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# Using anticipatory orders to manage disruption risk over a short product life cycle

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We study the impact of supply disruptions on sourcing strategies when product life cycles are short and future demand depends on current sales. We introduce the concept of order policies with anticipatory orders where some or all of orders with the unreliable supplier in future periods are moved to an earlier period. Using a 2-period model, we show that despite incurring additional holding cost when inventory is carried to a future period, anticipatory orders from the unreliable supplier are valuable when dealing with disruption risk when future demand depends on current sales. We find that anticipatory orders are less pronounced when future demand is more independent of current sales. We show that anticipatory orders from the unreliable supplier continue to be optimal for a range of disruption probabilities even when we include a responsive and reliable supplier.

*Key words:* Disruption, risk mitigation strategies, supply chain, demand sales dependence

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## 1. Introduction

In many industries like seasonal apparel and consumer electronics, products have a short life cycle and sales early in the life cycle are a strong indicator of demand in the rest of the life cycle. In such a setting, a supply disruption early in the life cycle results not only in current lost sales, but also in significant decline in future demand. For example, the selling season for fashion apparel is about three months and sales in the first few weeks are a strong indicator of future demand during the season. A supply disruption in the first month can be very damaging because the absence of inventory can result in customers shifting to a competitor for the rest of the season. Even in the case of staple products, customers who experience inventory stock-outs may switch to another firm in the future (e.g. Anderson et al., 2006; Straughn, 1991; Fitzsimons, 2000).

In the context of consumer electronics, Hu et al. (2019) use product life cycle curves to forecast demand for 170 Dell products. Product life cycle approaches define introduction, growth, maturity, and decline as four phases of a product life cycle. Hu et al. (2019) observe that most products hit maturity within 35% of the product life cycle. In such a setting, time to maturity is short and any supply disruption during introduction or growth is particularly damaging because it results in early lost sales, which in turn lowers future demand at maturity. Future demand is lower because of slower demand diffusion and reduced network externalities when the number of current adopters determines future attractiveness of a product (see e.g. Bass, 1969; Arthur, 1996; Nault and Vandenbosch, 2000).

This paper focuses on a scenario that applies directly to fashion apparel where life cycles are a few months, low cost suppliers are offshore and have long lead times (often larger than the duration of the selling season), and supply is subject to disruption. We consider a firm that is planning sourcing for a short life cycle product (e.g. a fashion product with a selling season of a few months). Our base model uses two periods as the planning horizon. The firm faces uncertain demand for the product and sources from a low cost, long lead time (a lead time of two periods or more), unreliable supplier subject to disruption. Given that the lead time from the unreliable supplier is two or more periods in our model, the firm must commit to order quantities for both periods before observing the supply and demand outcome for period 1. Demand in period 2 is assumed to depend on sales in period 1. If a disruption occurs in a given period, no supply arrives from the unreliable supplier in that period. A disruption limits available supply and thus sales in a given period, which in turn

can reduce future demand. We start the time horizon with zero initial inventory, which is reasonable for a new product. Our goal is to determine the optimal order quantities for each period. In a more general model, we also consider the availability of a responsive (lead time of one period) and reliable but high cost supplier.

When dealing with disruption risk, supply chain literature has proposed several risk mitigation strategies such as maintaining risk management inventory, contracting for reserve capacity, sourcing from multiple suppliers, or sourcing from a reliable and responsive supplier. In this paper, for a setting where future demand depends on current sales, we propose placing *anticipatory orders* with the unreliable supplier as another approach that helps maximize expected profits for a supply chain subject to supply disruptions. *Anticipatory orders* are defined relative to policies where an order  $q_i > 0$  is placed with the unreliable supplier for delivery in period  $i$  for  $i > 1$ . We define an *anticipatory order* to be an order where part or all of  $q_i$  is moved to an earlier period  $j$ ,  $j < i$  without changing the total quantity ordered over the planning horizon. To illustrate anticipatory orders, consider a policy  $P$  in a two period setting where we place an order  $q_2 > 0$  with the unreliable supplier for period 2. Relative to  $P$ , an anticipatory order moves some quantity  $A > 0$  from period 2 to period 1, increasing the order placed with the unreliable supplier for period 1 by  $A$  and decreasing the order placed with the unreliable supplier for period 2 by  $A$  to  $q_2 - A$ .

Anticipatory orders are interesting because they increase the quantity ordered in the earlier period, which at first seems harmful for a couple of reasons. If supply is disrupted in the earlier period, a larger quantity is disrupted if an anticipatory order is placed. If no disruption occurs, anticipatory orders incur the additional holding cost of carrying inventory to a future period. Despite these potential challenges, when future demand depends on current sales, we find anticipatory orders to be valuable in the presence of disruption risk because they provide the benefit of better matching supply with demand (compared to policies without anticipatory orders) in the event of a supply disruption in one of the two periods. If period 1 is disrupted (but not period 2), anticipatory orders result in less unsold inventory at the end of the life cycle. If period 2 is disrupted (but not period 1), anticipatory orders provide supply from period 1 to meet period 2 demand.

When future demand depends on current sales, we show the existence of a range of disruption probability over which placing anticipatory orders is optimal. If period 2 demand is independent of sales in period 1, however, anticipatory orders are not optimal for any

disruption probability. We also find that anticipatory orders are more pronounced when the demand in period 2 is more dependent on sales in period 1.

We extend our model to allow the firm a choice of two suppliers - a higher cost, short lead time, reliable supplier who is never disrupted and a lower cost, long lead time unreliable supplier who is subject to disruption. The reliable supplier is assumed to have a short lead time that allows the firm to place *contingent orders* with the reliable supplier for period 2 after observing the supply and demand outcome for the first period. We show that despite the presence of a responsive, reliable supplier where contingent orders can be placed, there exists a range of disruption probability over which placing anticipatory orders with the unreliable supplier is optimal when future demand depends on current sales.

In the next section we review relevant literature. In Section 3 we provide a numerical example and illustrate how the demand-sales dependency results in anticipatory orders from the unreliable supplier. In Section 4, we study how the sales-demand dependency influences the placement of anticipatory orders in five different settings -(1) the base model where demand for period 1 is deterministic, the supplier pays penalty fees for unmet orders, and disruption probabilities are different in the two periods, (2) disruption risk across the two periods is correlated, (3) the buyer has the option of recourse to reduce the period 2 order if period 1 is disrupted, (4) the buyer can also place orders with a responsive and reliable supplier, and (5) demand for period 1 is uncertain. Section 5 highlights conclusions and future research directions.

## 2. Literature Review

In the following, we review different streams in the literature that relate to this work, including papers on supply chain disruption risk management, papers that study the use of inventory for mitigating disruption risk, and papers that consider a dependence between present sales and future demand.

### Supply chain disruption risk management

Supply chain literature has proposed several approaches to build resilience into a supply chain such as maintaining risk management inventory, contracting for reserve capacity, sourcing from multiple suppliers, or sourcing from a reliable and responsive supplier (Tomlin, 2008; Chopra et al., 2007; Dada et al., 2007; Ang et al., 2017; Dong et al., 2018; Qi and Lee, 2015; Chopra and Sodhi, 2004). This literature has largely focused on single- and multi-period settings where future demand is independent of any current disruptions.

Consider the setting in Tomlin (2006) for the case when the duration of disruptions is constant and the reliable supplier has no volume flexibility (this is similar to our scenario if future demand is independent of current sales and the order from the reliable supplier is placed before the disruption uncertainty for a period is resolved). In this setting, it is optimal for the firm to single source from the unreliable supplier for low disruption probability, and switch to single sourcing from the reliable supplier for high disruption probability. In contrast, in our setting with demand-sales dependency, we show scenarios where it is optimal to place anticipatory orders from the unreliable supplier even when using the reliable supplier to meet period 1 demand.

Several papers that study disruptions in the presence of both reliable and unreliable suppliers find that increasing disruption risk tends to increase the use of the reliable supplier. For example, using a single period setting, Dada et al. (2007) find that increasing disruption risk for the unreliable supplier leads to a decrease in its utilization (as it is replaced by the reliable supplier). In our two-period setting, however, we find that placing an anticipatory order from the unreliable supplier (which increases quantity ordered from the unreliable supplier in period 1) is optimal, even in the presence of the reliable supplier.

### **Using inventory for mitigating disruption risk**

There is a broad literature that studies the value of inventory in mitigating disruption risks (Parlar and Perry, 1995; Parlar, 1997; Parlar et al., 1995; Parlar and Perry, 1996; Gürler and Parlar, 1997; Moynzadeh and Aggarwal, 1997). This work has also been extended to multi-location systems (Schmitt et al., 2015), as well as to settings where the cost of the waiting time after the disruption is quantified (Qi, 2013). None of this work considers a sales-demand dependency. For a general overview on inventory as a risk mitigation strategy we refer to the literature review by Snyder et al. (2016).

### **Future demand depends on present sales**

While new to the disruptions literature, dependence of future demand on current sales has previously been studied in the Operations literature. Our demand dynamics are inspired by a stream of papers that studies capacity decisions when future demand decreases if customers experience a stock-out (e.g. Schwartz, 1966; Hall and Porteus, 2000; Olsen and Parker, 2008; Olsen and Parker, 2014). Unlike our setting, stock-outs in these papers are not driven by supply disruptions and they focus on capacity decisions rather than risk-mitigation strategies when dual-sourcing.

There is also a stream of literature studying optimal production and sales policies when demand follows the Bass diffusion model (Bass, 1969) where future demand is driven by current sales (i.e. Ho et al., 2002; Kumar and Swaminathan, 2003). Ho et al. (2002) find that when additional capacity is expensive, it is optimal to stock up inventory before selling the product. While their result has a similar flavor as our findings, their problem does not account for disruption and focuses on re-allocating available capacity to meet as much demand as possible.

Sting and Huchzermeier (2012) consider the case where the uncertainties of demand and supply are correlated in a single-period setting. In contrast to our work, their model does not include disruption risk and supply uncertainty is in terms of one of two supply quantities. The authors find that the unreliable supplier becomes more attractive when there is a positive correlation of risk between demand and supply as there is an increased chance of supply matching demand under positive correlation.

To the best of our knowledge, our paper is the first to study supply disruption risk-mitigation strategies in short life cycle settings when future demand depends on current sales (which are influenced by current supply disruptions).

### 3. Demand-sales dependency and anticipatory orders

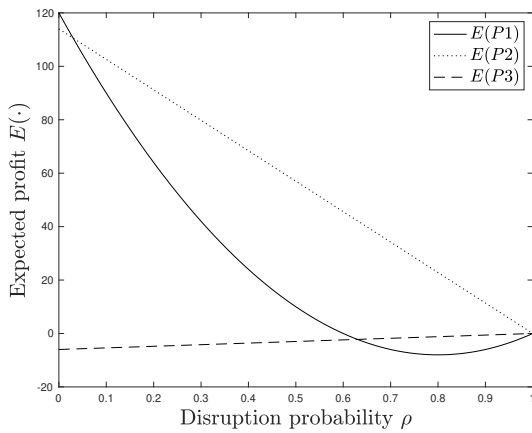
In this section, we use a simple numerical example to illustrate how the demand-sales dependency influences the ordering policy from an unreliable supplier. We illustrate that anticipatory orders may be optimal when future demand depends on present sales while anticipatory orders may never be optimal when future demand is independent of present sales. In the subsequent section, we show the validity of our observations using more general theoretical models.

We assume a two period horizon where a single firm (decision maker) sources from a long lead time, unreliable supplier subject to disruption. The unreliable supplier has a wholesale price  $c^u = 60$  per unit and is disrupted in any period with probability  $\rho \in [0, 1]$ . Disruption across both periods is independent and only applies to orders placed to arrive in that period. No supply arrives from the unreliable supplier in a given period if a disruption occurs. The unreliable supplier has a leadtime of 2 periods that requires the firm to commit to order quantities for periods 1 and 2 before the start of the sales season when supply outcomes in periods 1 and 2 are unknown. The sale price of the finished good is  $p = 100$ .

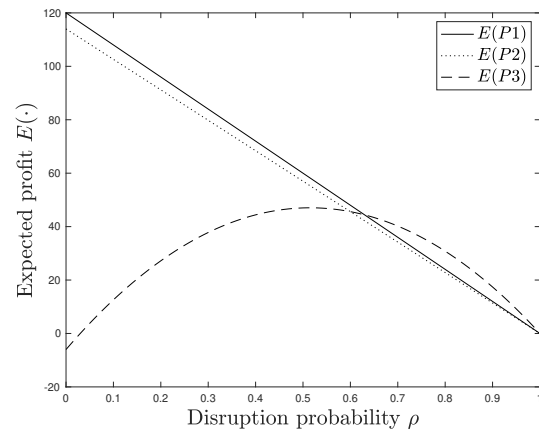
We consider two demand models. In the first demand model, period 2 demand depends on sales in period 1, which is affected by supply disruptions in period 1. If the unreliable supplier is disrupted in period 1, the inability to meet customer demand in period 1 limits demand in period 2. We set demand in period 1 to be  $D_1 = 1$  and the demand in period 2 to be  $\alpha = 2 \times (\text{sales in period 1})$ . In the second demand model, demand in period 1 is  $D_1 = 1$  and demand in period 2 is  $\alpha D_1 = 2$  independent of the occurrence of a disruption in period 1.

Order policies in our model are defined by the tuple  $(q_1, q_2)$  where  $q_1$  is the order quantity for period 1 and  $q_2$  is the order quantity for period 2. Any unsold inventory at the end of period 2 is assumed to have salvage value 0. The firm decides order quantities for delivery in period 1 ( $q_1$ ) and period 2 ( $q_2$ ) in order to maximize expected profits over the planning horizon. Let us consider the following order policies:

- *P1*: Place orders  $q_1 = 1$  and  $q_2 = 2$  where the ordered quantity matches expected demand in each period if no disruption occurs. We refer to *P1* as placing *current orders*.
- *P2*: Place orders  $q_1 = 3$  and  $q_2 = 0$ . The entire quantity to serve demand in periods 1 and 2 is ordered to arrive in period 1. We refer to *P2* as placing an *anticipatory order*.
- *P3*: Place orders  $q_1 = 3$  and  $q_2 = 2$ . The total quantity ordered exceeds expected demand if no disruption occurs and two orders are placed to serve demand in period 2. We refer to *P3* as placing a *double order*.



