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Supplement to “Forecasting Using Heterogeneous Panels with Cross-Sectional Dependence”

November 14, 2019

This document contains the material which are not reported in the paper in order to save space. These are the detailed description of the parameter estimation methods (Section S1) and additional Monte Carlo evidence obtained by considering a dynamic prediction model (Section S2).

S1 Estimation Methods

Consider the stationary predictive panel data model used in the paper. The h -steps ahead variable $y_{i,t+h}$, $h \geq 0$, $i = 1, 2, \dots, n$, $t = 1, 2, \dots, T$, is given by

$$y_{i,t+h} = \alpha_i + \beta_i' \mathbf{x}_{it} + \gamma_i^{y'} \mathbf{f}_t^y + u_{i,t+h}, \quad (\text{S1})$$

$$u_{i,t+h} = \sum_{j=1}^n r_{ij} \varepsilon_{j,t+h}, \quad (\text{S2})$$

where $\mathbf{x}_{it} = (x_{i1t}, x_{i2t}, \dots, x_{ik_x t})'$ is a $(k_x \times 1)$ vector of observed individual-specific regressors which can include predetermined variables, $\beta_i = (\beta_{i1}, \beta_{i2}, \dots, \beta_{ik_x})'$ represents the corresponding $(k_x \times 1)$ slope parameters, r_{ij} are unknown spatial weights, ε_{it} is an error term which is uncorrelated over time and individuals. $\mathbf{f}_t^y$ is a vector of unobservable common factors of size m_y , γ_i^y are the associated $(m_y \times 1)$ factor loadings. α_i are the unit specific time-invariant effects. Unless otherwise specified, β_i , γ_i^y and α_i are assumed to be fixed parameters.

S1.1 Estimators and Predictors with No Common Factors

The estimators which assume no common factors in the data generating process (DGP) set $\gamma_i^{y'} \mathbf{f}_t^y = 0$. In the paper, these estimators are used only in the empirical application and not in simulations. The predictions using these estimators take the form

$$\hat{y}_{i,T+h|T} = \hat{\alpha}_i + \hat{\beta}_i' \mathbf{x}_{iT}, \quad (\text{S3})$$

where for any estimator $\hat{\beta}_i$ the intercepts are estimated by $\hat{\alpha}_i = \frac{1}{T-h} \sum_{t=h+1}^T (y_{i,t} - \hat{\beta}_i' \mathbf{x}_{i,t-h})$. In the case of a pooled estimator $\hat{\beta}_i = \hat{\beta}$.

Let us define the matrices of deviations from the means as $\tilde{\mathbf{X}}_{i,-h} = \mathbf{M}_e \mathbf{X}_{i,-h}$, $\tilde{\mathbf{y}}_i = \mathbf{M}_e \mathbf{y}_i$, $\mathbf{X}_{i,-h} = (\mathbf{x}_{i1}', \mathbf{x}_{i2}', \dots, \mathbf{x}_{i,T-h}')'$, $\mathbf{y}_i = (y_{i,h+1}, y_{i2}, \dots, y_{iT})'$ and $\mathbf{M}_e = \mathbf{I}_{T-h} - (T-h)^{-1} \mathbf{e}_{T-h} \mathbf{e}_{T-h}'$, where $\mathbf{e}_{T-h}$ is a vector of ones of length $T-h$. The *OLS* estimator of the unit specific intercepts (*Ind. OLS*) is given by

$$\hat{\beta}_{OLS,i} = \left(\tilde{\mathbf{X}}_{i,-h}' \tilde{\mathbf{X}}_{i,-h} \right)^{-1} \tilde{\mathbf{X}}_{i,-h}' \tilde{\mathbf{y}}_i. \quad (\text{S4})$$

The predictions labeled by *Ind. OLS* are obtained by replacing (S4) in (S3).

Assume that the coefficients β_i follow a random coefficient model

$$\beta_i = \beta + \delta_i, \quad \delta_i \sim \text{IID}(\mathbf{0}, \Omega_\delta), \quad (\text{S5})$$

where $\beta = (\beta_1, \beta_2, \dots, \beta_{k_x})'$, $\delta_i = (\delta_{i1}, \delta_{i2}, \dots, \delta_{ik_x})'$, δ_i are distributed independently of u_{jt} and $\mathbf{x}_{jt}$, for each $i, j = 1, \dots, n$. Chamberlain (1982) and Pesaran and Smith (1995) suggest estimating the average coefficient vector β by the mean group (*MG*) estimator as

$$\hat{\beta}_{MG} = n^{-1} \sum_{i=1}^n \hat{\beta}_{OLS,i}. \quad (\text{S6})$$

Its asymptotic variance can be estimated by

$$\widehat{\text{Var}}(\hat{\beta}_{MG}) = \frac{1}{n(n-1)} \sum_{i=1}^n \left(\hat{\beta}_i - \hat{\beta}_{MG} \right) \left(\hat{\beta}_i - \hat{\beta}_{MG} \right)'. \quad (\text{S7})$$

Swamy (1970) derives the feasible *GLS* estimator of the average of the slope coefficients β_i under random coefficients assumption. The estimator of β is given by

$$\hat{\beta}_{SW} = \sum_{i=1}^n \mathbf{W}_i \hat{\beta}_{OLS,i}, \quad (\text{S8})$$

with $\mathbf{W}_i = \left[\sum_{j=1}^n \left(\hat{\Omega}_\delta + \hat{\Sigma}_{\hat{\beta}_{OLS,j}} \right)^{-1} \right]^{-1} \left(\hat{\Omega}_\delta + \hat{\Sigma}_{\hat{\beta}_{OLS,i}} \right)^{-1}$ where $\hat{\Sigma}_{\hat{\beta}_{OLS,i}} = \hat{\sigma}_i^2 \left(\tilde{\mathbf{X}}_{i,-h}' \tilde{\mathbf{X}}_{i,-h} \right)^{-1}$, $\hat{\sigma}_i^2 = \frac{1}{T-h} \tilde{\mathbf{e}}_i' \tilde{\mathbf{e}}_i$, $\tilde{\mathbf{e}}_i = \tilde{\mathbf{y}}_i - \tilde{\mathbf{X}}_{i,-h} \hat{\beta}_{OLS,i}$ and $\hat{\Omega}_\delta = n \widehat{\text{Var}}(\hat{\beta}_{MG})$. The variance of this estimator can be estimated

by

$$\widehat{\text{Var}}\left(\widehat{\beta}_{SW}\right) = \left[\sum_{i=1}^n \left(\widehat{\Omega}_\delta + \widehat{\Sigma}_{\widehat{\beta}_{OLS,i}} \right)^{-1} \right]^{-1}. \quad (\text{S9})$$

Conditional on β_i , the *Ind. OLS* estimator is the best linear unbiased predictor (BLUP) of the random coefficient β_i under the assumption of no cross-sectional dependence (CD). Lee and Griffiths (1979) show that, unconditionally, the feasible BLUP of β_i is given by

$$\widehat{\beta}_{SW,i} = \widehat{\beta}_{SW} + \widehat{\Omega}_\delta \widetilde{\mathbf{X}}'_{i,-h} \left(\widehat{\sigma}_i^2 \mathbf{I}_{T-h} + \widetilde{\mathbf{X}}_{i,-h} \widehat{\Omega}_\delta \widetilde{\mathbf{X}}'_{i,-h} \right)^{-1} \left(\widetilde{\mathbf{y}}_i - \widetilde{\mathbf{X}}_{i,-h} \widehat{\beta}_{SW} \right). \quad (\text{S10})$$

The predictions which are called *Ind. GLS* are obtained by replacing (S10) in (S3).

