11	11	0.130261670	01	0.135158520	01	0.834750870	00	0.909255850	00
12	12	0.124005970	01	0.124025580	01	0.733312080	00	0.733644700	00
16	16	0.120589160	01	0.121742340	01	0.673767280	00	0.694358360	00
17	17	0.134261540	01	0.130266310	01	0.895888390	00	0.842604780	00
18	18	0.149565900	01	0.149757060	01	0.109871870	01	0.111477250	01
19	19	0.134766580	01	0.132105590	01	0.903444070	00	0.863243160	00
20	20	0.127349300	01	0.120226960	01	0.788533150	00	0.667422090	00
21	21	0.124130860	01	0.122034360	01	0.735423110	00	0.699455790	00
22	22	0.121909710	01	0.122841220	01	0.697278900	00	0.713439880	00
23	23	0.120452530	01	0.1219447510	01	0.671476860	00	0.697939400	00
24	24	0.118038730	01	0.123327230	01	0.627148670	00	0.721775980	00
25	25	0.129077500	01	0.134011180	01	0.816149550	00	0.892132050	00
26	26	0.133799400	01	0.131201420	01	0.888947670	00	0.849341710	00
5	1	0.118257740	01	0.123078960	01	0.631260080	00	0.717535570	00
2	2	0.120552590	01	0.121843930	01	0.673270210	00	0.696128040	00
3	3	0.121860080	01	0.122870640	01	0.696677630	00	0.713944390	00
4	4	0.124297340	01	0.121946560	01	0.736645710	00	0.698235780	00
5	5	0.127264760	01	0.120296240	01	0.787166950	00	0.568205170	00
6	6	0.134555460	01	0.132315660	01	0.900287290	00	0.866301710	00
7	7	0.148437310	01	0.149829710	01	0.109697920	01	0.111654580	01
8	8	0.129053610	01	0.134032320	01	0.815850820	00	0.892467950	00
9	9	0.134203440	01	0.130823820	01	0.895017500	00	0.843497010	00
10	10	0.120476270	01	0.121849750	01	0.671902680	00	0.696229930	00
14	14	0.123937860	01	0.124093780	01	0.732160790	00	0.734796910	00
15	15	0.130261680	01	0.135158450	01	0.834751830	00	0.909214830	00
16	16	0.134798160	01	0.132087490	01	0.903310570	00	0.862965160	00
17	17	0.133845690	01	0.131154030	01	0.889644280	00	0.848609360	00
18	18	0.125334560	01	0.122332720	01	0.756391960	00	0.704688510	00
22	22	0.127695290	01	0.125989070	01	0.794108810	00	0.766311100	00
23	23	0.124051780	01	0.124085430	01	0.734087440	00	0.734653950	00
24	24	0.123561620	01	0.121310290	01	0.725773650	00	0.686615980	00
25	25	0.124159640	01	0.122036190	01	0.735908780	00	0.699467820	00
6	1	0.121880980	01	0.122870640	01	0.696776300	00	0.713924630	00
2	2	0.120552590	01	0.121843930	01	0.673270210	00	0.696128040	00
3	3	0.118257740	01	0.123078960	01	0.631260080	00	0.717535570	00
4	4	0.129058610	01	0.134032320	01	0.815850820	00	0.892467950	00
5	5	0.143437310	01	0.149829710	01	0.109697920	01	0.111654580	01
6	6	0.134555460	01	0.132305660	01	0.900287290	00	0.866301710	00
7	7	0.127264760	01	0.120226240	01	0.787166950	00	0.668705170	00
8	8	0.124203340	01	0.121905620	01	0.736645710	00	0.698235780	00
9	9	0.134203440	01	0.130823820	01	0.895017500	00	0.843497010	00
12	12	0.120476270	01	0.121849750	01	0.671902680	00	0.696229930	00
13	13	0.133845690	01	0.131154030	01	0.889644280	00	0.848609360	00
14	14	0.134798160	01	0.132087490	01	0.903310570	00	0.862965160	00
15	15	0.130261680	01	0.135158450	01	0.834751830	00	0.909214830	00
16	16	0.125334560	01	0.122332720	01	0.756391960	00	0.704688510	00
20	20	0.125384560	01	0.122332720	01	0.756391960	00	0.704688510	00
21	21	0.124159640	01	0.122036190	01	0.735908780	00	0.699467820	00
22	22	0.123561620	01	0.121310290	01	0.725773650	00	0.686615980	00
23	23	0.124051780	01	0.124085430	01	0.734087440	00	0.734653950	00
24	24	0.127695290	01	0.125989070	01	0.794108810	00	0.766311100	00

TABLE 5.13 (Continued)

PHO PT.		TAU		GAM	
NO	NO	x	y	x	y
7	3	0.123299000 01	0.122421670 01	0.754806880 00	0.706191500 00
	4	0.124241300 01	0.121958260 01	0.737295430 00	0.598127240 00
	5	0.123403230 01	0.121367800 01	0.724608740 00	0.587760340 00
	6	0.123986790 01	0.124150570 01	0.732988740 00	0.735755600 00
	7	0.123797710 01	0.126276870 01	0.789314680 00	0.771093280 00
	12	0.120580360 01	0.121742340 01	0.673767280 00	0.694348360 00
	13	0.134261540 01	0.130766310 01	0.895888390 00	0.842604780 00
	14	0.134619510 01	0.132256240 01	0.901244270 00	0.865546780 00
	15	0.130261620 01	0.135158520 01	0.834750870 00	0.909275850 00
	16	0.12005910 01	0.124025580 01	0.733312080 00	0.733644700 00
	18	0.121009710 01	0.122841220 01	0.697278900 00	0.713439280 00
	19	0.120452530 01	0.121947510 01	0.671747680 00	0.697939400 00
	20	0.118038780 01	0.123327230 01	0.627148670 00	0.721775980 00
	21	0.120077500 01	0.134011180 01	0.816149550 00	0.892132050 00
	22	0.148565900 01	0.149757060 01	0.109871870 01	0.111477250 01
	23	0.134766980 01	0.132105590 01	0.90344070 00	0.863243160 00
	24	0.127349300 01	0.120226960 01	0.788533150 00	0.667422090 00
	25	0.124130860 01	0.122034360 01	0.735423110 00	0.599455790 00
	26	0.133799400 01	0.131201420 01	0.888947670 00	0.849341710 00
8	1	0.123289000 01	0.122421670 01	0.754806880 00	0.706191500 00
	5	0.127297710 01	0.126276870 01	0.789314680 00	0.771093280 00
	6	0.123066790 01	0.124150570 01	0.732988740 00	0.735755600 00
	7	0.123403230 01	0.121367800 01	0.724608740 00	0.587760340 00
	8	0.124241880 01	0.121958260 01	0.737295430 00	0.598127240 00
	10	0.120580360 01	0.121742340 01	0.673767280 00	0.694348360 00
	14	0.124005910 01	0.124025580 01	0.733312080 00	0.733644700 00
	15	0.130261620 01	0.135158520 01	0.834750870 00	0.909275850 00
	16	0.134619510 01	0.132256240 01	0.901244270 00	0.865546780 00
	17	0.134261540 01	0.130766310 01	0.895888390 00	0.842604780 00
	18	0.118038780 01	0.123327230 01	0.627148670 00	0.721775980 00
	19	0.120452530 01	0.121947510 01	0.671747680 00	0.697939400 00
	20	0.121909710 01	0.122841220 01	0.697278900 00	0.713439280 00
	21	0.124130860 01	0.122034360 01	0.735423110 00	0.599455790 00
	22	0.127349300 01	0.120226960 01	0.788533150 00	0.667422090 00
	23	0.134766980 01	0.132105590 01	0.90344070 00	0.863243160 00
	24	0.148565900 01	0.149757060 01	0.109871870 01	0.111477250 01
	25	0.120077500 01	0.134011180 01	0.816149550 00	0.892132050 00
	26	0.133799400 01	0.131201420 01	0.888947670 00	0.849341710 00

TABLE 5.13 (Continued)

PHOTO POINT		TAU		DELTA(um)	
NO	HO	x	y	x	y
1					
1	1	0.150712570 01	0.151015610 01	0.21100 02	0.21140 02
2	2	0.135097200 01	0.132287600 01	0.18910 02	0.18520 02
3	3	0.128214610 01	0.125997090 01	0.17950 02	0.17640 02
4	4	0.124197470 01	0.122138260 01	0.17390 02	0.17100 02
5	5	0.125615280 01	0.126877030 01	0.17590 02	0.17760 02
6	6	0.120587630 01	0.121994700 01	0.16880 02	0.17080 02
7	7	0.122114120 01	0.125009600 01	0.17100 02	0.17500 02
8	8	0.129394880 01	0.134168940 01	0.18120 02	0.18780 02
9	9	0.133896920 01	0.131243390 01	0.18750 02	0.18370 02
10	10	0.134909630 01	0.132464490 01	0.18890 02	0.18550 02
11	11	0.130362090 01	0.135192190 01	0.18250 02	0.18930 02
12	12	0.124174140 01	0.124103270 01	0.17380 02	0.17370 02
16	16	0.120687020 01	0.121892050 01	0.16900 02	0.17060 02
17	17	0.134309520 01	0.130858330 01	0.18800 02	0.18320 02
18	18	0.128177330 01	0.126032500 01	0.17940 02	0.17640 02
19	19	0.124108480 01	0.124168920 01	0.17380 02	0.17380 02
20	20	0.131173040 01	0.127786390 01	0.18360 02	0.17890 02
24	24	0.125555700 01	0.126938520 01	0.17580 02	0.17770 02
25	25	0.124275260 01	0.122064420 01	0.17400 02	0.17090 02
2					
1	1	0.128172330 01	0.126032500 01	0.17940 02	0.17640 02
2	2	0.134909630 01	0.132464490 01	0.18890 02	0.18550 02
3	3	0.150712570 01	0.151015610 01	0.21100 02	0.21140 02
4	4	0.129394880 01	0.134168940 01	0.18120 02	0.18780 02
5	5	0.122114120 01	0.125009600 01	0.17100 02	0.17500 02
6	6	0.120687020 01	0.121892050 01	0.16900 02	0.17060 02
7	7	0.125555700 01	0.126938520 01	0.17580 02	0.17770 02
8	8	0.124275260 01	0.122064420 01	0.17400 02	0.17090 02
9	9	0.134309520 01	0.130858330 01	0.18800 02	0.18320 02
10	10	0.124108480 01	0.124168920 01	0.17380 02	0.17380 02
11	11	0.130362090 01	0.135192190 01	0.18250 02	0.18930 02
12	12	0.135097200 01	0.132287660 01	0.18910 02	0.18520 02
13	13	0.133896920 01	0.131243390 01	0.18750 02	0.18370 02
14	14	0.120587630 01	0.121994700 01	0.16880 02	0.17080 02
18	18	0.131173040 01	0.127786390 01	0.18360 02	0.17890 02
19	19	0.124174140 01	0.124103270 01	0.17380 02	0.17370 02
20	20	0.128214610 01	0.125997090 01	0.17950 02	0.17640 02
21	21	0.124197470 01	0.122138260 01	0.17390 02	0.17100 02
22	22	0.125615280 01	0.126877030 01	0.17590 02	0.17760 02
3					
1	1	0.131173040 01	0.127786390 01	0.18360 02	0.17890 02
2	2	0.124174140 01	0.124103270 01	0.17380 02	0.17370 02
3	3	0.128214610 01	0.125997090 01	0.17950 02	0.17640 02
4	4	0.124197470 01	0.122138260 01	0.17390 02	0.17100 02
5	5	0.125615280 01	0.126877030 01	0.17590 02	0.17760 02
10	10	0.124108480 01	0.124168920 01	0.17380 02	0.17380 02
11	11	0.130362090 01	0.135192190 01	0.18250 02	0.18930 02
12	12	0.135097200 01	0.132287660 01	0.18910 02	0.18520 02
13	13	0.133896920 01	0.131243390 01	0.18750 02	0.18370 02
14	14	0.120587630 01	0.121994700 01	0.16880 02	0.17080 02
18	18	0.128177330 01	0.126032500 01	0.17940 02	0.17640 02
19	19	0.134909630 01	0.132464490 01	0.18890 02	0.18550 02
20	20	0.150712570 01	0.151015610 01	0.21100 02	0.21140 02
21	21	0.129394880 01	0.134168940 01	0.18120 02	0.18780 02
22	22	0.122114120 01	0.125009600 01	0.17100 02	0.17500 02
23	23	0.120687020 01	0.121892050 01	0.16900 02	0.17060 02
24	24	0.125555700 01	0.126938520 01	0.17580 02	0.17770 02
25	25	0.124275260 01	0.122064420 01	0.17400 02	0.17090 02
26	26	0.134309520 01	0.130858330 01	0.18800 02	0.18320 02

TABLE 5.14. Undetected Gross Errors for case 9.
(at $\alpha = 0.05$ and $\gamma = 0.8$)

PHOTO POINT		TAU		DELTA(um)	
NO	NO	X	Y	X	Y
4	1	0.128177330 01	0.126032500 01	0.17940 02	0.17640 02
	2	0.124108480 01	0.124168920 01	0.17380 02	0.17380 02
	3	0.131173040 01	0.127786390 01	0.18360 02	0.17690 02
	7	0.125555700 01	0.126938520 01	0.17530 02	0.17770 02
	8	0.124275260 01	0.122064420 01	0.17400 02	0.17090 02
	10	0.134909630 01	0.132464490 01	0.18890 02	0.18550 02
	11	0.130362090 01	0.135192190 01	0.18250 02	0.18930 02
	12	0.124174140 01	0.124103270 01	0.17380 02	0.17370 02
	16	0.120687020 01	0.121892050 01	0.16900 02	0.17060 02
	17	0.134300520 01	0.130858330 01	0.18800 02	0.18320 02
	18	0.150712570 01	0.151015610 01	0.21100 02	0.21140 02
	19	0.135097200 01	0.132287660 01	0.18910 02	0.18520 02
	20	0.128214610 01	0.125997090 01	0.17950 02	0.17640 02
	21	0.124197470 01	0.122138260 01	0.17390 02	0.17100 02
	22	0.125615280 01	0.126877030 01	0.17590 02	0.17760 02
	23	0.120587630 01	0.121994700 01	0.16880 02	0.17080 02
	24	0.122114120 01	0.125009600 01	0.17100 02	0.17500 02
	25	0.129394880 01	0.134168940 01	0.18120 02	0.18780 02
	26	0.133896920 01	0.131243390 01	0.18750 02	0.18370 02
5	1	0.122114120 01	0.125009600 01	0.17100 02	0.17500 02
	2	0.120687020 01	0.121892050 01	0.16900 02	0.17060 02
	3	0.125555700 01	0.126938520 01	0.17580 02	0.17770 02
	4	0.124275260 01	0.122064420 01	0.17400 02	0.17090 02
	5	0.128177330 01	0.126032500 01	0.17940 02	0.17640 02
	6	0.134909630 01	0.132464490 01	0.18890 02	0.18550 02
	7	0.150712570 01	0.151015610 01	0.21100 02	0.21140 02
	8	0.129394880 01	0.134168940 01	0.18120 02	0.18780 02
	9	0.134300520 01	0.130858330 01	0.18800 02	0.18320 02
	10	0.120587630 01	0.121994700 01	0.16880 02	0.17080 02
	14	0.124108480 01	0.124168920 01	0.17380 02	0.17380 02
	15	0.130362090 01	0.135192190 01	0.16250 02	0.18930 02
	16	0.135097200 01	0.132287660 01	0.18910 02	0.18520 02
	17	0.133896920 01	0.131243390 01	0.18750 02	0.18370 02
	18	0.125615280 01	0.126877030 01	0.17590 02	0.17760 02
	22	0.131173040 01	0.127786390 01	0.18360 02	0.17890 02
	23	0.124174140 01	0.124103270 01	0.17380 02	0.17370 02
	24	0.128214610 01	0.125997090 01	0.17950 02	0.17640 02
	25	0.124197470 01	0.122138260 01	0.17390 02	0.17100 02
6	1	0.125555700 01	0.126938520 01	0.17580 02	0.17770 02
	2	0.120687020 01	0.121892050 01	0.16900 02	0.17060 02
	3	0.122114120 01	0.125009600 01	0.17100 02	0.17500 02
	4	0.129394880 01	0.134168940 01	0.18120 02	0.18780 02
	5	0.150712570 01	0.151015610 01	0.21100 02	0.21140 02
	6	0.134909630 01	0.132464490 01	0.18890 02	0.18550 02
	7	0.128177330 01	0.126032500 01	0.17940 02	0.17640 02
	8	0.124275260 01	0.122064420 01	0.17400 02	0.17090 02
	9	0.134309520 01	0.130858330 01	0.18800 02	0.18320 02
	12	0.120587630 01	0.121994700 01	0.16880 02	0.17080 02
	13	0.133896920 01	0.131243390 01	0.18750 02	0.18370 02
	14	0.135097200 01	0.132287660 01	0.18910 02	0.18520 02
	15	0.130362090 01	0.135192190 01	0.18250 02	0.18930 02
	16	0.124108480 01	0.124168920 01	0.17380 02	0.17380 02
	20	0.125615280 01	0.126877030 01	0.17590 02	0.17760 02
	21	0.124197470 01	0.122138260 01	0.17390 02	0.17100 02
	22	0.128214610 01	0.125997090 01	0.17950 02	0.17640 02
	23	0.124174140 01	0.124103270 01	0.17380 02	0.17370 02
	24	0.131173040 01	0.127786390 01	0.18360 02	0.17890 02

TABLE 5.14 (Continued)

PHOTO POINT		TAU		DELTA(um)	
NO	NO	x	y	x	y
7	3	0.125555700 01	0.126938520 01	0.17580 02	0.17770 02
	4	0.124275260 01	0.122064420 01	0.17400 02	0.17090 02
	5	0.128177330 01	0.126032500 01	0.17940 02	0.17640 02
	6	0.124108480 01	0.124168920 01	0.17380 02	0.17380 02
	7	0.131173040 01	0.127786390 01	0.18360 02	0.17890 02
	12	0.120687020 01	0.121892050 01	0.16900 02	0.17060 02
	13	0.134309520 01	0.130858330 01	0.18800 02	0.18320 02
14	14	0.134909630 01	0.132464490 01	0.18890 02	0.18550 02
	15	0.130362090 01	0.135192190 01	0.18230 02	0.18930 02
	16	0.124174140 01	0.124103270 01	0.17380 02	0.17370 02
	18	0.125615280 01	0.126877030 01	0.17500 02	0.17760 02
	19	0.120587630 01	0.121994700 01	0.16880 02	0.17080 02
	20	0.122114120 01	0.125009600 01	0.17100 02	0.17500 02
	21	0.129394880 01	0.134168940 01	0.18120 02	0.18780 02
	22	0.150712570 01	0.151015610 01	0.21100 02	0.21140 02
	23	0.135097200 01	0.132287660 01	0.18910 02	0.18520 02
	24	0.128214610 01	0.125997090 01	0.17950 02	0.17640 02
26	25	0.124197470 01	0.122138260 01	0.17300 02	0.17100 02
	26	0.133896920 01	0.131243390 01	0.18750 02	0.18370 02
8	1	0.125555700 01	0.126938520 01	0.17580 02	0.17770 02
	5	0.131173040 01	0.127786390 01	0.18360 02	0.17890 02
	6	0.124108480 01	0.124168920 01	0.17380 02	0.17380 02
	7	0.128177330 01	0.126032500 01	0.17940 02	0.17640 02
	8	0.124275260 01	0.122064420 01	0.17400 02	0.17090 02
	10	0.120687020 01	0.121892050 01	0.16900 02	0.17060 02
	14	0.124174140 01	0.124103270 01	0.17380 02	0.17370 02
	15	0.130362090 01	0.135192190 01	0.18230 02	0.18930 02
	16	0.134909630 01	0.132464490 01	0.18890 02	0.18550 02
	17	0.134309520 01	0.130858330 01	0.18800 02	0.18320 02
18	18	0.122114120 01	0.125009600 01	0.17100 02	0.17500 02
	19	0.120587630 01	0.121994700 01	0.16880 02	0.17080 02
	20	0.125615280 01	0.126877030 01	0.17500 02	0.17760 02
	21	0.124197470 01	0.122138260 01	0.17300 02	0.17100 02
	22	0.128214610 01	0.125997090 01	0.17950 02	0.17640 02
	23	0.135097200 01	0.132287660 01	0.18910 02	0.18520 02
	24	0.150712570 01	0.151015610 01	0.21100 02	0.21140 02
	25	0.129394880 01	0.134168940 01	0.18120 02	0.18780 02
	26	0.133896920 01	0.131243390 01	0.18750 02	0.18370 02

TABLE 5.14 (Continued)

PHOTO	POINT	TAU		DELTA (um)	
		x	y	x	y
1	1	0.148565900 01	0.149757060 01	0.20800 02	0.20970 02
	2	0.134766880 01	0.132105590 01	0.18870 02	0.18490 02
	3	0.127349300 01	0.120226960 01	0.17830 02	0.16830 02
	4	0.124130860 01	0.122034360 01	0.17380 02	0.17080 02
	5	0.121909710 01	0.122841220 01	0.17070 02	0.17200 02
	6	0.120452530 01	0.121947510 01	0.16860 02	0.17070 02
	7	0.118038780 01	0.123327230 01	0.16530 02	0.17270 02
	8	0.129077500 01	0.134011180 01	0.18070 02	0.18760 02
	9	0.133799400 01	0.131201420 01	0.18730 02	0.18370 02
	10	0.134619510 01	0.132256740 01	0.18850 02	0.18520 02
	11	0.130261620 01	0.135158520 01	0.18240 02	0.18920 02
	12	0.124005910 01	0.124025580 01	0.17360 02	0.17360 02
	16	0.120580360 01	0.121742340 01	0.16880 02	0.17040 02
	17	0.134261540 01	0.130766310 01	0.18800 02	0.18310 02
	18	0.123493230 01	0.121367800 01	0.17290 02	0.16990 02
	19	0.123986790 01	0.124150570 01	0.17360 02	0.17380 02
	20	0.127397710 01	0.126276070 01	0.17840 02	0.17680 02
	24	0.125289000 01	0.122421670 01	0.17540 02	0.17140 02
	25	0.124241880 01	0.121958260 01	0.17390 02	0.17070 02
2	1	0.127264760 01	0.120298240 01	0.17820 02	0.16840 02
	2	0.134555460 01	0.132305660 01	0.18840 02	0.18520 02
	3	0.143437310 01	0.149889110 01	0.20780 02	0.20980 02
	4	0.129058610 01	0.134032320 01	0.18070 02	0.18760 02
	5	0.118257740 01	0.123078960 01	0.16560 02	0.17230 02
	6	0.120552590 01	0.121843930 01	0.16880 02	0.17060 02
	7	0.121880980 01	0.122870640 01	0.17060 02	0.17200 02
	8	0.124203340 01	0.121965620 01	0.17390 02	0.17080 02
	9	0.134203440 01	0.130823820 01	0.18790 02	0.18320 02
	10	0.123937860 01	0.124093780 01	0.17350 02	0.17370 02
	11	0.130261680 01	0.135158450 01	0.18240 02	0.18920 02
	12	0.134798160 01	0.132087490 01	0.18870 02	0.18490 02
	13	0.133845690 01	0.131154030 01	0.18740 02	0.18360 02
	14	0.120476270 01	0.121849750 01	0.16870 02	0.17060 02
	18	0.127695290 01	0.125989070 01	0.17880 02	0.17640 02
	19	0.124051780 01	0.124085430 01	0.17370 02	0.17370 02
	20	0.123561620 01	0.121302990 01	0.17300 02	0.16980 02
	21	0.124159640 01	0.122036190 01	0.17380 02	0.17090 02
	22	0.125384560 01	0.122332720 01	0.17550 02	0.17130 02
3	1	0.127695290 01	0.125989070 01	0.17880 02	0.17640 02
	2	0.124051780 01	0.124085430 01	0.17370 02	0.17370 02
	3	0.123561620 01	0.121302990 01	0.17300 02	0.16980 02
	4	0.124159640 01	0.122036190 01	0.17380 02	0.17090 02
	5	0.125384560 01	0.122332720 01	0.17550 02	0.17130 02
	10	0.123937860 01	0.124093780 01	0.17350 02	0.17370 02
	11	0.130261680 01	0.135158450 01	0.18240 02	0.18920 02
	12	0.134798160 01	0.132087490 01	0.18870 02	0.18490 02
	13	0.133845690 01	0.131154030 01	0.18740 02	0.18360 02
	14	0.120476270 01	0.121849750 01	0.16870 02	0.17060 02
	18	0.127264760 01	0.120298240 01	0.17820 02	0.16840 02
	19	0.134555460 01	0.132305660 01	0.18840 02	0.18520 02
	20	0.143437310 01	0.149889110 01	0.20780 02	0.20980 02
	21	0.129058610 01	0.134032320 01	0.18070 02	0.18760 02
	22	0.118257740 01	0.123078960 01	0.16560 02	0.17230 02
	23	0.120552590 01	0.121843930 01	0.16880 02	0.17060 02
	24	0.121880980 01	0.122870640 01	0.17060 02	0.17200 02
	25	0.124203340 01	0.121965620 01	0.17390 02	0.17080 02
	26	0.134203440 01	0.130823820 01	0.18790 02	0.18320 02

TABLE 5.15. Undetected Gross Errors for case 10.
(at $\alpha = 0.05$ and $\gamma = 0.8$)

PHOTO POINT		TAU		DELTA(um)	
NO	HO	x	y	x	y
4	1	0.12349323D 01	0.12136780D 01	0.1729D 02	0.1699D 02
	2	0.12398679D 01	0.12415057D 01	0.1736D 02	0.1738D 02
	3	0.12739771D 01	0.12627687D 01	0.1784D 02	0.1768D 02
	7	0.12528900D 01	0.12242167D 01	0.1754D 02	0.1714D 02
	8	0.12424188D 01	0.12195826D 01	0.1739D 02	0.1707D 02
	10	0.13461951D 01	0.13225624D 01	0.1885D 02	0.1852D 02
	11	0.13026162D 01	0.13515852D 01	0.1824D 02	0.1892D 02
	12	0.12400591D 01	0.12402558D 01	0.1736D 02	0.1736D 02
	16	0.12058036D 01	0.12174234D 01	0.1688D 02	0.1704D 02
	17	0.13426154D 01	0.13076631D 01	0.1880D 02	0.1831D 02
	18	0.14856590D 01	0.14975706D 01	0.2080D 02	0.2097D 02
	19	0.13476688D 01	0.13210559D 01	0.1887D 02	0.1849D 02
	20	0.12734930D 01	0.12022696D 01	0.1783D 02	0.1683D 02
	21	0.12413086D 01	0.12203436D 01	0.1738D 02	0.1708D 02
	22	0.12190971D 01	0.12284122D 01	0.1707D 02	0.1720D 02
	23	0.12045253D 01	0.12194751D 01	0.1686D 02	0.1707D 02
	24	0.11803878D 01	0.12332723D 01	0.1633D 02	0.1727D 02
	25	0.12907750D 01	0.13401118D 01	0.1807D 02	0.1876D 02
	26	0.13379940D 01	0.13120142D 01	0.1873D 02	0.1837D 02
5	1	0.11325774D 01	0.12307896D 01	0.1656D 02	0.1723D 02
	2	0.12055259D 01	0.12184393D 01	0.1689D 02	0.1706D 02
	3	0.12188098D 01	0.12287064D 01	0.1706D 02	0.1720D 02
	4	0.12420334D 01	0.12196562D 01	0.1739D 02	0.1708D 02
	5	0.12726476D 01	0.12029824D 01	0.1722D 02	0.1684D 02
	6	0.13455546D 01	0.13230566D 01	0.1884D 02	0.1852D 02
	7	0.14843731D 01	0.14989116D 01	0.2078D 02	0.2098D 02
	8	0.12905861D 01	0.13403232D 01	0.1807D 02	0.1876D 02
	9	0.13420344D 01	0.13082332D 01	0.1879D 02	0.1832D 02
	10	0.12047627D 01	0.12184975D 01	0.1687D 02	0.1706D 02
	14	0.12393786D 01	0.12409378D 01	0.1735D 02	0.1737D 02
	15	0.13026168D 01	0.13515845D 01	0.1824D 02	0.1892D 02
	16	0.13479816D 01	0.13208749D 01	0.1887D 02	0.1849D 02
	17	0.13384569D 01	0.13115403D 01	0.1874D 02	0.1836D 02
	18	0.12538456D 01	0.12233272D 01	0.1755D 02	0.1713D 02
	22	0.12769529D 01	0.12592907D 01	0.1788D 02	0.1764D 02
	23	0.12405178D 01	0.12408543D 01	0.1737D 02	0.1737D 02
	24	0.12356162D 01	0.12130299D 01	0.1730D 02	0.1698D 02
	25	0.12415964D 01	0.12203619D 01	0.1738D 02	0.1709D 02
6	1	0.12188098D 01	0.12287064D 01	0.1706D 02	0.1720D 02
	2	0.12055259D 01	0.12184393D 01	0.1688D 02	0.1706D 02
	3	0.11325774D 01	0.12307896D 01	0.1656D 02	0.1723D 02
	4	0.12905861D 01	0.13403232D 01	0.1807D 02	0.1876D 02
	5	0.14843731D 01	0.14989116D 01	0.2078D 02	0.2098D 02
	6	0.13455546D 01	0.13230566D 01	0.1884D 02	0.1852D 02
	7	0.12726476D 01	0.12029824D 01	0.1782D 02	0.1664D 02
	8	0.12420334D 01	0.12196562D 01	0.1739D 02	0.1708D 02
	9	0.13420344D 01	0.13082382D 01	0.1879D 02	0.1832D 02
	12	0.12047627D 01	0.12184975D 01	0.1687D 02	0.1706D 02
	13	0.13384569D 01	0.13115403D 01	0.1874D 02	0.1836D 02
	14	0.13479816D 01	0.13208749D 01	0.1887D 02	0.1849D 02
	15	0.13026168D 01	0.13515845D 01	0.1824D 02	0.1892D 02
	16	0.12393786D 01	0.12409378D 01	0.1735D 02	0.1737D 02
	20	0.12538456D 01	0.12233272D 01	0.1755D 02	0.1713D 02
	21	0.12415964D 01	0.12203619D 01	0.1738D 02	0.1709D 02
	22	0.12356162D 01	0.12130299D 01	0.1730D 02	0.1698D 02
	23	0.12405178D 01	0.12408543D 01	0.1737D 02	0.1737D 02
	24	0.12769529D 01	0.12598907D 01	0.1788D 02	0.1764D 02

TABLE 5.15 (Continued)

PHOTO POINT		TAU		DELTA (um)	
NO	NO	x	y	x	y
7	3	0.125289000 01	0.122421670 01	0.17540 02	0.17140 02
	4	0.124241880 01	0.121958260 01	0.17390 02	0.17070 02
	5	0.123493230 01	0.121367800 01	0.17290 02	0.16990 02
	6	0.123986790 01	0.124150570 01	0.17360 02	0.17380 02
	7	0.127397710 01	0.126276870 01	0.17840 02	0.17660 02
	12	0.120580360 01	0.121742340 01	0.16880 02	0.17040 02
	13	0.134261540 01	0.130766310 01	0.18800 02	0.18310 02
	14	0.134619510 01	0.132256240 01	0.18850 02	0.18520 02
	15	0.130261620 01	0.135158520 01	0.16240 02	0.18920 02
	16	0.124005910 01	0.124025580 01	0.17360 02	0.17360 02
	18	0.121909710 01	0.122841220 01	0.17070 02	0.17200 02
	19	0.120452530 01	0.121947510 01	0.16860 02	0.17070 02
	20	0.118038780 01	0.123327230 01	0.16530 02	0.17270 02
	21	0.129077500 01	0.134011180 01	0.18070 02	0.18760 02
	22	0.148565900 01	0.149757060 01	0.20800 02	0.20970 02
	23	0.134766880 01	0.137105590 01	0.18870 02	0.18490 02
	24	0.127349300 01	0.120226960 01	0.17830 02	0.16830 02
	25	0.124130860 01	0.122034360 01	0.17380 02	0.17080 02
	26	0.133799400 01	0.131201420 01	0.18730 02	0.18370 02
8	1	0.125289000 01	0.122421670 01	0.17540 02	0.17140 02
	5	0.127397710 01	0.126276870 01	0.17840 02	0.17680 02
	6	0.123986790 01	0.124150570 01	0.17360 02	0.17380 02
	7	0.123493230 01	0.121367800 01	0.17290 02	0.16990 02
	8	0.124241880 01	0.121958260 01	0.17390 02	0.17070 02
	10	0.120580360 01	0.121742340 01	0.16880 02	0.17040 02
	14	0.124005910 01	0.124025580 01	0.17360 02	0.17360 02
	15	0.130261620 01	0.135158520 01	0.18240 02	0.18920 02
	16	0.134619510 01	0.132256240 01	0.18850 02	0.18520 02
	17	0.134261540 01	0.130766310 01	0.18800 02	0.18310 02
	18	0.118038780 01	0.123327230 01	0.16530 02	0.17270 02
	19	0.120452530 01	0.121947510 01	0.16860 02	0.17070 02
	20	0.121909710 01	0.122841220 01	0.17070 02	0.17200 02
	21	0.124130860 01	0.122034360 01	0.17380 02	0.17080 02
	22	0.127349300 01	0.120226960 01	0.17830 02	0.16830 02
	23	0.134766880 01	0.132105590 01	0.18870 02	0.18490 02
	24	0.148565900 01	0.147757060 01	0.20800 02	0.20970 02
	25	0.129077500 01	0.134011180 01	0.18070 02	0.18760 02
	26	0.133799400 01	0.131201420 01	0.18730 02	0.18370 02

TABLE 5.15 (Continued)

Chapter 3 and are listed in Tables (5.12) through (5.15) for cases 9,10. The maximum values of Tau, Gam and undetected gross errors for the different cases are summarised in Table (5.16) for the photogrammetric measurements. Those for slope distances are tabulated in Table (5.17).

