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Price Exuberance Episodes in Private Real Estate*

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July 12, 2024

Abstract

In this paper, we investigate price exuberance episodes in the main UK commercial real estate sectors – retail, offices and industrials - over the period December 1986 - April 2022. Using the Backward Supremum Augmented Dickey Fuller approach of Phillips et al. (2015a,b), we find that episodes of price explosiveness are asynchronous across sectors with only common phase being the period 2003-2007. We also conduct a multivariate probit analysis to identify factors that indicate the occurrence of price exuberance episodes and generate early signals for possible price bubble building. The predictors for price explosiveness differ by sector with more consistent signals obtained from the yield curve for retail and industrials, rent growth for offices and industrials, and inflation for retail and offices. A key implication of this study is that the study of price exuberance and bubbles in private real estate should be sector specific even within the same country.

Keywords: Commercial Real Estate, Price Exuberance, Backward Supremum Augmented Dickey Fuller Test, Warning Indicators.

J.E.L. Classification Numbers: C58, E30, E44, R30.

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1 Introduction

The real estate market plays an important role in financial and banking crises either precipitating or aggravating adverse developments, as in the economic slump and banking troubles in the UK in the early 1990s (Gentle et al., 1994), the Asian financial crisis in 1997 (Quigley, 2001; Collins & Senhadji, 2002), and the global financial crisis in 2008-09 (Bullard et al., 2009; Stanton & Wallace, 2010; Mishkin, 2011). During these crises, lenders and investors (particularly the latter in the global financial crisis) have heavy exposure to the property market. A significant and sudden fall in property values accompanied by a reduction in the disposable income generated by real estate proves damaging for the banking sector, construction companies, and for general investors exposed to real estate.

Episodes of overvaluations in the real estate sector and the potential destabilizing effects of price bubbles bursting are flagged up as a source of risk to the broader financial system and the economy (European Systemic Risk Board, 2015, ESRB; Mallbauer, 2022), and may lead Government interventions, across the globe, in order to reduce the probability of a housing market failure (Floro, 2019; Akbari & Krystyniak, 2021). Significant falls in property values and deficient cash flows from underlying real estate assets most likely leads to a credit crunch and cause stress to lenders not only through a breach of underwriting criteria but also through a rise in nonperforming loans. Investment portfolios of pension funds and sovereign funds (with allocations that can exceed 15%), insurance companies and property funds face large reductions in their portfolio valuations. Property funds may even be forced to cease trading as the case was in summer 2016 in the UK. Further, the impact on the construction industry is severe too as values may not cover replacement costs bringing construction activity to a halt with apparent repercussions on fixed capital investment and employment. Even if overvaluations in real estate are not per se the trigger for a financial crisis, they increase the vulnerability of the financial system to exogenous or internal shocks.

The potential destabilizing effect of overvaluations in the real estate market, with prices not supported by fundamentals creating conditions for price bubble formation, has long been acknowledged by scholars. Relevant research on price bubble in real estate is ongoing and follows the tradition of the broader literature on the subject. In line with this literature, real estate studies are concerned with: (i) generating sustainable or equilibrium values for real estate that reflect fundamentals and (ii) applying alternative methods to detect deviations of actual values from their fundamental path and hence establish the formation of price bubbles and their size. The pursuit of these objectives in empirical research informs the decision making of lenders, investors and developers.

There are two broad streams in price bubble work in real estate. The first stream focuses on the securitised segment of the market primarily represented by listed Real Estate Investment

Trusts (REITs) and related corporations such as real estate operating companies. The second stream examines price bubbles in the private sector, underlying, or direct market (representing the actual real estate assets). This distinction is important to be made as the causes for price bubble formation and the triggers for the potential burst of the bubble in the two segments can be diverse. There is plenty of research on real estate bubbles especially for US REITs reflecting the greater availability of data (Brooks et al., 2001; Jirasakuldech et al., 2006; Anderson et al., 2011; Nneji et al., 2013; Escobari & Jafarinejad, 2016; Xie & Chen, 2015; Huerta-Sanchez et al., 2020).

The private or underlying real estate market deserves attention as it is the segment that backs values and performance across the real estate investment universe. REITs valuations reflect (and expected to reflect) the real assets REITs are invested in (Waters & Payne, 2007; Hoesli & Oikarinen, 2021). On the debt side of the market, conditions in the private market underlie the underwriting of debt products. The global financial crisis demonstrates how complex derivatives such as collateralized debt obligation became worthless when the underlying market (primarily residential) run into trouble.

Within the private element of the real estate sector it is appropriate to make a further distinction between work on the residential and commercial sectors. Most of the work is on the residential sector (Goodman & Thibodeau, 2008; Baur & Heaney, 2017; Bourassa et al., 2019; Pavlidis et al., 2019). These studies develop fundamental models of the housing market and apply alternative methods to detect and quantify the size house price bubbles. In contrast, there is a dearth of studies on the private commercial sector the main components of which are offices, retail and warehouses. The commercial component in institutional portfolios is large. In the UK, based on Morgan Stanley Capital International (MSCI) data for 2021, the office component was 20.5%, the retail component 25% and the industrial/distribution warehouse allocation 32.6%, with residential representing a much smaller proportion of 8.3%. With a 78% allocation to the commercial sector, a collapse in commercial property prices will have significant repercussions on investor portfolio valuations and renders the early detection of price bubble incidents in commercial real estate a valuable exercise.

A small number of early studies addressed the topic of commercial price bubbles (Hendershott, 2000; Hendershott et al., 2003) but it is only recently that the topic is examined more systematically. An example is the work on price bubbles in the private commercial market by Deghi et al. (2021) and Investment Property Forum Research Report (IPF, 2020). The present study focuses on price bubbles in the private commercial real estate market and extends both the works of Pavlidis et al. (2016, 2019), and more recent works on commercial property sector by IPF (2020) and Deghi et al. (2021).

The presence of explosive prices does not necessarily imply bubbles as such behavior may

reflect changes in fundamentals.

The aim is to detect episodes of price exuberance in UK commercial real estate and identify their potentially predictors. More specifically, we investigate price exuberance episodes in the main UK private real estate sectors (Retail, Offices, Industrials), using the BSADF approach (Phillips et al., 2015a,b) over the period December 1986-April 2022. Following Pavlidis et al. (2016), we also investigate how economic, financial and real estate indicators are related to such explosiveness.

The empirical findings form part of an early warning system that key participants in this market -lenders, investors and developers - can use to detect the creation of bubbles on a continuing basis. For example, the findings are informative to real estate investors (property funds, pension funds, sovereign wealth funds, private equity, other), for possible portfolio rebalancing as the formation of a price bubble is detected. Similarly, lenders can adjust underwriting criteria and review risk assessments.

This study makes a number of empirical contributions. A well-known feature of real estate markets is that they are diverse, there is much heterogeneity among geographical markets and sectors alike. Countries have different levels of liquidity and display varied degrees of transparency, as documented in the surveys by JLL (2022). Hence, this factual geographical heterogeneity of real estate markets warrants the study of real estate price bubbles at country specific level. It should be noted that the degree of market transparency and information efficiency determine the quality of appraisals and market pricing. Similarly pricing is affected by different levels of liquidity (that is the volume of real estate assets transacted). In less transparent, less informationally efficient markets price bubble formation can be concealed or exuberance about real estate prices not assessed properly. In thin markets (lack of transactions) price evidence from market trades is sparse. In this paper we acknowledge that the study of bubbles is more appropriate to be country specific controlling for such non-cyclical factors that can distort pricing. Our focus is the UK market, the most sought-after country in cross-regional and global capital flows in real estate (CBRE, 2022). Further, the UK benefits from a comprehensive private real estate database with the concomitant data transparency and price discovery appealing to international investors.

A further contribution we make is the study of bubbles at sector level. Deghi et al. (2021) refer to the relevance of e-commerce in the study of bubbles in the retail sector. Indeed, the dynamics of price developments in retail differ from those in the office sector. The latter for example, as result of the COVID-19 pandemic and technology facilitating remote/smart working, faces a new occupation paradigm that is likely to shock demand for office space, rents and prices. In contrast, the business of logistics drives pricing in the distribution warehouse sector. A study that has made this distinction is Xiao & Tan (2007) for Hong Kong real estate:

authors break down the real estate market into three commercial parts (offices, retail, flatted factories) and residential, and deploy a state space model to spot bubbles. In conclusion these authors claim that the results do not clearly establish the bubble proportion in each sector. Still though, there is evidence to suggest that prices are not explained by fundamentals across the board and tentatively retail and flatted factories exhibit greater bubble proportions. Clearly, work at sector level brings additional information and improves our understanding of real estate price bubble dynamics. Another notable feature of the present investigation, is the monthly frequency of the data we use for the underlying market, as per the study of Xiao & Tan (2007), that allows the study of price bubbles by sector monthly. Monitoring bubble formation at monthly intervals is particularly valuable at times of uncertainty.

There are several approaches to test for the presence of exuberance behavior. In this paper, we apply the Backward Supremum Augmented Dickey Fuller (BSADF) approach, introduced by Phillips et al. (2015a,b), to find evidence of a exuberance behavior. This method detects bubbles in all sectors but the date of occurrence of bubble episodes do not coincide. This supports the argument that a more aggregated analysis of price bubbles in the private market conceals sector specific dynamics. We subsequently investigate how key indicators relate to the explosiveness of prices. We examine the role of a number of indicators the choice of which is guided both by existing studies and also time series that are closely monitored by real estate professionals in the UK when assessing property market trends. The results support the information content to analyze price exuberance of OECD's leading indicator, inflation, employment, the yield curve, consumer confidence and rent growth along with online sales in the retail sector. The findings also make clear that the relevance of the indicators differs by sector.

The paper is organized as follows. Section 2 present a review of studies with focus on the commercial property market. This review contains methods to spot exuberance behavior in real estate and methods to produce fundamental values. Section 3 briefly describes the data used and the econometric methodology implemented, and also reports the main empirical findings. Section 4 offers an in-sample forecasting assessment. Section 5 concludes.

2 Literature Review

In this section, we review some selected studies to illustrate how the price bubble research has evolved in real estate and how it should progress to assess risks to lenders, the economy and property investors. A workable distinction of this research area can be made between securitised and private real estate.

Most of the papers on bubbles in securitised real estate, specifically on REITs, focuses on

the United States (US). This is the result of readily available REITs data, spanning back to the early 1970s, and the straightforward application of methods deployed in similar studies in the stock market. The production of fundamental REITs share prices is broadly common across studies. The dividend growth model and the Gordon growth model are mostly used. Based on pertinent studies in the stock market, a range of tests is used to establish the presence of bubbles in REITs prices.

Brooks et al. (2001) and Brooks & Katsaris (2003) examine whether actual and fundamental REITs prices cointegrate. The presence of cointegration is indicative of bubbles (originally suggested by Diba & Grossman (1988a; 1988b)). Jirasakuldech et al. (2006) apply several tests to detect price bubbles. They consider the consumer price index, industrial production, the federal funds rate and the default risk premium (spread of Aaa and Baa bonds) as the fundamental variables determining equity REITs returns. The simple approach to detect bubbles is checking the integration order of the fundamental variables and actual prices. If the integration order of the latter is higher than that of the former it is a sign of a bubble. The second method comprises cointegration tests to study the presence of a bubbles in prices. A third approach is based on the duration dependence principle that underpins relevant work in the stock market (Cochran & Defina, 1995). This method utilises the relationship between the length of a positive run and the probability of the positive run ending.

Waters & Payne (2007) apply a momentum threshold autoregressive process to the residuals of the cointegrating relationship between REITs prices and dividends. The objective is to pick up possible asymmetric adjustments in the case of positive or negative bubbles. This method is appropriate to establish periodically collapsing bubbles.

Anderson et al. (2011) apply the van Norden-Schaller model (Van Norden & Schaller, 1993) to detect periodically collapsing bubbles and calculate the probability of a bubble collapsing. This methodology examines the different behaviour of returns in two regimes – a bubble surviving and a bubble collapsing regime. The probability of a bubble collapsing depends on the size of the bubble. A measure of the size of the bubble is used to determine the regime and regime switching. The fundamental values in this study are generated from the dividend multiple measure. These authors develop and evaluate trading strategies to profit from early signals about a bubble collapsing. These strategies are based on forecasting the time varying probabilities of a bubble collapsing. Periodically collapsing bubbles represent one of the focal points of Phillips et al. (2011) and subsequent work by these authors.

2.1 Private Real Estate

Price bubble research in the private real estate market is sparse and yet this is the segment of the market that can be a source of major risks. In price bubble investigations, the situation

in the direct market is opposite from that in the securitised market. There is dearth of studies on price bubbles much owing to the restricted availability of data unlike the REITs market. Still though there is data for such research in certain jurisdictions (UK, US). In other market the length and frequency of the available data is short of what scholars would desire. Another reason is the idiosyncrasies of the private market that must be considered when generating fundamental values. A factor than can complicate the outcome and interpretation of price bubble research in the private market is the range of models constructed to explain prices in the real estate market. There is much work on price determination in commercial real estate and the range of models estimated can result in different fundamental values. Nevertheless, the reality remains, that there is a notable gap in price bubble research in real to estate to be filled.

Two recent studies have addressed the deficiency of bubble research in real estate. The overall objective of the paper by Deghi et al. (2021) is to study the impact of commercial price bubbles on the economy and the banking system. Following the conventional structure in such research, these authors estimate fundamental (or fair) values. The deviation of fair values from the actual values determine the size of "misalignment" in prices. A structural VAR represents a general equilibrium framework to obtain fair commercial property values. The variables entering this VAR are net operating income growth, the spread between the capitalization rate (cap rate) and government bond yields, the output gap, yearly CPI inflation, non-financial corporate credit-to-output gap, broad money to output, the 3-month short-term interest rate and capital flow to output. The impact of price misalignment on future (commercial real estate) CRE prices is quantified with the construction and use of a commercial real estate price at risk measure. They find that a ten-basis point rise in misalignment corresponds to a 2.5% increase in downside risk in advanced economies. This impact on prices can amount to 10% over a three-year period.

The study by IPF (2020) takes a different approach. The generation of fundamental values is based both on naïve methods and simple empirical models. The empirical models in this study are reduced type regression equations. A key differentiation from the study of Deghi et al. (2021) is that bubbles are studied in capital values (prices) not in total returns. In terms of cycles there is little difference between the two (see Figure 1). Defining capital value as rent over the capitalization rate, price bubbles in the IPF study are examined through: (i) direct tests on the capital value series and (ii) based on trends in sustainable rents and sustainable cap rates. Income or rent in the IPF study is explicitly modeled to get equilibrium rent values whereas Deghi et al. (2021) use the actual net operating income.

Similarly, in the IPF (2020), cap rates are modeled to establish equilibrium cap rates. Based on the definition earlier, equilibrium/sustainable capital values are produced from sustainable

rents divided by sustainable cap rates.

A noteworthy feature of the IPF study is the use of different models for equilibrium rents and cap rates. The study then assesses whether implied price corrections (by all models/methodologies) accurately predict actual outturns and whether these results are sensitive to the different methodologies used. The evaluation is based on the conventional mean error, mean absolute error, root mean squared error and Theil U1 metrics. The overall picture from this forecast evaluation suggests that the method to obtain sustainable values and consequently implied corrections does matter. Model predictions can have different biases, the size of the error can differ with some models performing better on the mean error and mean absolute error criteria but worse so on root mean squared error and Theil U1 metrics which penalize for large errors. A further implication of this study is that results seem to be conditional on the actual data series used. In the IPF study the analysis is performed with MSCI UK Real Estate Indexes data along with data from Rent and Yield Monitor (CBRE) and JLL UK Property Index (JLL) (that is three sources but the MSCI data are dominant in acceptance and usage).

Given the importance of the private real estate market to the wider system, in this paper we focus on the private real estate market, which has important policy implications and/or warning signals for lenders and investors. The aims can extend to assess whether corrections implied by the estimated bubbles materialise (as in the IPF study).