The usual fixed effects (*FE*) estimator of the average slope parameter β which is used in the application is

$$\widehat{\beta}_{FE} = \left(\sum_{i=1}^n \widetilde{\mathbf{X}}'_{i,-h} \widetilde{\mathbf{X}}_{i,-h} \right)^{-1} \sum_{i=1}^n \widetilde{\mathbf{X}}'_{i,-h} \widetilde{\mathbf{y}}_i. \quad (\text{S11})$$

In the case of slope heterogeneity, the variance of this estimator can be estimated by

$$\widehat{\text{Var}}\left(\widehat{\beta}_{FE}\right) = n^{-1} \mathbf{Q}_0^{-1} \mathbf{\Lambda}_0 \mathbf{Q}_0^{-1}, \quad (\text{S12})$$

where

$$\mathbf{Q}_0 = n^{-1} \sum_{i=1}^n \left(\frac{\widetilde{\mathbf{X}}'_{i,-h} \widetilde{\mathbf{X}}_{i,-h}}{T-h} \right), \quad (\text{S13})$$

$$\mathbf{\Lambda}_0 = \sum_{i=1}^n \left(\frac{\widetilde{\mathbf{X}}'_{i,-h} \widetilde{\mathbf{X}}_{i,-h}}{T-h} \right) \widehat{\Omega}_\delta \left(\frac{\widetilde{\mathbf{X}}'_{i,-h} \widetilde{\mathbf{X}}_{i,-h}}{T-h} \right). \quad (\text{S14})$$

The predictions, called *FE*, are obtained by replacing (S11) in (S3).

Under general conditions, the assumption $\gamma_i^{y'} \mathbf{f}_t^y = 0$ and that the explanatory variables are strictly exogenous, $\widehat{\beta}_{OLS,i}$ is consistent for β_i as $T \rightarrow \infty$ and $\widehat{\beta}_{MG}$, $\widehat{\beta}_{SW}$ and $\widehat{\beta}_{FE}$ are consistent for β as $n, T \rightarrow \infty$. If the explanatory variables contain weakly exogenous regressors, Pesaran and Smith (1995) show that $\widehat{\beta}_{FE}$ is inconsistent in the case of heterogeneous slope parameters.

S1.2 Estimators and Predictors with Common Factors

The usual two-way fixed effects (*2WFE*) estimator assumes the existence of common factors, however, it is also assumed that their loadings are homogeneous across panel units. We use the standard formulation for this estimator and its variance. The details can be found, for instance, in Baltagi (2013). To forecast using

the $2WFE$, we use the coefficient on the last time dummy in the sample as the future value. Baltagi (2008) reviews alternative methods to forecast with the $2WFE$ estimator.

The predictions using the estimators which take into account the common factors and their heterogeneous can be written as

$$\hat{y}_{i,T+h|T}^P = \hat{\alpha}_i + \hat{\beta}_i' \mathbf{x}_{iT} + \hat{\gamma}_i^y \hat{\mathbf{f}}_T^y, \quad (\text{S15})$$

where the subscript $P \in \{R, A\}$ signifies one the two prediction approaches, the RBA or the AVA. The intercepts are estimated as before by $\hat{\alpha}_i = \frac{1}{T-h} \sum_{t=h+1}^T (y_{i,t} - \hat{\beta}_i' \mathbf{x}_{i,t-h})$ and in the case of a pooled estimator $\hat{\beta}_i = \hat{\beta}$.

When the unobserved common factors $\mathbf{f}_t^y$ are correlated with the explanatory variables $\mathbf{x}_{it}$ the estimators in the previous subsection turn inconsistent. We use two methods to estimate the unobserved common factors in order to achieve consistency. The first one uses simple cross-sectional averages of some observed variables and the second one uses principal component (PC) methods on observed or estimated variables.

Pesaran (2006) proposes to use the cross-sectional averages of the dependent variable and the independent variables as proxies for the common factors. The estimator is built on two crucial assumptions. First, it is assumed that the explanatory variables have a factor structure given by

$$\mathbf{x}_{it} = \mathbf{a}_i^x + \mathbf{\Gamma}_i^{x'} \mathbf{f}_t + \mathbf{v}_{it}. \quad (\text{S16})$$

Second, the following rank condition¹ on the factor loadings is satisfied

$$\text{rank}(\bar{\mathbf{C}}) = m \leq k_x + 1, \quad (\text{S17})$$

where $\bar{\mathbf{C}} = n^{-1} \sum_{i=1}^n \mathbf{C}_i$ and

$$\mathbf{C}_i = \begin{pmatrix} \gamma_i^y & \mathbf{\Gamma}_i^x \end{pmatrix} \begin{pmatrix} 1 & \mathbf{0} \\ \beta_i & \mathbf{I}_{k_x} \end{pmatrix}.$$

The estimator for the individual parameters, called *Ind. CCE* is given by

$$\hat{\beta}_{CCE,i} = (\mathbf{X}_{i,-h}' \mathbf{M}_{H_1} \mathbf{X}_{i,-h})^{-1} \mathbf{X}_{i,-h}' \mathbf{M}_{H_1} \mathbf{y}_i, \quad (\text{S18})$$

where $\mathbf{M}_{H_1} = \mathbf{I}_{T-h} - \mathbf{H}_1(\mathbf{H}_1' \mathbf{H}_1)^{-1} \mathbf{H}_1'$, $\mathbf{H}_1 = (\mathbf{e}_{T-h}, \bar{\mathbf{Z}})$, $\bar{\mathbf{Z}} = (\bar{\mathbf{z}}_{\cdot,1}', \bar{\mathbf{z}}_{\cdot,2}', \dots, \bar{\mathbf{z}}_{\cdot,T-h}')'$, $\bar{\mathbf{z}}_{\cdot,t}' = n^{-1} \sum_{i=1}^n \mathbf{z}_{it}'$ and $\mathbf{z}_{it} = (y_{it}, \mathbf{x}_{i,t-h}')'$, $t = h+1, \dots, T$.

In general, one can use cross-sectional averages of any variable correlated with the common factors. As an alternative we use the estimator called *Ind. CCEX* given by

$$\hat{\beta}_{CCEX,i} = (\mathbf{X}_{i,-h}' \mathbf{M}_{H_2} \mathbf{X}_{i,-h})^{-1} \mathbf{X}_{i,-h}' \mathbf{M}_{H_2} \mathbf{y}_i, \quad (\text{S19})$$

¹See Karabiyik et al. (2017) for a discussion on this topic.

where $\mathbf{M}_{H_2} = \mathbf{I}_{T-h} - \mathbf{H}_2(\mathbf{H}_2'\mathbf{H}_2)^{-1}\mathbf{H}_2'$, $\mathbf{H}_2 = (\mathbf{e}_{T-h}, \bar{\mathbf{W}})$, $\bar{\mathbf{W}} = (\bar{\mathbf{w}}'_{.1}, \bar{\mathbf{w}}'_{.2}, \dots, \bar{\mathbf{w}}'_{.T-h})'$, $\bar{\mathbf{w}}'_{.t} = n^{-1} \sum_{i=1}^n \mathbf{w}'_{it}$ with $\mathbf{w}_{it}$ being a vector of observed exogenous or weakly exogenous variables. The predictions which are called *Ind. CCE* and *Ind. CCEX* are obtained by replacing (S18) and (S19) in (S15), respectively. In our Monte Carlo simulations we used the right hand side variables and an additional exogenous variable to compute this estimator as described in the paper. In the empirical application this estimator uses all available variables in our data set, except the dependent variable. The advantage of this estimator is that it does not contain endogenous variables, hence, it is expected to perform better in terms of bias in the case of small n .

Chudik and Pesaran (2015) show that, when the model contains weakly exogenous variables the estimator (S18) cannot account for all the dependencies between the unobserved common factors and the right hand side variables. In this case sufficient number of lags of the cross-sectional averages of the dependent and independent variables should be added in the construction of the matrix $\mathbf{H}$. In our Monte Carlo simulations we added one lag of these cross-sectional averages to correct the *Ind. CCE* estimator. When there are numerous explanatory variables in the model, this can cause a serious loss of degrees of freedom. As *Ind. CCEX* already contains the cross-sectional averages of an additional variable, to avoid this problem, we do not correct this estimator.