Case	Tau		Gam		Maximum undetected gross error (μm)	
	x	y	x	y	x	y
1	1.9922311	2.8393368	1.7230743	2.6574110	27.89	39.75
2	1.9846228	2.8387769	1.7142778	2.6568128	27.78	39.74
3	1.9257858	2.8345602	1.6457980	2.6523068	26.96	39.68
4	1.9174364	2.8341615	1.6360202	2.6518807	26.84	39.68
5	1.8218268	3.0545315	1.5228437	2.8862021	25.51	42.76
6	1.8216204	3.0508413	1.5225967	2.8822964	25.50	42.71
7	1.6778018	1.7862221	1.3472264	1.4800640	23.49	25.01
8	1.6774491	1.7860925	1.3467870	1.4799076	23.48	25.01
9	1.5071257	1.5101561	1.1275762	1.1316233	21.10	21.14
10	1.4856590	1.4988911	1.0987187	1.1165458	20.80	20.98

TABLE (5.16) Max Tau, Gam and undetected gross errors for photogrammetric measurements (at probability levels $\alpha = 0.05$, $\gamma = 0.8$)

As anticipated, the reliability increases with the increase in number of cameras and higher internal reliability reflects higher external reliability as can be seen from the values shown in that table. Moreover, the more number of cameras used, the larger the chance to detect gross errors with less magnitudes.

Case	Tau	Gam	Maximum undetected gross error (mm)
2	1.9964749	1.7279792	2.795
4	1.9215369	1.6408242	2.690
6	1.8127073	1.5119219	2.538
8	1.6707385	1.3384196	2.339
10	1.5575812	1.1941771	2.181

TABLE (5.17) Max Tau, Gam and undetected gross errors for the 8 slope distances (at $\alpha = 0.05$, $\gamma = 0.8$)

5.6.2.3. Models for Sensitivity analysis

Simulated models to investigate the sensitivity of the photogrammetric network and its relation to the number of cameras and to the incorporation of slope distances are undertaken. In the design phase we assume, for simplicity, that we have deformations between two epochs only to be detected. Presumably, the deformation monitoring networks have the same datum and identical cofactor matrices. To assess the just-detectable deformations, we apply equation (3.59) given in Section (3.2.3.3).

Restating equation(3.59):

$$\omega^u = \frac{(\tilde{cd})^T Q_d^+ (\tilde{cd})}{\sigma_0^2} \quad (5.10)$$

in which $\tilde{cd}$ represents a just-detectable deformation and $\tilde{d}$ is a form vector which characterises the deformation model to be tested.

The models chosen are designated deflexion, expansion and settlement models. Before delving into the analysis of these models, a brief mention is to be given about the examination of the impact of single and multiple point movements on sensitivity. Table (5.18) reveals that the more the number of points in movement, the more sensitive the network is and vice versa.

Form vector description (units are in mm)	Sensitivity parameter c
$\tilde{d} = \begin{pmatrix} dx_1 & dz_1 \\ 10, 0, 5, 0, 0, 0, \dots, 0, 0, 0 \end{pmatrix}$	0.7807
$\tilde{d} = \begin{pmatrix} & dx_{24} & dz_{24} \\ 0, 0, 0, \dots, -10, 0, -5, \dots, 0, 0, 0 \end{pmatrix}$	0.5458
$\tilde{d} = \begin{pmatrix} dx_1 & dx_2 & dx_9 \\ 10, 0, 0, 10, 0, 0, \dots, 10, 0, 0, \dots, 0, 0, 0 \end{pmatrix}$	0.2104
$\tilde{d} = \begin{pmatrix} dx_1 & dx_2 & dx_9 & dx_{18} & dx_{19} & dx_{26} \\ 10, 0, 0, 10, 0, 0, \dots, 10, \dots, -10, 0, 0, -10, \dots, -10, 0, 0 \end{pmatrix}$	0.1569

TABLE (5.18) Values of sensitivity parameter (c) for case 1 (4 photos).

It is meant by deflexion model that the targets on both the upper and lower surfaces of the cube are subjected to 10 mm downward displacements. The same magnitude of displacement was assumed but in outward radial direction for all the targets representing what we called the expansion model. Unlike deflexion model, in the settlement model all the targets are supposed to be moving 10 mm downwards. It is important to notice that both expansion and

settlement models were applied to cases 5,6,7,8,9,10 only as cases 1-4 suffer from lack of determination of point 15. Deflexion model is applicable to all cases.

Table (5.19) lists the values of c for both expansion and settlement models while Table (5.20) displays these values for the deflexion model. The results summarised in such tables are graphically represented in Figures (5.14) and (5.15) respectively. These figures suggest that the sensitivity decreases with the increase of the number of cameras (larger values of (c) mean less sensitivity). This can be thought of as the effect of more correlation between the co-ordinates of different object points imaged on more photographs.

Number of cameras	Model designation	Photogrammetry c	Photo+ distances c
6	Expansion	0.2018	0.1003
6	Settlement	0.1531	0.1530
7	Expansion	0.2304	0.1076
7	Settlement	0.1736	0.1729
8	Expansion	0.2435	0.1130
8	Settlement	0.1899	0.1886

TABLE (5.19) Relation between sensitivity parameter (c) and number of cameras for Expansion and Settlement models (at $\alpha = 0.05$ and $\gamma = 0.8$)

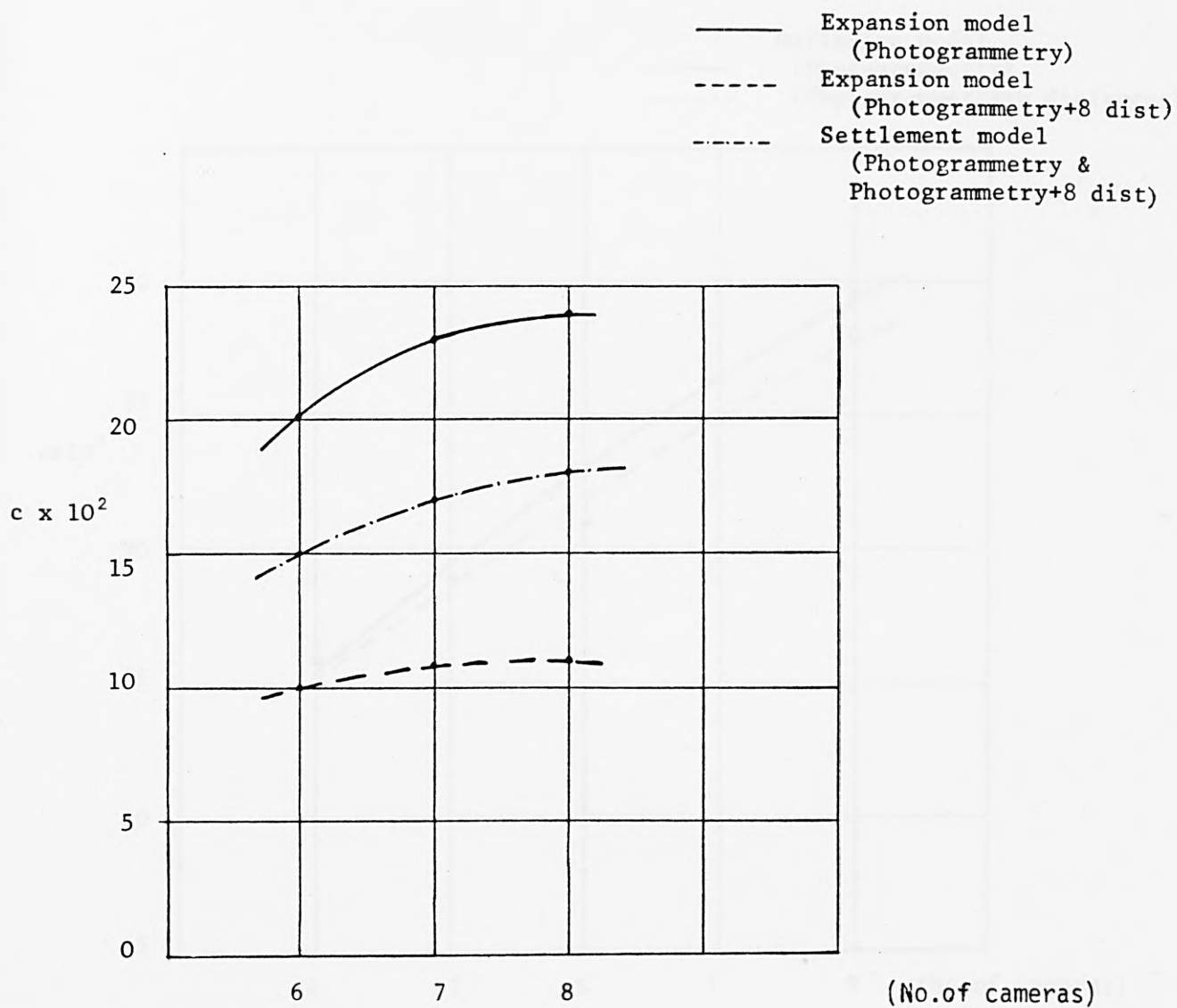


Figure 5.14. No. of cameras versus sensitivity parameter (c) for Expansion and Settlement models.

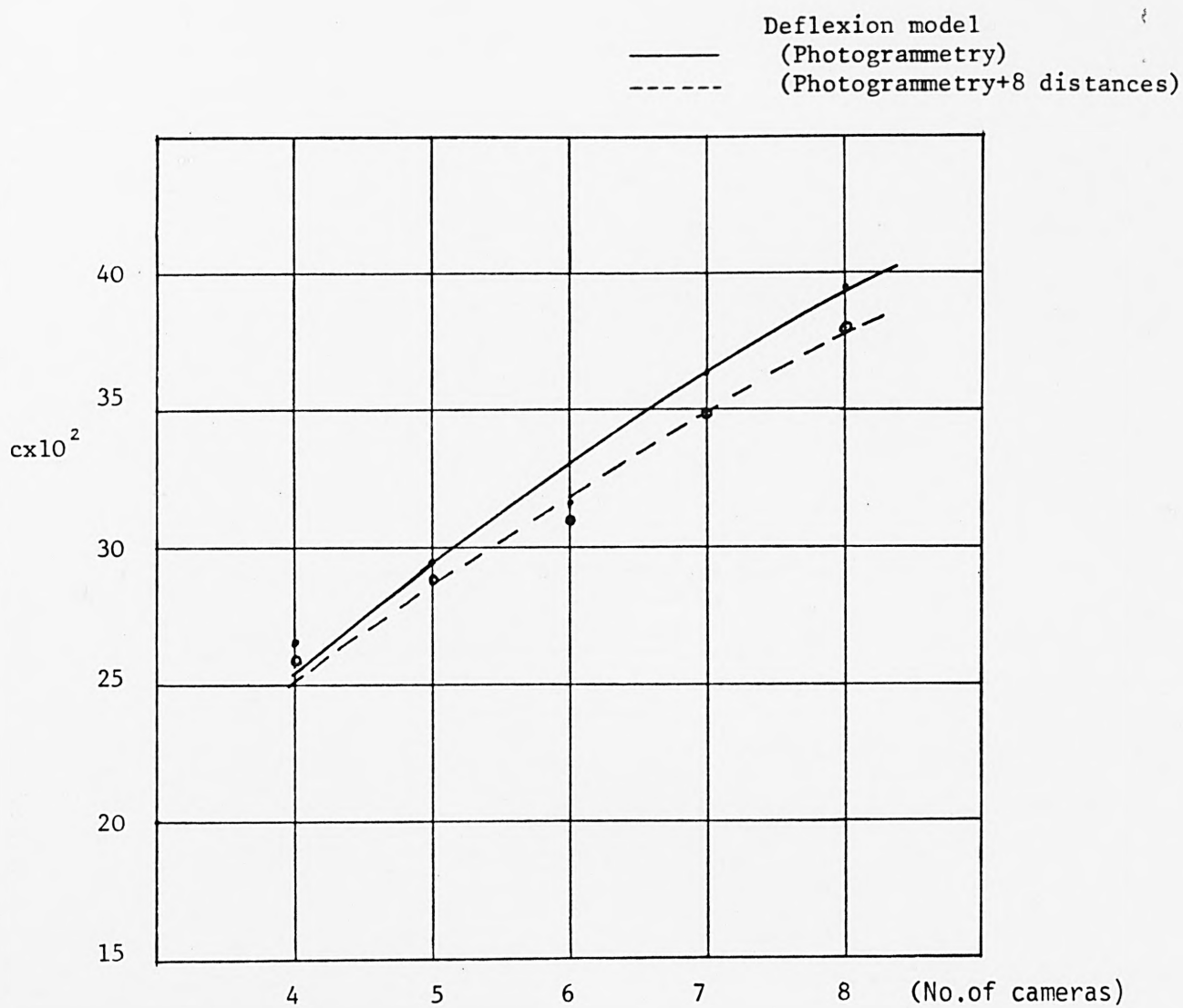


Figure 5.15. No. of cameras versus sensitivity parameter (c) for Deflexion model.

Number of cameras	Model designation	Photogrammetry c	Photos+distances c
4	Deflexion	0.2661	0.2599
5	Deflexion	0.2961	0.2877
6	Deflexion	0.3164	0.3064
7	Deflexion	0.3643	0.3494
8	Deflexion	0.3963	0.3781

TABLE (5.20) Relation between sensitivity parameter (c) and number of cameras for Deflexion model (at $\alpha = 0.05$ $\gamma = 0.8$)

Additionally, the inclusion of distances between object points, as anticipated, does not enhance the sensitivity for settlement model. On the other hand, it has slightly increased the sensitivity for both deflexion and expansion models.

5.6.3. Conclusions

On the basis of these simulations one can conclude that both the precision and reliability increase with the increase of the number of cameras used. However, sensitivity was found to decrease with the increase of that number, which can be thought of as being due to the increased correlation between the object point co-ordinates. This explanation is conformal with that of Fraser (1982a) where he states that the trace of the parameter covariance matrix is minimised at the expense of higher correlation between the parameters which is a well-known property of inner constraint adjustment. Comparing the global indicators $r_{\max}$ and σ_m^2 of the different cases it can be

noticed that good precision does correspond to good reliability.

With regard to precision, the more cameras the better precision, but the benefit gained decreases as the number of cameras increases. Should the number of cameras be doubled, an average percentage improvement of the order of 24 and 23 in purely photogrammetric and combined, which comprise slope distances, cases respectively would be achieved.

Addition of slope distances provides a marginal improvement "within" the cases of the same number cameras, while it considerably enhances the improvement "between" the different number of camera cases. All the cases produce submillimetre level for σ_m^2 but case 9 (8 photographs) and case 10 (same as case 9 but with 8 slope distances) exhibit the highest precision ($\sigma_{m(9)}^2 = 0.172 \text{ mm}^2$, $\sigma_{m(10)}^2 = 0.170 \text{ mm}^2$) which is attributed to the full coverage of the cube.

The main factor affecting the reliability is the number of images per object point. More intersecting rays from an object point give better reliability (larger redundancy numbers). However, it is noticed that improvement in redundancy numbers slows down considerably after 6 rays. Therefore, it can be said that it is not economically desirable to try to improve the redundancy if the object point already has 6 rays and its image co-ordinates display redundancy numbers of the order of 0.6 or more.

Distance observations between object points, in general, increase the reliability substantially when adjusted simultaneously with the photogrammetric data. Cases 9 and 10, again, show fully homogeneous and highest reliability ($r_{\max(9)}(x) = 0.688$, $r_{\max(9)}(y) = 0.673$;

$r_{\max}(10)^{(x)} = 0.718$, $r_{\max}(10)^{(y)} = 0.672$) which is justified by the symmetric arrangement of camera positions and distances measured on the cube's surface. Moreover, they can detect a gross error of the order of 4-fold the a priori standard error. It is notable that high internal reliability leads to higher external reliability, and the more the number of cameras the greater is the chance of detection of small gross errors.

Regarding the sensitivity it is worth mentioning that a network is least sensitive to a single point movement and most sensitive to multiple point displacements depending on the assumed pattern of movements of that cluster of points. Inclusion of distance observations between object points does not affect the sensitivity for the settlement model. On the other hand, such an addition has slightly increased the sensitivity for both the deflexion and expansion models.

Summarising the experiences gained by these investigations and indicated by the precision, reliability and sensitivity criteria, we have to prefer case 6 (6 cameras + slope distances) because in this case the x- and y- observations are of sufficient reliability ($r_{\max}(x) = 0.682$, $r_{\max}(y) = 0.670$) together with a satisfactory sensitivity (c (expansion, settlement models) = 0.10, 0.15 respectively; c (deflexion model) = 0.31) and precision ($\sigma_m^2 = 0.248 \text{ mm}^2$).

One more important point is the considerable saving in computer storage and processing time due to the strategies implemented in this research: first, the direct editing of the normal equations without need to form the observation equations and, secondly, the

CHAPTER 6

SIMULATED NETWORKS FOR DEFORMATION ANALYSIS (BRIDGE)

6.1. Introduction

The general need for monitoring structures is well known to the people of all walks of life due to the loss of human lives when large structures fail. Present methods of detecting and monitoring structural deformations can be divided into two basic groups: internal and external (Erlandson and Veress, 1975).

Internal methods generally utilize sophisticated instruments such as extensometers, inclinometers, and strain gauges. The main disadvantages of their use is their costly maintenance and the difficulty of interpretation of the large amount of data they supply. On the other hand, external methods utilize conventional field surveys (e.g. triangulation and/or trilateration, levelling) where a number of targets whose positions are determined by using some of these methods are placed on the surface of the structure. Such a survey is repeated at predetermined intervals, and positions of the targets each time are compared to quantify the deformations. If the number of targets is fairly large, these methods are considerably time consuming because an individual observation is required for each target on the structure. Erlandson and Veress (1975) claim that a complete survey of a large structure may take more than two weeks during which, deformation can occur but would be undetected.

Photogrammetry, with the inclusion of a few spatial distances, is thought to be the most economical alternative to survey methods as it can replace hundreds of angular measurements thus providing information about the structure as a whole and not to mention the common advantages such as near-instantaneous, complete and permanent recording of a situation. Bridge deformation presents itself as an ideal candidate for such kind of study to be presented in this Chapter.

6.2. Network Configurations

A plan of the bridge, under investigation, and the camera locations is shown in Figure (6.1). Both sides of the bridge (Figures 6.2a, 6.2b) and the columns (Figures 6.3a, b,c,d) with 41 targetted object points are based on real data which were made available to the author. Such data were considered as the approximate values required for the evaluation of the propagated covariance matrices. The wide angle metric camera Zeiss (Jena) UMK 10/1318 having a nominal focal length of 100 mm was assumed to be implemented with the use of glass plates with a negative format of 130 mm x 180 mm. Four camera stations were assumed to represent the start configuration. Subsequently, cameras 5 and 6 were added to provide the different network configurations as displayed in Table (6.2) keeping the number of targets unchanged. The orientation elements of the individual photographs are given in Table (6.1) assuming that the camera principal axis is directed towards the centre of the bridge. The photogrammetric co-ordinates

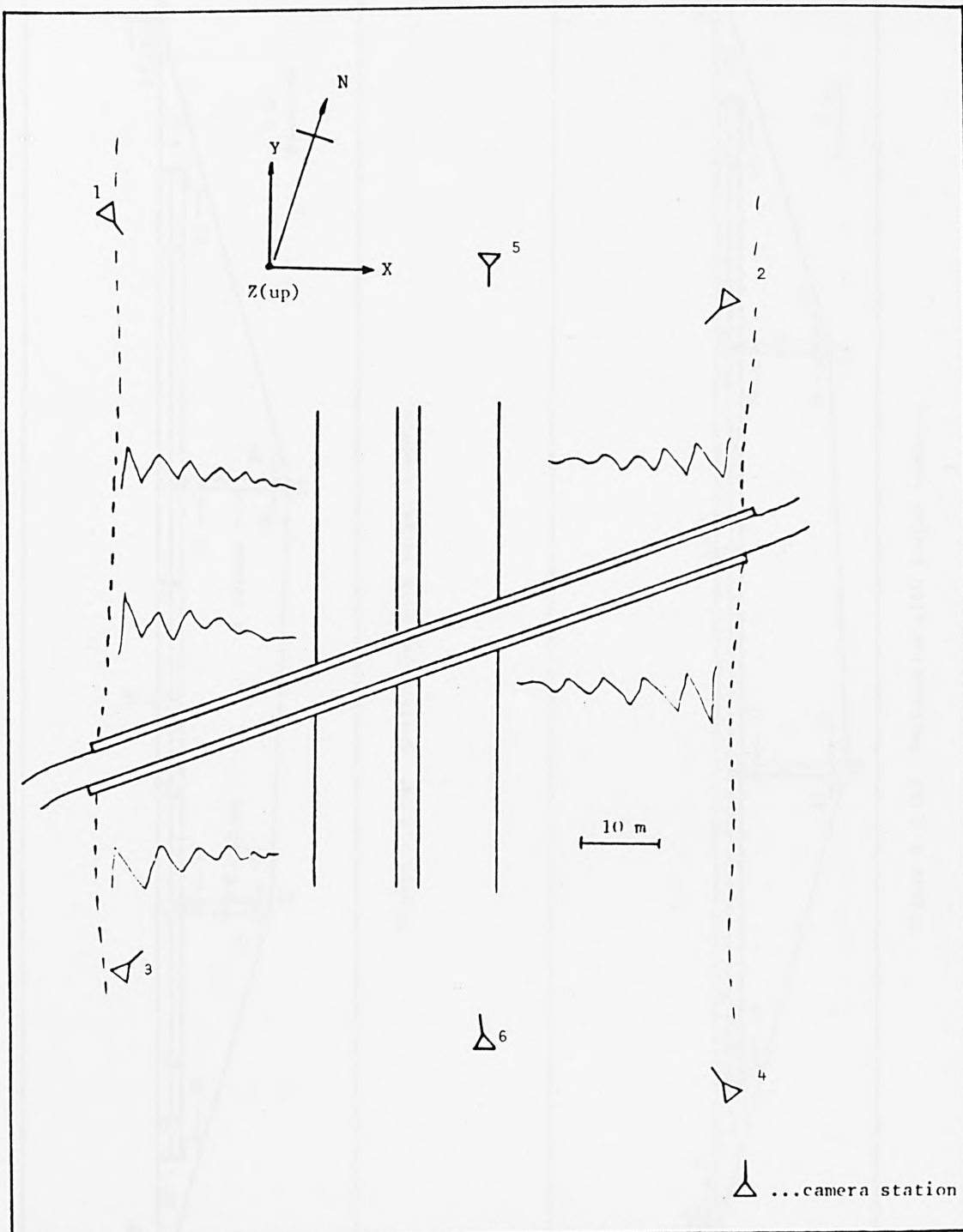


Figure 6.1. Camera station configurations (after Cooper, 1984)

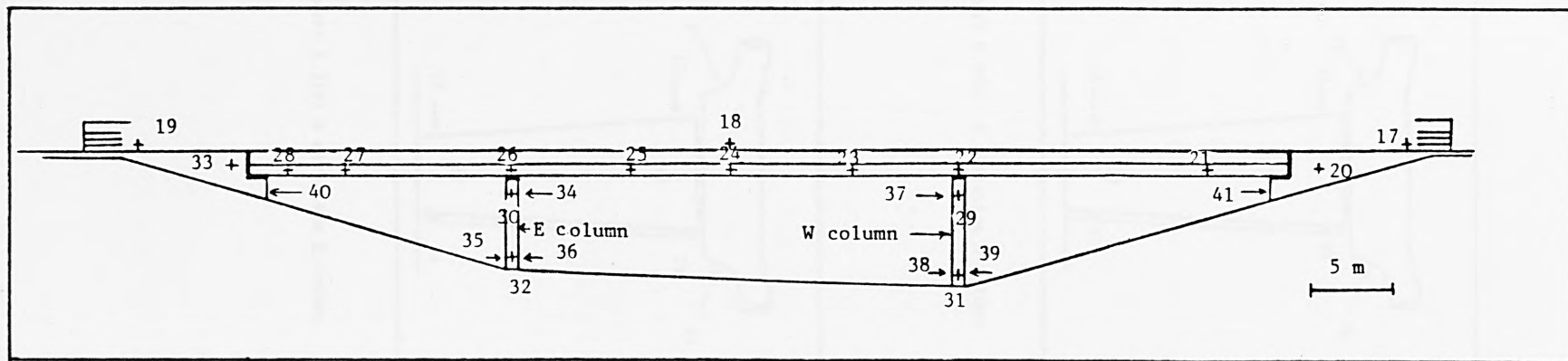


Figure 6.2.(a) S-elevation with target numbers.

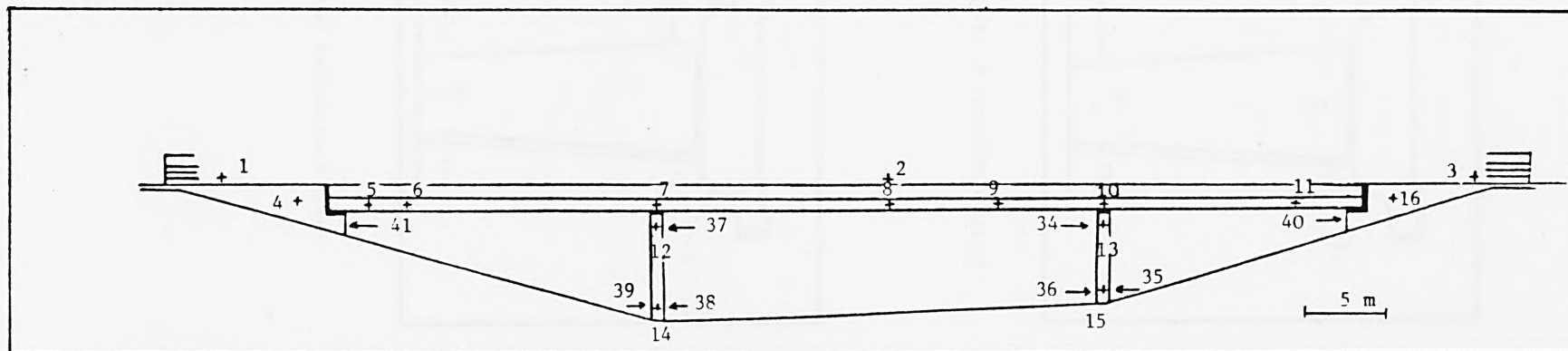


Figure 6.2.(b) N-elevation with target numbers.

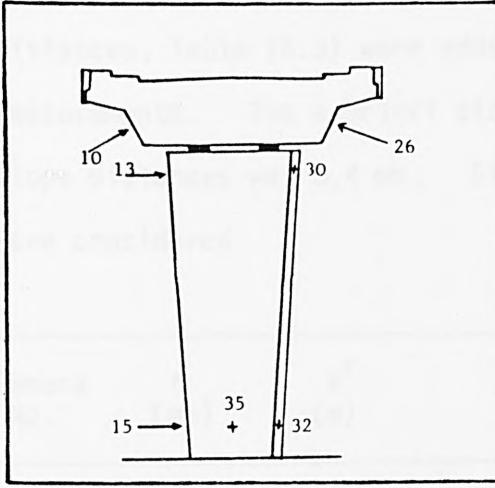


Figure 6.3(a) W elevation W column

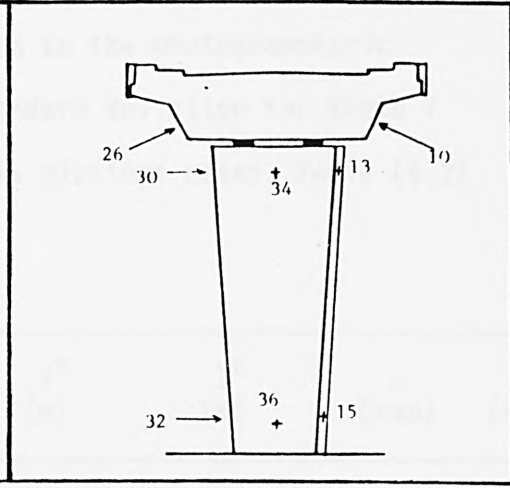


Figure 6.3(b) E elevation W column

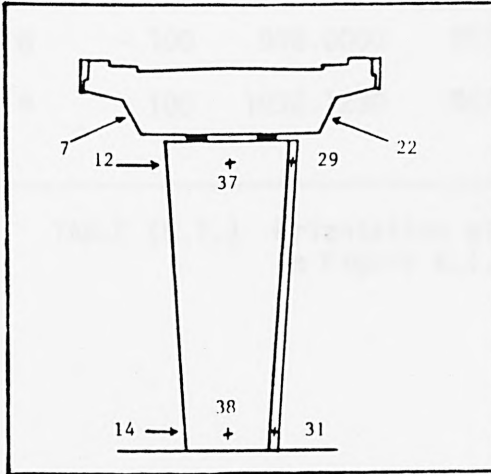


Figure 6.3(c) W elevation E column

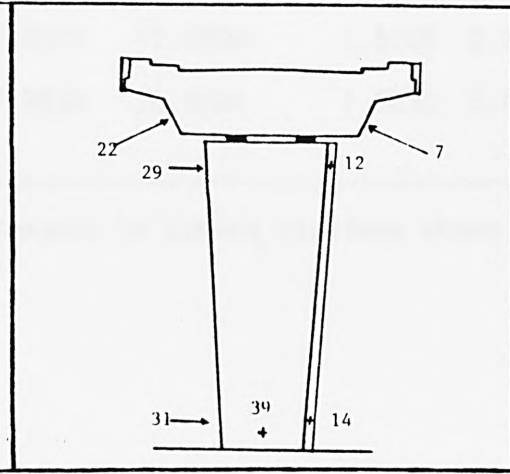


Figure 6.3(d) E elevation E column

were assigned a standard deviation of $\pm 3 \mu\text{m}$. Seven spatial distances, Table (6.3) were added to the photogrammetric measurements. The a priori standard deviation for these 7 slope distances was 0.4 mm. Six distinct cases, Table (6.2) were considered.

Camera No.	f (mm)	x^C (m)	y^C (m)	z^C (m)	ω (rad)	ϕ (rad)	κ (rad)
1	100	968.8800	1054.2146	18.6406	1.5728	3.6220	0.0040
5	100	1008.0000	1049.0000	11.0000	1.5700	3.1000	0.0000
2	100	1038.9038	1044.0665	18.9812	1.5719	2.6949	0.0063
3	100	964.1530	966.4781	19.7393	1.5752	-0.4506	-0.0077
6	100	998.0000	957.0000	12.0000	1.5700	0.0000	0.0000
4	100	1032.1230	947.9636	20.4986	1.5838	0.4517	-0.0088

TABLE (6.1.) Orientation elements of camera stations shown in Figure 6.1.

Case	Photo arrangement	Total number of Targets	Number of slope distances
1	1-2-3-4	41	-
2	Case 1 + slope dist.	41	7
3	1-2-3-4-5	41	-
4	Case 3 + slope dist.	41	7
5	1-2-3-4-5-6	41	-
6	Case 5 + slope dist.	41	7

TABLE (6.2) Photo arrangement, No.of targets and slope distances.

Observation No,	Target Object Point	
	From	To
1	1	3
2	1	2
3	17	19
4	17	18
5	1	17
6	2	18
7	3	19

TABLE 6.3. Simulated slope distances.

6.3. Results and Analysis

Both the observation equations and the normal equations were formed in the same way as has been previously discussed in Sections (5.3) and (5.4) respectively. The same inversion routine used for the cube cases in Section (5.5) was adopted. Again, results of cases 5 and 6 were selected to represent the different criteria of design. Figure (6.4) depicts the structure of the coefficient matrix of the normal equations for 6 photographs and 41 target object points. The same levels of probability adopted throughout this study, i.e. $\alpha = 0.05$ and $\beta = 0.20$ were applied.

6.3.1. Mean Variance of Object point co-ordinates

As has been previously discussed in Section (5.6.2.1), the mean variance, σ_m^2 , given in Equation (5.8) for each configuration is to be compared in order to show the influence of the number of cameras without and with the addition of spatial distances on the precision.

Restating Equation (5.8):

$$\sigma_m^2 = \frac{1}{3n_0} \text{tr} Q_{xx}^{(2)} \quad (6.1)$$

in which $Q_{xx}^{(2)}$ is the a priori cofactor matrix of the n_0 object points. The free network bundle adjustment was performed to compute the variances of the object point co-ordinates.

