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Multi-Variable Optimisation of Wet Vapour Organic Rankine Cycles with Twin-Screw Expanders

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ABSTRACT

A multi-variable optimization program has been developed to investigate the performance of Wet Organic Rankine Cycles for low temperature heat recovery applications. This cycle model contains a detailed thermodynamic model of the twin-screw expander, and the methods used to match the operation of the expander to the requirements of the cycle are described. The capability of the cycle model has been demonstrated for the case of heat recovery from a source of pressurized hot water at 120°C. There are two main findings from the paper. Firstly, power output can be increased by up to 50% by correct selection of the expander inlet and discharge conditions, compared to the case with dry vapour admission. Secondly, the maximum power output occurs at an expander inlet dryness of around 20%, but for lower dryness fractions the gain in net power output falls off due to the decreasing pressure ratio required to maintain good expander efficiency.

1. INTRODUCTION

The Organic Rankine Cycle (ORC) provides a means of recovering useful energy from low temperature heat sources. In comparison with conventional high temperature steam Rankine cycles, the low temperature of these heat sources means that the attainable cycle efficiency is much lower, while the required surface area of the heat exchangers per unit power output is much higher. The lower latent heat of evaporation of organic fluids relative to steam also means that the feed pump work required in ORCs is a significantly higher proportion of the gross power output.

Especially at lower source temperatures, up to approximately 120°C inlet, the only cycle normally considered is that where the working fluid enters the expander as dry saturated vapour, as shown in Figure 1. However, in most cases, this leads to the working fluid leaving the expander with some superheat, which must be removed before condensation begins.

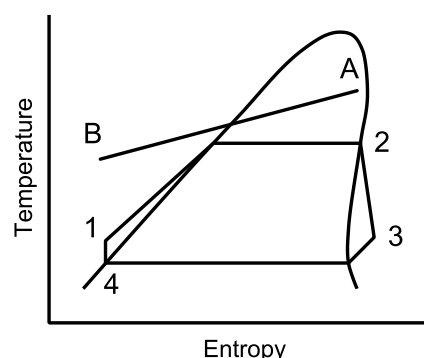


Figure 1: Illustrative T - s diagram showing conventional ORC with dry saturated vapour at the expander inlet

Maximising net power output is a compromise between increasing the mean temperature of heat addition (which, in accordance with Carnot's principle, can increase cycle efficiency) and increasing the amount of heat extracted from the source, which requires a lower evaporation temperature.

By the use of a screw expander, instead of the more conventional turbine, it is possible to admit the working fluid to the expander as wet vapour and thereby eliminate both the need to desuperheat the vapour after expansion and, simultaneously to raise the evaporation temperature, as shown in Figure 2, thus improving the cycle efficiency. The potential cost and performance benefits of using screw expanders in ORC systems have been extensively studied for geothermal applications (Smith et al. 2001, 2004, 2005).

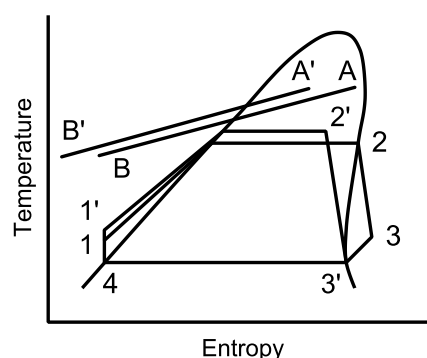


Figure 2: Illustrative T - s diagram showing how the need to cool superheated vapour in a conventional ORC system can be avoided by expanding wet vapour

However, screw expander efficiencies are more sensitive to expansion pressure ratio than turbines and the expansion ratio increases as the expander inlet vapour dryness fraction decreases. To determine the value of inlet dryness fraction that leads to the maximum system power output, it is therefore necessary to include estimates of how both the screw expander and the feed pump performance vary as the inlet dryness fraction of the working fluid is changed in such a wet ORC (WORC) system.