Further it is recommended that the geographical focus should be the UK. This reflects the availability and superior quality of real estate data in the UK. This enables a researcher to have choice of more methodologies to deploy. Within the UK it is appropriate to execute the analysis by main sector, that is office, retail and industrial. In this way the research will be able to address additional questions. For example are bubbles occurring simultaneously in every sector and are they of the same magnitude? This is important information for say lender to have as the industrial may experience lower price bubble risks compare to another sector. Collapsing all sectors into a general category masks different price bubble dynamics.

Our focus in bubbles is capital values. This is because the income return component of total returns is rather stable through time with the volatility of total returns mirroring that of capital values. Figure 1 below illustrates this point.

[INSERT SOMEWHERE HERE FIGURE 1]

All data in Figure 1 are in nominal terms and total return amounts to income return plus capital growth. The sample period is December 1986 to April 2022 (monthly frequency). The green bar series shows that income return (mostly income from rents net of any costs and charges such as ground rent) has displayed a downward trend though time but it does not exhibit much

of cyclicity. Capital growth on the other hand is the main source of the observed oscillations in total return data.

3 Empirical application

3.1 Data

Data for capital values in the sectors offices, retail and industrial are obtained from MSCI (MSCI Real Assets Analytics). MSCI assembles the data to construct performance indices from real estate investment portfolio managers. The data series are generated from open market valuations of actively managed portfolios and track prices in each sector among many property asset performance metrics calculated (MSCI, 2022). The MSCI database is used to monitor the different aspects of direct property market performance in the UK. The performance data compiled by MSCI exclude the impact of debt, hence they represent the true value of the underlying real asset. The MSCI data series are considered the standard benchmark for real estate investor. The data series used in this study are at UK level (national series) and of monthly frequency for the period December 1986 to April 2022.

[INSERT SOMEWHERE HERE FIGURE 2]

Figure 2 plots the capital value series for the three sectors and illustrates the relative cyclical pattern of prices in the three sectors. A distinctive trend emerges since 2013. Capital values in industrials largely deviate from the trajectory of the other two sectors exhibiting a strong upward trend. This owes to the popularity of distribution warehouses, a component of the industrial category, supported by e-commerce and online shopping.

The above data are also available for sub-sectors within retail (shopping centres, retail warehouses, standard unit shops), offices, industrial, and for regions in the UK (e.g., London, South East, West End, rest of UK). Table 1 shows the MSCI indexes composition.

[INSERT SOMEWHERE HERE TABLE 1]

3.2 Measuring price exuberance

Asset prices deviations from market fundamentals and the subsequent collapse are conventionally considered as financial bubbles (Blanchard, 1979; Diba & Grossman, 1988b; Evans, 1991; Vogel & Werner, 2015; Lee & Phillips, 2016; Roy & Kemme, 2020; Chen et al., 2022). In

financial markets, the bubble phenomena can be demonstrated using the present value theory of finance which also applies to the valuation and present value calculation of commercial real estate assets:

$$P_t = \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}(RE_{t+i} + U_{t+i}) + B_t \quad (1)$$

where P_t is the price of the asset at time t ; r is the discount rate, which in real estate is based on the risk free rate and risk premium and characterizes the investor's target rate of return; RE_{t+i} is the payoff received from the commercial asset in the form of rent (net of any costs); U_{t+i} represents the unobservable fundamentals at time t ; B_t is the bubble component at time t .

Equation (1) provides the basis for the valuation of commercial real estate assets. Equation (2) below represents a discounted cash flow valuation of a commercial assets that is a freehold investment, let on a long lease with regular rent reviews¹:

$$P_t \equiv CV_t = \sum_{m=1}^T \frac{RE_t}{(1+r)^m} + \sum_{m=1}^T \frac{RE_t(1+g)^T}{(1+r)^{T+m}} + \sum_{m=1}^T \frac{RE_t(1+g)^{2T}}{(1+r)^{2T+m}} + \dots + \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}(RE_{t+i} + U_{t+i}) + B_t \quad (2)$$

where, in addition, CV_t is the capital value of the real estate asset – a common term used for commercial prices, g is the growth rate of net rent at each rent review and for simplicity it is assumed constant, T is the rent review interval (for example 3 years). The bubble component B_t satisfies the following submartingale property:

$$\mathbb{E}_t(B_{t+1}) = (1+r)B_t \quad (3)$$

when $B_t = 0$, there is no presence of a bubble and the degree of nonstationarity of the asset price relies on the net rent series and unobservable fundamentals. For example, if D_t is an $I(1)$ process and U_t is either an $I(0)$ or an $I(1)$ process, then the asset price is at most an $I(1)$ process. Otherwise, given Equation (3), asset prices will be explosive in the presence of bubbles. When unobservable fundamentals are at most $I(1)$ and D_t is stationary after differencing, empirical evidence of explosive behavior in asset prices may be used to infer the existence of bubbles.

¹See, for example, Clayton (1996), Brown & Matysiak (2000), Martinez-Garcia & Grossman (2018), Hirano & Akira (2024). Additionally, see also the guidelines provided by the Royal Institution of Chartered Surveyors (2023).

3.3 Exuberance behavior in UK private real estate

Exuberance behavior in asset prices is a primary indicator of market exuberance during the phase of a bubble. These episodes may be subject to econometric testing using recursive testing procedures, like the right-sided unit root tests in Phillips et al. (2011). Recursive right-sided unit root tests seem to be particularly effective as real-time detection mechanisms for slightly explosive behavior and market exuberance.

As first step, we test whether the capital values series of the private real estate contain a unit root, against the alternative that they are explosive processes. We start with the following Augmented Dickey Fuller (ADF) regression equation:

$$\Delta y_t = \alpha_{r_1, r_2} + \beta_{r_1, r_2} y_{t-1} + \sum_{i=1}^k \delta_{r_1, r_2}^i \Delta y_{t-i} + \varepsilon_t \quad (4)$$

where Δ is the first-difference operator; y_t denotes the time series of interest at time t ; k is a scalar that denotes the number of lags of the dependent variable that are included to accommodate serial correlation; r_1 and r_2 denote the fraction of the total number of time periods in the sample that specify the starting and the ending points, respectively; α_{r_1, r_2} , β_{r_1, r_2} and δ_{r_1, r_2}^i are regression coefficients; ε_t is the error term. We test the null hypothesis of a unit root in y_t , $H_0 : \beta_{r_1, r_2} = 0$, against the alternative of exuberance behavior, $H_1 : \beta_{r_1, r_2} > 0$. The ADF test statistic corresponding to the null hypothesis is given by: $ADF_{r_1}^{r_2} = \frac{\hat{\beta}_{r_1, r_2}}{s.e.(\hat{\beta}_{r_1, r_2})}$.

Phillips et al. (2011) propose a methodology that is consistent with a single boom-bust episode. The methodology involves estimating Equation (4) using a forward expanding sample. In this setting, the beginning of the sub sample is held constant at $r_1 = 0$, while the end of the sub sample, r_2 , increases from r_0 (the minimum window size) to 1 (the entire sample period). Recursive estimation of Equation (4) yields a sequence of $ADF_0^{r_2}$ statistics. The supremum of this sequence is called the SADF and is defined as follows:

$$SADF_{r_0} = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2} \quad (5)$$

similar to the standard ADF test, when the SADF statistic exceeds the right-tailed critical value, the unit root hypothesis is rejected in favor of exuberance behavior. However, in contrast to the standard ADF test, the alternative hypothesis of the SADF test is that of exuberance dynamics in some parts of the sample. One potential limitation of the recursive approach suggested by Phillips et al. (2011) is that it provides consistent estimates of the origination and ending dates of the first bubble but not subsequent ones.

Phillips et al. (2015a,b) propose an extension of the SADF, the Generalized SADF (GSADF), which has the same alternative hypothesis as the SADF but which covers a larger number of sub

samples. The GSADF test involves an extensive set of regressions, in which the first observation varies from 0 to $r_2 - r_0$, while the last observation varies from r_0 to 1. In comparison to the SADF, the GSADF test shows a more flexibility on the estimation window and it is consistent with multiple exuberance periods, while the SADF test is consistent only with a single episode. The GSADF statistic is defined as:

$$GSADF_{r_0} = \sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2} \quad (6)$$

The rejection of the unit root hypothesis in favor of exuberance behavior requires that the test statistic exceeds the right-tailed critical value from its limit distribution.

If the null of a unit root in y_t is rejected, then the SADF and the GSADF methodologies can provide a sequence of episodes of exuberance. The inference for the ADF, SADF and GSADF statistics requires critical values computed using Monte Carlo simulations². Table 2 reports the ADF, the right-tail SADF and the GSADF statistics for UK private real estate during 1986:12 - 2022:4 time period.

[INSERT SOMEWHERE HERE TABLE 2]

For retail and industrial real estate sectors, SADF test provides evidence against the unit-root null hypothesis at the 1% significance level, while it fails to reject in the case of offices at all confidence levels. For all sectors, instead, GSADF test provides evidence of a sequence of episodes of exuberance at the 1% significance level³. The testing strategies based on recursive window and recursive flexible window estimation of Equation (4) provide useful guidance to date-stamp in real time the episodes of exuberance behavior if the null hypothesis is rejected. For completeness, Table A1 in Appendix A reports the sequences of t-statistics and corresponding critical values (95%) to identify the time periods during which episodes of explosive behavior occurred.

Consequently, Phillips et al. (2015a,b) introduce the the Backward Supremum Augmented Dickey Fuller (BSADF) statistic defined as follows:

²The finite sample critical values for 90%, 95% and 99% confidence levels are obtained from Monte Carlo simulations with 2,000 replications (see Phillips et al. (2015a,b) and Vasilopoulos et al. (2022)). The window size is given by $r_0 = \left(0.01 + 1.8/\sqrt{T}\right)$ as recommended by Phillips et al. (2015a,b). The time period is 1986:12 - 2022:4 (monthly frequency) and $T = 423$ observations.

³Table B1, in the Internet Appendix, reports the GSADF test statistics for sub-sectors within retail (shopping centres, retail warehouses, standard unit shops), offices, industrial, and for regions in the UK (e.g., London, South East, West End, rest of UK). In addition, we demonstrate that the GSADF test confirms the exuberance behavior for UK Private Real Estate using normalized price series, that is, capital values divided by rents. We also report the GSADF test for UK Real Estate Investment Trusts (REITs) in comparison to UK Private Real Estate. Tables C1 and D1, in the Internet Appendix, report the results.

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} \{BADF_{r_1}^{r_2}\} \quad (7)$$

where r_1 and r_2 denote the starting and the ending fraction of the sample, respectively, implying that $r_1 < r_2$; r_0 is the fractional threshold and it is chosen on a lower bound of 1% of the full sample with the following functional form: $r_0 = \left(0.01 + 1.8/\sqrt{T}\right)$, where T refers to the number of observations in the sample. We denote r_w , the window size of the regression, as $r_1 - r_2$. Appendix A describes the estimation approach. For completeness, Table A1 in Appendix A reports the sequences of t -statistics and corresponding critical values (95%) to identify the time periods during which episodes of explosive behavior occurred.

In our empirical analysis, given the potential presence of multiple periods of exuberance, we use the GSADF test and the BSADF date-stamping procedure. Figures 3, 4, and 5 reveal different episodes of exuberance behavior in private real estate. Regarding private real estate retail, we identify two episodes of exuberance behavior: (i) 2003:9 - 2007:8; and (ii) 2019:9-2021:3. For private real estate office, there are three episodes of exuberance behavior: (i) 1999:11 - 2001:12; (ii) 2005:11 - 2007:8; and (iii) 2014:3 - 2016:2. As far as private real estate industrial is concerned, we identify four episodes: (i) 1999:11 - 2001:4; (ii) 2003:11 - 2007:8; (iii) 2017:8 - 2019:12; and (iv) 2021:3 - 2022:4.

[INSERT SOMEWHERE HERE FIGURES 3, 4, AND 5]

3.4 *Determinants of prices exuberance in private real estate*

In this section, we study how UK economic, financial, and real estate indicators are related to the exuberance of prices in the UK's retail, offices and industrial sectors. We aim to generate evidence to address the question of what information is available to indicate the dawn of a price exuberance and bubble formation. Price explosiveness may not eventually prove harmful but the possibility of a price bubble building and collapsing with adverse consequences for the financial system and the economy is much greater. The analysis also alerts investors, lenders to real estate and policy makers about elevated risks from the detection of a possible bubble formation. We expect that the nature and significance of indicators will differ in each of the three sectors. This is warranted by their distinct cyclical activity and more importantly by the different structural changes these sectors have undergone. A prime example is the advent of e-commerce and online shopping affecting the retail sector.

To assess the information content of indicators to signal a forthcoming bubble episode, we deploy a probit as follows:

$$Pr(CVS_{j,t} = 1) = \Phi(X_{k,t-i}) \quad (8)$$

where *CVS* (*Capital Value State*) is a binary variable for price (capital value) exuberance in sector j (retail, offices, industrial). It takes the value 1 if BSADF tests indicate that prices in sector j are in an exuberance state and 0 otherwise; Φ is the cumulative normal distribution function; X_k is a vector of k potential bubble indicators. We examine both for contemporaneous and lagged effects ($t - i$) of these indicators on signaling price exuberance.

The monthly indicators we consider in the vector $X_{k,t-i}$ in Equation (8) relate both to the discount rate r in Equation (1) and to the growth rate g in Equation (2). Rent growth is the primary factor in driving net operating income of a property asset. A key component of the target rate of return is the risk premium, the required return over the risk-free rate in property investments. The risk premium reflects sentiment towards the sector, growth expectations and risk assessments. We consider the following monthly indicators, which, as noted earlier, are employed as gauges of potential bubbles in commercial real estate sectors⁴:

- $\Delta RETRE$, $\Delta OFFRE$ and $\Delta INDRE$: the monthly percentage change in nominal retail (RET), offices (OFF) and industrial (IND) rents, respectively. This is a metric to capture trends in rent growth (component g in Equations (2)). We expect strong rises or falls in rents to create conditions for price exuberance (source: MSCI's Real Estate Portal);
- ΔEMP : the monthly percentage growth in employment. This is an economic and business conditions indicator for all sectors that underlies the income growth in investment real estate. Strong employment growth rates can have the same impact, as rent growth, on price exuberance (source: Refinitiv);
- CLI : the OECD's composite leading indicator for the UK economy. The indicator comprises macroeconomic aggregates which precede movements in overall economic activity measured by GDP. Higher values of CLI can lead to price exuberance (source: OECD);

⁴Several authors used similar sets of variables to predict phases of price exuberance. For example, Deghi et al. (2021) include the growth in net operating income, the spread between the cap rate and the government bond yield, the output gap, CPI inflation, non-financial corporate credit-to-output gap, broad money-to-output and capital flow-to-output. In addition, Pavlidis et al. (2016) include the spread between the long- and short-term interest rates, long-term rates, changes in the stock market, credit growth to the private sector, real disposable income, unemployment, GDP growth, inflation quarter-on-quarter, oil prices and a global indicator of economic activity. Enoksen et al. (2020) include TED-spread as a proxy of the level of credit risk in the US economy, as the spread between the interbank interest rate and Treasury rate.

- *YC*: the yield curve measured by the difference between the 10-year UK government bond yield and the 1-year UK government bond yield. The yield curve is often cited as a leading indicator of economic activity. A positive and widening spread denotes stronger economic activity that can compress risk premia in anticipation of stronger rent growth and higher capital values. It has a positive effect on the possibility of price exuberance. Conversely, as the slope of the curve is a measure of monetary policy stance, a flatter yield curve does not point to major rises in future interest rates and therefore investors do not adjust their risk premia and discount rates. In a stable interest rate environment with threats to values from higher interest rates absent, they may even take more risks on pricing resulting in a positive contribution to the formation of a bubble (source: Bank of England);
- *PREM*: risk premium measure defined as the retail, offices and industrial yields minus the 10-year UK government bond yield. This is a metric for target rate of return. When the premium narrows it indicates more risk taking in the property investment accompanied by higher prices. Therefore, an inverse relationship is expected with price exuberance situations (source: MSCI and Bank of England);
- *CCI*: the *Consumer Confidence Index*. As an indicator that gauges consumer confidence about the state of the economy, this indicator is not only relevant to the retail sector and real estate market but also to office and industrial sectors. The association of *CCI* with episodes of price exuberance in commercial real estate is similar to *CLI* (source: OECD);
- *LEND*: the lending growth rate proxied by the M4 monetary aggregate. High growth rates in lending can generate conditions of price exuberance in the real estate market (source: Bank of England);
- ΔIRS : the annual changes in internet sales as a proportion of total retail sales for the retail sector. We can argue a negative relationship with episodes of price exuberance as a rise in internet sales affects demand for physical retail space putting pressure on income from retail properties. We need to consider another argument though for a possible positive link. The ever-rising trend of online retailing has led retailers to adapt and require property facilities for multi-level purposes (for example to support omni-channel retailing). This trend benefits assets such as retail warehouses (source: Office for National Statistics);
- *INFL*: the *Consumer Price Index* inflation. A long standing view is that real estate provides cushion against inflation as rental values increase in rent reviews to reflect inflation. The UK lease structure, of upward only rent reviews, supports this claim. This belief is

now challenged and inflation is seen as a source of uncertainty in real estate investment. Low inflation promotes confidence in economic conditions and encourages real estate investors to take on more risks accepting lower risk premia and putting pressure on prices. High inflation rates, and in particular cost push inflation, represent a risk to commercial real estate values (source: Refinitiv).

