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Complex system approach to language games

A. Baronchelli[†], E. Caglioti[‡], V. Loreto[†] and L. Steels[#]

[†] Physics Dept. and INFN-SMC, Università *La Sapienza*, P.le A. Moro 2, 00185 Roma, ITALY

[‡] Mathematics Dept., Università *La Sapienza*, P.le A. Moro 2, 00185 Roma, ITALY

[#] SONY CLS, 6 rue Amiot, 75005 Paris, FRANCE

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Abstract:

The mechanisms leading language conventions to be socially accepted and adopted by a group are object of an intense debate. The issue can be of course addressed by different points of view, and recently also complex system science has started to contribute, mainly by means of computer simulations and analytical approaches. In this paper we study a very simple multi agent model of convention spreading and investigate some of the crucial aspects of its dynamics, resorting, whenever possible, to quantitative analytic methods. In particular, the model is able to account for the emergence of global consensus out of local (pairwise) interactions. In this regard, a key question concerns the role of the size of the population. We investigate in detail how the cognitive efforts of the agents in terms of memory and the convergence time scale with the number of agents. We also point out the existence of an hidden timescale ruling a fundamental aspect of the dynamics, and we discuss the nature of the convergence process.

Corresponding Author:

Vittorio Loreto

Physics Dept. and INFN-SMC, Università *La Sapienza*, P.le A. Moro 2, 00185 Roma, ITALY

Tel: +39 06 4991 3437, e-mail: Vittorio.Loreto@roma1.infn.it

1 Introduction

How do linguistic conventions come to be accepted at a population level? What is the role of the population size as for the time needed to reach the global agreement? How and why does a specific word manage to impose itself defeating all the competitors? These are only few of the topic questions that one faces when dealing with the problem of the development and evolution of language. And they can be of course addressed by many different points of views, ranging from psychology to sociology, from evolutionary biology to artificial intelligence. Recently, however, there has been a growing effort to tackle them resorting to agent based models and mathematical approaches (cfr [1] for a review). The general issue has been often restricted, in this context, to that of the emergence of a shared vocabulary, and also in this paper we shall be focused on this problem.

In particular, we are interested here in the dynamics leading conventions to global sharing in all those cases in which semantics does not play any specific role. A real world example of such situations is given by the phenomenon of e-mail spamming. When it was at its beginning, many names had spontaneously emerged to indicate undesired mails, like, for instance, "junk mail" or "spam". After a while, however, and without any apparent reason, "spam" became universally accepted and now there is not actual competition and almost everybody talks about "spam" while other synonyms have almost been forgotten.

The more general issue of the emergence of a common vocabulary has been addressed, in the context of complex systems science, resorting to different models. Among these, a first distinction concerns evolutionary and cultural explanations. The evolutionary approach [2], to which belongs also the well known evolutionary language game [3], is based on the assumption that successful communicators, enjoying a selective advantage, are more likely to reproduce than worse communicators. Moreover, since communication strategies are innate, if one of them is better than the others then in an evolutionary time-span it will displace all the rivals, becoming the unique strategy of the population. The term strategy acquires a precise meaning in the context of each particular model. For instance, it can be a strategy for acquiring the lexicon of a language [2, 4, 5], or it can simply coincide with the lexicon of the parents [3], but other possibilities exist [1].

In this paper we discuss a model, first proposed in [6], that belongs to the cultural family [7, 8, 9]. Here, good strategies do not provide higher reproductive success, but only better communication abilities. Agents can then select better strategies exploiting cultural choice and direct feedback in communications. Moreover, innovations can be introduced due to the inventing ability of the agents. Thus, global coordination emerges over cultural timescales, and language is seen as an evolving and self-organized system [10]. Said in different words, while in the evolutionary approach cultural

transmission follows a vertical, 'genetic', line, in the cultural one it is horizontal and involves peer individuals [11].