Song (2013) considers an iterative PC estimator of the individual parameters based on the pooled estimator of Bai (2009). This estimator is defined by the following two nonlinear equations:

$$\hat{\beta}_{IPC,i} = (\mathbf{X}'_{i,-h} \mathbf{M}_{H_3} \mathbf{X}_{i,-h})^{-1} \mathbf{X}'_{i,-h} \mathbf{M}_{H_3} \mathbf{y}_{i.}, \quad (\text{S20})$$

$$\left[\frac{1}{nT} \sum_{i=1}^n (\mathbf{y}_{i.} - \mathbf{X}_{i,-h} \hat{\beta}_{IPC,i}) (\mathbf{y}_{i.} - \mathbf{X}_{i,-h} \hat{\beta}_{IPC,i})' \right] \hat{\mathbf{F}}^y = \hat{\mathbf{F}}^y \hat{\mathbf{V}}_{nT}, \quad (\text{S21})$$

where $\mathbf{M}_{H_3} = \mathbf{I}_{T-h} - \mathbf{H}_3(\mathbf{H}_3'\mathbf{H}_3)^{-1}\mathbf{H}_3'$, $\mathbf{H}_3 = (\mathbf{e}_{T-h}, \hat{\mathbf{F}}^y)$, $\hat{\mathbf{V}}_{nT}$ is a diagonal matrix containing the m_y largest eigenvalues of the matrix in the brackets on the left hand side of the equation and $\hat{\mathbf{F}}^y$ are the corresponding eigenvectors. The rows of these eigenvectors serve as estimates of the common factors $\mathbf{f}_t^{y'}$. To obtain the final estimator of the slope parameters, one can iterate between these two equations until numerical convergence.

The consistency of the PC estimator require stronger conditions than the boundedness of the of the spatial weight matrix $\mathbf{R} = [r_{ij}]$. As shown by Song, the estimator is consistent only if $E(u_{it}u_{js}) = \sigma_{ij,ts}$, $|\sigma_{ij,ts}| \leq \bar{\sigma}_{ij}$ for all (t, s) and $|\sigma_{ij,ts}| \leq \bar{\tau}_{ts}$ for all (i, j) such that $\frac{1}{n} \sum_{i,j=1}^n \bar{\sigma}_{ij} \leq M$, $\frac{1}{T} \sum_{t,s=1}^T \bar{\tau}_{ts} \leq M$, $\frac{1}{nT} \sum_{i,j,t,s=1}^n |\sigma_{ij,ts}| \leq M$. These conditions limit the serial and cross-sectional dependence and heteroskedasticity in the error terms.

A simpler estimator can be obtained by estimating the common factors from the explanatory variables, assuming that they have a common factor structure. Such an estimator is

$$\hat{\beta}_{PCX,i} = (\mathbf{X}'_{i,-h} \mathbf{M}_{H_4} \mathbf{X}_{i,-h})^{-1} \mathbf{X}'_{i,-h} \mathbf{M}_{H_4} \mathbf{y}_i, \quad (\text{S22})$$

where $\mathbf{M}_{H_4} = \mathbf{I}_{T-h} - \mathbf{H}_4(\mathbf{H}'_4 \mathbf{H}_4)^{-1} \mathbf{H}'_4$, $\mathbf{H}_4 = (\mathbf{e}_{T-h}, \hat{\mathbf{F}}^x)$ with $\hat{\mathbf{F}}^x$ being $\sqrt{T-h}$ times the first m_x eigenvectors of the matrix $\sum_{i=1}^n \mathbf{X}_{i,-h} \mathbf{X}'_{i,-h}$. This estimator uses the estimates only the common factors contained in the explanatory variables. As shown by Bai and Ng (2002) the consistent estimation of the common factors using the explanatory variables require weak serial and cross-sectional dependence and heteroskedasticity conditions as above.

In the case that the DGP for the dependent variable contains additional common factors, this estimator does not remove them. Alternatively, one can use

$$\hat{\beta}_{PCX2S,i} = (\mathbf{X}'_{i,-h} \mathbf{M}_{H_5} \mathbf{X}_{i,-h})^{-1} \mathbf{X}'_{i,-h} \mathbf{M}_{H_5} \mathbf{y}_i, \quad (\text{S23})$$

where $\mathbf{M}_{H_5} = \mathbf{I}_{T-h} - \mathbf{H}_5(\mathbf{H}'_5 \mathbf{H}_5)^{-1} \mathbf{H}'_5$, $\mathbf{H}_5 = (\mathbf{e}_{T-h}, \hat{\mathbf{F}}^x, \hat{\mathbf{F}}^u)$ with $\hat{\mathbf{F}}^u$ being $\sqrt{T-h}$ times the first m_y eigenvectors of the matrix $\sum_{i=1}^n (\mathbf{y}_i - \mathbf{X}_{i,-h} \hat{\beta}_{PCX,i}) (\mathbf{y}_i - \mathbf{X}_{i,-h} \hat{\beta}_{PCX,i})'$. The predictions which are called *Ind. IPC*, *Ind. PCX* and *Ind. PCX2S* are obtained by replacing (S20), (S22) and (S23) in (S15), respectively.

For the individual estimators given in (S18), (S19), (S20), (S22), (S23), we can compute the *MG* estimates of the average slope parameter as in (S6) and their variances can be estimated in the same spirit as (S7). For instance, we have

$$\hat{\beta}_{CCEMG} = n^{-1} \sum_{i=1}^n \hat{\beta}_{CCE,i}, \quad (\text{S24})$$

$$\widehat{\text{Var}}(\hat{\beta}_{CCEMG}) = \frac{1}{n(n-1)} \sum_{i=1}^n (\hat{\beta}_{CCE,i} - \hat{\beta}_{CCEMG}) (\hat{\beta}_{CCE,i} - \hat{\beta}_{CCEMG})'. \quad (\text{S25})$$

Another way to estimate the average slope parameter is to use the pooled estimators. The pooled estimator of Bai (2009) is obtained from the following two equations by iteration:

$$\hat{\beta}_{IPCP} = \left(\sum_{i=1}^n \mathbf{X}'_{i,-h} \mathbf{M}_{H_6} \mathbf{X}_{i,-h} \right)^{-1} \sum_{i=1}^n \mathbf{X}'_{i,-h} \mathbf{M}_{H_6} \mathbf{y}_i, \quad (\text{S26})$$

$$\left[\frac{1}{nT} \sum_{i=1}^n (\mathbf{y}_i - \mathbf{X}_{i,-h} \hat{\beta}_{IPCP}) (\mathbf{y}_i - \mathbf{X}_{i,-h} \hat{\beta}_{IPCP})' \right] \tilde{\mathbf{F}}^y = \tilde{\mathbf{F}}^y \tilde{\mathbf{V}}_{nT}, \quad (\text{S27})$$

where $\mathbf{M}_{H_6} = \mathbf{I}_{T-h} - \mathbf{H}_6(\mathbf{H}'_6 \mathbf{H}_6)^{-1} \mathbf{H}'_6$, $\mathbf{H}_6 = (\mathbf{e}_{T-h}, \tilde{\mathbf{F}}^y)$, $\tilde{\mathbf{V}}_{nT}$ is a diagonal matrix containing the m_y largest eigenvalues of the matrix in the brackets and $\tilde{\mathbf{F}}^y$ are the corresponding eigenvectors. The pooled counterparts

of other estimators given in (S18), (S19), (S22), (S23) are obtained in a standard way. For instance, *CCEP* and its heterogeneity-robust variance are computed by

$$\hat{\beta}_{CCEP} = \left(\sum_{i=1}^n \mathbf{X}'_{i,-h} \mathbf{M}_{H_1} \mathbf{X}_{i,-h} \right)^{-1} \sum_{i=1}^n \mathbf{X}'_{i,-h} \mathbf{M}_{H_1} \mathbf{y}_i, \quad (\text{S28})$$