We use the following selection criteria for specifying the probit model by sector: [i] indicators entering probit models are stationary, hence variables that are not stationary enter in first difference; [ii] variable should be statistically significant individually; [iii] variables should not exhibit collinearity based on the Variance Inflation Factors (VIFs) (Chatterjee & Hadi, 2015); and [iv] maximization of *McFadden's* R^2 if conditions are satisfied.

The probit models estimated are dynamic as lags of the variables in vector X are examined. We examine lagged effects up to six months in the past. It is assumed that if a variable conveys information about price exuberance this indication will manifest itself in commercial prices within six months.

We acknowledge the persistence in the two states of the binary variable CVS . We take into account the signals from this persistence and allow for momentum effects through the estimation of an autoregressive dynamic probit given by Equation (9):

$$Pr(CVS_{j,t} = 1) = \Phi(X_{k,t-i}) + \beta CVS_{j,t-1} \quad (9)$$

for both Equations (8) and (9) $i = 0, 1, \dots, 6$.

Tables 3, 4 and 5 report the probit results for retail, offices and industrial sectors, respectively. The results suggest that the set of factors which signal price exuberance differs by sector. That applies both to the dynamic and autoregressive probit models. Therefore, a more accurate prediction of price exuberance information in each of the main real estate sectors should be obtained from a specific set of variables. The findings do not establish common indicators across all three sectors although some factors are mutual in two sectors. These results strongly suggest that a study to identify the set of price exuberance indicators should be sector specific.

[INSERT SOMEWHERE HERE TABLES 3, 4 AND 5]

The economic interpretation of the results in Tables 3-5 relates to the formation of expectations about income (net rent) from commercial assets and changes to the discount rate (investor target rate of return) that can inflate/decrease commercial values. Growth in net rents is likely to create optimism about the performance of commercial assets and push values up to a state of exuberance in the office and industrial sectors. A similar impact is reasoned from employment growth on industrial prices (which includes prices of distribution warehouse and

logistics assets). In a prolonged period of low inflation – denoting a less risky macroeconomic environment - a risk-on investment approach is likely. Real estate investors can take more risks squeezing risk premia and hence discount rates pushing capital values higher, above the intrinsic value of assets, leading to price exuberance that can prove harmful. A rising leading indicator adds to the formation of positive expectations about the economy and can result in more optimistic pricing in commercial real estate. The results for office capital values support this line of reasoning. In the second model estimated for the retail sector (autoregressive probit), the coefficient on this variable takes a negative sign at lag six. This is counterintuitive as is the sign on consumer confidence indicator in the first retail probit model (dynamic probit) at lag five. These lagged terms may reflect some other dynamics in price exuberance in the retail sector.

The negative sign of the yield curve suggests more aggressive pricing in the absence of threats from forthcoming interest rate rises both in the retail and industrial sectors. In the former, this variable at lag six has a positive impact on the possibility of price exuberance, rationalized as optimism about future market conditions. The contribution of the yield curve variable towards the likelihood of price exuberance in the retail sector appears to be conditional to its lag length. The share of internet sales in overall sales appears to be a potential source of price exuberance in the retail sector. This variable is significant at the 10% level in only one of the two retail models.

The explanatory power of the base probits is strongest for offices, followed by retail and industrials. The Hosmer-Hemeshow tests point to the validity of the dynamic and autoregressive probit models. The exception is the dynamic probit model for industrials where model fitting issues are detected. The estimation of autoregressive probit models renders many of the lagged variables redundant across the board. These models include less variables than the dynamic probits. This is not surprising given the long periods of persistence of the dependent binary variable (*CVS*). Still though certain variables from vector X still enter these models. As expected, the explanatory power of autoregressive probits has risen significantly in all three sectors. The estimated probit models are parsimonious as the low values of VIFs measure for multicollinearity suggest.

4 Forecasting assessment

4.1 *In-sample forecasts*

In this section, we evaluate how the probit fitted values can be viewed as a reliable mechanism signaling price exuberance in commercial real estate prices and price bubble building. As

the probit models are rolled forward on a monthly frequency, they provide a constant update of whether an episode of price exuberance is emerging in the corresponding sector.

[INSERT SOMEWHERE HERE TABLE 6]

Table 6 reports the in-sample forecast performance of the estimated probit models. The assessment is based on the number of cases the probit models outperforms the naïve probit model in predicting both a normal state in prices and an episode of price exuberance (established by the BSADF procedure). The threshold probabilities denote the probabilities above which price exuberance events are established ($CVS = 1$). The results of the in-sample forecasting evaluation exercise show that all probit models outperform the naïve probit model by a considerable margin. The margin is greater for the autoregressive probit across the board. Regarding the dynamic probit, the greatest gain over the naïve model is obtained for offices, followed by retail and industrials. What matters more of course is the ability of the models to predict the state of price exuberance. For predicting this state in prices, the performance of the autoregressive model is more or less similar in all sectors. We should note though that the success of the dynamic models for predicting ($CVS = 1$) is quite close to that of the autoregressive probit models. We get a further picture of the in-sample forecast performance of the models from inspecting the graphical illustrations in Figures 6, 7, and 8.

[INSERT SOMEWHERE HERE FIGURES 6 - 8]

The shaded area in Figures 6 - 8 represents periods of overvaluations (price exuberance) as detected by the BSADF procedure. The solid lines are the fitted values of the probit models (dynamic and autoregressive dynamic probits) in each of the three sectors and represent the probability of price exuberance in the sector. Threshold values for the two probits by sector are reported. Fitted values above the threshold probability signals the formation of a bubble. The larger the fitted value (the higher the fitted line) the higher the probability of a price exuberance event (positive or negative) emerging. The visual inspection reveals a more successful autoregressive probit where the rising and falling probabilities coincide with the estimated periods of price exuberance and normal pricing.

This visual representation shows occasions of false signals generated by the dynamic probit. More specifically, the retail dynamic probit generates certain short-lived false signals (e.g., in 2012 and 2017) and it is slow to forecast the normal state of prices in 2001. On two other occasions though it indicates price exuberance earlier than the autoregressive probit (in 1998/1999 and 2019). In the offices sector, the dynamic probit makes a false call of price exuberance in 1997/1998 which is followed by an early call of the 2000/2001 price exuberance. The dynamic probit detects the event in 2005 a little later than the autoregressive probit,

however, in the latest episode of price exuberance in the sample (end of 2013 to 2016), it provides a signal marginally earlier than the latter (autoregressive probit). Finally, in the industrial sector an incorrect signal is obtained in 1998, however, the dynamic probit is persistent of a forthcoming price exuberance into the 1999 – 2001 episode. The autoregressive probit indicates faster than the dynamic probit the event that began in 2003 and lasted for several years. The price exuberance events in this sector at the end of the sample are signaled accurately by both probit specifications.

4.2 *Out-of-sample forecasts*

In this section we report the results of out-of-sample forecasts (Tables 7-9). The probit models are estimated recursively and, in each round, a one-, three- and six-month ahead forecast is obtained. The models are trained up to December 2006. Hence the first set of one-, three- and six-month ahead forecasts are obtained for January 2007, March 2007 and June 2007 respectively. The estimation of the models is then rolled forward by one month and the second set of forecasts for the three horizons is generated (February 2007, April 2007 and July 2007). We continue in this manner until the sample is exhausted. In the assessments the cutoff probabilities to indicate a period of exuberance are determined by the in-sample forecast exercise. A number of models contain contemporaneous and short lag length terms. We use actual values where appropriate as out-of-sample observations for the predictors. The benchmark model to compare the forecasts is like the constant probability model we use in the in-sample forecasts.

[INSERT SOMEWHERE HERE TABLES 7 - 9]

Several conclusions are gained from the out-of-sample exercise. Like the findings in the in-sample forecast evaluation, all estimated models outperform the benchmark forecasts in the one-, three- and six-month horizons (in the gain over naïve part of the tables). There is only partial evidence to suggest that the accuracy of the forecasts improves as the horizon increases. In some cases, the models are more successful predicting correctly six months ahead than a shorter horizon. The results do not establish large differences in the prediction accuracy by time horizon though – the use of actual values for some predictors in the longer horizons should contribute to this finding. A notable difference between the in-sample forecasts and the out-of-sample predictions is that the superior performance of the autoregressive probit in the former in all three sectors is not established in the latter where predicted values for the lagged state are used in the forecasts. Overall, the relative accuracy of the forecasts from the two probit specifications varies by sector and forecast horizon.

5 Conclusions

In this paper, we studied the existence of price exuberance episodes in the main commercial real estate sectors (retail, offices and industrials) in the UK, using the Backward Supremum Augmented Dickey Fuller approach over the period from December 1986 to April 2022. Price exuberance episodes represent risk warnings as they signal the formation of bubbles in commercial prices that can subsequent collapse hitting portfolio returns and leading to breaches in lending underwriting metrics. Our study showed the existence of periods of price exuberance in all sectors, though the chronology of these periods does not fully coincide across sectors with only common phase being the 2003-2007, exaggerated price behavior has sector specific elements. This is expected as offices, retail and industrials respond differently to macroeconomic conditions and also they are affected by distinct sector specific trends such as e-commerce impacting the retail sector. Therefore, the paper suggests that studies focusing on price exuberance and bubbles should take a sector specific view to better inform investors, lenders and developers.

We also examined the role of a number of indicators that signal price exuberance. Such indicators pick up phases of exaggerated pricing and provide early warning signals of a possible forthcoming bubbles. The choice of these indicators in our analysis is guided both by existing studies and data that are closely monitored by real estate professionals in the UK when assessing commercial property market trends. Hence, the monthly indicators we examine regularly appear in industry reports to gauge business conditions in the commercial real estate sector. The factors indicating price explosiveness differs by sector with more consistent signals obtained from the yield curve for retail and industrials, rent growth for offices and industrial, and inflation for retail and offices. The fact that empirical results reveal that the relevance and combination of the indicators differ by sector further support the earlier finding that explosive price behavior in the real estate sector is sector specific. A composite leading indicator, rent growth, the yield curve, employment growth and consumer confidence and internet sales (more relevant to the retail sector) are indicators pointing to price exaggeration in the UK commercial property sectors but as aforementioned the set of indicators differs by sector.

The use of monthly data in this study makes the analysis and results to be of significant relevance to risk management by investors and lenders. Moreover, it contributes to a real time assessment of pricing developments in the commercial property market. Institutional investors - pension funds, sovereign funds, property funds - and private equity can mitigate risks in the property component of their portfolio by constantly evaluating episodes of price exuberance and extracting advanced signals. The real time inspection of price overvaluations has apparent benefits to lenders who monitor adverse price developments across property sectors that can lead to the breach of underwriting ratios through the loan term. They can accordingly adjust

the level of loans and underwriting ratios to mitigate the possibility of a major correction of commercial real estate values in the UK.

The main findings in this paper suggest some interesting developments. Given the significant exposure of banks and investors to UK's commercial sector, this real time or frequent assessment of price developments provides authorities with more intelligence for the soundness of financial institutions and risks to financial stability. The UK's financial system has in the past come under significant strain due to adverse developments in commercial real estate. For example, in order to provide a better assessment of the risks posed by a rise in commercial real estate prices, policymakers and lending institutions ought tightening some macroprudential policies to reduce downside risks to real estate prices and to mitigate the negative effects of real estate price corrections on the real economy. This assessment in the UK can be partially informed from a sector specific focus and monthly detection of price exuberance episodes. Further, the study of price overvaluations in the commercial sector has also a broader implication for future research. Mostly the literature on bubbles in non-securitized real estate focuses on the residential sector. It would be interesting to evaluate whether bubbles in real estate are likely to migrate (contagion) to residential. Valuations and pricing in commercial real estate are more sophisticated than the residential sector as the participants differ (e.g., significant greater presence of institutional investors in commercial real estate). It should be expected that this more sophisticated pricing analysis in the commercial sector will pick up price exuberance signals earlier than the residential sector. Finally, the question arising from this study is to what degree price exuberance and bubbles are synchronized across geographies and regions within countries. Such an extension of the study will need to address data availability issues. The UK and just a few other countries benefit from monthly data, in most other countries data will be of quarterly frequency. The inconsistency of real estate data across countries and sectors poses a challenge to future research.

References

- Akbari, A., & Krystyniak, K. (2021). Government real estate interventions and the stock market. *International Review of Financial Analysis*, 75, 101742.
- Anderson, K., Brooks, C., & Tsolacos, S. (2011). Testing for periodically collapsing rational speculative bubbles in US REITs. *Journal of Real Estate Portfolio Management*, 17(3), 227–241.
- Baur, D. G., & Heaney, R. (2017). Bubbles in the Australian housing market. *Pacific-Basin Finance Journal*, 44, 113–126.
- Blanchard, O. J. (1979). Speculative bubbles, crashes and rational expectations. *Economics Letters*, 3(4), 387–389.
- Bourassa, S. C., Hoesli, M., & Oikarinen, E. (2019). Measuring house price bubbles. *Real Estate Economics*, 47(2), 534–563.
- Brooks, C., & Katsaris, A. (2003). Rational speculative bubbles: an empirical investigation of the London Stock Exchange. *Bulletin of Economic Research*, 55(4), 319–346.
- Brooks, C., Katsaris, A., McGough, T., & Tsolacos, S. (2001). Testing for bubbles in indirect property price cycles. *Journal of Property Research*, 18(4), 341–356.
- Brown, G. R., & Matysiak, G. A. (2000). *Real Estate Investment: A Capital Market Approach*. Financial Times Prentice Hall.
- Bullard, J., Neely, C. J., & Wheelock, D. C. (2009). Systemic risk and the financial crisis: A primer. *Federal Reserve Bank of St. Louis Review*, 91(5 Part 1), 403–17.
- CBRE Investment Intelligent Report. (2022). *Global Real Estate Capital Flows H1 2022*. September 2022.
- Chatterjee, S., & Hadi, A. S. (2015). *Regression Analysis by Example*. John Wiley & Sons.
- Chen, Y., Phillips, P. C., & Shi, S. (2022). Common bubble detection in large dimensional financial systems. *Journal of Financial Econometrics*.
- Clayton, J. (1996). Rational expectations, market fundamentals and housing price volatility. *Real Estate Economics*, 24(4), 441–470.
- Cochran, S. J., & Defina, R. H. (1995). Duration dependence in the US stock market cycle: a parametric approach. *Applied Financial Economics*, 5(5), 309–318.
- Collins, C., & Senhadji, A. (2002). Lending Booms, Real Estate Bubbles, and the Asian Crisis. Working Paper, N.02/20. IMF Working Paper Available at <https://ssrn.com/abstract=879360>.
- Deghi, A., Mok, J., & Tsuruga, T. (2021). Commercial Real Estate and Macrofinancial Stability During Covid-19. Working Paper, N. 2021/264. IMF Working Paper Available at <https://ssrn.com/abstract=4026494>.
- Diba, B. T., & Grossman, H. I. (1988a). The theory of rational bubbles in stock prices. *The Economic Journal*, 98(392), 746–754.
- Diba, B. T., & Grossman, H. I. (1988b). Explosive rational bubbles in stock prices? *The American Economic Review*, 78(3), 520–530.
- Enoksen, F. A., Landsnes, C. J., Lučivjanská, K., & Molnár, P. (2020). Understanding risk of bubbles in cryptocurrencies. *Journal of Economic Behavior & Organization*, 176, 129–144.
- Escobari, D., & Jafarinejad, M. (2016). Date stamping bubbles in real estate investment trusts. *The Quarterly Review of Economics and Finance*, 60, 224–230.