A further distinction among different models concerns the adopted mechanisms of social learning describing how stable dispositions are transmitted among individuals [12]. The two main approaches are the so called observational learning model and the operant conditioning model [13]. In the first, often associated with the evolutionary approach [2, 4, 5, 3], observation is the main ingredient of learning and statistical sampling of observed behaviors determines their acquisition. The second emphasizes the inferential nature of communication, in which the stimulus and the response to a stimulus play a central role. In our work, we adopt the operant conditioning approach, as in [7, 8, 9], according to which language learning, aiming at communicating meaning, is mainly functional.

In this paper we shall discuss a recently introduced model [6], inspired by the well known Naming Game [8], able to account for the emergence of a shared set of conventions in a population of agents. Individuals can only perform pairwise interactions, and no central control is needed for the emergence of a globally accepted common lexicon.

A crucial aspect of the proposed model is its simplicity. Usually, in fact, when defining a multi-agent model, the choice is between endowing agents with simple properties, so that one can hope to fully understand what happens in simulations, or with more complicated and realistic structures that yet risk to confuse experiments outputs. We choose to follow the first possibility since we are more interested in the global behavior of the population. In this perspective we do not seek answers to specific issues in the development of language, but rather we aim at analyzing deeply basic models that can constitute valuable starting points for more sophisticated investigations. Nevertheless, as we shall see, also extremely transparent agents and interaction rules can give rise to very complex and rich global behaviors and the study of simple models can help to shed light on general properties - a well known lesson in statistical physics. Finally, it is worth stressing that, very often, cultural frameworks lack of quantitative investigations, contrarily to what happens in the evolutionary approaches. In this paper, for instance, we shall discuss in great details how the main features of the process leading the population to a final convergence state scale with the population size, while in general other models concentrate on studying very small populations, even composed of only two players [14].

The paper is organized as follows. In section 2 we introduce the model and discuss the basic features of its dynamics. In section 3 we investigate how the main features of the process leading to convergence scale with the populations size. Section 4 is devoted to the analysis of an hidden timescale governing the transition between the initial phase with almost no communication to the final state of nearly perfect agreement. In section 5 we show that convergence always takes place. Finally conclusions are drawn in sec-

tion 6.

2 The model

Let us consider a population of N agents which perform pairwise games in order to agree on the name to assign to a single object. Each agent is characterized by its inventory or memory, i.e. a list of name-object associations that is empty at the beginning of the process and evolves dynamically during time. At each time step, two agents are randomly selected, one to play as speaker, the other one as hearer, and interact according to the following rules:

- The speaker has to transmit a name to the hearer. If its inventory is empty, it invents a new word, otherwise it selects *randomly* one of the names it knows;
- If the hearer has the uttered name in its inventory, the game is a success, and both agents delete all their words but the winning one;
- If the hearer does not know the uttered word, the game is a failure, and the hearer inserts the word in its inventory, i.e. it learns the new name.

A remark is now in order. The presence of a single object follows by the rather strong assumption of preventing homonymy, but does not imply any restriction on the environment in which the agents are ideally or (eventually physically) placed. In fact, once it is forbidden that the same name refers to more than one object, all different objects become independent, and one can work with only one of them without any loss of generality. Moreover, while the absence of homonymy allows for a strong reduction in the complexity of the model, it does not seem so drastic when thinking of artificial agents that assign randomly extracted real numbers to new objects.

It is also interesting to note that the problem of homonymy has been studied in great detail in the context of evolutionary game theory and [15] have shown that languages with homonymy are evolutionary unstable. However it is obvious that homonymy is an essential aspect of human languages, while synonymy seems less relevant. The two authors solve this apparent paradox noting that if we think of "words in a context" homonymy almost disappears while synonymy acquires a much grater role. This observation fits very well also with our inferential model of learning according to which we assume that agents are placed in a common environment and they are able to point referents. So, after a failure, the speaker is able to point the named object (or referent) to the hearer which in its turn can assign the new name to it.