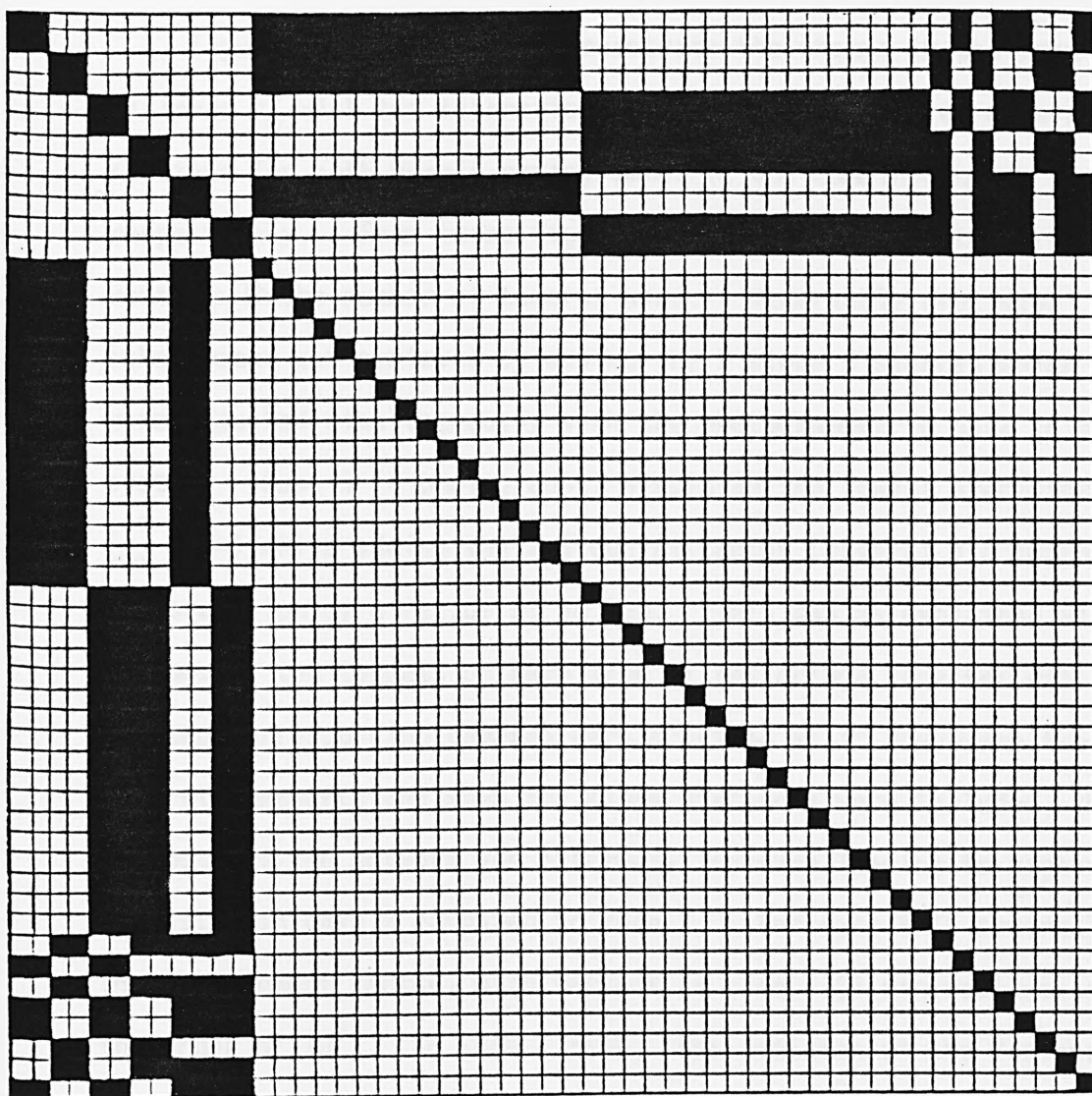


Figure 6.4. Structure of the coefficient matrix of the normal equations for 6 photos and 41 object points.

Figures (6.5) and (6.6) demonstrate the change in the mean variance σ_m^2 with the alteration of the configuration of camera stations for both purely photogrammetric and combined cases respectively. From these figures it is evident that the precision varies with the number of cameras used. Remarkable precision enhancement was achieved by adding the 7 slope distances which were adjusted simultaneously with the photogrammetric data in one system. Case 1 (4 photographs) displayed the lowest precision ($\sigma_m^2 \approx \infty$) while adding the 7 slope distances (case 2) did improve the precision by a factor of 30 000. This can be explained as follows: the targets on each side of the bridge together with the two corresponding cameras were adjusted as if they were two separate networks and when the slope distances were included, the strength of the network was extremely improved leading to higher precision in case 2 compared to case 1. For case 3 (5 cameras) the improvement factor, with respect to case 1 (4 cameras) was of the order of 10^5 . In such a case, the addition of the 7 slope distances improved the precision by 6-fold. The addition of the sixth camera improved the precision by only 3-fold and the same amount of improvement was attained when the 7 slope distances were added. Case 4 gave rise to 16-fold improvement in precision over case 2 whereas case 6 indicated the highest level of precision ($\sigma_m^2 = 2.644 \text{ mm}^2$) and the improvement compared to case 4 was only by 2-fold. Such higher precision can be thought of being due to the symmetric arrangements of the cameras around the bridge, in addition to the inclusion of the 7 slope distances.

POINT		VARIANCES			
NO	X (mm ²)	Y (mm ²)	Z (mm ²)		
1	0.241631520 02	0.299532400 02	0.196289690 01		
2	0.719866090 01	0.140060100 02	0.147579330 01		
3	0.444451330 02	0.416748340 02	0.206442920 01		
4	0.955495850 01	0.126460870 02	0.145283270 01		
5	0.537523950 01	0.804274650 01	0.110615160 01		
6	0.312151240 01	0.841362790 01	0.104674020 01		
7	0.368883580 01	0.936603100 01	0.104319780 01		
8	0.603082950 01	0.112681720 02	0.110399660 01		
9	0.582967540 01	0.106658710 02	0.115926370 01		
10	0.459367130 01	0.996202940 01	0.124134890 01		
11	0.424843680 01	0.994850220 01	0.126741550 01		
12	0.310606240 01	0.792278540 01	0.956852980 00		
13	0.405569110 01	0.891189170 01	0.122829220 01		
14	0.278551040 01	0.750663680 01	0.122900650 01		
15	0.426920110 01	0.864053140 01	0.130272180 01		
16	0.147410850 02	0.212943150 02	0.182704200 01		
17	0.337694440 02	0.287179220 02	0.213889920 01		
18	0.647278680 01	0.107407300 02	0.126539040 01		
19	0.260167970 02	0.327513710 02	0.168198600 01		
20	0.134548350 02	0.166991690 02	0.196389530 01		
21	0.438184060 01	0.776009440 01	0.119000350 01		
22	0.482419720 01	0.782837190 01	0.114440780 01		
23	0.562719140 01	0.823118650 01	0.102370100 01		
24	0.531075480 01	0.844323260 01	0.943985310 00		
25	0.430207360 01	0.812880740 01	0.895105810 00		
26	0.306711500 01	0.719154540 01	0.868125620 00		
27	0.285070140 01	0.683427670 01	0.868808580 00		
28	0.523988560 01	0.807060950 01	0.934666390 00		
29	0.414087090 01	0.674524170 01	0.115248510 01		
30	0.266702140 01	0.623943930 01	0.834133870 00		
31	0.414740050 01	0.743651260 01	0.127922710 01		
32	0.251643720 01	0.586923020 01	0.107141010 01		
33	0.101762570 02	0.135962610 02	0.130614350 01		
34	0.206513610 01	0.489026930 01	0.136533830 01		
35	0.316679630 01	0.135393430 02	0.194984100 01		
36	0.234891650 01	0.612555050 01	0.138845030 01		
37	0.226854150 01	0.416608630 01	0.130545710 01		
38	0.247428330 01	0.562516900 01	0.131422690 01		
39	0.360574410 01	0.117316260 02	0.259287490 01		
40	0.662607620 01	0.880358610 01	0.142609210 01		
41	0.608505670 01	0.775484350 01	0.139994300 01		

TABLE 6.4. Estimates of the variances of the 41 object point co-ordinates for case 5.

POINT		VARIANCES			
NO	X (mm ²)		Y (mm ²)		Z (mm ²)
1	0.227240830 01		0.326666490 01		0.116812760 01
2	0.150582430 01		0.212963900 01		0.109071400 01
3	0.213143100 01		0.364487940 01		0.171067860 01
4	0.207012290 01		0.294425280 01		0.106389920 01
5	0.232477120 01		0.281459630 01		0.893685380 00
6	0.248473000 01		0.406256450 01		0.875014310 00
7	0.233997330 01		0.440608840 01		0.887830110 00
8	0.207607740 01		0.512616600 01		0.985544910 00
9	0.227115610 01		0.514328270 01		0.105992550 01
10	0.264724010 01		0.510072480 01		0.115077610 01
11	0.276717740 01		0.511283800 01		0.117794800 01
12	0.218341890 01		0.430001950 01		0.897900860 00
13	0.263297680 01		0.520400740 01		0.117374390 01
14	0.216852830 01		0.637415900 01		0.956216240 00
15	0.287351490 01		0.746888710 01		0.120981950 01
16	0.411133670 01		0.689653230 01		0.157901390 01
17	0.173864220 01		0.307148460 01		0.184358500 01
18	0.147188770 01		0.212202090 01		0.999388820 00
19	0.182086570 01		0.374250000 01		0.104512630 01
20	0.373574630 01		0.621553000 01		0.170097180 01
21	0.241593310 01		0.410322050 01		0.111762320 01
22	0.231396170 01		0.407324380 01		0.107445710 01
23	0.208269720 01		0.402567020 01		0.957035930 00
24	0.203756810 01		0.393251490 01		0.870354020 00
25	0.216849510 01		0.370922800 01		0.808628190 00
26	0.246643490 01		0.322535360 01		0.761479090 00
27	0.257086450 01		0.299161550 01		0.752612720 00
28	0.238871620 01		0.227784280 01		0.777897890 00
29	0.228179570 01		0.409510370 01		0.109189120 01
30	0.234178730 01		0.318725140 01		0.767640760 00
31	0.251515120 01		0.645698620 01		0.117805300 01
32	0.233053480 01		0.464884140 01		0.843741740 00
33	0.188220870 01		0.259428480 01		0.988532280 00
34	0.108866380 01		0.448390390 01		0.107536940 01
35	0.244184120 01		0.111077790 02		0.144513970 01
36	0.179945960 01		0.565987070 01		0.117053610 01
37	0.113231380 01		0.363259820 01		0.100425790 01
38	0.164701870 01		0.497738930 01		0.109924600 01
39	0.282003570 01		0.102272890 02		0.196756170 01
40	0.161749460 01		0.530924560 01		0.111205720 01
41	0.171991450 01		0.425080050 01		0.109184840 01

TABLE 6.5. Estimates of the variances of the 41 object point co-ordinates for case 6.

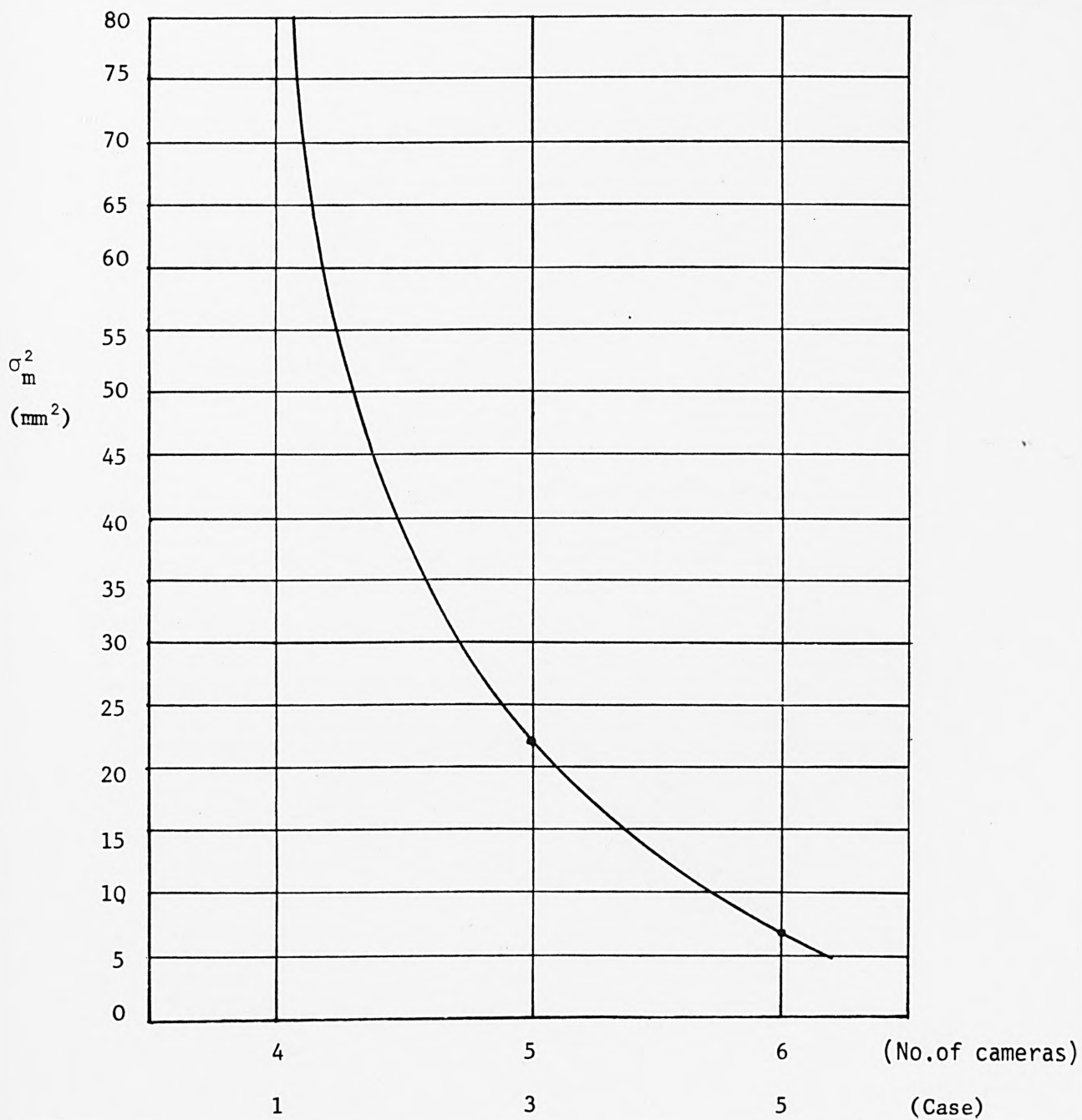


Figure 6.5. Relation between no.of cameras and precision
(Photogrammetry)

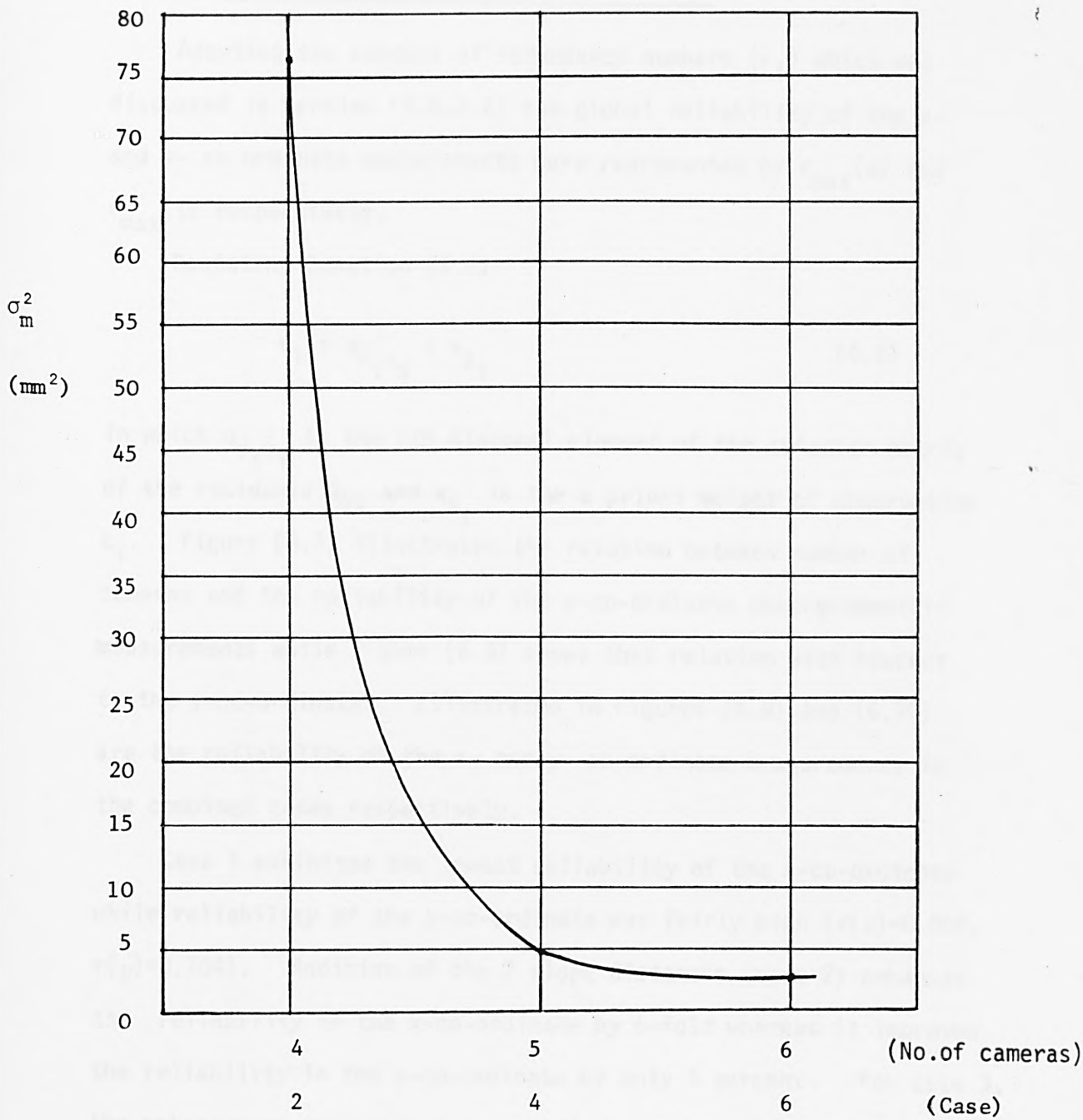


Figure 6.6. Relation between no. of cameras and precision
(Photogrammetry + 7 distances)

6.3.2. $r_{\max}$ and other reliability indicators

Adopting the concept of redundancy numbers (r_i) which was discussed in Section (5.6.2.2) the global reliability of the x- and y- co-ordinate measurements were represented by $r_{\max}(x)$ and $r_{\max}(y)$ respectively.

Restating Equation (5.9):

$$r_i = q_{\hat{v}_i \hat{v}_i} \times w_{\ell_i} \quad (6.2)$$

in which $q_{\hat{v}_i \hat{v}_i}$ is the i th diagonal element of the cofactor matrix of the residuals $Q_{\hat{v}}$ and w_{ℓ_i} is the a priori weight of observation ℓ_i . Figure (6.7) illustrates the relation between number of cameras and the reliability of the x-co-ordinate photogrammetric measurements while Figure (6.8) shows that relation with respect to the y-co-ordinate. Illustrated in Figures (6.9) and (6.10) are the reliability of the x- and y- co-ordinate measurements in the combined cases respectively.

Case 1 exhibited the lowest reliability of the x-co-ordinate while reliability of the y-co-ordinate was fairly high ($r(x)=0.002$, $r(y)=0.704$). Addition of the 7 slope distances (case 2) enhanced the reliability in the x-co-ordinate by 6-fold whereas it improved the reliability in the y-co-ordinate by only 3 percent. For case 3, the enhancement over case 1 x-co-ordinate was of the factor of 224 while adding the 7 slope distances (case 4) did improve the reliability in the same direction by 5.4 percent. As for the y-co-ordinate there was a slight improvement, 3.6 percent for the purely photogrammetric

PHOTO	POINT	$r(x)$	$r(y)$
1	1	0.280156770 00	0.589677730 00
	2	0.210042040 00	0.623627290 00
	3	0.420249630-01	0.378554360 00
	4	0.402188390 00	0.655424090 00
	5	0.412651360 00	0.730075230 00
	6	0.334026210 00	0.725710400 00
	7	0.309535740 00	0.718314800 00
	8	0.230107910 00	0.671185280 00
	9	0.195502560 00	0.637585570 00
	10	0.162420810 00	0.597965670 00
	11	0.153504900 00	0.586221040 00
	12	0.313196620 00	0.715779900 00
	13	0.162214980 00	0.589893610 00
	14	0.215820060 00	0.589046460 00
	15	0.110430070 00	0.487243890 00
	16	0.751608440-01	0.432023220 00
	35	0.228243880-01	0.365345000 00
	37	0.225536340 00	0.650457830 00
	38	0.126623710 00	0.650642410 00
	41	0.218174910 00	0.604297010 00
2	1	0.215696890-01	0.219492420 00
	2	0.216679160 00	0.582106200 00
	3	0.265080980 00	0.618481410 00
	4	0.510628660-01	0.307000260 00
	5	0.751168880-01	0.386351220 00
	6	0.137538160 00	0.509185390 00
	7	0.155196820 00	0.534700540 00
	8	0.231634840 00	0.610976860 00
	9	0.275938570 00	0.639920820 00
	10	0.314979300 00	0.659383780 00
	11	0.323621460 00	0.663079690 00
	12	0.161126890 00	0.539246130 00
	13	0.314939080 00	0.660493730 00
	14	0.110400550 00	0.393708610 00
	15	0.214418190 00	0.604179330 00
	16	0.331765130 00	0.646499160 00
	34	0.320084860 00	0.673310430 00
	36	0.268315170 00	0.635457680 00
	39	0.161413120-01	0.211108940 00
	40	0.241105920 00	0.653107480 00

TABLE 6.6. Computed values of redundancy numbers for case 5.

PHOTO	POINT	$r(x)$	$r(y)$
3	17	0.126946020 00	0.557576140 00
	18	0.956308260-01	0.480935070 00
	19	0.360277580-02	0.139488920 00
	20	0.160399020 00	0.575508080 00
	21	0.149500110 00	0.536250960 00
	22	0.143025780 00	0.578886000 00
	23	0.121350600 00	0.549481820 00
	24	0.101955350 00	0.512472590 00
	25	0.843835160-01	0.471146270 00
	26	0.652324090-01	0.415282370 00
	27	0.535345610-01	0.392433200 00
	28	0.356924920-01	0.293754530 00
	29	0.144126630 00	0.580814740 00
	30	0.674113500-01	0.415888070 00
	31	0.977913870-01	0.491932670 00
	32	0.531384530-01	0.317555600 00
	33	0.227051490-01	0.206412300 00
	35	0.934852380-02	0.149996840 00
	37	0.160523400 00	0.602570520 00
	38	0.112727360 00	0.550385100 00
	41	0.106827800 00	0.535289420 00
4	17	0.680462120-01	0.381063920 00
	18	0.313778580 00	0.672938000 00
	19	0.291912020 00	0.648181520 00
	20	0.115813340 00	0.428299060 00
	21	0.243001220 00	0.617178500 00
	22	0.256375430 00	0.633971080 00
	23	0.296922620 00	0.630804810 00
	24	0.335727890 00	0.716754320 00
	25	0.374352090 00	0.743514600 00
	26	0.421553330 00	0.765839930 00
	27	0.438058070 00	0.771110910 00
	28	0.470856170 00	0.772868930 00
	29	0.256763670 00	0.624966880 00
	30	0.422437530 00	0.758173460 00
	31	0.174363430 00	0.482468120 00
	32	0.330048360 00	0.647765110 00
	33	0.443466770 00	0.699680420 00
	34	0.299289690 00	0.655336430 00
	36	0.237629690 00	0.639417470 00
	39	0.198156310-01	0.339960960 00
	40	0.272523700 00	0.618328360 00

TABLE 6.6.(Continued)

PHOTO	POINT	r(x)		r(y)	
5	1	0.174779610	00	0.423769670	00
	2	0.392683180	00	0.492318470	00
	3	0.223782830	00	0.452606710	00
	4	0.305456430	00	0.491423190	00
	5	0.360754770	00	0.549963010	00
	6	0.404989360	00	0.537928610	00
	7	0.410957310	00	0.536000320	00
	8	0.433144390	00	0.538311100	00
	9	0.443033330	00	0.544263810	00
	10	0.441919840	00	0.546875310	00
	11	0.438824660	00	0.546594600	00
	12	0.429124000	00	0.555416730	00
	13	0.448873560	00	0.551686100	00
	14	0.295026620	00	0.535344920	00
	15	0.304490060	00	0.518936180	00
	16	0.333529090	00	0.477308270	00
	34	0.362808670	00	0.588197560	00
	36	0.326649700	00	0.591684540	00
	37	0.254684620	00	0.508034390	00
	38	0.169753660	00	0.489529710	00
	40	0.170557010	00	0.512331160	00
	41	0.127768310	00	0.346553720	00
6	17	0.289602760	00	0.487775870	00
	18	0.429331180	00	0.563158160	00
	19	0.163778170	00	0.481617020	00
	20	0.413590220	00	0.507183310	00
	21	0.511333030	00	0.601513480	00
	22	0.507152490	00	0.604490060	00
	23	0.487580970	00	0.610453420	00
	24	0.464635040	00	0.612519420	00
	25	0.442630710	00	0.614747100	00
	26	0.415825930	00	0.622347960	00
	27	0.404970030	00	0.627097590	00
	28	0.354577010	00	0.642571170	00
	29	0.515913100	00	0.609691320	00
	30	0.429185440	00	0.634583610	00
	31	0.347736520	00	0.570527620	00
	32	0.340138730	00	0.601129340	00
	33	0.297275600	00	0.564716120	00
	34	0.319508570	00	0.520721600	00
	36	0.273465600	00	0.517385360	00
	37	0.388433980	00	0.631444320	00
	38	0.372503400	00	0.622060440	00
	40	0.156498170	00	0.392683030	00
	41	0.139436100	00	0.542216960	00

TABLE 6.6.(Continued)

PHOTO	POINT	$r(x)$	$r(y)$
1	1	0.333331200 00	0.624794240 00
	2	0.272764180 00	0.641723880 00
	3	0.543059940-01	0.380893910 00
	4	0.411738630 00	0.665203010 00
	5	0.419338110 00	0.731132320 00
	6	0.336415360 00	0.730070660 00
	7	0.311926940 00	0.723537290 00
	8	0.231936000 00	0.677687990 00
	9	0.196520810 00	0.642719830 00
	10	0.163152910 00	0.601061110 00
	11	0.154307380 00	0.588818380 00
	12	0.314406460 00	0.722749870 00
	13	0.163373010 00	0.596476460 00
	14	0.249367010 00	0.657091770 00
	15	0.119337760 00	0.562812120 00
	16	0.796889620-01	0.437531830 00
	35	0.266652490-01	0.426824840 00
	37	0.389375340 00	0.675916480 00
	38	0.156795640 00	0.653420460 00
	41	0.467617490 00	0.607757760 00
2	1	0.302252230-01	0.224821120 00
	2	0.260190420 00	0.602373190 00
	3	0.368768220 00	0.654065870 00
	4	0.520870100-01	0.309758470 00
	5	0.760892270-01	0.387225460 00
	6	0.139254040 00	0.519864840 00
	7	0.157002980 00	0.545737660 00
	8	0.233230710 00	0.619049660 00
	9	0.277179500 00	0.645078520 00
	10	0.316685910 00	0.662879230 00
	11	0.325721840 00	0.666491620 00
	12	0.162214220 00	0.544043530 00
	13	0.317172040 00	0.664367740 00
	14	0.133584240 00	0.430451360 00
	15	0.232148230 00	0.651016770 00
	16	0.347963380 00	0.654136290 00
	34	0.407816860 00	0.683965570 00
	36	0.286152320 00	0.667137500 00
	39	0.189201660-01	0.247453000 00
	40	0.438317630 00	0.671207730 00

TABLE 6.7. Computed values of redundancy numbers for case 6.

PHOTO	POINT	r(x)	r(y)
3	17	0.240052580 00	0.589038220 00
	18	0.137143890 00	0.498612560 00
	19	0.125310840-01	0.144762490 00
	20	0.168542210 00	0.535789600 00
	21	0.150502980 00	0.589217930 00
	22	0.143914760 00	0.582142560 00
	23	0.122504660 00	0.554739610 00
	24	0.102571400 00	0.520569590 00
	25	0.851005100-01	0.481238080 00
	26	0.660752490-01	0.424888660 00
	27	0.593668730-01	0.400795560 00
	28	0.360649700-01	0.294459270 00
	29	0.144959310 00	0.534238610 00
	30	0.679663580-01	0.420949610 00
	31	0.106660730 00	0.554587900 00
	32	0.617341390-01	0.371499440 00
	33	0.231924420-01	0.211269340 00
	35	0.109216830-01	0.175238130 00
	37	0.245785590 00	0.620822410 00
	38	0.133861690 00	0.604946760 00
	41	0.349618410 00	0.607514780 00
4	17	0.806937230-01	0.333046640 00
	18	0.351580710 00	0.681640340 00
	19	0.363990720 00	0.666362630 00
	20	0.122912600 00	0.431837440 00
	21	0.244781440 00	0.618995100 00
	22	0.258369380 00	0.635908960 00
	23	0.299548000 00	0.632968680 00
	24	0.338535260 00	0.719077090 00
	25	0.376831960 00	0.745626520 00
	26	0.423636290 00	0.767394400 00
	27	0.440250810 00	0.772467680 00
	28	0.476230590 00	0.773869420 00
	29	0.258361910 00	0.630373630 00
	30	0.424511310 00	0.763247320 00
	31	0.186817460 00	0.565383600 00
	32	0.368186900 00	0.690252270 00
	33	0.451875610 00	0.708378430 00
	34	0.393004860 00	0.671298690 00
	36	0.252963820 00	0.642377390 00
	39	0.232270460-01	0.398487890 00
	40	0.491618490 00	0.625139280 00

TABLE 6.7.(Continued)

PHOTO	POINT	r(x)		r(y)	
5	1	0.182973870	00	0.456698090	00
	2	0.440049540	00	0.538030990	00
	3	0.252537030	00	0.478080230	00
	4	0.312315510	00	0.498669970	00
	5	0.366325470	00	0.550526380	00
	6	0.408264970	00	0.558847160	00
	7	0.414190480	00	0.558472170	00
	8	0.435902590	00	0.555468760	00
	9	0.444976870	00	0.553887470	00
	10	0.444090340	00	0.550149710	00
	11	0.441415000	00	0.548663300	00
	12	0.431091530	00	0.564944500	00
	13	0.452099970	00	0.552832670	00
	14	0.360109330	00	0.560194910	00
	15	0.330501200	00	0.542115920	00
	16	0.352278670	00	0.494440520	00
	34	0.404170210	00	0.596689310	00
	36	0.347853830	00	0.604449610	00
	37	0.361146880	00	0.546862600	00
	38	0.207192140	00	0.543809970	00
	40	0.305317720	00	0.520006160	00
	41	0.258269680	00	0.361388490	00
6	17	0.334869930	00	0.507944860	00
	18	0.486481320	00	0.605501300	00
	19	0.179055880	00	0.525955210	00
	20	0.437812030	00	0.521940560	00
	21	0.515119920	00	0.603670730	00
	22	0.510923740	00	0.607895900	00
	23	0.491178030	00	0.619292140	00
	24	0.467837780	00	0.627939770	00
	25	0.445552520	00	0.635258610	00
	26	0.419062440	00	0.643438210	00
	27	0.408492120	00	0.646021910	00
	28	0.358582620	00	0.643518900	00
	29	0.518944030	00	0.610133990	00
	30	0.431722430	00	0.642001300	00
	31	0.379977380	00	0.596323010	00
	32	0.398592310	00	0.632425590	00
	33	0.303798800	00	0.574565280	00
	34	0.377830710	00	0.544677390	00
	36	0.290747590	00	0.547902370	00
	37	0.455501400	00	0.641372520	00
	38	0.422137130	00	0.639614820	00
	40	0.264294940	00	0.403699170	00
	41	0.391346320	00	0.549578160	00

TABLE 6.7. (Continued)