**Figure 1.a** Demand in period 2 depends on sales in period 1



**Figure 1.b** Demand in period 2 is independent of sales in period 1

In Figure 1.a, we plot the profit of each ordering policy as a function of  $\rho$  when period 2 demand depends on period 1 sales. We observe that order policy  $P1$  (placing current orders) yields a higher profit than order policies  $P2$  (placing an anticipatory order) and  $P3$  (placing a double order) for low disruption probabilities when neither period is likely to be disrupted. Thus, placing current orders (policy  $P1$ ) is likely to be optimal for low disruption probabilities.

Placing an anticipatory order (order policy  $P2$ ) yields the highest profit when the disruption probability is somewhat higher. Observe that the total quantity ordered under policy  $P2$  is the same as that under policy  $P1$  (placing current orders). The only difference is that the quantity  $\alpha D_1 = 2$  is added to the first period order and subtracted from the second period order (relative to  $P1$ ). We refer to the quantity  $\alpha D_1 = 2$  as a *complete anticipatory order* because it moves the complete order from period 2 to period 1. Notice that the anticipatory order  $\alpha D_1 = 2$  is always sold if there is no disruption in period 1 because demand in period 2 is guaranteed when no disruption occurs in period 1. Placing an anticipatory order (policy  $P2$ ) is likely to be optimal for slightly larger probabilities of disruption (relative to  $P1$ ) because it does better if there is a disruption in one of periods 1 or 2, a more likely event when the probability of disruption is slightly higher. If there is a disruption in period 1, policy  $P2$  results in less unsold inventory at the end of period 2 relative to policy  $P1$ . If there is no disruption in period 1 but period 2 is disrupted, the quantity  $\alpha D_1$  ordered in period 1 adds value relative to policy  $P1$  because it can be used to serve period 2 demand. A slightly higher disruption probability makes it less likely that there will be no disruption in either period, thus increasing the value of policy  $P2$  relative to policy  $P1$ .

Placing a double order (policy  $P3$ ) is never optimal because there is no value in duplicating the anticipatory order  $\alpha D_1 = 2$  for delivery in the second period when period 2 demand depends on period 1 sales. If there is a disruption in period 1, there is no demand in period 2 and the duplicate order incurs cost without providing revenue. If there is no disruption in periods 1 and 2, then the duplicate order results in unsold inventory at the end of period 2.

In Figure 1.b, we plot the profit of each of the three ordering policy as a function of  $\rho$  when period 2 demand is independent of period 1 sales. Notice that placing current orders (policy  $P1$ ) is optimal when disruption probabilities are low, as in the previous demand

model. For very high disruption probabilities placing double orders (policy  $P3$ ) is optimal. Under this policy, the quantity  $\alpha D_1 = 2$  is ordered for delivery in periods 1 and 2 to serve demand  $\alpha D_1 = 2$  in period 2. This makes sense when one of the two periods is very likely to be disrupted. The double order allows period 2 demand to be served when either period 1 or period 2 is disrupted. Intuitively, this order policy is optimal for very high disruption probabilities.

Observe that placing an anticipatory order (policy  $P2$ ) is never optimal, i.e., no anticipatory orders are placed, when period 2 demand is independent of period 1 sales.

This numerical illustration suggests that dependence of future demand on current sales is a key driver of the use of anticipatory orders when facing disruption risk. In the next section, we prove these observations using theoretical models.

#### 4. Mathematical models linking sales-demand dependency to anticipatory orders

In the following, we study how the sales-demand dependency influences the placement of anticipatory orders using three different models:

1. Demand for period 1 is deterministic while single sourcing from an unreliable supplier with a long leadtime.
2. Demand for period 1 is deterministic while sourcing simultaneously from an unreliable supplier with long leadtime and a reliable supplier with short leadtime.
3. Demand for period 1 is stochastic while single sourcing from an unreliable supplier with a long leadtime.

All three models have the following assumptions in common:

Assumption 1. Sales occur over two periods. Any unsold inventory at the end of period 2 has salvage value 0.

Assumption 2. Inventory holding cost  $h$  satisfies  $h < \min\{p - c^u, c^u\}$ .

This assumption avoids scenarios where it is never optimal to carry inventory from period 1 to period 2.

In all three models, the unreliable supplier has a leadtime of at least 2 periods that requires the firm to commit to order quantities for periods 1 and 2 from the unreliable supplier before the beginning of the time horizon when supply outcomes in periods 1 and 2 are unknown.

For convenience, we summarize the fundamental notation defined in this section in Table 1. All additional variables used later are defined in the corresponding sections.

**Table 1** Fundamental notation defined in Section 2

| Notation     | Meaning                                                                                                                                                              |
|--------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $q_i \geq 0$ | Order quantities from the unreliable supplier for delivery in period $i \in 1, 2$ .<br>We add a superscript $r$ when the order is placed with the reliable supplier. |
| $D_i > 0$    | Deterministic demand in period $i \in 1, 2$                                                                                                                          |
| $\alpha > 0$ | Demand multiplier for demand in period 2                                                                                                                             |
| $\rho_i$     | Probability of disruption in period $i \in 1, 2$ with $0 \leq \rho_i < 1$                                                                                            |
| $p > 0$      | Sale price per unit                                                                                                                                                  |
| $c^u$        | Cost per unit from the unreliable supplier with $0 < c^u < p$                                                                                                        |
| $c^r$        | Cost per unit from the reliable supplier with $0 < c^u < c^r < p$                                                                                                    |
| $h$          | Cost for carrying inventory from period 1 to period 2 with $0 < h < \min\{p - c^u, c^u\}$ .                                                                          |
| $\beta$      | Sales multiplier for period 2 demand if a disruption occurs in period 2 with $0 \leq \beta \leq 1$ .                                                                 |
| $\xi \geq 0$ | Penalty fees for non-delivery received by buyer.                                                                                                                     |
| $r$          | Disruption correlation factor with $-1 \leq r \leq +1$ .                                                                                                             |

#### 4.1. Base model with deterministic demand for period 1, penalty fees, and different disruption probabilities in periods 1 and 2

In this subsection we develop our base model and include the following additional assumptions:

Assumption 3. Demand  $D_1$  in period 1 is deterministic. Demand in period 2 depends on the sales in period 1. If there is no disruption in period 1, demand in period 2 is  $\alpha \times$  (sales in period 1) with  $\alpha > 0$ . If there is a disruption in period 1 and no sales takes place, demand in period 2 is  $\beta \alpha \times$  (demand in period 1) and  $0 \leq \beta \leq 1$ .

Observe that the demand in period 2 depends on the supply outcome in period 1 and is lower if there is a disruption in period 1.

Assumption 4. The unreliable supplier is disrupted in period 1 with probability  $\rho_1$  and in period 2 with probability  $\rho_2$ . If a disruption occurs in period  $i \in \{1, 2\}$ , there is no supply from the unreliable supplier in that period. Each disruption lasts exactly one period. If no disruption occurs in period  $i$ , supply for period  $i$  arrives in time to meet demand for period  $i$ . The firm pays only for the quantity received in each period. The disruption probability in the first period is bound by  $\rho_1 < \frac{p - c^u}{p(1 - \beta)}$ .

Our assumption that  $\rho_1 < \frac{p - c^u}{p(1 - \beta)}$  ensures that placing an order to serve period 2 demand makes a positive contribution to expected profit. The expected revenue contribution of ordering  $q_2$  units for period 2 is given by  $(1 - \rho_1)(1 - \rho_2)pq_2 + \rho_1(1 - \rho_2)\beta q_2$  because revenue is collected if and only if neither period 1 nor period 2 is disrupted, or if only period 1 is

disrupted. The expected cost of placing an order  $q_2$  that arrives in period 2 is  $(1 - \rho_2)c^u q_2$ . Our assumption that  $\rho_1 < \frac{p-c^u}{p(1-\beta)}$  is equivalent to requiring that the expected revenue is larger than the expected cost of the period 2 order, i.e.  $(1 - \rho_2)((1 - \rho_1)p + \rho_1\beta p - c^u)q_2 > 0$ . This assumption assures that optimal order policies aim to serve demand in period 2 even though any left over inventory at the end of period 2 has a salvage value of 0.

Assumption 5. There is a penalty fee that the supplier pays the buyer for any non-delivery, i.e., the buyer receives payment  $\xi \geq 0$  per unit from the supplier for any ordered quantity that is not delivered to the buyer. We have  $\rho_i < \frac{c^u}{c^u + \xi} \forall i \in \{1, 2\}$ .

We limit the disruption probability to avoid scenarios where the buyer places large orders to collect penalty fees (when penalty fees are large) instead of goods.

Similar to Figure 1.a, we first show that there are only three potentially optimal order policies when future demand depends on current sales. The formal proof of Proposition 1 is provided in the appendix.

**PROPOSITION 1.** *Let  $\rho_1 < \frac{p-c^u}{p(1-\beta)}$ ,  $\rho_1 < \frac{c^u}{p}$ ,  $\rho_i < \frac{c^u}{c^u + \xi} \forall i \in \{1, 2\}$ , and  $h < p - c^u$ . Then, one of the following three order policies must be optimal:*

- *Policy P1:  $q_1 = D_1$  and  $q_2 = \alpha D_1$ .*
- *Policy P2:  $q_1 = D_1 + (1 - \beta)\alpha D_1$  and  $q_2 = \beta\alpha D_1$ .*
- *Policy P3:  $q_1 = D_1 + \alpha D_1$  and  $q_2 = \beta\alpha D_1$ .*

We discuss the three order policies identified in Proposition 1 and the disruption probabilities over which they are likely to be optimal.

Policy P1 sets the order for the first period  $q_1 = D_1$  and the order for the second period  $q_2 = \alpha D_1$ . The expected profit function is:

$$\begin{aligned}
 E(P1) = & \underbrace{(1 - \rho_1)(p - c^u)D_1}_{\text{Profit from first period}} \\
 & + \underbrace{(1 - \rho_1)(1 - \rho_2)p\alpha D_1 + \rho_1(1 - \rho_2)p\beta\alpha D_1 - (1 - \rho_2)c^u\alpha D_1}_{\text{Profit from second period}} \\
 & + \underbrace{\rho_1\xi D_1 + \rho_2\xi\alpha D_1}_{\text{Penalty fees received for non-delivery}}. \tag{1}
 \end{aligned}$$

Observe that revenue from the second period order  $q_2 = \alpha D_1$  depends on whether there is a disruption in either period. If a disruption occurs in the first period, there is reduced demand in the second period, which limits the revenue received. If there is a disruption in

the second period, no supply arrives in the second period and thus no revenue is received. Thus, policy  $P1$  is likely to be optimal for low disruption probabilities when the term  $(1 - \rho_1)(1 - \rho_2)$  is close to 1. In particular, policy  $P1$  is optimal when  $\rho_1 = \rho_2 = 0$ .

Observe that the total quantity ordered under Policy  $P2$  is the same as that under Policy  $P1$ . The only difference is that the quantity  $(1 - \beta)\alpha D_1$  is added to the first period order and subtracted from the second period order (relative to  $P1$ ). Recall that we refer to  $P2$  as an *anticipatory order* because it moves the quantity  $(1 - \beta)\alpha D_1$  from period 2 to period 1 relative to policy  $P1$ . The expected profit from Policy  $P2$  is as follows:

$$E(P2) = \underbrace{(1 - \rho_1)(p - c^u)D_1}_{\text{Profit from first period}} + \underbrace{(1 - \rho_1)(p - c^u - h)(1 - \beta)\alpha D_1}_{\text{Profit from anticipatory order}} + \underbrace{(1 - \rho_2)(p - c^u)\beta\alpha D_1}_{\text{Profit from period 2 order}} + \underbrace{\rho_1\xi(1 + (1 - \beta)\alpha)D_1 + \rho_2\xi\beta\alpha D_1}_{\text{Penalty fees received for non-delivery}}. \quad (2)$$

Observe that the anticipatory order  $(1 - \beta)\alpha D_1$  is always sold if there is no disruption in period 1 because demand in period 2 is guaranteed in this case. Policy  $P2$  is likely to be optimal for slightly larger probabilities of disruption (relative to  $P1$ ) because it does better if there is a disruption in one of periods 1 or 2, a more likely event when the probability of disruption is slightly higher. If there is a disruption in period 1, policy  $P2$  results in less unsold inventory at the end of period 2 relative to policy  $P1$ . If there is no disruption in period 1 but period 2 is disrupted, the quantity  $(1 - \beta)\alpha D_1$  ordered in period 1 adds value relative to policy  $P1$  because it can be used to serve period 2 demand. A slightly higher disruption probability makes it less likely that there will be no disruption in either period, thus increasing the value of policy  $P2$  relative to policy  $P1$ .

Observe that under the third order policy  $P3$ ,  $q_1 = D_1 + \alpha D_1$  and  $q_2 = \beta\alpha D_1$ , the quantity  $\beta\alpha D_1$  is ordered for delivery both in periods 1 and 2 to serve demand  $\beta\alpha D_1$  in period 2. This policy is likely to be optimal when disruption probabilities are very high making it very likely that at least one of the two periods is disrupted. The expected profit from policy  $P3$  is:

$$E(P3) = \underbrace{(1 - \rho_1)(p - c^u)D_1}_{\text{Profit from first period}} + \underbrace{(1 - \rho_1)(p - c^u - h)\alpha D_1}_{\text{Profit from p. 1 order carried over to p.2}} + \underbrace{(1 - \rho_2)(p\rho_1 - c^u)\beta\alpha D_1}_{\text{Profit from p.2 order}} + \underbrace{\rho_1\xi(1 + \alpha)D_1 + \rho_2\xi\beta\alpha D_1}_{\text{Penalty fees received for non-delivery}}. \quad (3)$$

Consistent with the above discussion, we now show that all three policies are optimal over a non-empty interval of disruption probabilities.