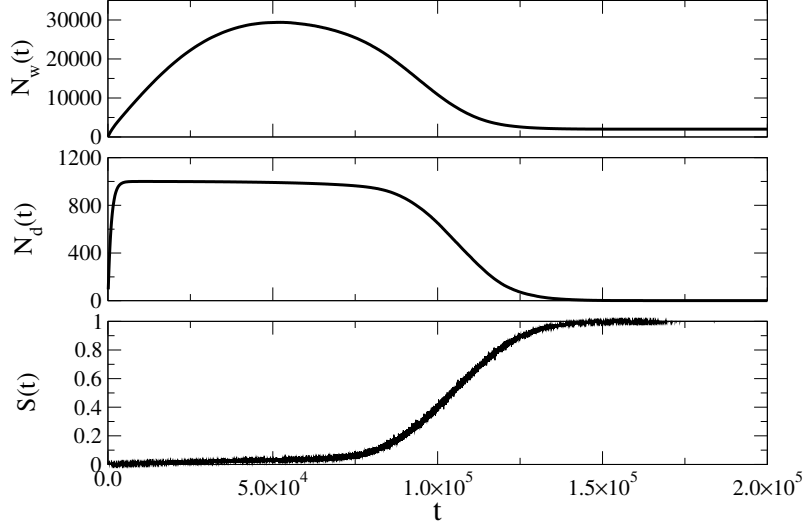


Figure 1: **Basic global dynamics:** The total number of words in the system, $N_w(t)$ (or total memory used), the number of different words, $N_d(t)$, and the success rate, $S(t)$, are plotted as a function of time. The final convergence state is characterized by the presence of the same unique word in the inventories of agents. Thus, at the end of the process, we have $N_w(t) = N$ and $N_d(t) = 1$, while the probability of a success is equal to 1 ($S(t) = 1$). The curves have been obtained averaging over 300 simulation runs for a population of 2×10^3 agents.

2.1 Convergence dynamics

To understand the behavior of the system, we report in Figure 1 three curves obtained averaging several runs of the process in a population of $N = 2 \times 10^3$ agents. They represent the evolution in time of the total number of words present in the system $N_w(t)$, which quantifies the total amount of memory used by the process, of the number of different words, $N_d(t)$, and of the success rate, $S(t)$, defined as the probability of a successful interaction between two agents at time t . The first thing to be noted is that the system reaches a final convergence state in which all agents have the same unique word, i.e. a final proto-communication system has been established. It is thus interesting to proceed with a more detailed analysis of how this final state of global communication emerges from purely binary interactions.

The process starts with a trivial phase in which the inventories are empty, so that the invention process is dominating and $N/2$ different words are created on average. This rapid transient is followed by a longer period of time in which most interactions are unsuccessful ($S(t) \simeq 0$), and the sizes of inventories keep growing. However the amount of memory used does not increase indefinitely, since correlations are progressively built up

