



City Research Online

City St George's, University of London

Citation: Cowell, R. (1997). Sampling without replacement in junction trees (Statistical Research Paper No. 15). London, UK: Faculty of Actuarial Science & Insurance, City University London.

This is the unspecified version of the paper.

This version of the publication may differ from the final published version. To cite this item please consult the publisher's version.

Permanent repository link: <https://city-ref-test.eprints-hosting.org/id/eprint/2364/>

Copyright and Reuse: Copyright and Moral Rights remain with the author(s) and/or copyright holders. Copies of full items can be used for personal research or study, educational, or not-for-profit purposes without prior permission or charge, unless otherwise indicated, provided that the authors, title and full bibliographic details are credited, a hyperlink and/or URL is given for the original metadata page and the content is not changed in any way. For full details of reuse please refer to [City Research Online policy](#).



**SAMPLING WITHOUT REPLACEMENT IN
JUNCTION TREES**

by

R COWELL

Statistical Research Paper No. 15

**Department of Actuarial Science and Statistics
City University
London**

June 1997

ISBN 1 901615 14 6

**Northampton Square
London EC1V 0HB
Telephone 0171-477 8000**

“Any opinions expressed in this paper are my/our own and not necessarily those of my/our employer or anyone else I/we have discussed them with. You must not copy this paper or quote it without my/our permission”.

Sampling without replacement in junction trees

Robert Cowell

School of Mathematics, Actuarial Science and Statistics

City University

Northampton Square

London EC1E 0HT

June 24, 1997

Abstract

It is shown that a highly efficient algorithm recently presented by Nilsson (1997) for finding the M-most probable configurations in a probabilistic expert system can be adapted for generating samples without replacement from a junction-tree representation of the joint distribution.

Keywords: Probabilistic expert system, Bayesian network, junction tree, max-propagation, sampling without replacement.

1 Introduction

For a probabilistic expert system, (also called a Bayesian network), of discrete variables Dawid (1992) showed how one could find the most-probable configuration of the variables by a single max-propagation on an associated junction tree. Recently Nilsson (1997) exploited this calculational method and combined it with a clever partitioning scheme to formulate an efficient method for finding the M most likely configurations of the joint distribution for any integer M . In his paper Dawid also showed how one can sample directly and efficiently from the posterior distribution represented by the marginal-potential representation on the junction tree. It is the purpose of this short paper to show that, using the partitioning scheme of Nilsson, one can sample without replacement directly and efficiently from the posterior distribution. The next section presents a summary of the key points of Nilsson's method. Sampling without replacement is developed by analogy in the subsequent section. We shall assume the reader is familiar with the papers of (Dawid 1992) and (Nilsson 1997), and we will borrow notations, definitions and results from these papers.

2 Finding the M -most likely configurations

Suppose we are given a connected¹ junction tree $\mathcal{T}$ having a potential representation of a probability function f in the discrete variables $X_u, u \in U$ given by

$$f = \frac{\prod_{C \in \mathcal{C}} a_C}{\prod_{S \in \mathcal{S}} b_S}, \quad (1)$$

where the clique and separator potentials a_C and b_S are known non-negative functions. Then by a process called *max-propagation* on the junction tree, consisting of a set of local messages passed between the cliques, the joint distribution f can be

¹This is not a problematic restriction, the disconnected case is readily treated.

expressed in the form

$$f = \frac{\prod_{C \in \mathcal{C}} \hat{f}_C}{\prod_{S \in \mathcal{S}} \hat{f}_S} \quad (2)$$

where $\hat{f}_C$ is the maximum of f over the subset of variables $x_C \subset U$. (See (Dawid 1992) for proof.) From this *max-marginal* representation one can readily find the configuration which has the highest probability, by the following search procedure (called *max-distribute*). First one designates a clique as the root, and orders the cliques starting with the root so that they obey the *running intersection property*. This is equivalent to making the junction tree directed, with all edges pointing away from the root, and then finding a topological ordering of the cliques beginning with the root. Such an ordering is always possible. For convenience label the ordered cliques sequentially thus $(C_0, C_1, \dots, C_k)$, with separators $(S_1, \dots, S_k)$ such that the separator S_j joins the clique C_j to the adjacent clique which is closer to the root. (see Figure 1 for an example). Denote the corresponding ordered *residual sets* by $(R_0, R_1, \dots, R_k)$ where $R_j \equiv C_j \setminus S_j$ for every $j > 0$ and we define $R_0 \equiv C_0$.