$$\widehat{\text{Var}}(\hat{\beta}_{CCEP}) = n^{-1} \mathbf{Q}_1^{-1} \mathbf{\Lambda}_1 \mathbf{Q}_1^{-1}, \quad (\text{S29})$$

where

$$\mathbf{Q}_1 = n^{-1} \sum_{i=1}^n \left(\frac{\mathbf{X}'_{i,-h} \mathbf{M}_{H_1} \mathbf{X}_{i,-h}}{T-h} \right), \quad (\text{S30})$$

$$\mathbf{\Lambda}_1 = \sum_{i=1}^n \left(\frac{\mathbf{X}'_{i,-h} \mathbf{M}_{H_1} \mathbf{X}_{i,-h}}{T-h} \right) \hat{\mathbf{\Omega}}_{\delta} \left(\frac{\mathbf{X}'_{i,-h} \mathbf{M}_{H_1} \mathbf{X}_{i,-h}}{T-h} \right). \quad (\text{S31})$$

where $\hat{\mathbf{\Omega}}_{\delta} = n^{-1} (\hat{\beta}_{CCE,i} - \hat{\beta}_{CCEMG}) (\hat{\beta}_{CCE,i} - \hat{\beta}_{CCEMG})'$.

When we have a dynamic model, as in the case of individual estimators, we modified the *CCEP* estimator by adding the lags of the related cross-sectional averages in the construction of the matrix $\mathbf{M}_{H_1}$. The estimator *CCEPX* is left unmodified for the same reasons mentioned above in the discussion of the *Ind*. *CCEX* estimator.

S2 Additional Monte Carlo Simulations For Dynamic Models

In the paper the simulation exercise focused on a static DGP for the variable of interest y_{it} . Here, we report results from a robustness check considering a dynamic model for the dependent variable as it is important to explore the consequences of having a lagged dependent variable in a forecasting study. We generated samples from the following DGP:

$$y_{i,t+h} = \alpha_i + \beta_{i1} y_{i,t} + \beta_{i2} x_{it} + \gamma_{i1} f_{1t} + \gamma_{i2} f_{2t} + u_{i,t+h}, \quad (\text{S32})$$

$$x_{it} = a_i + \gamma_{i21} f_{1t} + \gamma_{i23} f_{3t} + v_{it}, \quad (\text{S33})$$

where $i = 1, 2, \dots, n$, $t = 1, 2, \dots, T$, x_{it} is the observed explanatory variable, f_{jt} , $j = 1, 2, 3$, are the unobserved common factors with loadings γ_{ijk} , α_i and a_i are the fixed effects, β_{ij} are the slope coefficients. All variables and parameters except the coefficients β_{i1} are generated as in the paper. In all experiments, the parameter is generated from a uniform distribution with bounded support as $\beta_{i1} \sim \text{IIDU}(0.05, 0.95)$. We report the results for 1-step-ahead forecasts, hence h is set to 1. For the AVA approach we use the additional

variable x_{i3t} as generated in the paper and the estimators *Ind. CCEX* and *CCEPX* use the cross-sectional average of this variable as in the static model.

S2.1 Results

The results on the prediction performance of different estimators combined with the two forecast approaches for the case of a dynamic DGP are given in Tables S1-S8. Just as in the case reported in the paper, the forecast performance of each estimator is superior using the AVA compared to the RBA. Since in the case of a dynamic model the main theoretical expectation is that pooled estimators perform poorly, we focus on the comparison of the individual estimators and the pooled estimators. The results of the paper are confirmed with a few additional interesting results which we summarize as follows.

Table S1 reports the results for the case of low spatial dependence and low factor dependence (Case 1). The results show that, all individual estimators have lower relative RMSEs compared to their pooled counterparts. With a static model in Case 1, it was found that most of the individual estimators outperform the pooled estimators even when $T = 20$, the smallest size considered. The exceptions were *Ind. PCX* and its pooled version. In the case of a dynamic model, when $n, T = 20$ the relative RMSE of *Ind. PCX* is 0.920 and its pooled counterpart has a relative RMSE equal to 0.936 with the AVA approach. This shows that in a dynamic model individual estimator is always better. For other estimators the relative gain from using individual estimators is greater. An interesting finding is that the performance of these two estimators converge when n increases for a fixed T . For instance, when $T = 20$ and $n = 100$ the relative RMSEs for the *Ind. PCX* and its pooled version are 0.904 and 0.907, respectively, in the case of AVA approach. Whereas, as T gets larger the difference is more pronounced.

In the case of a dynamic predictive model, once more the conclusions do not change in the case of higher factor dependence, Case 2, reported in Table S2. Whereas, when the degree of spatial dependence is higher some changes are observed. This is Case 3 reported in Table S3. Again with only one exception, individual estimates outperform the pooled counterparts. The exception is *Ind. IPC* and its pooled version as before. When $n, T = 20$ with a small difference pooled estimator performs better than the individual estimate. Their relative RMSEs are 0.1014 and 0.999, respectively. However, this exception is valid only for the AVA. For the RBA still the individual estimator provides better forecasts than the pooled estimator. With the RBA their relative RMSEs are 1.117 and 1.154, respectively.

Most of these results are confirmed in the case of high heterogeneity (see Tables S5-S8). The only exceptions are *Ind. IPC* and its pooled version once more. To summarize, in the case of a dynamic model

the AVA is still the preferred forecasting approach over the RBA in all cases and individual estimators are preferred to pooled estimators.

References

- Bai, J. (2009). Panel data models with interactive fixed effects. *Econometrica*, 77(4):1229–1279.
- Bai, J. and Ng, S. (2002). Determining the number of factors in approximate factor models. *Econometrica*, 70(1):191–221.
- Baltagi, B. (2013). *Econometric Analysis of Panel Data*. John Wiley & Sons.
- Baltagi, B. H. (2008). Forecasting with panel data. *Journal of Forecasting*, 27(2):153–173.
- Chamberlain, G. (1982). Multivariate regression models for panel data. *Journal of the Econometrics*, 18(1):5–46.
- Chudik, A. and Pesaran, M. H. (2015). Common correlated effects estimation of heterogeneous dynamic panel data models with weakly exogenous regressors. *Journal of Econometrics*, 188(2):393–420.
- Karabiyik, H., Reese, S., and Westerlund, J. (2017). On the role of the rank condition in cce estimation of factor-augmented panel regressions. *Journal of Econometrics*, 197(1):60–64.
- Lee, L.-F. and Griffiths, W. E. (1979). The prior likelihood and best linear unbiased prediction in stochastic coefficient linear models. Center for Economic Research, Department of Economics, University of Minnesota.
- Pesaran, M. H. (2006). Estimation and inference in large heterogeneous panels with a multifactor error structure. *Econometrica*, 74(4):967–1012.
- Pesaran, M. H. and Smith, R. (1995). Estimating long-run relationships from dynamic heterogeneous panels. *Journal of Econometrics*, 68(1):79–113.
- Song, M. (2013). Asymptotic theory for dynamic heterogeneous panels with cross-sectional dependence and its applications. Department of Economics, Columbia University.
- Swamy, P. A. (1970). Efficient inference in a random coefficient regression model. *Econometrica*, 38(2):311–323.