**THEOREM 1.** *Let  $\rho_1 < \frac{p-c^u}{p(1-\beta)}$ ,  $\rho_1 < \frac{c^u}{p}$ ,  $\rho_i < \frac{c^u}{c^u+\xi} \forall i \in \{1,2\}$ , and  $h < p - c^u$ . Then, policy P1 is optimal for  $\rho_2 < \frac{h}{p} - \frac{c^u+\xi}{p} \frac{\rho_1-\rho_2}{1-\rho_1}$ . Policy P2 is optimal for  $\frac{h}{p} - \frac{c^u+\xi}{p} \frac{\rho_1-\rho_2}{1-\rho_1} < \rho_2 < \frac{c^u+h}{p} - \frac{\xi}{p} \frac{\rho_1}{1-\rho_1}$ . Policy P3 is optimal for  $\frac{c^u+h}{p} - \frac{\xi}{p} \frac{\rho_1}{1-\rho_1} < \rho_2 < \frac{c^u}{c^u+\xi}$ .*

The proof is provided in the appendix. Observe that policy P1 is optimal and no anticipatory orders are placed when disruption probability for period 2 is 0 (or very small). Theorem 1 shows that P2 is optimal and anticipatory orders are placed when disruption probability for period 2 increases above a threshold. For higher disruption probability, placing an anticipatory order increases expected profits because it adds value when there is a disruption in one of the two periods.

We make the following observation:

Observation 1: The larger the dependent demand component in period 2 (the lower the value of  $\beta$ ), the larger the size of the anticipatory order.

### Correlation of disruption probabilities

Next, we modify our model to discuss how correlation of disruption probabilities across both periods influences the placement of anticipatory orders. We observe that when correlation is negative, the range of disruption probability over which anticipatory orders are placed is large. As correlation increases towards 1, this range shrinks and no anticipatory orders are placed for correlation of 1.

To facilitate the analysis, we assume that disruption probabilities are identical across both periods ( $\rho = \rho_1 = \rho_2$ ), penalty fees are zero  $\xi = 0$ , and  $\beta = 0$ . Our insights remain valid when we relax these assumptions.

We introduce the correlation coefficient  $r \in [-1, 1]$ , where  $r = -1$  is the case where disruption probabilities across the two periods are perfectly negatively correlated,  $r = 0$  is the case where disruption probabilities across the two periods are uncorrelated, and  $r = +1$  is the case where disruption probabilities across the two periods are perfectly positively correlated. In the following we quantify the likelihood for the different disruption events:

First, with probability  $f_1(r)$ , we have the event that no disruption occurs in either period:

$$f_1(r) = (1 - \rho)^2 + \frac{1}{2}(1 - \rho)r + \frac{1}{2}(1 - \rho)(\rho - (1 - \rho))r^2.$$

Observe that  $f_1(r)$  satisfies the boundary conditions:  $f_1(-1) = 0$ ,  $f_1(0) = (1 - \rho)^2$  and  $f_1(+1) = (1 - \rho)$ .

Second, with probability  $f_2(r)$ , we have no disruption in period 1 and a disruption in period 2:

$$f_2(r) = (1 - \rho)\rho - \frac{1}{2}(1 - \rho)r + (1 - \rho)\left(\frac{1}{2} - \rho\right)r^2.$$

Observe that  $f_2(r)$  satisfies the boundary conditions:  $f_2(-1) = (1 - \rho)$ ,  $f_2(0) = (1 - \rho)\rho$  and  $f_2(+1) = 0$ .

Third, with probability  $f_3(r)$ , we have a disruption in period 1 and no disruption in period 2:

$$f_3(r) = \rho(1 - \rho) - \frac{1}{2}\rho r + \rho\left(\rho - \frac{1}{2}\right)r^2.$$

Observe that  $f_3(r)$  satisfies the boundary conditions:  $f_3(-1) = \rho$ ,  $f_3(0) = \rho(1 - \rho)$  and  $f_3(+1) = 0$ .

Fourth, with probability  $f_4(r)$ , we have a disruption in each of period 1 and period 2:

$$f_4(r) = \rho^2 + \frac{1}{2}\rho r + \rho\left(\frac{1}{2} - \rho\right)r^2.$$

Observe that  $f_4(r)$  satisfies the boundary conditions:  $f_4(-1) = 0$ ,  $f_4(0) = \rho^2$  and  $f_4(+1) = \rho$ .

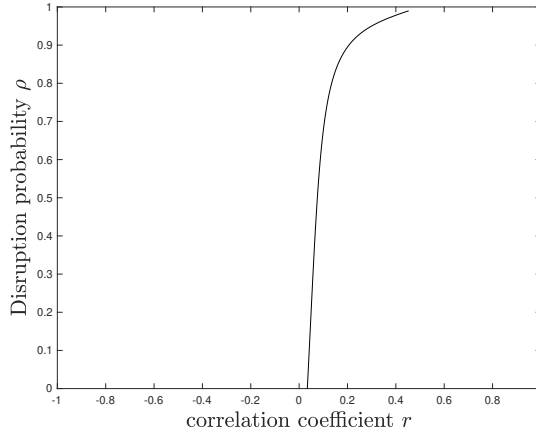
Define the policies  $P1$  ( $q_1 = D_1, q_2 = \alpha D_1$ ) and  $P2$  ( $q_1 = D_1 + (1 - \beta)\alpha D_1, q_2 = \beta\alpha D_1$ ) as in Proposition 1 in the previous section. We show that:

LEMMA 1. *Let  $\xi = 0$ ,  $\beta = 0$ ,  $\rho = \rho_1 = \rho_2$ ,  $\rho < \frac{p-c^u}{p}$ ,  $\rho < \frac{c^u}{p}$ , and  $h < p - c^u$ . Then, anticipatory orders are always optimal ( $P2$  has a higher expected profit than  $P1$ ) for  $r = -1$ . Anticipatory orders are never optimal ( $P1$  has a higher expected profit than  $P2$ ) for  $r = +1$ .*

If the disruption probabilities in period 1 and 2 are perfectly negatively correlated ( $r = -1$ ), it is always the case that one period is disrupted while the other is not for  $\rho > 0$ . Thus, placing anticipatory orders ( $P2$ ) always yields a higher profit than placing two independent orders for each period ( $P1$ ).

If the disruption probabilities in period 1 and 2 are perfectly positively correlated ( $r = +1$ ), there are only two cases: either there is no disruption in both periods or both periods are disrupted. In either case, placing two independent orders ( $P1$ ) yields a higher profit than placing anticipatory orders ( $P2$ ).

For values of  $r < 1$ , anticipatory orders continue to be optimal. As we plot in Figure 2.a ( $p = 100$ ,  $h = 7$ ,  $c^u = 70$ ), the disruption probability threshold when anticipatory orders are optimal increases with the value of  $r$ .



**Figure 2.a** Correlation of disruption probabilities across both periods

### Orders where the buyer has recourse in case of disruption

We consider recourse to be a setting where the firm has the right to reduce the order for period 2 if the order for period 1 has been disrupted. Given the long lead time, we do not allow orders to be increased in period 2 after observing the outcome of period 1. In this section we show that anticipatory orders continue to be optimal in the presence of recourse, although at a higher disruption probability than the case without recourse. To simplify the analysis, we assume that penalty fee  $\xi = 0$  and that disruption probabilities are identical across both periods ( $\rho = \rho_1 = \rho_2$ ). We have three potential order policies.

First, under policy  $P1r$ , the period 1 order is  $q_1 = D_1$  and period 2 order is  $q_2 = \alpha D_1$ . If there is no disruption in period 1, the period 2 order stays  $q_2 = \alpha D_1$ . If there is a disruption in period 1, the period 2 order is reduced by  $(1 - \beta)\alpha D_1$  to  $q_2 = \beta\alpha D_1$ . The expected profit under policy  $P1r$  is:

$$\begin{aligned} E(P1r) = & (1 - \rho)(p - c^u)D_1 + (1 - \rho)(1 - \rho)p\alpha D_1 + \rho(1 - \rho)p\beta\alpha D_1 - (1 - \rho)c^u\beta\alpha D_1 \\ & - (1 - \rho)(1 - \rho)c^u(1 - \beta)\alpha D_1. \end{aligned} \quad (4)$$

Second, under order policy  $P2r$  the same order quantities are placed as under order policy  $P2$  ( $q_1 = D_1 + (1 - \beta)\alpha D_1$  and  $q_2 = \beta\alpha D_1$ ).  $P2r$  contains an anticipatory order. There is no change in the period 2 order if period 1 is disrupted. The expected profit is:

$$E(P2r) = (1 - \rho)(p - c^u)D_1 + (1 - \rho)(p - c^u - h)(1 - \beta)\alpha D_1 + (1 - \rho)(p - c^u)\beta\alpha D_1. \quad (5)$$

Clearly,  $E(P2r) = E(P2)$  for  $\xi = 0$ . Observe that policy  $P2r$  does better than policy  $P1r$  when period 2 is disrupted but period 1 is not. Given recourse, they do equally well when period 1 is disrupted but period 2 is not.

Third, under order policy  $P3r$  the order quantities are:  $q_1 = D_1(1 + \alpha)$  and  $q_2 = \beta\alpha D_1$ . There is no change in the period 2 order if period 1 is disrupted. Policy  $P3r$  contains a duplicate order. The expected profit under policy  $P3r$  is:

$$E(P3r) = (1 - \rho)(p - c^u)D_1 + (1 - \rho)(p - c^u - h)\alpha D_1 + (1 - \rho)(\rho p - c^u)\beta\alpha D_1. \quad (6)$$

Observe that policy  $P3r$  does worse than policy  $P2r$  when neither period is disrupted because of unsold inventory at the end of the horizon. Policy  $P3r$ , however, does better than policy  $P2r$  when period 2 is disrupted but period 1 is not. They do equally well when period 1 is disrupted but period 2 is not or both periods are disrupted.

We find that in the presence of recourse, the optimal policy transitions from  $P1r$  to  $P2r$  to  $P3r$  as the disruption probability increases. Anticipatory orders (policy  $P2r$ ) continue to be optimal when buyers have recourse, albeit at a higher disruption probability compared to the case without recourse.

**LEMMA 2.** *Assume the buyer has recourse with the supplier, i.e., the buyer can reduce the second period order if the first period order is disrupted. Let  $\xi = 0$ ,  $\rho = \rho_1 = \rho_2$ ,  $\rho < \frac{p - c^u}{p(1 - \beta)}$ ,  $\rho < \frac{c^u}{p}$  and  $h < p - c^u$ . Then, policy  $P1r$  is optimal for  $\rho < \frac{h}{p - c^u}$ . Policy  $P2r$  is optimal for  $\frac{h}{p - c^u} < \rho < \frac{h + c^u}{p}$ . Policy  $P3r$  is optimal for  $\frac{h + c^u}{p} < \rho < \frac{p - c^u}{p(1 - \beta)}$ .*

#### 4.2. Introducing a responsive and reliable supplier

In this section we extend our findings to the context where a responsive and reliable supplier is also available along with the unreliable supplier. All assumptions 1-5 continue to hold in our model, but for ease of exposition, we set  $\rho := \rho_1 = \rho_2$ ,  $\xi = 0$  and  $\beta = 0$ . We show that anticipatory orders continue to be optimal, even when the reliable supplier is used in period 1.

We assume that the reliable supplier is never disrupted, and has a leadtime of one period. Further, in agreement with the existing literature (Qi, 2013; Tomlin, 2006), we assume that the reliable supplier costs  $c^r > c^u$ . For example, Zara sources goods at higher costs from reliable local suppliers in Spain while it also sources goods at lower costs from distant suppliers in Asia (Ferdows et al., 2003). Local suppliers are often assumed to be

more reliable than distant suppliers due to various risks associated with off-shoring such as transportation disruptions (e.g., the Suez Canal blockage).

Further, we assume that  $c^u + h < c^r$  to ensure that it may still be optimal to carry inventory ordered from the unreliable supplier from period 1 to period 2. The one period lead time allows the firm to observe the outcome of period 1 and place a contingent order with the reliable supplier for period 2 if needed. Let  $q_1^r$  ( $q_1^u$ ) and  $q_2^r$  ( $q_2^u$ ) be the orders placed with the reliable (unreliable) supplier for periods 1 and 2 respectively.

We identify the following possible order policies:

**PROPOSITION 2.** *Let  $c^u + h < c^r$  and  $\rho < \frac{p-c^u}{p}$ . Then, the following five ordering policies are potentially optimal:*

- *Policy P1R:*  $q_1^u = D_1$ ,  $q_2^u = \alpha D_1$ ,  $q_1^r = 0$  and  $q_2^r = 0$ .
- *Policy P2R:*  $q_1^u = D_1 + \alpha D_1$ ,  $q_2^u = 0$ ,  $q_1^r = 0$  and  $q_2^r = 0$ .
- *Policy P3R:*  $q_1^r = D_1$ ,  $q_2^u = \alpha D_1$ ,  $q_1^u = 0$  and  $q_2^r = 0$ .
- *Policy P4R:*  $q_1^r = D_1$ ,  $q_1^u = \alpha D_1$ ,  $q_2^u = 0$ , and a contingent order of  $q_2^r = \alpha D_1$  if there is a disruption in period 1.
- *Policy P5R:*  $q_1^r = D_1$ ,  $q_1^u = \alpha D_1$ ,  $q_2^u = \alpha D_1$ , and  $q_2^r = 0$ .

The formal proof of Proposition 2 is provided in the appendix. Let us better understand the five identified order policies in Proposition 2.

Policy *P1R* is identical to policy *P1* in Proposition 1.

Policy *P2R* is identical to policy *P2* in Proposition 1 and involves the placement of an anticipatory order.