To find the most likely configuration, one then proceeds outwards from the root as follows: First search the root clique C_0 for the configuration of its variables, $\hat{x}_{C_0}$ say, which maximizes the clique potential value (breaking ties arbitrarily). Distribute this as extra “evidence”, fixing successively the remaining variables in the cliques further from the root in their running intersection order by finding a maximal configuration consistent with the neighbouring cliques which have also been fixed, and including the states of the newly fixed variables as extra evidence. Thus for clique C_j one finds the configuration of the residual variables R_j which maximizes $\hat{f}_{C_j}$ subject to the separator variables S_j taking the configuration fixed by the earlier steps. The union of the “evidence” yields the most likely configuration. If there is “real” evidence then this will have been incorporated by the original max-propagation which put the

junction tree into the max-marginal representation.

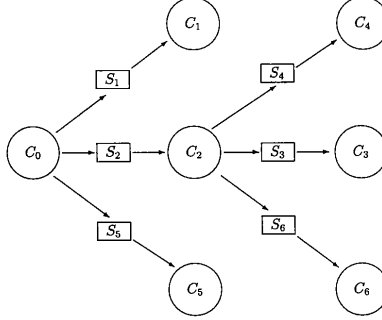


Figure 1: A junction tree as a directed acyclic graph.

The algorithm of (Nilsson 1997) for finding the M -most likely configurations is an iterative divide-and-conquer process built upon this algorithm for finding the most likely configuration. Following Nilsson, let H denote the complete set of configurations, and for $L = 1, 2$, etc let x^L denote the L th most probable configuration. The procedure for finding the $L + 1$ th configuration, which is essentially a search over the space $H \setminus \{x^1, \dots, x^L\}$, consists of three steps:

1. **Partition phase:** Partition $H \setminus \{x^1, \dots, x^L\}$ into disjoint representative subsets, which together are consistent with all remaining configurations.
2. **Candidate phase:** Search these partition-generated subsets for the set consistent with the $L + 1$ th most probable configuration.
3. **Identification phase:** Extract the configuration from the selected candidate.

First we discuss a simplified (and far less efficient) version to help explain Nilsson's algorithm. Denote the set of initial evidence (if any) by $\mathcal{E}$, and order the variables in some way, say $(X_1, X_2, \dots, X_n)$ where the j -th node has T_j states $(x_j^1, \dots, x_j^{T_j})$.

From the preceding discussion, one can find the most likely configuration, $x^1 = (x_1^{t_1^1}, \dots, x_n^{t_n^1})$. Now the second most likely configuration, x^2 must differ from x^1 in the state of at least one variable. So now perform a further n max-propagations, (each requires a re-initializing the junction tree), each with a distinct “pseudo-evidence” $\mathcal{E}_{j,\emptyset}^1$ defined by:

$$\begin{aligned}
\mathcal{E}_{1,\emptyset}^1 &= \mathcal{E} \text{ and } X_1 \neq x_1^{t_1^1}, \\
\mathcal{E}_{2,\emptyset}^1 &= \mathcal{E} \text{ and } X_1 = x_1^{t_1^1} \text{ and } X_2 \neq x_2^{t_2^1}, \\
\mathcal{E}_{3,\emptyset}^1 &= \mathcal{E} \text{ and } X_1 = x_1^{t_1^1} \text{ and } X_2 = x_2^{t_2^1} \text{ and } X_3 \neq x_3^{t_3^1}, \\
&\vdots \\
\mathcal{E}_{n,\emptyset}^1 &= \mathcal{E} \text{ and } X_1 = x_1^{t_1^1} \text{ and } \dots X_{n-1} = x_{n-1}^{t_{n-1}^1} \text{ and } X_n \neq x_n^{t_n^1}.
\end{aligned} \tag{3}$$

By this procedure we partition the search space for the set of all configurations, excluding the most likely one x^1 we have already found. Hence one of these must be consistent with the second most likely configuration x^2 , and only one if it is unique; which one can be found by calculating the max-normalizations of each and picking the one with the largest normalization.

If the second most likely configuration is found to arise from the j th set, then by a max-distribute operation it can found using initial evidence $\mathcal{E}_{j,\emptyset}^1$. Denote it by $x^2 = (x_1^{t_1^2}, \dots, x_n^{t_n^2})$, (where by the partition constraint $t_i^1 = t_i^2$ for all $i < j$).