Table S1: Relative RMSE (Dynamic Model) – Low Heterogeneity, Case 1: Low Spatial Dependence & Low Factor Dependence

		Individual								Pooled								
$T \backslash n$		Residual Based Approach				Auxiliary Variables Approach				Residual Based Approach				Auxiliary Variables Approach				
		20	30	50	100	20	30	50	100	20	30	50	100	20	30	50	100	
Ind. CCE										CCEP								
20		1.012	0.992	0.984	0.985	0.824	0.793	0.820	0.747	20	1.146	1.125	1.175	1.112	0.916	0.892	0.971	0.846
30		0.999	0.988	0.989	0.976	0.791	0.793	0.810	0.831	30	1.098	1.168	1.150	1.188	0.863	0.926	0.937	1.001
50		1.002	0.996	0.985	0.978	0.846	0.834	0.773	0.801	50	1.122	1.159	1.150	1.181	0.937	0.960	0.906	0.971
100		0.996	0.993	0.979	0.980	0.799	0.809	0.817	0.809	100	1.103	1.155	1.174	1.190	0.883	0.938	0.977	0.994
Ind. CCEX										CCEPX								
20		1.000	0.988	0.971	0.981	0.815	0.789	0.807	0.744	20	1.145	1.125	1.175	1.112	0.916	0.892	0.971	0.846
30		0.997	0.979	0.980	0.968	0.790	0.786	0.802	0.824	30	1.098	1.168	1.150	1.188	0.863	0.925	0.936	1.001
50		0.991	0.984	0.978	0.969	0.837	0.824	0.766	0.792	50	1.123	1.159	1.150	1.181	0.937	0.959	0.906	0.971
100		0.989	0.981	0.967	0.969	0.794	0.800	0.808	0.799	100	1.103	1.155	1.174	1.190	0.882	0.938	0.977	0.993
Ind. IPC										IPCP								
20		1.003	0.983	0.971	0.981	0.829	0.793	0.810	0.746	20	1.153	1.129	1.174	1.113	0.918	0.893	0.979	0.845
30		0.988	0.975	0.975	0.968	0.789	0.789	0.802	0.826	30	1.103	1.175	1.150	1.191	0.863	0.918	0.940	0.996
50		0.975	0.970	0.972	0.967	0.836	0.820	0.764	0.792	50	1.134	1.164	1.151	1.180	0.938	0.960	0.906	0.975
100		0.964	0.967	0.961	0.967	0.782	0.794	0.805	0.798	100	1.110	1.157	1.178	1.188	0.882	0.939	0.977	0.998
Ind. PCX										PCPX								
20		1.091	1.051	1.011	0.984	0.920	0.872	0.865	0.796	20	1.162	1.134	1.188	1.101	0.936	0.908	0.972	0.859
30		1.071	1.031	1.014	0.985	0.889	0.862	0.864	0.859	30	1.105	1.179	1.153	1.194	0.885	0.934	0.945	0.986
50		1.109	1.051	1.000	0.981	0.950	0.902	0.824	0.839	50	1.167	1.185	1.155	1.186	0.953	0.971	0.917	0.978
100		1.078	1.037	0.996	0.980	0.904	0.881	0.863	0.839	100	1.128	1.174	1.190	1.199	0.907	0.956	0.987	0.996
Ind. PCX2S										PCPX2S								
20		1.047	1.007	0.978	0.974	0.852	0.807	0.816	0.752	20	1.162	1.134	1.188	1.101	0.921	0.893	0.962	0.844
30		1.022	0.990	0.983	0.967	0.814	0.799	0.809	0.824	30	1.105	1.179	1.153	1.194	0.872	0.919	0.933	0.980
50		1.051	1.002	0.978	0.964	0.872	0.837	0.771	0.793	50	1.167	1.185	1.155	1.186	0.940	0.958	0.904	0.966
100		1.019	0.994	0.968	0.967	0.818	0.811	0.810	0.800	100	1.128	1.174	1.190	1.199	0.892	0.940	0.974	0.988

Table S2: Relative RMSE (Dynamic Model) – Low Heterogeneity, Case 2: Low Spatial Dependence & High Factor Dependence

Individual										Pooled								
$T \backslash n$		Residual Based Approach				Auxiliary Variables Approach				Residual Based Approach				Auxiliary Variables Approach				
		20	30	50	100	20	30	50	100	20	30	50	100	20	30	50	100	
Ind. CCE										CCEP								
	20	0.986	0.972	0.975	0.978	0.734	0.734	0.821	0.681	20	1.078	1.064	1.102	1.068	0.788	0.795	0.901	0.746
	30	0.973	0.976	0.980	0.973	0.659	0.731	0.805	0.849	30	1.045	1.092	1.092	1.110	0.709	0.813	0.879	0.941
	50	0.965	0.978	0.979	0.976	0.771	0.802	0.751	0.844	50	1.049	1.089	1.087	1.111	0.824	0.873	0.831	0.934
	100	0.964	0.977	0.974	0.982	0.688	0.760	0.799	0.854	100	1.039	1.088	1.098	1.117	0.742	0.837	0.886	0.951
Ind. CCEX										CCEPX								
	20	0.963	0.963	0.949	0.971	0.721	0.727	0.803	0.675	20	1.079	1.064	1.102	1.068	0.788	0.795	0.901	0.746
	30	0.970	0.958	0.961	0.955	0.656	0.719	0.794	0.841	30	1.045	1.093	1.092	1.110	0.709	0.812	0.879	0.941
	50	0.943	0.955	0.963	0.953	0.759	0.787	0.740	0.832	50	1.049	1.090	1.087	1.111	0.824	0.873	0.831	0.934
	100	0.950	0.954	0.948	0.956	0.679	0.746	0.786	0.839	100	1.039	1.088	1.098	1.117	0.742	0.836	0.886	0.951
Ind. IPC										IPCP								
	20	0.974	0.963	0.948	0.971	0.737	0.735	0.809	0.677	20	1.081	1.066	1.095	1.069	0.791	0.797	0.910	0.748
	30	0.961	0.957	0.955	0.953	0.659	0.726	0.795	0.843	30	1.051	1.105	1.086	1.114	0.708	0.808	0.885	0.943
	50	0.941	0.944	0.955	0.950	0.763	0.787	0.738	0.832	50	1.046	1.089	1.087	1.107	0.829	0.877	0.833	0.942
	100	0.928	0.940	0.942	0.953	0.673	0.743	0.785	0.839	100	1.041	1.086	1.099	1.111	0.743	0.839	0.888	0.958
Ind. PCX										PCPX								
	20	1.158	1.104	1.030	1.006	0.886	0.848	0.865	0.751	20	1.101	1.077	1.110	1.060	0.819	0.817	0.905	0.765
	30	1.145	1.087	1.043	1.003	0.833	0.836	0.865	0.875	30	1.057	1.120	1.096	1.123	0.744	0.829	0.888	0.931
	50	1.182	1.098	1.025	0.996	0.922	0.890	0.811	0.869	50	1.084	1.113	1.092	1.120	0.848	0.888	0.844	0.938
	100	1.161	1.084	1.029	0.995	0.861	0.860	0.857	0.865	100	1.058	1.105	1.119	1.126	0.776	0.858	0.900	0.950
Ind. PCX2S										PCPX2S								
	20	1.024	0.990	0.956	0.965	0.756	0.742	0.804	0.680	20	1.101	1.077	1.110	1.060	0.790	0.792	0.894	0.742
	30	1.001	0.974	0.965	0.953	0.682	0.729	0.794	0.835	30	1.057	1.120	1.096	1.123	0.711	0.803	0.875	0.924
	50	1.014	0.976	0.963	0.947	0.786	0.794	0.741	0.825	50	1.084	1.113	1.092	1.120	0.824	0.869	0.827	0.929
	100	0.986	0.969	0.950	0.953	0.702	0.754	0.784	0.833	100	1.058	1.105	1.119	1.126	0.745	0.835	0.881	0.946

Table S3: Relative RMSE (Dynamic Model) – Low Heterogeneity, Case 3: High Spatial Dependence & Low Factor Dependence