A further consideration that must be included, is the pump or fan power required to drive the coolant across the condenser coils. This increases with the coolant flow rate, but a higher coolant flow rate, permits a lower condensing temperatures for the same condenser pinch point temperature difference and, hence, a higher cycle efficiency and gross power output. The best combination of these conflicting factors to obtain the condensing temperature that yields the maximum net power output needs also to be estimated.

In reality there is some deviation from the idealised Rankine cycles shown in Figure 2 due to pressure drops in the heat exchangers and the requirement for sub-cooling of the condensed working fluid to avoid cavitation in the feed pump. While these effects do not generally have a large influence on the performance of individual components, the sensitivity of the cycle performance to the operating conditions means that they should also be considered in the analysis of low temperature heat recovery systems.

A further consideration is the size of the heat exchangers, which will depend on the heat transfer rate, the heat transfer coefficients of the fluids and the temperature difference between the fluids. This affects the overall system cost and therefore consideration must be given to it, overall evaluation of the system choice.

In practical applications of low temperature heat recovery there is usually a minimum allowable discharge temperature for the source fluid. Examples include engine jacket cooling, where excessive cooling can cause thermal deformation of the engine block, geothermal power generation, where solid precipitates can form and heat recovery from exhaust gasses, where corrosive condensates can form. If required, this lower limit must be included as the cut-off point in evaluating the whole system.

The potential of the WORC as a cost effective system for power recovery from low temperature heat sources was investigated by Leibowitz et al. (2006), but only limited cycle optimisation was performed using a simple expander

model with constant efficiency. The overall requirement for evaluating the potential of WORC systems is therefore for a computer program which simulates the performance of each of the components and then applies a multi variable optimisation procedure.

1.1 Screw Expanders for ORC Systems

The efficiency of the twin-screw expander depends on a range of factors including the expander dimensions, rotor profiles, inlet and discharge pressures, inlet dryness fraction, built-in volume ratio and rotational speed. All of these variables must also be chosen in to match the mass flow rate of the machine to that required in the cycle. It is therefore essential that the ORC system analysis should capture the operating characteristics of the expander to allow optimisation for a particular application; this can only be achieved through numerical modelling of the complete system using detailed thermodynamic models for key components.

2. ANALYSIS OF WET ORGANIC RANKINE CYCLES

A computational ORC model has been developed using a well-established quasi one dimensional model of twin-screw machines and with thermo-fluid properties obtained from the 'Reference Fluid Thermodynamic and Transport Properties Database' (REFPROP) program produced by the National Institute of Standards and Technology (NIST). Other cycle components such as the feed pump and motor have been characterised using performance data from manufacturers, and pressure loss in heat exchanger components is estimated. Multi-variable optimisation is implemented using an evolutionary algorithm; by defining a target function the optimum value of all the cycle variables identified. This optimisation program has been used to identify the cycle conditions that result in maximum net power output for a number of specific applications. This paper illustrates how the optimum cycle conditions can be identified for a given temperature and flow rate of the heat source fluid, and the effect that dryness fraction at the expander inlet has on the net power output, expander operation and required heat exchanger surface areas for the ORC system.

2.1 Thermodynamic Cycle Model

The performance of ORC systems has been assessed using a computational model of the cycle. This has been written as an object-oriented program in the C# language, which provides a convenient structure as it allows a generic description of heat sources, heat sinks and cycle components. Each of these cycle elements contains definitions for all the necessary input and output parameters along with the required calculations. Both simple cycles such as those shown in Figure 2, and more complex cases (including multiple heat source streams, multiple paths for the working fluid or varying working fluid composition) can be analysed by creating models of the required components and providing the necessary input parameter values.

The current study investigates a WORC system as illustrated in Figure 2. Simplified heat exchanger models have been used with specified pressure loss factors. The minimum allowable temperature differences in the boiler and condenser heat exchangers have also been specified as an input to the model. The efficiency of the feed pump has been characterised as a function of volumetric flow rate using data from manufacturers.