Now the third most likely configuration x^3 is either consistent with one of the set alternative evidences of (3) excluding the j th, or it has a configuration which disagrees with both the x^1 and x^2 configurations in at least one place for some variable having index $\geq j$ while agreeing for all indices less than j . To find out which perform a further partition of the search space to obtain an extra set of $n - j + 1$

disjoint evidences of the form:

$$\begin{aligned}
&\mathcal{E}_{j,j}^2 = \mathcal{E}_{j,\emptyset}^1 \text{ and } X_j \neq x_j^{t_j^2}, \\
&\mathcal{E}_{j+1,j}^2 = \mathcal{E}_{j,\emptyset}^1 \text{ and } X_j = x_j^{t_j^2} \text{ and } X_{j+1} \neq x_{j+1}^{t_{j+1}^2}, \\
&\vdots \\
&\mathcal{E}_{n,j}^2 = \mathcal{E}_{j,\emptyset}^1 \text{ and } \dots X_{n-1} = x_{n-1}^{t_{n-1}^2} \text{ and } X_n \neq x_n^{t_n^2}.
\end{aligned} \tag{4}$$

Finding the max-normalizations of these, and comparing with those of (3) (excluding the j th) we pick out that which attains the highest max-normalization and the use max-distribute to find the actual configuration. The representative evidence is then further partitioned, etc, until the desired number M of configurations have been found.

The partitioning the search space is one of the key steps in Nilsson's method, and will work with any ordering of the variables. However it does require a large number of junction tree initializations and propagations. The other key step is to select a specific ordering of the variables so that only one max-propagation is required, thus making the process much more efficient. Specifically, one orders the variables according to the ordered residual sets R , and in the partitioning phase excludes joint states of the residual variables (rather than states of individual variables). Thus, in terms of the residual sets, suppose the most likely configuration has the form $x^1 = (r_0^{t_0^1}, \dots, r_k^{t_k^1})$, (where now the ts range over the configurations of the residuals). Then as dummy evidence for finding the second most likely configuration, one could perform $k + 1$ propagations of the form:

$$\begin{aligned}
\mathcal{E}_{0,\emptyset}^1 &= \mathcal{E} \text{ and } R_0 \neq r_0^{t_0^1}, \\
\mathcal{E}_{1,\emptyset}^1 &= \mathcal{E} \text{ and } R_0 = r_0^{t_0^1} \text{ and } R_1 \neq r_1^{t_1^1}, \\
\mathcal{E}_{2,\emptyset}^1 &= \mathcal{E} \text{ and } R_0 = r_0^{t_0^1} \text{ and } R_1 = r_1^{t_1^1} \text{ and } R_2 \neq r_2^{t_2^1}, \\
&\vdots \\
\mathcal{E}_{k,\emptyset}^1 &= \mathcal{E} \text{ and } R_0 = r_0^{t_0^1} \text{ and } \dots R_{k-1} = r_{k-1}^{t_{k-1}^1} \text{ and } R_k \neq r_k^{t_k^1}.
\end{aligned} \tag{5}$$

The point about using this ordering is that, for example, if one were to do a max-collect using $\mathcal{E}_{j,\emptyset}^1$ then, because no new dummy-evidence has been entered beyond the j th residual set or clique, all messages received by the subtree induced by $(C_0, \dots, C_j)$ are identical to those which were sent in the original max-propagation using the evidence $\mathcal{E}$. This means that sending such messages is unnecessary, and one need only consider the max-collect operation on the subtree $(C_0, \dots, C_j)$ to find the max-normalization. But such an operation is trivial in that, because of the evidence $\mathcal{E}_{j,\emptyset}^1$ entered, only one non-zero value from each clique and separator (denoted by $x_{C_i}^*$ and $x_{S_i}^*$) survive for all indices $i < j$, thus the normalization can be found very efficiently by the simple formula:

$$\frac{\prod_{i=0}^{j-1} \hat{f}_{C_i}(x_{C_i}^*)}{\prod_{i=0}^{j-1} \hat{f}_{S_i}(x_{S_i}^*)} \max \hat{f}_{C_j}(x_{C_j}, \mathcal{E}_{j,\emptyset}^1) \tag{6}$$

which requires a simple search in the j th clique for the last term. This is Theorem 3 of (Nilsson 1997). $\mathcal{E}_{j,\emptyset}^1$ is then partitioned similarly to (4) into:

$$\begin{aligned}
\mathcal{E}_{j,j}^2 &= \mathcal{E}_{j,\emptyset}^1 \text{ and } R_j \neq r_j^{t_j^2}, \\
\mathcal{E}_{j+1,j}^2 &= \mathcal{E}_{j,\emptyset}^1 \text{ and } R_j = r_j^{t_j^2} \text{ and } R_{j+1} \neq r_{j+1}^{t_{j+1}^2}, \\
&\vdots \\
\mathcal{E}_{n,j}^2 &= \mathcal{E}_{j,\emptyset}^1 \text{ and } \dots R_{n-1} = r_{n-1}^{t_{n-1}^2} \text{ and } R_n \neq r_n^{t_n^2}.
\end{aligned} \tag{7}$$