Policy *P3R* is similar to policy *P1R* except that supply to serve demand in period 1 is ordered from the reliable supplier ( $q_1^r = D_1$ ). We have the following expected profit function:

$$E(P3R) = \underbrace{(p - c^r)D_1}_{\text{Profit from first period}} + \underbrace{(1 - \rho)(p - c^u)\alpha D_1}_{\text{Profit from second period}} \quad (7)$$

Policy *P3R* does not involve the placement of an anticipatory order.

Policy *P4R* is similar to policy *P2R* except that supply to serve demand in period 1 is ordered from the reliable supplier ( $q_1^r = D_1$ ). Policy *P4R* contains an anticipatory order. Using the reliable supplier to serve period 1 demand guarantees demand in period 2 even if a disruption occurs in period 1. If the unreliable supplier is not disrupted in period 1, the

anticipatory order  $q_1^u = \alpha D_1$  is used to meet demand. If the unreliable supplier is disrupted in period 1, the firm can place a contingent order  $q_2^r = \alpha D_1$  with the responsive and reliable supplier for period 2. We have the following expected profit function:

$$E(P4R) = \underbrace{(p - c^r)D_1}_{\text{Profit from serving low demand in p.1}} + \underbrace{(1 - \rho)(p - c^u - h)\alpha D_1}_{\text{Remaining p. 1 order carried over to p. 2}} + \rho(p - c^r)\alpha D_1$$

Finally, policy  $P5R$  sets the order for the first period  $q_1^r = D_1$  and  $q_1^u = \alpha D_1$ . The firm also commits to an order  $q_2^u = \alpha D_1$  for period 2 with the unreliable supplier (while not using the reliable supplier to serve period 2 demand  $q_2^r = 0$ ). Essentially, the firm double orders for demand in period 2 by placing  $q_1^u = \alpha D_1$  and  $q_2^u = \alpha D_1$ , hoping that one of the two orders arrives. Such a policy may potentially be optimal when the disruption probability is very high and when the reliable supplier is somewhat expensive.

In Figure 3.a and 3.b we present numerical results that show the optimality of policies  $P2R$  and  $P4R$  that contain anticipatory orders. We find the existence of anticipatory orders despite the use of the reliable supplier.

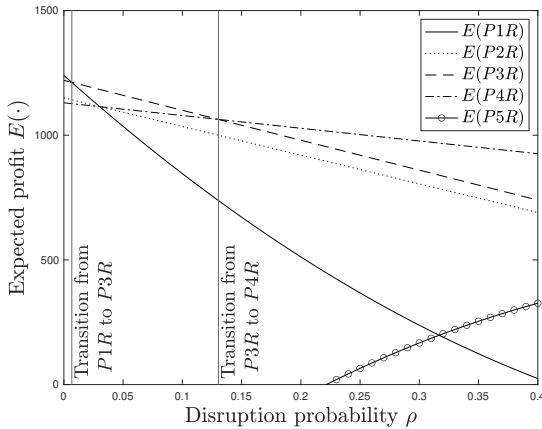


Figure 3.a Expected profit for a high value of  $\alpha$

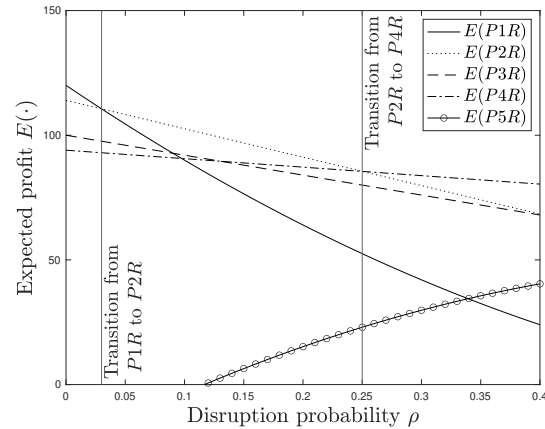


Figure 3.b Expected profit for a low value of  $\alpha$

In Figure 3.a, we plot the expected profit functions for the five order policies as a function of the disruption probability  $\rho$  for a high value of  $\alpha$ . We use the parameters:  $D = 1$ ,  $p = 100$ ,  $c^u = 60$ ,  $c^r = 80$ ,  $h = 3$ ,  $\alpha = 30$ . We observe that for very low disruption probabilities ( $\rho < 0.01$ , left of the first vertical line in the figure) order policy  $P1R$  is optimal. For somewhat higher disruption probabilities ( $0.01 < \rho < 0.13$ , between the two vertical lines in the figure), it is optimal to use order policy  $P3R$ . For even higher disruption probabilities

( $0.13 < \rho < 0.4$ , right of the second vertical line), it is optimal to use order policy  $P4R$ , which involves the placement of an anticipatory order. Observe that order policies  $P2R$  and  $P5R$  are never optimal for the chosen parameters for  $\rho \leq 0.4$ . Observe that for the given parameters, anticipatory orders (policy  $P4R$ ) are optimal despite the use of the reliable supplier.

In Figure 3.b, we plot the expected profit functions for the five order policies as a function of the disruption probability  $\rho$  for a low value of  $\alpha$ . We choose the same parameter set as before, except that  $\alpha$  is smaller:  $\alpha = 2$  (instead of  $\alpha = 30$ ). We observe that for low disruption probabilities ( $\rho < 0.03$ , left of the first vertical line in the figure) order policy  $P1R$  is optimal. For somewhat higher disruption probabilities ( $0.03 < \rho < 0.25$ , between the two vertical lines), it is optimal to use order policy  $P2R$ , which involves the placement of an anticipatory order without using the reliable supplier. For higher disruption probabilities ( $0.25 < \rho < 0.4$ , right of the second vertical line), it is optimal to use order policy  $P4R$ , which involves the placement of an anticipatory order while using the reliable supplier. Observe that order policies  $P3R$  and  $P5R$  are never optimal for the chosen parameters for  $\rho \leq 0.4$ .

Comparing both plots, we observe that it is optimal to place anticipatory orders for both low and high values of  $\alpha$  despite the use of the reliable supplier. For a high value of  $\alpha$ , notice that order policy  $P3R$  is optimal (and order policy  $P2R$  is not optimal) and transitions to  $P4R$  as  $\rho$  increases. In contrast, for a low value of  $\alpha$ , order policy  $P2R$  is optimal (and order policy  $P3R$  is not optimal) and transitions to  $P4R$  as  $\rho$  increases. This observation results from the expected profit functions of order policies  $P2R$  (Eq. 2), where the term  $(1 - \rho)(p - c^u - h)\alpha D_1$  dominates for large  $\alpha$ , and  $P3R$  (Eq. 7), where the term  $(1 - \rho)(p - c^u)\alpha D_1$  dominates for large  $\alpha$ .

We are particularly interested in understanding when the optimal order policy transitions from policy  $P3R$  to policy  $P4R$  with an anticipatory order placed with the unreliable supplier despite the use of the reliable supplier. Consistent with our numerical analysis, we next show that the optimal order policy transitions from  $P3R$  to  $P4R$  if the demand multiplication factor  $\alpha$  is sufficiently large. For large  $\alpha$ , we first show that order policy  $P3R$  is optimal over a range of disruption probability.

**LEMMA 3.** *Let  $c^u + h < c^r$ ,  $\rho < \frac{p - c^u}{p}$  and  $\alpha > \frac{c^r - c^u}{h}$ . Then order policy  $P3R$  is optimal for  $\frac{h}{p} < \rho < \min\{\frac{h}{p + h - c^r}, \frac{h + c^u}{p}, \frac{p - c^u}{p}\}$ .*

We next show that the optimal order policy transitions from  $P3R$  to order policy  $P4R$  at  $\rho = \frac{h}{p+h-c^r}$  if  $\frac{h}{p+h-c^r} < \min\{\frac{h+c^u}{p}, \frac{p-c^u}{p}\}$ :

**THEOREM 2.** *Let  $c^u + h < c^r$ ,  $\rho < \frac{p-c^u}{p}$ ,  $\alpha > \frac{c^r-c^u}{h}$ . Then order policy  $P3R$  is optimal for  $\frac{h}{p} < \rho < \min\{\frac{h}{p+h-c^r}, \frac{h+c^u}{p}, \frac{p-c^u}{p}\}$  and order policy  $P4R$  is optimal for  $\frac{h}{p+h-c^r} < \rho < \min\{\frac{h+c^u}{p}, \frac{p-c^u}{p}\}$ . This range is non-empty for  $\frac{h}{p+h-c^r} < \min\{\frac{h+c^u}{p}, \frac{p-c^u}{p}\}$ .*

Our results show that for large  $\alpha$ , as the disruption probability increases beyond  $\rho = \frac{h}{p+h-c^r}$ , despite the use of the reliable supplier to meet period 1 demand, it is optimal to place an anticipatory order with the unreliable supplier in period 1 to serve period 2. The anticipatory order is valuable when no disruption occurs in period 1 and the reliable supplier is used in period 2 for a contingent order if a disruption occurs in period 1. This is particularly helpful when the disruption probability is high.

Recall that we found in the numerical analysis that order policy  $P3R$  transitions to  $P4R$  at  $\rho = 0.13$  for  $\alpha = 30$ . This is consistent with Theorem 2 according to which the transition occurs at  $\rho = \frac{h}{p+h-c^r} \approx 0.13$ . Also the condition for a high  $\alpha$  value is satisfied:  $30 = \alpha > \frac{c^r-c^u}{h} \approx 7$ .

We conclude that anticipatory orders with the unreliable supplier may be optimal even when a firm has access to a reliable, quick-response supplier.

### 4.3. Stochastic demand model

In this section, we remove the reliable supplier but add demand uncertainty in the first period to understand its effect on anticipatory orders. We show that relative to the case without uncertainty, anticipatory orders are placed at even lower disruption probabilities when period 1 demand is uncertain. Assumptions 1, 2, 4, 5 continue to hold but for ease of exposition, we set  $\rho := \rho_1 = \rho_2$  and  $\xi = 0$ . Assumption 3 is replaced by the following:

**Assumption 3'.** Demand in period 1 follows a two-point distribution where demand is either  $D_1 + \Delta_1$  (high) with probability  $\delta_1$  (with  $0 < \delta_1 < 1$ ) or  $D_1 - \Delta_1$  (low) with probability  $1 - \delta_1$ . To ensure demand is positive, we assume  $\Delta_1 < D_1$ . Demand in period 2 depends on sales in period 1 and is given by

demand in period 2 = sales in period 1  $\times \alpha$ , where  $\alpha \geq 0$ .

In the online appendix, we further expand the model by adding uncertain demand to period 2 that is independent of period 1 sales making period 2 demand less dependent on period 1 sales (similar to the  $\beta$  parameter in the deterministic model). Consistent with

our findings in the absence of stochasticity, we find that the introduction of independent stochastic demand to period 2 results in smaller anticipatory orders being placed. Thus, the more period 2 demand is dependent on period 1 sales, the more pronounced anticipatory orders become (in agreement with the deterministic demand model).

Given that demand in period 1 can take only two discrete values, there is a finite set of potentially optimal order policies depending on the parameter values of the model. We show that there can be up to 6 order policies at optimality across all parameter values. We provide a complete discussion of all 6 order policies in the appendix.

Assumption 6. To simplify the analysis, in this section, we assume the parameter space to be constrained by the condition  $(1 - \rho)p\delta_1 < c^u$ .

This condition restricts attention to policies that do not serve high demand in period 2 because it ensures that serving a unit of high demand in period 2 provides negative expected contribution to profit. This occurs because  $(1 - \rho)c^u$ , the expected cost of serving a unit of high demand in period 2, is larger than  $(1 - \rho)(1 - \rho)\delta_1 p$ , the expected revenue from serving a unit of high demand in period 2. Note that revenue  $p$  in period 2 from a unit of high demand occurs with probability  $\delta_1$  and may only be collected if none of the two periods is disrupted (with probability  $(1 - \rho)(1 - \rho)$ ). We discuss the complementary parameter space defined by the condition  $(1 - \rho)p\delta_1 > c^u$  in the appendix because the proof techniques and insights are unchanged between the two parameter spaces.

We first show that there are only 3 potentially optimal order policies when  $(1 - \rho)p\delta_1 < c^u$  (or equivalently  $\frac{p\delta_1 - c^u}{p\delta_1} < \rho$ ). This condition results in a focus on serving low demand in period 2 because the expected cost of serving a unit of high demand in period 2 is larger than the expected revenue. The formal proof of Proposition 3 is provided in the appendix.

**PROPOSITION 3.** *Let  $\frac{p\delta_1 - c^u}{p\delta_1} < \rho < \frac{p - c^u}{p}$ ,  $h < p - c^u$  and  $\Delta_1 < D_1$ . Then, one of the following three order policies must be optimal:*

- *Policy P1S:  $q_1 = D_1 - \Delta_1$  and  $q_2 = \alpha(D_1 - \Delta_1)$ .*
- *Policy P2S:  $q_1 = D_1 + \Delta_1$  and  $q_2 = \alpha(D_1 - \Delta_1) - 2\Delta_1$  if  $\alpha(D_1 - \Delta_1) \geq 2\Delta_1$ .*
- *Policy P3S:  $q_1 = D_1 - \Delta_1 + \alpha(D_1 - \Delta_1) = (D_1 - \Delta_1)(1 + \alpha)$  and  $q_2 = 0$ .*

*If  $\alpha(D_1 - \Delta_1) < 2\Delta_1$ , policy P2S is not defined and can never be optimal.*

Let us better understand the three order policies identified in Proposition 3 and the disruption probabilities over which they are likely to be optimal.