2.2 Twin-Screw Expander Model

A full thermodynamic model of the expander has been created for integration with the cycle model. This is based on the quasi one dimensional analysis of twin-screw machines as described by Stosic and Hanjalic (1997a, 1997b), which has been extensively validated for compressors (and, to a lesser extent, expanders (Smith et al., 1996)) for a wide range of working fluids and operating conditions. Using this procedure, machine geometry and rotor profiles have been optimised for a particular set of operating conditions representative of those considered in this paper, and have been fixed for the purposes of the current study. The City University 'N' rotor profile described in Figure 3 has been used in the current analysis as this geometry is known to be superior to other well-known types.

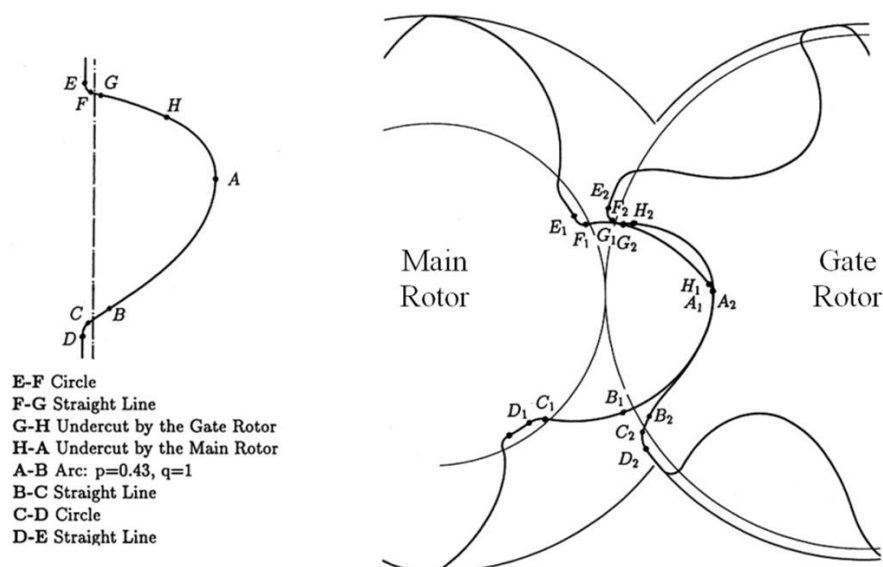


Figure 3: Description of the City University ‘N’ rotor profile for positive displacement screw machines

In principle, the screw machine geometry optimisation could be integrated with the cycle analysis described here to ensure the best profile is used for the required operating conditions, but this would be very computationally intensive and is not expected to significantly affect results. Furthermore, from a manufacturing perspective it would be prohibitively expensive to produce an optimised machine for every different application. For a specified geometry, the characteristics of the twin-screw machine such as the curve of working chamber volume against angular position, sealing line lengths, blowhole area and axial/radial clearances between the rotors and the casing are defined as fixed inputs for the expander model. The variable input parameters required for the expander model within the cycle analysis program are then limited to the main rotor speed, built-in volume ratio, inlet dryness fraction, and inlet and discharge pressures. In order to integrate this expander model with the main cycle model, two approaches can be taken to match the machine operation to the cycle conditions:

- i. The built-in volume ratio, $R_{v,bi}$, is specified and iterations are performed to find the rotor speed required to match the mass flow rate of expander to that of the working fluid calculated in the cycle model – no limits are imposed on rotor speed, which in some cases can become impractically high.
- ii. The rotor speed is fixed and iterations are performed to find the value of $R_{v,bi}$ required to match the mass flow rate – if $R_{v,bi}$ is greater than the limit for the chosen screw machine geometry then the expander cannot meet the requirements of the cycle conditions, and the expander efficiency is set to zero.

The expander efficiency calculated using either of these approaches is used in the thermodynamic cycle model to calculate overall cycle performance for specific operating conditions. An iterative numerical procedure can then be used to identify the optimum operating conditions for the cycle.