		Individual										Pooled							
$T \backslash n$		Residual Based Approach				Auxiliary Variables Approach						Residual Based Approach				Auxiliary Variables Approach			
		20	30	50	100	20	30	50	100			20	30	50	100	20	30	50	100
Ind. CCE										CCEP									
	20	1.033	1.018	1.013	1.006	0.941	0.911	0.923	0.860	20	1.118	1.115	1.148	1.102	0.983	0.964	1.017	0.921	
	30	1.018	1.008	1.012	1.003	0.923	0.907	0.925	0.916	30	1.078	1.136	1.125	1.166	0.951	0.984	0.999	1.033	
	50	1.032	1.021	1.006	1.000	0.947	0.942	0.886	0.901	50	1.099	1.130	1.123	1.147	0.986	1.009	0.967	1.015	
	100	1.025	1.014	0.996	0.994	0.919	0.917	0.906	0.896	100	1.074	1.120	1.134	1.145	0.948	0.991	1.009	1.021	
Ind. CCEX										CCEPX									
	20	1.064	1.041	1.013	1.009	0.964	0.926	0.923	0.861	20	1.120	1.116	1.150	1.102	0.982	0.963	1.016	0.920	
	30	1.042	1.016	1.015	1.003	0.939	0.912	0.927	0.915	30	1.079	1.137	1.126	1.166	0.951	0.983	0.999	1.032	
	50	1.038	1.023	1.008	0.998	0.953	0.943	0.887	0.898	50	1.100	1.130	1.124	1.147	0.986	1.009	0.966	1.014	
	100	1.028	1.012	0.992	0.990	0.922	0.916	0.903	0.892	100	1.075	1.121	1.135	1.145	0.948	0.991	1.009	1.021	
Ind. IPC										IPCP									
	20	0.991	0.989	0.990	0.997	0.925	0.900	0.910	0.855	20	1.123	1.112	1.145	1.101	0.988	0.968	1.025	0.924	
	30	0.987	0.983	0.988	0.992	0.904	0.892	0.913	0.908	30	1.078	1.142	1.120	1.167	0.953	0.981	1.006	1.033	
	50	0.981	0.984	0.988	0.986	0.922	0.921	0.875	0.891	50	1.110	1.132	1.124	1.145	0.988	1.013	0.968	1.020	
	100	0.985	0.986	0.979	0.984	0.894	0.897	0.892	0.888	100	1.085	1.123	1.138	1.145	0.949	0.991	1.008	1.023	
Ind. PCX										PCPX									
	20	1.117	1.082	1.041	1.013	1.014	0.969	0.949	0.889	20	1.154	1.134	1.174	1.101	0.999	0.976	1.017	0.929	
	30	1.082	1.053	1.037	1.013	0.990	0.953	0.954	0.930	30	1.090	1.154	1.139	1.177	0.967	0.991	1.004	1.021	
	50	1.106	1.061	1.019	1.001	1.014	0.979	0.915	0.918	50	1.142	1.156	1.131	1.154	0.999	1.017	0.975	1.017	
	100	1.072	1.041	1.007	0.994	0.978	0.957	0.930	0.911	100	1.099	1.137	1.147	1.153	0.966	1.003	1.015	1.023	
Ind. PCX2S										PCPX2S									
	20	1.062	1.031	1.009	0.999	0.957	0.920	0.918	0.861	20	1.154	1.134	1.174	1.101	0.985	0.964	1.011	0.919	
	30	1.033	1.014	1.007	0.996	0.935	0.909	0.922	0.908	30	1.090	1.154	1.139	1.177	0.956	0.980	0.998	1.017	
	50	1.071	1.029	1.000	0.987	0.967	0.941	0.884	0.893	50	1.142	1.156	1.131	1.154	0.991	1.009	0.967	1.011	
	100	1.045	1.018	0.990	0.985	0.934	0.919	0.900	0.890	100	1.099	1.137	1.147	1.153	0.959	0.995	1.008	1.018	

Table S4: Relative RMSE (Dynamic Model) – Low Heterogeneity, Case 4: High Spatial Dependence & High Factor Dependence

		Individual								Pooled								
$T \backslash n$		Residual Based Approach				Auxiliary Variables Approach				Residual Based Approach				Auxiliary Variables Approach				
		20	30	50	100	20	30	50	100	20	30	50	100	20	30	50	100	
Ind. CCE										CCEP								
20		1.010	0.995	0.994	0.993	0.846	0.829	0.888	0.777	20	1.081	1.074	1.099	1.069	0.879	0.872	0.950	0.826
30		0.993	0.990	0.997	0.990	0.790	0.820	0.888	0.902	30	1.046	1.086	1.090	1.109	0.816	0.881	0.943	0.979
50		0.992	0.999	0.992	0.988	0.857	0.886	0.834	0.900	50	1.050	1.086	1.080	1.103	0.886	0.936	0.894	0.975
100		0.992	0.994	0.983	0.986	0.797	0.846	0.863	0.899	100	1.036	1.082	1.090	1.104	0.825	0.903	0.933	0.980
Ind. CCEX										CCEPX								
20		1.020	1.007	0.981	0.992	0.854	0.836	0.881	0.775	20	1.082	1.074	1.099	1.069	0.879	0.871	0.950	0.826
30		1.011	0.987	0.991	0.981	0.799	0.818	0.885	0.898	30	1.046	1.087	1.090	1.108	0.816	0.880	0.943	0.979
50		0.986	0.991	0.986	0.975	0.855	0.881	0.829	0.893	50	1.051	1.087	1.081	1.103	0.886	0.936	0.894	0.974
100		0.988	0.983	0.969	0.971	0.795	0.840	0.856	0.890	100	1.037	1.082	1.090	1.105	0.824	0.903	0.933	0.980
Ind. IPC										IPCP								
20		0.975	0.971	0.963	0.984	0.834	0.820	0.873	0.772	20	1.087	1.074	1.093	1.070	0.882	0.874	0.956	0.828
30		0.966	0.967	0.968	0.974	0.777	0.810	0.876	0.896	30	1.049	1.097	1.086	1.113	0.817	0.877	0.948	0.979
50		0.952	0.961	0.971	0.968	0.839	0.868	0.821	0.889	50	1.053	1.089	1.082	1.100	0.889	0.938	0.895	0.980
100		0.950	0.962	0.959	0.967	0.778	0.829	0.851	0.889	100	1.041	1.083	1.093	1.101	0.824	0.904	0.932	0.984
Ind. PCX										PCPX								
20		1.155	1.107	1.037	1.011	0.955	0.912	0.919	0.825	20	1.108	1.087	1.111	1.064	0.900	0.886	0.952	0.839
30		1.123	1.078	1.039	1.008	0.912	0.897	0.925	0.917	30	1.056	1.113	1.096	1.122	0.840	0.891	0.947	0.968
50		1.146	1.079	1.019	0.994	0.964	0.942	0.871	0.911	50	1.085	1.107	1.085	1.109	0.903	0.943	0.902	0.974
100		1.114	1.062	1.014	0.989	0.913	0.911	0.899	0.903	100	1.054	1.095	1.105	1.112	0.848	0.916	0.940	0.977
Ind. PCX2S										PCPX2S								
20		1.043	1.012	0.981	0.982	0.860	0.835	0.875	0.776	20	1.108	1.087	1.111	1.064	0.880	0.869	0.945	0.822
30		1.016	0.993	0.986	0.976	0.806	0.820	0.879	0.891	30	1.056	1.113	1.096	1.122	0.818	0.873	0.940	0.964
50		1.033	1.001	0.981	0.967	0.868	0.880	0.826	0.885	50	1.085	1.107	1.085	1.109	0.888	0.933	0.892	0.969
100		1.008	0.991	0.967	0.967	0.808	0.841	0.851	0.884	100	1.054	1.095	1.105	1.112	0.829	0.902	0.929	0.976

Table S5: Relative RMSE (Dynamic Model) – High Heterogeneity, Case 1: Low Spatial Dependence & Low Factor Dependence