Policy  $P1S$  places orders to serve low demand in period 1 ( $q_1 = D_1 - \Delta_1$ ) and low demand in period 2 ( $q_2 = \alpha(D_1 - \Delta_1)$ ) without carrying any inventory from period 1 to period 2. The expected profit from policy  $P1S$  is as follows:

$$E(P1S) = \underbrace{(1-\rho)(p-c^u)(D_1 - \Delta_1)}_{\text{Profit from first period}} + \underbrace{((1-\rho)(1-\rho)p - (1-\rho)c^u)\alpha(D_1 - \Delta_1)}_{\text{Profit from second period}} \quad (8)$$

Observe that the second period order  $q_2 = \alpha(D_1 - \Delta_1)$  provides positive revenue only if there is no disruption in either period. If a disruption occurs in the first period, there is no demand in the second period. If there is a disruption in the second period, no supply arrives in the second period. Thus, policy  $P1$  is likely to be optimal for low disruption probabilities when the term  $(1-\rho)(1-\rho)$  is close to 1. In particular, policy  $P1S$  is optimal when disruption probability  $\rho = 0$ .

Observe that the total quantity ordered under Policy  $P2S$  is the same as that under policy  $P1S$ . The only difference is that a quantity  $2\Delta_1$  is added to the first period order and subtracted from the second period order (relative to  $P1S$ ). We refer to the quantity  $2\Delta_1$  as a *partial anticipatory order* because it partially moves some quantity from period 2 to period 1. We show that policy  $P2S$  is superior to policy  $P1S$  when disruption probability is somewhat higher. The expected profit from policy  $P2S$  is as follows:

$$E(P2S) = \underbrace{(1-\rho)(p-c^u)(D_1 - \Delta_1)}_{\text{Profit from serving low demand in p.1}} + \underbrace{(1-\rho)(p-c^u - (1-\delta_1)h)2\Delta_1}_{\text{Profit from selling quantity } 2\Delta_1 \text{ in p.1 or p.2}} + \underbrace{(1-\rho)(p(1-\rho) - c^u)(\alpha(D_1 - \Delta_1) - 2\Delta_1)}_{\text{Profit from selling } q_2 \text{ in p.2}} \quad (9)$$

Observe that the partial anticipatory order  $2\Delta_1$  is always sold if there is no disruption in period 1. It is either sold in period 1 (if high demand is realized in period 1), or in period 2 (if low demand is realized in period 1, an outcome that incurs holding cost). If there is no disruption in period 1, demand is at least  $\alpha(D_1 - \Delta_1)$  units in period 2. Policy  $P2S$  is likely to be optimal for slightly larger probability of disruption (relative to  $P1S$ ) because it does better if there is a disruption in one of periods 1 or 2, a more likely event when the probability of disruption is slightly higher. If there is a disruption in period 1, policy  $P2S$  results in less unsold inventory at the end of period 2 relative to policy  $P1S$ . If there is no disruption in period 1 but period 2 is disrupted, the extra  $2\Delta_1$  ordered in period 1 adds value relative to policy  $P1S$  because it can be used to serve some of period 2 demand

or high period 1 demand. The slightly higher disruption probability makes it less likely that there will be no disruption in either period, thus increasing the value of policy  $P2S$  relative to policy  $P1S$ .

Finally, order policy  $P3S$  also orders the same total quantity as policies  $P1S$  and  $P2S$  but places the entire order in period 1 ( $q_1 = (D_1 - \Delta_1)(1 + \alpha), q_2 = 0$ ). We refer to the quantity  $\alpha(D_1 - \Delta_1)$  as a *complete anticipatory order* because the entire period 2 order in policy  $P1S$  is moved to period 1 under this policy. The expected profit from policy  $P3S$  for  $\alpha(D_1 - \Delta_1) \geq 2\Delta_1$  is as follows:

$$E(P3S) = \underbrace{(1 - \rho)(p - c^u)(D_1 - \Delta_1)}_{\text{Profit from serving low demand in p.1}} + \underbrace{(1 - \rho)(p - c^u - (1 - \delta_1)h)2\Delta_1}_{\text{Profit from selling order } 2\Delta_1 \text{ in period 1 or 2}} + \underbrace{(1 - \rho)(p - c^u - h)(\alpha(D_1 - \Delta_1) - 2\Delta_1)}_{\text{Remaining period 1 order carried over to period 2}}$$

Policy  $P3S$  is superior to policy  $P2S$  when the probability of disruption is even larger because policy  $P3S$  provides higher profits than policy  $P2S$  when exactly one of the two periods 1 or 2 is disrupted. It provides lower profit than policy  $P2S$  when neither period is disrupted. However, if there is a disruption in period 1 and none in period 2, policy  $P3S$  is superior because it results in no unsold inventory in period 2. Similarly, if period 1 is not disrupted but period 2 is disrupted, policy  $P3S$  is superior to  $P2S$  because it serves more demand than policy  $P2S$  using the inventory carried over from period 1. Thus, policy  $P3S$  using the complete anticipatory order dominates policy  $P2S$  when disruption probabilities are higher (this decreases the probability of no disruption and increases the probability of one disruption).

Consistent with the above discussion, we now show that under mild assumptions, there is a range of disruption probability over which policies with anticipatory orders are optimal.

**THEOREM 3.** Let  $\frac{p\delta_1 - c^u}{p\delta_1} < \rho < \frac{p - c^u}{p}$ ,  $h < p - c^u$  and  $\Delta_1 < D_1$ . Then:

- Policy  $P1S$  is optimal for  $\frac{p\delta_1 - c^u}{p\delta_1} < \rho < \frac{h(1 - \delta_1)}{p}$ .
- Policy  $P2S$  is optimal for  $\frac{h(1 - \delta_1)}{p} < \rho < \frac{h}{p}$  if  $\alpha(D_1 - \Delta_1) \geq 2\Delta_1$ .
- Policy  $P3S$  is optimal for  $\frac{h}{p} < \rho < \frac{p - c^u}{p}$  if  $\alpha(D_1 - \Delta_1) \geq 2\Delta_1$ .

If  $\alpha(D_1 - \Delta_1) < 2\Delta_1$ , policy  $P2S$  is not optimal and policy  $P3S$  is optimal for  $\frac{h(1 - \delta_1)}{p} < \rho < \frac{p - c^u}{p}$ .

We discuss key insights that follow from Theorem 3.

First, observe that policy  $P1S$  is optimal when disruption probability is small. Thus, even when demand is uncertain, no anticipatory orders are placed when disruption probability is small (in particular 0). This shows that anticipatory orders add value when there is a risk of supply disruption.

Second, Theorem 3 shows that anticipatory orders are placed when disruption probability increases to  $\rho > \frac{h(1-\delta_1)}{p}$ . The size of the anticipatory order increases when disruption probability increases to the range  $\frac{h}{p} < \rho < \frac{p-c^u}{p}$ . Placing an anticipatory order increases expected profits because it adds value when there is a disruption in one of the two periods. As the probability of such an event increases (and the probability of no disruption decreases), we increase the size of the anticipatory order. This shows that even when demand is uncertain, there is a range of disruption probability where placing anticipatory orders is optimal.

## 5. Conclusion and future research

As recent supply chain disruptions such as COVID or the Suez canal blockage have drawn attention to increased disruption risk, most firms have focused on the need to develop more reliable supply sources. This choice is consistent with existing literature where a common recommendation is to increase use of a reliable supplier as the probability of disruption for the cheaper supplier increases. In this paper, however, we show that for products with short life cycles, when future demand depends on current sales, anticipatory orders from the cheaper but unreliable supplier can also be an effective risk mitigation strategy. The size of such anticipatory orders may increase in the probability of disruption.

We hope our paper raises the importance of understanding the relationship between sourcing policies and supply disruption in a variety of contexts. In particular, the role of the cheaper but unreliable supplier for risk mitigation in the context of short life cycle products merits further study. One possible extension of our work is to allow the unreliable supplier to improve its reliability as its total output grows (learning-by-doing effect). This results in a decrease in the probability of disruption over time. The learning effect may encourage both early and greater use of the unreliable supplier as a risk mitigation resource for the sake of “training.”

Another possible extension to our model is to include the dependency of price and demand in the model. In such an extension, the firm can change price for future periods based on whether previous periods have experienced a disruption or not.

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## Appendix

### A. Proofs in Section 4.1.

#### A.1. Proof Proposition 1

Proof. At optimality, we have:  $q_1 \in [D_1, D_1(1 + \alpha)]$ . The maximal order quantity in period 1 cannot be higher than  $D_1(1 + \alpha)$  because  $\rho_1 < \frac{c^u}{c^u + \xi}$ . If the order quantity was higher than  $D_1(1 + \alpha)$ , the ordering costs would marginally change by  $-(1 - \rho_2)c^u$  and the revenue (in the form of penalty fees for non-delivery) would increase marginally by  $\rho_2\xi$  (observe that there can never be more demand than  $D_1(1 + \alpha)$ ). Thus the marginal profit increase is negative for  $\rho_1 < \frac{c^u}{c^u + \xi}$ . The lowest order quantity in period 1 cannot be lower than  $D_1$  because  $(1 - \rho_1)(p - c^u) > 0$ .

The following lemma identifies the optimal order quantity in period 2:

LEMMA 4. Let  $\rho_1 < \frac{p - c^u}{p(1 - \beta)}$ ,  $\rho_1 < \frac{c^u}{p}$ ,  $\rho_i < \frac{c^u}{c^u + \xi} \forall i \in \{1, 2\}$  and  $h < p - c^u$ . Then:

- (i) If  $\exists a \in [0, (1 - \beta)\alpha D_1] : q_1 = D_1 + a$ , then  $q_2 = \alpha D_1 - a$ .
- (ii) If  $\exists a \in [(1 - \beta)\alpha D_1, \alpha D_1] : q_1 = D_1 + (1 - \beta)\alpha D_1 + a$ , then  $q_2 = \beta\alpha D_1$ .

Proof. We prove part (i) by contradiction. Let us assume that  $q_2 = \alpha D_1 - a + \epsilon$  for some  $\epsilon > 0$ . The marginal expected profit increase from increasing the order quantity in period 2 by  $\epsilon$  is:  $-(1 - \rho_2)c^u\epsilon + \rho_2\xi\epsilon$ , which is negative. Observe that even in the event that a disruption occurs in period 1, the increased order quantity in period 2 would not be helpful because there is no additional demand in period 2. Let us assume that  $q_2 = \alpha D_1 - a - \epsilon$  for some  $0 < \epsilon < \alpha D_1 - a$ . Demand in period 2 is  $\beta\alpha D_1$  if there is a disruption in period 1 or at least  $\alpha D_1$  if there is no disruption. Thus, expected profit can be increased by increasing  $q_2$ . Decreasing  $q_2$  is not optimal because  $\rho_1 < \frac{p - c^u}{p(1 - \beta)}$ . Regarding part (ii), observe that always  $q_2 \geq \beta\alpha D_1$  holds at optimality. Given that an anticipatory is placed, we also have  $q_2 \leq \beta\alpha D_1$ . The result thus follows. QED.

Recall that  $q_1 \in [D_1, D_1(1 + \alpha)]$ . We may split the range on which  $q_1$  may be optimal into two: 1)  $q_1 \in [D_1, D_1(1 + (1 - \beta)\alpha)]$  and 2)  $q_1 \in [D_1(1 + (1 - \beta)\alpha), D_1(1 + \alpha)]$ :

1)  $q_1 \in [D_1, D_1(1 + \alpha(1 - \beta))]$ . Let  $q_1 = D_1 + a$  with  $a \in [0, (1 - \beta)\alpha D_1]$ . Then,  $q_2 = \alpha D_1 - a$ . The expected profit function is:

$$E(\cdot) = (1 - \rho_1)(p - c^u)D_1 + \underbrace{(1 - \rho_1)(p - c^u - h)a}_{\text{Profit from selling anticipatory order}} + \underbrace{(1 - \rho_2)(p - c^u)\beta\alpha D_1 + (1 - \rho_2)((1 - \rho_1)p - c^u)((1 - \beta)\alpha D_1 - a)}_{\text{Profit from selling period 2 order}} + \underbrace{\rho_1\xi q_1 + \rho_2\xi q_2}_{\text{Penalty fees received}}$$

The expected profit function is linear in  $a$ , and hence in  $q_1$ .

2)  $q_1 \in [D_1(1 + \alpha(1 - \beta)), D_1(1 + \alpha)]$ . Let  $q_1 = D_1 + (1 - \beta)\alpha D_1 + a$  with  $a \in [0, \beta\alpha D_1]$ . Then,  $q_2 = \beta\alpha D_1$ . The expected profit function is:

$$E(\cdot) = (1 - \rho_1)(p - c^u)D_1 + \underbrace{(1 - \rho_1)(p - c^u - h)(1 - \beta)\alpha D_1}_{\text{Profit from selling anticipatory order}} + \underbrace{(1 - \rho_2)(p - c^u)q_2}_{\text{Profit from selling p.2 order}} + \underbrace{(1 - \rho_1)(p\rho_2 - c^u - h)a}_{\text{Profit from selling duplicate order}} + \underbrace{\rho_1\xi q_1 + \rho_2\xi q_2}_{\text{Penalty fees received}}$$

The expected profit function is linear in  $a$ , and hence in  $q_1$ .

Thus, we have no more than the following order policies at optimality:

1) Policy 1:  $q_1 = D_1$  and  $q_2 = \alpha D_1$ :

$$E(P1) = (1 - \rho_1)(p - c^u)D_1 + (1 - \rho_1)(1 - \rho_2)p\alpha D_1 + \rho_1(1 - \rho_2)p\beta\alpha D_1 - (1 - \rho_2)c^u\alpha D_1 + \rho_1\xi D_1 + \rho_2\xi\alpha D_1.$$

2) Policy 2:  $q_1 = D_1 + (1 - \beta)\alpha D_1$  and  $q_2 = \beta\alpha D_1$ :

$$\begin{aligned} E(P2) &= (1 - \rho_1)(p - c^u)D_1 + (1 - \rho_1)(p - c^u - h)(1 - \beta)\alpha D_1 + (1 - \rho_2)(p - c^u)\beta\alpha D_1 \\ &\quad + \rho_1\xi(1 + (1 - \beta)\alpha)D_1 + \rho_2\xi\beta\alpha D_1. \end{aligned}$$

3) Policy 3:  $q_1 = D_1 + \alpha D_1$  and  $q_2 = \beta\alpha D_1$ :

$$E(P3) = (1 - \rho_1)(p - c^u)D_1 + (1 - \rho_1)(p - c^u - h)\alpha D_1 + (1 - \rho_2)(p\rho_1 - c^u)\beta\alpha D_1 + \rho_1\xi(1 + \alpha)D_1 + \rho_2\xi\beta\alpha D_1.$$

The result thus follows. QED.