- European Systemic Risk Board. (2015). *Report on Commercial Real Estate and Financial Stability in the EU*. December 2015.
- Evans, G. W. (1991). Pitfalls in testing for explosive bubbles in asset prices. *The American Economic Review*, 81(4), 922–930.
- Floro, D. (2019). Testing the predictive ability of house price bubbles for macroeconomic performance: A meta-analytic approach. *International Review of Financial Analysis*, 62, 164–181.
- Gentle, C., Dorling, D., & Comford, J. (1994). Negative equity and British housing in the 1990s: cause and effect. *Urban Studies*, 31(2), 181–199.
- Goodman, A. C., & Thibodeau, T. G. (2008). Where are the speculative bubbles in US housing markets? *Journal of Housing Economics*, 17(2), 117–137.
- Hendershott, P. H. (2000). Property asset bubbles: evidence from the Sydney office market. *The Journal of Real Estate Finance and Economics*, 20(1), 67–81.
- Hendershott, P. H., Hendershott, R. J., & Ward, C. R. (2003). Corporate equity and commercial property market ‘bubbles’. *Urban Studies*, 40(5-6), 993–1009.
- Hirano, T., & Toda, A. A. (2024). Bubble economics. *Journal of Mathematical Economics*, 111, 102944.
- Hoesli, M., & Oikarinen, E. (2021). Does listed real estate behave like direct real estate? Updated and broader evidence. *Applied Economics*, 53(26), 3023–3042.
- Huerta-Sanchez, D., Jafarinejad, M., Kim, D., & Soyeh, K. W. (2020). Disentangling bubbles in equity reits. *The Quarterly Review of Economics and Finance*, 76, 357–367.
- Investment Property Forum. (2020). *Long-Term Value Methodologies in Commercial Real Estate Lending*. July 2020.
- Jirasakuldech, B., Campbell, R. D., & Knight, J. R. (2006). Are there rational speculative bubbles in REITs? *The Journal of Real Estate Finance and Economics*, 32(2), 105–127.
- JLL. (2022). *Global Real Estate Transparency Index*. August 2022.
- Lee, J. H., & Phillips, P. C. (2016). Asset pricing with financial bubble risk. *Journal of Empirical Finance*, 38, 590–622.
- Martinez-Garcia, E., & Grossman, V. (2018). Explosive dynamics in house prices? An exploration of financial market spillovers in housing markets around the world–globalization institute. Working paper N. 342. Federal Reserve Bank of Dallas.
- Mishkin, F. S. (2011). Over the cliff: From the subprime to the global financial crisis. *Journal of Economic Perspectives*, 25(1), 49–70.
- MSCI. (2022). *Property Indexes Methodology. Index Construction Objectives, Guiding Principles and Methodology for the MSCI Property Indexes*. June 2022. Available at <https://www.msci.com/documents/1296102/1311232/MSCI+Property+Indexes+Methodology.pdf/a33f5a6c-a5e4-4d18-f460-d8c537ded968>.
- Muellbauer, J. (2022). Real Estate Booms and Busts: Implications for Monetary and Macroprudential Policy in Europe. Paper presented at the *European Central Bank (ECB) Forum on Central Banking*, June.
- Nneji, O., Brooks, C., & Ward, C. (2013). Commercial real estate and equity market bubbles: are they contagious to reits? *Urban Studies*, 50(12), 2496–2516.
- Pavlidis, E., Martinez-Garcia, E., & Grossman, V. (2019). Detecting periods of exuberance: A look at the role of aggregation with an application to house prices. *Economic Modelling*, 80, 87–102.

- Pavlidis, E., Yusupova, A., Paya, I., Peel, D., Martínez-García, E., Mack, A., & Grossman, V. (2016). Episodes of exuberance in housing markets: in search of the smoking gun. *The Journal of Real Estate Finance and Economics*, 53, 419–449.
- Phillips, P. C., Shi, S., & Yu, J. (2015a). Testing for multiple bubbles: Historical episodes of exuberance and collapse in the S&P 500. *International Economic Review*, 56(4), 1043–1078.
- Phillips, P. C., Shi, S., & Yu, J. (2015b). Testing for multiple bubbles: Limit theory of real-time detectors. *International Economic Review*, 56(4), 1079–1134.
- Phillips, P. C., Wu, Y., & Yu, J. (2011). Explosive behavior in the 1990s Nasdaq: When did exuberance escalate asset values? *International Economic Review*, 52(1), 201–226.
- Quigley, J. M. (2001). Real estate and the Asian crisis. *Journal of Housing Economics*, 10(2), 129–161.
- Roy, S., & Kemme, D. M. (2020). The run-up to the global financial crisis: A longer historical view of financial liberalization, capital inflows, and asset bubbles. *International Review of Financial Analysis*, 69, 101377.
- Royal Institution of Chartered Surveyors. (2023). *Discounted Cash Flow Valuations*. November 2023.
- Stanton, R., & Wallace, N. (2010). CMBS subordination, ratings inflation, and the crisis of 2007-2009. Working Paper, N. 16206. NBER Working Paper Available at <http://www.nber.org/papers/w16206>.
- Van Norden, S., & Schaller, H. (1993). The predictability of stock market regime: evidence from the Toronto stock exchange. *The Review of Economics and Statistics*, 505–510.
- Vasilopoulos, K., Pavlidis, E., & Martínez-García, E. (2022). exuber: Recursive right-tailed unit root testing with R. *Journal of Statistical Software*, 103, 1–26.
- Vogel, H. L., & Werner, R. A. (2015). An analytical review of volatility metrics for bubbles and crashes. *International Review of Financial Analysis*, 38, 15–28.
- Waters, G. A., & Payne, J. E. (2007). Reit markets and rational speculative bubbles: an empirical investigation. *Applied Financial Economics*, 17(9), 747–753.
- Xiao, Q., & Randolph Tan, G. K. (2007). Signal extraction with kalman filter: A study of the Hong Kong property price bubbles. *Urban Studies*, 44(4), 865–888.
- Xie, Z., & Chen, S.-W. (2015). Are there periodically collapsing bubbles in the reit markets? new evidence from the US. *Research in International Business and Finance*, 33, 17–31.

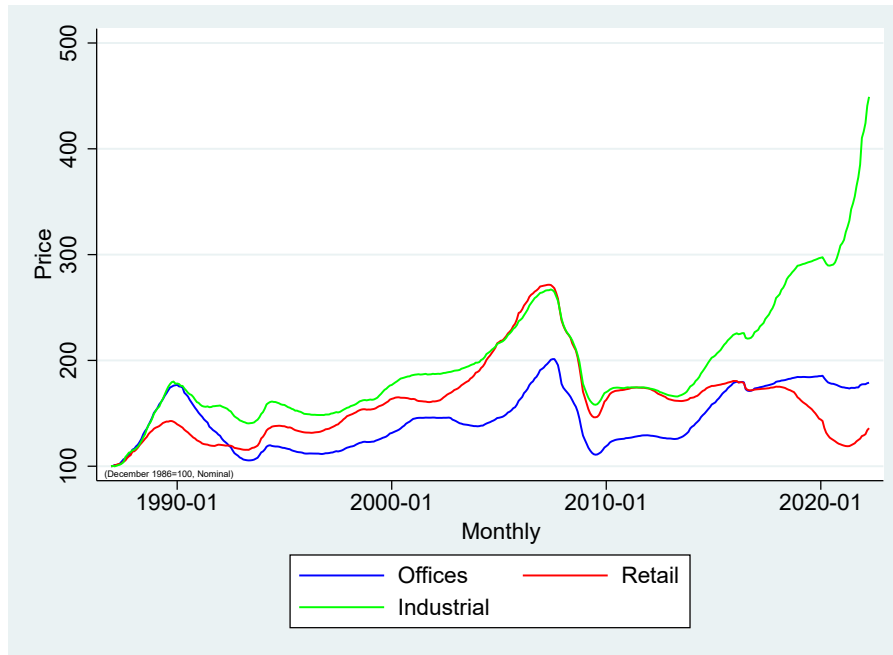
List of Figures

Figure 1: Components of total returns for Offices, Retail and Industrial in the UK.



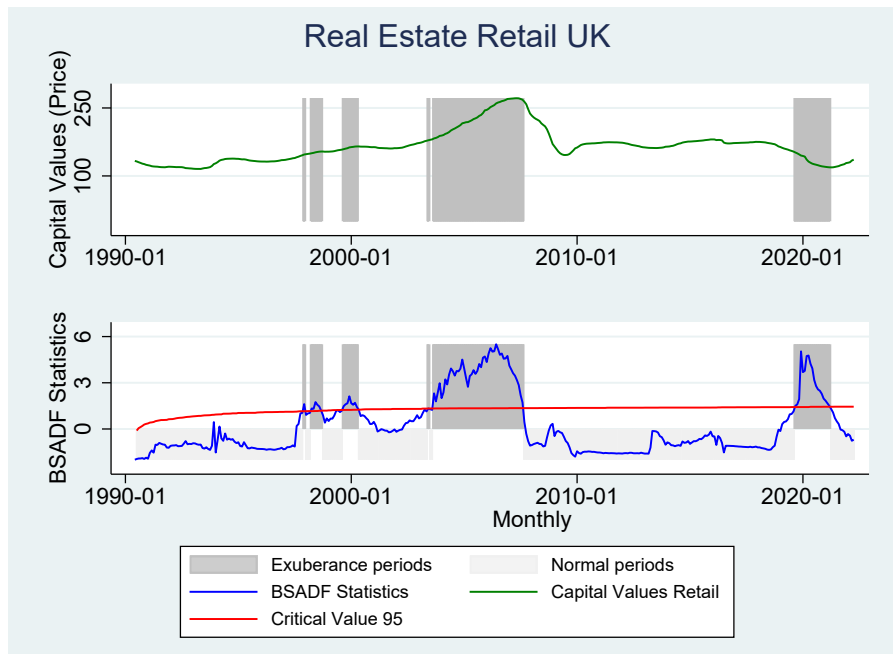
Source: Authors' elaboration with data from MSCI.

Figure 2: Capital values (prices) by main sector.



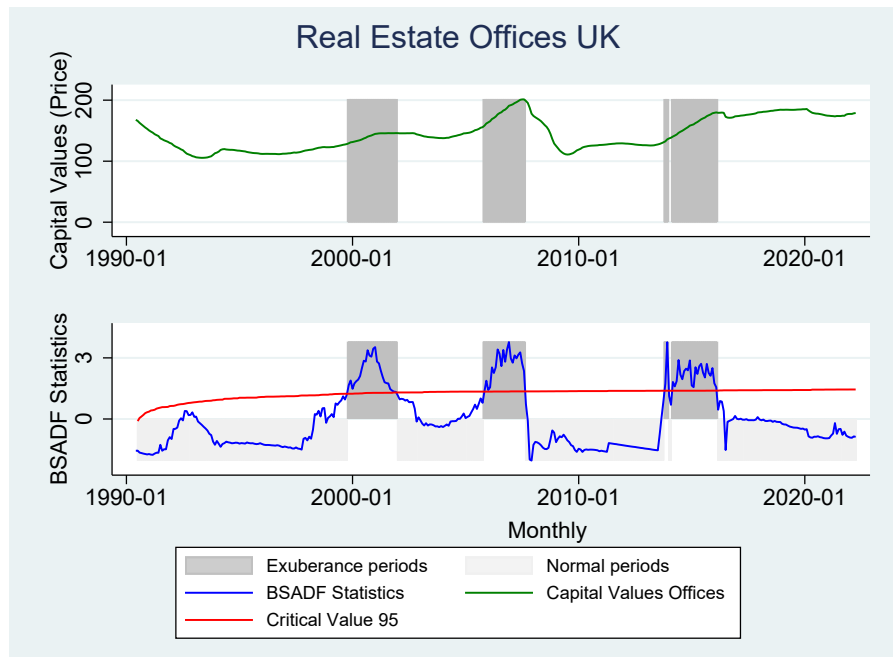
Source: Authors' elaboration with data from MSCI.

Figure 3: Detecting exuberance periods for Retail UK.



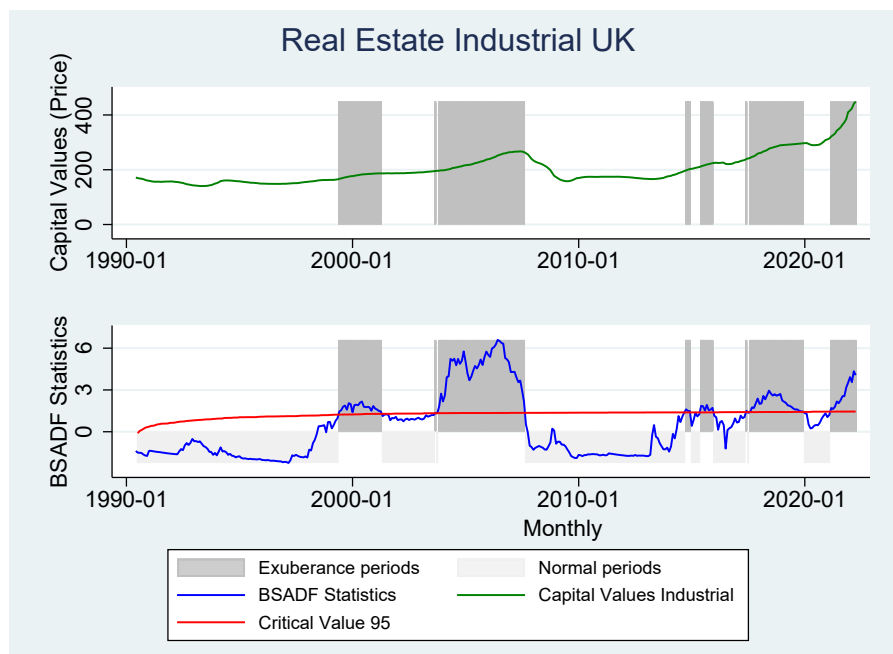
Source: Authors' calculation on Phillips et al. (2015a,b).

Figure 4: Detecting exuberance periods for Offices UK.



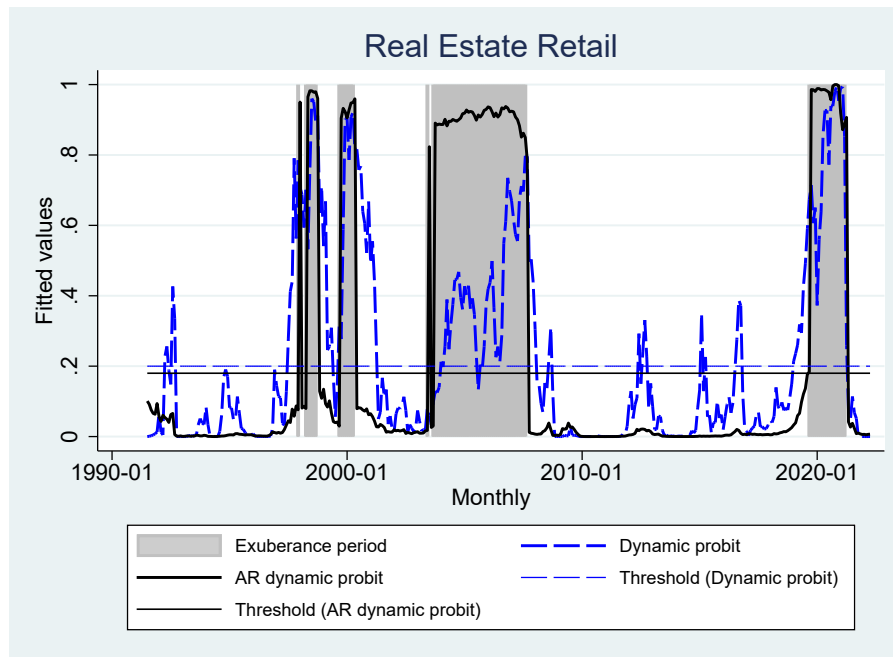
Source: Authors' calculation on Phillips et al. (2015a,b).