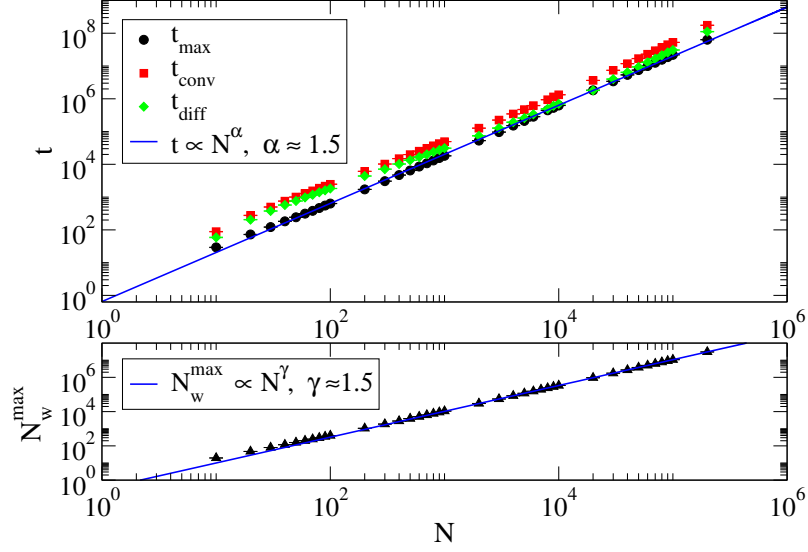


Figure 2: **Scaling with the population size N :** In the upper graph the scaling of the peak and convergence time, t_{max} and t_{conv} , is reported, along with their difference, t_{diff} . All curves scale with the power law $N^{1.5}$. The lower curve shows that the maximum number of words (peak height, $N_w^{max} = N_w(t_{max})$) obeys the same power law scaling.

among inventories and increase the probability of successful interactions. In particular, the $N_w(t)$ curve exhibits a well identified peak, whose height and occurrence time are important parameters to describe the process. Slightly after the peak, there is a quite abrupt transition from a disordered state in which communication among agents is difficult to a nearly optimal situation, which is captured by a jump of the success rate curve. The process then ends when the convergence state ($N_d(t) = 1$, $N_w(t) = N$) is reached. Finally, it is worth noting that the developed proto-communication system is not only effective (each agent understands all the others), but also efficient (no memory is wasted in the final state).

3 The role of the system size - scaling relations

Now that we have a qualitative picture of the dynamics leading the system to convergence, it is natural investigating the role played by the system size N . In particular, two fundamental aspects depend on N . The first is the time needed by the population to reach the final state, that we shall call the convergence time t_{conv} . The second concerns the cognitive effort in terms of memory required to each agent by the dynamics. This reaches its maximum in correspondence of the peak of the $N_w(t)$ curve. In Figure 2 it is shown

the scaling behavior of the convergence time t_{conv} , and the time and height of the peak of $N_w(t)$, namely t_{max} and $N_w^{max} \doteq N_w(t_{max})$. The difference time $(t_{conv} - t_{max})$ is also plotted. It turns out that all these quantities follow power law behaviors: $t_{max} \sim N^\alpha$, $t_{conv} \sim N^\beta$, $N_w^{max} \sim N^\gamma$ and $t_{diff} = (t_{conv} - t_{max}) \sim N^\delta$, with exponents $\alpha \approx \beta \approx \gamma \approx \delta \approx 1.5$.

The values for α and γ can be understood through simple analytical arguments. Indeed, assume that, when the total number of words is close to the maximum, each agent has on average cN^a words, so that it holds $\alpha = a + 1$. If we assume also that the distribution of different words in agents inventories is uniform, we have that the probability for the speaker to play a given word is $1/(cN^a)$, while the probability that the hearer knows that word is $2cN^a/N$ (where $N/2$ is the number of different words present in the system). The equation for the evolution of the number of words then reads:

$$\frac{dN_w(t)}{dt} \propto \frac{1}{cN^a} \left(1 - \frac{2cN^a}{N}\right) - \frac{1}{cN^a} \frac{2cN^a}{N} 2cN^a \quad (1)$$

where the first term is related to unsuccessful interactions (which increase N_w by one unit), while the second one to successful ones (which decrease N_w by $2cN^a$). At the maximum $\frac{dN_w(t_{max})}{dt} = 0$, so that, in the thermodynamic limit $N \rightarrow \infty$, the only possible value for the exponent is $a = 1/2$ which implies $\alpha = 3/2$ in perfect agreement with data from simulations.

For the exponent γ the procedure is analogous, but we have to use the linear behavior of the success rate and the relation $a = 1/2$ we have just obtained. The equation for $N_w(t)$ now can be written as:

$$\frac{dN_w(t)}{dt} \propto \frac{1}{cN^{1/2}} \left(1 - \frac{ct}{N^2}\right) - \frac{1}{cN^{1/2}} \frac{ct}{N^2} 2cN^{1/2} . \quad (2)$$

If we impose $\frac{dN_w(t)}{dt} = 0$, we find that the time of the maximum has to scale with the right exponent $\gamma = 3/2$ in the thermodynamic limit.