2.3 Optimisation Procedure

An evolutionary algorithm has been used to identify the optimum operating conditions for the cycle model. This is a flexible and stable numerical approach which allows for optimisation with any number of variables and is particularly good for distinguishing global from local maxima and coping with discontinuities in the target function. A population of solutions is defined in which each individual solution has a unique ‘gene’ consisting of a ‘chromosome’ for each of the cycle optimisation variables under consideration (e.g. boiler pressure, condenser pressure, dryness fraction at expander inlet and $R_{v,bi}$). The values of the chromosomes are initially randomly generated, and a function (in this study, the net power output of the cycle) is defined in order to calculate the ‘fitness’ of a particular solution. Over successive generations of the calculation procedure, ‘fitter’ genes are used to create new solutions through both combination and random mutation of the chromosomes. After a specified number of generations, the single best solution is identified as the optimum.

3. RESULTS OF CASE STUDY

In order to demonstrate the cycle analysis described in Section 2, a simple case study has been performed for the recovery of heat from a pressurised hot water source fluid. This liquid stream has an inlet temperature of 120°C and contains a recoverable heat content of 2.6MW if cooled to an ambient temperature of 15°C. No constraints have been applied to the outlet temperature in order to identify the maximum possible net power output from this source.

This study has investigated the generation of power from this heat source using a WORC with the following characteristics:

- The working fluid is refrigerant R245fa.
- The expander has a main rotor diameter of 204mm, and a maximum possible built-in volume ratio of 4.5.
- A water cooled condenser is used with 2°C of sub-cooling at the exit.
- Minimum pinch point temperature differences of 10°C have been applied for the boiler and condenser.
- An efficiency of 95% has been assumed for the electrical generator and 90% for pump motors.
- Expander inlet dryness fractions between 0.1 and 0.9 have been considered.

The key results from this study are the effect of expander inlet dryness fraction on the power inputs required at the feed and condenser pumps, the expander power and the net power output from the cycle. These values are shown in Figure 4, where dryness fraction is plotted on a logarithmic scale for clarity.

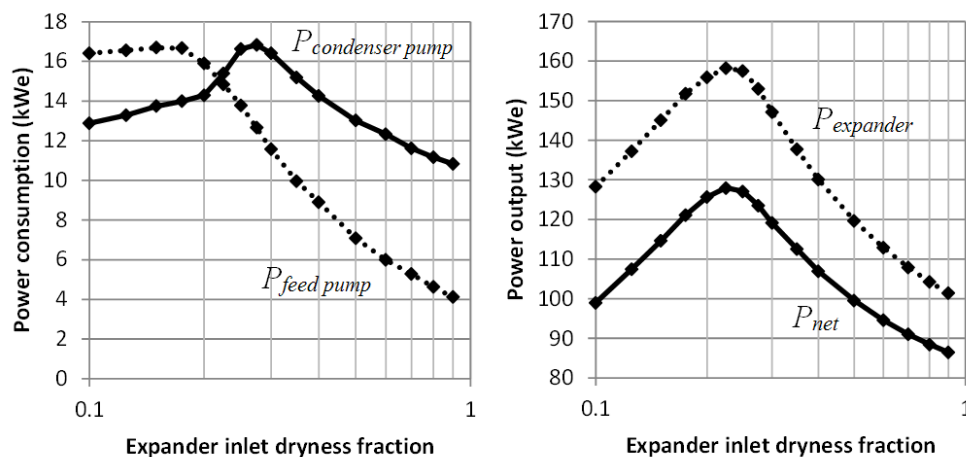


Figure 4: Power consumption and output for WORC components as a function of dryness fraction at expander inlet

The cycle operating conditions required to achieve maximum net power output as a function of expander inlet dryness fraction are given in Figures 5 and 6. This analysis uses the first of the two calculation methods described in Section 2.2, and the rotor speed is therefore unconstrained.