3 Sampling without replacement algorithm

Besides showing how one can find the most likely configuration, in his paper Dawid (1992) also showed how one can sample with replacement directly from the posterior distribution using the junction tree. The first stage is to put the junction tree potentials into their marginal-potential form by a sum-propagation of evidence, thus:

$$f(U, \mathcal{E}) = \frac{\prod_C f(C, \mathcal{E})}{\prod_S f(S, \mathcal{E})}. \quad (8)$$

Then to get a sample, one uses again the ordering of the cliques as for the previous maximization story. One begins by sampling a configuration in the root clique, drawing from $f(R_0, \mathcal{E})$ to obtain $r_0^{t_0^1}$ say. Next one samples from $f(R_1 | R_0 = r_0^{t_0^1}, \mathcal{E})$ found from clique C_1 , etc, processing the cliques in their running intersection ordering, so that when sampling for R_i from clique C_i the variables X_{S_i} will already have been fixed by earlier sampling. When all cliques have been so processed a configuration will have been generated with its correct probability. This process can be repeated to draw as many independent samples as may be required. We shall call such a search process a *sample-distribute* operation.

However, instead of sampling independent samples we can employ Nilsson's partitioning scheme to draw samples without replacement correctly from the posterior distribution. The differences are (1) sum-propagation rather than max-propagation is used to initialize the tree, and (2) sampling rather than maximization is performed in the candidate partition search stages.

Specifically, suppose that we have put the junction tree into sum-marginal representation by sum-propagation of evidence. Then one samples one configuration by the method described above - denote this first sample by x^1 writing it in terms of the residual sets as: $x^1 = (r_0^{t_0^1}, \dots, r_k^{t_k^1})$. Then to sample a second configuration, in analogy to finding the second most likely configuration one generates $k + 1$ candi-

dates using evidences given by (5), with the evidence defined by $\mathcal{E}_{j,\emptyset}^1$ being assigned a weight given by

$$\frac{\prod_{i=0}^{j-1} f_{C_i}(x_{C_i}^*)}{\prod_i^j f_{S_i}(x_{S_i}^*)} \sum f_{C_j}(x_{C_j}, \mathcal{E}_{j,\emptyset}^1) \quad (9)$$

(cf (6)). This weight could alternatively be found much more inefficiently using a sum-collect operation with evidence $\mathcal{E}_{j,\emptyset}^1$, but again in direct analogy to the max-collect operation, by sum-consistence the cliques and separators having indices greater than j have their contributions cancel.

Next from this set of weighted candidates one samples a candidate according to its weight, (max-distribute-search picks the highest weight configuration), and then proceed to complete the configuration by a sample-distribute operation, which ensures that the final configuration obtained is drawn from the distribution correctly.

Thus assume that in this process the j th term was sampled. Then in analogy to drawing a single sample, the configuration is completed by a sample-distribute-search using $\mathcal{E}_{j,\emptyset}^1$, (which can begin from the j clique as all previous cliques have completely specified residuals). Its probability can then be found easily. After completion, if a third configuration is required then this must also be partitioned to form new extra candidates. These candidates will have respective evidences as in (7), with weights given in analogy to (9).

4 Conclusions

We have shown informally how samples may be drawn without replacement using a variation of the top M -configuration search algorithm of Nilsson (1997), in which max-propagation is replaced with sum-propagation, and max-distribute-searching with sample-distribute-searching etc. With these simple replacements, plus sampling



ACTUARIAL RESEARCH CENTRE

DEPARTMENT OF ACTUARIAL SCIENCE AND STATISTICS

Actuarial Research Papers

1. Haberman S. Long Term Prognosis after Cerebral Infarction. 78 pages, 1984 (out of print).
ISBN 1 874 770 01 8
2. Haberman S. Long Term Prognosis after Subarachnoid Haemorrhage. 43 pages, 1984 (out of print).
ISBN 1 874 770 02 6
3. Haberman S. Age Variation in Incidence of Cerebrovascular Disease. 36 pages, 1985 (out of print).
ISBN 1 874 770 03 4
4. Kaye G.D. Taxation of Pension Funds. 11 pages, 1985.
ISBN 1 874 770 04 2