Before concluding this section, it is worth stressing that the possibility of resorting both to the massive numerical simulations needed to recover the right exponents and to the analytical results of eq. (1) and (2), is a direct consequence of the simplicity of the interaction rules defined in our model. It is also one of the most important contributions of our work, since, as we mentioned above, this kind of analysis was, as far as we know, completely lacking from models which adopt our point of view to investigate the spreading of conventions.

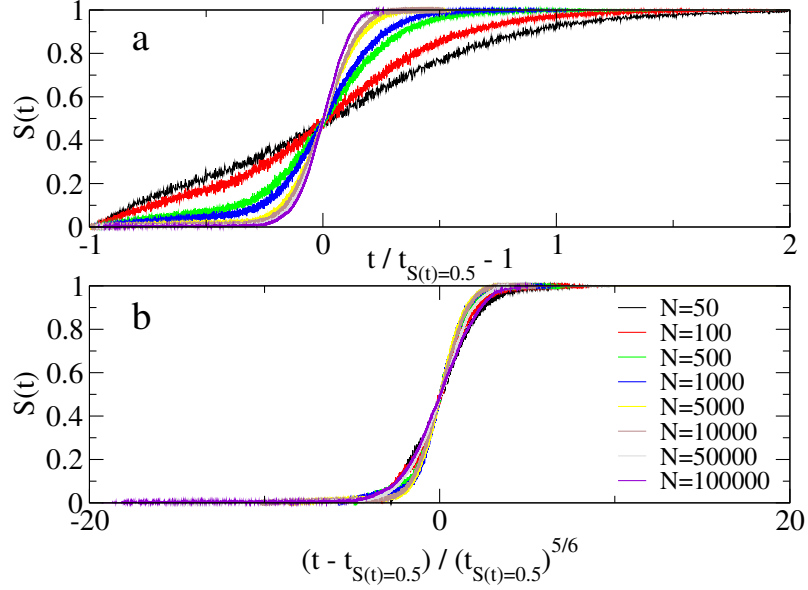


Figure 3: **Hidden timescale in the disorder/order transition.** a)(Top) The success rate, $S(t)$, curves do not collapse when the time is rescaled as $t \rightarrow t/t_{S(t)=0.5} - 1$, where $t_{S(t)=0.5}$ is the time in which the considered curve reaches the value 0.5. In particular, curves relative to larger populations present a faster jump towards the perfect communication regime ($S(t) = 1$). b)(Bottom) This is due to the fact that the transition happens on a timescale $O(N^{5/4})$. In Figure, a time rescaling of the form $t \rightarrow (t - t_{S(t)=0.5}) / t_{S(t)=0.5}^{5/6}$ [equivalent to $t \rightarrow (t - N^{3/2}) / N^{5/4}$] gives rise to a good collapse of the different curves.

4 The disorder/order transition

Now that we know that the characteristic time required by the system to reach convergence scales as $N^{1.5}$ we would expect a transformation of the form $t \rightarrow t/N^{3/2}$ to make collapse the curves of the total quantities, such as $S(t)$ or $N_w(t)$, relative to systems of different sizes. However, this is not true, as it is clear from Figure 3a, where the time has been rescaled following a transformation of the time $t \rightarrow t/t_{S(t)=0.5} - 1$, where $t_{S(t)=0.5} \sim N^{1.5}$ is the time in which the considered curve reaches the value 0.5¹ In particular, the rescaled curves become steeper and steeper as the population size grows. This means that the disorder/order transition from the initial situation in which almost no communication exists ($S(t) \simeq 0$) and the final one in which most interactions are successful ($S(t) \simeq 1$) happens on a different timescale,

¹This choice has been made in order to assure that all the different curves cross at the origin of the axis of the rescaled curve, thus preventing them from being slightly scattered on the graph.