		Individual								Pooled								
$T \backslash n$		Residual Based Approach				Auxiliary Variables Approach				Residual Based Approach				Auxiliary Variables Approach				
		20	30	50	100	20	30	50	100	20	30	50	100	20	30	50	100	
Ind. CCE										CCEP								
	20	1.013	0.992	0.984	0.985	0.831	0.804	0.832	0.755	20	1.183	1.167	1.220	1.155	0.964	0.948	1.028	0.898
	30	0.999	0.988	0.990	0.976	0.796	0.799	0.819	0.842	30	1.142	1.211	1.194	1.235	0.914	0.977	0.991	1.059
	50	1.003	0.996	0.985	0.978	0.856	0.845	0.783	0.815	50	1.166	1.201	1.194	1.227	0.991	1.012	0.960	1.031
	100	0.996	0.993	0.979	0.980	0.806	0.821	0.829	0.826	100	1.148	1.199	1.220	1.235	0.937	0.995	1.036	1.055
Ind. CCEX										CCEPX								
	20	1.000	0.988	0.971	0.981	0.822	0.799	0.818	0.751	20	1.183	1.168	1.220	1.155	0.964	0.948	1.027	0.897
	30	0.997	0.979	0.980	0.968	0.795	0.791	0.811	0.835	30	1.142	1.211	1.194	1.235	0.913	0.976	0.991	1.059
	50	0.992	0.984	0.978	0.969	0.847	0.834	0.776	0.807	50	1.166	1.201	1.195	1.228	0.991	1.012	0.960	1.031
	100	0.989	0.981	0.967	0.969	0.801	0.812	0.820	0.816	100	1.148	1.199	1.220	1.235	0.937	0.995	1.036	1.055
Ind. IPC										IPCP								
	20	1.004	0.985	0.974	0.981	0.834	0.803	0.822	0.753	20	1.191	1.171	1.220	1.158	0.966	0.949	1.037	0.899
	30	0.988	0.977	0.975	0.969	0.793	0.795	0.812	0.838	30	1.148	1.220	1.195	1.239	0.914	0.970	0.996	1.056
	50	0.974	0.970	0.972	0.967	0.844	0.830	0.774	0.807	50	1.179	1.206	1.196	1.227	0.992	1.013	0.961	1.037
	100	0.964	0.967	0.962	0.967	0.790	0.806	0.817	0.816	100	1.156	1.202	1.224	1.234	0.936	0.996	1.037	1.061
Ind. PCX										PCPX								
	20	1.090	1.051	1.012	0.984	0.925	0.880	0.874	0.802	20	1.201	1.178	1.234	1.146	0.983	0.961	1.026	0.909
	30	1.071	1.031	1.014	0.986	0.894	0.867	0.871	0.869	30	1.150	1.224	1.199	1.243	0.935	0.984	0.998	1.042
	50	1.109	1.051	0.999	0.981	0.956	0.910	0.832	0.850	50	1.212	1.230	1.201	1.235	1.005	1.022	0.970	1.036
	100	1.078	1.037	0.996	0.981	0.909	0.890	0.873	0.852	100	1.176	1.222	1.239	1.246	0.960	1.011	1.044	1.055
Ind. PCX2S										PCPX2S								
	20	1.047	1.008	0.978	0.973	0.858	0.816	0.826	0.758	20	1.201	1.178	1.234	1.146	0.969	0.947	1.018	0.895
	30	1.021	0.990	0.983	0.969	0.818	0.804	0.817	0.835	30	1.150	1.224	1.199	1.243	0.922	0.970	0.987	1.037
	50	1.051	1.002	0.978	0.964	0.880	0.847	0.781	0.807	50	1.212	1.230	1.201	1.235	0.994	1.010	0.957	1.025
	100	1.019	0.995	0.968	0.968	0.825	0.823	0.822	0.816	100	1.176	1.222	1.239	1.246	0.946	0.997	1.032	1.049

Table S6: Relative RMSE (Dynamic Model) – High Heterogeneity, Case 2: Low Spatial Dependence & High Factor Dependence

		Individual										Pooled							
$T \backslash n$		Residual Based Approach				Auxiliary Variables Approach						Residual Based Approach				Auxiliary Variables Approach			
		20	30	50	100	20	30	50	100			20	30	50	100	20	30	50	100
Ind. CCE										CCEP									
	20	0.986	0.971	0.974	0.978	0.746	0.750	0.834	0.704	20	1.096	1.084	1.123	1.089	0.821	0.835	0.937	0.793	
	30	0.973	0.976	0.980	0.972	0.670	0.743	0.821	0.862	30	1.065	1.112	1.112	1.131	0.746	0.849	0.919	0.978	
	50	0.965	0.978	0.979	0.976	0.786	0.819	0.769	0.863	50	1.069	1.108	1.108	1.133	0.861	0.912	0.871	0.976	
	100	0.964	0.977	0.973	0.982	0.702	0.778	0.817	0.871	100	1.060	1.109	1.119	1.138	0.780	0.878	0.929	0.990	
Ind. CCEX										CCEPX									
	20	0.963	0.963	0.949	0.971	0.733	0.743	0.817	0.698	20	1.096	1.084	1.123	1.088	0.821	0.834	0.937	0.793	
	30	0.969	0.958	0.961	0.955	0.667	0.731	0.810	0.854	30	1.065	1.112	1.112	1.131	0.745	0.848	0.918	0.977	
	50	0.942	0.955	0.963	0.953	0.774	0.804	0.757	0.851	50	1.070	1.109	1.108	1.133	0.861	0.912	0.871	0.976	
	100	0.950	0.954	0.948	0.955	0.692	0.764	0.805	0.856	100	1.060	1.109	1.120	1.138	0.780	0.878	0.929	0.990	
Ind. IPC										IPCP									
	20	0.976	0.964	0.949	0.970	0.751	0.751	0.823	0.700	20	1.098	1.085	1.116	1.091	0.825	0.837	0.947	0.796	
	30	0.961	0.956	0.954	0.953	0.670	0.738	0.811	0.857	30	1.072	1.123	1.107	1.136	0.745	0.845	0.925	0.981	
	50	0.941	0.944	0.955	0.950	0.777	0.804	0.756	0.851	50	1.067	1.108	1.108	1.128	0.866	0.916	0.874	0.985	
	100	0.928	0.941	0.942	0.953	0.686	0.761	0.804	0.856	100	1.061	1.106	1.120	1.133	0.781	0.881	0.931	0.998	
Ind. PCX										PCPX									
	20	1.160	1.103	1.031	1.005	0.895	0.858	0.876	0.769	20	1.119	1.097	1.133	1.081	0.849	0.852	0.939	0.809	
	30	1.144	1.087	1.043	1.004	0.841	0.844	0.876	0.886	30	1.078	1.140	1.118	1.145	0.778	0.863	0.925	0.966	
	50	1.182	1.098	1.025	0.997	0.931	0.903	0.824	0.884	50	1.107	1.134	1.113	1.141	0.882	0.924	0.881	0.978	
	100	1.159	1.085	1.029	0.996	0.870	0.873	0.870	0.879	100	1.080	1.128	1.142	1.147	0.812	0.896	0.939	0.987	
Ind. PCX2S										PCPX2S									
	20	1.025	0.991	0.957	0.964	0.767	0.756	0.817	0.701	20	1.119	1.097	1.133	1.081	0.822	0.831	0.931	0.788	
	30	1.001	0.973	0.966	0.954	0.692	0.740	0.809	0.848	30	1.078	1.140	1.118	1.145	0.748	0.839	0.914	0.961	
	50	1.015	0.977	0.963	0.948	0.799	0.811	0.757	0.844	50	1.107	1.134	1.113	1.141	0.861	0.908	0.867	0.971	
	100	0.987	0.970	0.950	0.953	0.714	0.771	0.801	0.850	100	1.080	1.128	1.142	1.147	0.783	0.876	0.924	0.985	

Table S7: Relative RMSE (Dynamic Model) – High Heterogeneity, Case 3: High Spatial Dependence & Low Factor Dependence