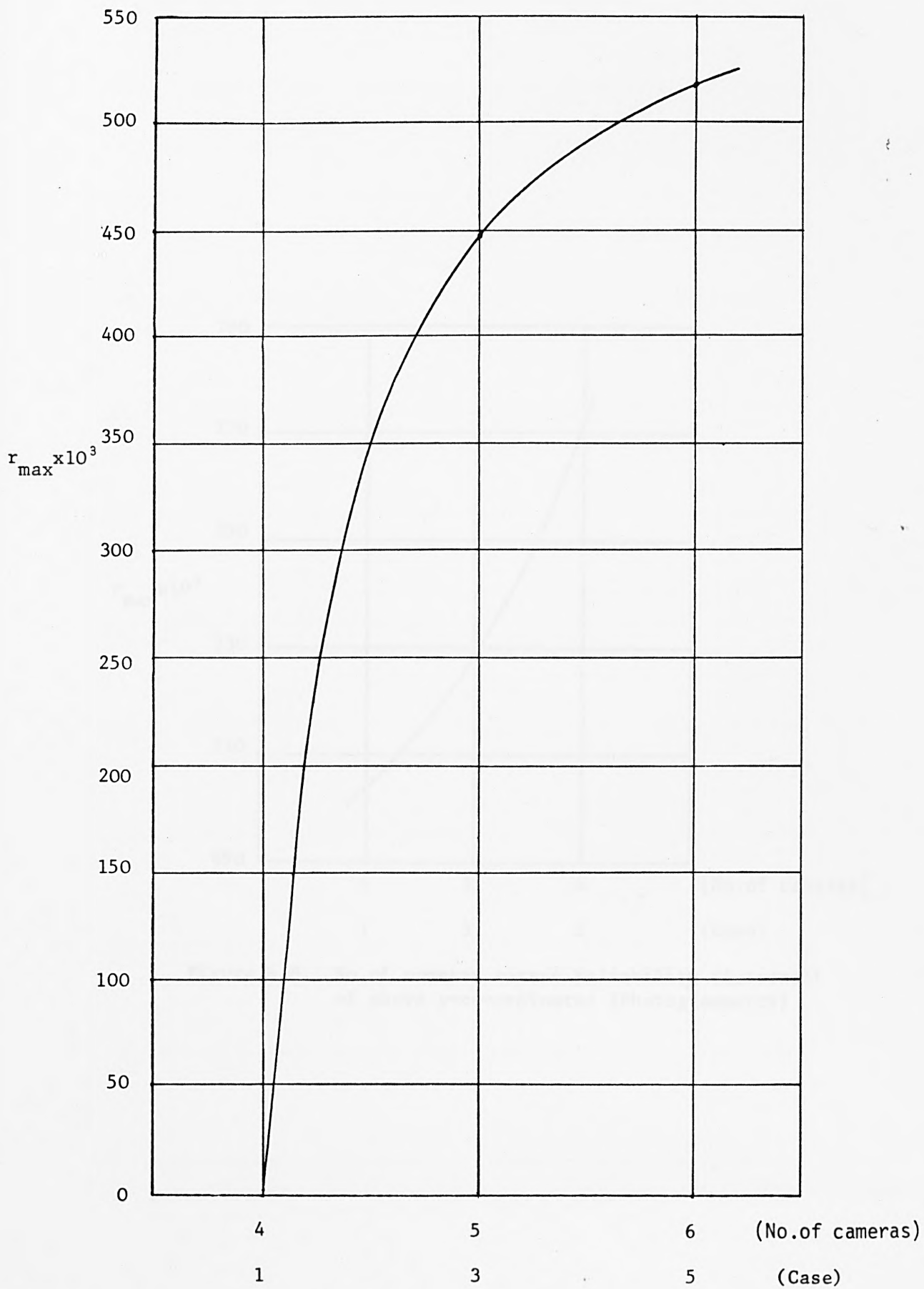


Figure 6.7. No. of cameras versus Reliability (internal) of photo x-co-ordinates (Photogrammetry)

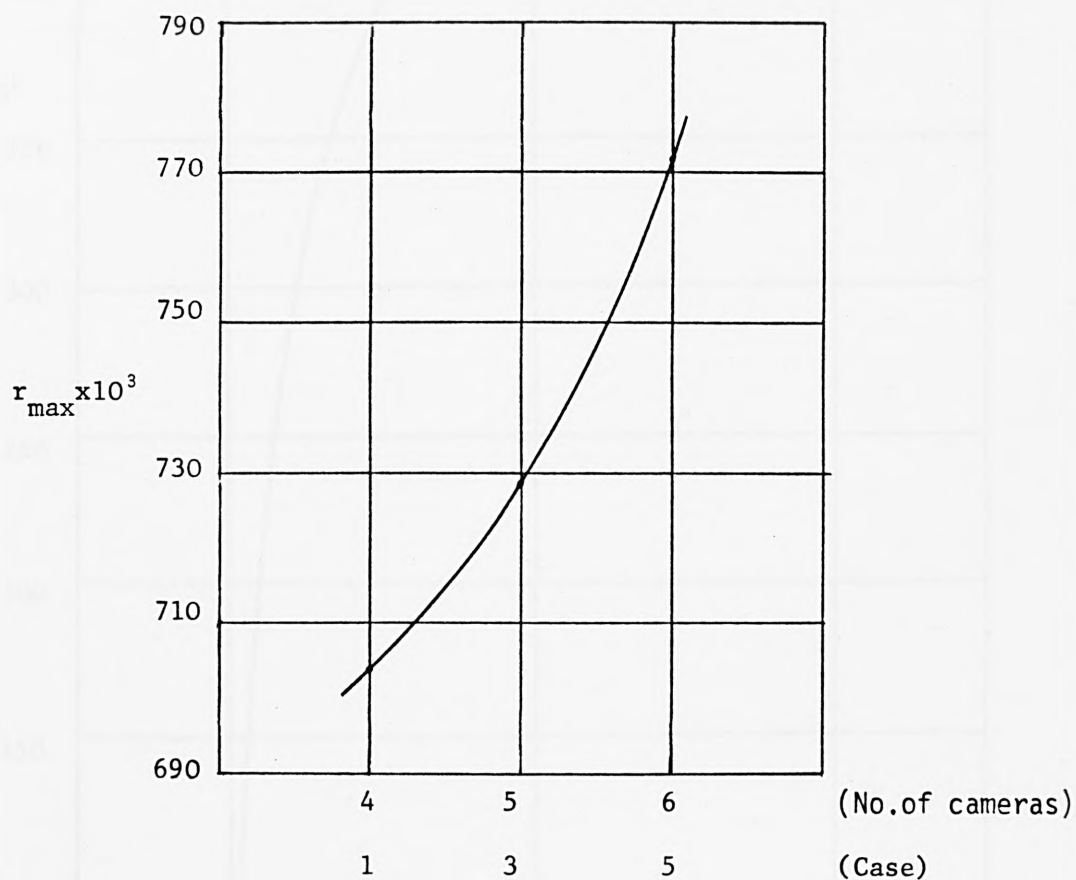


Figure 6.8. No. of cameras versus Reliability (internal) of photo y-co-ordinates (Photogrammetry)

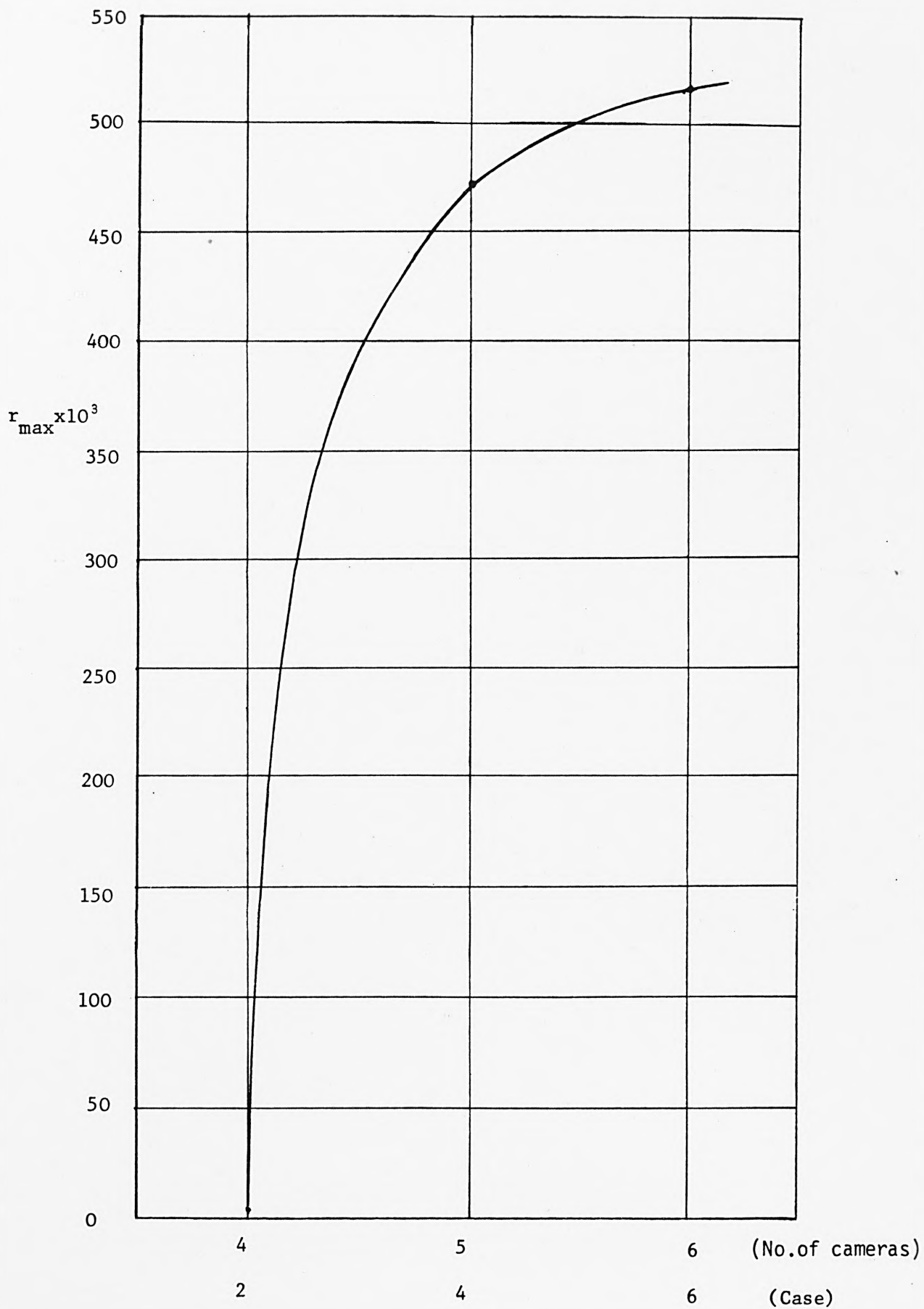


Figure 6.9. No. of cameras versus Reliability (internal) of photo x-co-ordinates (Photos + 7 distances)

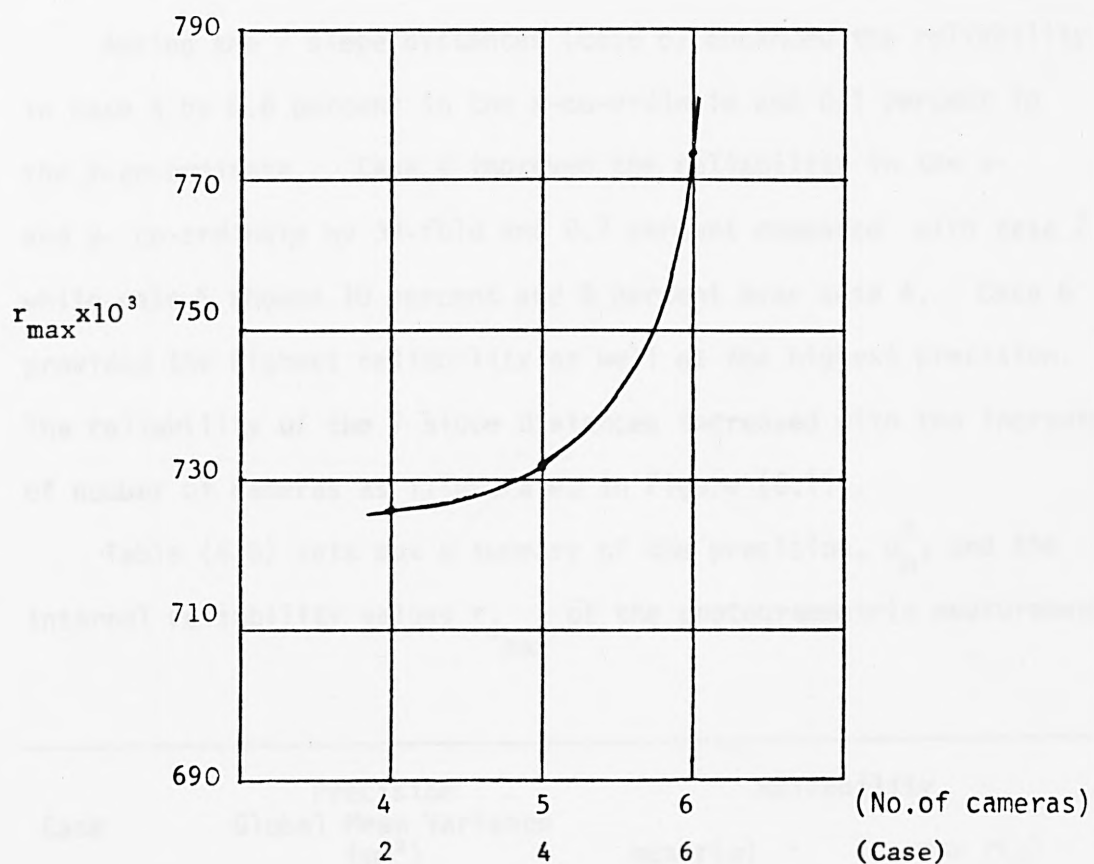


Figure 6.10. No. of cameras versus Reliability (internal) of photo y-co-ordinates (Photos + 7 distances)

measurements while only 0.3 percent for the combined case (case 4). 6 percent improvement was accomplished in both the x- and y-co-ordinate owing to the addition of the sixth camera (case 5) compared with case 3.

Adding the 7 slope distances (case 6) enhanced the reliability in case 5 by 0.6 percent in the x-co-ordinate and 0.1 percent in the y-co-ordinate. Case 4 improved the reliability in the x- and y- co-ordinate by 39-fold and 0.7 percent compared with case 2 while case 6 showed 10 percent and 6 percent over case 4. Case 6 provided the highest reliability as well as the highest precision. The reliability of the 7 slope distances increased with the increase of number of cameras as illustrated in Figure (6.11).

Table (6.8) sets out a summary of the precision, σ_m^2 , and the internal reliability values $r_{i_{\max}}$ of the photogrammetric measurements.

Case	Precision Global Mean Variance (mm ²)	Reliability	
		max $r(x)$	max $r(y)$
1-4 cameras	2312988.800000	0.002028	0.704284
2-4 cameras+ 7 distances	76.306048	0.011553	0.726046
3-5 cameras	22.382190	0.447481	0.728521
4-5 cameras+ 7 distances	4.887012	0.472039	0.730573
5-6 cameras	6.859595	0.515913	0.772869
6-6 cameras+ 7 distances	2.644175	0.518944	0.773869

TABLE (6.8) Summary of precision and reliability of photogrammetric measurements.

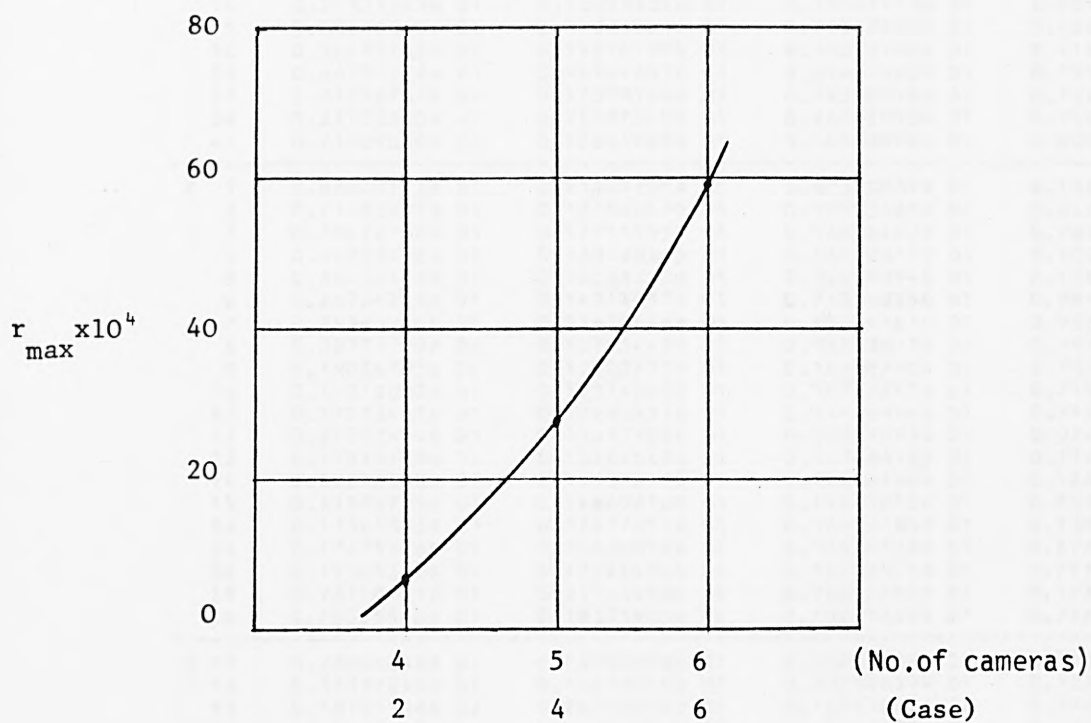


Figure 6.11. No. of cameras versus Reliability (internal) of 7 slope distances.

PHO PT.		TAU		GAM	
NO	NO	x	y	x	y
1	1	0.138929350 01	0.130224480 01	0.160294420 01	0.834171170 00
	2	0.218196050 01	0.126630240 01	0.193931730 01	0.776866700 00
	3	0.487205090 01	0.162530880 01	0.477445080 01	0.128126050 01
	4	0.157683130 01	0.123520430 01	0.121917880 01	0.725072200 00
	5	0.155671230 01	0.117035120 01	0.119304370 01	0.608047570 00
	6	0.173025350 01	0.117386550 01	0.141201170 01	0.614784620 00
	7	0.179739740 01	0.117989290 01	0.149353430 01	0.626213650 00
	8	0.208465520 01	0.122061520 01	0.182914930 01	0.699929690 00
	9	0.226164160 01	0.125236450 01	0.202855190 01	0.753934300 00
	10	0.248129940 01	0.129318860 01	0.227086910 01	0.819961470 00
	11	0.255234210 01	0.130607860 01	0.234828660 01	0.840143550 00
	12	0.172686390 01	0.118198030 01	0.148083840 01	0.630140860 00
	13	0.243287310 01	0.130200650 01	0.227258860 01	0.833799100 00
	14	0.215255420 01	0.130294240 01	0.190617150 01	0.835259820 00
	15	0.300923650 01	0.143260610 01	0.263822200 01	0.102584610 01
	16	0.364757450 01	0.152141070 01	0.350781980 01	0.114659950 01
	35	0.661912270 01	0.165443010 01	0.654314800 01	0.131800560 01
	37	0.210567690 01	0.123991080 01	0.185307180 01	0.733061170 00
	38	0.281023400 01	0.123973490 01	0.262629300 01	0.732763530 00
	41	0.214090600 01	0.128639630 01	0.189300780 01	0.809206650 00
2	1	0.680391730 01	0.213447090 01	0.673508390 01	0.188572690 01
	2	0.214823270 01	0.131066670 01	0.190134650 01	0.847289550 00
	3	0.194227500 01	0.127155950 01	0.166506220 01	0.785406560 00
	4	0.442534780 01	0.180480640 01	0.431088190 01	0.150244010 01
	5	0.364364160 01	0.160882520 01	0.350892940 01	0.126028510 01
	6	0.269642540 01	0.140139970 01	0.250413850 01	0.981794910 00
	7	0.253839140 01	0.136755460 01	0.233311610 01	0.932848160 00
	8	0.207777280 01	0.127934430 01	0.182130170 01	0.797949380 00
	9	0.190367930 01	0.125007730 01	0.161987500 01	0.750128870 00
	10	0.173130020 01	0.123148990 01	0.147472430 01	0.718726300 00
	11	0.175784810 01	0.122805310 01	0.144569360 01	0.712821410 00
	12	0.249124240 01	0.136177850 01	0.228172930 01	0.924359640 00
	13	0.178191390 01	0.123045480 01	0.147486180 01	0.716951120 00
	14	0.300963380 01	0.159372190 01	0.283864860 01	0.124094700 01
	15	0.215957950 01	0.128652160 01	0.191410120 01	0.809405780 00
	16	0.173613950 01	0.124370110 01	0.141921830 01	0.739454140 00
	34	0.176753260 01	0.121868740 01	0.145745380 01	0.696562290 00
	36	0.193053370 01	0.125445960 01	0.165135110 01	0.757409350 00
	39	0.737101210 01	0.217643990 01	0.780722950 01	0.193310390 01
	40	0.203655460 01	0.123739300 01	0.177413490 01	0.728794590 00
3	17	0.230666420 01	0.133920760 01	0.262247280 01	0.890773270 00
	18	0.323370980 01	0.144197180 01	0.307520390 01	0.103888540 01
	19	0.107815380 02	0.267750410 01	0.107350620 02	0.248375280 01
	20	0.249633850 01	0.131817870 01	0.226789250 01	0.858833550 00
	21	0.258630200 01	0.130604520 01	0.238515370 01	0.840091750 00
	22	0.264419110 01	0.131432710 01	0.244780450 01	0.852910220 00
	23	0.286474630 01	0.134903540 01	0.268454300 01	0.905481320 00
	24	0.313180700 01	0.139689790 01	0.296786370 01	0.975358330 00
	25	0.344247820 01	0.145687440 01	0.329403340 01	0.105947300 01
	26	0.391532930 01	0.155177320 01	0.378547270 01	0.118659180 01
	27	0.413327040 01	0.159630960 01	0.401047680 01	0.124426860 01
	28	0.529311780 01	0.184504820 01	0.519779730 01	0.155054920 01
	29	0.263407340 01	0.131214300 01	0.243687150 01	0.849540690 00
	30	0.385153130 01	0.155064280 01	0.371944850 01	0.118511310 01
	31	0.319773820 01	0.142576240 01	0.303740830 01	0.101626700 01
	32	0.433205990 01	0.177455760 01	0.422122780 01	0.146596550 01
	33	0.663648050 01	0.220106170 01	0.656070680 01	0.196078370 01
	35	0.103425700 02	0.258201610 01	0.102941130 02	0.238050560 01
	37	0.249592090 01	0.128823790 01	0.228683650 01	0.612131020 00
	38	0.297841590 01	0.134731610 01	0.280552330 01	0.902917810 00
	41	0.305955220 01	0.130711760 01	0.289151510 01	0.841757930 00

TABLE 6.9. Values of Tau and Gam for Case 5.
(at $\alpha = 0.05$ and $\gamma = 0.8$)

PHO PL.	TAU		GAM	
NO NO	X	Y	X	Y
4 17	0.38352260 01	0.161994800 01	0.370079660 01	0.127445350 01
18	0.178520610 01	0.121902460 01	0.147883760 01	0.697152060 00
19	0.105086190 01	0.124206600 01	0.1557746260 01	0.736734490 00
20	0.293846620 01	0.1528001090 01	0.276307500 01	0.115534290 01
21	0.202859700 01	0.127294090 01	0.176499450 01	0.787376530 00
22	0.197497590 01	0.125592950 01	0.170309420 01	0.759841450 00
23	0.183517870 01	0.121196610 01	0.153879200 01	0.664726070 00
24	0.172586290 01	0.118117660 01	0.140662810 01	0.628632000 00
25	0.163440570 01	0.115972560 01	0.129278070 01	0.587531970 00
26	0.154018790 01	0.112426960 01	0.117140040 01	0.552951970 00
27	0.151039460 01	0.113878460 01	0.113260870 01	0.544821350 00
28	0.145732320 01	0.113748870 01	0.106009000 01	0.542107410 00
29	0.197348220 01	0.126494460 01	0.170136180 01	0.774651400 00
30	0.153344420 01	0.114845960 01	0.116915920 01	0.564764900 00
31	0.239481680 01	0.143567910 01	0.217603940 01	0.103370060 01
32	0.174064900 01	0.124248520 01	0.142473120 01	0.737407260 00
33	0.150165260 01	0.119950150 01	0.112025020 01	0.655151830 00
34	0.182790710 01	0.123528690 01	0.153011250 01	0.725212920 00
36	0.205139670 01	0.125056930 01	0.179115280 01	0.750948400 00
39	0.710384700 01	0.175084300 01	0.703315080 01	0.139338230 01
40	0.191556930 01	0.127171680 01	0.163383160 01	0.785661310 00
5 1	0.239196390 01	0.153615510 01	0.217288930 01	0.116609280 01
2	0.159580140 01	0.142444800 01	0.124361660 01	0.101446770 01
3	0.209068350 01	0.148641300 01	0.183601670 01	0.109973800 01
4	0.180936160 01	0.142650130 01	0.150790880 01	0.101733030 01
5	0.166492230 01	0.134844510 01	0.133115220 01	0.904601630 00
6	0.157136910 01	0.135344520 01	0.121210590 01	0.926813210 00
7	0.155991760 01	0.136589550 01	0.119722290 01	0.930414170 00
8	0.151944030 01	0.136296070 01	0.114398350 01	0.926100350 00
9	0.150238690 01	0.135548670 01	0.112123430 01	0.915065180 00
10	0.150427850 01	0.135224640 01	0.112376770 01	0.910258420 00
11	0.150975730 01	0.135259360 01	0.113084680 01	0.910774110 00
12	0.152654140 01	0.1313180840 01	0.115339880 01	0.894678660 00
13	0.149253130 01	0.134633760 01	0.110806090 01	0.901457140 00
14	0.134106620 01	0.136673130 01	0.154580870 01	0.931640800 00
15	0.181223050 01	0.138810430 01	0.151135020 01	0.962721970 00
16	0.173154240 01	0.144743980 01	0.141359090 01	0.104646170 01
34	0.166020290 01	0.130338230 01	0.132524480 01	0.836725210 00
36	0.174963100 01	0.130003450 01	0.143575190 01	0.830715420 00
37	0.179152030 01	0.1429798630 01	0.171067950 01	0.984058280 00
38	0.242711540 01	0.142925750 01	0.221153550 01	0.102116450 01
40	0.242139260 01	0.139709070 01	0.220525530 01	0.975634440 00
41	0.279761260 01	0.168669230 01	0.261278320 01	0.137315530 01
6 17	0.185822650 01	0.143182470 01	0.156620750 01	0.102475460 01
18	0.152617310 01	0.133255400 01	0.115291120 01	0.880738390 00
19	0.247097570 01	0.146095060 01	0.225960620 01	0.103746740 01
20	0.155494440 01	0.140416300 01	0.119073600 01	0.985735110 00
21	0.139845360 01	0.128936930 01	0.977585080 00	0.813524530 00
22	0.140420570 01	0.128619090 01	0.985795890 00	0.808860050 00
23	0.143211080 01	0.127989320 01	0.102515430 01	0.798828280 00
24	0.146704700 01	0.127773290 01	0.107341830 01	0.79562370 00
25	0.150307010 01	0.127541570 01	0.112214960 01	0.791634500 00
26	0.155075370 01	0.126760330 01	0.118526470 01	0.778655340 00
27	0.157140650 01	0.126279380 01	0.121215450 01	0.771134320 00
28	0.167936350 01	0.124749060 01	0.134917080 01	0.745820260 00
29	0.137223240 01	0.128069290 01	0.968664510 00	0.800108390 00
30	0.152643220 01	0.125532330 01	0.115325420 01	0.758838900 00
31	0.169580060 01	0.133391980 01	0.136957680 01	0.867619450 00
32	0.171463610 01	0.128978120 01	0.139283050 01	0.814576880 00
33	0.183403880 01	0.133071460 01	0.153749210 01	0.877952860 00
34	0.176912590 01	0.135578930 01	0.145938570 01	0.959281040 00
36	0.191226750 01	0.139025010 01	0.162995920 01	0.965813300 00
37	0.160450620 01	0.128434390 01	0.125476650 01	0.763503560 00
38	0.163846640 01	0.129789620 01	0.129789800 01	0.77961880 00
40	0.252781550 01	0.159580170 01	0.232160330 01	0.124361700 01
41	0.267801120 01	0.135804280 01	0.248429950 01	0.918847220 00

TABLE 6.9 (Continued)

PHO PT.		TAU				GAM			
NO	NO	x		y		x		y	
1	1	0.173205640	01	0.126511930	01	0.141422040	01	0.774936710	00
	2	0.191472470	01	0.124831990	01	0.163284130	01	0.747196510	00
	3	0.429117390	01	0.162030950	01	0.417302930	01	0.127491290	01
	4	0.155343680	01	0.122609150	01	0.119529290	01	0.709436730	00
	5	0.154425030	01	0.116950480	01	0.117673720	01	0.606416910	00
	6	0.172409860	01	0.117035480	01	0.140446290	01	0.608054640	00
	7	0.179049680	01	0.117562700	01	0.148522010	01	0.618141390	00
	8	0.207642340	01	0.121474500	01	0.181976210	01	0.689641440	00
	9	0.225577470	01	0.124735230	01	0.202200880	01	0.745578890	00
	10	0.247572600	01	0.123985440	01	0.226477800	01	0.814692790	00
	11	0.254569670	01	0.130319480	01	0.234106210	01	0.835653370	00
	12	0.173342260	01	0.117626720	01	0.147668420	01	0.619358170	00
	13	0.247415730	01	0.129480200	01	0.226295430	01	0.822503560	00
	14	0.200253680	01	0.123363590	01	0.173497940	01	0.722397020	00
	15	0.289475000	01	0.133296360	01	0.271653780	01	0.881357970	00
	16	0.354242700	01	0.151180290	01	0.339835100	01	0.113382010	01
	35	0.612393720	01	0.153064740	01	0.604168800	01	0.115892760	01
	37	0.160256550	01	0.121633580	01	0.125228430	01	0.692439700	00
	38	0.252541640	01	0.123709670	01	0.231899290	01	0.728271260	00
	41	0.146236110	01	0.128272850	01	0.106700520	01	0.803363190	00
2	1	0.575195180	01	0.210992360	01	0.566435790	01	0.185687390	01
	2	0.196044360	01	0.128844350	01	0.168622030	01	0.812457180	00
	3	0.164673330	01	0.123648620	01	0.130833120	01	0.727253740	00
	4	0.438162500	01	0.179675310	01	0.426598700	01	0.149275640	01
	5	0.362525330	01	0.160700800	01	0.348460400	01	0.125796460	01
	6	0.267976120	01	0.133695070	01	0.248618580	01	0.961029080	00
	7	0.252374840	01	0.135365510	01	0.231717630	01	0.912349860	00
	8	0.207965210	01	0.127097570	01	0.181317400	01	0.784461150	00
	9	0.189941320	01	0.124506980	01	0.161485930	01	0.741753920	00
	10	0.177692260	01	0.122823870	01	0.146391210	01	0.713141230	00
	11	0.175217130	01	0.122490570	01	0.143878570	01	0.707385290	00
	12	0.243237890	01	0.135575490	01	0.227259490	01	0.915462350	00
	13	0.177563030	01	0.122686200	01	0.146726380	01	0.710767520	00
	14	0.273603960	01	0.144269750	01	0.254674550	01	0.103989240	01
	15	0.207547410	01	0.123937840	01	0.181867880	01	0.732160330	00
	16	0.167524670	01	0.123637230	01	0.136889060	01	0.727060220	00
	34	0.156591220	01	0.120915750	01	0.120502330	01	0.679751360	00
	36	0.186939620	01	0.122431260	01	0.157944370	01	0.706357820	00
	39	0.727005230	01	0.201026650	01	0.720094860	01	0.174349550	01
	40	0.143103020	01	0.122059480	01	0.102364420	01	0.699694080	00
3	17	0.204191790	01	0.130295150	01	0.177925660	01	0.835274030	00
	18	0.270029350	01	0.141617980	01	0.250830860	01	0.100277870	01
	19	0.893317150	01	0.262828210	01	0.887702390	01	0.243061040	01
	20	0.243582260	01	0.130655940	01	0.222108800	01	0.840890940	00
	21	0.257767030	01	0.130275280	01	0.237579180	01	0.834964020	00
	22	0.263601170	01	0.131064580	01	0.243896650	01	0.847226240	00
	23	0.285703360	01	0.134262710	01	0.267636980	01	0.895905990	00
	24	0.312238790	01	0.138599160	01	0.295792260	01	0.959673260	00
	25	0.342774560	01	0.144151780	01	0.327884290	01	0.103825510	01
	26	0.389027760	01	0.153413100	01	0.375955580	01	0.116342500	01
	27	0.410419430	01	0.157956880	01	0.398050380	01	0.122271730	01
	28	0.526571330	01	0.184233900	01	0.516988750	01	0.154791970	01
	29	0.262647720	01	0.130829260	01	0.242868020	01	0.843581310	00
	30	0.393577390	01	0.154129210	01	0.370312860	01	0.117285180	01
	31	0.306194730	01	0.134261070	01	0.289404930	01	0.896181160	00
	32	0.402473510	01	0.164066880	01	0.389852440	01	0.130068980	01
	33	0.656639130	01	0.217561360	01	0.648979930	01	0.193217350	01
	35	0.956375030	01	0.233383250	01	0.951635340	01	0.216945170	01
	37	0.201707380	01	0.126915980	01	0.175173820	01	0.781515570	00
	38	0.273320230	01	0.128570530	01	0.254369760	01	0.808107690	00
	41	0.169123070	01	0.128298500	01	0.136391400	01	0.803772660	00

TABLE 6.10. Values of Tau and Gam for Case 6.
(at $\alpha = 0.05$ and $\gamma = 0.8$)