Figure 5: Detecting exuberance periods for Industrial UK.



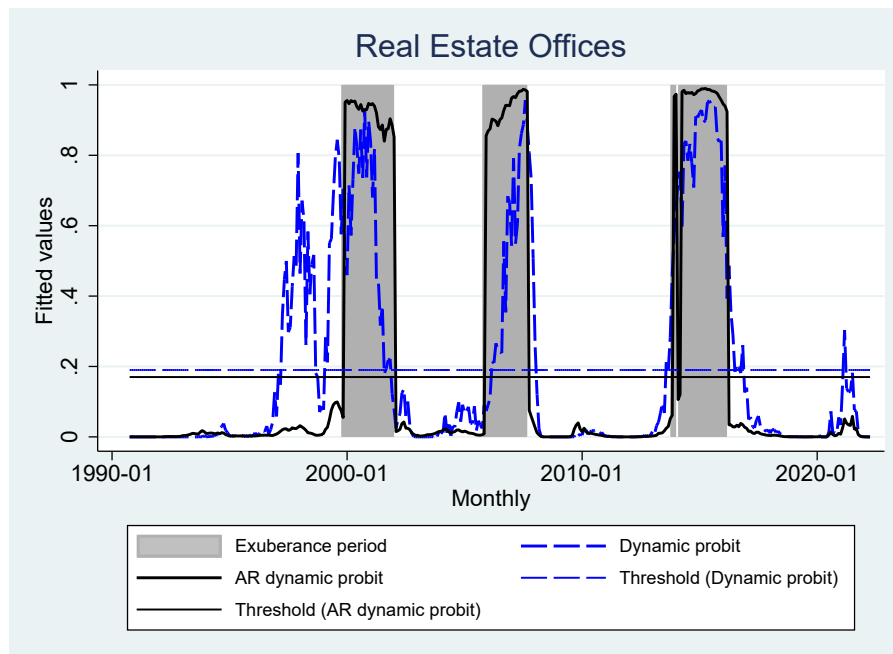
Source: Authors' calculation on Phillips et al. (2015a,b).

Figure 6: In-sample forecast performance - Retail UK.



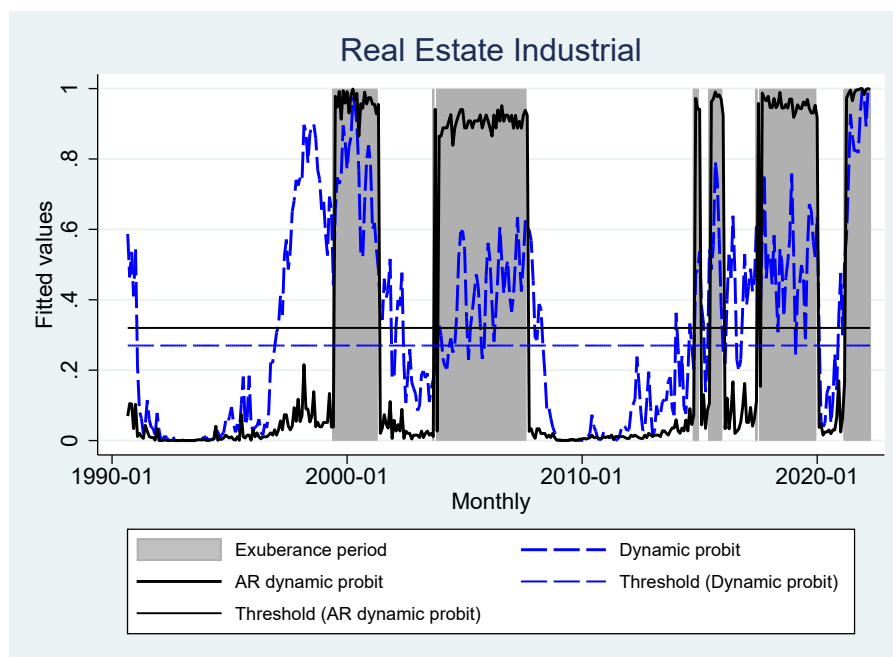
Source: Authors' elaboration with data from MSCI.

Figure 7: In-sample forecast performance - Offices UK.



Source: Authors' elaboration with data from MSCI.

Figure 8: In-sample forecast performance - Industrial UK.



Source: Authors' elaboration with data from MSCI.

List of Tables

Table 1: MSCI sub-indexes.

Current index	Components
UK Retail	- Standard retail in the South East (i.e., shop units, including high street); - Standard retail in the rest of UK; - Shopping centres (UK wide); - Retail warehouses (UK wide).
UK Offices	- City of London offices; - Mid-Town & West End offices; - Rest of the South East offices; - Rest of the UK offices.
UK Industrial	- South East industrials; - Rest of UK industrials.

The Table reports the MSCI sub indices.

Table 2: ADF, SADF, GSADF statistics for UK private real estate sectors.

Real Estate Sectors	Test		Critical values		
			90%	95%	99%
Retail	ADF	-2.42	-0.41	-0.04	0.54
	SADF	2.73	1.15	1.45	2.04
	GSADF	5.50	1.94	2.16	2.62
Offices	ADF	-2.48	-0.41	-0.04	0.54
	SADF	-0.74	1.15	1.45	2.04
	GSADF	3.78	1.94	2.16	2.62
Industrial	ADF	2.28	-0.41	-0.04	0.54
	SADF	2.67	1.15	1.45	2.04
	GSADF	6.59	1.94	2.16	2.62

The Table reports the right-tail Augmented Dickey Fuller (ADF), Supremum Augmented Dickey Fuller (SADF), and Generalized Supremum Augmented Dickey Fuller (GSADF) test statistics for exuberance behavior proposed by Phillips et al. (2011) and Phillips et al. (2015a,b). The finite sample critical values for 90%, 95% and 99% confidence levels are obtained from Monte Carlo simulations with 2,000 replications (see Phillips et al. (2015a,b) and Vasilopoulos et al. (2022)). The window size is given by $r_0 = (0.01 + 1.8/\sqrt{T})$ as recommended by Phillips et al. (2015a,b). The time period is 1986:12 - 2022:4 (monthly frequency) and $T = 423$ observations.

Table 3: Probit results for price exuberance - Retail.

Dependent variable: State of prices or capital values RetCVS Binary variable: RetCVS=0 (normal pricing); RetCVS=1 (price exuberance)					
Dynamic probit			Autoregressive dynamic probit		
Indicator	Coefficient (p-value)	VIFs	Indicator	Coefficient (p-value)	VIFs
Intercept	33.57*** (0.00)		Intercept	27.50** (0.04)	
$\Delta_{12}IRS_t$	0.10* (0.05)	1.3	YC_t	-0.61*** (0.00)	1.4
YC_{t-1}	-1.92*** (0.00)	4.6	CLI_{t-6}	-0.29** (0.03)	1.1
YC_{t-6}	0.70*** (0.00)	3.7	$RetCVS_{t-1}$	3.10*** (0.00)	1.4
CCI_{t-5}	-0.33*** (0.00)	2.1			
$INFL_t$	-0.49*** (0.00)	1.4			
N. Obs.	370		N. Obs.	370	
<i>Pseudo R</i> ²	0.43		<i>Pseudo R</i> ²	0.77	
<i>McFadden R</i> ²	0.42		<i>McFadden R</i> ²	0.79	
H-L test <i>p</i> -value	0.18		H-L test <i>p</i> -value	0.77	

The Table reports the results from the dynamic probit and autoregressive probit models for the UK retail sector. $\Delta_{12}IRS$ is the annual change (each month) in internet sales as a proportion of total retail sales; YC is the yield curve defined as the yield differential between the 10- and 1-year UK government bonds; CCI is the OECD's consumer confidence indicator for the UK; CLI is the OECD's composite leading indicator for the UK economy $INFL$ is the CPI inflation. *, **, *** denote the 10%, 5% and 1% significance level, respectively. All coefficients are statistically significant. The H-L test is the Hosmer-Lemeshow goodness of fit test (both probit models show no evidence of lack of fit computed p-values > 0.10). VIFs is the Variance Inflation Factors: we consider values lower than 4 indicating no multicollinearity with values close to 10 signalling strong multicollinearity (on one occasion some evidence of multicollinearity is present YC_{t-1}).

Table 4: Probit results for price exuberance - Offices.

Dependent variable: State of prices or capital values OffCVS					
Binary variable: OffCVS=0 (normal pricing); OffCVS=1 (price exuberance)					
Dynamic probit			Autoregressive dynamic probit		
Indicator	Coefficient (p-value)	VIFs	Indicator	Coefficient (p-value)	VIFs
Intercept	-67.08*** (0.00)		Intercept	-50.75*** (0.02)	
CLI_t	0.67*** (0.00)	1.1	CLI_t	0.49** (0.02)	1.1
$\Delta OFFRE_t$	1.12*** (0.00)	3.4	$INFL_t$	-0.39* (0.01)	1.1
$\Delta OFFRE_{t-2}$	0.89*** (0.01)	3.8	$OffCVS_{t-1}$	3.3*** (0.00)	1.3
$INFL_t$	-0.72*** (0.01)	1.1			
N. Obs.	379		N. Obs.	379	
<i>Pseudo R</i> ²	0.54		<i>Pseudo R</i> ²	0.82	
<i>McFadden R</i> ²	0.54		<i>McFadden R</i> ²	0.83	
H-L test <i>p</i> -value	0.5		H-L test <i>p</i> -value	0.72	

The Table reports the results from the dynamic probit and autoregressive probit models for the UK offices sector. *CLI* is the OECD's composite leading indicator for the UK economy; $\Delta OFFRE$ is the monthly percentage growth in nominal office rents; *INFL* is the CPI inflation. *, **, *** denote the 10%, 5% and 1% significance level, respectively. All coefficients are statistically significant. The H-L test is the Hosmer-Lemeshow goodness of fit test (both probit models show no evidence of lack of fit computed *p*-values>0.10). VIFs is the Variance Inflation Factors: we consider values lower than 4 indicating no multicollinearity with values close to 10 signalling strong multicollinearity.

Table 5: Probit results for price exuberance - Industrials.

Dependent variable: State of prices or capital values IndCVS					
Binary variable: IndCVS=0 (normal pricing); IndCVS=1 (price exuberance)					
Dynamic probit			Autoregressive dynamic probit		
Indicator	Coefficient (p-value)	VIFs	Indicator	Coefficient (p-value)	VIFs
Intercept	-0.81*** (0.00)		Intercept	-2.16*** (0.00)	
$\Delta INDRE_t$	1.34*** (0.00)	2.7	$\Delta INDRE_t$	1.92*** (0.00)	1.2
$\Delta INDRE_{t-1}$	1.21*** (0.01)	2.8	$IndCVS_{t-1}$	3.26*** (0.00)	1.2
ΔEMP_t	1.53*** (0.01)	1.4			
ΔEMP_{t-1}	1.10** (0.06)	1.4			
YC_t	-0.46*** (0.00)	1.3			
N. Obs.	380		N. Obs.	380	
<i>Pseudo R</i> ²	0.39		<i>Pseudo R</i> ²	0.84	
<i>McFadden R</i> ²	0.32		<i>McFadden R</i> ²	0.77	
H-L test <i>p</i> -value	0.02		H-L test <i>p</i> -value	0.76	

The Table reports the results from the dynamic probit and autoregressive probit models for the UK industrial sector. $\Delta INDRE$ is the monthly percentage growth in nominal industrial rents; ΔEMP is the monthly percentage growth in employment; YC is the yield curve (the yield on 10-year UK government bond minus yield on 1-year bond). *, **, *** denote the 10%, 5% and 1% significance level, respectively. All coefficients are statistically significant. The H-L test is the Hosmer-Lemeshow goodness of fit test (both probit models show no evidence of lack of fit computed *p*-values>0.10). VIFs is the Variance Inflation Factors: we consider values lower than 4 indicating no multicollinearity with values close to 10 signalling strong multicollinearity.

Table 6: In-sample forecast performance.

(a) Retail.

Description	Total N. Obs	N. Obs. in state 0	N. Obs. in state 1
		(normal)	(price exuberance)
Actual	370	287	83
Correct predictions			
Dynamic probit		230 (80.10%)	75 (90.40%)
Autoregressive dynamic probit		281 (97.90%)	78 (94.00%)
Total gain over naive			
Dynamic probit	60.0% (threshold probability 0.20)		
Autoregressive dynamic probit	74.6% (threshold probability 0.18)		

(b) Offices.

Description	Total N. Obs	N. Obs. in state 0	N. Obs. in state 1
		(normal)	(price exuberance)
Actual	379	305	74
Correct predictions			
Dynamic probit		261 (85.60%)	68 (91.90%)
Autoregressive dynamic probit		301 (98.70%)	70 (94.60%)
Total gain over naive			
Dynamic probit	67.3% (threshold probability: 0.19)		
Autoregressive dynamic probit	78.4% (threshold probability: 0.17)		

(c) Industrials.

Description	Total N. Obs	N. Obs. in state 0	N. Obs. in state 1
		(normal)	(price exuberance)
Actual	380	256	124
Correct predictions			
Dynamic probit		180 (70.30%)	112 (90.30%)
Autoregressive dynamic probit		249 (97.30%)	116 (93.60%)
Total gain over naive			
Dynamic probit	44.2% (threshold probability: 0.27)		
Autoregressive dynamic probit	63.4% (threshold probability: 0.32)		

In all sectors, both probit models outperform the naïve probability models. Threshold probabilities reported maximise the gain of the respective probit model over the naïve model. The naïve model is the constant probability model. It predicts all state 0 correctly and all state 1 wrongly.

Table 7: Out-of-sample forecast performance - Retail.

(a) One-month ahead forecasts.

Description	Total N.Obs	N.Obs. in state 0	N.Obs. in state 1
		(Normal state)	(Period of price exuberance)
Actual	184	157	27
Correct predictions			
Dynamic probit		125 (79.6%)	24 (88.9%)
Autoregressive dynamic probit		151 (96.2%)	18 (66.7%)
Total gain over naïve			
		Dynamic probit	Autoregressive dynamic probit
Overall correct predictions:		149	169
Total gain over naïve		66.30%	77.20%

(b) Three-month ahead forecasts.

Description	Total N.Obs	N.Obs. in state 0	N.Obs. in state 1
		(Normal state)	(Period of price exuberance)
Actual	182	157	25
Correct predictions			
Dynamic probit		113 (72%)	22 (88%)
Autoregressive dynamic probit		132 (84.1%)	19 (76%)
Total gain over naïve			
		Dynamic probit	Autoregressive dynamic probit
Overall correct predictions:		135	151
Total gain over naïve		60.4%	69.2%

(c) Six-month ahead forecasts.

Description	Total N.Obs	N.Obs. in state 0	N.Obs. in state 1
		(Normal state)	(Period of price exuberance)
Actual	179	157	22
Correct predictions			
Dynamic probit		111 (70.7%)	18 (81.8%)
Autoregressive dynamic probit		128 (81.5%)	19 (86.4%)
Total gain over naïve			
		Dynamic probit	Autoregressive dynamic probit
Overall correct predictions:		129	147
Total gain over naïve		59.8%	69.8%

The Table reports the out-of-sample forecast performance for Retail. The sample for the one-month forecasts comprises 184 observations obtained over the period January 2007 to April 2022. Over this period the sector was in normal state (state 0) for 157 months and in a period of price exuberance for 27 months. The dynamic probit correctly predicts 125 out of 157 months in state 0 (79.6% correct state 0 predictions) with the autoregressive probit predicting 151 months (96.2% correct state 0 predictions). The relative accuracy of the dynamic and autoregressive probits when the sector is in state 1 (24 months) is 24 and 18 months respectively. The calculation of the gain over the benchmark model follows in the results. The naïve model predicts all state 1 correctly and all state 0 wrongly. The total gain is the difference in the overall correct predictions of the two models. The overall correct predictions of the dynamic and autoregressive probits add up to 149 and 169 representing a respective gain over the benchmark forecast of 66.3% and 77.2% respectively (the actual number of overall correct predictions for the benchmark model is not reported in the table). The results for three- and six-month forecasts are interpreted in the same way.