Individual										Pooled								
$T \backslash n$		Residual Based Approach				Auxiliary Variables Approach				Residual Based Approach				Auxiliary Variables Approach				
		20	30	50	100	20	30	50	100	20	30	50	100	20	30	50	100	
Ind. CCE										CCEP								
	20	1.033	1.018	1.013	1.006	0.945	0.917	0.928	0.865	20	1.140	1.141	1.176	1.128	1.010	0.997	1.050	0.952
	30	1.018	1.008	1.012	1.003	0.926	0.910	0.930	0.923	30	1.103	1.162	1.151	1.196	0.979	1.014	1.031	1.069
	50	1.032	1.021	1.006	1.000	0.953	0.947	0.892	0.909	50	1.127	1.155	1.151	1.176	1.021	1.040	0.999	1.051
	100	1.025	1.014	0.996	0.994	0.923	0.924	0.913	0.905	100	1.103	1.149	1.164	1.174	0.981	1.027	1.045	1.059
Ind. CCEX										CCEPX								
	20	1.064	1.041	1.013	1.009	0.967	0.932	0.927	0.866	20	1.142	1.142	1.177	1.128	1.010	0.997	1.049	0.952
	30	1.042	1.016	1.015	1.002	0.942	0.915	0.933	0.921	30	1.104	1.162	1.151	1.196	0.979	1.013	1.031	1.069
	50	1.038	1.023	1.008	0.998	0.959	0.948	0.893	0.907	50	1.128	1.156	1.151	1.176	1.020	1.039	0.999	1.051
	100	1.028	1.012	0.992	0.990	0.926	0.924	0.910	0.901	100	1.103	1.149	1.164	1.174	0.980	1.026	1.045	1.058
Ind. IPC										IPCP								
	20	0.992	0.989	0.991	0.997	0.928	0.905	0.915	0.860	20	1.146	1.137	1.174	1.129	1.015	1.001	1.058	0.956
	30	0.987	0.984	0.988	0.993	0.906	0.895	0.919	0.915	30	1.104	1.168	1.146	1.197	0.982	1.012	1.039	1.071
	50	0.982	0.984	0.988	0.986	0.927	0.926	0.881	0.900	50	1.141	1.157	1.151	1.175	1.021	1.044	1.001	1.058
	100	0.985	0.986	0.979	0.984	0.898	0.905	0.899	0.897	100	1.115	1.153	1.168	1.174	0.981	1.027	1.045	1.061
Ind. PCX										PCPX								
	20	1.115	1.084	1.041	1.012	1.015	0.973	0.953	0.893	20	1.176	1.161	1.203	1.128	1.025	1.007	1.048	0.959
	30	1.081	1.052	1.037	1.013	0.992	0.955	0.958	0.935	30	1.117	1.182	1.167	1.207	0.995	1.020	1.035	1.056
	50	1.106	1.062	1.019	1.001	1.018	0.982	0.919	0.925	50	1.172	1.183	1.159	1.184	1.031	1.046	1.006	1.052
	100	1.072	1.042	1.007	0.994	0.981	0.963	0.936	0.918	100	1.131	1.168	1.179	1.183	0.998	1.037	1.050	1.059
Ind. PCX2S										PCPX2S								
	20	1.061	1.033	1.009	0.999	0.959	0.926	0.922	0.865	20	1.176	1.161	1.203	1.128	1.012	0.997	1.043	0.950
	30	1.032	1.013	1.006	0.998	0.937	0.911	0.927	0.915	30	1.117	1.182	1.167	1.207	0.985	1.010	1.029	1.054
	50	1.071	1.029	1.000	0.987	0.972	0.946	0.890	0.901	50	1.172	1.183	1.159	1.184	1.025	1.039	0.999	1.047
	100	1.044	1.018	0.990	0.985	0.937	0.925	0.907	0.898	100	1.131	1.168	1.179	1.183	0.992	1.030	1.044	1.055

Table S8: Relative RMSE (Dynamic Model) – High Heterogeneity, Case 4: High Spatial Dependence & High Factor Dependence

Individual										Pooled								
$T \backslash n$		Residual Based Approach				Auxiliary Variables Approach				Residual Based Approach				Auxiliary Variables Approach				
		20	30	50	100	20	30	50	100	20	30	50	100	20	30	50	100	
Ind. CCE										CCEP								
	20	1.010	0.994	0.994	0.993	0.852	0.842	0.897	0.796	20	1.094	1.089	1.115	1.085	0.900	0.902	0.976	0.862
	30	0.993	0.990	0.997	0.990	0.797	0.830	0.895	0.914	30	1.061	1.101	1.105	1.126	0.841	0.909	0.967	1.009
	50	0.992	0.999	0.992	0.987	0.866	0.894	0.845	0.912	50	1.067	1.101	1.097	1.119	0.913	0.960	0.923	1.004
	100	0.992	0.994	0.982	0.985	0.806	0.859	0.875	0.910	100	1.052	1.098	1.106	1.121	0.852	0.933	0.963	1.009
Ind. CCEX										CCEPX								
	20	1.019	1.007	0.981	0.992	0.860	0.849	0.889	0.794	20	1.095	1.089	1.115	1.085	0.900	0.902	0.975	0.861
	30	1.010	0.987	0.991	0.981	0.806	0.827	0.892	0.911	30	1.061	1.102	1.105	1.125	0.841	0.908	0.967	1.009
	50	0.986	0.991	0.985	0.975	0.865	0.889	0.841	0.906	50	1.067	1.101	1.097	1.119	0.913	0.960	0.923	1.004
	100	0.988	0.983	0.969	0.971	0.804	0.852	0.869	0.901	100	1.052	1.098	1.107	1.121	0.852	0.932	0.963	1.009
Ind. IPC										IPCP								
	20	0.976	0.972	0.964	0.983	0.842	0.833	0.882	0.790	20	1.099	1.089	1.109	1.086	0.903	0.904	0.982	0.864
	30	0.966	0.967	0.968	0.975	0.784	0.818	0.883	0.908	30	1.064	1.112	1.101	1.130	0.842	0.904	0.972	1.010
	50	0.952	0.961	0.971	0.968	0.849	0.876	0.833	0.901	50	1.069	1.103	1.098	1.116	0.916	0.962	0.924	1.010
	100	0.950	0.962	0.959	0.967	0.787	0.841	0.864	0.900	100	1.057	1.099	1.109	1.117	0.852	0.934	0.963	1.013
Ind. PCX										PCPX								
	20	1.154	1.106	1.038	1.010	0.958	0.921	0.925	0.839	20	1.121	1.103	1.128	1.080	0.919	0.914	0.976	0.872
	30	1.122	1.077	1.039	1.010	0.917	0.903	0.930	0.926	30	1.072	1.128	1.113	1.140	0.864	0.916	0.970	0.997
	50	1.146	1.080	1.018	0.994	0.970	0.948	0.880	0.921	50	1.104	1.124	1.102	1.126	0.928	0.966	0.929	1.003
	100	1.113	1.062	1.014	0.990	0.919	0.920	0.908	0.913	100	1.071	1.113	1.123	1.129	0.874	0.943	0.968	1.005
Ind. PCX2S										PCPX2S								
	20	1.043	1.012	0.982	0.981	0.865	0.847	0.883	0.793	20	1.121	1.103	1.128	1.080	0.901	0.899	0.970	0.858
	30	1.016	0.993	0.985	0.976	0.812	0.828	0.885	0.902	30	1.072	1.128	1.113	1.140	0.843	0.900	0.964	0.994
	50	1.033	1.000	0.981	0.967	0.877	0.888	0.837	0.897	50	1.104	1.124	1.102	1.126	0.915	0.956	0.920	1.000
	100	1.008	0.991	0.968	0.967	0.816	0.853	0.863	0.895	100	1.071	1.113	1.123	1.129	0.856	0.931	0.959	1.004