PHO PT.		TAU		GAM	
NO	NO	x	y	x	y
4	17	0.352030360 01	0.161575000 01	0.337528330 01	0.126911310 01
	18	0.168650440 01	0.121121810 01	0.135804900 01	0.683410060 00
	19	0.165750500 01	0.122502420 01	0.132186340 01	0.707590550 00
	20	0.285234330 01	0.152173790 01	0.267130350 01	0.114703360 01
	21	0.202120680 01	0.127103170 01	0.175649570 01	0.784551890 00
	22	0.196734030 01	0.125401440 01	0.169423370 01	0.756671760 00
	23	0.182711880 01	0.121003970 01	0.152917070 01	0.681319300 00
	24	0.171859200 01	0.117926730 01	0.139782050 01	0.625037180 00
	25	0.162931900 01	0.115808200 01	0.128596370 01	0.584083910 00
	26	0.153639690 01	0.114153880 01	0.116641140 01	0.550555070 00
	27	0.150712720 01	0.113778410 01	0.112757820 01	0.542726950 00
	28	0.144907660 01	0.113675310 01	0.104872450 01	0.540562360 00
	29	0.196736370 01	0.125950810 01	0.169426670 01	0.765741970 00
	30	0.153491260 01	0.114463590 01	0.116432380 01	0.556948200 00
	31	0.231361600 01	0.132992880 01	0.208634100 01	0.876761460 00
	32	0.164803270 01	0.120363850 01	0.130996640 01	0.669884840 00
	33	0.143761500 01	0.118813920 01	0.110136210 01	0.641618910 00
	34	0.158509690 01	0.122051210 01	0.122985050 01	0.699749860 00
	36	0.198324910 01	0.124768480 01	0.171346860 01	0.746134910 00
	39	0.656149820 01	0.158413590 01	0.648484840 01	0.122861160 01
	40	0.142621790 01	0.126477010 01	0.101690590 01	0.774366530 00
5	1	0.233778980 01	0.147774000 01	0.211311650 01	0.109070180 01
	2	0.150747190 01	0.136331540 01	0.112803870 01	0.926622360 00
	3	0.198922850 01	0.144627080 01	0.172041140 01	0.104484410 01
	4	0.173938270 01	0.141609830 01	0.148387680 01	0.100266360 01
	5	0.165221460 01	0.134775490 01	0.131522350 01	0.903572570 00
	6	0.156575260 01	0.133768380 01	0.120390600 01	0.888480740 00
	7	0.155311730 01	0.133813290 01	0.118926370 01	0.889156640 00
	8	0.151462550 01	0.134174560 01	0.113758100 01	0.894584420 00
	9	0.149910230 01	0.134365950 01	0.111682940 01	0.897452440 00
	10	0.15005790 01	0.134321620 01	0.111883600 01	0.904260490 00
	11	0.150513950 01	0.135004130 01	0.112491860 01	0.906979290 00
	12	0.152305380 01	0.133044560 01	0.114877890 01	0.877545090 00
	13	0.143724580 01	0.134474070 01	0.110086340 01	0.899369560 00
	14	0.165641360 01	0.133607370 01	0.133301700 01	0.886054720 00
	15	0.173745610 01	0.135816930 01	0.142327360 01	0.919034230 00
	16	0.163433290 01	0.142214200 01	0.135597260 01	0.101118150 01
	34	0.157296060 01	0.129457100 01	0.121416850 01	0.822139920 00
	36	0.169551470 01	0.123623390 01	0.136922270 01	0.808948470 00
	37	0.166401820 01	0.135226210 01	0.133002120 01	0.910281760 00
	38	0.217691550 01	0.135605220 01	0.195612830 01	0.915902650 00
	40	0.180977250 01	0.138674230 01	0.150840200 01	0.960757080 00
	41	0.196772000 01	0.166346180 01	0.169467450 01	0.132932510 01
6	17	0.172807240 01	0.140311000 01	0.140933820 01	0.984234550 00
	18	0.143372350 01	0.128511640 01	0.102741300 01	0.807170440 00
	19	0.236322450 01	0.137887730 01	0.214122600 01	0.949369570 00
	20	0.151131900 01	0.138417010 01	0.113317480 01	0.957040750 00
	21	0.139330380 01	0.128706340 01	0.970203860 00	0.810266740 00
	22	0.139901370 01	0.128258270 01	0.978386040 00	0.803130440 00
	23	0.142635730 01	0.127072690 01	0.101780240 01	0.784057900 00
	24	0.146201680 01	0.126194670 01	0.106653320 01	0.769746340 00
	25	0.149813360 01	0.125465610 01	0.111552870 01	0.757734820 00
	26	0.154475860 01	0.124665580 01	0.117740360 01	0.744413050 00
	27	0.156461740 01	0.124416040 01	0.120334020 01	0.740226390 00
	28	0.160975740 01	0.124657770 01	0.133744440 01	0.744282150 00
	29	0.138316070 01	0.128022820 01	0.962803240 00	0.799364910 00
	30	0.152194060 01	0.124805020 01	0.114730250 01	0.746745770 00
	31	0.162226250 01	0.129496850 01	0.127739410 01	0.822765770 00
	32	0.158392840 01	0.125746320 01	0.122834410 01	0.762373720 00
	33	0.181429110 01	0.131925970 01	0.151382040 01	0.960491880 00
	34	0.162636450 01	0.135497200 01	0.128323340 01	0.914302560 00
	36	0.185456450 01	0.135097840 01	0.156186090 01	0.908373650 00
	37	0.146168250 01	0.124866180 01	0.109333570 01	0.747767540 00
	38	0.153912260 01	0.125037630 01	0.116999930 01	0.750627030 00
	40	0.194516110 01	0.157387800 01	0.166842790 01	0.121535680 01
	41	0.159852480 01	0.134891710 01	0.124710920 01	0.905305150 00

TABLE 6.10 (Continued)

PHOTO	POINT	TAU		DELTA (um)	
		x	y	x	y
1	1	0.138929350 01	0.130224480 01	0.15870 02	0.10940 02
	2	0.218196050 01	0.126630240 01	0.18330 02	0.10640 02
	3	0.437805090 01	0.162530880 01	0.40980 02	0.13650 02
	4	0.157683130 01	0.123520430 01	0.13250 02	0.10380 02
	5	0.155671230 01	0.117035120 01	0.13080 02	0.98310 01
	6	0.173025350 01	0.117386550 01	0.14530 02	0.98600 01
	7	0.179739940 01	0.117989290 01	0.15100 02	0.99110 01
	8	0.208465520 01	0.122061520 01	0.17510 02	0.10250 02
	9	0.226164160 01	0.125236450 01	0.19000 02	0.10520 02
	10	0.248129940 01	0.129318860 01	0.20840 02	0.13860 02
	11	0.255224210 01	0.130607860 01	0.21440 02	0.10970 02
	12	0.178686390 01	0.118198030 01	0.15010 02	0.99290 01
	13	0.243237310 01	0.130200650 01	0.20860 02	0.10940 02
	14	0.215255420 01	0.130294240 01	0.18080 02	0.13940 02
	15	0.300923650 01	0.143260610 01	0.25280 02	0.12030 02
	16	0.364757450 01	0.152141070 01	0.30640 02	0.12780 02
	35	0.661912270 01	0.165443010 01	0.55600 02	0.13900 02
	37	0.210567690 01	0.123991080 01	0.17690 02	0.13420 02
	38	0.231023400 01	0.123973490 01	0.23610 02	0.10410 02
	41	0.214090600 01	0.128639630 01	0.17980 02	0.10810 02
2	1	0.630891730 01	0.213447090 01	0.57190 02	0.17930 02
	2	0.214828270 01	0.131068670 01	0.18050 02	0.11010 02
	3	0.194227500 01	0.127155950 01	0.16320 02	0.10680 02
	4	0.442534760 01	0.180480640 01	0.37170 02	0.15160 02
	5	0.364864160 01	0.160882520 01	0.30650 02	0.13510 02
	6	0.269642540 01	0.140139970 01	0.22650 02	0.11770 02
	7	0.253839140 01	0.136755460 01	0.21320 02	0.11490 02
	8	0.207777280 01	0.127934480 01	0.17450 02	0.10750 02
	9	0.190367930 01	0.125007730 01	0.15990 02	0.13500 02
	10	0.178180020 01	0.123148990 01	0.14970 02	0.10340 02
	11	0.175734810 01	0.122805310 01	0.14770 02	0.10320 02
	12	0.249124240 01	0.136177850 01	0.20930 02	0.11440 02
	13	0.178191390 01	0.123045480 01	0.14970 02	0.10340 02
	14	0.300963880 01	0.159372190 01	0.25280 02	0.13390 02
	15	0.215957950 01	0.128657160 01	0.18140 02	0.10810 02
	16	0.173613950 01	0.124370110 01	0.14580 02	0.10450 02
	34	0.176753260 01	0.121868740 01	0.14850 02	0.10240 02
	36	0.193053370 01	0.125445960 01	0.16220 02	0.10540 02
	39	0.787101210 01	0.217643990 01	0.66120 02	0.18280 02
	40	0.203655460 01	0.123739300 01	0.17110 02	0.10390 02
3	17	0.230666420 01	0.133920760 01	0.23580 02	0.11250 02
	18	0.323370980 01	0.144197180 01	0.27160 02	0.12110 02
	19	0.107815380 02	0.267750410 01	0.90560 02	0.22490 02
	20	0.249688050 01	0.131817870 01	0.20970 02	0.11070 02
	21	0.258630200 01	0.130604520 01	0.21720 02	0.10970 02
	22	0.264419110 01	0.131432710 01	0.22210 02	0.11040 02
	23	0.236474630 01	0.134903540 01	0.24060 02	0.11330 02
	24	0.313180700 01	0.139689790 01	0.26310 02	0.11730 02
	25	0.344247820 01	0.145687440 01	0.28920 02	0.12240 02
	26	0.391532930 01	0.155177320 01	0.32890 02	0.13030 02
	27	0.413327040 01	0.159630960 01	0.34720 02	0.13410 02
	28	0.529311780 01	0.184504820 01	0.44460 02	0.15500 02
	29	0.263407340 01	0.131214300 01	0.22130 02	0.11020 02
	30	0.385153180 01	0.155064280 01	0.32350 02	0.13030 02
	31	0.319778820 01	0.142576240 01	0.26860 02	0.11980 02
	32	0.433805990 01	0.177455760 01	0.36440 02	0.14910 02
	33	0.663648050 01	0.220106170 01	0.55750 02	0.18490 02
	35	0.103425700 02	0.258201610 01	0.86880 02	0.21690 02
	37	0.249592090 01	0.128823790 01	0.20970 02	0.10820 02
	38	0.297841590 01	0.134731610 01	0.25020 02	0.11320 02
	41	0.305955220 01	0.130711760 01	0.25700 02	0.10980 02

TABLE 6.11. Undetected Gross Errors for case 5.
(at $\alpha = 0.05$ and $\gamma = 0.8$)

PHOTO	POINT	TAU		DELTA(um)	
		x	y	x	y
4	NO				
	HO				
		x	y	x	y
	17	0.38335226	01	0.16199480	01
	18	0.17852061	01	0.12191624	01
	19	0.18506619	01	0.12420186	01
	20	0.29384662	01	0.15280109	01
	21	0.20285970	01	0.12729009	01
	22	0.19749755	01	0.12559295	01
	23	0.18351787	01	0.12119611	01
	24	0.17258629	01	0.11811766	01
	25	0.16344057	01	0.11597256	01
	26	0.15401879	01	0.11426968	01
	27	0.15108946	01	0.11367846	01
	28	0.14573232	01	0.11374887	01
	29	0.19734822	01	0.12649446	01
	30	0.15384642	01	0.11484596	01
	31	0.23948168	01	0.14396791	01
	32	0.17406490	01	0.12424852	01
	33	0.15016526	01	0.11955015	01
	34	0.18279071	01	0.12352869	01
	36	0.20513967	01	0.12505693	01
	39	0.71038870	01	0.17150843	01
	40	0.19155693	01	0.12717168	01
5		x	y	x	y
	1	0.23919639	01	0.15361551	01
	2	0.15958014	01	0.14244805	01
	3	0.20906835	01	0.14864130	01
	4	0.13093616	01	0.14265013	01
	5	0.16649223	01	0.13484451	01
	6	0.15713691	01	0.13634452	01
	7	0.15599176	01	0.13658955	01
	8	0.15194403	01	0.13629607	01
	9	0.15023869	01	0.13554867	01
	10	0.15042785	01	0.13522464	01
	11	0.15095743	01	0.13525936	01
	12	0.15265414	01	0.13416084	01
	13	0.14923313	01	0.13463376	01
	14	0.18410662	01	0.13667313	01
	15	0.18122305	01	0.13881043	01
	16	0.17315424	01	0.14474398	01
	34	0.16602029	01	0.13038823	01
	36	0.17496810	01	0.13000345	01
	37	0.19815208	01	0.14029683	01
	38	0.24271154	01	0.14292575	01
	40	0.24213926	01	0.13970907	01
	41	0.27976126	01	0.16986923	01
6		x	y	x	y
	17	0.18582265	01	0.14318247	01
	18	0.15261731	01	0.13325540	01
	19	0.24709579	01	0.14409506	01
	20	0.15549444	01	0.14041630	01
	21	0.13984536	01	0.12893693	01
	22	0.14042057	01	0.12861909	01
	23	0.14321103	01	0.12798932	01
	24	0.14670470	01	0.12777329	01
	25	0.15030701	01	0.12754157	01
	26	0.15507587	01	0.12676033	01
	27	0.15714065	01	0.12627938	01
	28	0.16793635	01	0.12474966	01
	29	0.13923324	01	0.12306929	01
	30	0.15264322	01	0.12553233	01
	31	0.16958008	01	0.13239180	01
	32	0.17146361	01	0.12897812	01
	33	0.18340388	01	0.13307146	01
	34	0.17691259	01	0.13857893	01
	36	0.19122675	01	0.13902501	01
	37	0.16045062	01	0.12584399	01
	38	0.16384564	01	0.12678962	01
	40	0.25278155	01	0.15958017	01
	41	0.26780112	01	0.13580428	01

TABLE 6.11 (Continued)

PHOTO	POINT	TAU		DELTA (um)	
		x	y	x	y
1	1	0.173205640 01	0.126511930 01	0.14550 02	0.10630 02
	2	0.171472470 01	0.124831990 01	0.16080 02	0.10490 02
	3	0.429117390 01	0.162030950 01	0.36050 02	0.13610 02
	4	0.155843680 01	0.122609150 01	0.13090 02	0.10300 02
	5	0.154425030 01	0.116950480 01	0.12970 02	0.98240 01
	6	0.172409360 01	0.117035430 01	0.14480 02	0.98310 01
	7	0.179047630 01	0.117562700 01	0.15040 02	0.98750 01
	8	0.207642340 01	0.121474500 01	0.17440 02	0.10200 02
	9	0.225577470 01	0.124735230 01	0.18950 02	0.10480 02
	10	0.247572600 01	0.126985440 01	0.20800 02	0.10830 02
	11	0.254569670 01	0.130319480 01	0.21380 02	0.10950 02
	12	0.178342260 01	0.117626720 01	0.14980 02	0.98810 01
	13	0.247405730 01	0.129480200 01	0.20780 02	0.10880 02
	14	0.200253680 01	0.123363590 01	0.16820 02	0.10360 02
	15	0.239475000 01	0.133296360 01	0.24320 02	0.11200 02
	16	0.354242700 01	0.151130290 01	0.29760 02	0.12700 02
	35	0.612308720 01	0.153064740 01	0.51440 02	0.12860 02
	37	0.160256550 01	0.121633580 01	0.13460 02	0.10220 02
	38	0.252541640 01	0.123709670 01	0.21210 02	0.10390 02
	41	0.146236110 01	0.123272850 01	0.12280 02	0.10770 02
2	1	0.575195180 01	0.210902360 01	0.48320 02	0.17720 02
	2	0.196044360 01	0.128844350 01	0.16470 02	0.10820 02
	3	0.164673330 01	0.123643620 01	0.13830 02	0.10390 02
	4	0.433162580 01	0.179675310 01	0.36810 02	0.15090 02
	5	0.362525380 01	0.160700800 01	0.30450 02	0.13500 02
	6	0.267976120 01	0.138693070 01	0.22510 02	0.11650 02
	7	0.252374840 01	0.135365510 01	0.21200 02	0.11370 02
	8	0.207065210 01	0.127097570 01	0.17390 02	0.10680 02
	9	0.189941320 01	0.124506980 01	0.15960 02	0.10460 02
	10	0.177699260 01	0.122823870 01	0.14930 02	0.10320 02
	11	0.175217130 01	0.122490570 01	0.14720 02	0.10290 02
	12	0.248287890 01	0.135575490 01	0.20860 02	0.11390 02
	13	0.177563030 01	0.122686200 01	0.14920 02	0.10310 02
	14	0.273003960 01	0.144269750 01	0.22980 02	0.12120 02
	15	0.207547410 01	0.123937840 01	0.17430 02	0.10410 02
	16	0.169524670 01	0.123637230 01	0.14240 02	0.10390 02
	34	0.156591220 01	0.120915750 01	0.13150 02	0.10160 02
	36	0.186939620 01	0.122431260 01	0.15700 02	0.10280 02
	39	0.727005230 01	0.201026650 01	0.61070 02	0.16890 02
	40	0.143103020 01	0.122059480 01	0.12020 02	0.10250 02
3	17	0.204101770 01	0.130295150 01	0.17140 02	0.10940 02
	18	0.270029850 01	0.141617930 01	0.22680 02	0.11900 02
	19	0.893317150 01	0.262828210 01	0.75040 02	0.22080 02
	20	0.243582260 01	0.130655940 01	0.20460 02	0.10980 02
	21	0.257767030 01	0.130275280 01	0.21650 02	0.10940 02
	22	0.263601170 01	0.131064580 01	0.22140 02	0.11010 02
	23	0.235708860 01	0.134262710 01	0.24000 02	0.11280 02
	24	0.312238790 01	0.138599160 01	0.26230 02	0.11640 02
	25	0.342794560 01	0.144151730 01	0.28790 02	0.12110 02
	26	0.339027760 01	0.153413100 01	0.32680 02	0.12890 02
	27	0.410419430 01	0.157956820 01	0.34480 02	0.13270 02
	28	0.526571330 01	0.184283900 01	0.44230 02	0.15480 02
	29	0.262649720 01	0.130829260 01	0.22060 02	0.10990 02
	30	0.383577390 01	0.154129210 01	0.32220 02	0.12950 02
	31	0.306194730 01	0.134281070 01	0.25720 02	0.11280 02
	32	0.402473510 01	0.164066880 01	0.33810 02	0.13780 02
	33	0.656639130 01	0.217561360 01	0.55160 02	0.18280 02
	35	0.956875030 01	0.238883250 01	0.80380 02	0.20070 02
	37	0.201707380 01	0.126915980 01	0.16940 02	0.10660 02
	38	0.273320230 01	0.128570530 01	0.22960 02	0.10600 02
	41	0.169123070 01	0.128298500 01	0.14210 02	0.10780 02

TABLE 6.12. Undetected Gross Errors for case 6.
(at $\alpha = 0.05$ and $\gamma = 0.8$)

PHOTO POINT		TAU		DELTA(um)	
NO	NO	X	Y	X	Y
4					
17	0.352030360	01	0.161575000	01	0.29570
18	0.163650440	01	0.121121810	01	0.14170
19	0.165750500	01	0.122502420	01	0.13920
20	0.285234330	01	0.152173790	01	0.23960
21	0.202120680	01	0.127103170	01	0.16980
22	0.196734030	01	0.125401440	01	0.16530
23	0.182711880	01	0.121003970	01	0.15350
24	0.171869200	01	0.117926730	01	0.14440
25	0.162901900	01	0.115808200	01	0.13680
26	0.153639690	01	0.114153680	01	0.12910
27	0.150712720	01	0.113778410	01	0.12660
28	0.144907660	01	0.113675310	01	0.12170
29	0.196736870	01	0.125950810	01	0.16530
30	0.153481260	01	0.114463590	01	0.12890
31	0.231361600	01	0.132992880	01	0.19430
32	0.164803270	01	0.120363850	01	0.13840
33	0.148761500	01	0.113813920	01	0.12500
34	0.158509690	01	0.122051210	01	0.13310
36	0.198824910	01	0.124768480	01	0.16700
39	0.656149820	01	0.158413590	01	0.55120
40	0.142621790	01	0.126477010	01	0.11980
5					
1	0.233778980	01	0.147974000	01	0.19640
2	0.150747190	01	0.136331540	01	0.12660
3	0.198992350	01	0.146270800	01	0.16720
4	0.178938270	01	0.141609830	01	0.15030
5	0.165221460	01	0.134775490	01	0.13880
6	0.156505260	01	0.133768380	01	0.13150
7	0.155381730	01	0.133813290	01	0.13050
8	0.151462550	01	0.134174560	01	0.12720
9	0.149910230	01	0.134365950	01	0.12590
10	0.150059790	01	0.134821620	01	0.12610
11	0.150513850	01	0.135004130	01	0.12640
12	0.152305380	01	0.133044560	01	0.12790
13	0.148724580	01	0.134494070	01	0.12490
14	0.166641360	01	0.136073700	01	0.14000
15	0.173945610	01	0.135816930	01	0.14610
16	0.168463290	01	0.142214200	01	0.14150
34	0.157290660	01	0.129457100	01	0.13210
36	0.169551490	01	0.128623390	01	0.14240
37	0.166401820	01	0.135226210	01	0.13980
38	0.219691550	01	0.135605220	01	0.18450
40	0.180977250	01	0.138674230	01	0.15200
41	0.196772000	01	0.166346180	01	0.16530
6					
17	0.172807240	01	0.140311000	01	0.14520
18	0.143372850	01	0.128511640	01	0.12040
19	0.236322850	01	0.137687730	01	0.19850
20	0.151131900	01	0.138417010	01	0.12700
21	0.139330380	01	0.128706340	01	0.11700
22	0.139901370	01	0.128258270	01	0.11750
23	0.142685730	01	0.127072690	01	0.11990
24	0.146201680	01	0.126194670	01	0.12280
25	0.149813360	01	0.125465610	01	0.12580
26	0.154475860	01	0.124665580	01	0.12980
27	0.156461740	01	0.124416040	01	0.13140
28	0.166995740	01	0.124657770	01	0.14030
29	0.138816070	01	0.128022820	01	0.11660
30	0.152194060	01	0.124805020	01	0.12780
31	0.162226250	01	0.129496850	01	0.13630
32	0.158392840	01	0.125746320	01	0.13300
33	0.181429110	01	0.131925970	01	0.15240
34	0.162686450	01	0.135497200	01	0.13670
36	0.185456450	01	0.135097840	01	0.15580
37	0.143168250	01	0.124866180	01	0.12450
38	0.153912260	01	0.125037630	01	0.12930
40	0.194516110	01	0.157387600	01	0.16340
41	0.159852480	01	0.134891710	01	0.13430

TABLE 6.12 (Continued)

In addition to the global indicators of reliability, the individual indicators Tau, Gam and Delta were computed (see Chapter 3) and are listed in Tables (6.9) through (6.12) for cases 5 and 6. The maximum values of Tau, Gam and undetected gross errors for the six cases are summarised in Table (6.13) for the photogrammetric measurements. Those for the 7 slope distances are set out in Table (6.14).

Case	Tau		Gam		Max.undetected gross errors (μm)	
	x	y	x	y	x	y
1	8767.473900	36.059799	8767.473900	36.045931	73650.00	302.90
2	7975.718700	4.491667	7975.718700	4.378935	67000.00	37.73
3	6922.358400	4.661124	6922.358400	4.552590	58150.00	39.15
4	162.169730	4.384509	162.1666650	4.268948	1362.00	36.83
5	10.781538	2.677504	10.735062	2.483753	90.56	22.49
6	9.568751	2.628282	9.516353	2.430610	80.38	22.08

TABLE (6.13) Maximum Tau, Gam and undetected gross errors for photogrammetric measurements (at $\alpha = 0.05$ and $\gamma = 0.8$)

Case	Tau	Gam	Max.undetected gross errors (mm)
2	87.636129	87.630424	98.153
4	38.869742	38.856877	43.534
6	21.673544	21.650462	24.274

TABLE (6.14) Maximum Tau, Gam and undetected gross errors for the 7 slope distances (at $\alpha = 0.05$ and $\gamma = 0.8$)

Conformal with the findings of Chapter 5, an insight into the latter two tables reveals that reliability increases with the increase of the number of cameras and higher internal reliability leads to higher external reliability. Moreover, detection of gross errors is more likely, the greater the number of cameras used.

6.3.3. Simulated models for sensitivity analysis

Only were the two models, namely the settlement and deflexion models mentioned in Section (5.6.2.3) investigated in this section. The just-detectable deformations were computed according to Equation (3.59) which reads:

$$\omega^u = \frac{(c\tilde{d})^T Q_d^+((c\tilde{d}))}{\sigma_0^2} \quad (6.3)$$

in which $c\tilde{d}$ indicates a just-detectable deformation and $\tilde{d}$ is a form vector characterising the deformation model to be tested.

Listed in Table (6.15) are the values of c for both settlement and deflexion models. The results summarised in such a table are graphically represented in Figures (6.12) and (6.13). These figures suggest that the sensitivity decreases when the number of cameras increases.

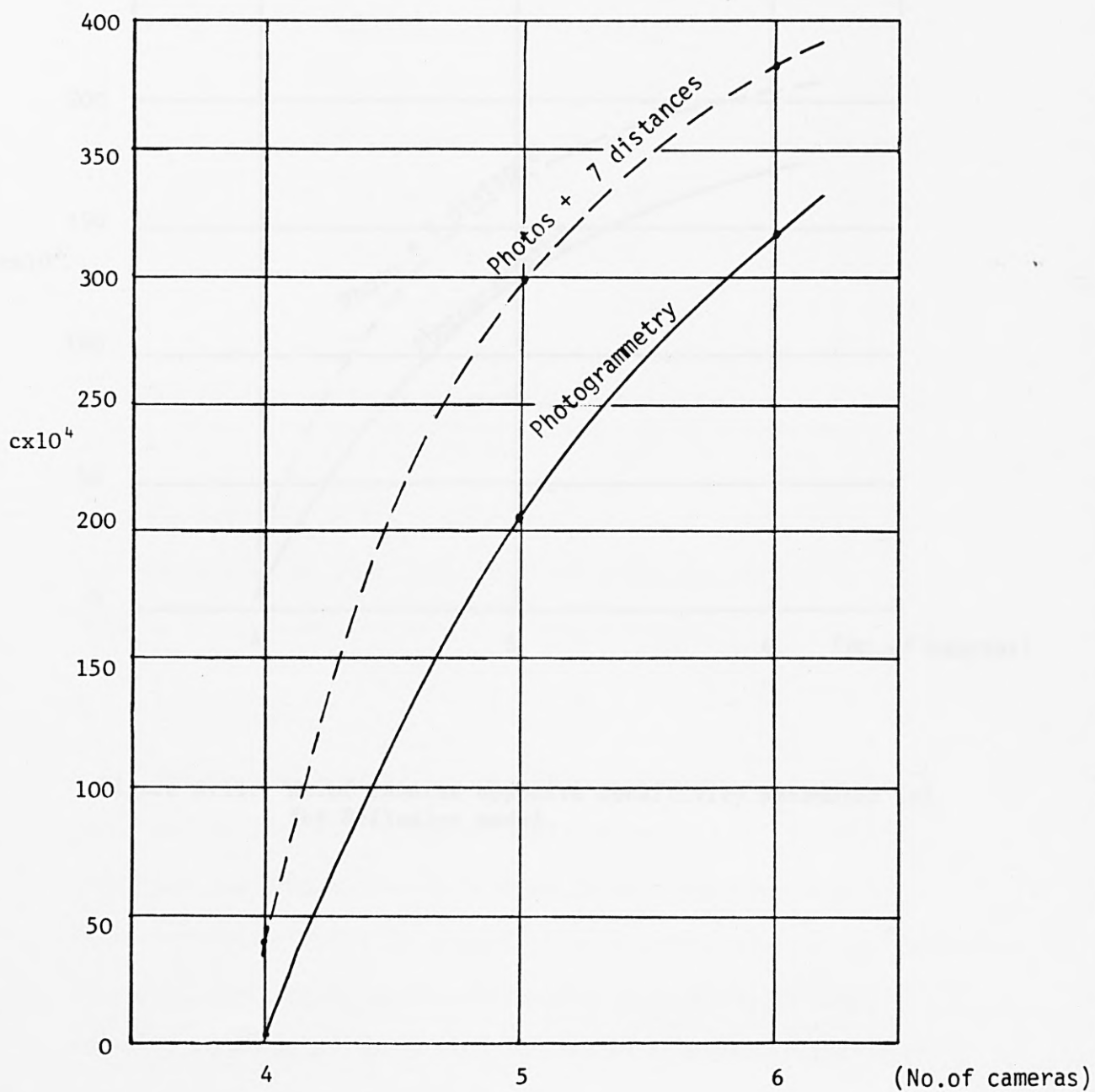


Figure 6.12. No. of cameras opposite sensitivity parameter (c) for Settlement model.

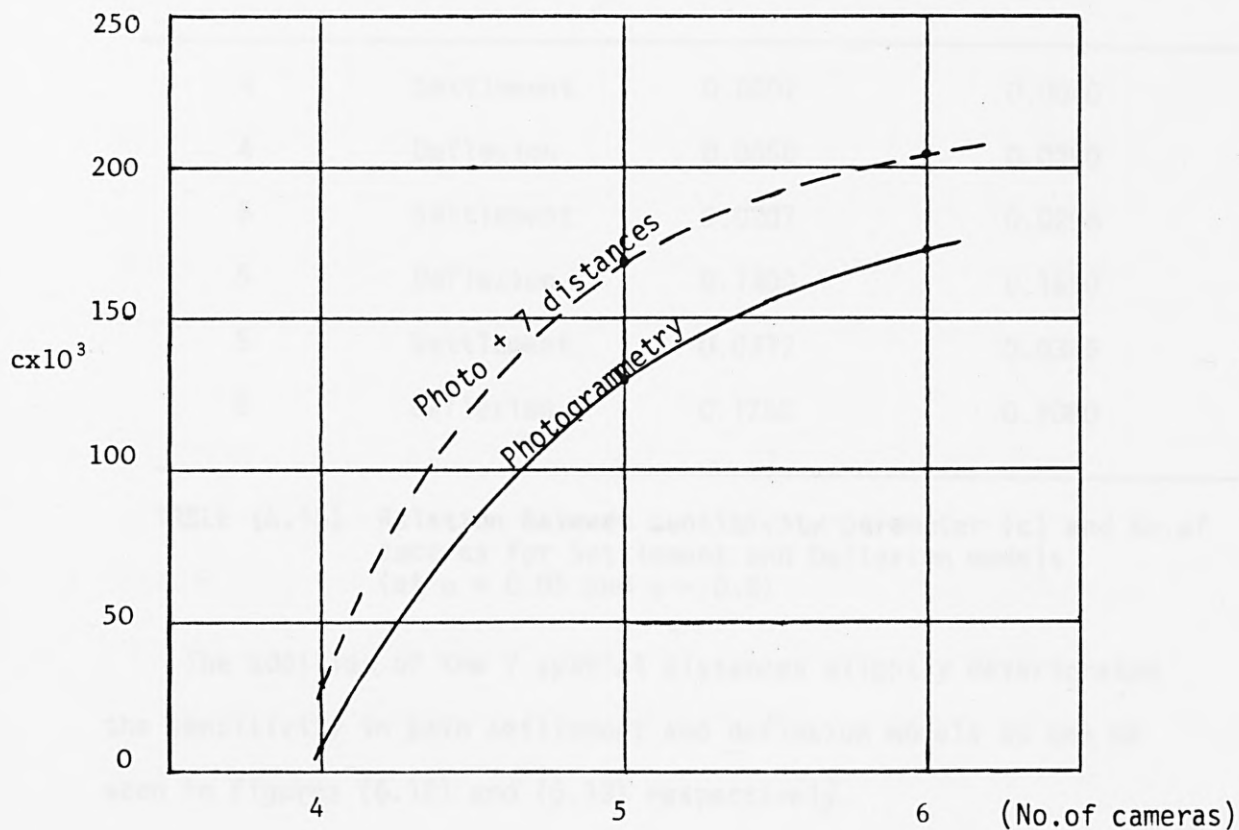


Figure 6.13. No. of cameras opposite Sensitivity parameter (c) for Deflexion model.

Number of cameras	Model	Photogrammetry	Photo+7 distances
		c	c
4	Settlement	0.0002	0.0040
4	Deflexion	0.0050	0.0290
5	Settlement	0.0207	0.0298
5	Deflexion	0.1300	0.1690
6	Settlement	0.0317	0.0385
6	Deflexion	0.1750	0.2060

TABLE (6.15) Relation between sensitivity parameter (c) and No.of cameras for Settlement and Deflexion models (at $\alpha = 0.05$ and $\gamma = 0.8$)

The addition of the 7 spatial distances slightly deteriorated the sensitivity in both settlement and deflexion models as can be seen in Figures (6.12) and (6.13) respectively.

6.4. Conclusions

Some of the conclusions mentioned in Section (5.6.3) were confirmed here in the bridge deformation analysis. First, the precision and reliability were found to be increasing with the number of cameras with the exception that sensitivity was decreasing. Secondly, good precision did correspond to good reliability. Thirdly, addition of the 7 slope distances provided extremely high precision improvement especially with case 1 when augmented with those distances (case 2). At last, the higher internal reliability

led to higher external reliability and the more the number of cameras the greater is the chance of detecting gross errors.

Case 6 (6 cameras + 7 slope distances) exhibited the highest precision ($\sigma_m^2 = 2.644 \text{ mm}$) as well as the highest reliability ($r_{\max}(x) = 0.519$, $r_{\max}(y) = 0.774$). Furthermore, in such a case gross errors of the order of 7- and 26-fold the a priori standard error in the x- and y-co-ordinates respectively can be detected. Consequently, the reliability in the y-co-ordinate direction was significantly much better than that of the x-co-ordinate.