Table 8: Out-of-sample forecast performance - Offices.

(a) One-month ahead forecasts.

Description	Total N.Obs	N.Obs. in state 0	N.Obs. in state 1
		(Normal state)	(Period of price exuberance)
Actual	184	150	34
Correct predictions			
Dynamic probit		130 (86.7%)	32 (94.1%)
Autoregressive dynamic probit		137 (91.3%)	28 (82.4%)
Total gain over naïve		Dynamic probit	Autoregressive dynamic probit
Overall correct predictions:		162	165
Total gain over naïve		69.6%	71.2%

(b) Three-month ahead forecasts.

Description	Total N.Obs	N.Obs. in state 0	N.Obs. in state 1
		(Normal state)	(Period of price exuberance)
Actual	182	150	32
Correct predictions			
Dynamic probit		123 (82%)	32 (100%)
Autoregressive dynamic probit		122 (81.3%)	28 (87.5%)
Total gain over naïve		Dynamic probit	Autoregressive dynamic probit
Overall correct predictions:		155	150
Total gain over naïve		67.6%	64.8%

(c) Six-month ahead forecasts.

Description	Total N.Obs	N.Obs. in state 0	N.Obs. in state 1
		(Normal state)	(Period of price exuberance)
Actual	179	150	29
Correct predictions			
Dynamic probit		122 (81.3%)	29 (100%)
Autoregressive dynamic probit		120 (80%)	24 (82.8%)
Total gain over naïve		Dynamic probit	Autoregressive dynamic probit
Overall correct predictions:		152	144
Total gain over naïve		68.8%	64.2%

The Table reports the out-of-sample forecast performance for Offices (See notes in Table 7). The 100% achieved accuracy of the probit model for offices (for forecast horizons three- and six-months) is the result of the model overpredicting state 1, there are false signals for state 1.

Table 9: Out-of-sample forecast performance - Industrials.

(a) One-month ahead forecasts.

Description	Total N.Obs	N.Obs. in state 0	N.Obs. in state 1
		(Normal state)	(Period of price exuberance)
Actual	184	122	62
Correct predictions			
Dynamic probit		97 (79.5%)	49 (79%)
Autoregressive dynamic probit		115 (94.3%)	38 (61.3%)
Total gain over naïve		Dynamic probit	Autoregressive dynamic probit
Overall correct predictions:		146	153
Total gain over naïve		45.7%	49.5%

(b) Three-month ahead forecasts.

Description	Total N.Obs	N.Obs. in state 0	N.Obs. in state 1
		(Normal state)	(Period of price exuberance)
Actual	182	122	60
Correct predictions			
Dynamic probit		97 (79.5%)	46 (76.7%)
Autoregressive dynamic probit		114 (93.4%)	31 (48.3%)
Total gain over naïve		Dynamic probit	Autoregressive dynamic probit
Overall correct predictions:		143	145
Total gain over naïve		45.6%	46.7%

(c) Six-month ahead forecasts.

Description	Total N.Obs	N.Obs. in state 0	N.Obs. in state 1
		(Normal state)	(Period of price exuberance)
Actual	179	122	57
Correct predictions			
Dynamic probit		91 (74.6%)	48 (84.2%)
Autoregressive dynamic probit		105 (86.1%)	34 (59.6%)
Total gain over naive			
		Dynamic probit	Autoregressive dynamic probit
Overall correct predictions:		139	139
Total gain over naïve		45.8%	45.8%

The Table reports the out-of-sample forecast performance for Industrials. See notes in Table 7.

Price Exuberance Episodes in Private Real Estate

INTERNET APPENDIX

July 12, 2024

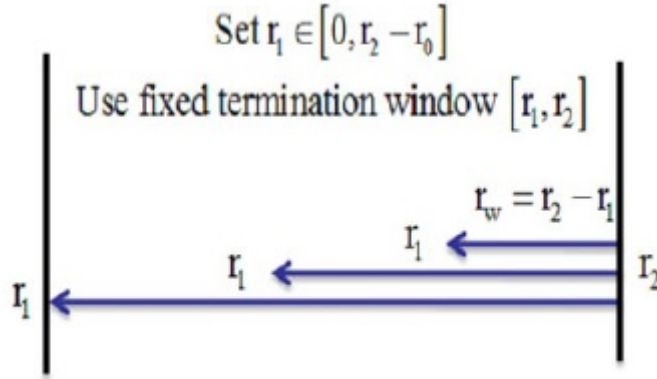
Appendix A Estimation of Bubble Episodes

The Backward Supremum Augmented Dickey Fuller (BSADF) is defined as follows:

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} \{BADF_{r_1}^{r_2}\} \quad (A1)$$

where r_1 and r_2 denote the starting and the ending fraction of the sample, respectively, implying that $r_1 < r_2$; r_0 is the fractional threshold and it is chosen on a lower bound of 1% of the full sample with the following functional form: $r_0 = (0.01 + 1.8/\sqrt{T})$, where T refers to the number of observations in the sample. We denote r_w , the window size of the regression, as $r_1 - r_2$. Figure A1 shows the flexibility of BSADF test in the detection of multiple bubbles.

Figure A1: Recursive nature of the BSADF test.



Source: Phillips et al. (2015a, p. 1052).

Phillips et al. (2015a) suggest to implement the BSADF test on a sample sequence where the end point is fixed at r_2 , and expands backwards to the starting point, r_1 , which varies between 0 and $(r_2 - r_0)$. Let r_e the fraction of the sample at which the bubble starts, r_f the

fraction of the sample at which it ends, and $\hat{r}_e$ and $\hat{r}_f$ the estimators of both. The origination and termination points of a bubble, i.e. r_e and r_f , are calculated according to the Equations (A2) and (A3):

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} [r_2 : BSADF_{r_2}(r_0) > scv_{r_2}^\beta] \quad (\text{A2})$$

$$\text{and } \hat{r}_f = \inf_{r_2 \in [\hat{r}_e + \delta \log(T), 1]} [r_2 : BSADF_{r_2}(r_0) < scv_{r_2}^\beta] \quad (\text{A3})$$

where T is the number of observations, $scv_{r_2}^\beta$ is the critical value of the BSADF statistic based on $[Tr_2]$ observations and confidence level β . $[Tr_2]$ refers to the largest integer smaller than or equal to Tr_2 . Phillips et al. (2015a) impose a condition that for a bubble to exist its duration must exceed a slowly varying (at infinity) quantity such as $L_T = \log(T)$. This condition helps to exclude short lived blips in the fitted autoregressive coefficient and can be adjusted to consider the data frequency. Thus, $\delta \log(T)$ is a minimal bubble length, and δ is a frequency-dependent parameter chosen freely.

Table A1: BSADF Statistics and BSADF Statistics 95% critical values.

Time (monthly)	Offices			Private Real Estate UK			Industrial		
	BSADFt	BSADFt 95% cv	Explosive behavior	BSADFt	BSADFt 95% cv	Explosive behavior	BSADFt	BSADFt 95% cv	Explosive behavior
1990-06	-1.560			-1.993			-1.381		
1990-07	-1.555	-0.102	0	-1.947	-0.102	0	-1.498	-0.102	0
1990-08	-1.649	0.052	0	-1.916	0.052	0	-1.509	0.052	0
1990-09	-1.672	0.123	0	-1.913	0.123	0	-1.522	0.123	0
1990-10	-1.699	0.200	0	-1.890	0.200	0	-1.633	0.200	0
1990-11	-1.719	0.289	0	-1.943	0.289	0	-1.708	0.289	0
1990-12	-1.738	0.333	0	-1.859	0.333	0	-1.737	0.333	0
1991-01	-1.720	0.362	0	-1.927	0.362	0	-1.374	0.362	0
1991-02	-1.748	0.417	0	-1.550	0.417	0	-1.371	0.417	0
1991-03	-1.758	0.430	0	-1.391	0.430	0	-1.395	0.430	0
1991-04	-1.698	0.454	0	-1.535	0.454	0	-1.420	0.454	0
1991-05	-1.656	0.509	0	-1.018	0.509	0	-1.439	0.509	0
1991-06	-1.656	0.536	0	-1.031	0.536	0	-1.456	0.536	0
1991-07	-1.267	0.559	0	-0.903	0.559	0	-1.470	0.559	0
1991-08	-1.558	0.583	0	-0.958	0.583	0	-1.497	0.583	0
1991-09	-1.485	0.583	0	-1.005	0.583	0	-1.510	0.583	0
1991-10	-1.486	0.588	0	-1.019	0.588	0	-1.533	0.588	0
1991-11	-1.047	0.593	0	-1.204	0.593	0	-1.549	0.593	0
1991-12	-0.890	0.633	0	-1.220	0.633	0	-1.567	0.633	0
1992-01	-0.982	0.642	0	-1.094	0.642	0	-1.582	0.642	0
1992-02	-0.489	0.645	0	-1.070	0.645	0	-1.595	0.645	0
1992-03	-0.481	0.666	0	-1.066	0.666	0	-1.610	0.666	0
1992-04	-0.421	0.696	0	-1.047	0.696	0	-1.621	0.696	0
1992-05	-0.077	0.716	0	-1.115	0.716	0	-1.414	0.716	0
1992-06	0.035	0.729	0	-1.110	0.729	0	-1.248	0.729	0
1992-07	-0.070	0.734	0	-1.098	0.734	0	-1.319	0.734	0
1992-08	0.393	0.750	0	-0.955	0.750	0	-0.977	0.750	0
1992-09	0.387	0.771	0	-0.786	0.771	0	-0.817	0.771	0
1992-10	0.188	0.787	0	-0.988	0.787	0	-0.952	0.787	0
1992-11	0.199	0.794	0	-0.966	0.794	0	-0.789	0.794	0
1992-12	0.328	0.808	0	-0.977	0.808	0	-0.521	0.808	0
1993-01	0.107	0.821	0	-0.924	0.821	0	-0.690	0.821	0
1993-02	0.116	0.830	0	-0.936	0.830	0	-0.666	0.830	0
1993-03	-0.123	0.841	0	-0.990	0.841	0	-0.740	0.841	0
1993-04	-0.205	0.853	0	-1.021	0.853	0	-0.690	0.853	0
1993-05	-0.287	0.853	0	-1.009	0.853	0	-0.864	0.853	0
1993-06	-0.382	0.871	0	-1.235	0.871	0	-1.033	0.871	0
1993-07	-0.466	0.885	0	-1.272	0.885	0	-1.102	0.885	0
1993-08	-0.565	0.889	0	-1.185	0.889	0	-1.218	0.889	0
1993-09	-0.724	0.892	0	-1.298	0.892	0	-1.436	0.892	0
1993-10	-0.790	0.898	0	-1.357	0.898	0	-1.451	0.898	0
1993-11	-1.109	0.918	0	-1.088	0.918	0	-1.656	0.918	0
1993-12	-1.266	0.924	0	0.449	0.924	0	-1.523	0.924	0
1994-01	-1.077	0.939	0	-1.525	0.939	0	-1.695	0.939	0
1994-02	-1.216	0.945	0	-0.720	0.945	0	-1.418	0.945	0
1994-03	-1.365	0.950	0	0.158	0.950	0	-1.120	0.950	0
1994-04	-1.226	0.950	0	-0.612	0.950	0	-1.414	0.950	0
1994-05	-1.185	0.991	0	-0.767	0.991	0	-1.395	0.991	0
1994-06	-1.112	0.992	0	-0.291	0.992	0	-1.479	0.992	0
1994-07	-1.142	0.993	0	-0.692	0.993	0	-1.657	0.993	0
1994-08	-1.161	0.996	0	-0.620	0.996	0	-1.590	0.996	0
1994-09	-1.191	1.002	0	-0.695	1.002	0	-1.717	1.002	0
1994-10	-1.180	1.016	0	-0.655	1.016	0	-1.748	1.016	0
1994-11	-1.163	1.016	0	-0.826	1.016	0	-1.841	1.016	0
1994-12	-1.169	1.016	0	-0.913	1.016	0	-1.785	1.016	0

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(a) Table A1 (*continued*).

Time (monthly)	Offices			Private Real Estate UK			Industrial		
	BSADFt	BSADFt 95% cv	Explosive behavior	BSADFt	BSADFt 95% cv	Explosive behavior	BSADFt	BSADFt 95% cv	Explosive behavior
1995-01	-1.207	1.042	0	-0.862	1.042	0	-1.761	1.042	0
1995-02	-1.183	1.042	0	-1.106	1.042	0	-1.880	1.042	0
1995-03	-1.193	1.042	0	-1.137	1.042	0	-1.925	1.042	0
1995-04	-1.206	1.042	0	-0.856	1.042	0	-1.909	1.042	0
1995-05	-1.151	1.048	0	-1.054	1.048	0	-1.946	1.048	0
1995-06	-1.181	1.048	0	-1.248	1.048	0	-1.962	1.048	0
1995-07	-1.192	1.048	0	-1.256	1.048	0	-1.941	1.048	0
1995-08	-1.189	1.048	0	-1.206	1.048	0	-1.975	1.048	0
1995-09	-1.216	1.048	0	-1.229	1.048	0	-1.967	1.048	0
1995-10	-1.233	1.050	0	-1.251	1.050	0	-1.989	1.050	0
1995-11	-1.221	1.070	0	-1.288	1.070	0	-1.973	1.070	0
1995-12	-1.189	1.077	0	-1.293	1.077	0	-1.962	1.077	0
1996-01	-1.230	1.083	0	-1.296	1.083	0	-2.028	1.083	0
1996-02	-1.263	1.083	0	-1.299	1.083	0	-2.033	1.083	0
1996-03	-1.277	1.094	0	-1.312	1.094	0	-2.052	1.094	0
1996-04	-1.274	1.097	0	-1.346	1.097	0	-2.028	1.097	0
1996-05	-1.278	1.097	0	-1.321	1.097	0	-2.063	1.097	0
1996-06	-1.290	1.097	0	-1.313	1.097	0	-2.087	1.097	0
1996-07	-1.297	1.097	0	-1.333	1.097	0	-2.075	1.097	0
1996-08	-1.307	1.100	0	-1.350	1.100	0	-2.097	1.100	0
1996-09	-1.288	1.102	0	-1.332	1.102	0	-2.108	1.102	0
1996-10	-1.305	1.103	0	-1.296	1.103	0	-2.124	1.103	0
1996-11	-1.349	1.111	0	-1.276	1.111	0	-2.128	1.111	0
1996-12	-1.380	1.111	0	-1.217	1.111	0	-2.190	1.111	0
1997-01	-1.351	1.111	0	-1.317	1.111	0	-2.162	1.111	0
1997-02	-1.374	1.119	0	-1.291	1.119	0	-2.179	1.119	0
1997-03	-1.403	1.119	0	-1.178	1.119	0	-2.243	1.119	0
1997-04	-1.393	1.119	0	-1.178	1.119	0	-2.058	1.119	0
1997-05	-1.445	1.119	0	-1.141	1.119	0	-1.695	1.119	0
1997-06	-1.450	1.119	0	-1.060	1.119	0	-1.739	1.119	0
1997-07	-1.394	1.124	0	-1.139	1.124	0	-1.658	1.124	0
1997-08	-1.450	1.127	0	0.194	1.127	0	-1.736	1.127	0
1997-09	-1.466	1.144	0	0.328	1.144	0	-1.795	1.144	0
1997-10	-1.502	1.154	0	1.016	1.154	0	-1.823	1.154	0
1997-11	-0.971	1.155	0	1.042	1.155	0	-1.839	1.155	0
1997-12	-0.926	1.155	0	1.616	1.155	1	-1.770	1.155	0
1998-01	-1.051	1.155	0	0.913	1.155	0	-1.852	1.155	0
1998-02	-0.869	1.161	0	1.013	1.161	0	-1.288	1.161	0
1998-03	-0.968	1.161	0	1.039	1.161	0	-1.348	1.161	0
1998-04	-0.529	1.161	0	1.332	1.161	1	-0.818	1.161	0
1998-05	0.241	1.161	0	1.313	1.161	1	-0.575	1.161	0
1998-06	0.216	1.167	0	1.746	1.167	1	-0.172	1.167	0
1998-07	0.402	1.167	0	1.572	1.167	1	0.392	1.167	0
1998-08	0.124	1.184	0	1.464	1.184	1	0.406	1.184	0
1998-09	0.524	1.190	0	1.193	1.190	1	0.437	1.190	0
1998-10	0.540	1.192	0	0.889	1.192	0	0.470	1.192	0
1998-11	-0.186	1.192	0	0.411	1.192	0	-0.036	1.192	0
1998-12	0.054	1.192	0	0.687	1.192	0	0.696	1.192	0
1999-01	0.129	1.209	0	0.517	1.209	0	0.703	1.209	0
1999-02	0.242	1.209	0	0.676	1.209	0	0.245	1.209	0
1999-03	0.092	1.222	0	0.671	1.222	0	0.473	1.222	0
1999-04	0.766	1.228	0	0.876	1.228	0	0.786	1.228	0
1999-05	0.695	1.228	0	1.174	1.228	0	0.963	1.228	0
1999-06	0.822	1.233	0	1.227	1.233	0	1.457	1.233	1
1999-07	0.913	1.235	0	1.097	1.235	0	1.548	1.235	1
1999-08	1.115	1.235	0	1.189	1.235	0	1.799	1.235	1
1999-09	0.956	1.238	0	1.485	1.238	1	1.881	1.238	1
1999-10	1.204	1.238	0	1.604	1.238	1	1.584	1.238	1
1999-11	1.665	1.240	1	1.670	1.240	1	2.052	1.240	1
1999-12	1.889	1.240	1	2.121	1.240	1	1.981	1.240	1

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(b) Table A1 (*continued*).