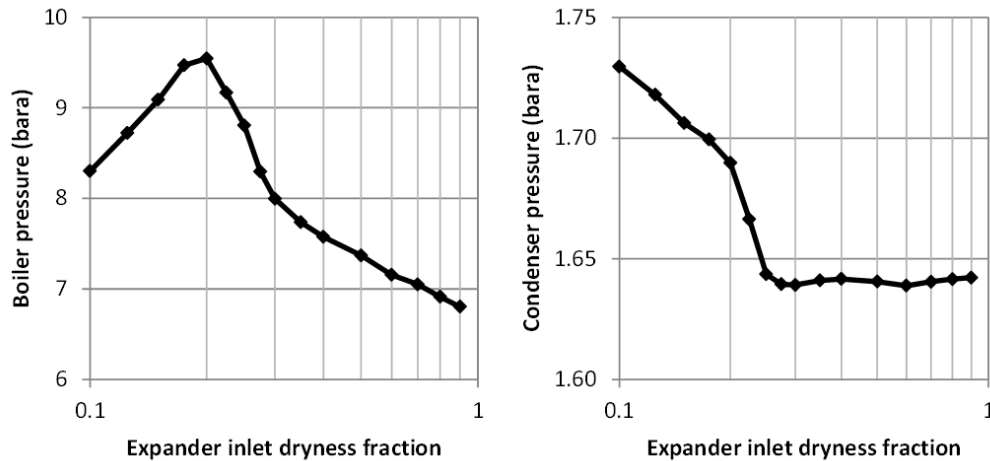


Figure 5: Boiler and condenser exit pressure for maximum net power output

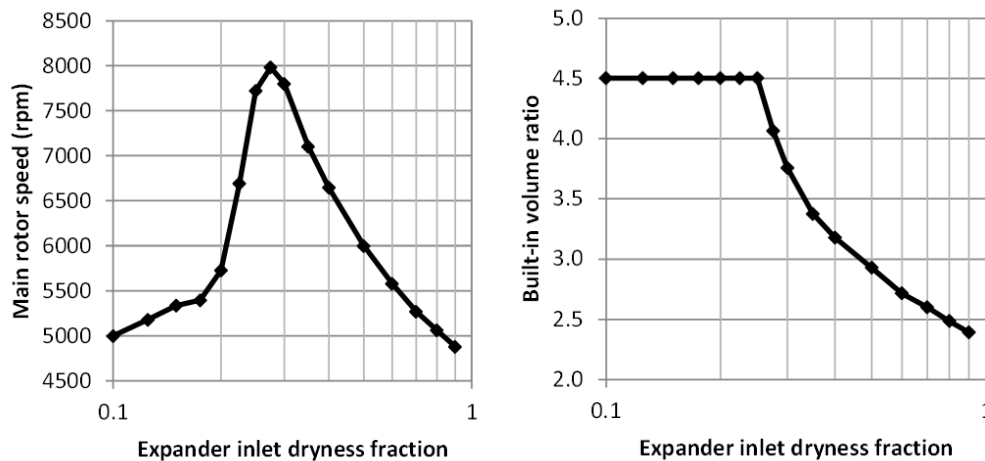


Figure 6: Expander rotor speed and built-in volume ratio for maximum net power output

The net power output shown in Figure 4 can be related to the required area of the heat exchangers. As all heat transfer in the system occurs between liquid water and either liquid or two-phase refrigerant, the heat transfer coefficients through the boiler and condenser will be similar. The total heat exchanger surface area required for the cycle can therefore be considered proportional to the total of the integrated values of $Q/LMTD$ for the heat exchangers, where Q is the rate of heat transfer and $LMTD$ is the log mean temperature difference between the fluids. This value can be used to illustrate how dryness fraction affects the net power output per unit heat transfer area for the cycle; this is shown in Figure 7, along with the required mass flow rate of the working fluid.

The cycle efficiency and overall energy conversion efficiency of the WORC system as a function of expander inlet dryness fraction are shown in Figure 8, while Figure 9 shows temperature-enthalpy diagrams for the optimum, maximum and minimum dryness fraction values.