With regard to sensitivity, the addition of the 7 slope distances, listed in Table (6.3), led to the results of slightly deteriorating the values of c in both settlement and deflexion models. On the contrary to the cube, where the slope distances were on each of its upright faces and no transverse distances, here in the bridge deformation investigation such transverse distances could be the reason for such a slight deterioration as it might have increased the correlation between the co-ordinates of the opposite linked targets.

CHAPTER 7

SIMULATED NETWORK DEFORMATION ANALYSIS (DAM)

7.1. Introduction

For safety reasons it is necessary to perform periodic control surveys of large engineering structures, such as dams, in order to determine whether the structure remains stable or whether it is subject to movements or local deformations as a function of time. Usually this is done by the classical methods mentioned in Section (6.1). Such methods are time consuming especially with large networks and therefore photogrammetric structural survey was chosen as an alternative. It offers an economical substitute and, rather, provides an instantaneous description due to the short time lapse between the data acquisition and data reduction.

During the preparation of this thesis (July 1985), the collapse of Starva dam, Italy, took place causing large scale losses and mortalities. It can be argued that the catastrophe happened, mainly, due to lack of surveillance and monitoring of that dam. Therefore, this investigation gets its utmost importance as a guard, if followed, against sudden failure of large structures among them are dams.

7.2. Network configurations

A typical concrete dam (Figure 7.1) was chosen to be under investigation. A total of 14 targets was placed in two rows on the face of the dam (Figure 7.3). A Zeiss (Jena) UMK 10/1318

camera, of nominal focal length of 100 mm, which accepts glass plates (130 mm x 180 mm) was assumed to be used for photography. An 3 μ m standard error has been postulated for the photogrammetric measurements.

Three camera stations were selected (Figure 7.1) to represent the first configuration. To strengthen the network, an additional camera was chosen to constitute the second configuration which almost takes the "L-letter" shape as shown in Figure (7.1). The third configuration which, also comprises four camera stations is illustrated in Figure (7.2). Consequently, three distinct cases were obtained taking into account that the number of targets is constant through all these cases. Table (7.1) sets out the orientation elements of the individual cameras along with the photo arrangements.

Case	Photo arrangement	Camera No.	X^C (m)	Y^C (m)	Z^C (m)	ω (deg)	ϕ (deg)	κ (deg)
1	1-2-3	1	326.0	94.0	55.0	87	55	0
		2	310.0	208.0	55.0	87	125	0
		3	444.0	208.0	55.0	38	95	0
2	1-2-3-4L	4L	344.0	148.0	50.0	88	90	0
3	1-2-3-4-	1	326.0	94.0	55.0	87	55	0
		2	312.0	130.0	50.0	88	109	0
		3	306.0	168.0	50.0	88	88	0
		4	310.0	208.0	55.0	87	125	0

TABLE (7.1) Orientation elements of camera stations shown in Figures 7.1 and 7.2.

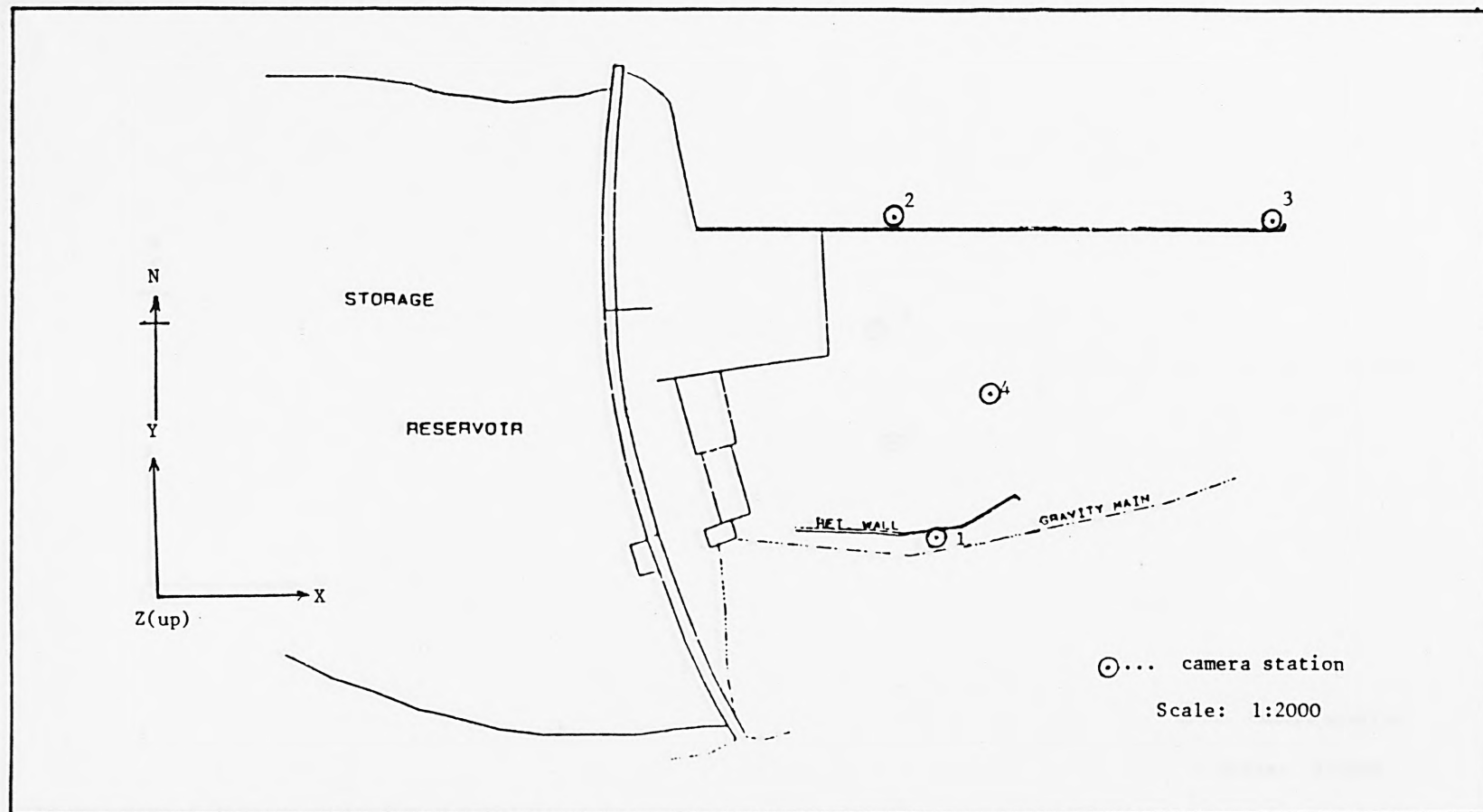


Figure 7.1. Camera Station Configurations for cases 1 and 2

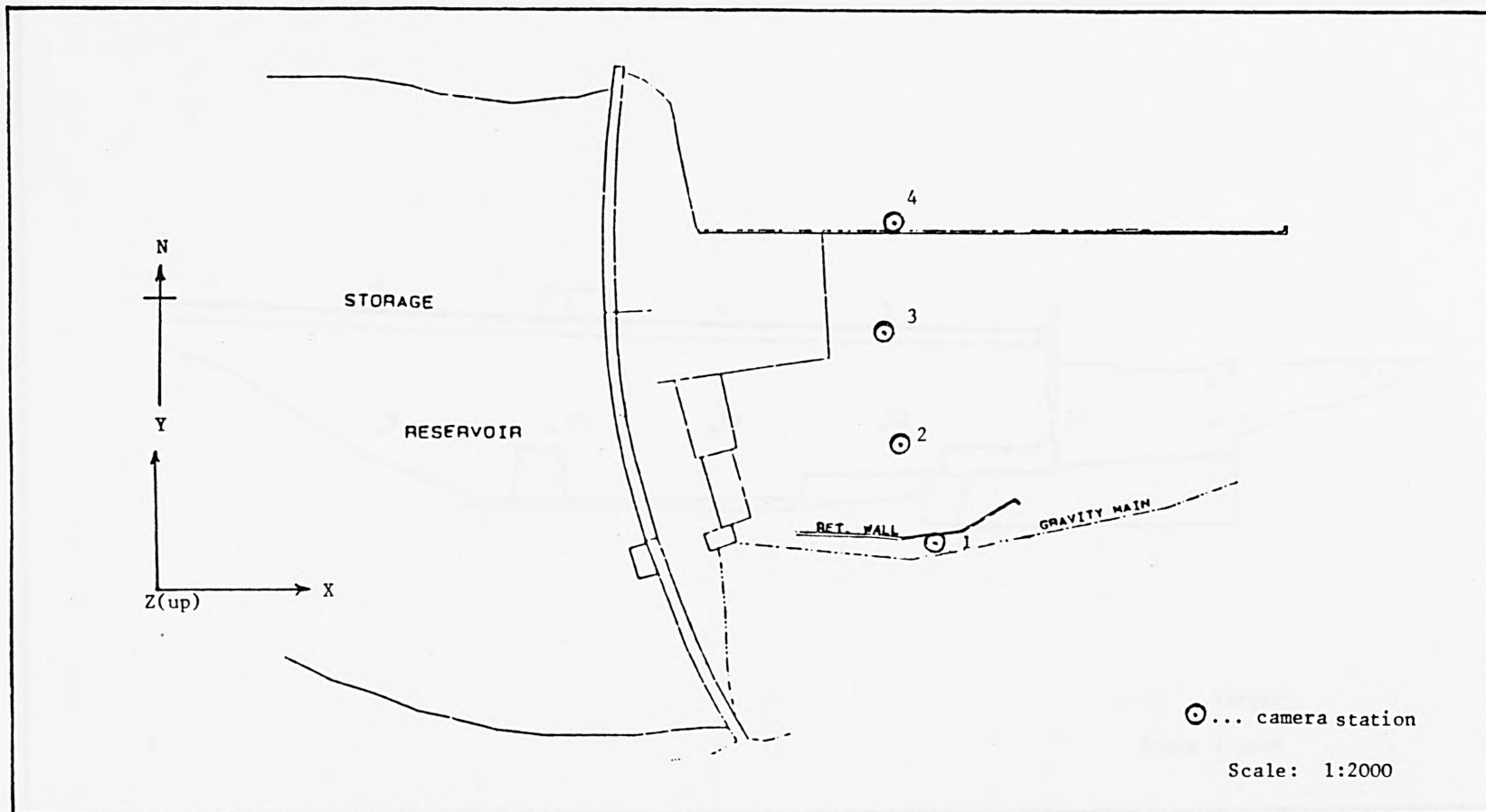


Figure 7.2. Camera station Configurations for case 3.

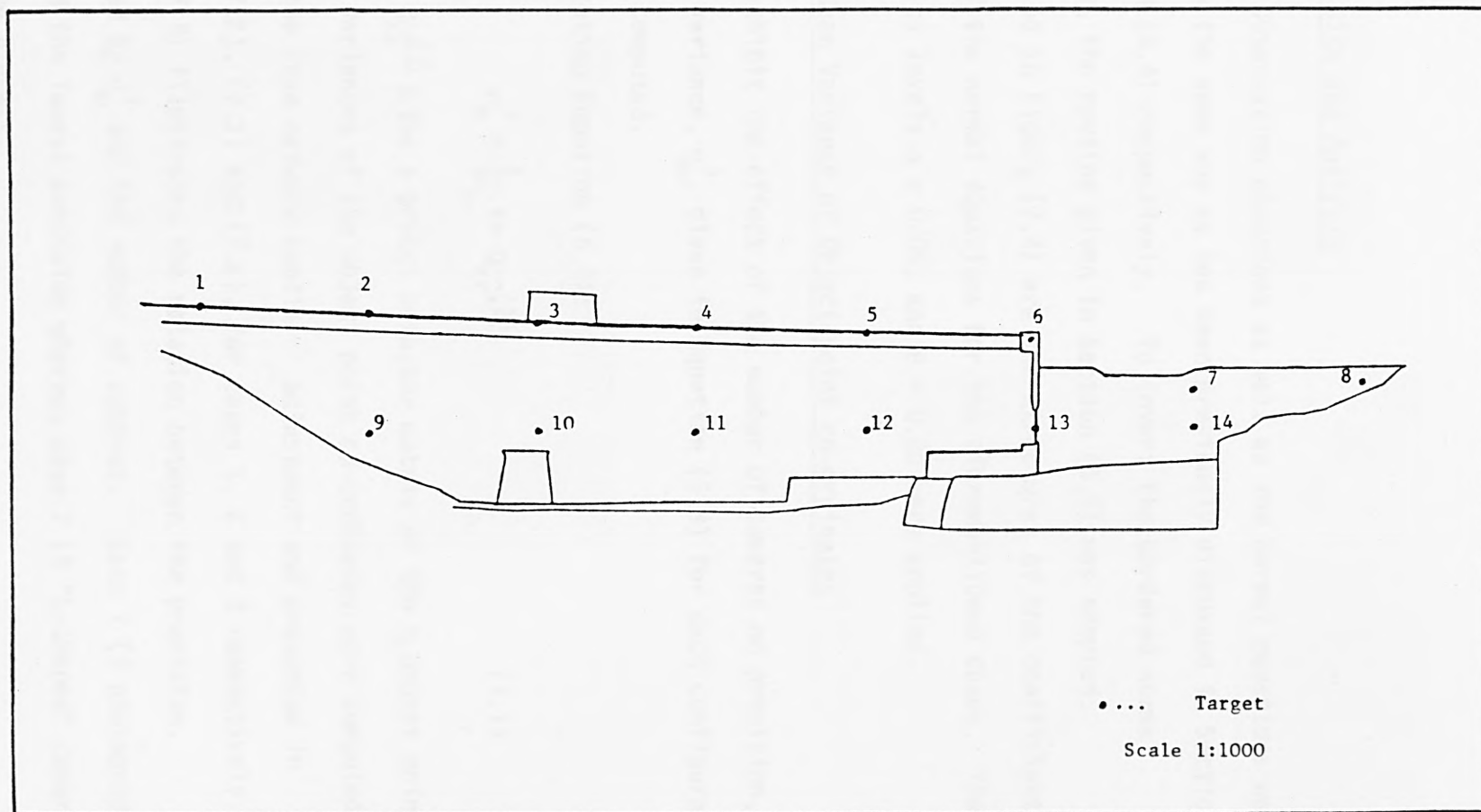


Figure 7.3. Dam Face Targets.

7.3. Results and Analysis

The observation equations as well as the normal equations were formed in the same way as has been previously discussed in Sections (5.3) and (5.4) respectively. To invert the bordered normal equations, the routine given in Section (5.5) was adopted.

Illustrated in Figure (7.4) are the structures of the coefficient matrix of the normal equations for the aforementioned cases. The probability levels $\alpha = 0.05$, and $\beta = 0.20$ were applied.

7.3.1. Mean Variance of Object-point co-ordinates

To exhibit the effect of the number of cameras on precision, the mean variance, σ_m^2 , given in Equation (5.8) for each configuration is to be computed.

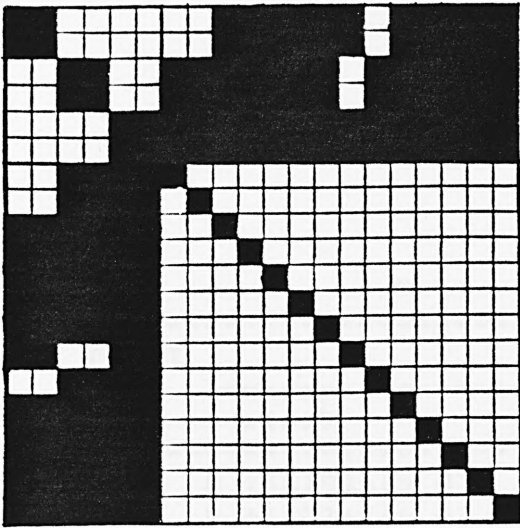
Restating Equation (5.8):

$$\sigma_m^2 = \frac{1}{3n_0} \text{tr } Q_{xx}^{(2)} \quad (7.1)$$

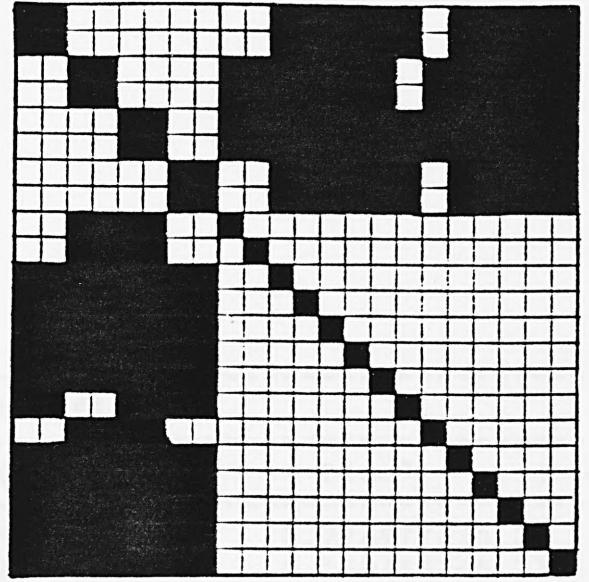
in which $Q_{xx}^{(2)}$ is the a priori cofactor matrix of the n_0 object points.

The variances of the object point co-ordinates were computed through the free network bundle adjustment and presented in Tables (7.2), (7.3) and (7.4), for cases 1, 2 and 3 respectively. Figure (7.5) illustrates the relation between the precision, designated by σ_m^2 and the number of cameras. Case 1 (3 photographs) exhibited the lowest precision whereas case 2 (4 "L-shaped" cameras) showed the highest.

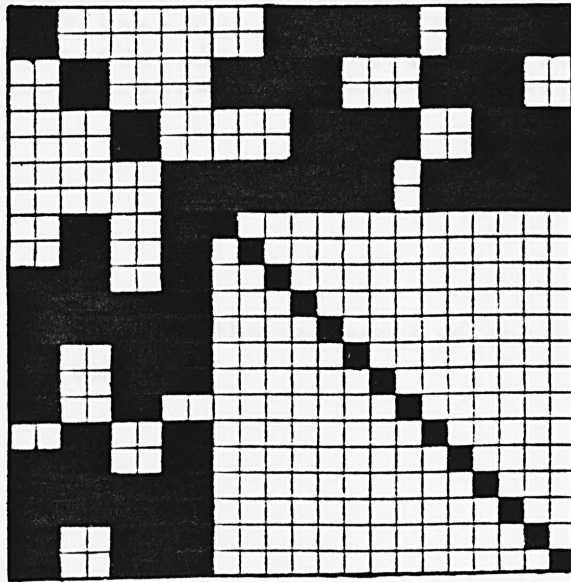
It is notable that the precision increases with the number of cameras, however it seems that the position of the cameras is of a



(a) Case 1-3 Photos and 14 object points.



(b) Case 2-4 Photos "L-Shaped" and 14 object points.



(c) Case 3-4 Photos and 14 object points.

Figure 7.4. Structure of coefficient matrix of the normal equations.

POINT		VARIANCES			
NO.	X (mm ²)	Y (mm ²)		Z (mm ²)	
1	0.155798640 03	0.758517940	03	0.276977230	02
2	0.641026360 02	0.129103900	03	0.169738760	02
3	0.670343860 02	0.122058080	03	0.173169380	02
4	0.934363330 02	0.813828690	02	0.874403730	01
5	0.112674000 03	0.503317200	02	0.352390450	01
6	0.776860390 02	0.183647230	02	0.110340780	02
7	0.702296120 02	0.283227160	02	0.955786630	01
8	0.242370270 03	0.604542920	03	0.208696460	02
9	0.939938640 02	0.121474820	03	0.149339400	02
10	0.420289430 02	0.139590500	03	0.882524610	01
11	0.676339940 02	0.130809680	03	0.632709760	01
12	0.813537740 02	0.106718200	03	0.705067740	01
13	0.540765460 02	0.497428750	02	0.332143890	01
14	0.939530780 02	0.207949360	02	0.994356720	01

TABLE 7.2. Estimates of the variances of the 14 object point co-ordinates for case 1.

POINT		VARIANCES			
NO.	X (mm ²)	Y (mm ²)		Z (mm ²)	
1	0.115493120 03	0.627440380 03		0.251743330 02	
2	0.578227530 02	0.123906810 03		0.166892350 02	
3	0.615007260 02	0.110296670 03		0.140727250 02	
4	0.745953270 02	0.717721390 02		0.766754500 01	
5	0.849302280 02	0.409818020 02		0.726938060 01	
6	0.578946930 02	0.133494500 02		0.802225580 01	
7	0.597353770 02	0.229424980 02		0.627433560 01	
8	0.197935550 03	0.451346980 03		0.137768540 02	
9	0.814260960 02	0.119543880 03		0.139043560 02	
10	0.365552290 02	0.120506780 03		0.684355260 01	
11	0.534126320 02	0.100390760 03		0.485536500 01	
12	0.664094360 02	0.741361430 02		0.508009470 01	
13	0.459717370 02	0.333847810 02		0.585585950 01	
14	0.734146780 02	0.175158770 02		0.721477450 01	

TABLE 7.3. Estimates of the variances of the 14 object point co-ordinates for case 2.

POINT		VARIANCES			
NO.	X (mm ²)		Y (mm ²)		Z (mm ²)
1	0.251139540 03		0.535889260 03		0.401177400 02
2	0.571609220 02		0.798373160 02		0.181670950 02
3	0.713587600 02		0.757575660 02		0.130486780 02
4	0.129425780 03		0.390820910 02		0.657161010 01
5	0.156068690 03		0.258947290 02		0.570966800 01
6	0.979249630 02		0.323084680 02		0.717199120 01
7	0.703962290 02		0.396258210 02		0.696048390 01
8	0.571571940 03		0.585682380 03		0.219189240 02
9	0.990774730 02		0.167464550 03		0.154009270 02
10	0.280870340 02		0.773824620 02		0.586290910 01
11	0.445433750 02		0.558618670 02		0.422801040 01
12	0.661443840 02		0.571486830 02		0.418840730 01
13	0.454475730 02		0.677456360 02		0.492286600 01
14	0.320127930 02		0.484547790 02		0.957148830 01

TABLE 7.4. Estimate of the variances of the 14 object point co-ordinates for case 3.

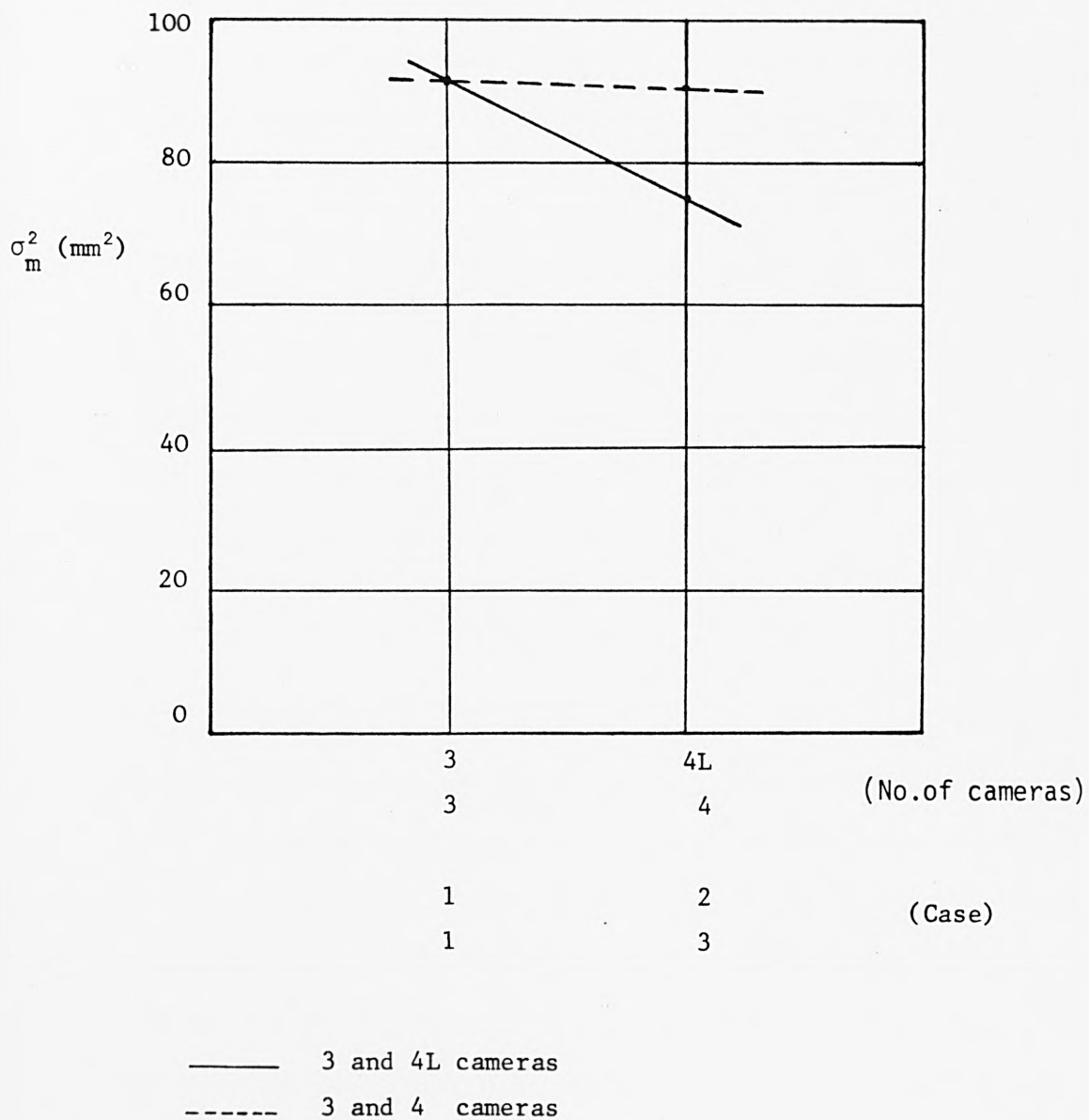


Figure 7.5. Relation between no. of cameras and precision

prime importance. This is evident from the comparison of the results of case 2 to those of case 3 where the presence of camera No.3 which contains images of all object points in the former case gave rise to considerable higher precision compared with the latter.

7.3.2. $r_{\max}$ and other reliability indicators

The global reliability of the x- and y- co-ordinate measurements represented by $r_{\max}(x)$ and $r_{\max}(y)$ respectively where r is the redundancy number mentioned in Section (5.6.2.2.) and computed as given by Equation (5.9) which reads:

$$r_i = q_{\hat{v}_i \hat{v}_i} \times w_{\ell_i} \quad (7.2)$$

in which $q_{\hat{v}_i \hat{v}_i}$ is the i th diagonal element of the cofactor matrix of the residuals $Q_{\hat{v}\hat{v}}$ and w_{ℓ_i} is the a priori weight of observation ℓ_i . The values of $r_{\max}(x)$ and $r_{\max}(y)$ for case 1 are displayed in Table (7.5) while Tables (7.6) and (7.7) set out these values for cases 2 and 3 respectively. Figure (7.6) shows the relation between number of cameras and the reliability of the x-co-ordinate photogrammetric measurements whereas Figure (7.7) demonstrates that relation with regard to the y-co-ordinate.

Agreeable with what has been mentioned in Section (7.3.1) the reliability does increase with number of cameras, yet such an increase depends on the location of the camera and number of imaged points caught by such a camera.

PHOTO	POINT	$r(x)$	$r(y)$
1	3	0.148125510-01	0.653224130-01
	4	0.320948780-01	0.315126700 00
	5	0.308577360-01	0.378861780 00
	6	0.222772670-01	0.420155740 00
	7	0.949586490-02	0.526284490 00
	8	0.727016310-03	0.171373420 00
	10	0.146898000-01	0.680054740-01
	11	0.243645160-01	0.313951640 00
	12	0.223532460-01	0.379267800 00
	13	0.120542370-01	0.509789330 00
	14	0.220485870-02	0.463145360 00
2	1	0.272703340-02	0.179369830 00
	2	0.542703340-02	0.236329440 00
	3	0.689864080-01	0.331469430 00
	4	0.113324890 00	0.469415330 00
	5	0.105010420 00	0.352500500 00
	6	0.827244460-01	0.241446020 00
	7	0.430590810-01	0.155561340 00
	9	0.783813840-04	0.223831480 00
	10	0.618941330-01	0.362930570 00
	11	0.911258000-01	0.478783630 00
	12	0.892237090-01	0.362993300 00
	13	0.781223330-01	0.282192690 00
	14	0.459872800-01	0.131833090 00
3	1	0.141663700 00	0.196525310 00
	2	0.212535890 00	0.338017220 00
	3	0.396570620 00	0.627349460 00
	4	0.692579620 00	0.724648820 00
	5	0.685587500 00	0.721575010 00
	6	0.720120840 00	0.720504970 00
	7	0.653085960 00	0.530565920 00
	8	0.214066960 00	0.912478890-01
	9	0.274347460 00	0.229029980 00
	10	0.579980470 00	0.544053900 00
	11	0.702987060 00	0.655582930 00
	12	0.718113260 00	0.647356430 00
	13	0.744603280 00	0.670517120 00
	14	0.594442320 00	0.578776360 00

TABLE 7.5. Computed values of redundancy numbers for case 1.

PHOTO	POINT	$r(x)$	$r(y)$
1	3	0.479464980-01	0.116831040 00
	4	0.127635130 00	0.434206520 00
	5	0.176217880 00	0.499788170 00
	6	0.220044130 00	0.555067260 00
	7	0.253693600 00	0.642850860 00
	8	0.124112100 00	0.428395140 00
	10	0.468385910-01	0.132847260 00
	11	0.121214730 00	0.437385520 00
	12	0.167702370 00	0.538451070 00
	13	0.222794660 00	0.622088130 00
	14	0.224275220 00	0.583085700 00
2	1	0.277055260-02	0.182232290 00
	2	0.546231760-02	0.237865940 00
	3	0.156763570 00	0.454344540 00
	4	0.211650980 00	0.558420120 00
	5	0.168609000 00	0.468753220 00
	6	0.122070030 00	0.340557230 00
	7	0.651399370-01	0.243973710 00
	9	0.792377320-04	0.226276930 00
	10	0.141118320 00	0.473689660 00
	11	0.188782560 00	0.561090610 00
	12	0.149105480 00	0.430051960 00
	13	0.120638230 00	0.397516130 00
	14	0.671923930-01	0.210281330 00
3	1	0.143924430 00	0.199661540 00
	2	0.213917700 00	0.340214860 00
	3	0.457848000 00	0.656756390 00
	4	0.720319120 00	0.763778660 00
	5	0.720139240 00	0.752784100 00
	6	0.755287590 00	0.745372760 00
	7	0.712097760 00	0.610586680 00
	8	0.332060800 00	0.135353720 00
	9	0.277344810 00	0.231532220 00
	10	0.626277710 00	0.591980880 00
	11	0.752490830 00	0.698322700 00
	12	0.763946100 00	0.693289410 00
	13	0.777297400 00	0.700517620 00
	14	0.668844480 00	0.629237190 00
4	3	0.296480330 00	0.430542060 00
	4	0.544313080 00	0.559689500 00
	5	0.541387200 00	0.538183830 00
	6	0.507814430 00	0.560965230 00
	7	0.445544670 00	0.567171810 00
	8	0.122141540 00	0.290313910 00
	10	0.290349110 00	0.339369700 00
	11	0.512220960 00	0.597510510 00
	12	0.492675030 00	0.633925040 00
	13	0.484305230 00	0.637081130 00
	14	0.401202710 00	0.523999280 00

TABLE 7.6. Computed values of redundancy numbers for case 2.

PHOTO	POINT	$r(x)$	$r(y)$
1	3	0.813487940-01	0.130560280 00
	4	0.250010800 00	0.510162200 00
	5	0.259340340 00	0.577542000 00
	6	0.219372200 00	0.573914570 00
	7	0.251055540 00	0.657596070 00
	8	0.466069460-01	0.404810310 00
	10	0.577904310-01	0.148615670 00
	11	0.183816120 00	0.518403360 00
	12	0.233704610 00	0.618329120 00
	13	0.186526340 00	0.621701660 00
	14	0.258372770 00	0.551964440 00
2	1	0.588503510-02	0.566923040-01
	2	0.142971140-01	0.188406140 00
	3	0.326719000 00	0.306756810 00
	4	0.478830330 00	0.454344920 00
	5	0.184734140 00	0.336574920 00
	9	0.358491340-02	0.111202030 00
	10	0.300883230 00	0.316447440 00
	11	0.315718980 00	0.505075670 00
	12	0.146998180 00	0.307346340 00
3	4	0.180856220 00	0.355685790 00
	5	0.407873430 00	0.416588690 00
	6	0.395106260 00	0.364347890 00
	7	0.257825040 00	0.416079890 00
	8	0.301242110-02	0.120421780 00
	11	0.136682880 00	0.283017710 00
	12	0.320407230 00	0.526078930 00
	13	0.319774490 00	0.479943530 00
4	14	0.260941370 00	0.358607570 00
	1	0.421922060-01	0.153966790 00
	2	0.849767070-01	0.460372500 00
	3	0.180108200 00	0.553253000 00
	4	0.298682260 00	0.640852440 00
	5	0.317874830 00	0.548933520 00
	6	0.169692190 00	0.337086670 00
	7	0.540041560-01	0.285191570 00
	9	0.176976340-01	0.275999940 00
	10	0.153162240 00	0.542693040 00
	11	0.271314420 00	0.618871240 00
	12	0.248416840 00	0.566337080 00
	13	0.131351930 00	0.453416970 00
	14	0.454723230-01	0.237285460 00

TABLE 7.7. Computed values of redundancy numbers for case 3.