[Also: Haberman S. and Bloomfield D.S.F. Social Class Differences in Mortality Around 1981. SSRU Working Paper No. 55. 41 pages, 1987.]
5. Kaye G.D. Life Offices' Expenses Regression Analyses I, 98 pages, 1988. (Limited circulation).
ISBN 1 874 770 05 0
6. Kaye G.D. Life Assurance Taxation - Comments in Response to the Board of the Inland Revenue's Consultative Document Concerning the Taxation of Life Insurance. 11 pages. 1988.
ISBN 1 874 770 06 9
7. Verrall R.J. Bayesian Linear Models and the Claims Run-Off Triangle. 32 pages. 1988.
ISBN 1 874 770 07 7
8. Kaye G.D. Life Offices' Expenses Regression Analyses II. 12 pages. 1989. (Limited circulation).
ISBN 1 874 770 08 5
9. Bloomfield D.S.F. and Haberman S. The Assessment of Damages and the Measurement of Work time Lost to Sickness, Unemployment and Stoppages. 100 pages. 1989.
ISBN 1 874 770 09 3
10. Adams A.T. Pricing Property Investment Company Shares. 14 pages. 1989.
ISBN 1 874 770 10 7

11. Booth P.M. History of Exchange Controls in the EC with Particular Reference to the United Kingdom. 39 pages. 1989.
ISBN 1 874 770 11 5
12. Verrall R.J. Chain Ladder and Maximum Likelihood. 12 pages. 1989.
ISBN 1 874 770 12 3
13. Verrall R.J. On the Unbiased Estimation of Reserves from Loglinear Models. 13 pages, 1989.
ISBN 1 874 770 13 1
14. Adams A.T. An Analysis of Investment Trust Shares, 1990. 31 pages.
ISBN 1 874 770 14 X
15. Haberman S. Demographic Trends Towards 2000. 16 pages. 1989.
ISBN 1 874 770 15 8
16. Haberman S. British Life Insurance: The Role of the Actuary and 1992. 30 pages. 1990.
ISBN 1 874 770 16 6
17. Verrall R.J. Bootstrapping, Saddlepoint Approximations and the Probability of Ruin. 10 pages.
July 1990.
ISBN 1 874 770 17 4
18. Booth P.M. Theoretical Aspects of the Taxation of Real Returns from Short Term Deposits.
26 pages. June 1990.
ISBN 1 874 770 18 2
19. Booth P.M. Practical Aspects of the Taxation of Real Interest. 38 pages, 1990.
ISBN 1 874 770 19 0
20. Puzey A.S. Actuarial Funding in Unit-linked Life Assurance - Derivation of the Actuarial
Funding Factors. 19 pages. July 1990.
ISBN 1 874 770 20 4
21. Kaye G.D. Economies of Scale in the United Kingdom Insurance Industry. A report for the
Association of British Insurers. Limited Circulation. October 1990.
ISBN 1 874 770 21 2
22. Adams A.T. & Venmore-Rowland P. Direct or Indirect Property Investment. 71 pages.
September 1990.
ISBN 1 874 770 22 0
23. England P.D. Statistical Modelling of Excess Mortality. 124 pages. September 1990.
ISBN 1 874 770 23 9
24. Verrall R.J. & Li Z. Negative Incremental Claims: Chain Ladder and Linear Models.
November 1990.
ISBN 1 874 770 24 7
25. Haberman S. State Pension Provision and Social Security in the United Kingdom. Presented at a
Conference in Lisbon on "Social Security and Private Provision; What are the Prospects?".
40 pages. October 1990.
ISBN 1 874 770 25 5

26. Renshaw A.E. Graduation by Generalized Linear Modelling Techniques. 42 pages. February 1991.
ISBN 1 874 770 26 3
27. England P.D. Statistical Modelling of Excess Mortality No. 2. 170 pages. June 1991.
ISBN 1 874 770 27 1
28. Adams, A.T. The Market Pricing of Composite Insurance Shares. 37 pages. July 1991.
ISBN 1 874 770 28 X
29. Haberman, S. The Effect of Autoregressive Rates of Return on the Variability of Pension Contributions and Fund Levels. 58 pages. September 1991.
ISBN 1 874 770 29 8
30. Puzey, A.S. An Investigation into the Effect of Averaging Age Nearest Birthday at the Beginning of a Rate Year to Exact Age. October 1991. 33 pages.
ISBN 1 874 770 30 1
31. Chadburn, R.G. Bias in Select Mortality Investigations where Data are Subdivided by Age at Death. November 1991. 69 pages.
ISBN 1 874 770 31 X
32. England P.D. and Verrall R.J. Dynamic Estimation for Models of Excess Mortality. January 1992. 19 pages.
ISBN 1 874 770 32 8
33. Verrall R.J. A State Space Formulation of Whittaker-Henderson Graduation, with Extensions. January 1992. 13 pages.
ISBN 1 874 770 33 6
34. Verrall R.J. Graduation by Dynamic Regression Methods. January 1992. 37 pages.
ISBN 1 874 770 34 4
35. Gerrard R.G. and Haberman S. Stability of Pension Systems when Gains/Losses are Amortized and Rates of Return are Autoregressive. March 1992. 12 pages.
ISBN 1 874 770 35 2
36. Haberman S. HIV, AIDS and the Approximate Calculation of Life Insurance Functions, Annuities and Net Premiums. April 1992. 28 pages.
ISBN 1 874 770 36 0
37. Cooper D.R. Savings and Loans: An Historical Perspective. May 1992. 29 pages.
ISBN 1 874 770 37 9
38. Verrall R.J. Dynamic Bayes Estimation for AIDS Delay Tables. May 1992. 16 pages.
ISBN 1 874 770 38 7
39. Kaye G.D. Data Sources Relating to Life Assurance Expenses. May 1992. 39 pages. Presented to the Staple Inn Actuarial Society and Royal Statistical Society - February 1992.
ISBN 1 874 770 39 5
40. Renshaw A.E., and Haberman S. On the Graduation Associated with a Multiple State Model for Permanent Health Insurance. May 1992.
ISBN 1 874 770 40 9