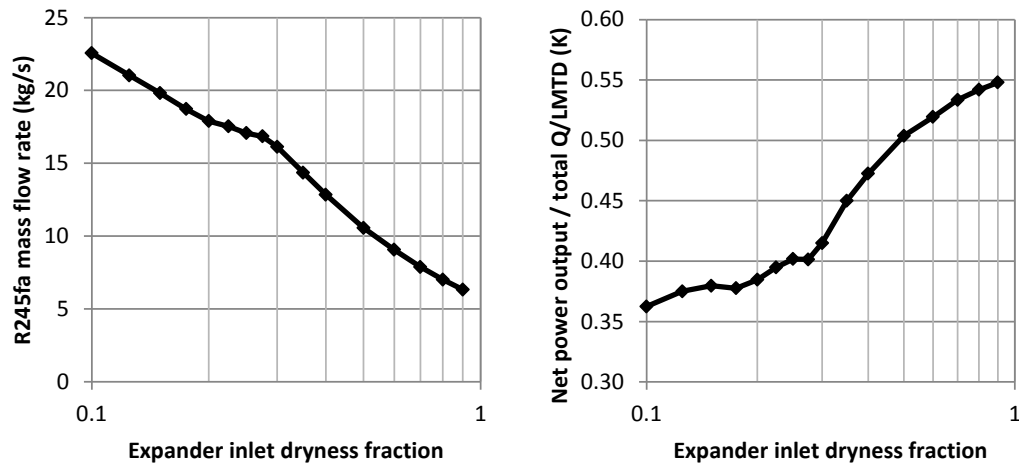


Figure 7: Mass flow rate of working fluid and net power per unit heat exchanger area at maximum net power output

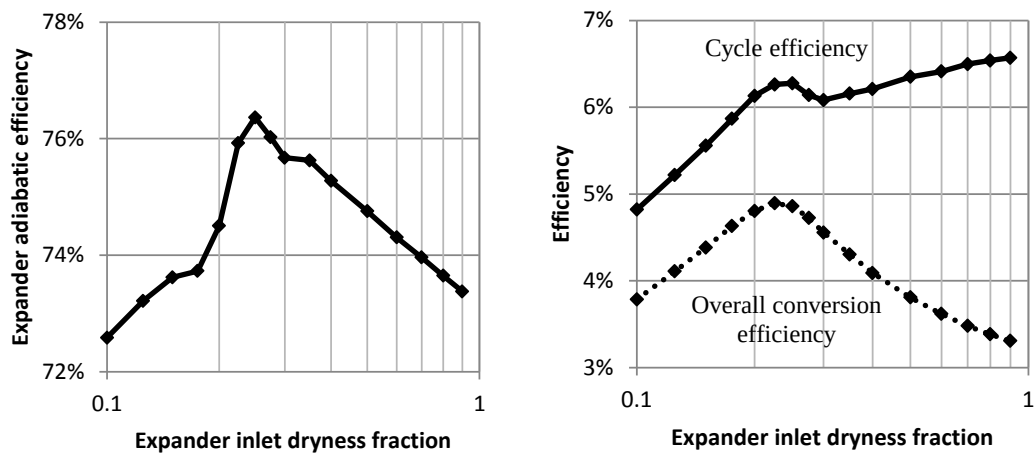


Figure 8: Expander, cycle and overall conversion efficiencies at maximum net power output

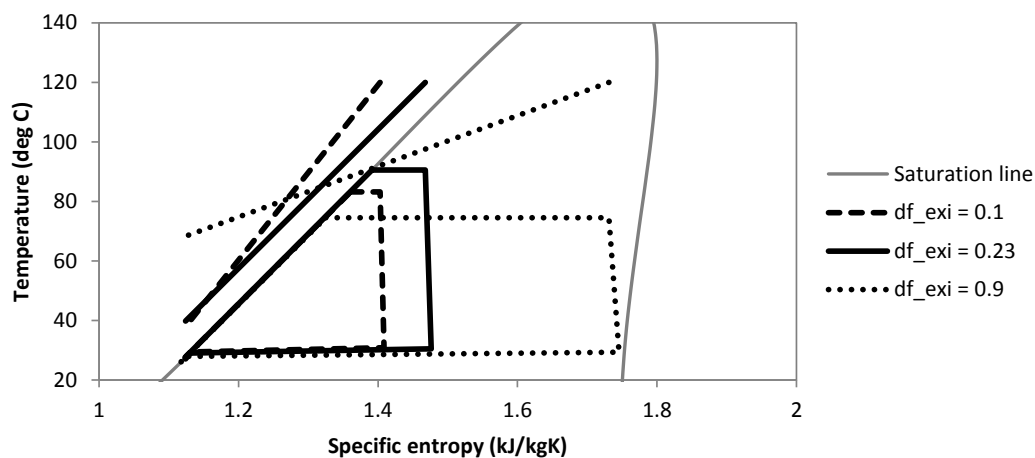


Figure 9: T-s diagrams showing source and working fluids for a WORC producing maximum net power output with a fixed expander inlet dryness fraction of 0.1, 0.23 and 0.9