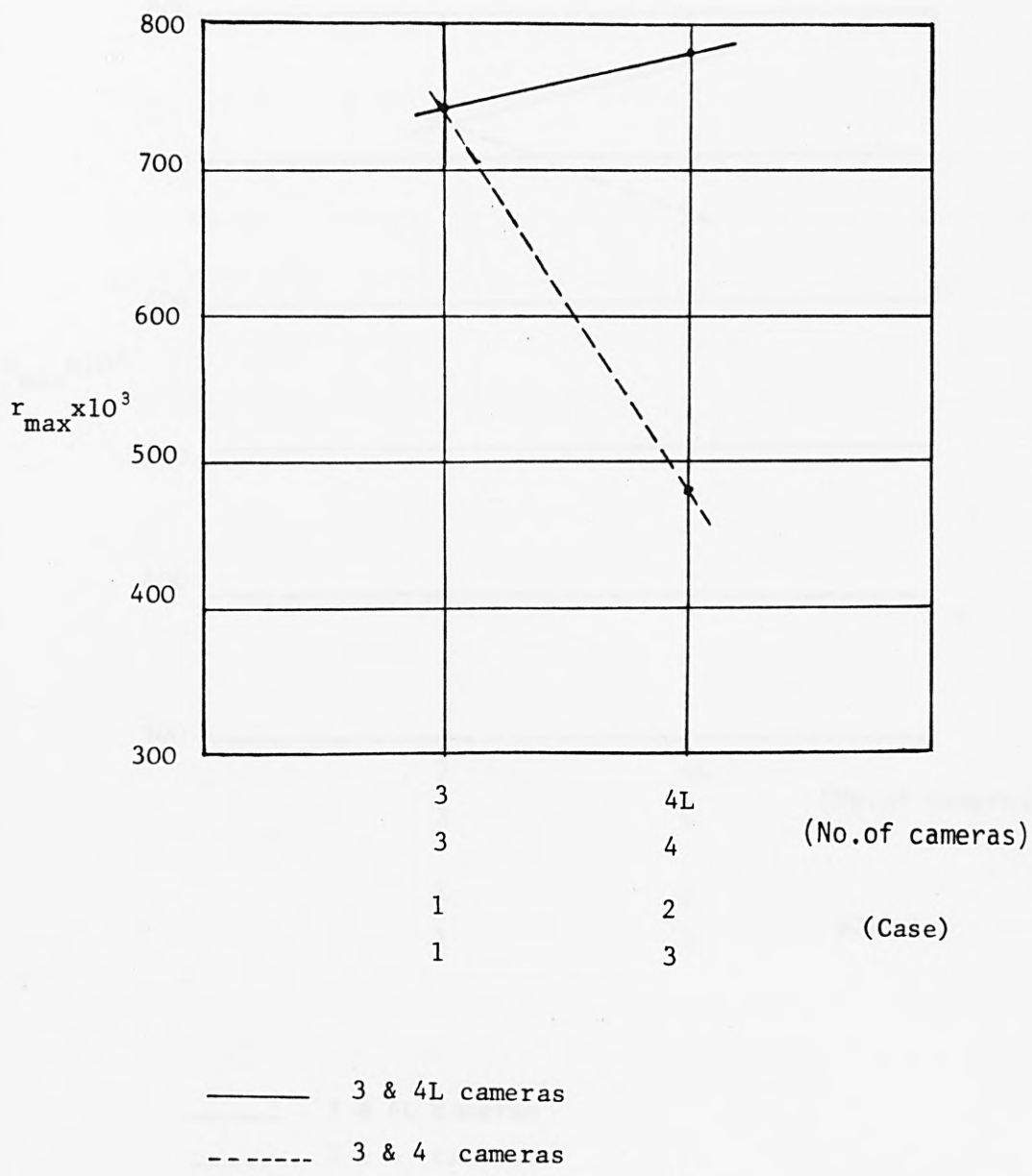
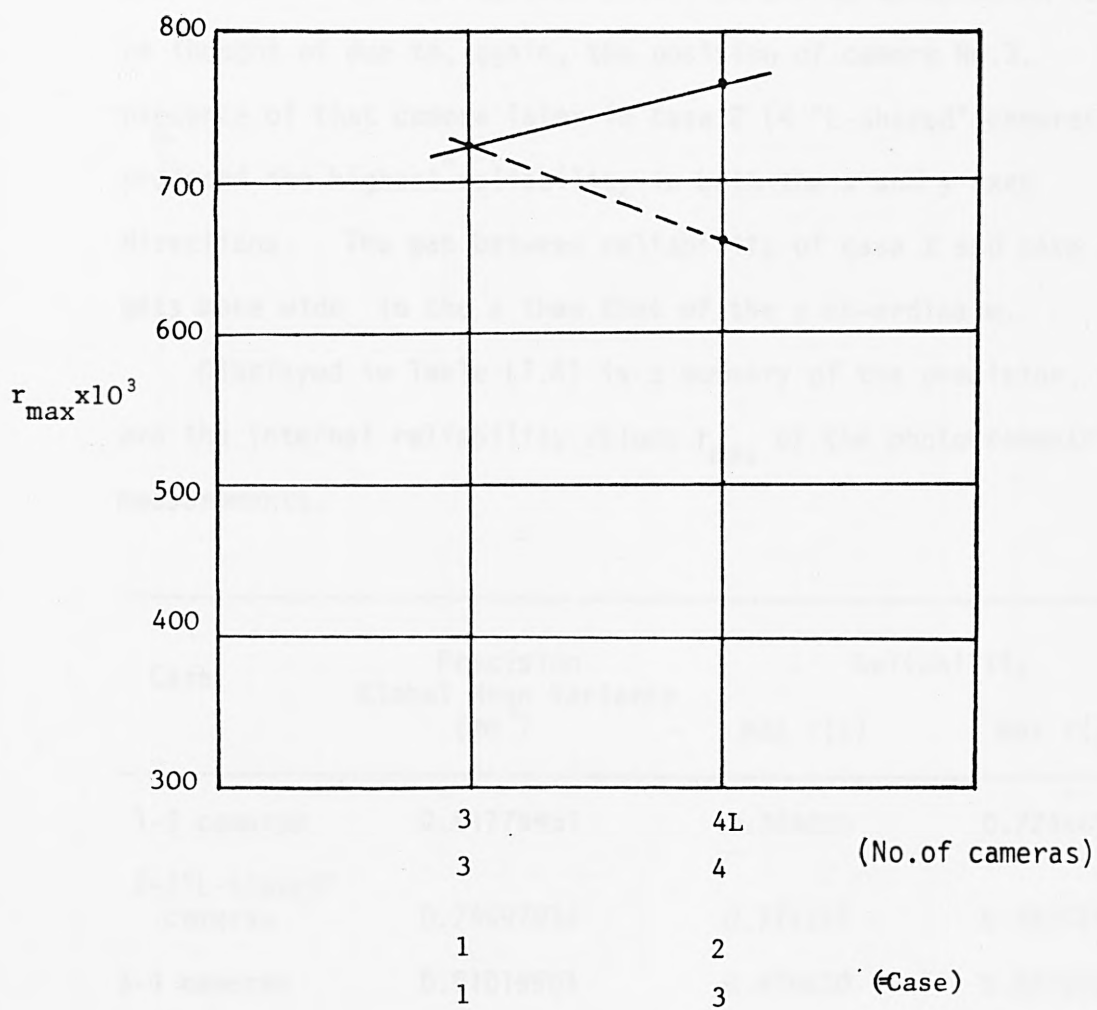


Figure 7.6. No.of cameras versus Reliability (internal) of photo x-co-ordinates.



——— 3 & 4L cameras
 - - - - 3 & 4 cameras

Figure 7.7. No. of cameras versus Reliability (internal) of the photo y-co-ordinates.

It is surprising and notable that case 3 (4 photographs) showed lower reliability than case 1 (3 photographs) which can be thought of due to, again, the position of camera No.3. The presence of that camera later in case 2 (4 "L-shaped" cameras) produced the highest reliability in both the x and y axes directions. The gap between reliability of case 2 and case 3 gets more wide in the x than that of the y co-ordinate.

Displayed in Table (7.8) is a summary of the precision, σ_m^2 , and the internal reliability values $r_{\max}$ of the photogrammetric measurements.

Case	Precision Global Mean Variance (mm ²)	Reliability	
		max $r(x)$	max $r(y)$
1-3 cameras	0.91779957	0.744603	0.724649
2-4 "L-shaped" cameras	0.74697933	0.777297	0.763779
3-4 cameras	0.91019901	0.478830	0.657596

TABLE (7.8) Summary of precision and reliability of photogrammetric measurements

Tau and Gam as individual indicators of reliability, were computed and are presented in Tables (7.9.) through (7.14) for cases 1, 2 and 3.

The maximum values of Tau, Gam and the undetected gross errors for the three cases, at hand, are shown in Table (7.15).

PHO PT.		TAU		GAM	
NO	NO	x	y	x	y
1	3	0.021646610 01	0.391263100 01	0.315533570 01	0.376263170 01
	4	0.553190110 01	0.178138340 01	0.549159540 01	0.147422070 01
	5	0.569262580 01	0.162464920 01	0.560417570 01	0.123042380 01
	6	0.662921120 01	0.154274750 01	0.662486300 01	0.117476370 01
	7	0.102620170 02	0.137844590 01	0.102131780 02	0.948742850 00
	8	0.370875310 02	0.241561800 01	0.370740470 02	0.219891120 01
	10	0.825072400 01	0.333467060 01	0.318989900 01	0.370193580 01
	11	0.640650410 01	0.178471400 01	0.632797720 01	0.147824350 01
	12	0.663851500 01	0.162377930 01	0.661333750 01	0.127931990 01
	13	0.910314930 01	0.140056940 01	0.905308700 01	0.980609300 00
	14	0.212965680 02	0.146940440 01	0.212730770 02	0.107663800 01
2	1	0.121423320 02	0.236115930 01	0.191232540 02	0.213894210 01
	2	0.135743410 02	0.205703220 01	0.135374570 02	0.179760430 01
	3	0.380731000 01	0.173691380 01	0.367363700 01	0.142016530 01
	4	0.27055330 01	0.145955800 01	0.279717480 01	0.106316020 01
	5	0.303591380 01	0.168430260 01	0.291939450 01	0.135531380 01
	6	0.347632690 01	0.203511980 01	0.332991370 01	0.177248760 01
	7	0.431211130 01	0.253541560 01	0.471422370 01	0.232987320 01
	9	0.112951900 03	0.211368090 01	0.112947470 03	0.186216190 01
	10	0.401952280 01	0.165992410 01	0.389315040 01	0.132489550 01
	11	0.331267880 01	0.144520300 01	0.315313880 01	0.104337250 01
	12	0.334730280 01	0.165978060 01	0.319496220 01	0.132471570 01
	13	0.357776900 01	0.188246590 01	0.343517640 01	0.159489120 01
	14	0.466316380 01	0.275415120 01	0.455468370 01	0.256619350 01
3	1	0.265337250 01	0.225574090 01	0.246149700 01	0.202198000 01
	2	0.216912140 01	0.172000850 01	0.192486040 01	0.139943390 01
	3	0.152726060 01	0.126254030 01	0.123353920 01	0.770719090 00
	4	0.120161450 01	0.117472500 01	0.666241180 00	0.616424190 00
	5	0.120772640 01	0.117722440 01	0.577202350 00	0.621174130 00
	6	0.117841240 01	0.117809820 01	0.623422660 00	0.622823520 00
	7	0.123741340 01	0.137287290 01	0.723829210 00	0.940627410 00
	8	0.216135040 01	0.331046130 01	0.191609910 01	0.315581330 01
	9	0.123919170 01	0.203755510 01	0.162634950 01	0.163473180 01
	10	0.131303640 01	0.135574820 01	0.350997060 00	0.915452440 00
	11	0.112263680 01	0.123505470 01	0.650001000 00	0.724817240 00
	12	0.113005350 01	0.124287730 01	0.626528530 00	0.735067780 00
	13	0.115837750 01	0.122122320 01	0.585659490 00	0.700909460 00
	14	0.129701540 01	0.131445160 01	0.825983650 00	0.853102040 00

TABLE 7.9. Values of Tau and Gam for case 1. (at $\alpha = 0.05$ and $\gamma = 0.8$)

PRO PT.		TAU		GAM	
NO	NO	α	γ	α	γ
1	3	0.447453060 01	0.292563990 01	0.436135570 01	0.274943060 01
	4	0.279907720 01	0.151758080 01	0.261435140 01	0.114154260 01
	5	0.230213240 01	0.141423030 01	0.216212700 01	0.100002370 01
	6	0.213170230 01	0.134223080 01	0.188269560 01	0.595311950 00
	7	0.193538730 01	0.124721750 01	0.171515680 01	0.745353210 00
	8	0.233852640 01	0.152703950 01	0.265654510 01	0.115511620 01
	10	0.462059700 01	0.274361840 01	0.451108820 01	0.255488590 01
	11	0.237224930 01	0.151205570 01	0.269254380 01	0.113415720 01
	12	0.244171420 01	0.136270350 01	0.222776680 01	0.925839610 00
	13	0.211859340 01	0.126786800 01	0.186773610 01	0.779415910 00
	14	0.211158330 01	0.130958530 01	0.185978690 01	0.845584860 00
2	1	0.139983900 02	0.234254170 01	0.189720540 02	0.211837240 01
	2	0.135304230 02	0.205037770 01	0.134934230 02	0.178998570 01
	3	0.252567480 01	0.148275200 01	0.231927430 01	0.109473460 01
	4	0.217365120 01	0.133319520 01	0.192996360 01	0.889250480 00
	5	0.243534010 01	0.146058850 01	0.222055830 01	0.106457440 01
	6	0.206217030 01	0.171358220 01	0.268179390 01	0.139153300 01
	7	0.391810740 01	0.202454990 01	0.378834600 01	0.176034150 01
	9	0.112339390 03	0.210222020 01	0.112335440 03	0.184915210 01
	10	0.266200150 01	0.145295800 01	0.246703300 01	0.105403100 01
	11	0.230154290 01	0.133499730 01	0.207294470 01	0.884430850 00
	12	0.250972230 01	0.144329760 01	0.238806200 01	0.104072470 01
	13	0.237910450 01	0.158607100 01	0.269705930 01	0.123110570 01
	14	0.335780210 01	0.218071870 01	0.372594110 01	0.193792000 01
3	1	0.263592310 01	0.223796240 01	0.243887080 01	0.200211780 01
	2	0.216210430 01	0.171444420 01	0.191694940 01	0.139259440 01
	3	0.147733060 01	0.123395080 01	0.108017730 01	0.722934710 00
	4	0.117784150 01	0.114423770 01	0.622342820 00	0.556129340 00
	5	0.117839740 01	0.115256330 01	0.623394210 00	0.573063810 00
	6	0.115065150 01	0.115789090 01	0.569209090 00	0.583704770 00
	7	0.118503230 01	0.127975350 01	0.635847130 00	0.798604480 00
	8	0.173536640 01	0.232270200 01	0.141827240 01	0.209641230 01
	9	0.189884700 01	0.207323320 01	0.161419330 01	0.182182590 01
	10	0.126362010 01	0.129970910 01	0.772486720 00	0.830207050 00
	11	0.115273790 01	0.119666320 01	0.573515340 00	0.657269130 00
	12	0.114411230 01	0.120099920 01	0.555371270 00	0.665130360 00
	13	0.113424370 01	0.119478690 01	0.535265230 00	0.653846950 00
	14	0.122274930 01	0.126064500 01	0.703644710 00	0.767610470 00
4	3	0.183654700 01	0.152402540 01	0.154042360 01	0.115006670 01
	4	0.135542540 01	0.133667680 01	0.914974300 00	0.686963890 00
	5	0.135908310 01	0.136312180 01	0.920384070 00	0.926337490 00
	6	0.140329020 01	0.133515600 01	0.984491380 00	0.884670370 00
	7	0.149314680 01	0.132793060 01	0.111554640 01	0.873575510 00
	8	0.236133230 01	0.185594920 01	0.268089960 01	0.156350480 01
	10	0.185583670 01	0.171657770 01	0.156337130 01	0.139522010 01
	11	0.139724100 01	0.129368110 01	0.975849620 00	0.820737920 00
	12	0.142468790 01	0.125597510 01	0.101475380 01	0.759916810 00
	13	0.143694590 01	0.125286020 01	0.103189800 01	0.754757440 00
	14	0.157076710 01	0.136144840 01	0.122168150 01	0.953099980 00

TABLE 7.10. Values of Tau and Gam for case 2 (at $\alpha = 0.05$ and $\gamma = 0.8$)

PHO PT.		TAU		GAM	
NO	NO	x	y	x	y
1	3	0.350610110 01	0.276754360 01	0.336046800 01	0.258056150 01
	4	0.199995680 01	0.140005750 01	0.173200090 01	0.979878010 00
	5	0.196365400 01	0.131535550 01	0.168995170 01	0.855263590 00
	6	0.213505570 01	0.132000740 01	0.188638880 01	0.561637750 00
	7	0.199579110 01	0.123316270 01	0.172718910 01	0.721583780 00
	8	0.463206540 01	0.157171650 01	0.452283420 01	0.121255630 01
	10	0.415979600 01	0.259393640 01	0.403780920 01	0.239348400 01
	11	0.233242780 01	0.138888440 01	0.210718220 01	0.963546390 00
	12	0.206355160 01	0.127171600 01	0.181077490 01	0.785660030 00
	13	0.231542070 01	0.126826200 01	0.208834220 01	0.780056770 00
	14	0.196732740 01	0.134599310 01	0.169421870 01	0.900950000 00
2	1	0.130354330 02	0.419939030 01	0.129970200 02	0.407910260 01
	2	0.836326390 01	0.230384080 01	0.330326340 01	0.207549570 01
	3	0.174949540 01	0.180552240 01	0.143552570 01	0.150330010 01
	4	0.144513750 01	0.143356700 01	0.104327490 01	0.109588320 01
	5	0.232662510 01	0.172368980 01	0.210075810 01	0.140396110 01
	9	0.167016990 02	0.299377330 01	0.166717350 02	0.282712600 01
	10	0.132306020 01	0.177766210 01	0.152431900 01	0.146772190 01
	11	0.177971170 01	0.140708970 01	0.147220030 01	0.989399570 00
	12	0.260821830 01	0.160379000 01	0.240890130 01	0.150121900 01
3	4	0.235143660 01	0.167674390 01	0.212820440 01	0.134590570 01
	5	0.156580360 01	0.154933630 01	0.120486210 01	0.118340580 01
	6	0.159090760 01	0.165669240 01	0.123732160 01	0.132084430 01
	7	0.196941590 01	0.155028530 01	0.169664350 01	0.118464540 01
	8	0.182197390 02	0.283169150 01	0.181922760 02	0.270261330 01
	11	0.270434350 01	0.187972010 01	0.251320620 01	0.159164940 01
	12	0.176664320 01	0.137371510 01	0.145637500 01	0.949134050 00
	13	0.176839020 01	0.144346060 01	0.145849370 01	0.104095070 01
	14	0.195762060 01	0.166989930 01	0.168293750 01	0.133737190 01
4	1	0.486837350 01	0.250811130 01	0.476456300 01	0.230013520 01
	2	0.343044180 01	0.147302330 01	0.328145250 01	0.108157180 01
	3	0.235631450 01	0.134442970 01	0.213359280 01	0.398605210 00
	4	0.182976490 01	0.124916840 01	0.153233140 01	0.748613120 00
	5	0.177366640 01	0.134970890 01	0.146488650 01	0.996484540 00
	6	0.242755500 01	0.172238090 01	0.221201790 01	0.140235380 01
	7	0.430314930 01	0.187254240 01	0.418534270 01	0.158316610 01
	9	0.751696260 01	0.190346770 01	0.745014950 01	0.161962620 01
	10	0.255517550 01	0.135744700 01	0.235136770 01	0.917966330 00
	11	0.191983350 01	0.127115890 01	0.163882900 01	0.784757920 00
	12	0.200636290 01	0.132830810 01	0.173939410 01	0.875060580 00
	13	0.275919100 01	0.149508430 01	0.257160160 01	0.109794150 01
	14	0.468949880 01	0.205238410 01	0.458163710 01	0.179285620 01

TABLE 7.11. Values of Tau and Gam for case 3. (at $\alpha = 0.05$ and $\gamma = 0.8$)

PHOTO	POINT	TAU		DELTA (um)	
		x	y	x	y
1	3	0.821646610 01	0.391263100 01	0.69020 02	0.32870 02
	4	0.553170110 01	0.178138340 01	0.46890 02	0.14960 02
	5	0.569269530 01	0.162464920 01	0.47820 02	0.13850 02
	6	0.669991120 01	0.154274750 01	0.56280 02	0.12960 02
	7	0.102620170 02	0.137844590 01	0.86200 02	0.11580 02
	8	0.370375310 02	0.241561800 01	0.31150 03	0.20290 02
	10	0.325072400 01	0.383467060 01	0.69310 02	0.32210 02
	11	0.640650410 01	0.176471400 01	0.53810 02	0.14990 02
	12	0.668351500 01	0.162377930 01	0.56120 02	0.13640 02
	13	0.913314930 01	0.140056940 01	0.76510 02	0.11760 02
	14	0.212965620 02	0.146940440 01	0.17890 03	0.12340 02
2	1	0.191493320 02	0.236115930 01	0.16090 03	0.19830 02
	2	0.135743410 02	0.205703220 01	0.11400 03	0.17280 02
	3	0.300731000 01	0.173691380 01	0.31980 02	0.14590 02
	4	0.277055330 01	0.145955800 01	0.24950 02	0.12260 02
	5	0.303591330 01	0.163430260 01	0.25920 02	0.14150 02
	6	0.347682690 01	0.203511980 01	0.29210 02	0.17100 02
	7	0.431911870 01	0.253541560 01	0.40480 02	0.21300 02
	9	0.112951900 03	0.211368090 01	0.94880 03	0.17750 02
	10	0.401952930 01	0.165992410 01	0.33760 02	0.13940 02
	11	0.331267380 01	0.144520800 01	0.27830 02	0.12140 02
	12	0.334730280 01	0.165978060 01	0.28120 02	0.13940 02
	13	0.357776980 01	0.188246590 01	0.30050 02	0.15810 02
	14	0.466316080 01	0.275415120 01	0.39170 02	0.23130 02
3	1	0.265687250 01	0.225574890 01	0.22320 02	0.18950 02
	2	0.216912140 01	0.172000850 01	0.18220 02	0.14450 02
	3	0.153796060 01	0.126254030 01	0.13340 02	0.10610 02
	4	0.120161450 01	0.117472500 01	0.10090 02	0.98680 01
	5	0.120772640 01	0.117722440 01	0.10140 02	0.98590 01
	6	0.117841240 01	0.117809820 01	0.98990 01	0.98960 01
	7	0.123741340 01	0.137237290 01	0.10390 02	0.11530 02
	8	0.216135040 01	0.331046180 01	0.18160 02	0.27610 02
	9	0.199919170 01	0.208955510 01	0.16040 02	0.17550 02
	10	0.131308640 01	0.135574320 01	0.11030 02	0.11390 02
	11	0.119263660 01	0.123505470 01	0.10020 02	0.10370 02
	12	0.113005350 01	0.124287730 01	0.99120 01	0.10440 02
	13	0.115837750 01	0.122122320 01	0.97350 01	0.10260 02
	14	0.129701540 01	0.131445160 01	0.10890 02	0.11040 02

TABLE 7.12. Undetected Gross Errors for case 1. (at $\alpha = 0.05$ and $\gamma = 0.8$)

PHOTO		POINT		TAU		DELTA (um)			
NO	NO	x		y		x		y	
1	3	0.447453060	01	0.292563990	01	0.37590	02	0.24580	02
	4	0.279907720	01	0.151758000	01	0.23510	02	0.12750	02
	5	0.238213240	01	0.141423030	01	0.20010	02	0.11880	02
	6	0.213170330	01	0.134223080	01	0.17910	02	0.11270	02
	7	0.193538730	01	0.124721750	01	0.16680	02	0.10480	02
	8	0.233852640	01	0.152783950	01	0.23240	02	0.12830	02
	10	0.462059700	01	0.274361840	01	0.38810	02	0.23050	02
	11	0.287224930	01	0.151205570	01	0.24130	02	0.12700	02
	12	0.244191420	01	0.136278350	01	0.20510	02	0.11450	02
	13	0.211859340	01	0.126786800	01	0.17800	02	0.10650	02
	14	0.211158830	01	0.130953530	01	0.17740	02	0.11000	02
2	1	0.139983900	02	0.234254170	01	0.15960	03	0.19680	02
	2	0.135304200	02	0.205037770	01	0.11370	03	0.17220	02
	3	0.252567430	01	0.148275200	01	0.21220	02	0.12460	02
	4	0.217365120	01	0.133819520	01	0.18260	02	0.11240	02
	5	0.243534010	01	0.146058650	01	0.20460	02	0.12270	02
	6	0.236217030	01	0.171358220	01	0.24040	02	0.14390	02
	7	0.321310740	01	0.202454990	01	0.32910	02	0.17010	02
	9	0.112330390	03	0.210222820	01	0.04370	03	0.17660	02
	10	0.266200150	01	0.145295800	01	0.22360	02	0.12230	02
	11	0.230154290	01	0.133499730	01	0.19330	02	0.11210	02
	12	0.253972230	01	0.144329760	01	0.21750	02	0.12120	02
	13	0.207210450	01	0.158607100	01	0.24180	02	0.13320	02
	14	0.335730210	01	0.218071870	01	0.32410	02	0.18320	02
3	1	0.263592310	01	0.223796240	01	0.22140	02	0.18500	02
	2	0.216210430	01	0.171444420	01	0.18160	02	0.14400	02
	3	0.147738060	01	0.123395080	01	0.12410	02	0.10370	02
	4	0.117734150	01	0.114423770	01	0.08940	01	0.06120	01
	5	0.117339740	01	0.115256330	01	0.08990	01	0.06820	01
	6	0.115065150	01	0.115739090	01	0.06650	01	0.07250	01
	7	0.118503230	01	0.127975350	01	0.09540	01	0.10750	02
	8	0.173536640	01	0.232270200	01	0.14530	02	0.19510	02
	9	0.139384700	01	0.207823320	01	0.15950	02	0.17460	02
	10	0.126362010	01	0.129970910	01	0.10610	02	0.10920	02
	11	0.115278790	01	0.119666320	01	0.06830	01	0.10050	02
	12	0.114411230	01	0.120099920	01	0.06110	01	0.10090	02
	13	0.113424370	01	0.119478690	01	0.09520	01	0.10040	02
	14	0.122274930	01	0.126064500	01	0.10270	02	0.10590	02
4	3	0.133654700	01	0.152402540	01	0.15430	02	0.12800	02
	4	0.135542540	01	0.133667680	01	0.11390	02	0.11230	02
	5	0.135908310	01	0.136312180	01	0.11420	02	0.11450	02
	6	0.140329020	01	0.133515600	01	0.11790	02	0.11220	02
	7	0.142314620	01	0.132783060	01	0.12530	02	0.11150	02
	8	0.236133230	01	0.135594920	01	0.24040	02	0.15590	02
	10	0.135583670	01	0.171657770	01	0.15590	02	0.14420	02
	11	0.139724100	01	0.129368110	01	0.11740	02	0.10870	02
	12	0.142463790	01	0.125597510	01	0.11970	02	0.10550	02
	13	0.143674590	01	0.125286020	01	0.12070	02	0.10520	02
	14	0.157376710	01	0.138144840	01	0.13260	02	0.11600	02

TABLE 7.13. Undetected Gross Errors for case 2. (at $\alpha = 0.05$ and $\gamma = 0.8$)

PHOTO	POINT	TAU		DELTA (um)	
		x	y	x	y
1	3	0.350610110 01	0.276754360 01	0.29450 02	0.23250 02
	4	0.199955680 01	0.140005750 01	0.16800 02	0.11760 02
	5	0.196365400 01	0.131585550 01	0.16490 02	0.11050 02
	6	0.213505570 01	0.132000740 01	0.17930 02	0.11090 02
	7	0.199579110 01	0.123316270 01	0.16760 02	0.10360 02
	8	0.463206540 01	0.157171650 01	0.38910 02	0.13200 02
	10	0.415979600 01	0.259398640 01	0.34940 02	0.21790 02
	11	0.233242790 01	0.138888440 01	0.19590 02	0.11670 02
	12	0.206655160 01	0.127171600 01	0.17380 02	0.10680 02
	13	0.231542070 01	0.126826200 01	0.19450 02	0.10650 02
	14	0.196732740 01	0.134599810 01	0.16530 02	0.11310 02
2	1	0.130354330 02	0.419989030 01	0.10950 03	0.35250 02
	2	0.836326390 01	0.230384080 01	0.70250 02	0.19350 02
	3	0.174949540 01	0.180552240 01	0.14700 02	0.15170 02
	4	0.144513750 01	0.148356700 01	0.12140 02	0.12460 02
	5	0.232662510 01	0.172368980 01	0.19540 02	0.14480 02
	9	0.167016990 02	0.299877330 01	0.14030 03	0.25190 02
	10	0.162306020 01	0.177766210 01	0.15310 02	0.14930 02
	11	0.177271170 01	0.140708970 01	0.14950 02	0.11520 02
	12	0.260321230 01	0.180379000 01	0.21910 02	0.15150 02
3	4	0.235143660 01	0.167674390 01	0.19750 02	0.14080 02
	5	0.156520360 01	0.154933830 01	0.13150 02	0.13010 02
	6	0.159090060 01	0.165669240 01	0.13360 02	0.13920 02
	7	0.126941590 01	0.155028530 01	0.16540 02	0.13020 02
	8	0.132197390 02	0.288169150 01	0.15300 03	0.24210 02
	11	0.270484850 01	0.187972010 01	0.22720 02	0.15790 02
	12	0.176664320 01	0.137871510 01	0.14840 02	0.11560 02
	13	0.176839020 01	0.144346060 01	0.14850 02	0.12130 02
	14	0.195762060 01	0.166989930 01	0.16440 02	0.14030 02
4	1	0.436337350 01	0.250811130 01	0.40890 02	0.21070 02
	2	0.343044180 01	0.147302330 01	0.28820 02	0.12370 02
	3	0.235631450 01	0.134442970 01	0.19790 02	0.11290 02
	4	0.182976490 01	0.124916840 01	0.15370 02	0.10490 02
	5	0.177366640 01	0.134970890 01	0.14900 02	0.11340 02
	6	0.242755500 01	0.172238090 01	0.20390 02	0.14470 02
	7	0.430314930 01	0.137254240 01	0.36150 02	0.15730 02
	9	0.751696260 01	0.190346770 01	0.63140 02	0.15990 02
	10	0.255519550 01	0.135744700 01	0.21460 02	0.11400 02
	11	0.191983350 01	0.127115890 01	0.16130 02	0.10680 02
	12	0.200636290 01	0.132880810 01	0.16350 02	0.11160 02
	13	0.275919100 01	0.143503430 01	0.23130 02	0.12470 02
	14	0.463949800 01	0.205288410 01	0.39390 02	0.17240 02

TABLE 7.14. Undetected Gross Errors for case 3. (at $\alpha = 0.05$ and $\gamma = 0.8$)

Case	Tau		Gam		Max.undetected gross errors (μm)	
	x	y	x	y	x	y
1	112.951900	3.912631	112.947470	3.782682	948.80	32.87
2	112.339890	2.925640	112.335440	2.749431	943.70	24.58
3	18.219739	4.199890	18.192276	4.079103	153.00	35.28

TABLE (7.15) Maximum Tau, Gam and Undetected Gross Errors for photogrammetric measurements (at $\alpha = 0.05$, and $\gamma = 0.8$)

An insight into Table (7.15) reveals that, agreeable with the findings of Chapters 5 and 6, higher internal reliability reflects higher external reliability. With one exception, all the results given in Table (7.15) are in line with computed values of $r_{\max}(x)$ and $r_{\max}(y)$ for the different cases. Case 3 provides the best chance of detecting gross errors in the x-axis direction, yet it manifests the lowest level of detection in the y-axis direction.

7.3.3. Simulated models for sensitivity analysis

In addition to the settlement model discussed in Section (5.6.2.3) the drift model was regarded. By drift model it is meant all the targets were to undergo displacement of the order of 10 mm in the positive direction of the X-axis, i.e. in a direction which is nearly perpendicular to the dam wall. Recalling Equation (3.59) to compute the just-detectable deformation ($\tilde{cd}$):

$$\omega^u = \frac{(\tilde{c}\tilde{d})^T Q_d^+ (\tilde{c}\tilde{d})}{\sigma_0^2} \quad (7.3)$$

where $\tilde{d}$ is a form vector which characterises the deformation model under test.

The values of the sensitivity parameter, c , for both Settlement and Drift models are shown in Table (7.16) and graphically represented in Figures (7.8) and (7.9).