Time (monthly)	Offices			Private Real Estate UK			Industrial		
	BSADFt	BSADFt 95% cv	Explosive behavior	BSADFt	BSADFt 95% cv	Explosive behavior	BSADFt	BSADFt 95% cv	Explosive behavior
2000-01	1.484	1.240	1	1.674	1.240	1	1.401	1.240	1
2000-02	1.745	1.248	1	1.573	1.248	1	1.920	1.248	1
2000-03	1.855	1.250	1	1.703	1.250	1	1.939	1.250	1
2000-04	1.934	1.250	1	1.430	1.250	1	1.902	1.250	1
2000-05	2.108	1.257	1	1.246	1.257	0	2.091	1.257	1
2000-06	2.502	1.257	1	0.838	1.257	0	2.167	1.257	1
2000-07	2.860	1.269	1	0.898	1.269	0	1.796	1.269	1
2000-08	2.822	1.277	1	0.842	1.277	0	1.769	1.277	1
2000-09	3.374	1.280	1	0.701	1.280	0	1.784	1.280	1
2000-10	3.116	1.280	1	0.579	1.280	0	1.758	1.280	1
2000-11	3.060	1.282	1	0.323	1.282	0	1.510	1.282	1
2000-12	3.450	1.284	1	0.329	1.284	0	1.845	1.284	1
2001-01	3.531	1.284	1	0.462	1.284	0	1.665	1.284	1
2001-02	2.818	1.284	1	0.239	1.284	0	1.595	1.284	1
2001-03	2.731	1.284	1	-0.163	1.284	0	1.492	1.284	1
2001-04	2.374	1.284	1	-0.095	1.284	0	1.461	1.284	1
2001-05	2.066	1.284	1	-0.007	1.284	0	1.120	1.284	0
2001-06	1.806	1.287	1	-0.049	1.287	0	1.236	1.287	0
2001-07	1.761	1.287	1	-0.105	1.287	0	1.246	1.287	0
2001-08	1.747	1.287	1	-0.110	1.287	0	1.235	1.287	0
2001-09	1.472	1.287	1	-0.216	1.287	0	0.821	1.287	0
2001-10	1.393	1.287	1	-0.193	1.287	0	0.991	1.287	0
2001-11	1.343	1.303	1	-0.119	1.303	0	0.962	1.303	0
2001-12	1.330	1.303	1	-0.029	1.303	0	1.072	1.303	0
2002-01	1.157	1.303	0	-0.181	1.303	0	0.835	1.303	0
2002-02	0.965	1.303	0	-0.108	1.303	0	0.881	1.303	0
2002-03	0.992	1.303	0	-0.054	1.303	0	0.887	1.303	0
2002-04	0.972	1.303	0	-0.033	1.303	0	0.749	1.303	0
2002-05	0.902	1.303	0	0.209	1.303	0	0.885	1.303	0
2002-06	0.940	1.304	0	0.384	1.304	0	0.942	1.304	0
2002-07	0.824	1.304	0	0.406	1.304	0	0.838	1.304	0
2002-08	0.833	1.304	0	0.559	1.304	0	0.915	1.304	0
2002-09	0.832	1.304	0	0.505	1.304	0	0.906	1.304	0
2002-10	0.510	1.304	0	0.503	1.304	0	0.868	1.304	0
2002-11	-0.126	1.304	0	0.679	1.304	0	0.945	1.304	0
2002-12	0.084	1.304	0	0.897	1.304	0	1.008	1.304	0
2003-01	-0.058	1.305	0	0.688	1.305	0	0.916	1.305	0
2003-02	-0.225	1.305	0	0.768	1.305	0	0.881	1.305	0
2003-03	-0.317	1.324	0	1.038	1.324	0	0.954	1.324	0
2003-04	-0.207	1.324	0	1.264	1.324	0	1.185	1.324	0
2003-05	-0.321	1.324	0	1.143	1.324	0	1.081	1.324	0
2003-06	-0.264	1.331	0	1.332	1.331	1	1.131	1.331	0
2003-07	-0.271	1.331	0	1.281	1.331	0	1.200	1.331	0
2003-08	-0.307	1.331	0	1.240	1.331	0	1.194	1.331	0
2003-09	-0.364	1.331	0	2.313	1.331	1	1.341	1.331	1
2003-10	-0.361	1.335	0	1.794	1.335	1	1.334	1.335	0
2003-11	-0.399	1.335	0	2.368	1.335	1	1.838	1.335	1
2003-12	-0.347	1.335	0	2.971	1.335	1	2.735	1.335	1
2004-01	-0.395	1.335	0	2.005	1.335	1	2.179	1.335	1
2004-02	-0.334	1.335	0	2.370	1.335	1	2.606	1.335	1
2004-03	-0.267	1.337	0	3.220	1.337	1	3.918	1.337	1
2004-04	-0.331	1.337	0	2.892	1.337	1	3.980	1.337	1
2004-05	-0.124	1.341	0	3.534	1.341	1	5.233	1.341	1
2004-06	-0.106	1.341	0	3.928	1.341	1	5.095	1.341	1
2004-07	-0.107	1.341	0	3.748	1.341	1	5.245	1.341	1
2004-08	-0.095	1.341	0	3.467	1.341	1	4.774	1.341	1
2004-09	0.016	1.341	0	3.767	1.341	1	5.228	1.341	1
2004-10	0.006	1.345	0	3.740	1.345	1	4.875	1.345	1
2004-11	0.127	1.345	0	3.915	1.345	1	5.091	1.345	1
2004-12	0.271	1.345	0	4.506	1.345	1	5.766	1.345	1

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(c) Table A1 (*continued*).

Time (monthly)	Offices			Private Real Estate UK			Industrial		
	BSADFt	BSADFt 95% cv	Explosive behavior	BSADFt	BSADFt 95% cv	Explosive behavior	BSADFt	BSADFt 95% cv	Explosive behavior
2005-01	0.044	1.345	0	3.929	1.345	1	5.034	1.345	1
2005-02	0.104	1.345	0	3.308	1.345	1	4.178	1.345	1
2005-03	0.150	1.345	0	2.739	1.345	1	3.698	1.345	1
2005-04	0.324	1.345	0	3.440	1.345	1	3.949	1.345	1
2005-05	0.490	1.345	0	3.554	1.345	1	4.367	1.345	1
2005-06	0.705	1.345	0	3.821	1.345	1	4.727	1.345	1
2005-07	0.494	1.345	0	3.577	1.345	1	4.541	1.345	1
2005-08	0.746	1.345	0	3.766	1.345	1	4.792	1.345	1
2005-09	1.010	1.345	0	4.218	1.345	1	5.187	1.345	1
2005-10	0.795	1.345	0	4.011	1.345	1	4.796	1.345	1
2005-11	1.363	1.345	1	4.630	1.345	1	5.351	1.345	1
2005-12	1.878	1.345	1	4.706	1.345	1	5.756	1.345	1
2006-01	1.425	1.345	1	4.347	1.345	1	5.526	1.345	1
2006-02	1.559	1.345	1	4.896	1.345	1	5.372	1.345	1
2006-03	2.533	1.345	1	5.246	1.345	1	5.982	1.345	1
2006-04	2.261	1.348	1	5.023	1.348	1	6.063	1.348	1
2006-05	2.539	1.348	1	5.045	1.348	1	6.253	1.348	1
2006-06	3.413	1.348	1	5.499	1.348	1	6.592	1.348	1
2006-07	3.205	1.348	1	5.181	1.348	1	6.521	1.348	1
2006-08	2.596	1.348	1	4.815	1.348	1	6.386	1.348	1
2006-09	3.390	1.348	1	4.899	1.348	1	6.317	1.348	1
2006-10	2.811	1.348	1	4.562	1.348	1	5.295	1.348	1
2006-11	3.416	1.348	1	4.581	1.348	1	5.192	1.348	1
2006-12	3.775	1.348	1	4.743	1.348	1	4.996	1.348	1
2007-01	2.972	1.356	1	4.114	1.356	1	4.285	1.356	1
2007-02	2.765	1.356	1	3.871	1.356	1	4.295	1.356	1
2007-03	3.121	1.356	1	3.619	1.356	1	4.299	1.356	1
2007-04	2.977	1.356	1	3.461	1.356	1	3.891	1.356	1
2007-05	3.159	1.356	1	3.182	1.356	1	3.561	1.356	1
2007-06	3.271	1.356	1	2.852	1.356	1	3.670	1.356	1
2007-07	2.728	1.356	1	2.194	1.356	1	2.948	1.356	1
2007-08	2.358	1.356	1	1.726	1.356	1	2.147	1.356	1
2007-09	0.738	1.356	0	0.453	1.356	0	0.542	1.356	0
2007-10	-0.086	1.356	0	-0.312	1.356	0	-0.057	1.356	0
2007-11	-1.988	1.356	0	-0.785	1.356	0	-0.999	1.356	0
2007-12	-2.040	1.356	0	-1.077	1.356	0	-1.100	1.356	0
2008-01	-1.268	1.356	0	-1.042	1.356	0	-1.295	1.356	0
2008-02	-1.122	1.356	0	-0.976	1.356	0	-1.160	1.356	0
2008-03	-1.148	1.356	0	-0.933	1.356	0	-1.088	1.356	0
2008-04	-1.202	1.356	0	-0.892	1.356	0	-1.019	1.356	0
2008-05	-1.289	1.359	0	-0.970	1.359	0	-1.089	1.359	0
2008-06	-1.404	1.359	0	-0.984	1.359	0	-1.152	1.359	0
2008-07	-1.469	1.359	0	-1.134	1.359	0	-1.274	1.359	0
2008-08	-1.477	1.359	0	-1.096	1.359	0	-1.291	1.359	0
2008-09	-1.414	1.360	0	-0.558	1.360	0	-1.041	1.360	0
2008-10	-1.017	1.360	0	-0.087	1.360	0	-0.773	1.360	0
2008-11	-0.561	1.360	0	0.232	1.360	0	0.219	1.360	0
2008-12	-0.668	1.360	0	0.339	1.360	0	0.089	1.360	0
2009-01	-1.136	1.362	0	-0.459	1.362	0	-0.859	1.362	0
2009-02	-0.965	1.362	0	-0.185	1.362	0	-0.716	1.362	0
2009-03	-0.849	1.362	0	-0.124	1.362	0	-0.718	1.362	0
2009-04	-0.894	1.362	0	-0.212	1.362	0	-0.925	1.362	0
2009-05	-0.770	1.362	0	-0.259	1.362	0	-0.932	1.362	0
2009-06	-0.897	1.362	0	-0.505	1.362	0	-1.041	1.362	0
2009-07	-1.046	1.362	0	-0.781	1.362	0	-1.272	1.362	0
2009-08	-1.132	1.367	0	-0.909	1.367	0	-1.475	1.367	0
2009-09	-1.286	1.367	0	-1.193	1.367	0	-1.745	1.367	0
2009-10	-1.334	1.367	0	-1.565	1.367	0	-1.820	1.367	0
2009-11	-1.520	1.367	0	-1.697	1.367	0	-1.885	1.367	0
2009-12	-1.615	1.367	0	-1.793	1.367	0	-1.887	1.367	0

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(d) Table A1 (*continued*).

Time (monthly)	Offices			Private Real Estate UK			Industrial		
	BSADFt	BSADFt 95% cv	Explosive behavior	BSADFt	BSADFt 95% cv	Explosive behavior	BSADFt	BSADFt 95% cv	Explosive behavior
2010-01	-1.460	1.370	0	-1.443	1.370	0	-1.660	1.370	0
2010-02	-1.496	1.370	0	-1.561	1.370	0	-1.688	1.370	0
2010-03	-1.616	1.370	0	-1.607	1.370	0	-1.749	1.370	0
2010-04	-1.520	1.370	0	-1.500	1.370	0	-1.680	1.370	0
2010-05	-1.494	1.370	0	-1.472	1.370	0	-1.673	1.370	0
2010-06	-1.512	1.370	0	-1.476	1.370	0	-1.622	1.370	0
2010-07	-1.503	1.370	0	-1.456	1.370	0	-1.629	1.370	0
2010-08	-1.504	1.370	0	-1.465	1.370	0	-1.608	1.370	0
2010-09	-1.510	1.370	0	-1.486	1.370	0	-1.607	1.370	0
2010-10	-1.520	1.370	0	-1.482	1.370	0	-1.619	1.370	0
2010-11	-1.536	1.370	0	-1.505	1.370	0	-1.632	1.370	0
2010-12	-1.582	1.370	0	-1.525	1.370	0	-1.693	1.370	0
2011-01	-1.556	1.370	0	-1.527	1.370	0	-1.687	1.370	0
2011-02	-1.578	1.370	0	-1.543	1.370	0	-1.649	1.370	0
2011-03	-1.593	1.370	0	-1.573	1.370	0	-1.694	1.370	0
2011-04	-1.612	1.370	0	-1.551	1.370	0	-1.682	1.370	0
2011-05	-1.182	1.381	0	-1.562	1.381	0	-1.555	1.381	0
2011-06	-1.197	1.382	0	-1.573	1.382	0	-1.575	1.382	0
2011-07	-1.210	1.382	0	-1.566	1.382	0	-1.593	1.382	0
2011-08	-1.225	1.382	0	-1.572	1.382	0	-1.617	1.382	0
2011-09	-1.235	1.382	0	-1.576	1.382	0	-1.637	1.382	0
2011-10	-1.253	1.382	0	-1.592	1.382	0	-1.655	1.382	0
2011-11	-1.267	1.382	0	-1.580	1.382	0	-1.672	1.382	0
2011-12	-1.280	1.382	0	-1.587	1.382	0	-1.689	1.382	0
2012-01	-1.297	1.382	0	-1.593	1.382	0	-1.700	1.382	0
2012-02	-1.311	1.382	0	-1.572	1.382	0	-1.724	1.382	0
2012-03	-1.325	1.382	0	-1.564	1.382	0	-1.734	1.382	0
2012-04	-1.339	1.382	0	-1.577	1.382	0	-1.730	1.382	0
2012-05	-1.352	1.382	0	-1.542	1.382	0	-1.714	1.382	0
2012-06	-1.365	1.382	0	-1.547	1.382	0	-1.728	1.382	0
2012-07	-1.375	1.382	0	-1.575	1.382	0	-1.672	1.382	0
2012-08	-1.391	1.382	0	-1.578	1.382	0	-1.728	1.382	0
2012-09	-1.404	1.382	0	-1.571	1.382	0	-1.708	1.382	0
2012-10	-1.420	1.382	0	-1.583	1.382	0	-1.717	1.382	0
2012-11	-1.429	1.382	0	-1.539	1.382	0	-1.712	1.382	0
2012-12	-1.448	1.382	0	-1.571	1.382	0	-1.726	1.382	0
2013-01	-1.460	1.386	0	-1.596	1.386	0	-1.750	1.386	0
2013-02	-1.470	1.386	0	-1.593	1.386	0	-1.746	1.386	0
2013-03	-1.483	1.386	0	-1.601	1.386	0	-1.657	1.386	0
2013-04	-1.501	1.386	0	-1.206	1.386	0	-0.253	1.386	0
2013-05	-1.516	1.386	0	-0.130	1.386	0	0.478	1.386	0
2013-06	-1.526	1.386	0	-0.137	1.386	0	-0.362	1.386	0
2013-07	-1.536	1.386	0	-0.158	1.386	0	-0.464	1.386	0
2013-08	-0.749	1.386	0	-0.229	1.386	0	-0.819	1.386	0
2013-09	0.097	1.389	0	-0.464	1.389	0	-1.043	1.389	0
2013-10	0.895	1.389	0	-0.528	1.389	0	-1.260	1.389	0
2013-11	1.739	1.389	1	-0.740	1.389	0	-1.263	1.389	0
2013-12	3.769	1.389	1	-1.021	1.389	0	-1.142	1.389	0
2014-01	1.133	1.389	0	-0.867	1.389	0	-1.427	1.389	0
2014-02	0.696	1.389	0	-0.921	1.389	0	-1.066	1.389	0
2014-03	1.819	1.389	1	-1.051	1.389	0	-0.152	1.389	0
2014-04	1.600	1.389	1	-1.064	1.389	0	-0.476	1.389	0
2014-05	1.860	1.389	1	-1.116	1.389	0	0.196	1.389	0
2014-06	2.893	1.389	1	-1.071	1.389	0	1.127	1.389	0
2014-07	2.178	1.389	1	-1.049	1.389	0	0.700	1.389	0
2014-08	1.953	1.389	1	-1.094	1.389	0	1.075	1.389	0
2014-09	2.353	1.389	1	-0.810	1.389	0	1.370	1.389	0
2014-10	2.471	1.389	1	-0.899	1.389	0	1.609	1.389	1
2014-11	2.388	1.389	1	-0.875	1.389	0	1.491	1.389	1
2014-12	2.875	1.389	1	-0.783	1.389	0	1.489	1.389	1

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(e) Table A1 (*continued*).