41. England P.D. Statistical Modelling of Excess Mortality Number 3. June 1992. 163 pages.
ISBN 1 874 770 41 7
42. Bloomfield D.S.F. and Haberman S. Male Social Class Mortality Differences Around 1981: An Extension to Include Childhood Ages. 21 pages. June 1992.
ISBN 1 874 770 42 5
43. Berg M.P and Haberman S. Trend Analysis and Prediction Procedures for Time Nonhomogeneous Claim Processes. 33 pages. June 1992.
ISBN 1 874 770 43 3
44. Booth P.M. The Single Market for Insurance, Free Capital Movements and attempts to Fix Exchange Rates. October 1992. 28 pages.
ISBN 1 874 770 44 1
45. Verrall R.J. Chain Ladder with Varying Run-off Evolutions. February 1993. 15 pages.
ISBN 1 874 770 45 X
46. Gavin J., Haberman S. and Verrall R.J. Moving Weighted Average Graduation using Kernel Estimation. November 1992. 14 pages.
ISBN 1 874 770 46 8
47. Gavin J., Haberman S. and Verrall R.J. On the Choice of Bandwidth for Kernel Graduation. November 1992. 21 pages.
ISBN 1 874 770 47 6
48. S. Haberman. Pension Funding with Time Delays and the Optimal Spread Period. May 1993. 13 pages.
ISBN 1 874 770 48 4
49. S. Haberman. Stochastic Investment Returns and the Present Value of Future Contributions in a Defined Benefit Pension Scheme. May 1993. 22 pages.
ISBN 1 874 770 49 2
50. A. Zimbidis and S. Haberman. Delay, Feedback and Variability of Pension Contributions and Fund Levels. May 1993. 19 pages.
ISBN 1 874 770 50 6
51. S. Haberman. Pension Funding: The Effect of Changing The Frequency Valuations. June 1993. 19 pages.
ISBN 1 874 770 51 4
52. S Haberman. HIV, AIDS Markov Chains and PHI. June 1993. 15 pages.
ISBN 1 874 770 52 2
53. S Haberman. A Consideration of Pension Credit and Termination Insurance. June 1993. 22 pages.
ISBN 1 874 770 53 0
54. M Z Khorasane. Survey of Actuarial Practice in the Funding of UK Defined Benefit Pension Schemes. July 1993. 19 pages.
ISBN 1 874 770 54 9