Number of cameras	Model designation	Sensitivity parameter c
3	Settlement	0.020460
3	Drift	0.005428
4L	Settlement	0.023120
4L	Drift	0.006224
4	Settlement	0.022920
4	Drift	0.004877

TABLE (7. 16) Number of cameras against the sensitivity parameter (c) for Settlement and Drift models. (at $\alpha = 0.05$ and $\gamma = 0.8$)

In general, the two figures pertain to the outcomes of Chapters 5 and 6 concerning the sensitivity analysis in such a way that the more the number of cameras, the less the sensitivity of the photogrammetric network.

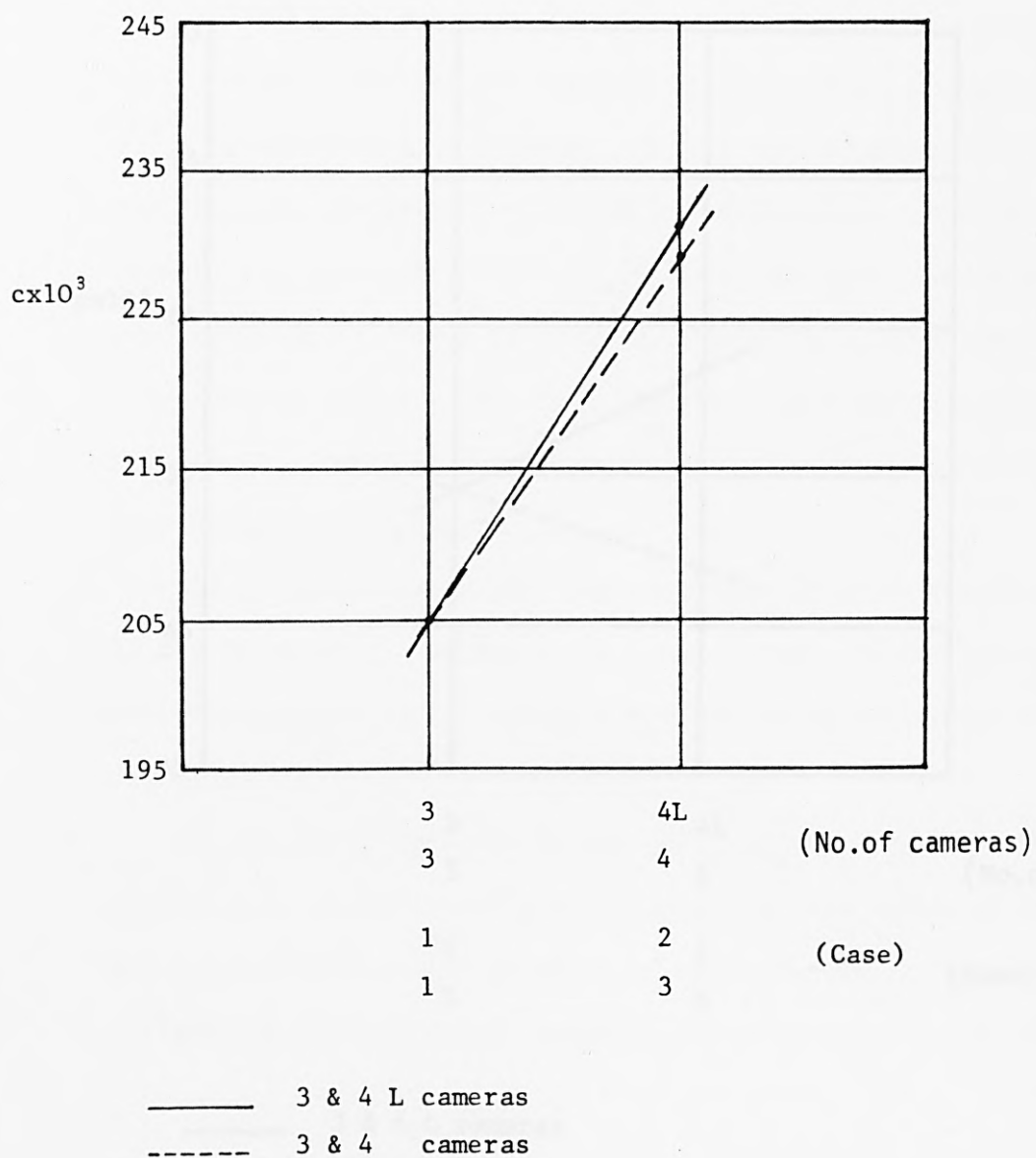


Figure 7.8. Relation between sensitivity parameter (c) and no. of cameras (Settlement model).

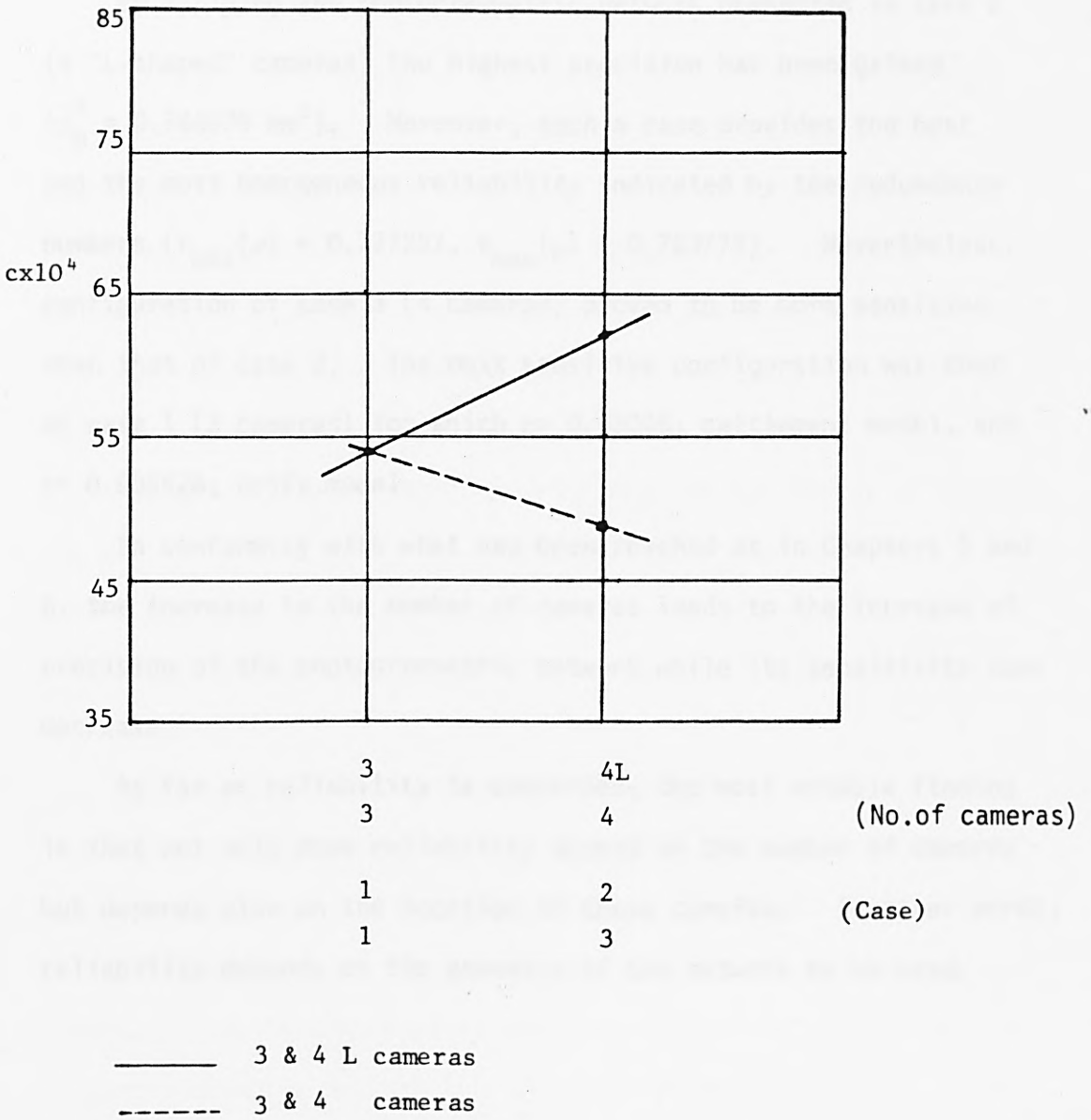


Figure 7.9. Relation between sensitivity parameter (c) and no. of cameras (Drift model).

7.4. Conclusions

By applying the photogrammetric network presented in case 2 (4 "L-shaped" cameras) the highest precision has been gained ($\sigma_m^2 = 0.746979 \text{ mm}^2$). Moreover, such a case provides the best and the most homogeneous reliability indicated by the redundancy numbers ($r_{\max}(x) = 0.777297$, $r_{\max}(y) = 0.763779$). Nevertheless, configuration of case 3 (4 cameras) proved to be more sensitive than that of case 2. The most sensitive configuration was that of case 1 (3 cameras) for which $c = 0.02046$; settlement model, and $c = 0.005428$; drift model.

In conformity with what has been reached at in Chapters 5 and 6, the increase in the number of cameras leads to the increase of precision of the photogrammetric network while its sensitivity does decrease.

As far as reliability is concerned, the most notable finding is that not only does reliability depend on the number of cameras but depends also on the location of these cameras. In other words, reliability depends on the geometry of the network to be used.

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

Detailed discussion of results and conclusions are given in Chapters 5, 6 and 7. The following is a summary of the important findings pertaining to all aspects of the research.

1. The techniques which have been developed for design of a photogrammetric network for precision, reliability and sensitivity can be adapted to the required specifications in any deformation survey.
2. The precision is almost proportional to the number of cameras in use. However, there is an insignificant improvement in precision especially after implementing six cameras as in the case study of the cube. Such an increase cannot be justified economically.
3. The redundancy number, which lies between 0 and 1, as an indicator of reliability has been used successfully in testing the reliability of the simulated networks in Chapters 5, 6 and 7.
4. The inclusion of additional observations through increasing the number of cameras used in order to improve the reliability of a photogrammetric network is a precocyped misconception. Such an increase of number of cameras does lead to the degradation of the reliability unless the cameras are properly configured. Therefore the geometrical configuration or disposition of the cameras, is a very important factor in reliability which should be accounted for when designing networks.

5. A network, whether it be photogrammetric or geodetic, must be precise as well as reliable. In practice, one could encounter a very precise network which is not reliable at all.

6. Based on the simulation studies it was found that the main factor which affects the reliability is the number of images per object point. More intersecting rays from an object point give better reliability, i.e. larger redundancy numbers. Nevertheless it is noticed that improvement in reliability slows down considerably after 6 rays (cube case study). Therefore it is advisable not to try to improve the redundancy, in turn the reliability, if the object point already has 6 rays and its image co-ordinates display redundancy numbers of the order of 0.6 or more.

7. It is proved that at 95% confidence the networks are least sensitive to a single point movement and most sensitive to multiple point displacements depending on the hypothetical pattern of movement of that cluster of points.

8. The more cameras used the less sensitive the networks would be at 95% confidence. This can be thought of as being due to the increasing correlation between the object point co-ordinates when more cameras are used.

9. The addition of survey measurements namely, slope distances, provides a marginal improvement in precision "within" the cases of the same number cameras, while it considerably enhances the improvement in both precision and reliability "between" the different number of cameras cases. As for its effect on sensitivity it can be exhibited as follows:

- (a) It does not affect the sensitivity for the settlement model of the cube case study whereas it has slightly increased the sensitivity for both the deflexion and expansion models postulated in that case.
- (b) On the contrary, for the bridge case study the settlement and deflexion models lead to slightly worse values of the sensitivity parameter (c). This can be explained as follows: the slope distances are measured on the upright faces of the cube but in the bridge it is the presence of transverse distances which led to what might be an increase in the correlation between the co-ordinates of the opposite linked targets.

10. The photogrammetric monitoring system is both independent of the structure, which is a necessity when hazardous deformations occur, and economical in a sense that if the number of targets were increased it would not affect the costs too much. In addition, sometimes unexpected deformation occurs. In this case, the photographic record is a valuable source of information about the state of the object at the time of photography.

11. The set of equations used in the pre-analysis in this research is a subset of the equations used in the least squares adjustment.

12. Considerable savings in computer storage and processing time has been obtained by:

- (a) The direct formation of the normal equations matrix without the need to form the observation equations.
- (b) The computation and storage of only the diagonal elements of the cofactor matrix of the residuals.

Moreover, the software accepts a normal equations matrix of any size. The size would be limited by the hardware only. In other words, any number of cameras, object points, or slope distances can be accommodated.

With these thoughts in mind, the following recommendations are made.

1. It is highly recommended that after arriving at the suitable design for the three criteria, the resulting network configuration is to be executed, adjusted and checked for the goodness of the adjustment. This can be achieved by applying χ^2 and F tests. Above all, a posteriori analysis must be carried out to emphasise that the functional analysis complies with the assumptions made before.
2. The banded bordered structure of the normal equations matrix has not been fully exploited as the number of targets in this investigation is too small. If it happened that such a number is large (more than a few hundred) then further work should be oriented towards the full utilisation of that structure which would save a great deal of computer storage.
3. The effect of incorporating slope distances between the camera stations and the object points, rather than distances between the object points only, on the three design criteria is an important aspect for future research. However, it is difficult with metric cameras in current production to locate the perspective centre. It might be undertaken with the Centrax camera, produced by the National Physical Laboratory, London.

4. In-situ measurements (inclinometers, extensometers, strain gauges, etc.) can be usefully incorporated with the photogrammetric and survey measurements in a widened deformation study. This is a useful but undeveloped area of further research into design of monitoring deformation. In such a case, structural deformation models will play an important part.

APPENDIX A

SOME PROPERTIES OF GENERALISED MATRIX INVERSES

A.1. Significance of generalised inverses

Recent years have seen development for rectangular and singular matrices of an analogue of the inverse of non-singular matrices. The resulting matrices, known generally as generalised inverses, play an important role in understanding the solutions to linear equations $AX=Y$ when A has no inverse (Searle, 1982).

Given here is a brief account of generalised inverses in addition to their properties, especially as they pertain to the least squares estimation.

It has been shown that the column rank of A , the design matrix, and hence that of N , the normal equations matrix, is deficient by 7, in a three-dimensional space, due to the orientation, translation and scale defects so that:

$$\text{rank}(A) = 6n_s + 3n_o - 7$$

in which n_s denotes the number of photographs and n_o is the number of object points. Consequently, the normal equations matrix cannot be inverted using the standard Cayley inverse. Then, generalised matrix algebra is required, particularly to search for an inverse so that the solution is unique, the primary pre-requisite property.

A.2. Definitions

Penrose (1955) on foundations laid by Moore (1920) shows that for any matrix A there is a unique matrix K which satisfies the following conditions:

$$(i) \quad AKA = A \quad (A.1)$$

$$(ii) \quad KAK = K \quad (A.2)$$

$$(iii) \quad (KA)^T = KA \quad (A.3)$$

$$(iv) \quad (AK)^T = AK \quad (A.4)$$

Such a matrix K exists and is unique no matter what the form of A is, be it square (singular or non-singular) or rectangular (Boullion and Odell, 1971; Searle, 1971). It was named after Penrose as Moore-Penrose inverse and Greville (1957) used instead the term "pseudoinverse". Table (A.1) presents some names for matrices satisfying some or all of the Penrose conditions (equations A.1 through A.4)

Conditions satisfied	Name of matrix	Symbol
i	Generalised inverse	A^-
i and ii	Reflexive generalised inverse	A^r
i,ii and iii	Left weak generalised inverse	A^w
i,ii and iv	Right weak generalised inverse	A^n
i,ii,iii and iv	Moore-Penrose inverse (pseudoinverse)	A^+

TABLE A.1. Names of matrices satisfying some or all of Penrose conditions.

A.3. Useful identities related to the Moore-Penrose inverses

The following well-known identities involving the Moore-Penrose inverses will be listed without proof (e.g. Rao and Mitra, 1971; Blaha, 1982a, 1982b, 1982c).

$$(A^+)^+ = A \quad (A.5)$$

$$(A^T)^+ = (A^+)^T \quad (A.6)$$

$$(AA^T)^+ = (A^+)^T A^+ \quad (A.7)$$

$$A^+ A = AA^+ \text{ iff } A \text{ is normal, i.e. } A^T A = AA^T \quad (A.8)$$

$$(A^n)^+ = (A^+)^n \text{ iff } A \text{ is normal} \quad (A.9)$$

$$(AA^T)^+ AA^T = AA^+ \quad (A.10)$$

$$A^+ = (A^T A)^+ A^T = A^T (AA^T)^+ \quad (A.11)$$

$$A^+ = A^+ (A^+)^T A^T = A^T (A^+)^T A^+ \quad (A.12)$$

$$A^T = A^T AA^+ = A^+ AA^T \quad (A.13)$$

$$A^T = A^T AA^+ = A^+ AA^T \quad (A.14)$$

$$A = (A^+)^T A^T A = AA^T (A^+)^T \quad (A.15)$$

$$(A^T A)^+ = A^+ (A^+)^T \quad (A.16)$$

$$A^+ A = (A^T A)^+ A^T A = A^T A (A^T A)^+ \quad (A.17)$$

$$\text{It is very important to notice that: } A^+ A \neq I \quad (A.18)$$

$$(A^-)^- \neq A \quad (A.19)$$

A.4. Some computational methods of Moore-Penrose inverses

A rapid review of some of the theorems for computing the Moore-Penrose inverses is provided. They are to be construed as neither the best nor the only possible. Strictly speaking, not all of which may be suitable for computer coding.

A.4.1. Computation of A^+ when independent rows or columns are identifiable

Let, if possible, A be partitioned in the form:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

by a permutation of rows and columns, if necessary, such that A_{11} is an $r \times r$ matrix of rank r ; A_{12} , A_{21} and A_{22} are matrices of suitable orders such that the order of A is $m \times n$.

In such a case $A_{22} = A_{21} A_{11}^{-1} A_{12}$

Then one choice of a generalised inverse of A is:

$$A^- = \begin{bmatrix} A_{11}^{-1} & 0 \\ 0 & 0 \end{bmatrix} \quad (\text{A.20})$$

and the Moore-Penrose inverse A^+ (Graybille, 1969; Rao and Mitra, 1971) is as follows:

$$A^+ = \begin{bmatrix} A_{11}^T B A_{11} & A_{11}^T B A_{21} \\ A_{12}^T B A_{11} & A_{12}^T B A_{21} \end{bmatrix} \quad (\text{A.21})$$

where

$$B = (A_{11} A_{11}^T + A_{12} A_{12}^T)^{-1} A_{11} (A_{11}^T A_{11} + A_{21}^T A_{21})^{-1}$$

A.4.2. Computation of A^+ based on factorisation of matrices

(a) Rank factorisation

If the $m \times n$ matrix A is of rank r then it can be factorised in the form:

$$A = BC \quad (\text{A.22})$$

in which B is an m x r matrix and c an r x n matrix and both of rank r.

Although the factorisation is not unique since if Y is any non-singular r x r matrix, BY^{-1} are also factors of the specific type.

The Moore-Penrose inverse (Greville, 1960; Peters and Wilkinson, 1970; Rao and Mitra, 1971) is given by:

$$A^+ = C^T(CC^T)^{-1}(B^TB)^{-1}B \quad (A.23)$$

(b) The LU and related factorisations (Square matrices only)

According to Peters and Wilkinson (1970) the simplest factorisation used in solving linear systems are those related to Gaussian elimination technique. These effectively give a decomposition of A of the form $A = LU$ where L is lower triangular and U is upper triangular matrices. Either L or U can be chosen to have a unit diagonal or alternatively one can have $A = LDU$ where D is a diagonal matrix and both L and U have unit diagonals, then:

$$A^+ = U^T(L^T L U U^T)^{-1} L^T \quad (A.24)$$

The matrix $Y = (L^T L U U^T)$ is of dimension r x r, $L^T L$ and $U U^T$ are symmetric positive definite.

(c) Singular value decomposition

Any matrix A of order m x n can be written (Rao and Mitra, 1971) in the following form:

$$A = \lambda_1 v_1 u_1^T + \dots + \lambda_r v_r u_r^T \quad (A.25)$$

in which $\lambda_1^2, \dots, \lambda_r^2$ are the non-zero eigenvalues of AA^T or $A^T A$ and the vectors $v_1, \dots, v_r$ are orthonormal eigenvectors of AA^T and $u_1, \dots, u_r$ are orthonormal eigenvectors of $A^T A$ corresponding to the eigenvalues $\lambda_1^2, \dots, \lambda_r^2$. Then:

$$A^+ = \lambda_1^{-1} u_1 v_1^T + \dots + \lambda_r^{-1} u_r v_r^T \quad (A.26)$$

(d) Diagonal reduction

Searle (1971) defines A^- as a generalised inverse which satisfies the first condition of Penrose (equation A.1) such an inverse is not unique as will be shown below.

If a matrix A has order $m \times n$ then it can be reduced to a diagonal form as follows:

$$\begin{matrix} P & A & Q \\ m \times m & m \times n & n \times n \end{matrix} = \begin{matrix} \Delta \\ m \times n \end{matrix} = \begin{bmatrix} D_{r \times r} & 0_{r \times (n-r)} \\ 0_{(m-r) \times r} & 0_{(m-r) \times (n-r)} \end{bmatrix} \quad (A.27)$$

or simply, as:

$$PAQ = \Delta = \begin{bmatrix} D_r & 0 \\ 0 & 0 \end{bmatrix}$$

where P and Q are products of elementary matrices (non-singular), r is the rank of A and D_r is a diagonal matrix of order r .

Analogous to Δ , Δ^- can be defined (Searle, 1971) as:

$$\Delta^- = \begin{bmatrix} D_r^{-1} & 0 \\ 0 & 0 \end{bmatrix} \quad (\text{A.28})$$

then

$$A^- = Q\Delta^-P \quad (\text{A.29})$$

Clearly A^- as given by (A.29) is not unique because neither P nor Q by their definition is unique, neither Δ nor Δ^- . However, the Moore-Penrose inverse A^+ can be established (Searle, 1972) as:

$$A^+ = FA^T \quad (\text{A.30})$$

According to Searle (1971) the derivation of F can be done as follows.

Consider $A^T A$, for some integer t there will be a series of scalars $\lambda_1, \lambda_2, \dots, \lambda_t$ not all zero, such that:

$$\lambda_1 A^T A + \lambda_2 (A^T A)^2 + \dots + \lambda_t (A^T A)^t = 0$$

If λ_r is the first λ in this identity that is non-zero, then F is defined as (Searle, 1971):

$$F = (-1/\lambda_r) [\lambda_{r+1} I + \lambda_{r+2} (A^T A) + \dots + \lambda_t (A^T A)^{t-r-1}]$$

APPENDIX B

INNER CONSTRAINTS APPROACH

B.1. Minimal adjustment constraints

In a free network adjustment because the columns of the design matrix A are linearly dependent the system of normal equations is singular. Any solution vector will be biased statistically as shown below.

Let the functional model be given by the linearised observation equations:

$$A\Delta x = v + b \quad (B.1)$$

in which $\Delta x, v$ and b are the n -vector of unknown parameter corrections, the n -dimensional vector of residuals and the n -vector related to the observations, respectively.

With the W matrix as the weight matrix of the observations, the classical least squares solution is:

$$\hat{\Delta x} = (A^T W A)^{-1} A^T W b \quad (B.2)$$

$$\begin{aligned} E\{\hat{\Delta x}\} &= E\{(A^T W A)^{-1} A^T W b\} \\ &= (A^T W A)^{-1} A^T W E\{A\Delta x - v\} \end{aligned}$$

with $E\{v\} = 0$, then

$$\begin{aligned} E\{\hat{\Delta x}\} &= (A^T W A)^{-1} A^T W A E\{\Delta x\} \\ &= E\{\Delta x\} \end{aligned} \quad (B.3)$$

On the other hand if A is not of full (column) rank, then the standard Cayley inverse will be replaced by the Moore-Penrose inverse (see Appendix A). Then:

$$\Delta \hat{x} = (A^T W A)^+ A^T W b \quad (B.4)$$

$$\begin{aligned} E\{\Delta \hat{x}\} &= E\{(A^T W A)^+ A^T W b\} \\ &= (A^T W A)^+ A^T W A E\{\Delta x\} \end{aligned}$$

with $(A^T W A)^+ A^T W A \neq I$, then:

$$E\{\Delta \hat{x}\} \neq E\{\Delta x\} \quad (B.5)$$

However, as only the shape defined by the co-ordinates, and not the co-ordinates themselves, is important, such a bias (equation B.5) is not of significance.

The singularity of the normal equations matrix will disappear if information about the definition of the co-ordinate system is introduced (Leick, 1982). This is accomplished by imposing as many constraints upon the adjustment as there is rank deficiency.

Adjustments which incorporate no more and no less conditions than are necessary to define the reference system lead to the so-called minimal constraint solutions. Each specific choice of constraints results in a different adjustment. Some quantities remain invariant with respect to each choice; others vary. The adjusted observations represent an invariant set (Cooper, 1980; Blaha, 1982b) while the vector of unknown parameters x , and their covariance matrix $\sigma_0^2 Q_{xx}^{\wedge}$ are identified as variants.

One widely used scheme for imposing minimal constraints to overcome the defect inherent in the normal equations matrix is to suppress d , rank deficiency, appropriate columns from the design matrix. Nevertheless, there is one pitfall in such an approach,

the reference system may be ill-defined by some particular set of minimal constraints.

B.2. Inner adjustment constraints

It is advisable to define the reference system the best way bringing the inherent numerical difficulties ((ill- definition) pointed out in Section (B.1), to a minimum. In this sense "best" is interpreted as resulting the smallest trace of the covariance matrix of the unknown parameters (co-ordinates) (Blaha, 1971) and minimises $\Delta x^T \Delta x$. The set of minimal constraints which materialise this "best" co-ordinate system is called inner adjustment constraints; first developed by Meissl (1962) for geodetic networks (Ashkenazi, 1973) and reported in detail by Blaha (1971, 1982a, 1982b, 1982c).

According to Meissl (1966) a free network can be obtained from a given arbitrary network by a Helmert transformation (Papo and Perelmutter, 1982) G^T matrix. This transformation is to be applied to the corrections to the approximate co-ordinates so that $G^T \Delta x = 0$ (inner constraints). Such a transformation can be arrived at by subjecting the object point network to three translations, three rotations and a scale change all differentially small (Fraser, 1982a) What follows is a detailed discussion concerning the construction of the G matrix.

Let $(X_i^0)^T = [X^0 \ Y^0 \ Z^0]_i$ denotes the co-ordinates of point i in an original co-ordinate system; and
 $X_i^T = [X \ Y \ Z]_i$ its co-ordinates in a new system;

and

$$dt^T = [dt_1 \quad dt_2 \quad dt_3]$$

where

$$dt_1^T = [\delta X \quad \delta Y \quad \delta Z] \quad \text{differential translations; and}$$

$$dt_2^T = [\delta \omega \quad \delta \phi \quad \delta \kappa] \quad \text{differential rotations; and}$$

$$dt_3 = \delta L \quad \text{change in scale}$$

$$\text{Let } G \text{ be partitioned as } G = [G_t \quad G_r \quad G_s]$$

where t, r and s refer to translation, rotation and scale respectively.

For the differential translations, one can write:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i = \begin{bmatrix} X^0 \\ Y^0 \\ Z^0 \end{bmatrix}_i + \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} \quad (B.6)$$

thus the differential changes due to the translations $\delta X, \delta Y, \delta Z$ are as follows:

$$\begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix}_{\delta X, \delta Y, \delta Z} = \underset{3 \times 3}{I} \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} \quad (B.7)$$

which gives:

$$G_{t_i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad dt_1 = \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} \quad (B.8)$$

The differential orientations can be found as follows:

As is known, it is possible to relate the co-ordinates of two Cartesian co-ordinate systems having the same origin by the equation:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R^T \begin{bmatrix} X^0 \\ Y^0 \\ Z^0 \end{bmatrix} \quad (B.9)$$

where the rotation matrix R can be written as follows (Ghosh, 1979):

$$R = R_\omega R_\phi R_\kappa$$

$$R = \begin{bmatrix} \cos\phi \cos\kappa & \cos\omega \sin\kappa + \sin\omega \sin\phi \cos\kappa & \sin\omega \sin\kappa - \cos\omega \sin\phi \cos\kappa \\ -\cos\phi \sin\kappa & \cos\omega \cos\kappa - \sin\omega \sin\phi \sin\kappa & \sin\omega \cos\kappa + \cos\omega \sin\phi \sin\kappa \\ \sin\phi & -\sin\omega \cos\phi & \cos\omega \cos\phi \end{bmatrix} \quad (B.10)$$

Now assuming differentially small rotations $\delta\omega$, $\delta\phi$, $\delta\kappa$ around X^0, Y^0, Z^0 axes respectively it follows:

$$\begin{aligned} \sin\omega &= \delta\omega \\ \sin\phi &= \delta\phi \\ \sin\kappa &= \delta\kappa \\ \cos\omega &= \cos\phi = \cos\kappa \approx 1 \end{aligned} \quad (B.11)$$

and neglecting second-order terms, the rotation matrix becomes:

$$R = \begin{bmatrix} 1 & \delta\kappa & -\delta\phi \\ -\delta\kappa & 1 & \delta\omega \\ \delta\phi & -\delta\omega & 1 \end{bmatrix} \quad (B.12)$$

It should be noted that R (equation B.12) is orthogonal only up to first-order terms. Therefore, the differential changes due to the rotations $\delta\omega$, $\delta\phi$, $\delta\kappa$ are given by:

$$\begin{bmatrix} X - X^0 \\ Y - Y^0 \\ Z - Z^0 \end{bmatrix}_i = R_i^T \begin{bmatrix} X^0 \\ Y^0 \\ Z^0 \end{bmatrix}_i - \begin{bmatrix} X^0 \\ Y^0 \\ Z^0 \end{bmatrix} = (R_i^T - I) \begin{bmatrix} X^0 \\ Y^0 \\ Z^0 \end{bmatrix} \quad (B.13)$$

or in a compact form:

$$\begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix}_i = \begin{bmatrix} 0 & -\delta\kappa & \delta\phi \\ \delta\kappa & 0 & -\delta\omega \\ -\delta\phi & \delta\omega & 0 \end{bmatrix} \begin{bmatrix} X^0 \\ Y^0 \\ Z^0 \end{bmatrix} \quad (B.14)$$

$\delta\omega, \delta\phi, \delta\kappa$

Thus,

$$G_{r_i} = \begin{bmatrix} 0 & Z^0 & -Y^0 \\ -Z^0 & 0 & X^0 \\ Y^0 & -X^0 & 0 \end{bmatrix}; \quad dt_2 = \begin{bmatrix} \delta\omega \\ \delta\phi \\ \delta\kappa \end{bmatrix} \quad (B.15)$$

For the change in scale, one has:

$$\begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix}_i = \begin{bmatrix} X^0 \delta L \\ Y^0 \delta L \\ Z^0 \delta L \end{bmatrix} = \delta L \begin{bmatrix} X^0 \\ Y^0 \\ Z^0 \end{bmatrix} \quad (B.16)$$

δL

Hence,

$$G_{s_i} = \begin{bmatrix} X^0 \\ Y^0 \\ Z^0 \end{bmatrix} \quad \text{and} \quad dt_3 = \delta L \quad (B.17)$$

Finally, if we put $C = G^T = \begin{bmatrix} G_t^T \\ G_r^T \\ G_s^T \end{bmatrix}$ then C constitutes a set of

minimal constraints, namely the set of inner constraints (Blaha, 1971).

Then:

$$G_i^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -Z^0 & Y^0 \\ Z^0 & 0 & -X^0 \\ -Y^0 & X^0 & 0 \\ X^0 & Y^0 & Z^0 \end{bmatrix} \quad (B.18)$$

(7x3)

This pattern of G-matrix repeats for each object point.

B.3. Geometrical interpretation of inner constraints

Let x_i^0 denote the set of approximate co-ordinates of the i th unknown point, Δx_i denote the corrections to these co-ordinates, and x_i denote the adjusted co-ordinates, i.e.

$$x_i = x_i^0 + \Delta x_i \quad (B.19)$$

For those inner adjustment constraints which correspond to G_t^T it holds that:

$$G_t^T \Delta x = 0 \quad (B.20)$$

In vector notation this can be expressed as:

$$\sum_{i=1}^{n_0} \delta x_i = 0 \quad (B.21)$$

in which n_0 = number of object points in a network.

The geometrical interpretation of these conditions is that the centre of gravity of all points will not change after adjustment, i.e.

$$\sum_{i=1}^{n_0} x_i = \sum_{i=1}^n x_i^0 \quad (\text{B.22})$$

The second set of constraint equations,

$$G_r^T \Delta x = 0 \quad (\text{B.23})$$

corresponds to the conditions:

$$\sum_{i=1}^{n_0} x_i^0 \times \delta x_i = 0 \quad (\text{B.24})$$

If the centre of the system remains fixed, then the cross products $x_i^0 \times \delta x_i$ reflect rotations of the points around the fixed centre. These constraint equations ensure that the sums of the rotations around all three co-ordinate axes are zero. Geometrically, this means that the mean orientation of the system of points involved in forming G_r will not change after adjustment either.

Finally, the part

$$G_s^T \Delta x = 0 \quad (\text{B.25})$$

with the previous notations and interpretations yields:

$$\sum_{i=1}^{n_0} (x_i^0 \cdot \delta x) = 0 \quad (\text{B.26})$$

which means that the average scale of the network will be held fixed.

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