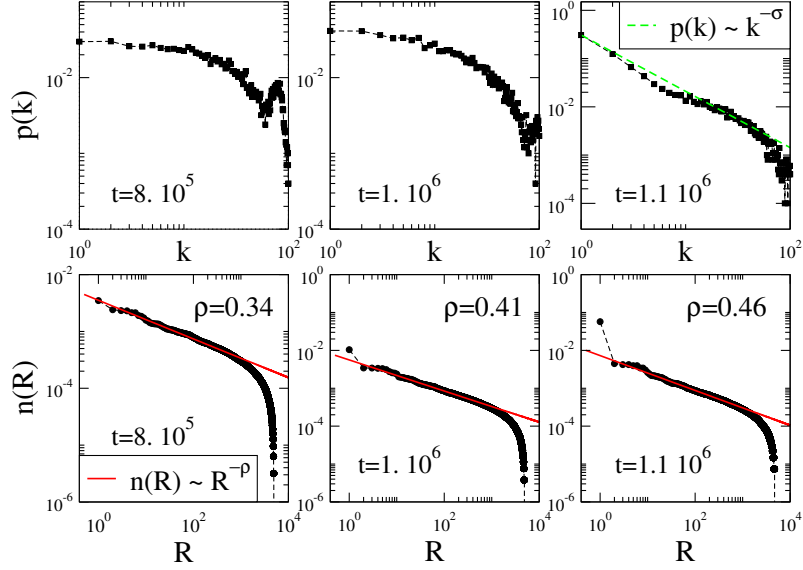


Figure 4: **Microscopic analysis:** Top) Distribution $p(k)$ of inventory sizes k . Close to convergence the distribution is well described by a power law $p(k) \sim k^{-7/6}$. Bottom) Distribution $n(R)$ of words of rank R . The most popular word has rank $R = 1$, the second $R = 2$, etc. The distribution follows a power law behavior $n(R) \sim N^{-\rho}$ with an exponent that varies in time. Close to the disorder/order transition the most popular word breaks the symmetry and leaves the power law distribution, which, however, continues to describe well the behavior of less common words. In both cases data refer to a population size of $N = 10^4$ agents, for which $t_{max} = 6.2 \times 10^5$ and $t_{conv} = 1.3 \times 10^6$ time-steps.

which disappears as the population size diverges ($N \rightarrow \infty$).

Indeed, the hidden timescale governing the transition turns out to be $O(N^{5/4})$, as it is clear from Figure 3b where different curves collapse as the time has been rescaled with a transformation equivalent to $t \rightarrow \frac{t - N^{3/2}}{N^{5/4}}$.

To understand the origin of this new temporal scale, we must look closer at the process leading to convergence. In particular it is important investigating the microscopic properties of both the agents and the words. First of all it is relevant looking at the distribution, $p(k)$, of the agents inventory sizes k . In particular, as it is shown in Figure 4, just before the transition this distribution follows a power law behavior, so that we can write:

$$p(k) \sim k^{-\sigma} f(k/\sqrt{N}) \quad (3)$$

where $f(x) = 1$ for $x \ll 1$ and $f(x) = 0$ for $x \gg 1$. Numerically, it turns

out that $\sigma \approx 7/6$, as it is shown in Figure 4, and this value is irrespective of the system size (data not shown).

The process leading the population to agree on the same unique convention consists in negotiations among the agents and competition among the words. Indeed, only one convention will survive in the final state, and an interesting question concerns the mechanisms determining its victory. At each time step one can ask which is the most popular word, the second most common and so on. In Figure 4 it is plotted the rank distribution, $n(R)$, describing which is the fraction of agents possessing each word of rank R . We find that also in this case the histograms are well fitted by a power law distribution. Interestingly, however, close to the transition the most popular word breaks the symmetry, abandons the distribution which continue to describe well the remaining words, and prepares itself to become dominating. Thus, we can write:

$$n(R) = n(1)\delta_{R,1} + \frac{N_w/N - n(1)}{(1 - \rho)((N/2)^{1-\rho} - 2^{1-\rho})} R^{-\rho} f\left(\frac{R}{N/2}\right), \quad (4)$$

where δ is the Kronecker delta function ($\delta_{a,b} = 1$ iff $a = b$ and $\delta_{a,b} = 0$ if $a \neq b$) and the normalization factors are derived imposing that $\int_1^\infty n(R)dR = N_w/N^2$.