4. DISCUSSION

To aid understanding of the results presented above, the three particular cases shown in Figure 9 will be highlighted for the following discussion; low, optimum and high dryness are used to describe expander inlet dryness fractions of 0.1, 0.23 and 0.9 respectively. The optimum dryness case achieves the maximum overall net power output, while the high and low dryness cases represent the maximum and minimum expander inlet dryness fractions considered in this study. The results for these cases are summarised in Table 1.

Table 1: Cycle analysis results for the specified cases with low, optimum and high dryness

Dryness fraction case:		Low	Optimum	High
Expander inlet dryness fraction	-	0.10	0.23	0.90
Expander built-in volume ratio	-	4.50	4.50	2.39
Expander main rotor speed	rpm	5001	6693	4881
Boiler pressure	bara	8.30	9.17	6.81
Condenser pressure	bara	1.73	1.67	1.64
Working fluid mass flow rate	kg/s	22.6	17.5	6.3
Expander adiabatic efficiency	-	72.6%	75.9%	73.4%
Thermodynamic cycle efficiency	-	4.8%	6.3%	6.6%
Expander power output	kWe	128.3	158.2	101.4
Condenser pump power input	kWe	12.9	15.4	10.8
Feed pump power input	kWe	16.4	14.8	4.1
Net power output	kWe	99.0	128.0	86.5

These results illustrate that while cycle efficiency is maximised in the high dryness case, this limits the amount of heat that is extracted from the source fluid. The optimum power is achieved at the much lower dryness fraction of 0.23, increasing net power output by almost 50%. This large increase in power is achieved using exactly the same expander unit but with a different built-in volume ratio and rotor speed. The optimised solution achieves the best compromise between the conflicting requirements of increasing boiler pressure, decreasing condenser pressure, maximising heat recovery from the source fluid and maximising individual component efficiencies. Interestingly, it can be seen from Figure 5 that the optimum cycle conditions coincide with the point at which the built-in volume ratio reaches the maximum allowable value. This illustrates the sensitivity of the twin-screw expander to the expansion pressure ratio; for dryness fractions lower than the optimum, the built-in volume ratio is limited to 4.5, and high expander efficiency can only be maintained by reducing boiler pressure and increasing condenser pressure. Consequently, the cycle efficiency and net power output are significantly reduced in the low dryness case.

The function of net power output per unit heat transfer area shown in Figure 7 gives an indication of the compromise between increasing power output and increasing the cost of heat exchangers for the WORC system. The results suggest that heat transfer area required per unit power output for the optimum dryness case is around 38% larger than for the high dryness case. The size and cost of the heat exchangers for the optimum case are therefore likely to be significantly higher. This trade-off between increasing power output and increasing initial system cost can only be rigorously investigated with accurate component and system costing information and a detailed technical and economic analysis of the particular heat recovery application.

For all dryness fractions the expander adiabatic efficiency was found to be over 70%. The high dryness case has the lowest net power output due to the high exit temperature of the source fluid resulting in less heat input to the cycle. The low overall specific volume ratio in the expansion, resulting from the low liquid content of the two-phase fluid, means that a relatively low built-in volume ratio of 2.39 is required for the expander, increasing the volumetric capacity per revolution of the machine. This combined with the lower flow rate of working fluid required (due to reduced heat input and the increased specific enthalpy change during heat addition) mean that the required expander speed is relatively low at just over 4700rpm. When maximum net power output is achieved in the optimum dryness case, the required mass flow rate is significantly higher. The built-in volume ratio must also be higher in order to achieve good efficiency with the larger specific volume ratio for the expansion of the wetter two-phase fluid. This reduces the volumetric capacity per revolution of the expander, and the rotational speed must increase to around 6700rpm in order to achieve the required mass flow rate. This rotational speed corresponds to a rotor tip speed of

around 72 m/s, which is higher than the tip speed limit generally applied to oil-injected gas compressors in order to limit viscous losses. It is therefore important to consider how practical operating limits of the expander influence the cycle performance.