## A.2. Proof of Theorem 1

From  $E(P1) = E(P2)$  follows:

$$\begin{aligned} &(1 - \rho_1)(1 - \rho_2)p\alpha D_1 + \rho_1(1 - \rho_2)p\beta\alpha D_1 - (1 - \rho_2)c^u\alpha D_1 + \rho_1\xi D_1 + \rho_2\xi\alpha D_1 \\ &= (1 - \rho_1)(p - c^u - h)(1 - \beta)\alpha D_1 + (1 - \rho_2)(p - c^u)\beta\alpha D_1 + \rho_1\xi(1 + (1 - \beta)\alpha)D_1 + \rho_2\xi\beta\alpha D_1 \end{aligned}$$

This is equivalent to:

$$\begin{aligned} &(1 - \rho_1)(1 - \rho_2)p + \rho_1(1 - \rho_2)p\beta - (1 - \rho_2)c^u + \rho_2\xi(1 - \beta) \\ &= (1 - \rho_1)(p - c^u - h)(1 - \beta) + (1 - \rho_2)(p - c^u)\beta + \rho_1\xi(1 - \beta) \end{aligned}$$

This is equivalent to:

$$\begin{aligned} (1 - \rho_1)(1 - \rho_2)p + \rho_1(1 - \rho_2)p\beta &= (1 - \rho_1)(p - c^u - h)(1 - \beta) + (1 - \rho_2)p\beta + (1 - \rho_2)c^u(1 - \beta) \\ &\quad + (\rho_2 - \rho_1)\xi(1 - \beta) \end{aligned}$$

This is equivalent to:

$$\begin{aligned} (1 - \rho_1)(1 - \rho_2)p &= (1 - \rho_1)(p - c^u - h)(1 - \beta) + (1 - \rho_2)p\beta(1 - \rho_1) + (1 - \rho_2)c^u(1 - \beta) \\ &\quad + (\rho_2 - \rho_1)\xi(1 - \beta) \\ (1 - \rho_1)(1 - \rho_2)p(1 - \beta) &= (1 - \rho_1)(p - c^u - h)(1 - \beta) + (1 - \rho_2)c^u(1 - \beta) + (\rho_2 - \rho_1)\xi(1 - \beta) \\ (1 - \rho_1)(1 - \rho_2)p &= (1 - \rho_1)(p - c^u - h) + (1 - \rho_2)c^u + (\rho_2 - \rho_1)\xi \\ (1 - \rho_1)p - \rho_2(1 - \rho_1)p &= (1 - \rho_1)(p - h) + c^u(\rho_1 - \rho_2) + (\rho_2 - \rho_1)\xi \\ -\rho_2(1 - \rho_1)p &= -(1 - \rho_1)h + c^u(\rho_1 - \rho_2) + (\rho_2 - \rho_1)\xi \\ \rho_2 &= \frac{h}{p} - \frac{c^u}{p} \frac{\rho_1 - \rho_2}{1 - \rho_1} - \frac{\rho_2 - \rho_1}{1 - \rho_1} \frac{\xi}{p} \\ \rho_2 &= \frac{h}{p} - \frac{c^u + \xi}{p} \frac{\rho_1 - \rho_2}{1 - \rho_1}. \end{aligned}$$

(10)

From  $E(P2) = E(P3)$  follows:

$$\begin{aligned} & -\beta(1-\rho_1)(p-c^u-h)\alpha D_1 + (1-\rho_2)p\beta\alpha D_1 + \rho_1\xi(1+(1-\beta)\alpha)D_1 + \rho_2\xi\beta\alpha D_1 \\ & = (1-\rho_2)p\rho_1\beta\alpha D_1 + \rho_1\xi(1+\alpha)D_1 + \rho_2\xi\beta\alpha D_1. \end{aligned}$$

This is equivalent to:

$$\begin{aligned} & -\beta(1-\rho_1)(p-c^u-h) + (1-\rho_2)(1-\rho_1)p\beta + \rho_1\frac{\xi}{\alpha}(1+(1-\beta)\alpha) = \rho_1\frac{\xi}{\alpha}(1+\alpha) \\ & -\beta(1-\rho_1)(p-c^u-h) + (1-\rho_2)(1-\rho_1)p\beta - \rho_1\xi\beta = 0 \\ & -(p-c^u-h) + (1-\rho_2)p - \xi\frac{\rho_1}{1-\rho_1} = 0 \\ & c^u + h - \xi\frac{\rho_1}{1-\rho_1} = \rho_2p \\ & \rho_2 = \frac{c^u + h}{p} - \frac{\xi}{p}\frac{\rho_1}{1-\rho_1}. \end{aligned} \quad (11)$$

QED.

### A.3. Proof of Lemma 1

We consider the two ordering policies  $P1$  and  $P2$ . First, consider  $P1$  with  $q_1 = D_1$  and  $q_2 = \alpha D_1$ . We have four cases. First case: no disruption in either period. The profit function is:

$$C_1 = (p-c^u)(D_1 + \alpha D_1)$$

Second case: no period 1 disruption, but period 2 disruption. The profit function is:

$$C_2 = (p-c^u)D_1$$

Third case: Period 1 disruption, but no period 2 disruption. The profit function is:

$$C_3 = -c^u\alpha D_1$$

Fourth case: Both periods disrupted. The profit function is:

$$C_4 = 0$$

The expected profit function across all four cases is:

$$\begin{aligned} E(P1) &= f_1(r)C_1 + f_2(r)C_2 + f_3(r)C_3 + f_4(r)C_4 \\ &= f_1(r)(p-c^u)(D_1 + \alpha D_1) + f_2(r)(p-c^u)D_1 - f_3(r)c^u\alpha D_1 \end{aligned}$$

Second, consider  $P2$  with  $q_1 = D_1(1+\alpha)$  and  $q_2 = 0$ . We have four cases. First case: no disruption in either period. The profit function is:

$$C_1 = (p-c^u)(D_1 + \alpha D_1) - h\alpha D_1$$

Second case: no period 1 disruption, but period 2 disruption. The profit function is:

$$C_2 = (p-c^u)(D_1 + \alpha D_1) - h\alpha D_1$$

Third case: Period 1 disruption, but no period 2 disruption. The profit function is:

$$C_3 = 0$$

Fourth case: Both periods disrupted. The profit function is:

$$C_4 = 0$$

The expected profit function across all four cases is:

$$\begin{aligned} E(P2) &= f_1(r)C_1 + f_2(r)C_2 + f_3(r)C_3 + f_4(r)C_4 \\ &= f_1(r)((p - c^u)(D_1 + \alpha D_1) - h\alpha D_1) + f_2(r)((p - c^u)(D_1 + \alpha D_1) - h\alpha D_1) \end{aligned}$$

The condition  $E(P1) = E(P2)$  is equivalent to:

$$\begin{aligned} &f_1(r)(p - c^u)(D_1 + \alpha D_1) + f_2(r)(p - c^u)D_1 - f_3(r)c^u\alpha D_1 \\ &= f_1(r)((p - c^u)(D_1 + \alpha D_1) - h\alpha D_1) + f_2(r)((p - c^u)(D_1 + \alpha D_1) - h\alpha D_1) \end{aligned}$$

This is equivalent to:

$$\begin{aligned} -f_3(r)c^u\alpha D_1 &= f_1(r)(-h\alpha D_1) + f_2(r)((p - c^u)\alpha D_1 - h\alpha D_1) \\ -f_3(r)c^u &= -f_1(r)h + f_2(r)((p - c^u) - h) \end{aligned}$$

This is equivalent to:

$$-pf_2(r) - c^u(f_3(r) - f_2(r)) + h(f_1(r) + f_2(r)) = 0 \quad (12)$$

We have:

$$\begin{aligned} f_1(r) + f_2(r) &= (1 - \rho)(1 - \rho) + \frac{1}{2}(1 - \rho)r + \frac{1}{2}(1 - \rho)(\rho - (1 - \rho))r^2 \\ &\quad + (1 - \rho)\rho - \frac{1}{2}(1 - \rho)r + (1 - \rho)(\frac{1}{2} - \rho)r^2 \\ &= 1 - \rho \end{aligned}$$

We have:

$$\begin{aligned} f_3(r) - f_2(r) &= \rho(1 - \rho) - \frac{1}{2}\rho r + \rho(\rho - \frac{1}{2})r^2 \\ &\quad - (1 - \rho)\rho + \frac{1}{2}(1 - \rho)r - (1 - \rho)(\frac{1}{2} - \rho)r^2 \\ &= (\frac{1}{2} - \rho)r - (\frac{1}{2} - \rho)r^2 = (\frac{1}{2} - \rho)(r - r^2) \end{aligned}$$

Thus, the condition  $E(P1) = E(P2)$  is equivalent to:

$$-p((1 - \rho)\rho - \frac{1}{2}(1 - \rho)r + (1 - \rho)(\frac{1}{2} - \rho)r^2) - c^u(\frac{1}{2} - \rho)(r - r^2) + h(1 - \rho) = 0. \quad (13)$$

For  $r = 0$ , we have:

$$-p(1 - \rho)\rho + h(1 - \rho) = 0 \quad \rightarrow \quad \rho = \frac{h}{p}.$$

For  $r = +1$ , we have

$$\begin{aligned} -p((1-\rho)\rho - \frac{1}{2}(1-\rho) + (1-\rho)(\frac{1}{2} - \rho)) + h(1-\rho) &= 0 \\ -p(\rho - \frac{1}{2} + (\frac{1}{2} - \rho)) + h &= 0 \\ h &= 0. \end{aligned}$$

Thus, order policy P1 always dominates over order policy P2 when  $r = +1$ .

For  $r = -1$ , we have:

$$\begin{aligned} -p((1-\rho)\rho + \frac{1}{2}(1-\rho) + (1-\rho)(\frac{1}{2} - \rho)) + 2c^u(\frac{1}{2} - \rho) + h(1-\rho) &= 0 \\ -p((1-\rho)\rho + (1-\rho) - (1-\rho)\rho) + 2c^u(\frac{1}{2} - \rho) + h(1-\rho) &= 0 \\ -p(1-\rho) + 2c^u(\frac{1}{2} - \rho) + h(1-\rho) &= 0 \\ -p + p\rho + c^u - 2c^u\rho + h - h\rho &= 0 \\ -p + c^u + h + \rho(p - 2c^u - h) &= 0 \rightarrow \rho = \frac{p - c^u - h}{p - 2c^u - h}. \end{aligned}$$

Thus, policy P2 always dominates over P1 when  $r = -1$ . The result thus follows. QED.

#### A.4. Proof of Lemma 2

Proof: We have  $E(P1r) = E(P2r)$  iff

$$\begin{aligned} (1-\rho)p + \rho p\beta - c^u\beta - (1-\rho)c^u(1-\beta) &= (p - c^u - h)(1-\beta) + (p - c^u)\beta \\ (1-\rho)p + \rho p\beta - (1-\rho)c^u(1-\beta) &= (p - c^u - h)(1-\beta) + p\beta \\ (1-\rho)(p - c^u(1-\beta) - p\beta) &= (p - c^u - h)(1-\beta) \\ (1-\rho)(p - c^u) &= p - c^u - h \\ \rho &= \frac{h}{p - c^u} \end{aligned} \tag{14}$$

We have  $E(P2r) = E(P3r)$  iff

$$\begin{aligned} (p - c^u - h)(1-\beta) + (p - c^u)\beta &= (p - c^u - h) + (\rho p - c^u)\beta \\ -(p - c^u - h) + (p - c^u) &= (\rho p - c^u) \\ \rho &= \frac{h + c^u}{p} \end{aligned} \tag{15}$$

The result thus follows. QED.

## B. Proofs in Section 4.2.

### B.1. Proof Proposition 2

Let us consider the following order policies. First, policy  $P1R$  places orders to serve demand in period 1 using the unreliable supplier ( $q_1^u = D_1$  and  $q_1^r = 0$ ) and demand in period 2 using the unreliable supplier ( $q_2 = \alpha D_1$  and  $q_2^r = 0$ ) without carrying any inventory from period 1 to period 2. The expected profit from policy  $P1R$  is as follows:

$$E(P1R) = (1-\rho)(p - c^u)D_1 + ((1-\rho)(1-\rho)p - (1-\rho)c^u)\alpha D_1.$$

This order policy is identical to order policy  $P1$  from the base model. The marginal value of increasing  $q_2^u$  is:  $-(1-\rho)c^u < 0$ . The marginal value of increasing  $q_2^r$  is negative.

Second, policy  $P2R$ :  $q_1^u = D_1 + \alpha D_1$ ,  $q_1^r = 0$ . The expected profit function is:

$$E(P2R) = (1-\rho)(p-c^u)D_1 + (1-\rho)(p-c^u-h)\alpha D_1.$$

The marginal value of increasing  $q_2^u$  is:  $-(1-\rho)c^u < 0$ . The marginal value of increasing  $q_2^r$  is negative. The marginal value of increasing  $q_1^u$  is negative.

Third, consider policy  $P3R$ :  $q_1^r = D_1$ ,  $q_1^u = 0$ ,  $q_2^u = \alpha D_1$  and  $q_2^r = 0$ . The expected profit function is:

$$E(P3R) = (p-c^r)D_1 + (1-\rho)(p-c^u)\alpha D_1.$$

The marginal value of increasing  $q_2^u$  is:  $-(1-\rho)c^u < 0$ . The marginal value of increasing  $q_2^r$  is negative.