55. P M Booth, R G Chadburn and A S K Ong. A Utility Maximisation Approach to Asset Allocation. September 1993. 40 pages.
ISBN 1 874 770 55 7
56. R G Chadburn. Bias in Select Mortality Investigations. August 1993. 62 pages.
ISBN 1 874 770 56 5
57. M Z Khorasane. A Comparison of Pension Funding Strategies by means of Simulations for a Model Scheme. August 1993. 43 pages.
ISBN 1 874 770 57 3
58. A E Renshaw, P Hatzopolous and S Haberman. Recent Mortality Trends in Male Assured Lives. June 1993. 23 pages.
ISBN 1 874 770 58 1
59. E Pitacco. Disability Risk Models: Towards A Unifying Approach. September 1993. 33 pages.
ISBN 1 874 770 59 X
60. M Boskov and R J Verrall. Premium Rating by Geographic Area Using Spatial Models. September 1993. 14 pages.
ISBN 1 874 770 60 3
61. R G Chadburn. Managing Mutual Life Offices: The Use of an Expense Ratio in New Business Decision Making and Expense Control. October 1993. 21 pages.
ISBN 1 874 770 61 1
62. Haberman S. Pension Funding Modelling and Stochastic Investment Returns. 56 pages. March 1994.
ISBN 1 874 770 62 X
63. Renshaw A E and Verrall R J. The Stochastic Model Underlying the Chain-Ladder Technique. 25 pages. April 1994.
ISBN 1 874 770 63 8
64. Haberman S and Sung J-H. Dynamic Approaches to Pension Funding. 22 pages. April 1994.
ISBN 1 874 770 64 6
65. Renshaw A.E. On the Second Moment Properties and the Implementation of Certain GLIM Based Stochastic Claims Reserving Models. 36 pages. September 1994.
ISBN 1 874 770 65 4
66. Booth P.M., J.N. Allan, and J.W. Jang. An Evaluation of the UK Life Insurance Mismatching Test. September 1994.
ISBN 1 874 770 66 2
67. Booth P.M. and Stroinski K. Insurance and Investment Markets in Poland. September 1994. 35 pages.
ISBN 1 874 770 67 0
68. Ong A. A Stochastic Model for Treasury-Bills: An Extension to Wilkie's Model. September 1994. 12 pages.
ISBN 1 874 770 68 9

69. Bloomfield D.S.F. Moving on from Undergraduate Exams to Professional Exams: Actuaries. November 1994. 22 pages.
ISBN 1 874 770 69 7
70. Huber P. A Review of Wilkie's Stochastic Investment Model. January 1995. 22 pages.
ISBN 1 874 770 70 0
71. Renshaw A.E. On the Graduation of 'Amounts'. January 1995. 24 pages.
ISBN 1 874 770 71 9
72. Renshaw A.E. Claims Reserving by Joint Modelling. December 1994. 26 pages.
ISBN 1 874 770 72 7
73. Renshaw A.E. Graduation and Generalised Linear Models: An Overview. February 1995. 40 pages.
ISBN 1 874 770 73 5
74. Khorasane M.Z. Simulation of Investment Returns for a Money Purchase Fund. June 1995. 20 pages.
ISBN 1 874 770 74 3
75. Owadally M.I. and Haberman S. Finite-time Pension Fund Dynamics with Random Rates of Return. June 1995. 28 pages.
ISBN 1 874 770 75 1
76. Owadally M.I. and Haberman S. Stochastic Investment Modelling and Optimal Funding Strategies. June 1995. 25 pages.
ISBN 1 874 770 76 X
77. Khorasane M.Z. Applying the Defined Benefit Principle to a Defined Contribution Scheme. August 1995. 30 pages.
ISBN 1 874 770 77 8
78. Sebastiani P. and Settini R. Experimental Design for Non-Linear Problems. September 1995. 13 pages.
ISBN 1 874 770 78 6
79. Verrall R.J. Whittaker Graduation and Parametric State Space Models. November 1995. 23 pages.
ISBN 1 874 770 79 4
80. Verrall R.J. Claims Reserving and Generalised Additive Models. November 1995. 17 pages.
ISBN 1 874 770 80 8
81. Nelder J.A. and Verrall R.J. Credibility Theory and Generalized Linear Models. November 1995. 15 pages.
ISBN 1 874 770 81 6
82. Renshaw A.E., Haberman S. and Hatzopoulos P. On The Duality of Assumptions Underpinning The Construction of Life Tables. December 1995. 17 Pages.
ISBN 1 874 770 82 4