On the other hand from equation (3) one gets, by a simple integration, the relation $N_w/N \sim N^{1-\sigma/2}$ which, substituted into eq. (4) gives

$$n(R)|_{R>1} \sim \frac{1}{N^{\sigma/2-\rho}} R^{-\rho} f\left(\frac{R}{N/2}\right). \quad (5)$$

It follows that $n(R)|_{R>1} \rightarrow 0$ as $N \rightarrow \infty$, so that, in the thermodynamic limit $n(1)$, i.e. the number of players with the most popular word, is a finite fraction of the whole population.

Now, in order to understand better what happens at the transition, we can profitably map the agents in the nodes of a network. Then, we connect two agents each time that they have a word in common, so that multiple links are allowed. In the network, a word is represented by a fully connected subgraph, i.e. by a clique, and the final coherent state corresponds to a fully connected network with all pairs connected by a only one link. When two players interact, a failure determines the propagation of a word, while a success can result in the elimination of a certain number of words competing with the one used. In network view, this translates into a clique that grows when one of its nodes is represented by a speaker that takes part in a failure,

²We use integrals instead of discrete sums, an approximation valid in the limit of large systems.

and is diminished when one (or two) of its nodes are involved in a successful interaction with a competing word.

If we make the hypothesis that, when N is large, just before the transition all the agents have the word that will dominate, the problem of convergence reduces to the study of the rate at which competing words disappear. Said in different words, the crucial information is how the number of deleted links in the network, M_d , scales with N . It holds:

$$M_d = \frac{N_w}{N} \int_2^\infty n^2(R) N dR \sim N^{3-\frac{3}{2}\sigma} \quad (6)$$

where $\frac{N_w}{N}$ is the average number of words known by each agents (i.e. the average number of cliques entered by a node), $n(R)$ is the probability of having a word of rank R (i.e. the probability for a given clique to be involved in the deletion process), and $n(R)N$ is the number of agents that have that word (i.e. the size of the subgraph corresponding to the word). The integration starts from the second most popular word that is first one that can be eliminated according to our assumption.

Inserting the value for σ obtained from simulations, $\sigma \simeq 7/6$, we obtain that $M_d \sim N^{5/4}$. Thus, as we expected, we find that, for large systems, $M_d/N^{3/2} \rightarrow 0$, and this explains the greater slope, on the system timescale $N^{3/2}$, of the success rate curves for large populations (Figure 3).

5 Convergence properties

So far we have looked at all the timescales involved in the process leading the population to the final agreement state. Yet, we have not investigated *whether* this convergence state is always reached. Actually, this is the case, and trivial considerations allow to clarify this point. First of all, it must be noticed that, according to the interaction rules of the agents, the agreement condition constitutes the only possible absorbing state of our model. The proof that convergence is always reached is then straightforward. Indeed, from any possible state there is always a non-zero probability to reach an absorbing state in, for instance, $2(N-1)$ interactions. For example, a possible sequence is as follows. A given agent speaks twice with all the other $(N-1)$ agents using always the same word A . After these $2(N-1)$ interactions all the agents have only the word A . Denoting with p the probability of the sequence of $2(N-1)$ steps, the probability that the system has not reached an absorbing state after $2(N-1)$ iterations is smaller or equal to $(1-p)$. Therefore, iterating this procedure, the probability that, starting from any state, the system has not reached an absorbing state after $2k(N-1)$ iterations, is smaller than $(1-p)^k$ which vanishes exponentially with k .