Fourth, policy  $P4R$ :  $q_1^r = D_1$ ,  $q_1^u = \alpha D_1$  and the following contingent order:  $q_2^r = \alpha D_1$  if there is a disruption in period 1. The expected profit function is:

$$E(P4R) = (p-c^r)D_1 + (1-\rho)(p-c^u-h)\alpha D_1 + \rho(p-c^r)\alpha D_1.$$

The marginal value of increasing  $q_2^r$  is negative.

Fifth, policy  $P5R$ :  $q_1^r = D_1$ ,  $q_1^u = \alpha D_1$ ,  $q_2^u = \alpha D_1$  and  $q_2^r = 0$ . The expected profit function is:

$$E(P5R) = (p-c^r)D_1 + (1-\rho)(p-c^u-h)\alpha D_1 - (1-\rho)c^u\alpha D_1 + \rho(1-\rho)p\alpha D_1.$$

The marginal value of increasing  $q_2^r$  or  $q_2^u$  is negative. The marginal value of increasing  $q_1^r > D_1$  is negative because it is never optimal to place an anticipatory order with the reliable supplier. The result thus follows. QED.

## B.2. Proof of Lemma 3

Order policy  $P2R$  dominates over order policy  $P1R$  for  $\rho > \frac{h}{p}$ .

We have  $E(P2R) = E(P3R)$  iff

$$(1-\rho)(p-c^u)D_1 + (1-\rho)(p-c^u-h)\alpha D_1 = (p-c^r)D_1 + (1-\rho)(p-c^u)\alpha D_1$$

$$(1-\rho)(p-c^u) - (1-\rho)h\alpha = p-c^r$$

$$c^r - c^u - h\alpha = \rho(p-c^u-h\alpha)$$

$$\rho = \frac{c^r - c^u - h\alpha}{p - c^u - h\alpha}.$$

Thus, order policy  $P3R$  dominates over order policy  $P2R$  for  $\rho > \frac{c^r - c^u - h\alpha}{p - c^u - h\alpha}$ . Observe that  $\frac{c^r - c^u - h\alpha}{p - c^u - h\alpha} < 0$  for  $\frac{c^r - c^u}{h} < \alpha < \frac{p - c^u}{h}$ . Further, observe that order policy  $P3R$  dominates over order policy  $P2R$  for  $\alpha \geq \frac{p - c^u}{h}$ .

Thus, order policy  $P2R$  is never optimal for  $\alpha > \frac{p - c^u}{h}$ .

Because order policy  $P2R$  dominates over order policy  $P1R$  for  $\rho > \frac{h}{p}$ , we know that order policy  $P3R$  also dominates over order policy  $P1R$  if  $\rho > \frac{h}{p}$  and  $\alpha > \frac{c^r - c^u}{h}$ .

We have  $E(P3R) = E(P4R)$  iff

$$\begin{aligned}(p - c^r)D_1 + (1 - \rho)(p - c^u)\alpha D_1 &= (p - c^r)D_1 + (1 - \rho)(p - c^u - h)\alpha D_1 + \rho(p - c^r)\alpha D_1 \\ 0 &= -(1 - \rho)h + \rho(p - c^r) \\ h &= \rho(p + h - c^r) \\ \rho &= \frac{h}{p + h - c^r} > 0.\end{aligned}$$

Thus, order policy  $P4R$  dominates over order policy  $P3R$  for  $\rho > \frac{h}{p + h - c^r}$ .

Next, we show that policy  $P5R$  is never optimal for  $\rho < \frac{h + c^u}{p}$ . We have  $E(P3R) = E(P5R)$  iff

$$\begin{aligned}(p - c^r)D_1 + (1 - \rho)(p - c^u)\alpha D_1 &= (p - c^r)D_1 + (1 - \rho)(p - c^u - h)\alpha D_1 - (1 - \rho)c^u\alpha D_1 + \rho(1 - \rho)p\alpha D_1 \\ 0 &= -h - c^u + \rho p \\ \rho &= \frac{h + c^u}{p}.\end{aligned}$$

Thus, policy  $P3R$  always dominates over policy  $P5R$  for  $\rho < \frac{h + c^u}{p}$ . Thus, policy  $P5R$  can never be optimal for  $\rho < \frac{h + c^u}{p}$ .

We conclude that order policy  $P3R$  is optimal on the range  $\rho \in (\frac{h}{p}, \min\{\frac{h}{p + h - c^r}, \frac{h + c^u}{p}, \frac{p - c^u}{p}\})$  and order policy  $P4R$  is optimal on the range  $\rho \in (\frac{h}{p + h - c^r}, \min\{\frac{h + c^u}{p}, \frac{p - c^u}{p}\})$  if  $\alpha > \frac{c^r - c^u}{h}$ . The result thus follows. QED.

### B.3. Proof of Theorem 2

Note that the interval  $(\frac{h}{p + h - c^r}, \min\{\frac{h + c^u}{p}, \frac{p - c^u}{p}\})$  is non-empty if  $\frac{h}{p + h - c^r} < \min\{\frac{h + c^u}{p}, \frac{p - c^u}{p}\}$ . Theorem 2 then follows from the proof of Lemma 3. The result thus follows. QED.

## C. Proofs in Section 4.3.

### C.1. Proof Proposition 3

Proof. At optimality, we have:  $q_1 \in [0, (D_1 - \Delta_1)(1 + \alpha)]$ . The following lemma identifies the optimal order quantity in period 2:

LEMMA 5. Let  $\frac{p\delta_1 - c^u}{p\delta_1} < \rho < \frac{p - c^u}{p}$ ,  $h < p - c^u$  and  $\Delta_1 < D_1$ . Then:

- (i) If  $q_1 \leq D_1 - \Delta_1$ , then  $q_2 = \alpha q_1$ .
- (ii) If  $\exists a > 0$ :  $q_1 = D_1 - \Delta_1 + a$ , then  $q_2 = \alpha(D_1 - \Delta_1) - a$ .

Proof. The first statement follows from the fact that demand in period 2 is  $\alpha q_1$  if no disruption occurs in period 1.  $q_2 > 0$  because  $\rho < \frac{p - c^u}{p}$ . The second statement is proven by contradiction. Let us assume that  $q_2 = \alpha(D_1 - \Delta_1) - a + \epsilon$  for some  $\epsilon > 0$ . The marginal expected profit increase from increasing the order quantity in period 2 by  $\epsilon$  is  $(\delta_1 p(1 - \rho) - c^u)\epsilon$ , which is negative because we have assumed  $\rho > \frac{p\delta_1 - c^u}{p\delta_1}$ . Let us assume that  $q_2 = \alpha(D_1 - \Delta_1) - a - \epsilon$  for some  $0 < \epsilon < \alpha(D_1 - \Delta_1) - a$ . Demand in period 2 is 0 if there is a disruption in period 1 or at least  $\alpha(D_1 - \Delta_1)$  if there is no disruption. Thus, expected profit can be increased by increasing  $q_2$ . Decreasing  $q_2$  to  $q_2 = 0$  is not optimal because  $\rho < \frac{p - c^u}{p}$ . The result thus follows. QED.

We know that  $q_1 \in [0, (D_1 - \Delta_1)(1 + \alpha)]$  at optimality. We divide the interval  $[0, (D_1 - \Delta_1)(1 + \alpha)]$  into three regions:  $[0, D_1 - \Delta_1]$ ,  $[D_1 - \Delta_1, D_1 + \Delta_1]$ ,  $[D_1 + \Delta_1, (D_1 - \Delta_1)(1 + \alpha)]$ . We show that the expected profit functions are linear in  $q_1$  on each interval.

Let  $\alpha(D_1 - \Delta_1) \geq 2\Delta_1$ . Then:

1)  $q_1 \in [0, D_1 - \Delta_1]$ . Then, we have  $q_2 = \alpha q_1$ . The expected profit function is:

$$E(\cdot) = \underbrace{(1-\rho)(p-c^u)q_1}_{\text{Profit from first period}} + \underbrace{((1-\rho)(1-\rho)p - (1-\rho)c^u)q_2}_{\text{Profit from second period}}.$$

This function is linear in  $q_1$ .

2)  $q_1 \in [D_1 - \Delta_1, D_1 + \Delta_1]$ : Let  $q_1 = D_1 - \Delta_1 + a$  with  $a \in [0, 2\Delta_1]$ . Then,  $q_2 = \alpha(D_1 - \Delta_1) - a$ . The expected profit function is:

$$E(\cdot) = \underbrace{(1-\rho)(p-c^u)(D_1 - \Delta_1)}_{\text{Profit from serving low demand in p.1}} + \underbrace{(1-\rho)(p-c^u - (1-\delta_1)h)a}_{\text{Profit from selling quantity } a \text{ in period 1 or 2}} + \underbrace{(1-\rho)(p(1-\rho) - c^u)(\alpha(D_1 - \Delta_1) - a)}_{\text{Profit from selling period 2 order } q_2}. \quad (16)$$

Observe that the quantity  $a$  is always sold if it arrives in period 1: it is sold in period 1 if high demand is realized in period 1; it is sold in period 2 if low demand is realized in period 1 (incurring inventory holding costs). A minimum of low demand is always realized in period 2 if the order in period 1 arrives. The expected profit function is linear in  $a$ , and hence in  $q_1$ .

3)  $q_1 \in [D_1 + \Delta_1, (D_1 - \Delta_1)(1 + \alpha)]$ . Let  $q_1 = D_1 - \Delta_1 + a$  with  $a \in [2\Delta_1, (D_1 - \Delta_1)(1 + \alpha) - (D_1 - \Delta_1)]$ , equivalently:  $a \in [2\Delta_1, \alpha(D_1 - \Delta_1)]$ . Then,  $q_2 = \alpha(D_1 - \Delta_1) - a$ . The expected profit function is:

$$E(\cdot) = (1-\rho)(p-c^u)(D_1 - \Delta_1) + \underbrace{(1-\rho)(p-c^u - (1-\delta_1)h)2\Delta_1}_{\text{Profit from selling order } 2\Delta_1 \text{ in period 1 or 2}} + \underbrace{(1-\rho)(p-c^u - h)(a - 2\Delta_1)}_{\text{Remaining period 1 order carried over to period 2}} + \underbrace{(1-\rho)(p(1-\rho) - c^u)(\alpha(D_1 - \Delta_1) - a)}_{\text{Profit from selling period 2 order}}.$$

The expected profit function is linear in  $a$ , and hence in  $q_1$ .

Thus, we have no more than the following order policies at optimality:

1) Policy 1:  $q_1 = D_1 - \Delta_1$  and  $q_2 = \alpha(D_1 - \Delta_1)$ :

$$E(P1) = (1-\rho)(p-c^u)(D_1 - \Delta_1) + (1-\rho)((1-\rho)p - c^u)\alpha(D_1 - \Delta_1).$$

2) Policy 2:  $q_1 = D_1 + \Delta_1$  and  $q_2 = \alpha(D_1 - \Delta_1) - 2\Delta_1$ :

$$E(P2) = (1-\rho)(p-c^u)(D_1 - \Delta_1) + (1-\rho)(p-c^u - (1-\delta_1)h)2\Delta_1 + (1-\rho)(p(1-\rho) - c^u)(\alpha(D_1 - \Delta_1) - 2\Delta_1).$$

3) Policy 3:  $q_1 = (D_1 - \Delta_1)(1 + \alpha)$  and  $q_2 = 0$ :

$$E(P3) = (1-\rho)(p-c^u)(D_1 - \Delta_1) + (1-\rho)(p-c^u - (1-\delta_1)h)2\Delta_1 + (1-\rho)(p-c^u - h)(\alpha(D_1 - \Delta_1) - 2\Delta_1).$$

Let  $\alpha(D_1 - \Delta_1) < 2\Delta_1$ . The expected profit from policy  $P3S$  ( $q_1 = (D_1 - \Delta_1)(1 + \alpha)$  and  $q_2 = 0$ ) is then:

$$E(P3S) = \underbrace{(1-\rho)(p-c^u)(D_1 - \Delta_1)}_{\text{Profit from serving low demand in p.1}} + \underbrace{(1-\rho)(p-c^u - (1-\delta_1)h)\alpha(D_1 - \Delta_1)}_{\text{Profit from selling order } \alpha(D_1 - \Delta_1) \text{ in period 1 or 2}}$$

Observe that policy  $P2S$  is not defined for  $\alpha(D_1 - \Delta_1) < 2\Delta_1$ .

The result thus follows. QED.

### C.2. Proof of Theorem 3

In the following we identify conditions when one order policy dominates over another one. Let  $\alpha(D_1 - \Delta_1) \geq 2\Delta_1$  (which ensures that policy  $P2S$  is defined). We have  $E(P1S) = E(P2S)$  iff

$$\begin{aligned} 0 &= p - c^u - (1 - \delta_1)h - (p(1 - \rho) - c^u) \\ p(1 - \rho) - c^u &= p - c^u - h(1 - \delta_1) \\ \rho &= \frac{h(1 - \delta_1)}{p}. \end{aligned}$$

Thus, order policy  $P2S$  dominates over order policy  $P1S$  for  $\rho > \frac{h(1 - \delta_1)}{p}$ .

Further, we have  $E(P2S) = E(P3S)$  iff

$$\begin{aligned} p(1 - \rho) - c^u &= p - c^u - h \\ \rho &= \frac{h}{p}. \end{aligned}$$

Thus, order policy  $P3S$  dominates over order policy  $P2S$  for  $\rho > \frac{h}{p}$ . We have  $\frac{h(1 - \delta_1)}{p} < \frac{h}{p}$ .

Let  $\alpha(D_1 - \Delta_1) < 2\Delta_1$ . The expected profit from policy  $P3S$  ( $q_1 = (D_1 - \Delta_1)(1 + \alpha)$  and  $q_2 = 0$ ) is:

$$E(P3S) = \underbrace{(1 - \rho)(p - c^u)(D_1 - \Delta_1)}_{\text{Profit from serving low demand in p.1}} + \underbrace{(1 - \rho)(p - c^u - (1 - \delta_1)h)\alpha(D_1 - \Delta_1)}_{\text{Profit from selling order } \alpha(D_1 - \Delta_1) \text{ in period 1 or 2}}.$$

We have  $E(P1S) = E(P3S)$  iff  $\rho = \frac{h(1 - \delta_1)}{p}$ . The result thus follows. QED.