Time (monthly)	Offices			Private Real Estate UK			Industrial		
	BSADFt	BSADFt 95% cv	Explosive behavior	BSADFt	BSADFt 95% cv	Explosive behavior	BSADFt	BSADFt 95% cv	Explosive behavior
2015-01	1.647	1.389	1	-0.965	1.389	0	0.424	1.389	0
2015-02	1.564	1.389	1	-0.878	1.389	0	0.840	1.389	0
2015-03	2.541	1.389	1	-0.787	1.389	0	1.372	1.389	0
2015-04	2.213	1.389	1	-0.794	1.389	0	1.101	1.389	0
2015-05	2.588	1.389	1	-0.741	1.389	0	1.246	1.389	0
2015-06	2.710	1.389	1	-0.643	1.389	0	1.859	1.389	1
2015-07	2.272	1.389	1	-0.660	1.389	0	1.831	1.389	1
2015-08	2.039	1.389	1	-0.563	1.389	0	1.509	1.389	1
2015-09	2.704	1.392	1	-0.412	1.392	0	1.926	1.392	1
2015-10	2.259	1.392	1	-0.450	1.392	0	1.550	1.392	1
2015-11	2.123	1.392	1	-0.373	1.392	0	1.583	1.392	1
2015-12	2.499	1.392	1	-0.171	1.392	0	1.716	1.392	1
2016-01	1.752	1.393	1	-0.469	1.393	0	1.146	1.393	0
2016-02	1.558	1.393	1	-0.485	1.393	0	1.035	1.393	0
2016-03	0.456	1.393	0	-1.033	1.393	0	0.143	1.393	0
2016-04	0.883	1.403	0	-0.450	1.403	0	0.688	1.403	0
2016-05	0.854	1.403	0	-0.592	1.403	0	0.771	1.403	0
2016-06	0.387	1.408	0	-0.907	1.408	0	0.470	1.408	0
2016-07	-1.523	1.408	0	-1.521	1.408	0	-1.209	1.408	0
2016-08	-0.138	1.408	0	-1.091	1.408	0	0.124	1.408	0
2016-09	-0.091	1.408	0	-1.090	1.408	0	0.251	1.408	0
2016-10	-0.049	1.408	0	-1.095	1.408	0	0.331	1.408	0
2016-11	0.001	1.408	0	-1.100	1.408	0	0.588	1.408	0
2016-12	0.150	1.408	0	-1.103	1.408	0	0.956	1.408	0
2017-01	0.008	1.408	0	-1.117	1.408	0	0.662	1.408	0
2017-02	-0.012	1.411	0	-1.128	1.411	0	0.702	1.411	0
2017-03	0.015	1.411	0	-1.139	1.411	0	1.158	1.411	0
2017-04	-0.034	1.411	0	-1.143	1.411	0	0.972	1.411	0
2017-05	-0.036	1.411	0	-1.156	1.411	0	1.058	1.411	0
2017-06	-0.008	1.411	0	-1.163	1.411	0	1.499	1.411	1
2017-07	-0.059	1.411	0	-1.170	1.411	0	1.360	1.411	0
2017-08	-0.057	1.411	0	-1.178	1.411	0	1.422	1.411	1
2017-09	-0.057	1.411	0	-1.153	1.411	0	1.745	1.411	1
2017-10	-0.060	1.411	0	-1.184	1.411	0	1.709	1.411	1
2017-11	-0.031	1.411	0	-1.168	1.411	0	2.040	1.411	1
2017-12	0.093	1.411	0	-1.143	1.411	0	2.382	1.411	1
2018-01	-0.022	1.414	0	-1.193	1.414	0	2.000	1.414	1
2018-02	-0.065	1.414	0	-1.192	1.414	0	2.036	1.414	1
2018-03	-0.026	1.418	0	-1.235	1.418	0	2.569	1.418	1
2018-04	-0.094	1.418	0	-1.258	1.418	0	2.328	1.418	1
2018-05	-0.101	1.418	0	-1.286	1.418	0	2.628	1.418	1
2018-06	-0.084	1.418	0	-1.336	1.418	0	2.952	1.418	1
2018-07	-0.078	1.418	0	-1.355	1.418	0	2.739	1.418	1
2018-08	-0.141	1.418	0	-1.328	1.418	0	2.598	1.418	1
2018-09	-0.097	1.418	0	-1.180	1.418	0	2.695	1.418	1
2018-10	-0.139	1.418	0	-1.084	1.418	0	2.591	1.418	1
2018-11	-0.146	1.418	0	-0.511	1.418	0	2.643	1.418	1
2018-12	-0.149	1.418	0	-0.045	1.418	0	2.714	1.418	1
2019-01	-0.238	1.418	0	-0.134	1.418	0	2.246	1.418	1
2019-02	-0.267	1.418	0	0.317	1.418	0	2.156	1.418	1
2019-03	-0.271	1.418	0	0.457	1.418	0	2.066	1.418	1
2019-04	-0.325	1.418	0	0.491	1.418	0	1.889	1.418	1
2019-05	-0.371	1.418	0	0.631	1.418	0	1.861	1.418	1
2019-06	-0.396	1.418	0	0.954	1.418	0	1.775	1.418	1
2019-07	-0.419	1.424	0	0.976	1.424	0	1.660	1.424	1
2019-08	-0.425	1.424	0	1.140	1.424	0	1.592	1.424	1
2019-09	-0.430	1.424	0	1.507	1.424	1	1.604	1.424	1
2019-10	-0.455	1.424	0	1.598	1.424	1	1.506	1.424	1
2019-11	-0.484	1.424	0	1.938	1.424	1	1.465	1.424	1
2019-12	-0.469	1.424	0	5.040	1.424	1	1.445	1.424	1

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(f) Table A1 (*continued*).

Time (monthly)	Offices			Private Real Estate UK			Industrial		
	BSADFt	BSADFt 95% cv	Explosive behavior	BSADFt	BSADFt 95% cv	Explosive behavior	BSADFt	BSADFt 95% cv	Explosive behavior
2020-01	-0.521	1.424	0	3.690	1.424	1	1.335	1.424	0
2020-02	-0.499	1.424	0	3.775	1.424	1	1.287	1.424	0
2020-03	-0.853	1.424	0	4.738	1.424	1	0.534	1.424	0
2020-04	-0.904	1.424	0	4.766	1.424	1	0.239	1.424	0
2020-05	-0.919	1.441	0	4.237	1.441	1	0.280	1.441	0
2020-06	-0.869	1.441	0	3.932	1.441	1	0.437	1.441	0
2020-07	-0.872	1.441	0	3.181	1.441	1	0.487	1.441	0
2020-08	-0.849	1.441	0	2.785	1.441	1	0.468	1.441	0
2020-09	-0.890	1.441	0	2.570	1.441	1	0.659	1.441	0
2020-10	-0.906	1.441	0	2.481	1.441	1	0.951	1.441	0
2020-11	-0.937	1.441	0	2.284	1.441	1	1.158	1.441	0
2020-12	-0.949	1.444	0	1.972	1.444	1	1.319	1.444	0
2021-01	-0.948	1.444	0	1.841	1.444	1	1.049	1.444	0
2021-02	-0.936	1.444	0	1.673	1.444	1	1.206	1.444	0
2021-03	-0.811	1.444	0	1.509	1.444	1	1.702	1.444	1
2021-04	-0.714	1.444	0	1.298	1.444	0	1.634	1.444	1
2021-05	-0.200	1.444	0	1.074	1.444	0	1.800	1.444	1
2021-06	-0.728	1.448	0	0.709	1.448	0	2.173	1.448	1
2021-07	-0.498	1.448	0	0.497	1.448	0	2.049	1.448	1
2021-08	-0.487	1.448	0	0.327	1.448	0	2.266	1.448	1
2021-09	-0.778	1.448	0	-0.038	1.448	0	2.554	1.448	1
2021-10	-0.620	1.448	0	-0.076	1.448	0	2.556	1.448	1
2021-11	-0.815	1.448	0	-0.179	1.448	0	3.180	1.448	1
2021-12	-0.872	1.448	0	-0.480	1.448	0	3.525	1.448	1
2022-01	-0.912	1.449	0	-0.329	1.449	0	3.920	1.449	1
2022-02	-0.921	1.449	0	-0.432	1.449	0	3.564	1.449	1
2022-03	-0.856	1.449	0	-0.760	1.449	0	4.342	1.449	1
2022-04	-0.877	1.449	0	-0.699	1.449	0	4.065	1.449	1

The Table reports $BSADF_t$ and $BSADF_t$ the 95% critical values (cv). The $BSADF_t$ 95% critical values are based on a window size given by $r_0 = (0.01 + 1.8/\sqrt{T})$, where T refers to the number of observations in the sample. Explosive behavior is an indicator variable equal to 1 when $BSADF_t$ is above its 95% critical value and 0 otherwise.

Appendix B Exuberance behavior in UK Private Real Estate regions and sub-sectors

In this section, we report the right-tail Generalized Supremum Augmented Dickey Fuller (GSADF) test statistics for exuberance behavior computed for UK Private Real Estate regions and sub-sectors. In particular, the analysis focuses on sub-sectors such as retail (shopping centres, retail warehouses, standard unit shops), offices, industrial, and for regions in the UK (e.g., London, South East, West End, rest of UK).

Table B1 shows that for all sub-sectors the GSADF tests provide evidence against the unit-root null hypothesis at the 1% significance level and thus there is evidence of a sequence of episodes of exuberance.

Table B1: GSADF statistics for UK Private Real Estate regions and sub-sectors.

Real Estate Sectors	Components	GSADF Test	Critical values		
			90%	95%	99%
Retail	Standard retail in the South East	4.97	1.94	2.16	2.62
	Standard retail in the rest of UK	4.35	1.94	2.16	2.62
	Shopping centres (UK wide)	6.58	1.94	2.16	2.62
	Retail warehouses (UK wide)	5.38	1.94	2.16	2.62
Offices	City of London offices	4.90	1.94	2.16	2.62
	Mid-Town & West End offices	5.06	1.94	2.16	2.62
	Rest of the South East offices	3.62	1.94	2.16	2.62
	Rest of the UK offices	6.42	1.94	2.16	2.62
Industrial	South East industrials	6.71	1.94	2.16	2.62
	Rest of UK industrials	6.13	1.94	2.16	2.62

The Table reports the Generalized Supremum Augmented Dickey Fuller (GSADF) test statistics for exuberance behavior proposed by Phillips et al. (2011) and Phillips et al. (2015a,b) for UK private real estate sub-sectors. The finite sample critical values for 90%, 95% and 99% confidence levels are obtained from Monte Carlo simulations with 2,000 replications (see Phillips et al. (2015a,b) and Vasilopoulos et al. (2022)). The window size is given by $r_0 = (0.01 + 1.8/\sqrt{T})$ as recommended by Phillips et al. (2015a,b). The time period is 1986:12 - 2022:4 (monthly frequency) and $T = 423$ observations.

Appendix C Exuberance behavior in UK Private Real Estate - normalized price series

In this section, we report the GSADF test statistics for exuberance behavior for UK private real estate using normalized price series. Since asset price bubbles are often thought of as price deviations from fundamental values, we additionally consider the normalized price series by considering, for each real estate sector, the ratio between capital values and rents.

Table C1 shows that for all sectors, the GSADF test provides evidence of a sequence of episodes of exuberance at the 1% significance level. Thus, the results confirm the presence of explosiveness periods both using capital values and capital values over rents.

Table C1: GSADF statistics for UK Private Real Estate sectors - normalized price series.

Real Estate Sectors	GSADF Test	Critical values		
		90%	95%	99%
Retail	5.46	1.94	2.16	2.62
Offices	4.94	1.94	2.16	2.62
Industrial	6.52	1.94	2.16	2.62

The Table reports the Generalized Supremum Augmented Dickey Fuller (GSADF) test statistics for exuberance behavior proposed by Phillips et al. (2011) and Phillips et al. (2015a,b) using normalized price series as the ratio between capital values and price rent. The finite sample critical values for 90%, 95% and 99% confidence levels are obtained from Monte Carlo simulations with 2,000 replications (see Phillips et al. (2015a,b) and Vasilopoulos et al. (2022)). The window size is given by $r_0 = (0.01 + 1.8/\sqrt{T})$ as recommended by Phillips et al. (2015a,b). The time period is 1986:12 - 2022:4 (monthly frequency) and $T = 423$ observations.

Appendix D Exuberance behavior in UK Real Estate Investment Trusts (REITs)

In this section, we report and we compare the GSADF test statistics for exuberance behavior for UK Private Real Estate and for Real Estate Investment Trusts (REITs). Table D1 shows that for all sectors, the GSADF test provides evidence of a sequence of episodes of exuberance at the 1% significance level. Thus, the results confirm the presence of explosiveness periods both for UK Private Real Estate and REITs.

Table D1: GSADF statistics for UK Private Real Estate and REITs.

Description	GSADF Test	Critical values		
		90%	95%	99%
Real Estate Investment Trusts (REITs)	3.54	1.91	2.15	2.61
Retail	5.50	1.94	2.16	2.62
Offices	3.78	1.94	2.16	2.62
Industrial	6.59	1.94	2.16	2.62

The Table reports the Generalized Supremum Augmented Dickey Fuller (GSADF) test statistics for exuberance behavior proposed by Phillips et al. (2011) and Phillips et al. (2015a,b) for UK Private Real Estate and UK Real Estate Investment Trusts (REITs). The finite sample critical values for 90%, 95% and 99% confidence levels are obtained from Monte Carlo simulations with 2,000 replications (see Phillips et al. (2015a,b) and Vasilopoulos et al. (2022)). The window size is given by $r_0 = \left(0.01 + 1.8/\sqrt{T}\right)$ as recommended by Phillips et al. (2015a,b). For UK Private Real Estate, the time period is 1986:12 - 2022:4 (monthly frequency) and $T = 423$ observations. For REITs, the time period is 1990:1 - 2022:4 (monthly frequency) and $T = 388$ observations.