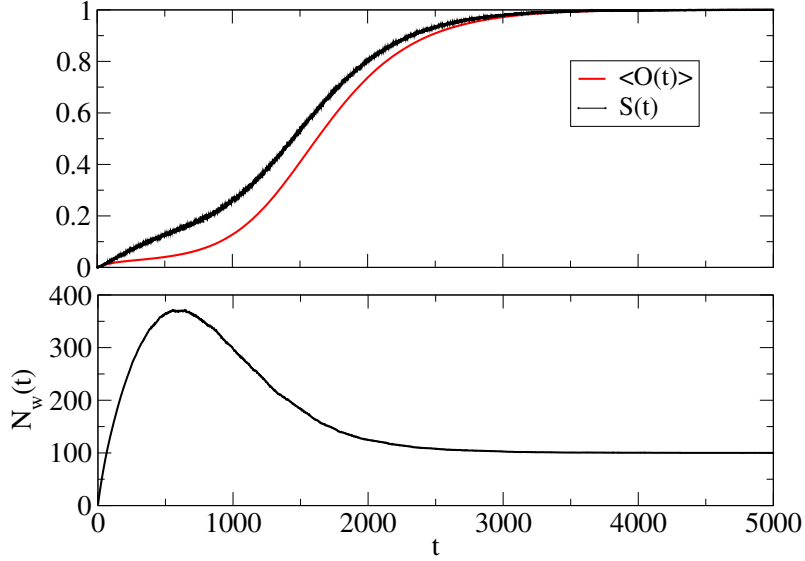


Figure 5: **Overlap functional $O(t)$.** In Figure it is shown the evolution in time of the overlap functional averaged on different runs (for a population of $N = 10^3$ agents). Curves for the success rate $S(t)$ and total number of words $N_w(t)$ are included for reference. From simulations we have that holds $\langle O(t+1) \rangle > \langle O(t) \rangle$, which, along with the stronger $\langle O(t+1) \rangle > O(t)$ valid for almost all configurations, indicate that the system will reach the final state of convergence where $O(t) = 1$.

Another perspective to address the problem of convergence consists in monitoring the lexical coherence of the system. To this purpose, we introduce the overlap functional O :

$$O = \frac{2}{N(N-1)} \sum_{i>j} \frac{|a_i \cap a_j|}{k_i k_j} \quad (7)$$

where a_i is the i^{th} agent's inventory, whose size is k_i , and $|a_i \cap a_j|$ is the number of words in common between a_i and a_j . The overlap functional is a measure of the lexical coherence in the system and it is bounded, $O(t) \leq 1$. At the beginning of the process it is equal to zero, $O(t=0) = 0$, while at convergence it reaches its maximum, $O(t=t_{conv}) = 1$.

From simulations it turns out that, averaged over several runs, the functional always grows, i.e. $\langle O(t+1) \rangle > \langle O(t) \rangle$ (see Figure 5). Moreover, looking at the single realization, this function grows almost always, i.e. $\langle O(t+1) \rangle > O(t)$. The monotonicity of the overlap, combined with the fact that it is bounded to be not larger than 1, strongly suggests that the system will converge. Contrarily to the previous absorbing-state argument,

this is not a rigorous proof of the convergence, since we lack of a rigorous proof of what we observe, nonetheless the overlap approach allows to gain a deeper insight into the way in which the final state is reached.

6 Conclusion

In this paper we have discussed a very simple model of convention spreading. Out of transparent pairwise interactions, a population is able to converge on the name to assign to an object. We have then investigated, both with simulations and analytical approaches, the role of the system size, N , on the convergence dynamics. In particular we have focused on this scaling with N of the amount memory required to single agents and the time needed to reach the final agreement state. Having identified the system timescale, which is of order $O(N^{3/2})$, we have also identified a different timescale, of order $O(N^{5/4})$, on which the transition between the initial situation (in which there is approximately no communication) and the final one (in which there is an almost perfect agreement) takes place. Finally, we have shown, both with a rigorous simple proof and with a more intuitive argument, that the convergence state is always reached by the system.

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