### C.3. Analysis of the parameter space $\delta_1 p(1 - \rho) > c^u$

In the following, we extend Section 4.3 by analyzing the complementary parameter space defined by the condition  $p\delta_1(1 - \rho) > c^u$  (while all other assumptions in Section 4.3 continue to hold). If this condition is satisfied, it is economical to serve high demand in both periods. We have the following order policies:

PROPOSITION 4. *Let  $\rho < \frac{p\delta_1 - c^u}{p\delta_1}$ ,  $\rho < \frac{p - c^u}{p}$ ,  $h < p - c^u$  and  $\Delta_1 < D_1$ . Then no more than three order policies may be optimal:*

- Policy  $P1S'$ :  $q_1 = D_1 + \Delta_1$  and  $q_2 = \alpha(D_1 + \Delta_1)$ .
- Policy  $P2S'$ :  $q_1 = (D_1 - \Delta_1)(1 + \alpha)$  and  $q_2 = 2\Delta_1(1 + \alpha)$  if  $\alpha(D_1 - \Delta_1) - 2\Delta_1 \geq 0$ .
- Policy  $P3S'$ :  $q_1 = (D_1 + \Delta_1)(1 + \alpha)$  and  $q_2 = 0$ .

If  $\alpha(D_1 - \Delta_1) - 2\Delta_1 < 0$ , order policy  $P2S'$  is not optimal.

The formal proof is provided further below. These order policies are quite similar to those identified in Proposition 3. Observe that the total quantity ordered across the two periods is the same under all three order policies. Order policy  $P1S'$  orders supply to meet high demand in each period without intending to carry any inventory from period 1 to period 2. Order policy  $P2S'$  places an anticipatory order by moving  $\alpha(D_1 - \Delta_1) - 2\Delta_1$  from period 2 to period 1 relative to policy  $P1S'$  (for  $\alpha(D_1 - \Delta_1) - 2\Delta_1 \geq 0$ ). Order policy  $P3S'$  places a larger anticipatory order by moving  $\alpha(D_1 + \Delta_1)$  from period 2 to period 1 relative to policy  $P1S'$ .

In the following, we show that under mild assumptions policy  $P2S'$  and policy  $P3S'$  are optimal over a non-empty interval of disruption probabilities:

PROPOSITION 5. *Let  $\rho < \frac{p\delta_1 - c^u}{p\delta_1}$ ,  $\rho < \frac{p - c^u}{p}$ ,  $h < p - c^u$  and  $\Delta_1 < D_1$ . Then, policy  $P1S'$  is optimal for  $0 < \rho < \frac{h}{p}$ . Policy  $P3S'$  is optimal for  $\frac{h}{p} < \min\{\frac{p\delta_1 - c^u}{p\delta_1}, \frac{p - c^u}{p}\}$ . Policy  $P2S'$  is never optimal.*

The formal proof is provided below.

### Proof of Proposition 4

The proof is similar to the proof of Proposition 3. At optimality, we have:  $q_1 \in [D_1 - \Delta_1, (D_1 + \Delta_1)(1 + \alpha)]$ . The following lemma identifies the optimal order quantity in period 2:

LEMMA 6. Let  $\rho < \frac{p\delta_1 - c^u}{p\delta_1}$ ,  $\rho < \frac{p - c^u}{p}$ ,  $h < p - c^u$  and  $\Delta_1 < D_1$ . Then:

- (i) If  $q_1 \leq D_1 + \Delta_1$ , then  $q_2 = \alpha q_1$ .
- (ii) If  $\exists a > 0$ :  $q_1 = D_1 + \Delta_1 + a$ , then  $q_2 = \alpha(D_1 + \Delta_1) - a$ .

Proof. The first statement follows from the fact that demand in period 2 is  $\alpha q_1$  if no disruption occurs in period 1. For  $q_1 \geq D_1 - \Delta_1$ , high demand in period 2 is met because  $\delta_1 p(1 - \rho) - c^u > 0$  (which is equivalent to  $\rho < \frac{p\delta_1 - c^u}{p\delta_1}$ ). The second statement is proven by contradiction. Let us assume that  $q_2 = \alpha(D_1 + \Delta_1) - a + \epsilon$  for some  $\epsilon > 0$ . The marginal profit increase upon increasing  $q_2$  by  $\epsilon$  is  $-(1 - \rho)c^u\epsilon$ , which is negative. Let us assume that  $q_2 = \alpha(D_1 + \Delta_1) - a - \epsilon$  for some  $0 < \epsilon < \alpha(D_1 + \Delta_1) - a$ . The expected profit can be increased by increasing  $q_2$  because  $\delta_1 p(1 - \rho) - c^u > 0$  (which is equivalent to  $\rho < \frac{p\delta_1 - c^u}{p\delta_1}$ ). The result thus follows. QED.

Let  $\alpha(D_1 - \Delta_1) \geq 2\Delta_1$ . We know that  $q_1 \in [D_1 - \Delta_1, (D_1 + \Delta_1)(1 + \alpha)]$  at optimality. We divide the interval  $[D_1 - \Delta_1, (D_1 + \Delta_1)(1 + \alpha)]$  into three regions:  $[D_1 - \Delta_1, D_1 + \Delta_1]$ ,  $[D_1 + \Delta_1, (D_1 - \Delta_1)(1 + \alpha)]$ ,  $[(D_1 - \Delta_1)(1 + \alpha), (D_1 + \Delta_1)(1 + \alpha)]$ . We then show that the expected profit functions are linear in  $q_1$  on each interval.

1)  $q_1 \in [D_1 - \Delta_1, D_1 + \Delta_1]$ . Let us rewrite  $q_1 = D_1 - \Delta_1 + a$  with  $a \in [0, 2\Delta_1]$ . Then, we have  $q_2 = \alpha q_1$ . The expected profit function is:

$$\begin{aligned}
 E(\cdot) = & \underbrace{(1 - \rho)(p - c^u)(D_1 - \Delta_1)}_{\text{Profit from serving low demand in p.1}} + \underbrace{(1 - \rho)(p - c^u - (1 - \delta_1)h)a}_{\text{Profit from selling remaining order that arrives in period 1}} \\
 & + \underbrace{(1 - \rho)(p(1 - \rho) - c^u)(\alpha(D_1 - \Delta_1) - a)}_{\text{Profit from serving further demand in p.2}} \\
 & + \underbrace{(1 - \rho)((1 - \rho)\delta_1 p - c^u)(q_2 - (\alpha(D_1 - \Delta_1) - a))}_{\text{Profit from serving further demand in p.2}}
 \end{aligned} \tag{17}$$

Observe that the quantity  $q_2 - (\alpha(D_1 - \Delta_1) - a)$  (last term of the objective function) is only sold if there is no disruption in either period and if high demand is realized. If high demand is realized in period 1, there is also high demand in period 2. The objective function is linear in  $q_1$ .

2)  $q_1 \in [D_1 + \Delta_1, (D_1 - \Delta_1)(1 + \alpha)]$ . Let us rewrite  $q_1 = D_1 + \Delta_1 + a$  with  $a \in [0, (D_1 - \Delta_1)(1 + \alpha) - (D_1 + \Delta_1)]$ . Then  $q_2 = \alpha(D_1 + \Delta_1) - a$ . The expected profit function is:

$$\begin{aligned}
 E(\cdot) = & \underbrace{(1 - \rho)(p - c^u)(D_1 - \Delta_1)}_{\text{Profit from serving low demand in p.1}} + \underbrace{(1 - \rho)(p - c^u - (1 - \delta_1)h)2\Delta_1}_{\text{Profit from selling order } 2\Delta_1 \text{ in period 1 or 2}} \\
 & + \underbrace{(1 - \rho)(p - c^u - h)a}_{\text{Remaining period 1 order carried over to period 2}} + \underbrace{(1 - \rho)((1 - \rho)\delta_1 p - c^u)q_2}_{\text{Profit from selling period 2 order}}
 \end{aligned}$$

The objective function is linear in  $q_1$ .

3)  $q_1 \in [(D_1 - \Delta_1)(1 + \alpha), (D_1 + \Delta_1)(1 + \alpha)]$ . Let us rewrite  $q_1 = D_1 + \Delta_1 + a$  with  $a \in [\alpha(D_1 - \Delta_1) - 2\Delta_1, \alpha(D_1 + \Delta_1)]$ . Then  $q_2 = \alpha(D_1 + \Delta_1) - a$ . The expected profit function is:

$$\begin{aligned} E(\cdot) = & \underbrace{(1 - \rho)(p - c^u)(D_1 - \Delta_1)}_{\text{Profit from serving low demand in p.1}} + \underbrace{(1 - \rho)(p - c^u - (1 - \delta_1)h)2\Delta_1}_{\text{Profit from selling order } 2\Delta_1 \text{ in period 1 or 2}} \\ & + \underbrace{(1 - \rho)(p - c^u - h)(\alpha(D_1 - \Delta_1) - 2\Delta_1)}_{\text{Profit from serving low demand in p.1}} \\ & + (1 - \rho)(\delta_1 p - c^u - \delta_1 h)(a - (\alpha(D_1 - \Delta_1) - 2\Delta_1)) \\ & + (1 - \rho)(\delta_1 p(1 - \rho) - c^u)q_2 \end{aligned}$$

The objective function is linear in  $q_1$ .

Thus, there can only be the following three order policies at optimality:

1') Policy  $P1S'$ :  $q_1 = D_1 + \Delta_1$  and  $q_2 = \alpha(D_1 + \Delta_1)$ :

$$\begin{aligned} E(P1S') = & (1 - \rho)(p - c^u)(D_1 - \Delta_1) + (1 - \rho)(p - c^u - (1 - \delta_1)h)2\Delta_1 \\ & + (1 - \rho)(p(1 - \rho) - c^u)(\alpha(D_1 - \Delta_1) - 2\Delta_1) + (1 - \rho)\left((1 - \rho)\delta_1 p - c^u\right)2\Delta_1(1 + \alpha) \end{aligned}$$

2') Policy  $P2S'$ :  $q_1 = (D_1 - \Delta_1)(1 + \alpha)$  and  $q_2 = 2\Delta_1(1 + \alpha)$ :

$$\begin{aligned} E(P2S') = & (1 - \rho)(p - c^u)(D_1 - \Delta_1) + (1 - \rho)(p - c^u - (1 - \delta_1)h)2\Delta_1 \\ & + (1 - \rho)(p - c^u - h)(\alpha(D_1 - \Delta_1) - 2\Delta_1) + (1 - \rho)\left((1 - \rho)\delta_1 p - c^u\right)2\Delta_1(1 + \alpha) \end{aligned}$$

3') Policy  $P3S'$ :  $q_1 = (D_1 + \Delta_1)(1 + \alpha)$  and  $q_2 = 0$ :

$$\begin{aligned} E(P3S') = & (1 - \rho)(p - c^u)(D_1 - \Delta_1) + (1 - \rho)(p - c^u - (1 - \delta_1)h)2\Delta_1 \\ & + (1 - \rho)(p - c^u - h)(\alpha(D_1 - \Delta_1) - 2\Delta_1) + (1 - \rho)(\delta_1 p - c^u - \delta_1 h)2\Delta_1(1 + \alpha). \end{aligned}$$

Let  $\alpha(D_1 - \Delta_1) < 2\Delta_1$ . The expected profit from policy  $P1S'$  is then:

$$\begin{aligned} E(P1S') = & (1 - \rho)(p - c^u)(D_1 - \Delta_1) + (1 - \rho)(p - c^u - (1 - \delta_1)h)\alpha(D_1 - \Delta_1) + (1 - \rho)(p\delta_1 - c^u)(2\Delta_1 - \alpha(D_1 - \Delta_1)) \\ & + (1 - \rho)\left((1 - \rho)\delta_1 p - c^u\right)(\alpha 2\Delta_1 + \alpha(D_1 - \Delta_1)). \end{aligned}$$

Policy  $P2S'$  is never optimal for  $\alpha(D_1 - \Delta_1) < 2\Delta_1$ .

The expected profit from policy  $P1S'$  is then:

$$\begin{aligned} E(P3S') = & (1 - \rho)(p - c^u)(D_1 - \Delta_1) + (1 - \rho)(p - c^u - (1 - \delta_1)h)\alpha(D_1 - \Delta_1) + (1 - \rho)(p\delta_1 - c^u)(2\Delta_1 - \alpha(D_1 - \Delta_1)) \\ & + (1 - \rho)\left(\delta_1 p - c^u - h\delta_1\right)(\alpha 2\Delta_1 + \alpha(D_1 - \Delta_1)). \end{aligned}$$

The result thus follows. QED.

### Proof of Proposition 5

The proof is very similar to the proof of Proposition 3. Let  $\alpha(D_1 - \Delta_1) \geq 2\Delta_1$ . In the following, we identify conditions when one order policy dominates over another one. We have:  $E(P1S') = E(P2S')$  iff

$$\begin{aligned} p(1 - \rho) - c^u &= p - c^u - h \\ \rho &= \frac{h}{p} \end{aligned} \tag{18}$$

Thus, order policy  $P2S'$  dominates order policy  $P1S'$  for  $\rho > \frac{h}{p}$ .

Further, we have:  $E(P2S') = E(P3S')$  iff

$$\begin{aligned} \delta_1 p(1 - \rho) - c^u &= \delta_1 p - c^u - \delta_1 h \\ \rho &= \frac{h}{p} \end{aligned} \tag{19}$$

Thus, order policy  $P3S'$  dominates order policy  $P2S'$  for  $\rho > \frac{h}{p}$ . Let  $\alpha(D_1 - \Delta_1) < 2\Delta_1$ . We then have that  $E(P1S') = E(P3S')$  iff  $\rho > \frac{h}{p}$ . The result thus follows. QED.