83. Chadburn R.G. Use of a Parametric Risk Measure in Assessing Risk Based Capital and Insolvency Constraints for With Profits Life Insurance. March 1996. 17 Pages.
ISBN 1 874 770 84 0
84. Haberman S. Landmarks in the History of Actuarial Science (up to 1919). March 1996.
62 Pages.
ISBN 1 874 770 85 9
85. Renshaw A.E. and Haberman S. Dual Modelling and Select Mortality. March 1996.
30 Pages.
ISBN 1 874 770 88 3
86. Booth P.M. Long-Term Care for the Elderly: A Review of Policy Options. April 1996.
45 Pages.
ISBN 1 874 770 89 1
87. Huber P.P. A Note on the Jump-Equilibrium Model. April 1996. 17 Pages.
ISBN 1 874 770 90 5
88. Haberman S and Wong L.Y.P. Moving Average Rates of Return and the Variability of Pension Contributions and Fund Levels for a Defined Benefit Pension Scheme. May 1996. 51 Pages.
ISBN 1 874 770 91 3
89. Cooper D.R. Providing Pensions for Employees with Varied Working Lives. June 1996.
25 Pages.
ISBN 1 874 770 93 X
90. Khorasanee M.Z. Annuity Choices for Pensioners. August 1996. 25 Pages.
ISBN 1 874 770 94 8
91. Verrall R.J. A Unified Framework for Graduation. November 1996. 25 Pages.
ISBN 1 874 770 99 9
92. Haberman S. and Renshaw A.E. A Different Perspective on UK Assured Lives Select Mortality.
November 1996. 61 Pages.
ISBN 1 874 770 00 X
93. Booth P.M. The Analysis of Actuarial Investment Risk. March 1997. 43 Pages.
ISBN 1 901615 03 0
94. Booth P.M., Chadburn R.G. and Ong A.S.K. Utility-Maximisation and the Control of Solvency for Life Insurance Funds. April 1997. 39 Pages.
ISBN 1 901615 04 9
95. Chadburn R.G. The Use of Capital, Bonus Policy and Investment Policy in the Control of Solvency for With-Profits Life Insurance Companies in the UK. April 1997. 29 Pages.
ISBN 1 901615 05 7
96. Renshaw A.E. and Haberman S. A Simple Graphical Method for the Comparison of Two Mortality Experiences. April 1997. 32 Pages.
ISBN 1 901615 06 5

97. Wong C.F.W. and Haberman S. A Short Note on Arma (1, 1) Investment Rates of Return and Pension Funding. April 1997. 14 Pages.
ISBN 1 901615 07 3
98. Puzey A S. A General Theory of Mortality Rate Estimators. June 1997. 26 Pages.
ISBN 1 901615 08 1
99. Puzey A S. On the Bias of the Conventional Actuarial Estimator of q_x . June 1997. 14 Pages.
ISBN 1 901615 09 X
100. Walsh D. and Booth P.M. Actuarial Techniques in Pricing for Risk in Bank Lending. June 1997. 55 Pages.
ISBN 1 901615 12 X

Statistical Research Papers

1. Sebastiani P. Some Results on the Derivatives of Matrix Functions. December 1995. 17 Pages.
ISBN 1 874 770 83 2
2. Dawid A.P. and Sebastiani P. Coherent Criteria for Optimal Experimental Design. March 1996. 35 Pages.
ISBN 1 874 770 86 7
3. Sebastiani P. and Wynn H.P. Maximum Entropy Sampling and Optimal Bayesian Experimental Design. March 1996. 22 Pages.
ISBN 1 874 770 87 5
4. Sebastiani P. and Settini R. A Note on D-optimal Designs for a Logistic Regression Model. May 1996. 12 Pages.
ISBN 1 874 770 92 1
5. Sebastiani P. and Settini R. First-order Optimal Designs for Non Linear Models. August 1996. 28 Pages.
ISBN 1 874 770 95 6
6. Newby M. A Business Process Approach to Maintenance: Measurement, Decision and Control. September 1996. 12 Pages.
ISBN 1 874 770 96 4
7. Newby M. Moments and Generating Functions for the Absorption Distribution and its Negative Binomial Analogue. September 1996. 16 Pages.
ISBN 1 874 770 97 2
8. Cowell R.G. Mixture Reduction via Predictive Scores. November 1996. 17 Pages.
ISBN 1 874 770 98 0
9. Sebastiani P. and Ramoni M. Robust Parameter Learning in Bayesian Networks with Missing Data. March 1997. 9 Pages.
ISBN 1 901615 00 6

10. Newby M.J. and Coolen F.P.A. Guidelines for Corrective Replacement Based on Low Stochastic Structure Assumptions. March 1997. 9 Pages.
ISBN 1 901615 01 4.
11. Newby M.J. Approximations for the Absorption Distribution and its Negative Binomial Analogue. March 1997. 6 Pages.
ISBN 1 901615 02 2
12. M. Ramoni and P. Sebastiani. The Use of Exogenous Knowledge to Learn Bayesian Networks from Incomplete Databases. June 1997. 11 Pages.
ISBN 1 901615 10 3
13. M. Ramoni and P. Sebastiani. Learning Bayesian Networks from Incomplete Databases. June 1997. 14 Pages.
ISBN 1 901615 11 1
14. P. Sebastiani and H.P. Wynn. Risk Based Optimal Designs. June 1997. 10 Pages.
ISBN 1 901615 13 8
15. R. Cowell. Sampling without Replacement in Junction Trees. June 1997. 10 Pages.
ISBN 1 901615 14 6

ARJ-ARP/aredsk

ISBN 1 901615 14 6