In order to investigate the effect of reducing the rotor speed, the maximisation of net power output as a function of expander inlet dryness fraction has been calculated with an imposed maximum allowable tip speed of 50 m/s. This has been achieved by applying the approach described in Section 2.2 for matching cycle and expander mass flow rates with a specified rotor speed. Figure 10 shows how this speed limitation affects power output and Table 2 provides data for the overall performance compared to the case with no tip speed limit.

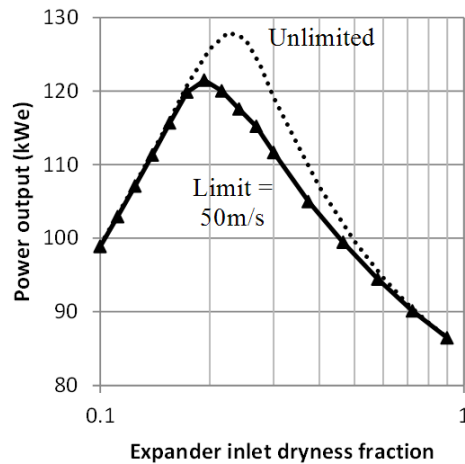


Figure 10: Effect of tip speed limit on net power output as a function of expander inlet dryness fraction

Table 2: Cycle analysis results for optimum conditions with and without tip speed limit

Expander tip speed limit:		Unlimited	50 m/s
Expander inlet dryness fraction	-	0.23	0.19
Expander built-in volume ratio	-	4.50	4.50
Expander main rotor speed	rpm	6693	4685
Boiler pressure	bara	9.17	10.18
Condenser pressure	bara	1.67	1.71
Working fluid mass flow rate	kg/s	17.5	17.2
Expander adiabatic efficiency	-	0.759	0.720
Total (Q/LMTD) for heat exchangers	kW/K	324.2	317.8
Expander power output	kWe	158.2	150.7
Condenser pump power input	kWe	15.4	12.9
Feed pump power input	kWe	14.8	16.3
Net power output	kWe	128.0	121.5

Limiting the tip speed of the rotors to 50 m/s reduces the optimum expander inlet dryness fraction slightly from 0.23 to 0.19, although the optimum built-in volume ratio remains at the maximum allowable value of 4.5. The lower rotor speed reduces the fluid velocity at the expander inlet, reducing the pressure drop that occurs during filling of the working chamber. Combined with the effects of higher inlet pressure and lower inlet dryness fraction, this means that the density of the fluid when the inlet port closes is significantly higher than in the case with unlimited tip speed. The required mass flow rate for the cycle can therefore be achieved at the lower rotor speed. The tip speed limit does lead to a slight reduction in the optimum net power output, but this remains over 40% greater than for the high dryness case.

5. CONCLUSIONS

The key finding of this study is that the net power output from ORC systems designed for low temperature heat recovery can be maximised by allowing the expansion of wet working fluid. For the particular case presented, the expansion of two-phase fluid with an optimum dryness of around 20% was found to generate up to 50% more power than the case with dry saturated vapour at the outlet from the expander. For lower expander inlet dryness fractions the gain in net power output falls off. This is caused by the decreasing pressure ratio required to maintain good expander efficiency, which leads to a reduction in cycle efficiency.

While the results of the case study do not present a definitive design for a heat recovery application, they demonstrate the capability of the current cycle analysis and optimisation method. In reality, additional factors need to be considered. These include the minimum allowable source fluid temperature, the choice of working fluid, available standard machine sizes and operating limits, the sizing and design of the heat exchangers and the cost of components. In addition, variations in the temperature of heat source and sink fluids and off-design system performance will have an influence on the best solution for a particular application. These issues will be addressed in future developments of the ORC model.

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